Getting it Right for HPC:

An Introduction to Testing and Model Checking

Model Checking I

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Motivation: The limitations of testing

1. **lack of coverage** of input space
   - only a tiny fraction of inputs can be tested
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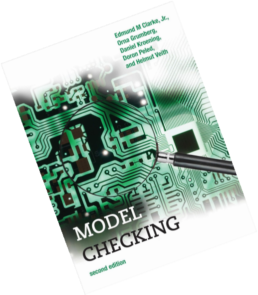
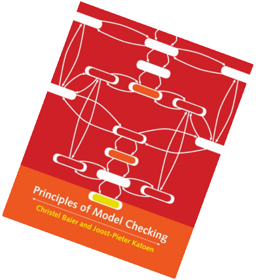
1. **lack of coverage** of input space
   - only a tiny fraction of inputs can be tested
   - no sure way to select the tests guaranteed to find all bugs
2. **nondeterminism**
   - a program can behave differently on two executions with the same input
     - concurrency: interleaving of statements from different threads/processes may impact observable behavior
     - compilers, MPI implementation, OpenMP runtime, . . . : behavior is (intentionally) not fully specified
   - correct result on one execution does not even guarantee correct result on another execution with the same input
   - repeating test failures is non-trivial

"...testing can be used to show the presence of bugs, but never to show their absence!"

— E. Dijkstra

What is Model Checking?





A wide variety of techniques, all involving three essential tasks:

1. Represent the program as a mathematically precise finite state model
   - automata, program graph, labeled transition system, . . .
2. Formulate program properties as logical properties of the model
   - first order logic, temporal logic, . . .
3. Use automated algorithmic techniques to check the model satisfies the properties
   - graph search algorithms, computer algebra, automated theorem proving, . . .

Model Checking Example: Shared Resource

```
  boolean x;
  proc rw0 {
    while (true) {
0     x := 0;
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2     if (x == 0)
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Model Checking Example: Shared Resource

Property 1: Freedom from deadlock
The program does not deadlock.

Property 2: Mutual exclusion
It is never the case that both processes use the resource at the same time.

Property 3: Liveness
The resource will eventually be used.

State: $[x, \text{pc}_0, \text{pc}_1]$

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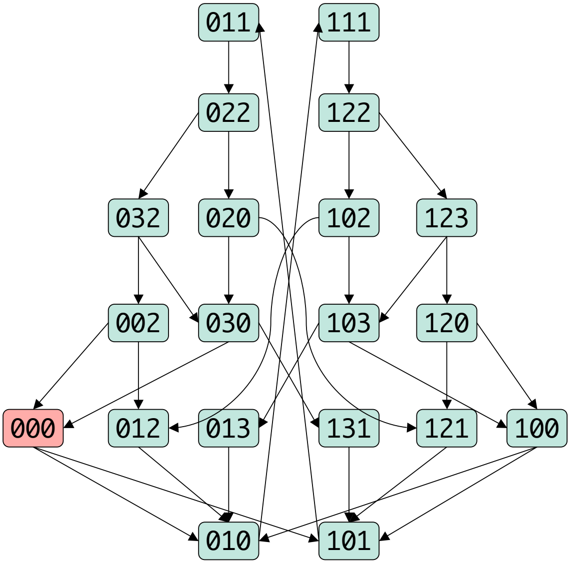


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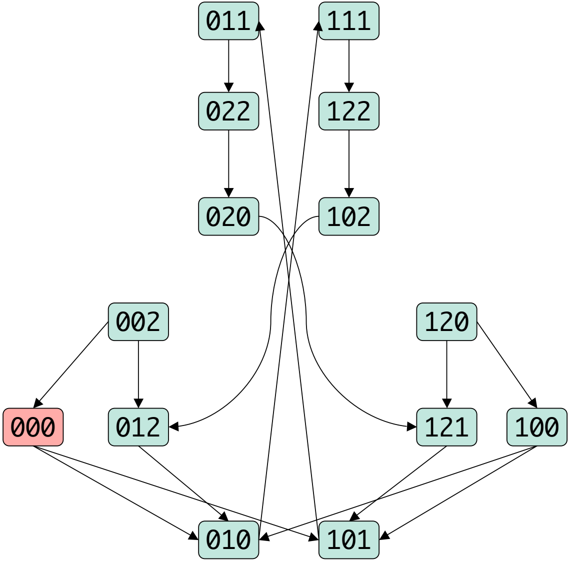


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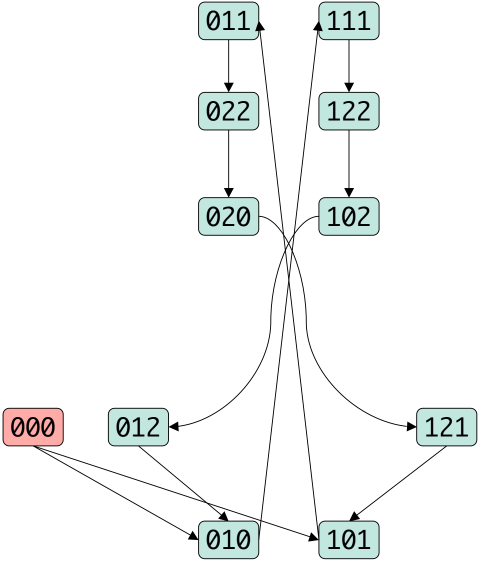


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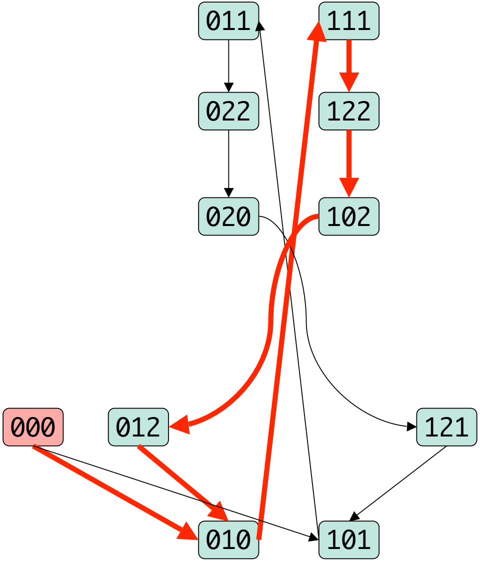


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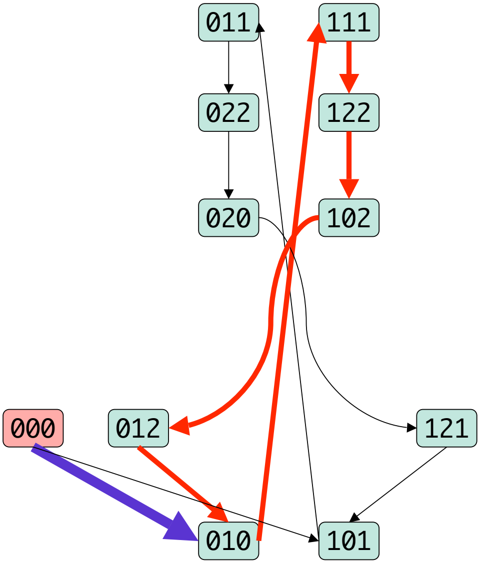


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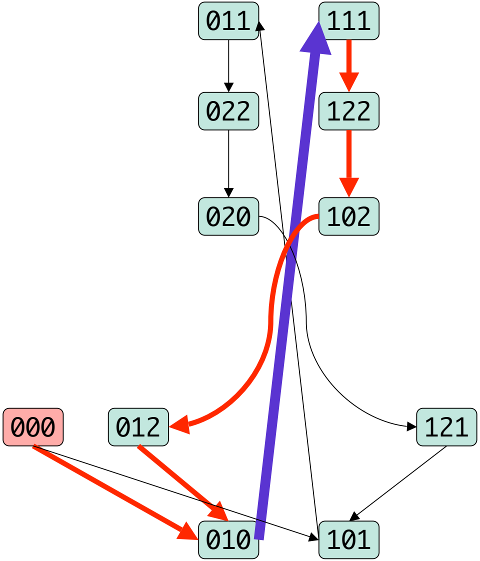


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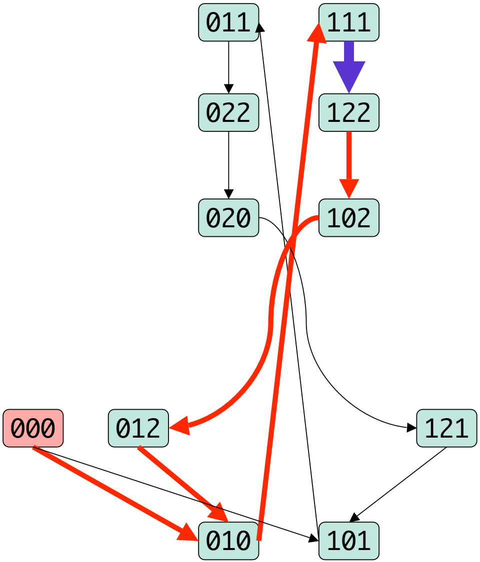


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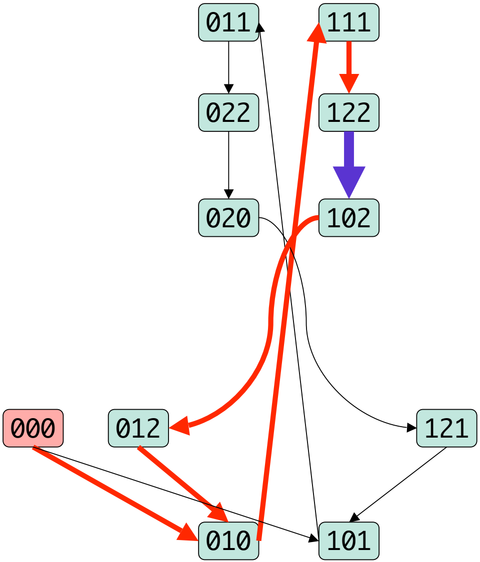


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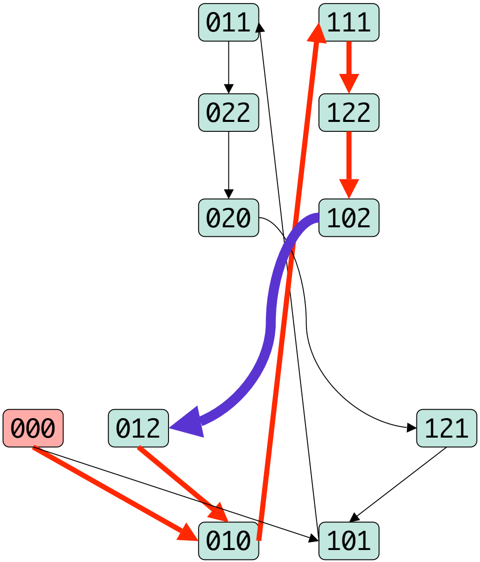


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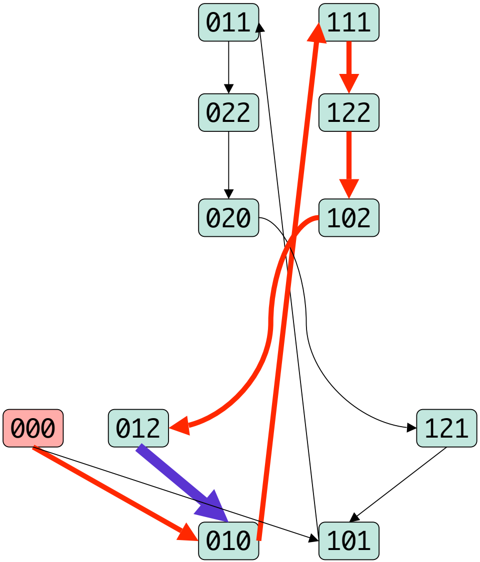


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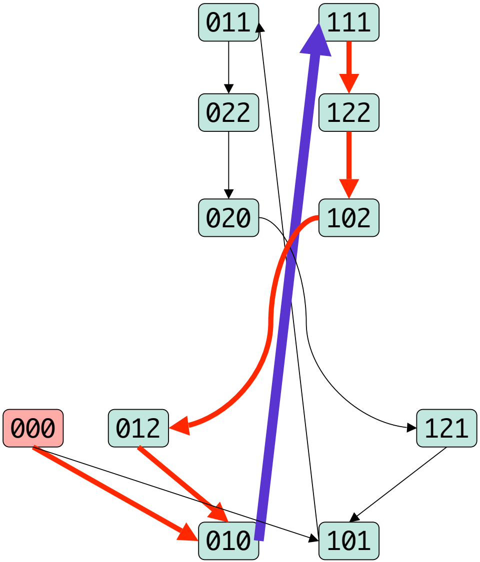


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  - at a branch on condition e, split into two cases
    1. pc $\leftarrow$ pc $\wedge\, e$, follow *true* branch
    2. pc $\leftarrow$ pc $\wedge\, \neg e$, follow *false* branch

Symbolic Execution Example

```
   input real a, b;
1  if (a>b)
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   else
3     b=b-a;
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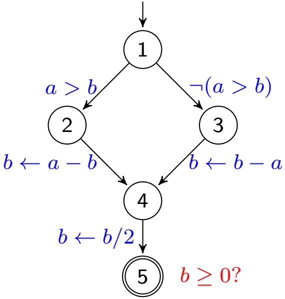

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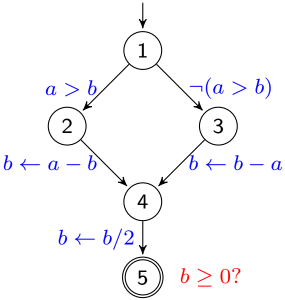

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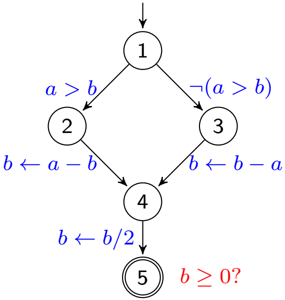

State:
$\langle \mathsf{pc}, \mathsf{location}, a, b \rangle$

$$\langle \mathsf{true}, 1, X_1, X_2 \rangle$$

$$\langle X_1 > X_2, 2, X_1, X_2 \rangle \qquad \langle \neg(X_1 > X_2), 3, X_1, X_2 \rangle$$

$$\langle X_1 > X_2, 4, X_1, X_1 - X_2 \rangle \qquad \langle \neg(X_1 > X_2), 4, X_1, X_2 - X_1 \rangle$$

$$\langle X_1 > X_2, 5, X_1, (X_1 - X_2)/2 \rangle \qquad \langle \neg(X_1 > X_2), 5, X_1, (X_2 - X_1)/2 \rangle$$

$$X_1 > X_2 \implies (X_1 - X_2)/2 \geq 0? \qquad \neg(X_1 > X_2) \implies (X_2 - X_1)/2 \geq 0?$$

Some properties checked by CIVL

- generic properties of correct C programs: absence of ...
  - out-of-bound array indexing
  - dereferences of a NULL or undefined pointer
  - pointer arithmetic beyond the bounds of an object
  - reading a variable before it is initialized
  - memory leaks, double frees, divisions by zero, integer overflow for signed integer types

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  - any message received will fit in the specified receive buffer
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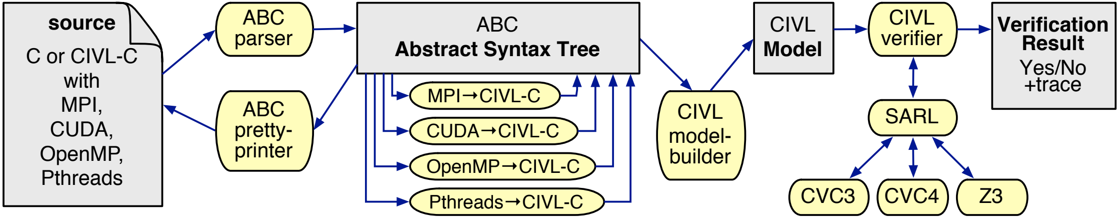
  - in OpenMP, shared variable accesses will avoid data-races
- application-specific properties
  - on any execution of a program, no assertion will be violated
  - given two programs with the same input-output signature:
    - on any execution of the two programs starting from the same inputs
    - the two programs will produce the same output

The CIVL framework



- an intermediate verification language for concurrency: CIVL-C
- an analysis and verification framework centered around that language
- front-end: ABC (extended C compiler front-end) and transformers for each dialect
- back-end: a CIVL-C verifier based on model checking and symbolic execution
- Open source (L/GPL), 100% Java 17, documentation!, unit tests, coverage analysis, . . .