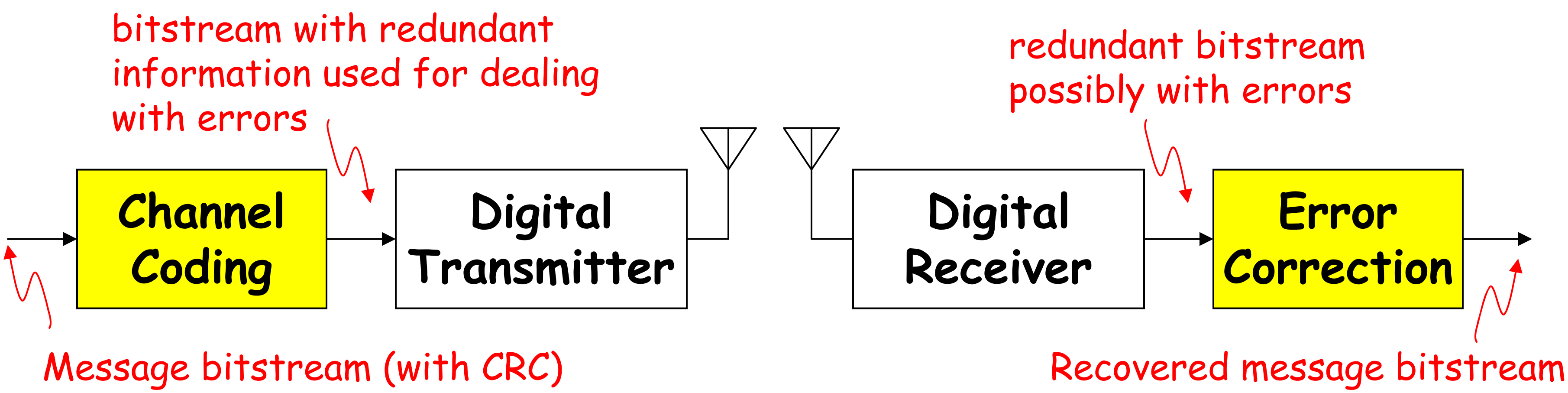
Source Coding

- **Information & Entropy**
- **Variable-length codes: Huffman's algorithm**
- **Adaptive variable-length codes: LZW**

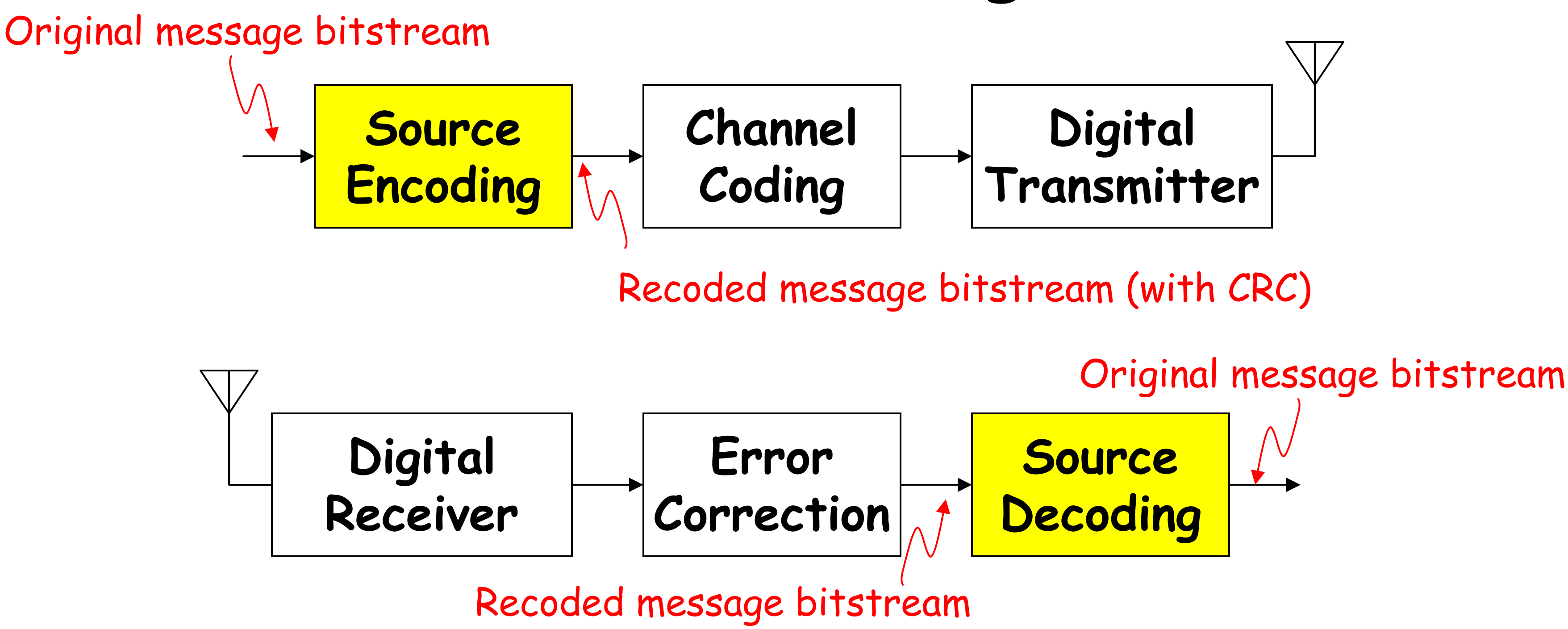
Where we've gotten to…

With channel coding (along with block numbers and CRC), we have a way to reliably send bits across a channel:



Next step: think about recoding the message bitstream to send the **information** it contains in as few bits as possible.

Source coding



Many message streams use a "natural" fixed-length encoding: 7-bit ASCII characters, 8-bit audio samples, 24-bit color pixels.

If we're willing to use variable-length encodings (message symbols of differing lengths) we could assign short encodings to common symbols and longer encodings to other symbols… this should shorten the average length of a message.

Measuring information content

Suppose you're faced with N equally probable choices, and I give you a fact that narrows it down to M choices. Claude Shannon offered the following formula for the information you've received.

$\log_2(N/M)$ *bits* of information

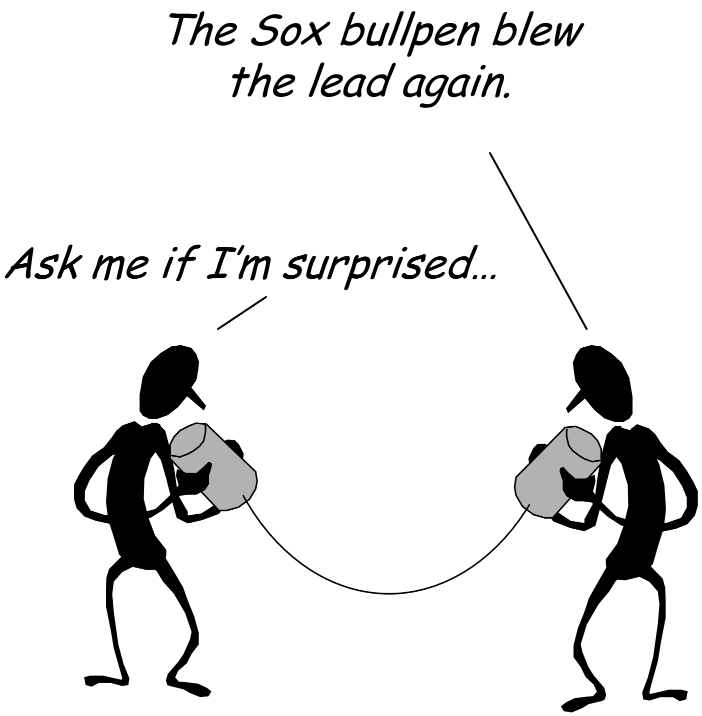
Information is measured in bits (binary digits) which you can interpret as the number of binary digits required to encode the choice(s)

Examples:
- **information in one coin flip: $\log_2(2/1) = 1$ bit**
- **roll of 2 dice: $\log_2(36/1) = 5.2$ bits**
- **outcome of a Red Sox game: 1 bit**

(well, actually, are both outcomes equally probable?)

What is "Information"?

information, *n.* **Knowledge communicated or received *concerning a particular fact or circumstance.***



Information resolves uncertainty. Information is simply that which cannot be predicted.

The less predictable a message is, the more information it conveys!

When choices aren't equally probable

When the choices have different probabilities (p_i), you get more information when learning of a unlikely choice than when learning of a likely choice

Information from $\text{choice}_i = \log_2(1/p_i)$ bits

We can use this to compute the average information content taking into account all possible choices:

Average information content in a choice $= \sum p_i \cdot \log_2(1/p_i)$

This characterization of the information content in learning of a choice is called the *information entropy* or *Shannon's entropy*.

Example

$choice_i$	p_i	$log_2(1/p_i)$
"A"	1/3	1.58 bits
"B"	1/2	1 bit
"C"	1/12	3.58 bits
"D"	1/12	3.58 bits

Average information content in a choice
= (.333)(1.58) + (.5)(1) + (2)(.083)(3.58)
= 1.626 bits

Can we find an encoding where transmitting 1000 choices is close to 1626 bits on the average?

The "natural" fixed-length encoding uses two bits for each choice, so transmitting the results of 1000 choices requires 2000 bits.

Variable-length encodings

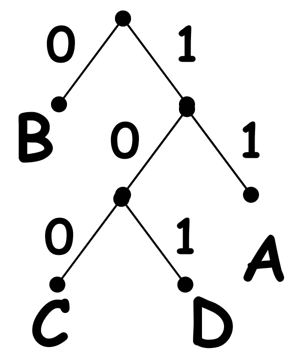
(David Huffman, MIT 1950)

Use shorter bit sequences for high probability choices, longer sequences for less probable choices

choice$_i$	*p_i*	*encoding*
"A"	1/3	11
"B"	1/2	0
"C"	1/12	100
"D"	1/12	101

B C A B A D

010011011101



Huffman Decoding Tree

Average information
=(.333)(2)+(.5)(1)+(2)(.083)(3)
= 1.666 bits

Transmitting 1000 choices takes an average of 1666 bits… better but not optimal

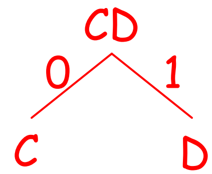
To get a more efficient encoding (closer to information content) we need to encode sequences of choices, not just each choice individually. This is the approach taken by most file compression algorithms…

Huffman's Coding Algorithm

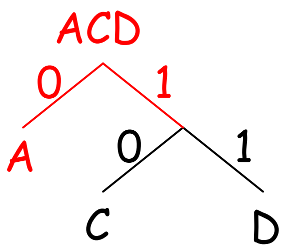
- Begin with the set S of symbols to be encoded as binary strings, together with the probability P(x) for each symbol x. The probabilities sum to 1 and measure the frequencies with which each symbol appears in the input stream. In the example from the previous slide, the initial set S contains the four symbols and their associated probabilities from the table.
- Repeat the following steps until there is only 1 symbol left in S:
  - Choose the two members of S having lowest probabilities. Choose arbitrarily to resolve ties.
  - Remove the selected symbols from S, and create a new node of the decoding tree whose children (sub-nodes) are the symbols you've removed. Label the left branch with a "0", and the right branch with a "1".
  - Add to S a new symbol that represents this new node. Assign this new symbol a probability equal to the sum of the probabilities of the two nodes it replaces.

Huffman Coding Example

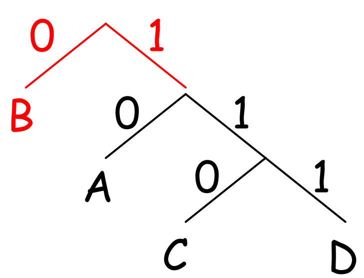
- Initially S = { (A, 1/3) (B, 1/2) (C, 1/12) (D, 1/12) }
- First iteration
  - Symbols in S with lowest probabilities: C and D
  - Create new node
  - Add new symbol to S = { (A, 1/3) (B, 1/2) (CD, 1/6) }



- Second iteration
  - Symbols in S with lowest probabilities: A and CD
  - Create new node
  - Add new symbol to S = { (B, 1/2) (ACD, 1/2) }



- Third iteration
  - Symbols in S with lowest probabilities: B and ACD
  - Create new node
  - Add new symbol to S = { (BACD, 1) }



- Done

Huffman Codes – the final word?

- Given static symbol probabilities, the Huffman algorithm creates an optimal encoding when each symbol is encoded separately.
- Huffman codes have the biggest impact on average message length when some symbols are substantially more likely than other symbols.
- You can improve the results by adding encodings for symbol pairs, triples, quads, etc. But the number of possible encodings quickly becomes intractable.
- Symbol probabilities change message-to-message, or even within a single message.
- Can we do adaptive variable-length encoding?

Adaptive Variable-length Codes

- Algorithm first developed by Lempel and Ziv, later improved by Welch. Now commonly referred to as the "LZW Algorithm"
- As message is processed a "string table" is built which maps symbol sequences to a fixed-length code
  - When processing byte streams, the first 256 table entries are initialized with the single character strings.
  - Table size = 2 ^ (size of fixed-length code)
- Note: String table can be reconstructed by the decoder based on information in the encoded stream – the table, while central to the encoding and decoding process, is never transmitted!

LZW Encoding

```
STRING = get input symbol

WHILE there are still input symbols DO
   SYMBOL = get input symbol
   IF STRING + SYMBOL is in the string table THEN
       STRING = STRING + SYMBOL
   ELSE
       output the code for STRING
       IF string table is full THEN
           output code for reinitializing table
           reinitialize table
       END
       add STRING + SYMBOL to the string table
       STRING = SYMBOL
   END
END

output the code for STRING
```


From http://marknelson.us/1989/10/01/lzw-data-compression/

Example: CHRIS_ repeated

- End of first repeat
  - Transmitted: C H R I S
  - Table: CH HR RI IS S_
  - Current String: _
- End of second repeat
  - Transmitted: _ [CH] [RI]
  - Table: CH HR RI IS S_ _C CHR
  - Current String: S_
- End of third repeat
  - Transmitted: [S_] [CHR] [IS]
  - Table: CH HR RI IS S_ _C CHR S_C CHRI IS_
  - Current String: _
- End of fourth repeat
  - Transmitted: [_C] [HR]
  - Table: CH HR RI IS S_ _C CHR S_C CHRI IS_ _CH HRI
  - Current String: IS_
- End of fifth repeat
  - Transmitted: [IS_] [CHRI]
  - Table: CH HR RI IS S_ _C CHR S_C CHRI IS_ _CH HRI IS_C CHRIS
  - Current String: S_

LZW Decoding

```
Read OLD_CODE
output OLD_CODE
SYMBOL = OLD_CODE

WHILE there are still input characters DO
    Read NEW_CODE
    IF NEW_CODE is not in the translation table THEN
        STRING = get translation of OLD_CODE
        STRING = STRING + SYMBOL
    ELSE
        STRING = get translation of NEW_CODE
    END
    output STRING
    SYMBOL = first character in STRING
    add OLD_CODE + SYMBOL to the translation table
    OLD_CODE = NEW_CODE
END
```

From http://marknelson.us/1989/10/01/lzw-data-compression/

Summary

- Source coding: recode message stream to remove redundant information, aka compression. Our goal: match data rate to actual information content.
- Information content from $\text{choice}_i = \log_2(1/p_i)$ bits
- Shannon's Entropy: average information content on learning a choice $= \sum p_i \cdot \log_2(1/p_i)$
- Huffman's encoding algorithm builds optimal variable-length codes when symbols encoded individually
- LZW algorithm implements adaptive variable-length encoding