

# Implementation of 1-D Vascular Model using Structured Tree Outflow Conditions

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# Overview

## 1 Introduction

- Project Goals

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- Discretization
- Inflow
- Outflow

## 3 Results

- Our Implementation
- Olufsen Implementation

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# Project Goals

- Use Olufsen's thesis to implement similar 1-D code in Matlab
- Investigate the effect of root impedance on blah
- Investigate blah on wave reflections

# One-dimensional flow of blood - governing equations

## Continuity

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

## Conservation of X Momentum

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left( \frac{q^2}{A} \right) + \frac{A}{p} \frac{\partial p}{\partial x} = - \frac{2R\pi\nu q}{\delta A} \quad (2)$$

A

Two-Step Lax-Wendroff (Richtmyer) scheme is used. Equations ?? and ?? can be written in the following form:

$$\frac{\partial}{\partial t} \bar{U} + \frac{\partial}{\partial x} \bar{R}(\bar{U}) = \bar{S} \quad (3)$$

# Hyperbolic system and finite difference discretization

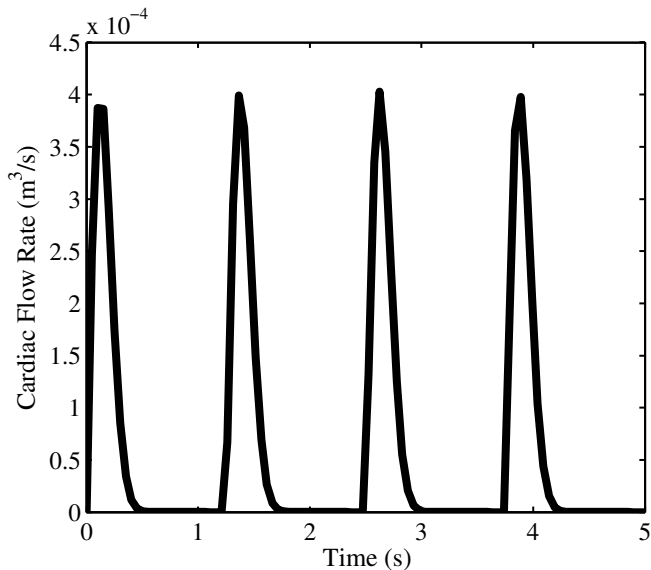
1

$$\bar{U}_j^{T+1} = \bar{U}_j^T - \frac{k}{h} \left( \bar{R}_{j+1/2}^{T+1/2} - \bar{R}_{j-1/2}^{T+1/2} \right) + \frac{k}{2} \left( \bar{S}_{j+1/2}^{T+1/2} + \bar{S}_{j-1/2}^{T+1/2} \right) \quad (4)$$

2

$$\begin{aligned} \bar{U}_j^{T+1/2} &= \frac{\bar{U}_{j+1/2}^T + \bar{U}_{j-1/2}^T}{2} + \frac{k}{2h} \left( -\bar{R}_{j+1/2}^T - \bar{R}_{j-1/2}^T \right) \\ &\quad + \frac{k}{4} \left( \bar{S}_{j+1/2}^T + \bar{S}_{j-1/2}^T \right) \end{aligned}$$

# Inflow profile and inlet boundary condition



# The structured tree

## Olufsen's

She retains no structure of the vessels within the tree and recalculates all this information for each structured tree.

$$(r_0)(n, k) = R_{root} \alpha^k \beta^{n-k} \quad (5)$$

## Our's

We retain a structure of the tree by a vector of nodes containing certain information about the tree.

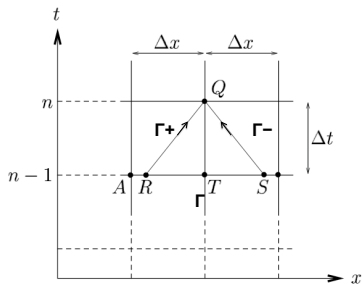
$$[ \textit{Daughter 1} \quad \textit{Daughter 2} \quad \textit{Parent} \quad \textit{Radius} ] \quad (6)$$



# Outflow boundary condition using structured tree

## Method of Characteristics

$$\Gamma_{+/-} : A_Q - A_{R/S} + \frac{Q_Q - Q_{R/S}}{-(Q_{R/S}/A_{R/S}) + C_R} = H_{R/S}^{+/-} \Delta t \quad (7)$$



# Toy problem for comparison

## Single Tube with Resistance Boundary Condition

The pressure profile follows the same curve. The dicrotic notch is less visible in our implementation, and the magnitude is slightly higher.

**Flow analysis: What information can we get from such 1-dimensional, reduced order models ?**

# Wave reflection phenomena - comparison with Windkessel models

# Varying terminal resistance of tree for flow regulation

# Radius of large vessels - insights on stenosis/anuerysms ?

# Concluding remarks

- Currently implemented way is not always the best way.
- Something else