

1. Suppose that in an instance of the Stable Matching problem where a man m ranks a woman w as his first choice, and the woman w also ranks the man m as her first choice. Prove that in any stable matching, m and w will be matched.

A stable matching of men and women is achieved if there is no man-woman pair where both of them prefer each other over their current partners. Let's assume that m and w are not paired with each other. This means that m is paired with some other woman w' and w is paired with some other man m' . Since m has ranked w as their top choice, m prefers w over his current partner w' . Likewise, w prefers m over her current partner m' . As such, there exists a man-woman pair in this configuration where both the man and the woman prefer each other over their current partners. This matching is hence unstable. Since (m and w are not paired) \rightarrow (matching is unstable), by contrapositive, a stable matching always implies that m and w are paired.

2. In this problem, you will investigate what happens when participants cheat when reporting preferences to the Gale-Shapley algorithm. Consider the example discussed in lecture, where the men are Ashish, Bao, and Charlie, and the women are Xinyu, Yashoda, and Zuzu. Recall that Gale-Shapley results in the men-optimal stable matching: (Ashish, Xinyu), (Bao, Yashoda), (Charlie, Zuzu). Show that Xinyu can misreport her preferences so as to force Gale-Shapley to output the women-optimal stable matching: (Ashish, Yashoda), (Bao, Xinyu), (Charlie, Zuzu).

Xinyu can choose: Bao, Charlie, Ashish

Since Gale-Shapley algorithm returns the same configuration no matter what order, WLOG, we shall go with the order [Ashish, Bao, Charlie]. Following the algorithm, the following processes take place:

Ashish pairs with Xinyu.

Bao pairs with Yashoda.

Charlie pairs with Xinyu, as Xinyu prefers him over Ashish. Ashish is now free.

Ashish pairs with Yashoda, as Yashoda prefers him over Bao. Bao is now free.

Bao pairs with Xinyu, as Xinyu prefers him over Charlie. Charlie is now free.

Charlie pairs with Zuzu.

Thus, this leads to the women-optimal stable matching: (Ashish, Yashoda), (Bao, Xinyu), (Charlie, Zuzu).

3. Read this article about how bias and unfairness can arise in algorithmic decisions. Write a paragraph summarizing the problem the article discusses.

The article discusses the overreliance of the justice system on risk scores, which attempts to predict the likelihood of committing a repeated offence for any convicted individual.

According to the article, the current risk score was flawed and showed very little success when predicting which defendants would re-offend. This was stated to be likely because the algorithm inappropriately used certain fields such as socioeconomic status and race while missing out on more immediate factors such as criminal history. Despite being flawed, the system would still heavily influence the sentence and bond amounts that defendants would receive, which leads to an unjust criminal justice system.