# PID Control

## Terms

* Feedback
* Continuous
* Most popular
* Weighted sum of 3 terms

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| **P** | Proportional to the error | The larger the error, the larger the control action. | Eliminate the basic error |
| **I** | Proportional to the accumulated error | The more error is accumulated, the larger the control action. | Eliminate the residual steady-state error (steady disturbance) (found in P only control). If the current control action is not enough to bring error to zero, the control action will be increased as time passes. |
| **D** | Proportional to rate of change of error | The more rapid the error changes, the greater the damping effect. | Reduce the overshoot and oscillation while keeping fast convergence. It will try to bring error rate to zero. Flatten the error trajectory into a horizontal line (so reduces overshoot) |

I term: Try to eliminate residual error left after applying P term

D term: estimate the future trand of error based on current its rate of change. The more rapid the change, the greater the damping effect.

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| **PID** | **:** | Proportional-Integral-Differential |
| **Function** | **:** | To eliminate the error  To stabilize a state variable at a desired set-point  To track a time-varying reference |
| **Remark** | **:** | Most popular control method |
| **Tuning Parameters** | **:** |  |

* Error 🡪

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|  |  | To eliminate the (basic) error | |  | To eliminate **stead-state** error and **bias** | |  | To reduce **overshot** while keeping fast **convergence** | |

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|  | **Code for PID Control** |
| 1  2  3  4  5  6  7  8  9  10  11  12  13  14 | **def** PID**(**error**,** error\_last**,** Integral**,** dt**,** PID\_Gains**,** u\_bound**):**  KP **=** PID\_Gains**[**0**]**  KD **=** PID\_Gains**[**2**]**  KI **=** PID\_Gains**[**1**]**  u\_min **=** u\_bound**[**0**]**  u\_max **=** u\_bound**[**1**]**  Proportional **=** KP**\***error  Integral **+=** KI**\***error**\***dt # Forward Euler  Integral **=** clamp**(**Integral**,**u\_min**,**u\_max**)** # Intergral Anti-Windup  Differential **=** KD**\*(**error**-**error\_last**)/**dt  u **=** Proportional **+** Integral **+** Differential  u **=** clamp**(**u**,**u\_min**,**u\_max**)**  **return** u**,** Integral |

## PID Extension

* Control Saturation/Bounds (Clamping between min and max)
* Intergral Anti-windup (to eliminate the accumulation of integral term over bounds when a steady-state error occurred for a long time) - 3 solution methods
  1. Disable Integration until control action has entered the allowable range
  2. Prevent the integral term from accumulating above and below bounds (of control action)
  3. Back calculating the integral term to constrain the output within feasible bounds. (If output is over the bound, substract the extra output from the integral term)
* Fitered Derivative (If the error signal has noise (e.g. measurement noise, sudden change of reference signal, the will be very large. To eliminate that, the should be filtered.
* Sometimes, instead of (rate of change of error), difference of rate of change of state(process) variable and that of reference or just (assume ) is also used. Because the derivatives of state variable is always continuous (no step change when ref is changed). Useful when is nicely measurable
* and : The idea is to make the controller more aggrasive or conservative by just changing one parameter . Chaning will affect all three terms.

## PID Modifications

* P only, PD, PI, I only (D only can not drive the error to zero, it derives the error rate to zero so the process signal become flat at set-point), PI: when derivative noise amplification is unacceptable
* Feedforward-PID
* Cascaded PID
* Gain-sheduled PID
* Fractional PID: increase the DOF from 3 to 5 (Advanced)
* Deadband: to reduce auturator wear
* Set-point change (reference generator: not to suddenly change the error when set-point is changed)
* Set-point weighting
  + Adjustable factors (0-1) in the setpoint of Proportional and Derivative terms.
  + Integral term use no weight, so the real error is gunranteed to be eliminated.
  + Can tune two additional parameters to improve the set-point response.

## PID Limitations

* Linear (Linearize the plant when tuning)
* Amplification of noise in derivative term
  + Solution: Low-pass filter, Low noise sensor, state observer(e.g. KF(less delay, need plant model, sensor fusion OK))
  + Problem: low-pass filter introduces delay, low-pass filter and derivative action can cancel out each other (just using PI may be simpler), Check phase margin or disc margin for stability.
* Not optimal in general
* Difficulties in case of
  + Process measurement dealy
  + Control action delay
  + In the presence of non-linearities
* Does not leverage the knowledge of process/plant/vehicle to be controlled

Cautions: The signs of the PID terms should be in accordance with the physical meaning of control variable (reverse control action, e.g. water level control (if error is negative(below the desired level), valve opening is positive))

## PID Tuning Methods

* Manual (when tune with real system, need to minimize how many time the trial and error is done, may be Bayesian optimization?)
* Ziegler-Nichols
* Tyreus Luyben
* Cohen-Coon
* Astrom-Hagglund
* Software Tool

Desired: Critically damped, balance between robustness and fast convergence

Stability (no bounded oscillation) is essential requirement.

Other two basic requirements:

* Regulation (disturbance rejection – staying at set-point)
* Tracking (respond to the set-point changes: rise time, settling time)
  + Some processes may not allow overshoot (due to the safety)
  + Other processes must minimize the energy when trying to reach a new set-point

Other requirements may conflict with one another.

## PID Robustness and Stability

* Gain and Phase Margins
* Disc Margin
* Nyquist Stability Criterion
* Pole Placement