NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2021/22) Semester 2 ST2334 Probability and Statistics Tutorial 9

1. Let X be the time between two successive arrivals at the drive-up window of a fast-food restaurant. Suppose X has an exponential distribution with $\lambda = 1$. (i.e. $f_X(x) = e^{-x}$ for $x > 0$ and 0 otherwise.)
   (a) Compute the expected time between two successive arrivals.
   (b) Find the standard deviation of the time between successive arrivals.
   (c) Compute $\Pr(X \leq 4)$ and $\Pr(2 \leq X \leq 5)$.

2. Extensive experience with fans of a certain type used in diesel engines has suggested that the exponential distribution provides a good model for time until failure. Suppose the mean time until failure is 25000 hours.
   (a) What is the probability that a randomly selected fan will last 20000 hours? At most 30000 hours? Between 20000 and 30000 hours?
   (b) What is the probability that the lifetime of a fan exceeds the mean value by more than 2 standard deviations?

3. The life, in years, of a certain type of electrical switch has an exponential distribution with an average life of 2 years.
   (a) Find the variance of the lifetimes of those electrical switches.
   (b) If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year?

4. According to Chebyshev's theorem, the probability that any random variable assumes a value within 3 standard deviations of the mean is at least 8/9. If it is known that the probability distribution of a random variable X is normal with mean μ and variance σ^2, what is the exact value of $\Pr(\mu - 3\sigma < X < \mu + 3\sigma)$?

5. A soft-drink machine is regulated so that it discharges an average of 200 ml per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 ml, find
   (a) The probability that the cups will contain more than 224 ml.
   (b) The probability that a cup contains between 191 and 209 ml.
   (c) How many cups will probably overflow if 230 ml cups are used for the next 1000 drinks?
   (d) Below what value do we get the smallest 25% of the drinks?

6. A lawyer commutes daily from his suburban home to his midtown office. The average time for a one-way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of trip times to be normally distributed.
   (a) What is the probability that a trip will take at least half an hour?
   (b) If the office opens at 9.00 a.m. and he leaves his house at 8.45 a.m. daily, what percentage of the time is he late for work?
   (c) Find the probability that 2 of the next 3 trips will take at least ½ hour.

7. A coin is tossed 400 times, Use the normal approximation to find the probability of obtaining
   (a) Between 185 and 210 heads inclusive
   (b) Exactly 205 heads
   (c) Less than 176 or more than 227 heads.

8. Statistics released by the authority show that on an average weekend night, 1 out of every 10 drivers on the road is drunk. Suppose that 400 drivers are randomly checked next Saturday night. Use the normal approximation to find the probability that the number of drunk drivers will be
   (a) Less than 32?
   (b) More than 49?
   (c) At least 35 but less than 47?

9. A company produces component parts for an engine. Parts specifications suggest that 95% of items meet specifications. The parts are shipped to customers in lots of 100. Use the normal approximation to find the probability that
   (a) More than 2 items will be defective in a given lot?
   (b) More than 10 items will be defective in a lot?

10. The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
$\Pr(X = x)$	0.2	0.4	0.3	0.1

   (a) Find the mean μ and the variance σ^2 of X.
   (b) Find the mean $\mu_{\bar{x}}$, and the variance ${\sigma_{\bar{x}}}^2$ of the mean $\bar{X}$ for random samples of 36 cherry puffs from the above probability distribution.
   (c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

11. The chemical benzene is highly toxic to humans. However, it is used in the manufacture of many medicine dyes, leather, and many coverings. In any production process involving benzene, the water in the output of the process must not exceed 7950 parts per million (ppm) of benzene because of government regulations. For a particular process of concern the water sample was collected by a manufacturer 25 times randomly and the sample average was 7960 ppm. It is known from historical data that the standard deviation σ is 100ppm.
   (a) What is the probability that the sample average in this experiment would exceed the government limit if the population mean is equal to the limit? Use the central limit theorem.
   (b) Is an observed sample average of 7960 in this experiment firm evidence that the population mean for the process exceeds the government limit? Answer by computing $\Pr(\bar{X} \geq 7960 \mid \mu = 7950)$. Assume that the distribution of benzene concentration is normal.

Answers to selected problems

1. (a) $X \sim Exp(1), E(X) = 1$
   (b) $\sigma = 1$
   (c) 0.9817, 0.1286.
2. (a) $X \sim Exp(1/25000)$. 0.4493, 0.6988, 0.1481
   (b) $\Pr(X > 75000) = 0.0498$
3. (a) 4
   (b) 0.03347 (exact), 0.0350 (approx.), 0.0351 (approx., statistical table)
4. 0.9973
5. (a) 0.05480
   (b) 0.4515
   (c) 23
   (d) 189.88 ml
6. (a) 0.05717
   (b) 99.11%
   (c) 0.00925
7. (a) 0.7926 (exact), 0.7926 (approx.), 0.7925 (approx., statistical table)
   (b) 0.0352 (exact), 0.0352 (approx.), 0.0352 (approx., statistical table)
   (c) 0.0100 (exact), 0.0101 (approx.), 0.0101 (approx., statistical table)
8. (a) 0.0746 (exact), 0.0783 (approx.), 0.0778 (approx., statistical table)
   (b) 0.0601 (exact), 0.0567 (approx.), 0.0571 (approx., statistical table)
   (c) 0.6796 (exact), 0.6810 (approx.), 0.6811 (approx., statistical table)
9. (a) 0.8817 (exact), 0.8743 (approx.), 0.8749 (approx., statistical table)
   (b) 0.0115 (exact), 0.0058 (approx.), 0.0059 (approx., statistical table)
10. (a) $\mu = 5.3, \sigma^2 = 0.81$
    (b) 5.3; 0.0225
    (c) 0.90879
11. (a) 0.5
    (b) 0.30854; No