Problems for Exercise of Laplace Transform

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Instruction:

- Each objective question has 4 choices for its answer $(A), (B), (C)$ and (D). Only ONE of them is the correct answer. Select the correct answer of question.
- Unless otherwise mentioned symbols and notations have their usual standard meanings.
- Any essential Result, Formula or Theorem assumed for anwering must be clearly stated.
- Use of any kind of electronic device is NOT allowed.

1. If $f(t)$ is function defined for all $t \geq 0$, its Laplace Transform $F(s)$ is defined as

   (A) $\int_0^\infty e^{st} f(t)dt$ (B) $\int_0^\infty e^{-st} f(t)dt$ (C) $\int_0^\infty e^{ist} f(t)dt$ (D) $\int_0^\infty e^{-ist} f(t)dt$

2. Which of the following function is NOT of the exponential order?

   (A) $f(t) = t^{2020}$ (B) $g(t) = e^{t^2}$ (C) $h(t) = e^{-t^2}$ (D) $p(t) = \sin(t)$

3. Which of the following function is NOT of the exponential order?

   (A) $f(t) = 2020$ (B) $g(t) = e^{2020t}$ (C) $h(t) = \cos(t)$ (D) $p(t) = \tan(t)$

4. The Laplace transform of the function $f(t) = e^{-t}$ is given by

   (A) $\frac{1}{(s+1)^2}$ (B) $\frac{1}{s-1}$ (C) $\frac{1}{s+1}$ (D) $\frac{1}{(s-1)^2}$

5. The Laplace transform of the function e^{-2t} is

   (A) $\frac{1}{2s}$ (B) $\frac{2}{s}$ (C) $\frac{1}{s+2}$ (D) e^{-2s}

6. The Laplace transform of $f(t) = 1 - e^{-2t}$ is

   (A) $\frac{2}{s(s+2)}$ (B) $\frac{1}{s(s+2)}$ (C) $\frac{2}{s(s-2)}$ (D) $\frac{1}{s(s-2)}$

7. Laplace transform of $\cos(\omega t)$ is

(A) $\frac{s}{s^2+\omega^2}$ (B) $\frac{\omega}{s^2+\omega^2}$ (C) $\frac{s}{s^2-\omega^2}$ (D) $\frac{\omega}{s^2-\omega^2}$

8. Laplace transform for the function $f(t) = \cosh(at)$ is

(A) $\frac{a}{s^2-a^2}$ (B) $\frac{s}{s^2-a^2}$ (C) $\frac{a}{s^2+a^2}$ (D) $\frac{s}{s^2+a^2}$

9. The Laplace transform of $f(t) = 2t + 6$ is

(A) $\frac{1}{s}+\frac{2}{s^2}$ (B) $\frac{3}{s}-\frac{6}{s^2}$ (C) $\frac{6}{s}+\frac{2}{s^2}$ (D) $-\frac{6}{s}+\frac{2}{s^2}$

10. The Laplace transformation of the function $f(t) = t^2 + 2t + 1$ is

(A) $\frac{1}{s^3}+\frac{3}{s^2}+\frac{1}{s}$ (B) $\frac{4}{s^3}+\frac{4}{s^2}+\frac{1}{s}$ (C) $\frac{1}{s^3}+\frac{2}{s^2}+\frac{1}{s}$ (D) $\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}$

11. Let $\mathcal{L}$ be the Laplace transform. Then $\mathcal{L}(\sqrt{t})$ is

(A) $\sqrt{\frac{\pi}{s}}, \quad s>0$ (B) $\frac{\sqrt{\pi}}{2}$ (C) $\frac{\sqrt{\pi}}{2s^{3/2}}, \quad s>0$ (D) $\frac{\pi}{s}, \quad s>0$

12. The Laplace transform of $\frac{1}{\sqrt{t}}$ is

(A) $\sqrt{\frac{\pi}{s}}$ (B) $\frac{1}{\sqrt{s}}$ (C) $\frac{1}{s^{3/2}}$ (D) does not exist

13. The Laplace transform of $f(t) = \sqrt{1/\pi t}$ is

(A) $3s^{-5/2}/2$ (B) $s^{-1/2}$ (C) $s^{1/2}$ (D) $s^{3/2}$

14. The Laplace transform of function $6t^3 + 3\sin 4t$ is

(A) $\frac{36}{s^4}+\frac{12}{s^2+16}$ (B) $\frac{36}{s^4}+\frac{12}{s^2-16}$ (C) $\frac{18}{s^4}+\frac{12}{s^2-16}$ (D) $\frac{36}{s^3}+\frac{12}{s^2+16}$

15. Given that the Laplace transform, $\mathcal{L}(e^{at}) = \frac{1}{s-a}$ then $\mathcal{L}(3e^{5t}\sinh 5t)$ is equal to

(A) $\frac{3s}{s^2-10s}$ (B) $\frac{15}{s^2-10s}$ (C) $\frac{3s}{s^2+10s}$ (D) $\frac{15}{s^2+10s}$

16. The Laplace transform $e^{-2t}\cos(4t)$ is

(A) $\frac{s-2}{(s-2)^2+16}$ (B) $\frac{s+2}{(s-2)^2+16}$ (C) $\frac{s-2}{(s+2)^2+16}$ (D) $\frac{s+2}{(s+2)^2+16}$

17. The Laplace Transform of $f(t) = e^{2t}\sin(5t)u(t)$ is

(A) $\dfrac{5}{s^2-4s+29}$ (B) $\dfrac{5}{s^2+5}$ (C) $\dfrac{s-2}{s^2-4s+29}$ (D) $\dfrac{5}{s+5}$

18. The Laplace transform of function te^t is

(A) $\dfrac{s}{(s+1)^2}$ (B) $\dfrac{1}{(s-1)^2}$ (C) $\dfrac{1}{(s+1)^2}$ (D) $\dfrac{s}{(s-1)}$

19. The Laplace transform of $f(t) = t^n e^{-\alpha t} u(t)$ is

(A) $\dfrac{(n+1)!}{(s+\alpha)^{n+1}}$ (B) $\dfrac{n!}{(s+\alpha)^n}$ (C) $\dfrac{(n-1)!}{(s+\alpha)^{n+1}}$ (D) $\dfrac{n!}{(s+\alpha)^{n+1}}$

20. Given that the Laplace transform of $y(t) = e^{-t}(2\cos 2t - \sin 2t)$ is $Y(s) = \dfrac{2s}{(s+1)^2+4}$, the Laplace transform of $y_1(t) = e^t(2\cos 2t - \sin 2t)$ is

(A) $\dfrac{2(s-2)}{(s-1)^2+4}$ (B) $\dfrac{2(s+2)}{(s+3)^2+4}$ (C) $\dfrac{2(s+2)}{(s+1)^2+4}$ (D) $\dfrac{2(s-1)}{(s-1)^2+4}$

21. The Laplace Transform of $f(t) = t\cos ht$ is

(A) $\dfrac{1+s^2}{(s^2-1)^2}$ (B) $\dfrac{st}{s^2-1}$ (C) $\dfrac{1-s^2}{(s^2-1)^2}$ (D) $\dfrac{1+s^2}{1-s^2}$

22. The Laplace transform of $f(t)$ is $\dfrac{1}{s^2+s+1}$. The Laplace transform of $tf(t)$ is

(A) $\dfrac{-s}{(s^2+s+1)^2}$ (B) $\dfrac{-(2s+1)}{(s^2+s+1)^2}$ (C) $\dfrac{s}{(s^2+s+1)^2}$ (D) $\dfrac{2s+1}{(s^2+s+1)^2}$

23. If $F(s)$ is the Laplace transform of function of function $f(t)$, then Laplace transform of $\displaystyle\int_0^t f(\tau)d\tau$ is

(A) $\dfrac{1}{s}F(s)$ (B) $\dfrac{1}{s}F(s) - f(0)$ (C) $sF(s) - f(0)$ (D) $\displaystyle\int F(s)ds$

24. The Laplace transform of function $f(t) = \begin{cases} t & \text{if } 0 < t < 1 \\ 1 & \text{if } \quad t > 1 \end{cases}$ is

(A) $\dfrac{-(1-e^{-s})}{s^2}$ (B) $\dfrac{1-e^{-s}}{s^2}$ (C) $\dfrac{1+e^{-s}}{s^2}$ (D) $\dfrac{1-e^{s}}{s^2}$

25. If $f(x) = \begin{cases} 0 & \text{for } \quad x < 3 \\ x-3 & \text{for } x \geq 3 \end{cases}$ then the Laplace transform of $f(x)$ is

(A) $s^{-2}e^{3s}$ (B) s^2e^{-3s} (C) s^{-2} (D) $s^{-2}e^{-3s}$

26. Which of the following pairs of the given function $f(t)$ and its Laplace transform $F(s)$ is NOT CORRECT ?

(A) $f(t) = 1,\ F(s) = \dfrac{1}{s},\quad (s > 0)$

(B) $f(t) = \delta(t),\ F(s) = 1,$ (Singularity at $+0$)

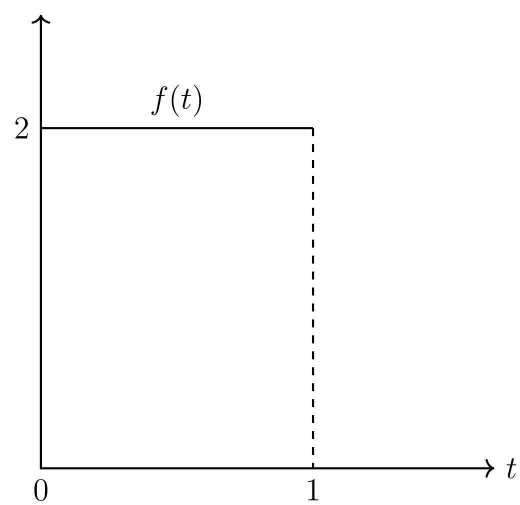
(C) $f(t) = \sin kt,\ F(s) = \dfrac{s}{s^2 + k^2},\quad (s > 0)$

(D) $f(t) = te^{kt},\ F(s) = \dfrac{1}{(s-k)^2},\quad (s > k, s > 0)$

27. Let $u(t)$ represents the unit step function. The Laplace transform of $u(t-a)$ is (where $a > 0$)

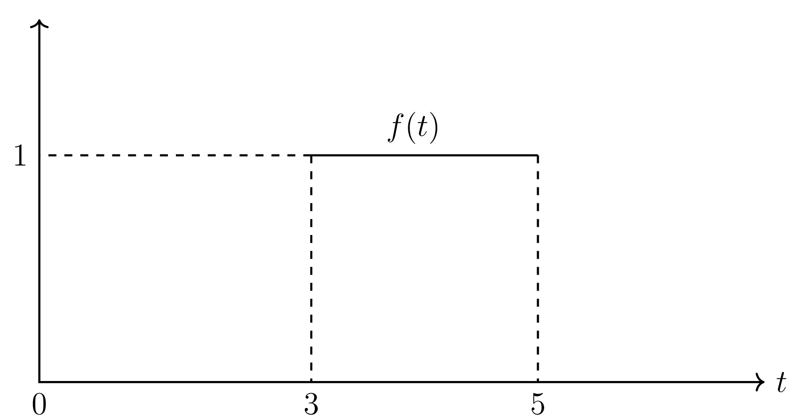
(A) $\dfrac{1}{sa}$ (B) $\dfrac{1}{s-a}$ (C) $\dfrac{e^{-sa}}{s}$ (D) e^{-sa}

28. Laplace transform of the function shown below is given by



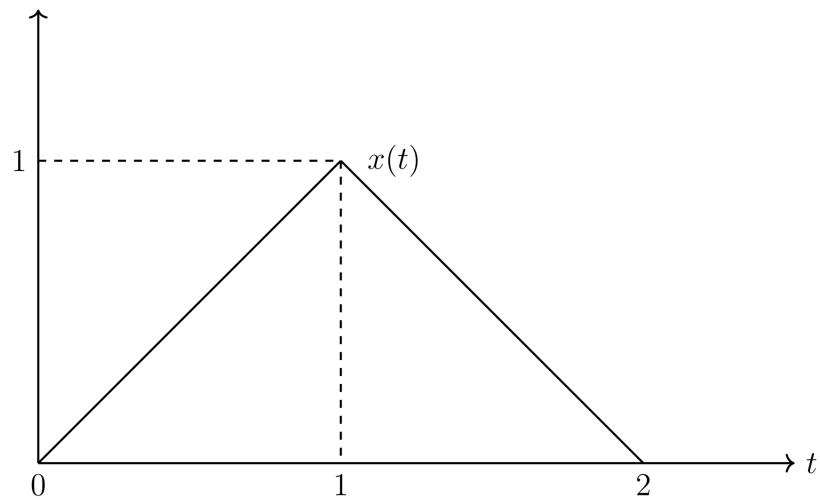
(A) $\dfrac{1-e^{-2s}}{s}$ (B) $\dfrac{1-e^{-s}}{2s}$ (C) $\dfrac{2-2e^{-s}}{s}$ (D) $\dfrac{1-2e^{-s}}{s}$

29. Given $f(t)$ as shown below, then Laplace transform of $f(t)$ is



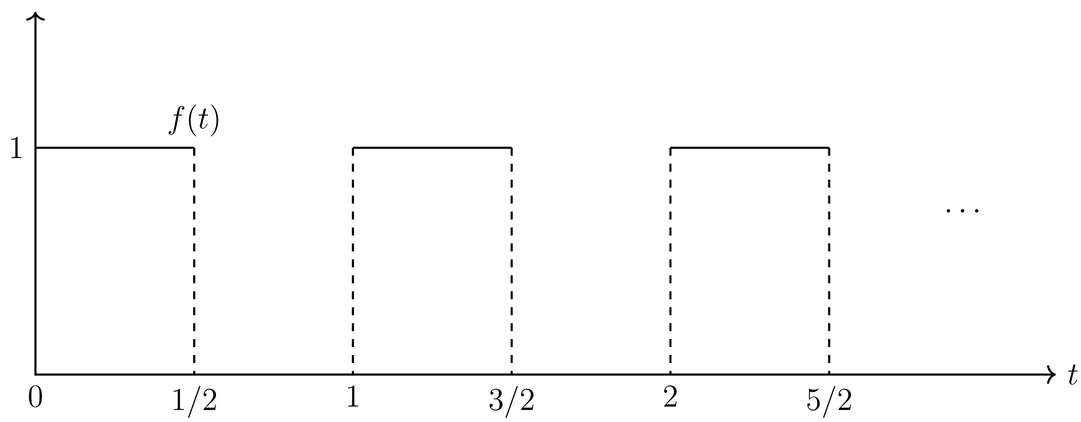
(A) $\frac{1}{s}(e^{-3s} - e^{-5s})$ (B) $\frac{1}{s}(e^{-5s} - e^{3s})$ (C) $\frac{e^{3s}}{s}(1 - e^{-2s})$ (D) $\frac{1}{s}(e^{5s} - e^{3s})$

30. The Laplace Transform representation of the triangular pulse shown below is



(A) $\frac{1}{s^2}[1 + e^{-2s}]$ (B) $\frac{1}{s^2}[1-e^{-s}+e^{-2s}]$ (C) $\frac{1}{s^2}[1-e^{-s}+2e^{-2s}]$ (D) $\frac{1}{s^2}[1-2e^{-s}+e^{-2s}]$

31. The Laplace transform of the periodic square wave of period 1 shown in the figure below is



(A) $\frac{1}{1 + e^{-s/2}}$ (B) $\frac{1}{s\,(1 + e^{-s/2})}$ (C) $\frac{1}{s(1 - e^{-s})}$ (D) $\frac{1}{1 - e^{-s}}$

32. Let $F(s) = \frac{(s+10)}{(s+2)(s+20)}$ The inverse Laplace transform of $F(s)$ is

(A) $2e^{-2t} + 20e^{-20t}$ (B) $5e^{-2t} + 2e^{-20t}$ (C) $\frac{4}{9}e^{-2t} + \frac{5}{9}e^{-20t}$ (D) $\frac{9}{4}e^{-2t} + \frac{9}{5}e^{-20t}$

33. The inverse Laplace transform of $F(s) = \frac{(s+1)}{(s+4)(s-3)}$ is

(A) $\frac{3}{7}e^{4t}+\frac{4}{7}e^{-3t}$ (B) $\frac{3}{7}e^{-4t}+\frac{4}{7}e^{3t}$ (C) $\frac{5}{7}e^{-4t}+\frac{6}{7}e^{3t}$ (D) $\frac{5}{7}e^{4t}+\frac{6}{7}e^{-3t}$

34. The Laplace transform of a function $f(t)$ is $\frac{1}{s^2(s+1)}$. The function $f(t)$ is

(A) $t-1+e^{-t}$ (B) $t+1+e^{-t}$ (C) $-1+e^{-t}$ (D) $2t+e^{-t}$

35. The inverse Laplace transform of $\frac{1}{s^2+s}$ is

(A) $1+e^{t}$ (B) $1-e^{t}$ (C) $1-e^{-t}$ (D) $1+e^{-t}$

36. The inverse Laplace transform of $H(s)=\frac{s+3}{s^2+2s+1}$ for $t\geq 0$ is

(A) $3te^{-t}+e^{-t}$ (B) $3e^{-t}$ (C) $2te^{-t}+e^{-t}$ (D) $4te^{-t}+e^{-t}$

37. The inverse Laplace transform of $\frac{2s^2-4}{(s-3)(s^2-s-2)}$ is

(A) $(1+t)e^{-t}+\frac{7}{2}e^{-3t}$ (B) $\frac{e^{t}}{3}+te^{-t}+2t$ (C) $\frac{7}{2}e^{3t}-\frac{e^{-t}}{6}-\frac{4}{3}e^{2t}$ (D) $\frac{7}{2}e^{-3t}-\frac{e^{t}}{6}-\frac{4}{3}e^{-2t}$

38. The Laplace transform of a function $f(t)$ is $F(s)=\frac{s+2}{(s+2)^2+(10)^2}$ The values of $f(0)$

(A) -1 (B) 0 (C) 1 (D) 2

39. If $F(s)=\frac{2(s+1)}{s^2+4s+7}$ then the initial and finial values of $f(t)$ are respectively

(A) 0, 2 (B) 2, 0 (C) 0, 2/7 (D) 2/7, 0

40. The Laplace transform of a function is $\frac{s+1}{s(s+2)}$. The initial and final values, respectively, of the function are

(A) 0 and 1 (B) 1 and $\frac{1}{2}$ (C) $\frac{1}{2}$ and 1 (D) $\frac{1}{2}$ and 0

41. Let y be the solution of the initial value problem $\frac{d^2y}{dx^2}+y=6\cos 2x,\ \ y(0)=3,\ y'(0)=1$. Let the Laplace transform of y be $Y(s)$. Then the value of $Y(1)$ is

(A) $\frac{17}{5}$ (B) $\frac{13}{5}$ (C) $\frac{11}{5}$ (D) $\frac{9}{5}$

42. The Laplace transform $\mathcal{L}(u(t))=U(s)$, for the solution $u(t)$ of the problem $\frac{d^2u}{dt^2}+2\frac{du}{dt}+u=1,\ \ t>0$ with initial conditions $u(0)=0$, $\frac{du(0)}{dt}=5$. Then $U(s)$ is:

(A) $\dfrac{6}{(s+1)^2}$ (B) $\dfrac{5s+1}{s(s+1)^2}$ (C) $\dfrac{1-5s}{s(s+1)^2}$ (D) $\dfrac{5s^2+1}{s(s+1)^2}$

43. A system is described by the differential equation $\dfrac{d^2y}{dt^2}+5\dfrac{dy}{dt}+6y(t)=x(t)$. Let $x(t)$ be a rectangular pulse given by $x(t)=\begin{cases}1 & 0<t<2\\ 0 & \text{otherwise}\end{cases}$ Assuming that $y(0)=0$ and $\dfrac{dy}{dt}=0$ at $t=0$, the Laplace transform of $y(t)$ is

(A) $\dfrac{e^{-2s}}{s(s+2)(s+3)}$ (B) $\dfrac{1-e^{-2s}}{s(s+2)(s+3)}$ (C) $\dfrac{e^{-2s}}{(s+2)(s+3)}$ (D) $\dfrac{1-e^{-2s}}{(s+2)(s+3)}$

44. Let $y(x)$ be the solution of the differential equation $\dfrac{dy}{dx}+y=f(x)$, for $x\ \geq\ 0,\ \ y(0)=0$, where $f(x)=\begin{cases}2, & 0\ \leq\ x<1\\ 0, & x\ \geq\ 1\end{cases}$ Then $y(x)$ is equal to

(A) $2(1-e^{-x})$ when $0\ \leq\ x<1$ *and* $2(e-1)e^{-x}$ when $x\ \geq\ 1$

(B) $2(1-e^{-x})$ when $0\ \leq\ x<1$ *and* 0 when $x\ \geq\ 1$

(C) $2(1-e^{-x})$ when $0\ \leq\ x<1$ *and* $2(1-e^{-1})e^{-x}$ when $x\ \geq\ 1$

(D) $2(1-e^{-x})$ when $0\ \leq\ x<1$ *and* $2e^{1-x}$ when $x\ \geq\ 1$

45. The solution to the initial value problem $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+5y=3e^{-t}\sin t,\quad y(0)=0$ and $\dfrac{dy}{dt}(0)=3$, is

(A) $y(t)=e^{t}(\sin t+\sin 2t)$ (B) $y(t)=e^{-t}(\sin t+\sin 2t)$

(C) $y(t)=3e^{t}\sin t$ (D) $y(t)=3e^{-t}\sin t$

46. If $Y(s)$ is the Laplace transform of $y(t)$, which is the solution of the initial value problem $\dfrac{d^2y}{dt^2}+y(t)=\begin{cases}0 & 0<t<2\pi\\ \sin t & t>2\pi\end{cases}$, with $y(0)=1$ and $y^{'}(0)=0$, then $Y(s)$ equals

(A) $\dfrac{s}{s^2+1}+\dfrac{e^{-2\pi s}}{(s^2+1)^2}$ (B) $\dfrac{s+1}{s^2+1}$ (C) $\dfrac{s}{s^2+1}+\dfrac{e^{-2\pi s}}{(s^2+1)}$ (D) $\dfrac{s(s^2+1)+1}{(s^2+1)^2}$

47. Consider the initial value problem: $\dfrac{d^2y}{dt^2}+4\dfrac{dy}{dt}+6y=f(t);\quad y(0)=2,\quad \left(\dfrac{dy}{dt}\right)_{t=0}=1$

If $Y(s)=\displaystyle\int_0^{\infty}y(t)e^{-st}dt$ and $F(s)=\displaystyle\int_0^{\infty}f(t)e^{-st}dt$ are the Laplace transform of $y(t)$ and $f(t)$ respectively, then $Y(s)$ is given by:

(A) $\dfrac{F(s)}{s^2+4s+6}$ (B) $\dfrac{F(s)+2s+9}{s^2+4s+6}$ (C) $\dfrac{F(s)}{-s^2+4s+6}$ (D) $\dfrac{F(s)-2s+9}{s^2+4s+6}$

48. A system is discribed by the following differential equation, where $u(t)$ is the input to the system and $y(t)$ is the output of the system $\dot{y}(t)+5y(t)=u(t)$. When $y(0)=1$ and $u(t)$ is a unit step function, $y(t)$ is

(A) $0.2+0.8e^{-5t}$ (B) $0.2-0.2e^{-5t}$ (C) $0.8+0.2e^{-5t}$ (D) $0.8-0.8e^{-5t}$

49. A function $y(t)$ satisfies the differential equation: $\dfrac{dy(t)}{dt}+y(t)=\delta(t)$. Assuming zero initial condition, The solution of given differential equation $y(t)$ is

(A) e^t (B) e^{-t} (C) $e^t u(t)$ (D) $e^{-t}u(t)$

50. The solution of the differential equation, for $t>0$, $y''(t)+2y'(t)+y(t)=0$ with initial conditions $y(0)=0$ and $y'(0)=1$, is

(A) $te^{-t}u(t)$ (B) $(e^{-t}-te^{-t})u(t)$ (C) $(-e^{-t}+te^{-t})u(t)$ (D) $e^{-t}u(t)$

51. The solution of the differential equation $\dfrac{d^2y}{dt^2}-y=2\cosh(t)$ subject to the initial conditions $y(0)=0$ and $\left.\dfrac{dy}{dx}\right|_{t=0}=0$, is

(A) $\dfrac{1}{2}\cosh(t)+t\sinh(t)$ (B) $-\sinh(t)+t\cosh(t)$

(C) $t\cosh(t)$ (D) $t\sinh(t)$

52. Solution of system of differentaial equation represented by $\begin{bmatrix} dx/dt \\ dy/dt \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ with initial condition $x(0)=1$ and $y(0)=-1$ is

(A) $x(t)=-1,\ \ y(t)=2$ (B) $x(t)=-e^{-t},\ \ y(t)=2e^{-t}$

(C) $x(t)=e^{-t},\ \ y(t)=-e^{-2t}$ (D) $x(t)=-e^{-t},\ \ y(t)=-2e^{-t}$

53. Consider the integral equation $y(x)=x^3+\int_0^x \sin(x-t)y(t)dt,\quad x\in[0,2\pi]$. Then the values of $y(1)$ is

(A) $\dfrac{19}{20}$ (B) 1 (C) $\dfrac{17}{20}$ (D) $\dfrac{21}{20}$

54. The solution of the integral equation $\varphi(x)=x+\int_0^x \sin(x-t)\varphi(t)dt$ is

(A) $x^2+\dfrac{x^3}{3}$ (B) $x-\dfrac{x^3}{3!}$ (C) $x+\dfrac{x^3}{3!}$ (D) $x^2-\dfrac{x^3}{3!}$

55. Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t) * y(t)$ is

(A) $e^{-t} - e^{-2t}$ (B) e^{-3t} (C) e^{t} (D) $e^{-t} + e^{-2t}$

56. Find the Laplace transform of the signal $x(t) = e^{-3t}u(t) + e^{-2t}u(t)$

57. Find the Laplace transform of function: $f(t) = 2e^{-t}\cos(10t) - t^4 + 6e^{-(t-10)}$ for $t > 0$.

58. Find the Laplace transform of function $f(t) = e^{-t^2}$ for $t \geq 0$.

59. Find the Inverse Laplace Transform of $\dfrac{s+2}{(s+1)(s+3)^2}$.

60. Solve the following initial value problem using Laplace transform:

$$\frac{d^2y}{dx^2} + 9y = r(x),\ y(0) = 0,\ y'(0) = 4 \ \text{ where } \ r(x) = \begin{cases} 8\sin x & \text{if } \ 0 < x < \pi \\ 0 & \text{if } \ x \geq \pi \end{cases}$$

61. Let system of differential equations is given as $\dfrac{dx}{dt} = y$ and $\dfrac{dy}{dt} = -x$ with the initial conditions $x(0) = 1$ and $y(0) = 1$ then find the solution of the system.

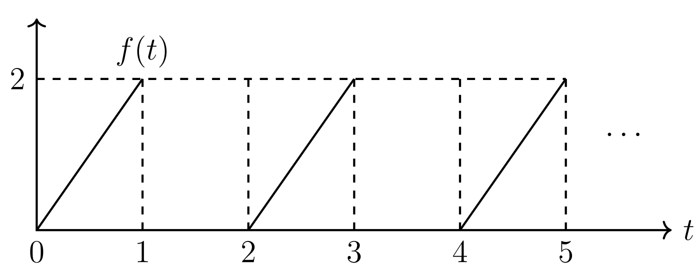
62. Find the general solution of the linear system $\dfrac{dx}{dt} = x - y$, $\dfrac{dy}{dt} = x + y$, $t > 0$ with initial condition $x(0) = 1 = y(0)$

63. Consider the system of linear differential equations $\dfrac{dx}{dt} = 5x - 2y$ and $\dfrac{dy}{dt} = 4x - y$ with the initial conditions $x(0) = 0$, $y(0) = 1$ then find the solution of the system.

64. Solve the integral equations $f(x) = x + \displaystyle\int_0^x \sin(x-t)f(t)\,dt$

65. Let $f(x) = \begin{cases} 1 & \text{if } \ 0 \leq x \leq 1 \\ 0 & \text{if } \ otherwise \end{cases}$ find convolution $f * f$

66. Find the Laplace transform of the periodic function $f(t)$ shown in following figure



If you found any mistake(s) please report me at `dng.maths@coep.ac.in`.

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