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PERFORMANCE COMPARISON BETWEEN LQR AND PID CONTROLLER FOR AN INVERTED PENDULUM SYSTEM

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Abstract:

The objective of this paper is to compare the time specification performance between two conventional controllers for an inverted pendulum system. The goal is to determine which control strategy delivers better performance with respect to pendulum's angle and cart's position. The inverted pendulum represents a challenging control problem, which continually moves toward an uncontrolled state. Two controllers are presented such as Linear-Quadratic-Regulator (LQR) and Proportional-Integral-Derivatives (PID) controllers for controlling the linearized system of inverted pendulum model. Simulation study has been done in Matlab simulink environment shows that both controllers are capable to control multi output inverted pendulum system successfully. The result shows that LQR produced better response compared to PID control strategies and is presented in time domain.

Keywords: Proportional-integral-derivative (PID), Linear Quadratic Regulator (LQR), Inverted Pendulum System, PACS: 60

Introduction

One-dimensional inverted pendulum is a nonlinear problem, which has been considered by many researchers (Omatu and Yashioka, 1998; Magana and Holzapfel, 1998; Nelson and Kraft, 1994; Anderson, 1989), most of which have used linearization theory in their control schemes. In general, the control of this system by classical methods is a difficult task (Lin and Sheu, 1992). This is mainly because this is a nonlinear problem with two degrees of freedom (i.e. the angle of the inverted pendulum and the position of the cart), and only one control input [1].

The inverted pendulum is used for control engineers to verify a modern control theory since its characteristics as marginally stable as a control. This system is popularly known as a model for the attitude control especially in aerospace field. However, it also has its' own deficiency due to its principles; highly non-linear and open-loop unstable system [2]. Thus, causing the pendulum falls over quickly whenever the system is simulated due to the failure of standard linear techniques to model the non-linear dynamics of the system. Moreover, it makes the identification and control become more challenging [3].

The common control approaches to overcome the problem by this system namely linear quadratic regulator (LQR) control and PID control that require a good knowledge of the system and accurate tuning to obtain good performance. Nevertheless, it attribute to difficulty in specifying an accurate mathematical model of the process [4].

This paper presents investigations of performance comparison between conventional (PID) and modern control (LQR) schemes for an inverted pendulum system. The dynamic model and design requirement have been taken from Carnegie Mellon, University of Michigan [5]. Performance of both control strategy with respect to pendulum's angle and cart's position is examined. Comparative assessment of both control schemes to the system performance is presented and discussed.

Modeling of the inverted pendulum system

This section provides a brief description on the modeling of the inverted pendulum system, as a basis of a simulation environment for development and assessment of both control schemes. The system consists of an inverted pole hinged on a cart which is free to move in the *x* direction as shown in Figure 1.

In order to obtain the dynamic model of the system, the following assumptions have been made:

I. The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.
II. The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.
III. A step input is applied

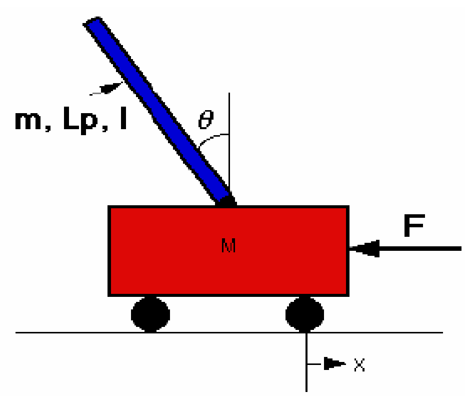

FIGURE 1. Free body diagram of the Inverted Pendulum System

The parameters of the system are shown in Table 1.

TABLE 1. Parameter of the system

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.5	kg
m	Mass of the pendulum	0.5	kg
B	Friction of the cart	0.1	N/m/s
L	Length of the pendulum	0.3	m
I	Inertia of the pendulum	0.006	kgm^2
g	Gravity	9.8	m/s^2

Figure 2 shows the free body diagram of the system. From the free body diagram, the following dynamic equations in horizontal direction in (1) and vertical direction in (2) are determined.

$$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F \quad (1)$$

$$(I+ml^2)\ddot{\theta}+mgl\sin\theta=-ml\ddot{x}\cos\theta \quad (2)$$

The dynamic equations in (1) and (2) should be linearized about $\theta = .\pi$

After linearization, the dynamic equations in (3) and (4) are obtained:

$$(I+ml^2)\ddot{\phi}-mgl\phi=ml\ddot{x} \quad (3)$$

$$(M+m)\ddot{x}+b\dot{x}-ml\ddot{\phi}=u \quad (4)$$

By manipulating the dynamics equations in (3) and (4), and substituting the parameter values of the cart and pendulum, the following transfer function of the pendulum's angle and transfer function of cart's position are obtained as shown in (5) and (6) respectively.

$$\frac{\Phi(s)}{u(s)}=\frac{4.5455s}{s^3+0.1818s^2-31.1818s-4.4545} \quad (5)$$

$$\frac{x(s)}{u(s)}=\frac{1.8182s-44.5455}{s^3+0.1818s^2-31.1818s-4.4545} \quad (6)$$

The transfer functions can be represented in state–space form and output equation as stated in (7) and (8)

$$\begin{bmatrix}\dot{x}(t)\\ \ddot{x}(t)\\ \dot{\phi}(t)\\ \ddot{\phi}(t)\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -0.1818 & 2.6727 & 0\\ 0 & 0 & 0 & 1\\ 0 & -0.4545 & 31.1818 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\\ \phi(t)\\ \dot{\phi}(t)\end{bmatrix}+\begin{bmatrix}0\\ 1.8182\\ 0\\ 4.5455\end{bmatrix}u(t) \quad (7)$$

$$y(t)=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x(t)\\ \dot{x}(t)\\ \phi(t)\\ \dot{\phi}(t)\end{bmatrix}+\begin{bmatrix}0\\ 0\end{bmatrix}u(t) \quad (8)$$

From (7), the stability of the system can be determined by calculating the open-loop poles using (9)

$$det\ (SI-A) \quad (9)$$

where A is a system matrix. By solving (9), the open-loop poles are determined as follows:

Open-loop poles: 0 -0.1428 5.5651 -5.6041

As can be seen, one of the four poles, 5.5651 lies on right hand side of the s-plane which stated that the system is unstable. Therefore, a controller has to be designed in order to stabilize the inverted pendulum system.

Controller design and simulation

In this section, two control schemes (LQR and PID) are proposed and described in detail. Furthermore, the following design requirements have been made to examine the performance of both control strategies.

I. The system overshoot (*%OS*) of cart position, x is to be at most 22.5%.
II. The Rise time (T_r) of cart position, x less than 1 s.
III. The settling time (T_s) of cart position, x and cart angle θ is to be less than 5 s.
IV. Steady-state error is within 2%.

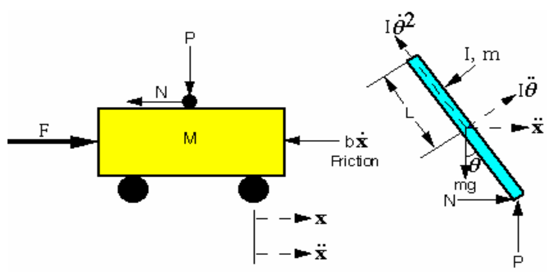

FIGURE 2. Free body diagram of the System

LQR Controller

LQR is a method in modern control theory that uses state-space approach to analyze such a system. Using state-space methods it is relatively simple to work with a multi-output system [6]. The system can be stabilized using full state feedback. The schematic of this type of control system is shown in Figure 3.

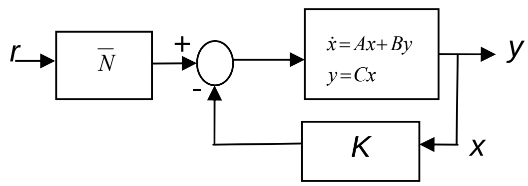

FIGURE 3. The LQR Control Structure

In designing LQR controller, LQR function in matlab m-file can be used to determine the value of the vector K which determines the feedback control law. This is done by choosing two parameter values, input *(R)=1* and $Q=C'xC$ where C is from state equation in (8). The controller can be tuned by changing the nonzero x and y elements in Q matrix which is done in m-file code. Consequently, by tuning the values of x = 5000 and y =100, the following values of matrix K are obtained.

$$K = [-70.7107 \quad -37.8345 \quad 105.5298 \quad 20.9238]$$

In order to reduce the steady state error of the system output, a value of constant gain, *Nbar* should be added after the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, multiply that by the chosen gain K, and use a new value as the reference for computing the input. *Nbar* can be found using the user-defined function which can be used in m-file code. The value of constant gain, *Nbar* are found to be:

$$Nbar = -70.7107$$

PID Controller

PID stands for Proportional-Integral-Derivative [7]. This is a type of feedback controller whose output, a control variable (CV), is generally based on the error (e) between some user-defined set point (SP) and some measured process variable (PV). Each element of the PID controller refers to a particular action taken on the error.

• Proportional: error multiplied by a gain, K_p. This is an adjustable amplifier. In many systems K_p is responsible for process stability: too low and the PV can drift away; too high and the PV can oscillate.

• Integral: the integral of error multiplied by a gain, K_i. In many systems K_i is responsible for driving error to zero, but to set K_i too high is to invite oscillation or instability.

• Derivative: the rate of change of error multiplied by a gain, K_d. In many systems K_d is responsible for system response: too high and the PV will oscillate; too low and the PV will respond sluggishly.

This paper uses a Ziegler and Nichols approach to design PID controller. The values of K_p, K_i, and K_d can be determined using the 'ultimate cycle method' which has been introduced by Ziegler-Nichols [7]. As shown in the Figure 4, the system consists of two PID controllers. 'PID Controller 1' controls the Pendulum's angle while 'PID Controller 2' controls the cart's position. The output of the controllers are summed together to be an input force to the inverted pendulum system. Using the Ziegler and Nichols technique, the parameter values of K_p, K_i, and K_d for both PID are successfully determined. For the 'PID Controller 1', the parameter values are K_P = -50, K_i = -110, K_d = -3.5 while for 'PID Controller 2' are K_P = -0.0001 K_i = -10, K_d = 0.3.

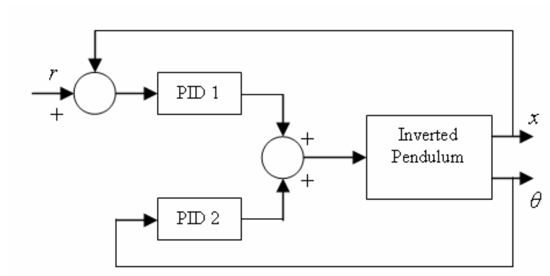

FIGURE 4. The PID Control Structure

Results and Analysis

In this section, the simulation results of the proposed controller, which is performed on the model of an inverted pendulum, are presented. Comparative assessment of both control strategies to the system performance also is discussed in this section.

Both systems with LQR and PID controller block diagram produced two responses, pendulum's angle and cart's position. Figure 5 shows the comparison of step response of the pendulum's angle between LQR and PID controller graphically. In this figure, the response for the pendulum's angle of the PID controller is in red color and the response for the pendulum's angle of the LQR controller is in blue color. Table 2 shows the summary of the performance characteristics of the step response of the pendulum's angle between LQR and PID controller quantitatively. Based on the Table 2, LQR has the fastest settling time of 3.34 s while PID has the slowest settling time of 4.48 s. However, for the percentage overshoot range, PID controller has the best range which is the lowest range between two controllers from 0.50° to -4.39°. The LQR controller has exceeded the maximum range of 20 degrees from 19.6° to -46.5°. Despite the large initial values for angles (19.6° to -46.5°) the proposed LQR controller is able to bring the pendulum to the vertical position. Also, the responses have acceptable overshoot and undershoot. It should be mentioned, however, that such large initial values for angles are not practical because the initial amount of forces would be also very large in order to bring the pendulum to the vertical position. This can be an indication of the high degree of the robustness of the controller. Compared to LQR controller, PID controller has very good ability to control the pendulum's angle from the starting state. For the last characteristic, both PID and LQR controllers have no steady state error. We can say that the LQR controller able to response faster than PID controller.

Figure 6 shows the comparison of step response of the cart's position between LQR and PID controller. In this figure, the response for the cart's position of the PID controller is in red color and the response for the cart's position of the LQR controller is in blue color. Table 3 shows the summary of the performance characteristics of the step response of the cart's position between LQR and PID controller. It shows that LQR control method has better performance as compared to PID control method. From Table 3, we can clearly see that LQR has the fastest rising time (T_r) of 0.41 s while PID has the rising time of 1.50 s. In addition, PID has the larger value of settling time, (T_s) of 3.59 s compared to the value of settling time for the LQR controller which is 2.04 s. From both of these characteristics, we can say that the LQR controller able to response faster than PID controller. Furthermore, for the percent overshoot (%OS), LQR has 0% overshoots while the PID controller has the larger percent overshoot of 0.32%. The last characteristic is the steady state error, (e_{ss}).

TABLE 2. Summary of the performance characteristics for pendulum's angle

Time response specifications	LQR	PID
Settling Time (T_s)	3.34 s	4.48 s
Maximum Overshoot Range	19.6° → -46.5°	0.50° → -4.39°
Steady State Error (e_{ss})	0	0

TABLE 3. Summary of the performance characteristics for cart's position

Time response specifications	LQR	PID
Rise Time (T_r)	0.41 s	1.50 s
Settling Time (T_s)	2.04 s	3.59 s
Percent Overshoot (%os)	0.00 %	0.32 %
Steady State Error (e_{ss})	0	0

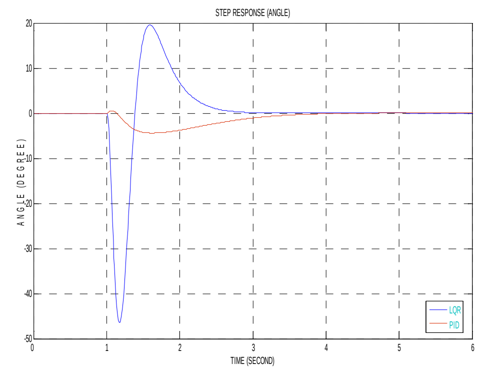

FIGURE 5. Pendulum angle response

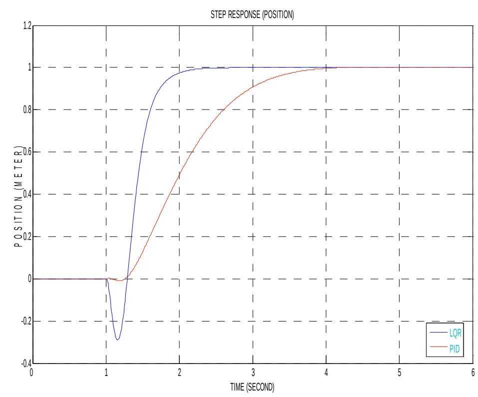

FIGURE 6. Cart position response

Both PID controller and LQR controller has the zero steady state error, (e_{ss}). Comparing all the characteristics from Table 3, LQR controller has higher ability to control cart's position than PID controller. We can clearly see the performance by referring to the bar chart as shown in Figure 7.

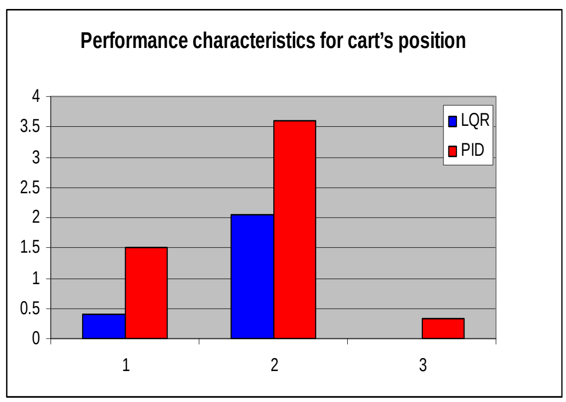

FIGURE 7. Performance characteristics for cart's position

However, for the overall performance by considering both pendulum's angle and cart's position, both controllers are successfully designed and in fact LQR controller has the best response and better performance which satisfy the design criteria very much.

Conclusion

In this paper, two controllers such as LQR and PID are successfully designed. Based on the results and the analysis, a conclusion has been made that both of the control method, modern controller (LQR) and conventional controller (PID) are capable of controlling the inverted pendulum's angle and the cart's position of the linearized system. All the successfully designed controllers were compared. The responses of each controller were plotted in one window and are summarized in Table 2 and Table 3. Simulation results show that LQR controller has better performance compared to PID controller in controlling the inverted pendulum system. Further improvement need to be done for both of the controllers. LQR controller should be improved so that the percentages overshoot for the pendulum's angle does not have very high range as required by the design criteria. On the other side, PID controller can be improved so that it's settling time might be reduced as faster as LQR controller.

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