Implementation of 1-D Vascular Model using Structured Tree Outflow Conditions

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Overview

Project Goals

- Use Olufsen's thesis to implement similar 1-D code in Matlab
- Investigate the effect of root impedance on blah
- Investigate blah on wave reflections

One-dimensional flow of blood - governing equations

Continuity

$$\frac{\partial A}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad (1)$$

Conservation of X Momentum

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{A}\right) + \frac{A}{p}\frac{\partial p}{\partial x} = -\frac{2R\pi\nu q}{\delta A} \qquad (2)$$

Numerical Solution & Development of a Code

A Two-Step Lax-Wendroff (Richtmyer) scheme is used. Equations 1 and 2 can be written in the following form:

$$\frac{\partial}{\partial t}\bar{U} + \frac{\partial}{\partial x}\bar{R}(\bar{U}) = \bar{S} \tag{3}$$

$$\bar{U}_j^{T+1} = \bar{U}_j^{T} - \frac{k}{h}\left(\bar{R}_{j+1/2}^{T+1/2} - \bar{R}_{j-1/2}^{T+1/2}\right) + \frac{k}{2}\left(\bar{S}_{j+1/2}^{T+1/2} + \bar{S}_{j-1/2}^{T+1/2}\right)$$

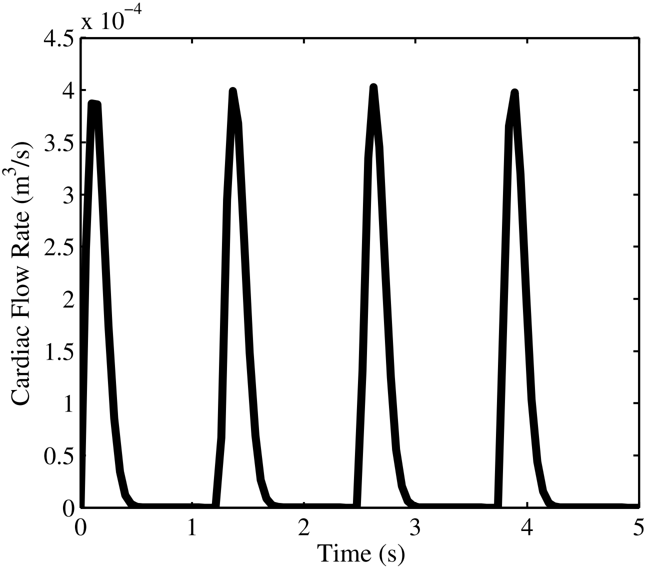
$$\bar{U}_j^{T+1/2} = \frac{\bar{U}_{j+1/2}^{T} + \bar{U}_{j-1/2}^{T}}{2} + \frac{k}{2h}\left(-\bar{R}_{j+1/2}^{T} - \bar{R}_{j-1/2}^{T}\right) + \frac{k}{4}\left(\bar{S}_{j+1/2}^{T} + \bar{S}_{j-1/2}^{T}\right)$$

Hyperbolic system and finite difference discretization

$$\bar{U}_j^{T+1} = \bar{U}_j^T - \frac{k}{h}\left(\bar{R}_{j+1/2}^{T+1/2} - \bar{R}_{j-1/2}^{T+1/2}\right) + \frac{k}{2}\left(\bar{S}_{j+1/2}^{T+1/2} + \bar{S}_{j-1/2}^{T+1/2}\right)$$

$$\bar{U}_j^{T+1/2} = \frac{\bar{U}_{j+1/2}^T + \bar{U}_{j-1/2}^T}{2} + \frac{k}{2h}\left(-\bar{R}_{j+1/2}^T - \bar{R}_{j-1/2}^T\right) + \frac{k}{4}\left(\bar{S}_{j+1/2}^T + \bar{S}_{j-1/2}^T\right)$$

Inflow profile and inlet boundary condition



The structured tree

Olufsen's

She retains no structure of the vessels within the tree and recalculates all this information for each structured tree.

$$(r_0)_(n,k) = R_{root}\alpha^k\beta^{n-k} \tag{4}$$

Our's

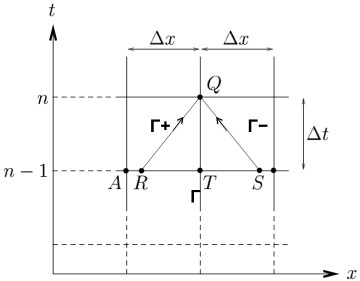
We retain a structure of the tree by a vector of nodes containing certain information about the tree.

$$\begin{bmatrix} Daughter\,1 & Daughter\,2 & Parent & Radius \end{bmatrix} \tag{5}$$

Outflow boundary condition using structured tree

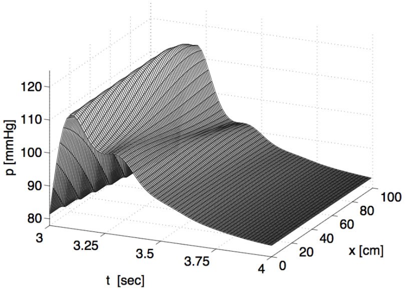
Method of Characteristics

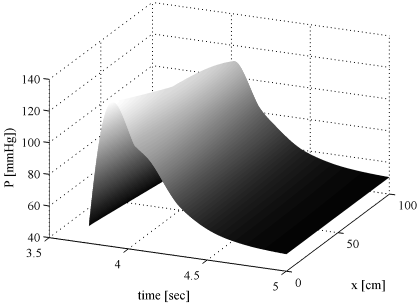
$$\Gamma_{+/-} \ : A_Q - A_{R/S} + \frac{Q_Q - Q_{R/S}}{-(Q_{R/S}/A_{R/S}) + C_R} = H^{+/-}_{R/S}\Delta t \quad (6)$$



Single Tube with Resistance Boundary Condition

The pressure profile follows the same curve. The dicrotic notch is less visible in our implementation, and the magnitude is slightly higher.

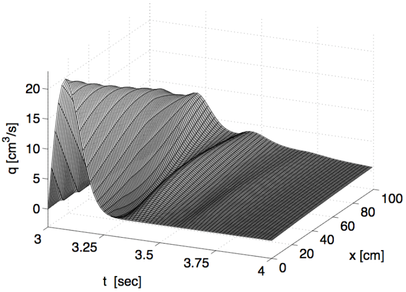


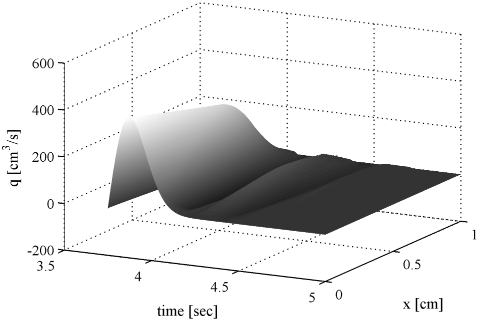


Toy problem for comparison

Single Tube with Structured Tree Boundary Condition

The pressure profile follows the same curve. The dicrotic notch is less visible in our implementation, and the magnitude is slightly higher.



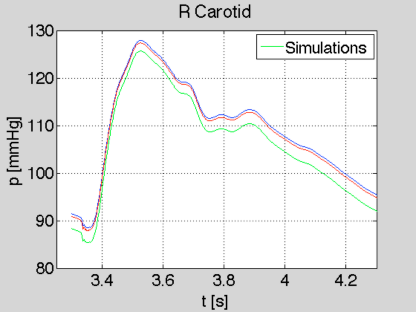


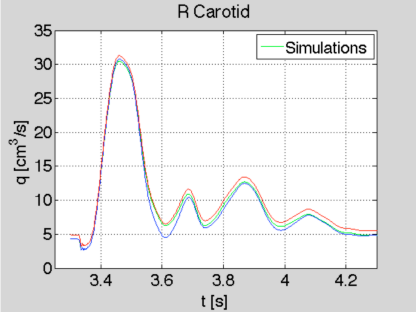
Flow analysis: What information can we get from such 1-dimensional, reduced order models ?

Wave reflection phenomena - comparison with Windkessel models

This was discussed in detail during the Journal Club review of the paper

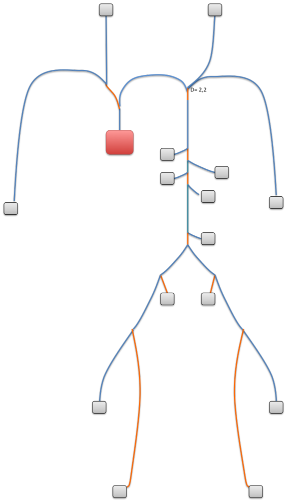
Varying terminal resistance of tree for flow regulation





increased terminal resistance from 0 (in green) to 230000 $\text{dynes} - \text{s}/\text{cm}^5$ (in blue) in all vascular beds, and then reduced terminal resistance to 0 in R. Carotid to regulate flow (in red).

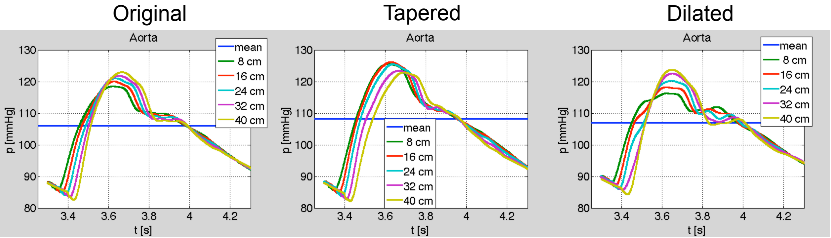
Radius of large vessels - insights on stenosis/aneurysms ?



A modified version of the vascular tree for studying vessel radius effects

Modifications to the vessel radii

Radius of large vessels - insights on stenosis/aneurysms



Concluding remarks

- Currently implemented way is not always the best way.
- Something else