

Course	Thing	Explanation	Date	Important	Index
MA1521	Precise Definition of limit	$\forall \epsilon > 0, \exists \delta \epsilon > 0$ st $ f(x) - L < \epsilon$ whenever $0 < x - a < \delta \epsilon$	18/08/2021	Important	43
MA1521	Composite limit	$\lim_{x \rightarrow a} f(g(x)) = f(\lim_{x \rightarrow a} g(x))$.	18/08/2021		44
MA1521	Continuous functions	A function is continuous on an interval if it is continuous at every number in the interval.	18/08/2021		45
MA1521	Limit laws	Preserved under addition, multiplication, division, exponentiation	18/08/2021		46
MA1521	Prove continuous	$\lim_{h \rightarrow 0} f(x+h) = f(x)$	25/08/2021		86
MA1521	Prove differentiable	$\lim_{h \rightarrow 0} (f(x+h) - f(x))/h$	25/08/2021		87
MA1521	Extreme Value Theorem	If f continuous in interval, then it contains an absolute max and min value; so there are numbers in the interval and the function is between the max and min for every other in the interval	27/08/2021	Important	99
MA1521	First Derivative Theorem	$f'(x) = 0$ at the max or min value	27/08/2021		100
MA1521	Rolle's theorem	If f continuous and differentiable and $f(a) = f(b)$, then $\exists c$ such that $f'(c) = 0$	01/09/2021	Important	135
MA1521	Mean Value Theorem	If f continuous and differentiable, then $\exists c$ $f'(c) = (f(b) - f(a))/(b - a)$	01/09/2021	Important	136
MA1521	Cauchy Mean Value Theorem	if f and g continuous and differentiable, then $\exists c$ $f'(c)/g'(c) = (f(b) - f(a))/(g(b) - g(a))$	01/09/2021		137
MA1521	Riemann Sum	$\sum f(c)(\text{size of interval})$	03/09/2021		141
MA1521	Limit comparison test	Instead of comparing by inequality, you can take the quotient of the two limits to be tested; If the new limit is not zero or ∞ , then the two smaller limits are both convergent or both divergent; $\lim \sum f(x) = \lim (f(x)/g(x)) \Rightarrow$ if zero or ∞ , then nature of $f(x)$ is the nature of $g(x)$	06/10/2021		262
MA1521	Comparison test	$\lim \sum f(x) \Rightarrow$ compare the size of $f(x)$ vs $g(x) \Rightarrow$ can corral the $f(x)$ using the nature of $g(x)$	06/10/2021		263
MA1521	Absolute convergence	$\sum f(x) \Rightarrow \sum f(x)$	06/10/2021		264
MA1521	Conditionally convergent	$\sum a_n$ convergent and $\sum a_n $ divergent	06/10/2021		266
MA1521	Monotonic sequence theorem	If the terms > 0 and have upper bound, then limit is the upper bound; If the terms < 0 and have lower bound, then limit is lower bound	06/10/2021		267
MA1521	Integral test	$\sum f(x) \approx \int f(x)dx \Rightarrow$ the integral from 1 to ∞ will decide whether the sum is convergent or divergent	06/10/2021		268
MA1521	Ratio test	$\lim a_n / a_{n+1} = L$; $ L < 1 \Rightarrow$ absolutely convergent; $L > 1$ or infinite \Rightarrow divergent; $L = 1 \Rightarrow$ inconclusive	06/10/2021		269
MA1521	Root test	$\lim (n^{\text{th}} \sqrt[n]{a_n}) = L$; $ L < 1 \Rightarrow$ absolutely convergent; $L > 1$ or infinite \Rightarrow divergent; $L = 1 \Rightarrow$ inconclusive	06/10/2021		270
MA1521	Alternating series test	$\sum (-1)^n u_n$ convergent if: $\rightarrow u_n > 0$ $\rightarrow u_{n+1} \leq u_n$ $\rightarrow u_n \rightarrow 0$	10/11/2021		270.5
MA1521	Absolute convergence test	$\sum a_n $ convergent $\rightarrow \sum a_n$ convergent	10/11/2021		270.75
MA1521	Find radius of convergence of power series	$R = \lim c_n / c_{n+1} = \lim 1/c_n^{1/(n+1)} $; $ x - a < R$ $\Leftrightarrow \sum c_n (x - a)^n$ convergent $\Leftrightarrow \sum c_n (x - a)^{n+1}$ convergent $R = 0 \Rightarrow \sum c_n (x - a)^n$ convergent only at a $R = \infty \Rightarrow \sum c_n (x - a)^n$ convergent everywhere	08/10/2021	Important	311
MA1521	Open/closed region	Closed region contains all boundary points; Open region contains only interior points	08/10/2021		312
MA1521	Derivative of inverse function	$f^{-1}(x)' = 1 / f'(f^{-1}(x))$	09/10/2021	Important	331
MA1521	Trigo identities	$\sec^2 x = 1 + \tan^2 x$; $\csc^2 x = 1 + \cot^2 x$; $\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$; $\sin \text{AcosB} = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$; $\sin \text{A sinB} = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$; $\cos \text{AcosB} = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$; $(\tan x)' = \sec^2 x$; $(\cot x)' = -\csc^2 x$; $(\sec x)' = \tan x \sec x$; $(\csc x)' = -\cot x \csc x$; $(\sin^{-1} x)' = 1/\sqrt{1-x^2}$; $(\cos^{-1} x)' = -1/\sqrt{1-x^2}$; $(\tan^{-1} x)' = 1/(1+x^2)$; $(\cot^{-1} x)' = -1/(1+x^2)$; $(\sec^{-1} x)' = 1/(x \sqrt{x^2-1})$; $(\csc^{-1} x)' = -1/(x \sqrt{x^2-1})$; $\int 1/(x^2+d^2)dx = \tan^{-1}(x/d)/d$; $\int \csc(x)dx = -\ln(\csc(x) + \cot(x))$; $\int \cot(x)dx = \ln(\sin(x))$; $\int \sec(x)dx = \ln(\sec(x) + \tan(x))$; $\int 1/(a^2-x^2)dx = \ln(a+x/a-x)/2a$; $\int 1/\sqrt{a^2-x^2}dx = \sin^{-1}(x/a)$; $\int 1/\sqrt{x^2 \pm a^2}dx = \ln(x + \sqrt{x^2 \pm a^2})$;	09/10/2021	Important	332
MA1521	Mean Value Theorem for Definite Integrals	$\exists c$ s.t. $f(c) = \int f(x)dx / (b-a)$; There is a value $f(c)$ that equals to the average y of the integral	12/10/2021	Important	335
MA1521	Series	Limit of the sequence of partial sums (limit of \sum)	15/10/2021		371
MA1521	Precise definition of limit in 2 dimensions	The limit at a point (x_0, y_0) exists if $\exists \delta, \epsilon$ such that $\sqrt{(x-x_0)^2 + (y-y_0)^2} < \delta \Rightarrow f(x,y) - L < \epsilon$	15/10/2021		372

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MA1521	Find limit of multivariable expressions	Use substitution; Use composite functions;	15/10/2021		373
MA1521	Partial Derivative	$f_x(x_0, y_0) = (\partial f / \partial x)(x_0, y_0) = \lim_{h \rightarrow 0} [f(x_0 + h, y_0) - f(x_0, y_0)] / h$	15/10/2021	Important	374
MA1521	Mixed Derivative Theorem	$f_{xy} = f_{yx}$ iff continuous	15/10/2021		375
MA1521	Clairaut's theorem	$f_{xy} = f_{yx}$ iff continuous	20/10/2021		406
MA1521	Partial Derivatives and Continuity	Partial derivatives may exist even if the function is not continuous, as long as the x and y directions are both continuous; (1, 1) direction may be discontinuous	20/10/2021	Important	407
MA1521	Prove multivariable function is continuous	Prove that f_x and f_y is continuous	20/10/2021		408
MA1521	Find rate of change of f	$df/dt = (\partial f / \partial x)(dx/dt) + (\partial f / \partial y)(dy/dt)$	20/10/2021	Important	409
MA1521	Saddle point	Critical point with some neighbours more than it and some neighbours less than it	20/10/2021		410
MA1521	Second derivative test for multivariable function	Saddle point: $f_{xx}f_{yy} - f_{xy}^2 < 0$ Maximum point: $f_{xx}f_{yy} - f_{xy}^2 > 0$ and $f_{xx} < 0$ Minimum point: $f_{xx}f_{yy} - f_{xy}^2 > 0$ and $f_{xx} > 0$	20/10/2021	Important	411
MA1521	Critical points at boundary	Sub in the equation for the boundary, then find the critical point using one variable	20/10/2021		412
MA1521	Prove divergent	Use n^{th} term test	21/10/2021		418
MA1521	Find Lagrange multiplier (with one constraint)	Find the maximum value of f when travelling across g → Let f, g be continuous and differentiable, and $g(a, b) = 0$ → Then, $(g_x(a, b), g_y(a, b)) \neq (0, 0) \Rightarrow (f_x(a, b), f_y(a, b)) = \lambda(g_x(a, b), g_y(a, b))$ → Sub into g to solve for λ	22/10/2021	Important	431
MA1521	Find maxima and minima of closed regions	→ Find interior critical points using f_x and f_y → Find boundary critical points by subbing in $(x, 9-x)$ for $y = 9 - x$	09/11/2021		467
MA1521	Volume of revolution (disc)	$\pi f^2 dx$	09/11/2021		468
MA1521	Volume of revolution (shell)	$2\pi x y dx$	09/11/2021		469
MA1521	Arc length	$\int \sqrt{1 + y'^2} dx$	09/11/2021		470
MA1521	Surface area of revolution	$2\pi y \sqrt{1 + y'^2} dx$	09/11/2021		471