Implementation of 1-D Vascular Model using Structured Tree Outflow Conditions

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May 16, 2014

Overview

Project Goals

One-dimensional flow of blood - governing equations

Continuity

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{1}$$

Conservation of X Momentum

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{A}\right) + \frac{A}{p}\frac{\partial p}{\partial x} = -\frac{2R\pi\nu q}{\delta A} \tag{2}$$

Numerical Solution & Development of a Code

A

Two-Step Lax-Wendroff (Richtmyer) scheme is used. Equations 1 and 2 can be written in the following form:

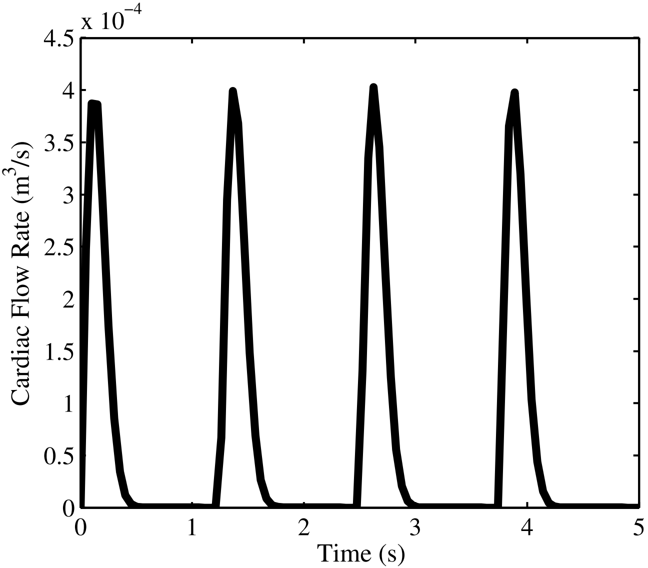
$$\frac{\partial}{\partial t}\bar{U} + \frac{\partial}{\partial x}\bar{R}(\bar{U}) = \bar{S} \tag{3}$$

Hyperbolic system and finite difference discretization

1.
$$\bar{U}_j^{T+1} = \bar{U}_j^{T} - \frac{k}{h}\left(\bar{R}_{j+1/2}^{T+1/2} - \bar{R}_{j-1/2}^{T+1/2}\right) + \frac{k}{2}\left(\bar{S}_{j+1/2}^{T+1/2} + \bar{S}_{j-1/2}^{T+1/2}\right) \quad (4)$$

2.
$$\bar{U}_j^{T+1/2} = \frac{\bar{U}_{j+1/2}^{T} + \bar{U}_{j-1/2}^{T}}{2} + \frac{k}{2h}\left(-\bar{R}_{j+1/2}^{T} - \bar{R}_{j-1/2}^{T}\right) + \frac{k}{4}\left(\bar{S}_{j+1/2}^{T} + \bar{S}_{j-1/2}^{T}\right)$$

Inflow profile and inlet boundary condition



The structured tree

Olufsen's

She retains no structure of the vessels within the tree and recalculates all this information for each structured tree.

$$(r_0)_(n, k) = R_{root}\alpha^k \beta^{n-k} \tag{5}$$

Our's

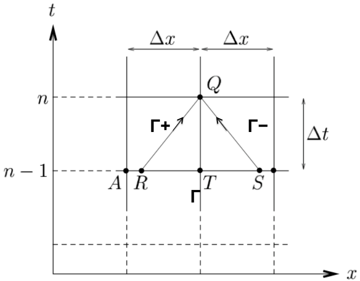
We retain a structure of the tree by a vector of nodes containing certain information about the tree.

$$\begin{bmatrix} Daughter\,1 & Daughter\,2 & Parent & Radius \end{bmatrix} \tag{6}$$

Outflow boundary condition using structured tree

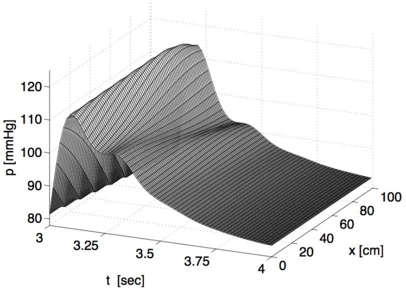
Method of Characteristics

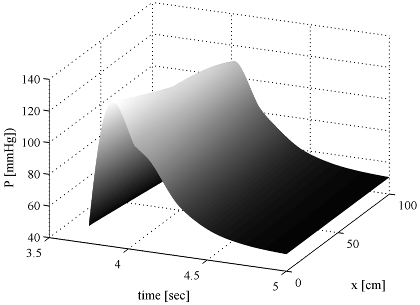
$$\Gamma_{+/-} \ : A_Q - A_{R/S} + \frac{Q_Q - Q_{R/S}}{-(Q_{R/S}/A_{R/S}) + C_R} = H^{+/-}_{R/S}\Delta t \qquad (7)$$



Single Tube with Resistance Boundary Condition

The pressure profile follows the same curve. The dicrotic notch is less visible in our implementation, and the magnitude is slightly higher.





Flow analysis: What information can we get from such 1-dimensional, reduced order models ?

Wave reflection phenomena - comparison with Windkessel models

Varying terminal resistance of tree for flow regulation

Radius of large vessels - insights on stenosis/anuerysms ?

Concluding remarks