L3: Higher-order Functions; Scope of Names

CS1101S: Programming Methodology

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Announcements

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- Quests: they are "extra homework"! If you are struggling with your time management or work/life balance: Don't touch the quests!

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- Contest "Beautiful Runes" opened on Monday! Closes next week on Tuesday

Some Words of Advice

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- Use the substitution model (and stepper tool, if needed)

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- Read the textbook
- Use the substitution model (and stepper tool, if needed)
- Think, then program
- Less is more

1. Announcements
2. Nested constant declarations
3. More fun with recursion
4. Higher-order functions
5. Scope of names

Names can refer to intermediate values

Example from SICP JS 1.3.2

$$f(x,y) = x(1+xy)^2 + y(1-y) + (1+xy)(1-y)$$

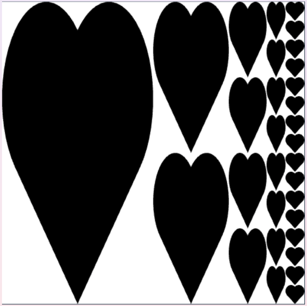
Compute $f(2,3)$

```
function f(x, y) {
    const a = 1 + x * y;
    const b = 1 - y;
    return x * square(a) + y * b + a * b;
}

f(2, 3);
```

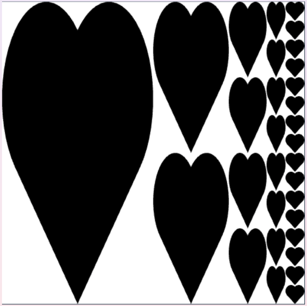
Example from Mission "Rune Reading": `fractal`

Define a function `fractal` that returns pictures like this:



Example from Mission "Rune Reading": `fractal`

Define a function `fractal` that returns pictures like this:



when we call it like this: `fractal(heart, 5)`

Example from "Rune Reading": fractal, Solution 1

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
                 stack(fractal_1(rune, n - 1),
                       fractal_1(rune, n - 1)));
}
```

Can we avoid tree recursion?

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
              stack(fractal_1(rune, n - 1),
                    fractal_1(rune, n - 1)));
}
```

Can we avoid tree recursion?

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
              stack(fractal_1(rune, n - 1),
                    fractal_1(rune, n - 1)));
}
```

Question

Can we implement this function with linear recursion?

Is this a good idea?

```
function fractal_2(rune, n) {
    const smaller_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune,
            stack(smaller_frac, smaller_frac));
}
```

Is this a good idea?

```
function fractal_2(rune, n) {
    const smaller_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune,
            stack(smaller_frac, smaller_frac));
}
```

Can we declare a const...

Is this a good idea?

```
function fractal_2(rune, n) {
    const smaller_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune,
            stack(smaller_frac, smaller_frac));
}
```

Can we declare a const...

...**just for the alternative of the conditional**?

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.
- A block can have local names, only visible inside the block.

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.
- A block can have local names, only visible inside the block.
- Remember to return a result *in each branch*.
  (Otherwise `undefined` is returned.)

Excursion 1: Fractals with iterative process?

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```
function iter(rune, n, current_fractal) {
    return n === 1
        ? current_fractal
        : iter(rune, n - 1,
               beside(rune, stack(current_fractal,
                                  current_fractal)));
}
function fractal_4(rune, n) {
     return iter(rune, n, rune);
}
```

Excursion 2: a "divine" solution

```
function fractal_5(rune, n) {
    return n === 1
           ? rune
           : beside(rune,
                    fractal_5(stack(rune, rune),
                              n - 1));
}
```

Example: Rick the Rabbit

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- Rick the rabbit needs to climb a flight of stairs.

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- Rick the rabbit needs to climb a flight of stairs.
- Given: Rick can **hop** (1 step), **skip** (2 steps) or **jump** (3 steps).

Example: Rick the Rabbit

- Rick the rabbit needs to climb a flight of stairs.
- Given: Rick can **hop** (1 step), **skip** (2 steps) or **jump** (3 steps).
- Let's consider the problem of how many different ways can Rick climb a flight of `n` stairs?

Example: Rick the Rabbit

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs):

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop skip

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop skip
- How about 3 stairs?

Example: Rick the Rabbit

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Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop
  skip

- How about 3 stairs? hop hop hop
  skip hop
  hop skip

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop
  skip
- How about 3 stairs? hop hop hop
  skip hop
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Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop
  skip

- How about 3 stairs? hop hop hop
  skip hop
  hop skip
  jump

Example: Rick the Rabbit

- Consider the case of `n = 2` (two stairs): hop hop
  skip
- How about 3 stairs? hop hop hop
  skip hop
  hop skip
  jump
- What about 0 stairs?
- What about -1 stairs?

Example: Rick the Rabbit

Example: Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining

Example: Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining
- If Rick skips, we have `n - 2` stairs remaining

Example: Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining
- If Rick skips, we have `n - 2` stairs remaining
- If Rick jumps, we have `n - 3` stairs remaining

We now have a smaller problem (the number of stairs is decreasing)

Example: Rick the Rabbit Source

```
function rick_the_rabbit(n) {
    return n < 0
           ? 0
           : n === 0
           ? 1
           : rick_the_rabbit(n - 1) // Rick hops
           +
           rick_the_rabbit(n - 2) // Rick skips
           +
           rick_the_rabbit(n - 3); // Rick jumps
}
```

Example: coin change (1.2.2)

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- Given: Different kinds of coins (unlimited supply)

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Step 1

Read the problem *very carefully*

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?
- How about 20 cents?

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?
- How about 20 cents?
- 5 cents?

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?
- How about 20 cents?
- 5 cents?
- 0 cents?

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?
- How about 20 cents?
- 5 cents?
- 0 cents?
- -1 cents?

Play

- Given: Different kinds of coins (unlimited supply)
- Given: Amount of money in cents
- Wanted: Number of ways to change amount into coins
- How about 10 cents?
- How about 20 cents?
- 5 cents?
- 0 cents?
- -1 cents?

Step 2

Play with examples

Think

Think

- Example: Number of ways to change 120 cents

Think

- Example: Number of ways to change 120 cents
- Idea: Highest coin is either 100 or not 100

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- In the first case: Smaller problem

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- In the second case: Smaller problem

Think

- Example: Number of ways to change 120 cents
- Idea: Highest coin is either 100 or not 100
- In the first case: Smaller problem
- In the second case: Smaller problem
- Base cases?

Think

- Example: Number of ways to change 120 cents
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- In the first case: Smaller problem
- In the second case: Smaller problem
- Base cases?

Step 3

Think: existing solution? divide-and-conquer? "wishful thinking"?

Representing the kinds of coins

```
function first_denomination(kinds_of_coins) {
    return kinds_of_coins === 1 ?   5 :
           kinds_of_coins === 2 ?  10 :
           kinds_of_coins === 3 ?  20 :
           kinds_of_coins === 4 ?  50 :
           kinds_of_coins === 5 ? 100 : 0;
}
```

Idea in Source

```
function cc(amount, kinds_of_coins) {
  return ...
      ? /* base cases */
      : cc(amount -
           first_denomination(kinds_of_coins),
           kinds_of_coins)
        +
        cc(amount, kinds_of_coins - 1);

}
```

Idea in Source

```
function cc(amount, kinds_of_coins) {
  return ...
      ? /* base cases */
      : cc(amount -
           first_denomination(kinds_of_coins),
           kinds_of_coins)
        +
        cc(amount, kinds_of_coins - 1);

}
```

Step 4

Program using Source

Adding base cases

```
function cc(amount, kinds_of_coins) {
    return amount === 0
           ? 1
           : amount < 0 || kinds_of_coins === 0
           ? 0
           : cc(amount -
                   first_denomination(kinds_of_coins),
                   kinds_of_coins)
             +
             cc(amount, kinds_of_coins - 1);
}
```

Adding base cases

```
function cc(amount, kinds_of_coins) {
    return amount === 0
           ? 1
           : amount < 0 || kinds_of_coins === 0
           ? 0
           : cc(amount -
                   first_denomination(kinds_of_coins),
                   kinds_of_coins)
             +
             cc(amount, kinds_of_coins - 1);
}
```

Step 5

Test your program

Passing functions to functions

```
function f(g, x) {
    return g(x);
}

function g(y) {
    return y + 1;
}

f(g, 7);
```

Passing more functions to functions

```
function f(g, x) {
    return g(g(x));
}

function g(y) {
    return y + 1;
}

f(g, 7);
```

Abstraction: Recall `repeat_pattern`

Repeating the pattern *n* times

```
repeat_pattern(4, make_cross, rcross);
```

Remember Mission "Rune Trials"

```
function transform_mosaic(p1, p2, p3, p4,
                          transform) {
    return transform(mosaic(p1, p2, p3, p4));
}
```

Look at this function (1.3.1)

```
function sum_integers(a, b) {
    return a > b
        ? 0
        : a + sum_integers(a + 1, b);
}
```

...and this one

```
function cube(x) {
    return x * x * x;
}

function sum_skip_cubes(a, b) {
    return a > b
        ? 0
        : cube(a) + sum_skip_cubes(a + 2, b);
}
```

Abstraction (1.3.1)

```
function sum(a, b) {
    return a > b ? 0
                 : ⟨compute value with a⟩
                   +
                   sum(⟨next value from a⟩, b);
}
```

Abstraction (1.3.1)

```
function sum(a, b) {
    return a > b ? 0
                 : ⟨compute value with a⟩
                   +
                   sum(⟨next value from a⟩, b);
}
```

in Source:

```
function sum(term, a, next, b) {
    return a > b ? 0
                 : term(a)
                   +
                   sum(term, next(a), next, b);
}
```

sum_integers using sum

```
function identity(x) {
    return x;
}
function plus_one(x) {
    return x + 1;
}
function sum_integers(a, b) {
    return sum(identity, a, plus_one, b);
}
```

sum_skip_cubes using sum

```
function cube(x) {
    return x * x * x;
}
function plus_two(x) {
    return x + 2;
}
function sum_skip_cubes(a, b) {
    return sum(cube, a, plus_two, b);
}
```

sum_skip_cubes using sum

```
function cube(x) {
    return x * x * x;
}
function plus_two(x) {
    return x + 2;
}
function sum_cubes(a, b) {
    return sum(cube, a, plus_two, b);
}
```

Visibility

sum_skip_cubes using sum

```
function cube(x) {
    return x * x * x;
}
function plus_two(x) {
    return x + 2;
}
function sum_cubes(a, b) {
    return sum(cube, a, plus_two, b);
}
```

Visibility

Can we "hide" cube and plus_two inside of sum_skip_cubes?

Yes, we can! (see also SICP JS 1.1.8)

```
function sum_skip_cubes(a, b) {
    function cube(x) {
        return x * x * x;
    }
    function plus_two(x) {
        return x + 2;
    }
    return sum(cube, a, plus_two, b);
}
```

Another look at such local functions

```
function sum_skip_cubes(a, b) {
    function cube(x) {
        return x * x * x;
    }
    function plus_two(x) {
        return x + 2;
    }
    return sum(cube, a, plus_two, b);
}
```

Another look at such local functions

```
function sum_skip_cubes(a, b) {
    function cube(x) {
        return x * x * x;
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}
```

This is still quite verbose

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This is still quite verbose

Do we need all these words such as `function`, `return`?

Another look at such local functions

```
function sum_skip_cubes(a, b) {
    function cube(x) {
        return x * x * x;
    }
    function plus_two(x) {
        return x + 2;
    }
    return sum(cube, a, plus_two, b);
}
```

This is still quite verbose

Do we need all these words such as `function`, `return`?
Do we need to even give names to these functions?

No, we don't! Lambda expressions

```
function sum_skip_cubes(a, b) {
    function cube(x) {
        return x * x * x;
    }
    function plus_two(x) {
        return x + 2;
    }
    return sum(cube, a, plus_two, b);
}
// instead just write:

function sum_skip_cubes(a, b) {
    return sum(x => x * x * x, a, x => x + 2, b);
}
```

Lambda expressions (1.3.2)

New kinds of expressions

(*parameters*) => *expression*

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If there is only one parameter, you can write

parameter => *expression*

Lambda expressions (1.3.2)

New kinds of expressions

(*parameters*) => *expression*

If there is only one parameter, you can write

parameter => *expression*

Meaning

The expression evaluates to a function value.
Function has given *parameters* and `return` *expression* ; as body.

An alternative syntax for function declaration

```
function plus4(x) {
    return x + 4;
}
```

can be written as

```
const plus4 = x => x + 4;
```

Returning Functions from Functions (1.3.4)

```
function make_adder(x) {
    function adder(y) {
        return x + y;
    }
    return adder;
}

const adder_four = make_adder(4);
adder_four(6);
```

...or with the new lambda expressions

```
function make_adder(x) {
    return y => x + y;
}

const adder_four = make_adder(4);
adder_four(6);
```

Returning Functions from Functions

```
function make_adder(x) {
    return y => x + y;
}

( make_adder(4) )(6);

// you can also write:
//
// make_adder(4)(6);
```

Returning Functions from Functions

```
function make_adder(x) {
    return y => x + y;
}

const adder_1 = make_adder(1);
const adder_2 = make_adder(2);

adder_1(10); // returns 11

adder_2(20); // returns 22
```

Summary of new constructs today

- Nested constant and function declaration statements
- Conditional statements and blocks
- Lambda expressions

1. Announcements
2. Nested constant declarations
3. More fun with recursion
4. Higher-order functions
5. Scope of names
   - Examples
   - Overview of scoping rules
   - The details

Scope of names: an example

```
const z = 2;
function f(g) {
    const z = 4;
    return g(z);
}

f( y => y + z );
```

Scope of names: an example

```
const z = 2;
function f(g) {
    const z = 4;
    return g(z);
}

f( y => y + z );
```

Questions about scope

What names are declared by this program?

Scope of names: an example

```
const z = 2;
function f(g) {
    const z = 4;
    return g(z);
}

f( y => y + z );
```

Questions about scope

What names are declared by this program?

Which declaration does each name occurrence refer to?

Scope of names: another example

```
const x = 10;
function square(x) {
    return x * x;
}
function addx(y) {
    return y + x;
}
square(x + 5) * addx(x + 20);
```

Questions about scope

Which declaration does each occurrence of `x` refer to?

Scope of names: yet another example

```
const pi = 3.141592653589793;
function circle_area_from_radius(r) {
    const pi = 22 / 7;
    return pi * square(r);
}
```

Questions about scope

Which declaration does the occurrence of `pi` refer to?

Scope of names: hypotenuse example

```
function square(x) {
    return x * x;
}
function hypotenuse(a, b) {
    function sum_of_squares() {
        return square(a) + square(b);
    }
    return math_sqrt(sum_of_squares());
}
```

Scope of names: hypotenuse example

```
function square(x) {
    return x * x;
}
function hypotenuse(a, b) {
    function sum_of_squares() {
        return square(a) + square(b);
    }
    return math_sqrt(sum_of_squares());
}
```

Names can refer to declarations outside of the immediately surrounding function declaration.

Overview of scoping rules

Declarations mandatory

All names in Source must be declared.

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Forms of declaration

1. Pre-declared constants
2. Constant declarations
3. Parameters of function declarations and lambda expressions
4. Function name of function declarations

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Scoping rule

A name occurrence refers to the *closest surrounding* declaration.

Forms of Declaration

(1) Pre-declared names

The Source §1 pages tell us what names are pre-declared, e.g. `math_floor`.

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(1) Pre-declared names

The Source §1 pages tell us what names are pre-declared, e.g. `math_floor`.
We can also `import` further pre-declared names from modules. For example, from the rune module:

```
import { heart, quarter_turn_right } from "rune";
```

Forms of Declaration

(2) Constant declarations

The scope of a constant declaration is the closest surrounding pair of {...}, or the whole program, if there is none.

Example

```
function f(x, y) {
    if (x > 0) {
        const z = x * y;
        return math_sqrt(z);
    } else {
        ...
    }
}
```

Forms of Declaration

(3) Parameters

The scope of the parameters of a lambda expression or function declaration is the body of the function.

```
function f(x, y, z) {
    ... x ... y ... z ...
}

(v, w, u) => ... v ... w ... u ...
```

Forms of Declaration

(4) Function name

The scope of the function name of a function declaration is as if the function was declared with `const`.

```
function f(x) {
    ...
}
```

as if we wrote

```
const f = ...;
```

Lexical scoping

Scoping rule

A name occurrence refers to the *closest surrounding* name declaration.

```
function f(x) {
    function g(w) {
        const x = ...;
        const y = x + 1;
        return x =>
            ...x...;
    }
    return g(x);
}
```

Important Ideas

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- Higher-order functions are *useful* for building abstractions

Important Ideas

- *Hiding* can be a useful abstraction technique
- *Recursion* is an elegant pattern of problem solving
- Functions can be *passed to* functions
- Functions can be *returned from* functions
- Higher-order functions are *useful* for building abstractions
- With nested functions and conditional statements, we need to understand the *scope* of names

Important Ideas

- *Hiding* can be a useful abstraction technique
- *Recursion* is an elegant pattern of problem solving
- Functions can be *passed to* functions
- Functions can be *returned from* functions
- Higher-order functions are *useful* for building abstractions
- With nested functions and conditional statements, we need to understand the *scope* of names
- The CS1101S 5-step Method of Problem Solving™

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- Look out for Unit 2: Building Abstractions with Data, starting with L4