

COMPSCI 4X03
Assignment 3

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Problem 1

$$a = 0$$

$$b = 1$$

$$r = \# \text{ interval}$$

$$h = \frac{1}{r}$$

$$f(x) = \sin(\pi x^2/3)$$

$$f'(x) = -\frac{4x}{3} \sin \frac{\pi x^2}{3} + \frac{2x}{3} \cos \frac{\pi x^2}{3}$$

$$M = \max_{x \in [0,1]} |f'(x)| = |f'(1)| = \left| -\frac{2\sqrt{3}}{3} + \frac{2}{3} \right| \approx 2.752$$

$$UBE = \frac{M}{12} (b-a) h^2 = \frac{2.752}{12} \times \frac{1}{r^2}$$

$$\text{To let } \frac{2.752}{12} \times \frac{1}{r^2} \leq 10^{-8},$$

$$\text{that is } r \geq \sqrt{\frac{2.752}{12}} \times 10^4$$

$$\Rightarrow r \geq 4789.9 \Rightarrow r_{\min} = 4789$$

$$\# \text{ points} = r+1 = 4790$$

Ans: We need 4790 points to achieve this accuracy.

PI

Problem 2

$$a = -1$$

$$b = 1$$

$$r = 5 - 1 = 4$$

$$h = \frac{b-a}{r} = \frac{1}{5}$$

$$t_i = a + ih = \frac{1}{5}i - 1$$

$$t_0 = -1, t_r = 1$$

$$f(x) = (x - 0.5)^2$$

$$f^{(4)}(x) = 0$$

using composite Simpson rule,

$$\int_{-1}^1 (x - 0.5)^2 dx \approx \frac{1}{6} \left[\frac{9}{4} + 2 * \frac{1}{4} + 4 * 1 + \frac{1}{4} \right]$$

$$= \frac{7}{6}$$

$$\text{Error} \leq M = \max_{x \in [-1, 1]} |f^{(4)}(x)| = 0$$

$$\text{Error} = 0$$

p2
part 2

Problem 3

Process

$$A = \begin{bmatrix} 1 & 0 & 0 & 0; & 0 & 1 & 0 & 0; & 0 & 0 & 1 & 0; & 0 & 0 & 0 & 1; & 1 & -1 & 0 & 0; & 1 & 0 & -1 & 0; & 1 & 0 & 0 & -1; & 0 & 1 & -1 & 0; & 0 & 1 & 0 & -1; & 0 & 0 & 1 & -1 \end{bmatrix};$$

$$b = [2.95; 1.74; -1.45; 1.32; 1.23; 4.45; 1.61; 3.21; 0.45; -2.75];$$

$$x = A \setminus b;$$

$$\text{fprintf}('x1 = \%6f \backslash n', x(1));$$

$$\text{fprintf}('x2 = \%6f \backslash n', x(2));$$

$$\text{fprintf}('x3 = \%6f \backslash n', x(3));$$

$$\text{fprintf}('x4 = \%6f \backslash n', x(4));$$

Output

$$x1 = 2.960000$$

$$x2 = 1.746000$$

$$x3 = -1.460000$$

$$x4 = 1.314000$$

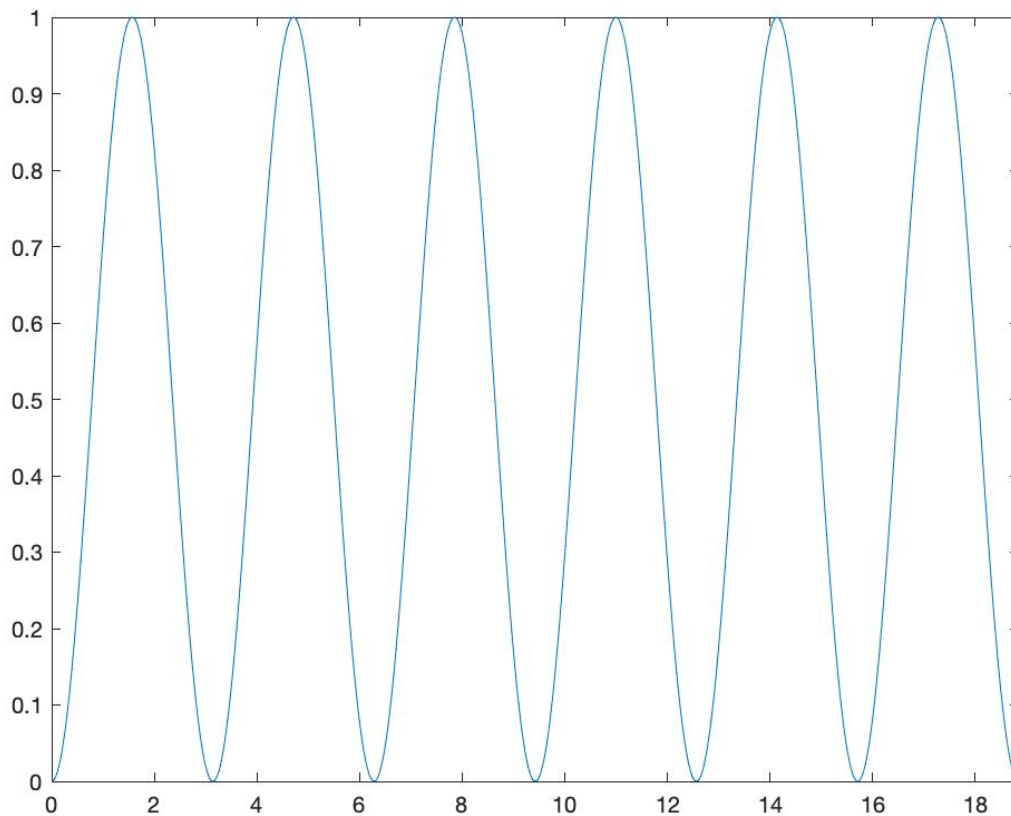
Compare

They are similar but slightly different from direct measurement, because we also take into consideration the possible bias of only one reference value (we add more reference values for each x , so it could be more accurate).

Problem 4

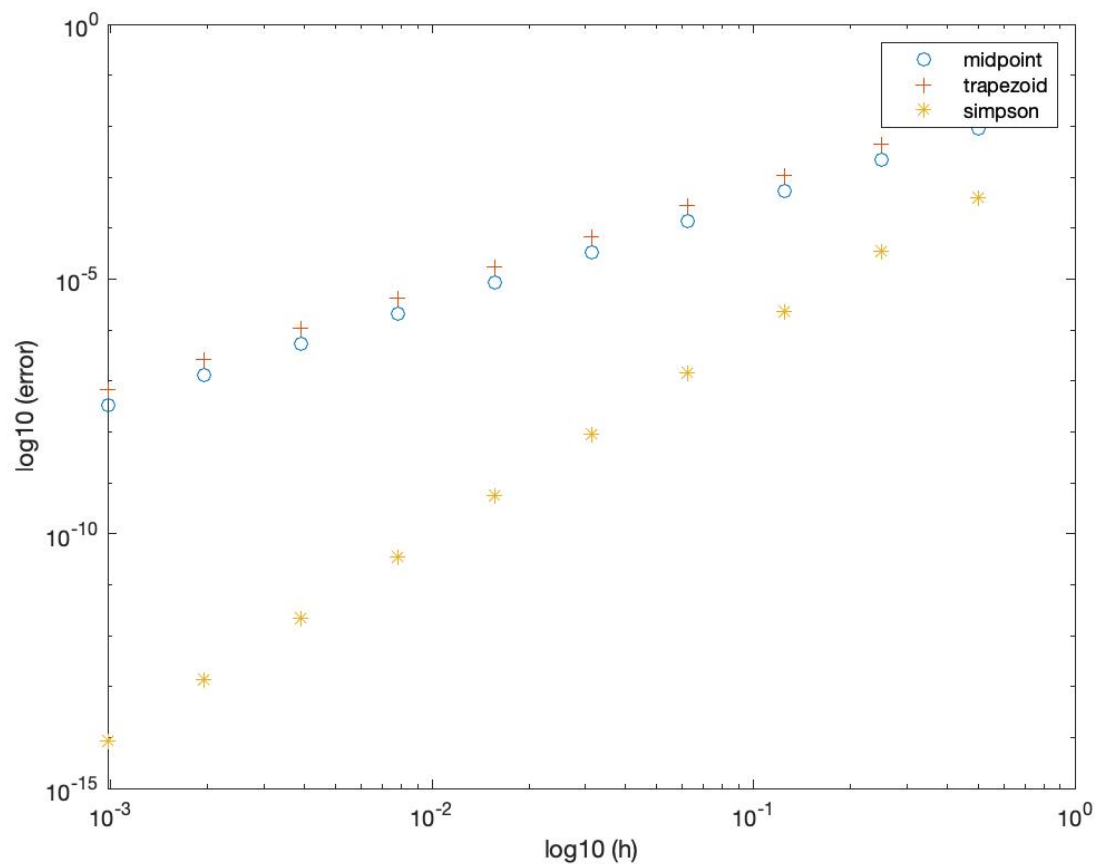
Function

$$\int_0^{6\pi} \sin^2(x) dx$$



composite Simpson **error**: 1.684973e-07
C1: 5703
adaptive Simpson **error**: 3.552714e-15
C2: 35

Problem 5



midpoint $3.48e-02 \cdot h^{2.00}$
trapezoid $6.95e-02 \cdot h^{2.00}$
simpson $7.87e-03 \cdot h^{3.97}$

Problem 6

	a	b	c	d	e
Jupiter	-1.185397	0.022029	-0.495039	-0.145054	26.982216
Saturn	-1.166745	0.035963	0.116729	-1.089852	90.381602
Uranus	-1.194134	0.011627	1.827051	-0.259256	367.268144
Neptune	-1.167128	0.020704	-0.392687	-0.423158	903.808671
Pluto	-1.003337	0.238833	11.847098	12.717063	1290.679928