List Processing (2.2.1)
Continuation-Passing Style
Higher-order List Processing (2.2.1–2.2.3)
Trees and Tree Processing (2.2.2)

### L5: List and Tree Processing

CS1101S: Programming Methodology

Martin Henz

September 8, 2021



- 1 List Processing (2.2.1)
- 2 Continuation-Passing Style
- 3 Higher-order List Processing (2.2.1–2.2.3)
- 4 Trees and Tree Processing (2.2.2)

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### Where are we?

#### Module overview

• Unit 1—Functional abstraction: SICP Chapter 1

- Unit 1—Functional abstraction: SICP Chapter 1
- Unit 2—Data abstraction: SICP Chapter 2

- Unit 1—Functional abstraction: SICP Chapter 1
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- Unit 3—State: SICP Chapter 3

- Unit 1—Functional abstraction: SICP Chapter 1
- Unit 2—Data abstraction: SICP Chapter 2
- Unit 3—State: SICP Chapter 3
- Unit 4—Beyond: SICP Chapter 4

#### Lectures and Briefs

• Intro to data abstraction: L4

- Intro to data abstraction: L4
- List and Tree Processing: L5

- Intro to data abstraction: L4
- List and Tree Processing: L5
- Language Processing: B5

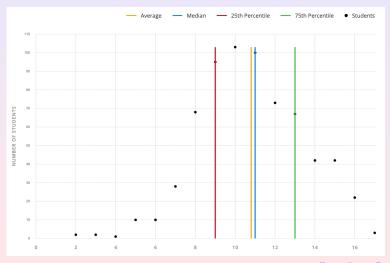
- Intro to data abstraction: L4
- List and Tree Processing: L5
- Language Processing: B5
- Symbolic Processing: L6

- Intro to data abstraction: L4
- List and Tree Processing: L5
- Language Processing: B5
- Symbolic Processing: L6
- Guest Lecture by Joel Low: B6

# Some stock-taking

- Reading Assessment 1
- Unit 1 Survey

## RA1: Results per student



## RA1: Results per question



# Reading Assessment 1: The hardest question

```
const x = 3;
function fun(x) {
    if (x \% 2 === 0) {
        const z = 20;
    } else {
        const z = 30;
    return x + y + z;
}
const y = 5;
const z = 10;
fun(x + y);
```

Anonymous

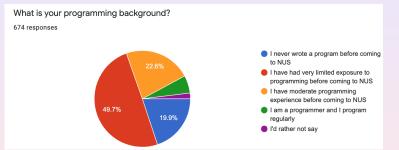
- Anonymous
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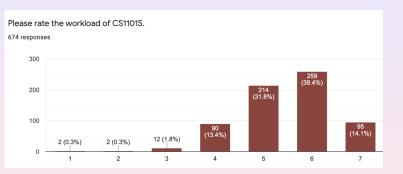
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- Candid feedback on module and staff
- We hear you!
- Many thanks for participating!

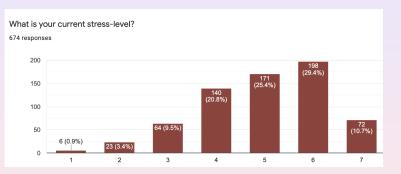
# Unit 1 Survey: Programming Background



## Unit 1 Survey: Workload



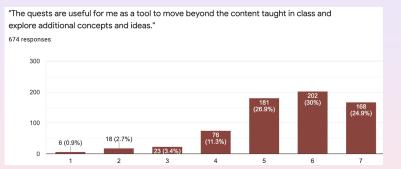
# Unit 1 Survey: Stress Level



# Unit 1 Survey: Preparation for Missions?

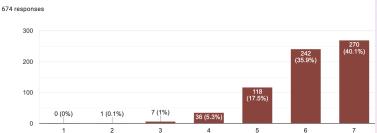
"The lectures, reflections and studios prepare me appropriately for successfully completing the missions on time." 674 responses 200 194 (28.8%) 195 (28.9%) 150 112 (16.6%) 100 94 (13.9%) 50 47 (7%) 9 (1.3%) 23 (3.4%) 2 3 4 5 6

# Unit 1 Survey: Quests

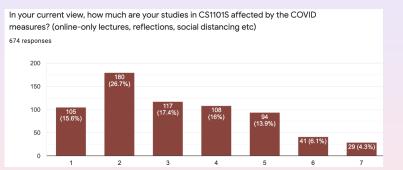


## Unit 1 Survey: Paths

"The paths are useful for me as a tool to understand my own grasp of the material, and what I should ideally understand at a particular point in time."



## Unit 1 Survey: COVID



List Processing (2.2.1)
Continuation-Passing Style
Higher-order List Processing (2.2.1–2.2.3)
Trees and Tree Processing (2.2.2)

• No "curve"! Your individual learning counts!

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- Make use of our resources!
- Learning attitude

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# Mastery Checks

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- Conducted in 3-way meetings: 2 students, 1 Avenger, mostly of the same Studio

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- pass/fail
- repetition allowed, also per topic



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- Other staff may check as well for repeated attempts.
- Try finish MC1 b4 Recess Week. (Great prep for Midterm!)



List Processing (2.2.1)
Continuation-Passing Style
Higher-order List Processing (2.2.1–2.2.3)
Trees and Tree Processing (2.2.2)

#### Contest: Beautiful Runes

- 1 List Processing (2.2.1)
  - Review: The length of a list
  - append
  - reverse
- 2 Continuation-Passing Style
- 3 Higher-order List Processing (2.2.1–2.2.3)
- 4 Trees and Tree Processing (2.2.2)

# Computing the length of a list (2.2.1)

Review: The length of a list append reverse

# Computing the length of a list (2.2.1)

```
The length of...
```

Review: The length of a list append reverse

# Computing the length of a list (2.2.1)

The length of...

...the empty list is 0

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#### The length of...

...the empty list is 0, and the length of a non-empty list is one more than the **length of its tail**.

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...the empty list is 0, and the length of a non-empty list is one more than the **length of its tail**.

```
function length(xs) {
  return is_null(xs)
     ? 0
     : 1 + length(tail(xs));
}
```

#### We can do this with an iterative process!

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```
function length(xs) {
                             // recursive
  return is_null(xs)
      : 1 + length(tail(xs));
}
function length_iter(xs) { // iterative
  function len(xs, counted_so_far) {
    return is_null(xs)
        ? counted_so_far
        : len(tail(xs),
              counted_so_far + 1);
  }
  return len(xs, 0);
```

# Appending two lists

Append list(1, 3, 5) and list(2, 4) results in

list(1, 3, 5, 2, 4)

Assumption: Both arguments are lists

If xs is empty, return ys.

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Otherwise, wishful thinking!

If xs is empty, return ys.

Otherwise, wishful thinking!

Append the tail of xs to ys

If xs is empty, return ys.

Otherwise, wishful thinking!

Append the tail of xs to ys

Form a pair of the head of xs and the result.

#### The same in Source

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## Reversing a list: First attempt

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#### Reversing a list: First attempt

```
function reverse(xs) {
     return is_null(xs)
             ? null
               pair(reverse(tail(xs)),
                      head(xs));
}
What's wrong?
Result not a list.
We simply reverse
roles of head and tail.
```

### Reversing a list: Correct but naive

#### Reversing a list: Correct but naive

## Reversing a list efficiently

## Reversing a list efficiently

```
function reverse(xs) {
  function rev(original, reversed) {
    return is_null(original)
            ? reversed
            : rev(tail(original),
                   pair (head (original),
                        reversed));
  return rev(xs, null);
}
Order of growth? Time? Space?
```

- 1 List Processing (2.2.1)
- 2 Continuation-Passing Style
  - Closer Look at append
  - Iterative process with reverse
  - Iterative process with Continuation Passing
- 3 Higher-order List Processing (2.2.1–2.2.3)
- 4 Trees and Tree Processing (2.2.2)

#### A closer look in Source at append

#### A closer look in Source at append

Can we do this using an iterative process?



### Iterative process, first attempt

#### Iterative process using reverse

#### Iterative process using reverse

Can we do this without reverse?



#### An iterative version of append

```
function append(xs, ys) {
                                // recursive proc
  return is_null(xs)
         ? ys
         : pair(head(xs),
                append(tail(xs), ys));
function app(current_xs, ys, c) { // iter. proc
  return is_null(current_xs)
         ? c(ys)
         : app(tail(current_xs), ys,
             x => c(pair(head(current_xs), x)));
}
function append_iter(xs, ys) {
    return app(xs, ys, x => x);
```

## Continuation-Passing Style

#### Programming Pattern: CPS

Passing the deferred operation as a function in an extra argument is called "Continuation-Passing Style" (CPS).

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#### Programming Pattern: CPS

Passing the deferred operation as a function in an extra argument is called "Continuation-Passing Style" (CPS). We can convert *any* recursive function this way!

200

#### Recall L3-Excursion 2: a "divine" solution

#### Recall L3-Excursion 2: a "divine" solution with CPS

```
function fractal_5(rune, n) {
   return n === 1
       ? rune
       : beside(rune, fractal_5(stack(rune, rune),
                                n - 1)):
}
function frac(rune, n, c) {
   return n === 1
      ? c(rune)
      : frac(stack(rune, rune), n - 1,
             res => c(beside(rune, res)));
}
function fractal_5_iter(rune, n) {
   return frac(rune, n, rune => rune);
```

- List Processing (2.2.1)
- 2 Continuation-Passing Style
- 3 Higher-order List Processing (2.2.1–2.2.3)
  - map
  - accumulate
  - filter
- 4 Trees and Tree Processing (2.2.2)

Advantage of functional programming

Results only depend on arguments.

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FP has achieved widespread use in software industry.

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Set of abstractions that forms the core of many big data processing engines, such as Apache Hadoop.

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Set of abstractions that forms the core of many big data processing engines, such as Apache Hadoop.

#### **Terminology**

Following SICP, we say accumulate instead of reduce.



# Scaling a list (2.2.1)

Let us scale all elements of a list of numbers by a factor f.

### Squaring a list

Let us square all elements of a list of numbers.

Mapping means applying a given function  ${\tt f}$  element-wise to a given list  ${\tt xs}$ .

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The result is a list consisting of the results of applying f to each element of xs.

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```
function scale_list(xs, factor) {
  return map(x => factor * x, xs);
}
```

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```
function scale_list(xs, factor) {
  return map(x => factor * x, xs);
}

function square_list(xs) {
  return map(x => x * x, xs);
}
```

# Definition of map

# Definition of map

```
function map(fun, xs) {
  return is_null(xs)
         ? null
          : pair(fun(head(xs)),
                 map(fun, tail(xs)));
}
function scale_list(xs, factor) {
  return map(x => factor * x, xs);
}
function square_list(xs) {
  return map(x \Rightarrow x * x, xs);
}
```

# Example: summing the elements of a list (2.2.3)

#### Problem

Compute the sum of all elements of a given list of numbers

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## Example: summing the elements of a list (2.2.3)

#### **Problem**

Compute the sum of all elements of a given list of numbers

#### Right-to-left folding

```
list_sum(list(1,2,3,4)) computes 1 + (2 + (3 + (4 + 0))).
```



#### accumulate: an abstraction for right-to-left folding

### accumulate: an abstraction for right-to-left folding

```
function list_sum(xs) { // programmed "by hand"
    return is_null(xs)
        7 ()
        : head(xs) + list_sum(tail(xs));
}
function accumulate(f, initial, xs) {
  return is_null(xs)
    ? initial
    : f(head(xs),accumulate(f,initial,tail(xs)));
}
function list_sum(xs) { // using accumulate
    return accumulate((x, y) \Rightarrow x + y, 0, xs);
}
```

# filter (2.2.3)

Problem: take only even elements of list of numbers

## filter (2.2.3)

Problem: take only even elements of list of numbers

filter(x => x 
$$\%$$
 2 === 0, list(1, 2, 3, 4, 5, 6));

## filter (2.2.3)

Problem: take only even elements of list of numbers

- 1 List Processing (2.2.1)
- 2 Continuation-Passing Style
- 3 Higher-order List Processing (2.2.1–2.2.3)
- 4 Trees and Tree Processing (2.2.2)
  - What are trees?
  - Higher-order Tree Processing
  - Counting Data Items
  - Syntax Trees

List Processing (2.2.1)
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What are trees? Higher-order Tree Processing Counting Data Items Syntax Trees

Trees (2.2.2)

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 $\ldots$ is either null or a pair whose head is of that type and whose tail is a list of that type

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... is a list whose elements are of that type, or trees of that type.

# Trees (2.2.2)

#### A list of a certain type

...is either null or a pair whose head is of that type and whose tail is a list of that type

#### A tree of a certain type

... is a list whose elements are of that type, or trees of that type.

#### Consider:

```
const tree = list(0, list(1,2), list(3,4), 5)
```

## Trees (2.2.2)

#### A list of a certain type

...is either null or a pair whose head is of that type and whose tail is a list of that type

#### A tree of a certain type

... is a list whose elements are of that type, or trees of that type.

#### Consider:

```
const tree = list(0, list(1,2), list(3,4), 5)
```

#### Caveat

Cannot consider null or pair as "certain type" for trees.



## Scaling trees (2.2.2)

Example: scale each data item by a factor 10

```
const my_tree =
    list(1, list(2, list(3, 4), 5), list(6, 7));

scale_tree(my_tree, 10);

should have the same result as:
list(10, list(20, list(30, 40), 50), list(60, 70));
```

## Scaling trees: Idea

Recall: A tree is a list whose elements are data items, or trees.

## Scaling trees: Idea

Recall: A tree is a list whose elements are data items, or trees.

Idea: Map over the list. If element is a data item, scale element.

If not: scale tree

## Scaling trees: Implementation using list map

## Abstraction: Mapping over trees

# Scaling trees: Implementation using map\_tree

## Counting Data Items (2.2.2)

```
Task: Compute the number of data items in a given tree?

const tree = pair(list(1, 2), list(3, 4));

What is count_data_items(tree)?
```

### Counting Data Items: Idea

Every tree is a list.

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## Counting Data Items: Idea

Every tree is a list.

That list can be empty, in which case we return 0.

If the list is not empty, we add the *number of data items* of the head to the *number of data items* of the tail.

The head can be a tree, in which case we need to count *its* data items. If it's not a tree, it's a data item and we count 1.

# Counting Data Items

## Counting Data Items

#### Exercise

Design an abstraction that lets you count the elements using (x, y) => x + y and that lets you compute a list of elements using append

## How to Implement Programming Language Tools?

Goal: implement language processing tools

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#### Teaser for Lecture L11

The evaluator, which determines the meaning of statements in a programming language, is just another program.



## Summary

• length, reverse, append

- length, reverse, append
- Continuation-Passing Style for iterative processes

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- Higher-order list processing with map, accumulate, filter

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- Outlook: Brief B5: Overview of programming language processing