CS/SE 4X03 Scientific Computation

Midterm Exam
29 October 2021
11:30–12:30

Instructions:

1. You <u>must not</u> communicate with anybody during this exam.
2. You <u>must not</u> use the Internet.
3. You <u>can use</u> a calculator
4. You <u>must not</u> use Matlab.
5. Textbooks <u>are allowed</u>.
6. Write in the provided space. You can write on separate pages if you wish.
7. Submit one PDF. Name your file `Lastname-Firstname-studentnumber.pdf`.
8. Submit your PDF file to Avenue by **12:45**p.m.
9. If you have a SAS accommodation, please email me your PDF within the time allowed.
10. Sign the next page electronically and you must submit it with your exam.

    NO PDFs WILL BE ACCEPTED IF SENT BY EMAIL

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You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

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"By signing this document I agree to follow the McMaster University Policy on Academic Integrity. My signature below confirms that the work submitted for this exam is my own and did not involve the use of unauthorized aids."

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P1.

(a) $x = 1 \times 10^{178}$

$y = 0$

$z = 0$

(b)

P3. ~~when $x \not\approx y$~~

~~when with $x \approx y$,~~

when $x \approx y$.

when $x \approx y$, both expression will have cancellation error. But the callcellation error in $(x^3 - y^3)$ has a ~~more~~ larger magnitude, which will enlarge this error, while $(x-y)$ will has a smaller error because we loose less ~~bits~~ digits.

解.

assume $F_{n\times n}$ $A_{s\times s}$ $B_{t\times t}$. where $s+t=n$.
$C_{t\times t}$

$T(Fx=b) = O(n^3)$.

$T(Ax_1=b_1) = O(s^3)$

$T(Cx_2=b_2-Bx_1) = O(t^3)$.

$$\frac{t_A}{t_B} \# = \frac{O(n^3)}{O(s^3)+O(t^3)} = \frac{O(n^3)}{\max(O(s^3), O(t^3))} \gg 1$$

P5.

x_i	y_i	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
1	1			
2	1	0		
3	1	0	0	
4	6	5	$\frac{5}{2}$	$\frac{5}{6}$

(1) $p(x) = f[x_0] + f[x_0, x_1](x - x_0)$

$= 1 + 0$ $\qquad \hat{T}(1.5) = 1$

$= 1.$

(2) $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_1)(x - x_1)$

$= 1 + 0 + \frac{5}{2}(x - \cancel{0} 1)(x - 2)$

$= \frac{5}{2}x^2 - 10x + 6.$ $\qquad \hat{T}(1.5) = -\frac{27}{8}.$

(3) $p(x) = 1 + 0 + \frac{5}{2}(x-1)(x-2) + \frac{5}{6}(x-1)(x-2)(x-3)$

$\hat{T}(1.5) = p(1.5) = -\frac{27}{8} + \frac{5}{6} \times 0.5 \times (-0.5) \times (-1.5)$

$= \frac{-147}{8}.$

P6.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \times -1 \quad \times 0$$

$$M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \times -1$$

$$M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L = M_1^{-1} M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

P7

(a) $A = LU$

~~(b)~~ use route for solving a triangle system.

solve

$Ly = b$

then

solve

$y = Ux$.

to get p.

$\cancel{A}\, y^T \cdot A = c^T$

$A = LU$.

$y^T \cdot LU = c^T$.

~~solve~~ $\cancel{(y^T L)\, U = c^T}$

for U.

~~then solve~~

P8. $fl(x^2+y)$ where $|\epsilon| \le u = \epsilon_{mach}/2$

$= fl(fl(x^2) + fl(y))$

$= (fl(x^2) + fl(y))(1+\epsilon_1)$

$= (x^2(1+\epsilon_2) + y(1+\epsilon_3))(1+\epsilon_1)$

$= x^2(1+\epsilon_1)(1+\epsilon_2) + y(1+\epsilon_1)(1+\epsilon_3).$

$= x^2(1+\epsilon_1+\epsilon_2+\epsilon_1\epsilon_2) + y(1+\epsilon_1+\epsilon_3+\epsilon_1\epsilon_3)$

$\epsilon_1\epsilon_2 << u$

$\epsilon_2\epsilon_3 << u$

$\approx x^2(1+\epsilon_1+\epsilon_2) + y(1+\epsilon_1+\epsilon_3)$

$\cancel{= (1+\epsilon_1)(x^2+y) + \epsilon(\epsilon_1+\epsilon_2)x^2 + \epsilon_1+\epsilon_3}$

$\cancel{\epsilon_2 x^2 + \epsilon_3 y}$

denote $\delta_{12} = \epsilon_1+\epsilon_2$

denote $\delta_{13} = \epsilon_1+\epsilon_2$

$= x^2(1+\delta_{12}) + y(1+\delta_{13})$ where $\delta \le 2u$