NATIONAL UNIVERSITY OF SINGAPORE

ANSWERS

CS1231S – DISCRETE STRUCTURES

(Semester 1: AY2019/20)

ANSWER BOOKLET

Time Allowed: 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This answer booklet consists of **SIX (6)** printed pages.
2. Fill in your Student Number **with a pen clearly** below. Do NOT write your name.
3. You may write your answers in pencil (2B or above).

STUDENT NUMBER (fill in with a pen):

A								

For examiner's use only		
Question	***Total***	***Marks***
MCQs (Q1-15)	30	
Q16	10	
Q17	12	
Q18	20	
Q19	20	
Q20	8	
Total	**100**	

Section A: MCQs

1.	D	2.	B	3.	C	4.	A	5.	B
6.	B	7.	C	8.	D	9.	C	10.	B
11.	A	12.	D	13.	E	14.	B	15.	C

MCQs total: /30

Section B:

16a. $\gcd(98069, 30629)$.

[2]

281

b. Base 9 representation of $(53292)_{10}$.

[2]

$(81083)_9$

c. A positive integer that has exactly 5 positive divisors.

[2]

p^4 where p is a prime (eg: 16, 81, 625, etc.)

d.

[2]

5, 7 or 11.

e. $34x \equiv 89 \pmod{113}$

[2]

$x \equiv 99 \pmod{113}$

$34(10) \equiv 1 \pmod{113}$

$x \equiv 890 \pmod{113} \equiv 99 \pmod{113}$

Q16: /10

17. Partial order.

a.
[6]

$\tilde{R} \subseteq T$ and $\tilde{R}$ is reflexive, so T is reflexive.

T is transitive since it is the transitive closure.

Note that P is a transitive relation containing $\tilde{R}$ as a subset, so $T \subseteq P$ by the minimality of T. In particular, since P is anti-symmetric, so is T.

Thus T is a partial order on A.

b.
[6]

If T' is another partial order on A such that $R \subseteq T'$, then $\tilde{R} \subseteq T'$ since $\tilde{R}$ is the smallest reflexive relation on A containing R, and hence $T \subseteq T'$ since T is the smallest transitive relation on A containing $\tilde{R}$.

Q17: /12

18a. $a + b + c = 100$.

[2] 5151

b. Selecting children

[3] 209

c. "I", "CAN", "DO", "IT"

[4] 72

d. 3×3 grid

[3]

There are 7 possible values (pigeonholes): -3, -2, -1, 0, 1, 2 and 3.

The number of row-, column- and diagonal-sums (pigeons) is 8.

Therefore, by the Pigeonhole Principle, there must be two sums with the same value.

e. Conditional probability. Write your answers correct to 3 significant figures.

[8] (i) 5.28% or 0.0528

(ii) 5.57% or 0.0557

(iii) 4.99% or 0.0499

Q18: /20

Workings:

a. Multiset problem: $n = 3, r = 100$; $\binom{r+n-1}{r} = \binom{102}{100} = \binom{102}{2} = \mathbf{5151}$.

b. N(1 boy) + N(2 boys) + N(3 boys) + N(4 boys)

$= \left(\binom{6}{1} \times \binom{4}{3}\right) + \left(\binom{6}{2} \times \binom{4}{2}\right) + \left(\binom{6}{3} \times \binom{4}{1}\right) + \left(\binom{6}{4} \times \binom{4}{0}\right) = \mathbf{209}$.

or, N(4 children) – N(4 children with no boy) = $\binom{10}{4} - \binom{4}{4} = 210 - 1 = \mathbf{209}$.

c. There are two cases. (1) When "I" and "IT" are opposite of each other; then there are 2 × 3! ×2! ×2! = 48 ways. (2) When "I" and "IT" are next to each other; then the only valid arrangement between them is "ITI", and "CAN" and "DO" can be swapped, hence 2 × 3! ×2! = 24. Therefore, total = 48 + 24 = **72**.

d. There are 7 possible values (pigeonholes): -3, -2, -1, 0, 1, 2, 3. The number of row-, column- and diagonal-sums (pigeons) is 8. Therefore, by the Pigeonhole Principle, there must be two sums with the same value.

e. Let T = "tested positive", S = "sufferer". $P(S) = 0.003$, $P(T|S) = 0.98$, $P(T|\bar{S}) = 0.05$.

(i) $P(T) = P(T|S) \cdot P(S) + P(T|\bar{S}) \cdot P(\bar{S}) = (0.98 \times 0.003) + (0.05 \times 0.997)$

$= 0.05279 = \mathbf{5.28\%}$

(ii) $P(S|T) = \frac{P(T|S) \cdot P(S)}{P(T)} = \frac{0.98 \times 0.003}{0.05279} = 0.05569 = \mathbf{5.57\%}$

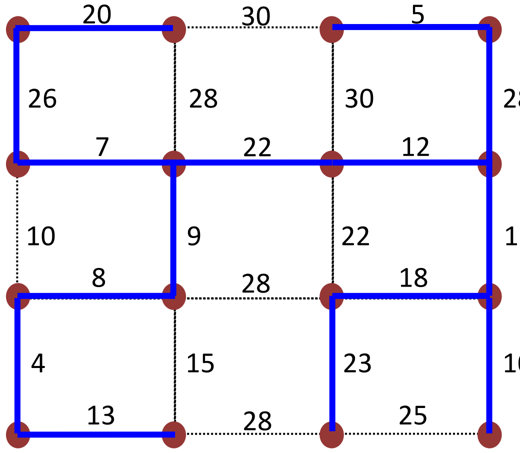
(iii) $P(misclassified) = P(T \cap \bar{S}) + P(\bar{T} \cap S) = P(T|\bar{S}) \cdot P(\bar{S}) + P(\bar{T}|S) \cdot P(S)$
$= (0.05 \times 0.997) + (0.02 \times 0.003) = 0.04991 = \mathbf{4.99\%}$

19a.
[2]

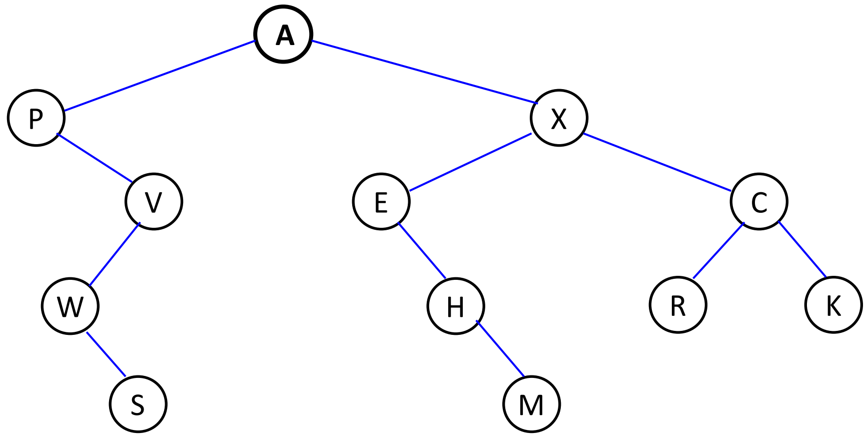
25

From formula: $f = e - v + 2$.

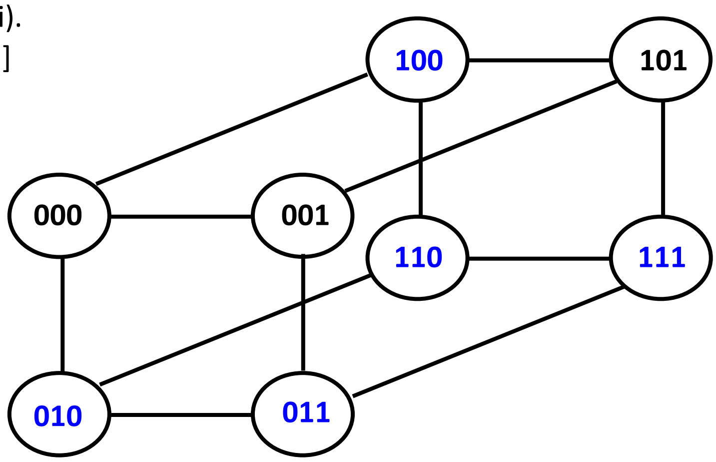
b. Mark out the MST of the graph below.
[4]



c. Draw the binary tree.
[4]



d. (i).
[2]



(ii) What is $N_v(Q_n)$?
[3]

2^n

(iii) What is $N_e(Q_n)$?
[5]

$\frac{n2^n}{2}$

Q19: /20

20.
[8]

Let $f: A \to A$ be defined by $f(a) = a^2 \textbf{ mod } n$.

Then $f(n-1) = (n-1)^2 \textbf{ mod } n = (n^2 - 2n + 1) \textbf{ mod } n = 1 \textbf{ mod } n = f(1)$.

Since $n \geq 3$, $n - 1 \neq 1$, and so f is not injective, and hence not surjective.

Thus there exists $m \in A - f(A)$, i.e. $m \in A$ such that $m \not\equiv a^2 \ (mod\ n)$ for any $a \in \mathbb{Z}$.

Q20: /8

=== END OF PAPER ===