Problem 1 (6 points**)** For each of the following expressions, explain when cancellations can occur and how to avoid them.

(a) $\sqrt{x+1} - 1$

(b) $(e^x - e^{-x})/2$

(c) $(1 - \cos x)/\sin x$

Problem 2 (4 points**)** The following Matlab program

```
x = 1;
while  (x+1)-x == 1
    x = 2*x;
end
x
```

outputs `9.007199254740992e+15`. Explain why this loop terminates and explain how this value is produced.

Problem 3 (4 points**)** Consider $f(x) = x\sin(x)$. Assume that you are given values for $f(x)$ at $x = 0, \pi/8, \pi/4, 3\pi/8$. Denote by $p(x)$ the polynomial interpolating these values. Derive a bound for $|f(x) - p(x)|$ for any $x \in [0, 3\pi/8]$.

Problem 4 (4 points**)** Let A be an $n \times n$ nonsingular matrix and let B be an $n \times m$ matrix, where $m \geq 1$. How can you compute efficiently an $n \times m$ matrix X such that

$$AX = B$$

What is the complexity of your approach in big-O notation?

Problem 5 (4 points**)** Let x and y be floating-point numbers. Assume that you have the log and exp functions available and you want to compute x^y using them. That is, you compute x^y by evaluating the expression $e^{y \ln x}$ using $\exp(y*\log(x))$, which is x^y in exact arithmetic.

Assume that $\text{fl}(\log(x)) = (\ln x)(1 + \epsilon)$, where $|\epsilon| \leq \eta$ for some η. Ignore the errors in the multiplication and the exp function, that is, assume they produce exact results.

What is the relative error in $\exp(y * \log(x))$. Can this error be large and why?