Mathematics is a Science

George Hu

April 19, 2022

Mathematics, the area of study that has shaped the world from the grandest theories of the fabric of the universe to everyday modern currency systems, unsurprisingly still finds difficulty in categorization provided its grand apparatus of all things quantitative. Because mathematical seems so primitive and such an innate way to communicate numbers and related logical structures, some would contend it is the language of numbers. Or given the sort of abstractions with intricate connections between various sub-fields of mathematics that inspire an aesthetic notion, some would argue that mathematics is a form of art. But while these comparisons are valid, I will contend that mathematics is best described to be a science. This claim arises from the necessity of experimentation in mathematics, the centrality of *reductionist abstraction* in mathematics, and the seeming inevitability of mathematical industry applications.

The engines of mathematics, mathematicians and the structures supporting them, rely upon a unique form of experimentation analogous to the scientific method, indicating the scientific quality of mathematics. In the Enlightenment era derived viewpoint of the scientific method, the scientist first reviews relevant background material and proposes some hypotheses to answer an open question. Then, they perform relevant experiments, measuring the response of some dependent variable, to determine a conclusion on the validity of the hypotheses. This is perhaps the most distinguishing facet of sciences compared to other areas of human endeavor. The humanities and the arts do not this same strict structure of experimentation; new groundbreaking forms of art do have an experimental part, but Heisenberg[2] notes that "even the questions are uncertain" for the artist versus the well defined questions the scientist ponders. While mathematics with its complex abstractions may seem aimlessly wandering to the layperson, one can note that the distinguishing factor of science is the *mode of approach* through clearly defined questions. One way or another, mathematicians process certain logical hierarchies of possibilities for relating various mathematical quantities as Poincare[3]

describes, iteratively doing experimentation as such. On the other hand this methodological experimentation is justifiably frowned upon within the arts and humanities—ill-defined objectives are what distinguishes artists and their ability to move human emotions in no similar well-constrained way. Moreover, any conception of experimentation in language quickly reduce to literature or a more macroscopic linguistic phenomena. For the former, the arguments regarding the humanities hold, and for the latter, there do not exist the language equivalent of singular mathematicians, indicating that language is a poor descriptor of mathematics. Hence the experimental nature of mathematics can best be described as scientific experimentation.

Another foundation of mathematics that best relates to the sciences is what I will call *reductionist abstraction*. By this I mean that abstractions in mathematics, from elementary sets and groups to larger abstract systems, have a bent towards reductionism, wherein we prefer to reduce the exact numerical or logical manifestations of such abstractions to something smaller in complexity. The multivariate Gaussian reduces to mean and the covariance matrix, various geometries reduce to the topological torus, and so on. In the broadest interpretation, this reductionist abstraction ca be interpreted as a model, which has parlance particularly in applied mathematics, where the reductionist abstraction is in an epidemiological model or a set of rules for a dynamical system. These models hence relate analogously to those in science: hard sciences deal with physical models such as general relativity or polymerization in chemistry, and soft sciences deal with social models such as economic systems or cultural interactions. These models along with the notion of reductionist abstraction are central to the sciences and mathematics. To talk about measure theory is to talk about probability measures, and to talk about ecology is to talk about food webs and environmental interactions. In contrast, it would be heresy to the arts to primarily view Monet as an impressionist without emphasizing the unique artworks of lily pads and the like. Reducing Bach to Baroque counterpoint is not fundamental to music compared to reducing algorithms to Turing Machines in computational theory. The emphasis on reductionist abstraction is therefore a significant indicator of mathematics being a science.

Mathematics also seemingly always leads to industry applications, and this phenomena further indicates how mathematics is a science. Although much of mathematical theory originates with no industrial affiliate in mind—what Hardy[1] considers the artistic nature of mathematics—the progress of time and the expanding forays of human industries has converted theory to application far too many times. Weiner[4] rightfully points out the usage of mathematics for particle physics, once some amalgamation of purely theoretical equations for quantum field theory, but now considered fundamental to actual nuclear

devices. Turing's and Von Neumann's models of computation were originally non-physically realizable but are now the basis for modern hardware and software, the largest economic sector in the world. The Fourier transform, once just a neat property of sinusoidal bases, provides groundwork for almost all signal processing for digital communication systems today. No matter what, it seems like mathematics cannot avoid direct usage for new human technologies and industries; today's completely abstract category theory could be tomorrow's next technology revolution. This strong inclination towards applicability is mirrored in the sciences; physical sciences almost have exact applications in engineering—physics to mechanical engineering and chemistry to chemical engineering, and social sciences are inherently application driven due to their focus upon peoples. Now one could say that the arts and humanities are similarly focused upon peoples so they are applied in a sense, but in this case I will consider the question: what is being applied? For the sciences it is the previously described models, *reductionist abstractions*, which are, but that is not present for the arts and humanities. Thus, the application tendency of mathematics indicates its nature as a science.

The peculiar categorization of mathematics as a scientific study perhaps connotes too simple of a synthesis, but the simplicity of the analogies between mathematics and the sciences only further proves the claim. Outside the simple tautology of *math is math* which is undoubtedly attractive to me, science as the hypernym of mathematics is the most factual.

References

[1] G.H. Hardy, H. Hardy, C.P. Snow, and Cambridge University Press. *A Mathematician's Apology*. Cambridge paperbacks: Mathematics. Cambridge University Press, 1992.

[2] E.T. Heise, W. Heisenberg, and A. Pomerans. *Physics and Beyond: Encounters and Conversations*. Harper torchbooks. Harper & Row, 1971.

[3] P. Henri. *Science and Method - Scholar's Choice Edition*. Creative Media Partners, LLC, 2015.

[4] N. Wiener. *The Human Use Of Human Beings: Cybernetics And Society*. Hachette Books, 1988.