P1 (2) would be more accurate.

Because there will be floating point error accumated in the evaluation of each x_i in (1); while for (2), we calculate every term x_i directly based on a, i, and h.

P2 a. when $x \approx 1$. there will be cancellation error both in $\sqrt{x^2-1}$ and $x - \sqrt{x^2-1}$.

b. $\log\left(\frac{1}{x+\sqrt{x^2-1}}\right)$

P3. Write $x = \frac{1}{R^2}$, $f(x) = \cancel{1-R^2x}\ \frac{1}{x} - R^2$.

Apply Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$\cancel{= x_n - \frac{1-R^2 x_n}{-R^2}}$$

$$\cancel{= \frac{1}{R^2} \cdot x_n}$$

~~Set x_0 = some initial guess, apply $x_{n+1} = \frac{1}{R^2} x_n$ iteratively~~

$$x_{n+1} = x_n - \frac{\frac{1}{x_n} - R^2}{-\frac{1}{x_n^2}}.$$

we get

$$x_{n+1} = -R^2 x_n^2 + 2x_n \quad (*)$$

Approach:

step 1. set x_0 = some initial guess,

step 2. apply (*) until get the accurate result we want.

~~P4~~

P4. if $F'(x) = \begin{pmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{pmatrix}$ is singular, it will break down.

That is, $\begin{vmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{vmatrix} = 4x_1^2 + 4x_2^2 = 0$.

when $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

P5. We will use newton's interpolation for this question.

i	x_i	$f[x_i]$.	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	0	1			
1	1	9	8		
2	2	23	14	3	
3	4	93	35	7	1

$$P_3(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)$$

$$= 1 + 8(x-0) + 3\cdot(x-0)(x-1) + 1\cdot(x-0)(x-1)(x-2)$$

$$= x^3 + 7x + 1.$$

$$f(2.5) = 2.5^3 + 7 \times 2.5 + 1 = 34.1250$$

P6. Assume we have n points for interpolation, i.e. we have $x_1, x_2, \cdots, x_n$, $y_1, y_2, \cdots, y_n$.

solve $Ax = b$ for x.

where

$$A = \begin{bmatrix} 1 & \cos(x_1) & \sin(x_1) \\ 1 & \cos(x_2) & \sin(x_2) \\ \vdots & \vdots & \vdots \\ 1 & \cos(x_n) & \sin(x_n) \end{bmatrix}.$$

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$b = \begin{bmatrix} \cancel{f(x_1)}\ y_1 \\ \cancel{f(x_2)}\ y_2 \\ \vdots \\ \cancel{f(x_n)}\ y_n \end{bmatrix}.$$

P7. (a) set $x = A^{-1}(B+C)d$,

then $Ax = (B+C)d$

Approach:

step 1. Calculate $y = (B+C) \cdot d$

step 2. Use Guess Elimination solve $Ax = y$ for x.

step 3. Use backward substitution for solving x in $Ax = y$.

(b). In first step,

for, $(B+C)$, there is n^2 addition.

then for $(B+C) \cdot d$, there is n^2 multiplication and $n(n-1)$ addition.

In second step,

the Guess Elimination takes $2n^3/3 - n^2 + n/6$ time.

In third step,

the backward substitution takes. n^2 time.

So the total time complexity is $O(n^3)$.

P8. If $Ax = b$ is an over determined system: more equations than variables. i.e. if $A \in \mathbb{R}^{m \times n}$, then $m > n$.

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P9. let $n = \#$ points.

$r = n - 1$

$a = 0$

$b = \pi$

$$h = \frac{b-a}{r} = \frac{\pi}{n-1}$$

(a) for trapezoid composite rule,

$$\text{error} = -\frac{f''(\mu)}{12}(b-a)h^2$$

$$= -\frac{-\sin(\mu)}{12} \cdot \pi \cdot \left(\frac{\pi}{n-1}\right)^2$$

let $$|\text{error}_{max}| = \left|\frac{\pi^3}{12} \cdot \frac{1}{(n-1)^2}\right| \leq 10^{-6}.$$

$n_{min} = 1609.$

(b) for Simpson's composite rule,

$$\text{error} = -\frac{f^{(4)}(\xi)}{180}(b-a)h^4.$$

$$= -\frac{\sin(\xi)}{180} \cdot \pi \cdot \left(\frac{\pi}{n-1}\right)^4$$

let $$|\text{error}_{max}| = \left|\frac{\pi^5}{180} \cdot \frac{1}{(n-1)^4}\right| \leq 10^{-6}.$$

$n_{min} = 38$

P10.

This is explicit trapezoidal method.

$$y_{i+1} = y_i + \frac{h}{2} \cdot [\lambda y_i + \lambda y_{i+1}]$$

that is,

$$(1 - \frac{h\lambda}{2}) y_{i+1} = (1 + \frac{h\lambda}{2}) y_i.$$

we want. $|y_{i+1}| \leq |y_i|$, that is.

$$\left| \frac{y_{i+1}}{y_i} \right| = \left| \frac{1+\frac{h\lambda}{2}}{1-\frac{h\lambda}{2}} \right| \leq 1.$$

because $\lambda < 0$

that is.
$$\frac{|2+h\lambda|}{2-h\lambda} \leq 1.$$

$$h \leq 0.$$

the condition is $h \leq 0$.

P11. Based on theorem of covergence,

we have

$$|e_{n+1}| \leq c(\delta) |e_n|^2.$$

where

$$c(\delta) = \frac{1}{2} \cdot \frac{\max_{|r-x| \leq \delta} |f''(x)|}{\min_{|r-x| \leq \delta} |f'(x)|}.$$

$$f'(x_n) = \frac{f'(x_n) g(x_n) - f(x_n) g'(x_n)}{g^2(x_n)}$$

$$f''(x_n) = .$$

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