Linear Algebra, F2023	Posted: September 6, 2023
Project 0	
Lecturer: Hyang-Won Lee	**Due: September 20, 2023**

Important Notes

- This is a programming project for basic matrix operations.

- You need to write a computer program in C (no other languages are allowed).

- Keep in mind that your code will be examined using "VS2022 on Windows10" regardless of the platform you worked on.

- Code with compile error, segmentation fault, etc. will get zero score.

- We consider only real matrices, vectors and scalars. The numbers of columns and rows are all integer types.

- Take a look at the sample code uploaded (matOperations.c).

- Upload your report (containing the screenshot of the results of your code) and code at ecampus

1. (**50 points**) Write the following functions (choose the return/input types on your own decision):

    (a) transposeMatrix(A, m, n): transpose the $m \times n$ matrix A and return the result

    (b) normalizeVector(v, n): normalize the n-dimensional vector v and return the result

    (c) calculateLength(v, n): calculate the length of the n-dimensional vector v and return the result

    (d) scaleMatrix(A, m, n, c): scale the $m \times n$ matrix A with scalar c

    (e) multiplyTwoMatrices(A, m, n, B, l, k): for $m \times n$ matrix A and $l \times k$ matrix B, calculate and return AB. Return null if multiplication is impossible.

    (f) addTwoMatrices(A, m, n, B, l, k): for $m \times n$ matrix A and $l \times k$ matrix B, calculate and return $A + B$. Return null if addition is impossible.

2. (**50 points**) Write a computer program in C that performs the following:

    (a) Test the correctness of each of the function you wrote in 1.

    (b) For given $n \times n$ matrices A and $\tilde{H}$, normalize each column of $\tilde{H}$ (let H be this normalized matrix). Then, calculate $B = H^T A H$, and then, $C = H B H^T$.

- For example, if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\tilde{H} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$, then $H = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$ and

$$
\begin{aligned}
B &= H^T A H \\
&= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \\
&= \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
C &= H B H^T \\
&= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}
\end{aligned}
$$