Application

In[ ]:=
```
ext0 = (* { {{p[i], m[i][x], t[i]}} , {l[i]} }
   meaning : forall i, for x≥ l[i],
  p[i][x+1]= p[i][x]*m[i][x+1]+t[i] *){{p[1], -13 122 (x + 1) (x + 1 + 1) / (x + 1 + 3)^3, 0},
   {p[2], 26 244 (x + 1)^2 * (x + 1 + 2)^2 / (x + 1 + 3)^2, 0},
   {p[3], I (x + 1) (x + 1 + 2)^3 / (729 (x + 1 + 5)), 0}, {p[4], -162 (x + 1) (x + 1 + 2) / (x + 1 + 5), 0}}
bounds0 = {1, 1, 1, 1}
tow0 = {{x, 1, 1}}
```

Out[ ]=
$$\left\{\left\{p[1], -\frac{13\,122\,(1+x)\,(2+x)}{(4+x)^3}, 0\right\}, \left\{p[2], \frac{26\,244\,(1+x)^2\,(3+x)^2}{(4+x)^2}, 0\right\},\right.$$
$$\left.\left\{p[3], \frac{i\,(1+x)\,(3+x)^3}{729\,(6+x)}, 0\right\}, \left\{p[4], -\frac{162\,(1+x)\,(3+x)}{6+x}, 0\right\}\right\}$$

Out[ ]=
```
{1, 1, 1, 1}
```

Out[ ]=
```
{{x, 1, 1}}
```

In[ ]:=
```
{outputRep0, outputTow0, outputBounds0} = ProductsSetReduce[tow0, ext0, bounds0, True]
```

```
tower={{x, 1, 1}}
```

```
r=4
```

$$\text{alphas}=\left\{-\frac{13\,122\,(1+x)\,(2+x)}{(4+x)^3},\ \frac{26\,244\,(1+x)^2\,(3+x)^2}{(4+x)^2},\ \frac{i\,(1+x)\,(3+x)^3}{729\,(6+x)},\ -\frac{162\,(1+x)\,(3+x)}{6+x}\right\}$$

```
M={{6, 0, 4, -6}, {0, 1, 0, -2}}
```

```
u=2
```

$$A=\begin{pmatrix}0 & 1\\ -1 & 2\end{pmatrix}\quad D=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 2 & 0 & 0\end{pmatrix}\quad B=\begin{pmatrix}-1 & 1 & -2 & 1\\ 1 & 0 & 0 & 2\\ 2 & -2 & 3 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\quad \text{invB}=\begin{pmatrix}0 & 1 & 0 & -2\\ -3 & 1 & -2 & 1\\ -2 & 0 & -1 & 2\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$\text{betas}=\left\{\frac{(6+x)^2}{(4+x)^2},\ -\frac{(4+x)^7\,(6+x)}{(1+x)^2\,(2+x)^3\,(3+x)^3},\ -\frac{i\,(4+x)^6}{9\,(1+x)\,(2+x)^2\,(3+x)\,(6+x)},\ -\frac{162\,(1+x)\,(3+x)}{6+x}\right\}$$

$$\text{qis}=\left\{\frac{p[2]}{p[4]^2},\ \frac{p[2]\times p[4]}{p[1]^3\,p[3]^2},\ \frac{p[4]^2}{p[1]^2\,p[3]},\ p[4]\right\}$$

```
his={1, 1, 1, 1}
```

```
lmax=1
```

$$\text{gis}=\left\{-(4+x)^2\,(5+x)^2,\ -(1+x)^2\,(2+x)^5\,(3+x)^8\,(4+x)\,(5+x)\,\iota_{-1}\right\}$$

$$\text{cjs}=\left\{-\frac{1}{900},\ \frac{1}{1\,911\,029\,760},\ \frac{81\,i}{512},\ -\frac{1}{81}\right\}$$

$$\text{ffis}=\left\{-\frac{6561}{16},\ \frac{59\,049}{4},\ \frac{i}{162},\ -81\right\}$$

$$\text{RPitower}=\left\{\{x, 1, 1\},\ \{\iota_{-1}, -1, 0\},\ \left\{q[\iota_3],\ -\frac{i\,(4+x)^6}{9\,(1+x)\,(2+x)^2\,(3+x)\,(6+x)},\ 0\right\},\ \left\{q[\iota_4],\ -\frac{162\,(1+x)\,(3+x)}{6+x},\ 0\right\}\right\}$$

```
Representations=
```

$$\left\{\frac{5\,(1+x)^2\,(2+x)^5\,(3+x)^8\,\iota_{-1}\,q[\iota_4]}{52\,488\,(4+x)\,(5+x)\,q[\iota_3]^2},\ \frac{1}{400}\,(4+x)^2\,(5+x)^2\,q[\iota_4]^2,\ \frac{2\,754\,990\,144\,(4+x)^2\,(5+x)^2\,q[\iota_3]^3}{25\,(1+x)^4\,(2+x)^{10}\,(3+x)^{16}\,\iota_{-1}^2},\ q[\iota_4]\right\}$$

```
returns : {representations,RPitower,newBounds for RPi-monomials}
```

Out[•]=

$$\left\{\left\{\frac{5\,(1+x)^2\,(2+x)^5\,(3+x)^8\,\iota_{-1}\,q[\iota_4]}{52\,488\,(4+x)\,(5+x)\,q[\iota_3]^2},\ \frac{1}{400}\,(4+x)^2\,(5+x)^2\,q[\iota_4]^2,\ \frac{2\,754\,990\,144\,(4+x)^2\,(5+x)^2\,q[\iota_3]^3}{25\,(1+x)^4\,(2+x)^{10}\,(3+x)^{16}\,\iota_{-1}^2},\ q[\iota_4]\right\},\right.$$

$$\left\{\{x, 1, 1\},\ \{\iota_{-1}, -1, 0\},\ \left\{q[\iota_3],\ -\frac{i\,(4+x)^6}{9\,(1+x)\,(2+x)^2\,(3+x)\,(6+x)},\ 0\right\},\right.$$

$$\left.\left.\left\{q[\iota_4],\ -\frac{162\,(1+x)\,(3+x)}{6+x},\ 0\right\}\right\},\ \{1, 1\}\right\}$$

In[•]:=
```
t1 = SigmaProduct[ext0[[1, 2]] /. x → (x - 1), {x, bounds0[[1]], 3}]
t2 = outputRep0[[1]] /. IndexName[-1] → (-1)^(x) /. x → 3 /.
    q[IndexName[4]] → SigmaProduct[-162 (1 + x) (3 + x) / (6 + x) /. x → (x - 1), {x, 1, 3}] /.
  q[IndexName[3]] → SigmaProduct[-i (4 + x)^6 / (9 (1 + x) (2 + x)^2 (3 + x) (6 + x)) /. x → (x - 1), {x, 1, 3}]
Ev[{outputRep0[[1]]}, 3, outputTow0, outputBounds0, True][[1]]
```

Out[•]=
$$-\frac{94\,143\,178\,827}{500}$$

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$$-\frac{94\,143\,178\,827}{500}$$

$$\left\{x \to I_0,\ I_{-1} \to (-1)^{I_0},\ q[I_3] \to \prod_{I_{-2}=1}^{I_0} \cdot \left(-\frac{i\,(3+I_{-2})^6}{9\,I_{-2}\,(1+I_{-2})^2\,(2+I_{-2})\,(5+I_{-2})}\right),\ q[I_4] \to \prod_{I_{-2}=1}^{I_0} \cdot \left(-\frac{162\,I_{-2}\,(2+I_{-2})}{5+I_{-2}}\right)\right\}$$

Out[•]=
$$-\frac{94\,143\,178\,827}{500}$$