CS/SE 4X03 — Assignment 2

5 October, 2021

Due date: 25 October

Instructions

- If you write your solutions by hand, please ensure your handwriting is legible. We may subtract marks for hard-to-read solutions.
- Submit to Avenue a **PDF file** containing your solutions and the **required** MATLAB **files**.

  Assignments in other formats, e.g. IMG, PNG, **will not be marked**.

  Name your MATLAB files **exactly** as specified.
- Name your PDF file **Lastname-Firstname-studentnumber.pdf**.
- Submit **only what is required**.
- Do not submit zipped files. We will **ignore any compressed file** containing your files.

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Problem 1 [3 points] Solve the system

$$\begin{bmatrix} 3 & 4 & 3 \\ 1 & 5 & -1 \\ 6 & 3 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 7 \\ 15 \end{bmatrix}$$

using Gaussian elimination

- without pivoting
- with partial pivoting

Show all steps. In your calculations, you can carry four digits after the decimal point, and a nonzero digit before the decimal point.

Problem 2 [3 points] Consider the linear system

$$\begin{bmatrix} 1 & 1+\epsilon \\ 1-\epsilon & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+(1+\epsilon)\epsilon \\ 1 \end{bmatrix}.$$

The exact solution is $x = [1, \epsilon]^T$ for any value of ϵ. Solve this system in MATLAB with various values for ϵ, especially for values near $\sqrt{\epsilon_{\text{mach}}}$.

- For each value of ϵ you try, compute the condition number of the matrix and the relative error in each component of the solution. Submit in the PDF results for 4 different values of ϵ in the form

ϵ	$\lvert x_1 - 1\rvert$	$\lvert x_2 - \epsilon\rvert/\epsilon$	cond(A)
$\vdots$			

- What conclusions can you draw from this experiment?

Problem 3 [12 points]

(a) [2,3,1 points] Implement in Matlab the following functions:

```
function B = GE(A)
```

performs LU factorization of an $n \times n$ matrix A without pivoting and stores the L and U factors in the output B. U is stored in the upper-triangular part of B. The diagonal of L is not stored, and the part of L below the main diagonal is stored below the main diagonal of B. For example, for a 3×3 matrix A, B is

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21} & u_{22} & u_{23} \\ l_{31} & l_{32} & u_{33} \end{bmatrix}$$

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```
function [B, ipivot] = GEPP(A)
```

performs LU factorization with partial pivoting. The L and U factors are stored in the output `B` as in `GE`. `ipivot` is permutation of the vector `1:n` recording the row permutations. When computing the LU factorization you **must not swap rows** in memory.

```
function x = backward(B, b, ipivot)
```

performs backward substitution. `B` contains the L and U factors as computed by `GE` or `GEPP`; `ipivot` is an index vector storing row permutations. `b` is an n-vector.

Then you can solve a linear system $Ax = b$ as

```
[B,ipivot] = GEPP(A);
x=backward(B,b,ipivot);
```

Store these functions in files `GE.m`, `GEPP.m`, and `backward.m`.

(b) [2 points] Write a main program `main_ge.m` that

- generates an $n \times n$ random matrix A and vector x of ones
- computes $b = Ax$
- solves $Ax = b$ using Gauss elimination with and without partial pivoting and with MATLAB's `A\b`
- produces output in the format

	`A\b`	no pivoting	pivoting	cond(A)
1	7.3e-13	6.5e-11	6.2e-13	1.9e+04

The first column is experiment number, the next three columns are the relative errors in the solutions computed with `A\b`, Gauss elimination without pivoting, and with pivoting, respectively. The last column is the condition number of A.

(c) [2 points] Generate a table for 5 systems with matrices of size 2000×2000 each.

(d) [2 points] How do the condition numbers relate to the accuracy of the computed solutions?

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- PDF: the Matlab files, table, (d)
- Avenue: the Matlab files

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Problem 4 [4 points] You have to interpolate e^x by a polynomial of degree five using equally spaced points in $[0, 1]$. What error would you expect if you use this polynomial?

Using equally spaced points, what degree polynomial would you use to achieve a maximum error of 10^{-8}?

Problem 5 [5 points] You are given the data points

x_i	0	0.1	0.2	0.3
y_i	1.0000	1.0488	1.0954	1.1402

where $y_i \approx \sqrt{x_i + 1}$, $i = 0, 1, 2, 3$.

a. (2 points) Using these data calculate approximations for $\sqrt{0.05}$ and $\sqrt{0.15}$

b. (2 points) Derive a bound for the error in these approximations.

c. (1 point) How does it compare to the actual errors?

Problem 6 [4 points] Let $f(x) = |x|$ where $x \in [-1, 1]$. Use the `polyfit` function to interpolate $f(x)$ at 21 equally spaced points $-1 = x_0 < x_1 < \cdots x_{20} = 1$. Denote the interpolating polynomial by $p(x)$.

(a) Plot on the same plot $f(x)$ and $p(x)$ versus x at 41 equally spaced points in $[-1, 1]$.

(b) Plot the error $|f(x) - p(x)|$ versus x at these points.

(c) Repeat (a) and (b), but now use Chebyshev points for the interpolation.

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- PDF: the four plots

Problem 7 [3 points] Redo Problem 6, with $f(x) = \sin(x)$ and interval $[-\pi, \pi]$.

Explain the differences in the errors when interpolating $|x|$ and $\sin(x)$.

Submit

- PDF: the four plots, your explanation

Problem 8 [4 points] Write a routine for computing the Newton polynomial interpolant for a given set of data points, and a second routine for evaluating the Newton interpolant at a given argument value using the Horner's rule. Name your files `newton.m` and `hornerN.m`. Write a main program `main_interp.m` that uses your Newton and Horner routines and produces the plots in Problem 6(a).

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- PDF: newton.m and hornerN.m
- Avenue: newton.m, hornerN.m, main_interp.m

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