Due Thursday April 8th at 11:59pm

Problem 1

Create the following matrices and vectors in MATLAB or python. (Make sure that they are the correct shape, especially in python.)

$$A = \begin{pmatrix} 3 & -0.5 \\ 3.14 & e^3 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 & -4 \\ \pi & 6 & 3 & -1.4 \end{pmatrix} \qquad C = \begin{pmatrix} 2.7 & -3.4 & 0 \\ 1 & 5.5 & -3.7 \\ 4.5 & -1.1 & 6.7 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ \cos(4) \\ -2 \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} 3 & -5 \end{pmatrix} \qquad \mathbf{z} = \begin{pmatrix} 2 \\ 0 \\ \tan(2) \\ -3.6 \end{pmatrix}$$

For all of the following questions, you must use the variables you defined above. Do not type out the answers directly.

(1) Calculate $3\mathbf{x}$ and save the result in a variable named `A1`.
(2) Calculate $\mathbf{z}^T B^T + \mathbf{y}$ and save the result in a variable named `A2`. (Remember that the T indicates the transpose operation, which changes rows to columns and vice versa.)
(3) Only one of the following is defined: $C\mathbf{x}$, $C\mathbf{y}$ or $C\mathbf{z}$. Calculate the one that is defined and save the result in a variable named `A3`.
(4) Only one of the following is defined: AB or BA. Calculate the one that is defined and save the result in a variable named `A4`.
(5) Only one of the following is defined: $A^T B^T$ or $B^T A^T$. Calculate the one that is defined and save the result in a variable named `A5`.

Problem 2

In the following parts, you should use predefined MATLAB or python commands (such as the colon operator `:` or `linspace` in MATLAB or `np.arange` or `np.linspace` in python) to create any needed vectors. Do not just type the values in directly. (Especially in python, make sure that your vectors are the right shape.)

(1) Make a row vector $\mathbf{x}$ with 73 evenly spaced elements beginning with -4 and ending with 1. Save the resulting vector in a variable named `A6`.
(2) Make a row vector $\mathbf{y}$ of the following form:

$$\mathbf{y} = \begin{pmatrix} \cos(0) & \cos(1) & \cos(2) & \cdots & \cos(71) & \cos(72) \end{pmatrix}.$$

That is, the ith entry of the vector (counting from 1) is $\cos(i-1)$. Save the resulting vector in a variable named `A7`.
(3) Calculate a vector whose elements are the product of the elements of $\mathbf{x}$ and $\mathbf{y}$. That is, the ith entry of your result should be $x_i y_i$. Save the resulting vector in a variable named `A8`.
(4) Calculate a vector whose elements are the ratio of the elements of $\mathbf{x}$ and $\mathbf{y}$. That is, the ith entry of your result should be x_i/y_i. Save the resulting vector in a variable named `A9`.
(5) Calculate a vector whose elements are the difference between the elements of $\mathbf{x}$ cubed and the elements of $\mathbf{y}$. That is, the ith entry of your result should be $x_i^3 - y_i$. Save the resulting vector in a variable named `A10`.

Problem 3

(1) The logistic map is a model that is often used in ecology to model population growth. It is defined by

$$P(t+1) = rP(t)\left(1 - \frac{P(t)}{K}\right).$$

Here, $P(t)$ represents the population density of a given species at year t, the parameter r is a growth rate and the parameter K is the maximum possible population density (known as the carrying capacity). This equation says that if we know the density at one year, we can substitute it into the right hand side to find the population at the next year. For instance, if we knew the population at year 0 (given by $P(0)$), we could calculate

$$P(1) = rP(0)\left(1 - \frac{P(0)}{K}\right).$$

Once we had the population density at year 1, we could then find the density at year 2 using

$$P(2) = rP(1)\left(1 - \frac{P(1)}{K}\right).$$

(a) Suppose that $P(0) = 5$, $K = 10$ and $r = 2$. Calculate $P(3)$ and save it in a variable named `A11`.

(b) Now suppose that $P(0) = 10$, $K = 15$ and $r = 3$. Calculate $P(4)$ and save it in a variable named `A12`.

(2) A slightly different version of this model, known as the Ricker model, is often used in fisheries biology. The Ricker model uses the formula

$$P(t+1) = P(t)e^{r\left(1 - \frac{P(t)}{K}\right)}.$$

(The function `exp` or `np.exp` might be useful in the following calculations.)

(a) Suppose that $P(0) = 5$, $K = 12$ and $r = 2$. Calculate $P(3)$ and save it in a variable named `A13`.

(b) Now suppose that $P(0) = 2$, $K = 25$ and $r = 2.5$. Calculate $P(4)$ and save it in a variable named `A14`.

(c) Now suppose that $P(0) = 0$, $K = 20$ and $r = 3.1$. "Calculate" $P(500)$. (There won't be any calculations necessary for this one.) Save your answer in a variable named `A15`.