

CS3230: Assignment for Week 4

Due: Monday, 5th Sep 2022, 7 pm SGT.

Please upload a PDF file containing your solution (hand-written & scanned, or typed) by 5th Sep, 7 pm on Canvas. You may discuss the problems with your classmates, though you should write up your solutions on your own. Please note the names of your collaborators in your submission. You may want to refer to the plagiarism policy from Lecture 0.

1. (a) There are 3230 students, each with 1 student card. All of them place their student cards into a box, then a professor randomly distributes 1 card to each student. What is the expected number of students that receive their own card back?
(b) Let X be a discrete random variable which can take 3 distinct values in $[0, 1]$. Suppose X can take each of those 3 values with nonzero probability. Prove that $\mathbb{E}[X] > \mathbb{E}[X^2]$.
2. Assume that 2% of the machinery produced by a company are faulty. A testing method developed by Professor M is able to detect the presence of the fault. Professor M claims that the test method has a high accuracy of detection in terms of the following conditional probabilities obtained from their quality control testing:

$$Pr[\text{the method shows positive} | \text{the machinery is faulty}] = 0.998$$

$$Pr[\text{the method shows negative} | \text{the machinery is not faulty}] = 0.996$$

- (a) Given that a machinery is tested to be positive using this method, what is the probability that this machinery is not faulty?
- (b) Given that a machinery is tested to be negative using this method, what is the probability that this machinery is faulty?

3. You are given the following algorithms:

Algorithm 1 *LAUGH()*

return a random integer in range $[1,7]$ uniformly.

Algorithm 2 *HAPPY()*

```
while True do
     $a = LAUGH()$ 
     $b = LAUGH()$ 
     $c = (a - 1) \times 7 + b$ 
    if  $c < 41$  then
        return  $a \% 10 + 1$ 
    end if
end while
```

- (a) What does the algorithm *HAPPY()* do? You do not need a formal proof in this part.
- (b) Determine the expected number of *LAUGH()* being called on one execution of *HAPPY()*.