CS/SE 4X03 Final Examination

DAY CLASS — Dr. N. Nedialkov
DURATION OF EXAMINATION: 2 hours
MCMASTER UNIVERSITY FINAL EXAMINATION — 14 December, 2021, 16:00–18:00

Special Instructions:

1. You must not communicate with anybody during this exam.
2. You must not use the Internet.
3. Matlab and similar programs are not allowed. You can use a calculator.
4. Textbooks are allowed.
5. Write your solutions in the space provided in this PDF, or on separate paper.
6. Sign the next page and you must submit it. Electronic signature, or signing a printed copy and scanning it, or signing the PDF using a tablet is fine.

Submission. You will have 10 minutes extra time for submission. This time is not for writing the exam. There will be 3 drop boxes.

Dropbox 1 will be open from 16:00 to 18:09:59

Dropbox 2 will be open from 18:10 to 18:24:59. A submission in it will have a 20% penalty.

Dropbox 3 will be open from 18:25 to 18:40. A submission in it will have a 40% penalty.

Only exams on Avenue will be marked.

If you have a SAS accommodation, please email me your exam within the time indicated in your SAS letter + 10min.

COURSE: CS/SE4X03

EXAM DATE: 14 December, 2021

McMaster University Statement on Academic Integrity:
You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: "Grade of F assigned for academic dishonesty") and/or suspension of expulsion from the university.

"By signing this document I agree to follow the McMaster University Policy on Academic Integrity. My signature below confirms that the work submitted for this exam is my own and did not involve the use of unauthorized aids."

STUDENT NAME:

STUDENT ID:

STUDENT SIGNATURE:

Problem 1 [3 points] Suppose you need to generate $n+1$ equally spaced points in the interval $[a,b]$ with spacing $h=(b-a)/n$, $n>1$. You can generate them using

$$x_0 = a, \quad x_i = x_{i-1} + h, \quad \text{or} \tag{1}$$

$$x_i = a + ih, \tag{2}$$

for $i = 1,\ldots,n-1$, $x_0 = a$ and $x_n = b$. Which of (1) and (2) would be more accurate and why?

Problem 2 [5 points] You need to evaluate $\log(x-\sqrt{x^2-1})$ in floating-point arithmetic.

a. [2 points] What numerical inaccuracies can occur and for what values of x?

b. [2 points] Obtain an equivalent formula to avoid such inaccuracies.

Problem 3 [3 points] Given a scalar R, describe a method for computing $1/R^2$ using only addition (or subtraction) and multiplication operations.

Problem 4 [3 points] Consider Newton's method applied to the system of nonlinear equations

$$
\begin{aligned}
x_1^2 - x_2^2 &= 0 \\
2x_1x_2 &= 1
\end{aligned}
$$

Is there an initial guess x_0 for which Newton would break down? If so, give such a guess.

Problem 5 [3 points] Given the data

x	0	1	2	4
$f(x)$	1	9	23	93

find an approximation for $f(2.5)$ by evaluating the polynomial interpolating these data points.

Problem 6 [3 points] Consider $f(x) = e^x$ over $[0, \pi]$. Suppose we approximate $f(x)$ by a trigonometric polynomial of the form $p(x) = a + b\cos(x) + c\sin(x)$. What is the linear system to be solved for determining the least squares fit of p to f?

Problem 7 [4 points] Let A, B, and C be $n \times n$ matrices with A nonsingular, and let d be an n-vector.

A. [2 points] How would you efficiently compute the product $A^{-1}(B+C)d$?

B. [2 points] Derive the complexity of your approach in terms of big-O notation.

Problem 8 [2 points] Under what circumstances does a small residual vector $r = b - Ax$ imply that x is an accurate solution to the linear system $Ax = b$?

Problem 9 [4 points] What is the smallest number of points that are needed to compute $\int_0^\pi \sin x \, dx$ with accuracy 10^{-6} using the following methods with equally spaced points:

(a) trapezoid composite rule

(b) Simpson's composite rule

Problem 10 [4 points] Consider the following method

$$y_{i+1} = y_i + \frac{h}{2}[f(t_i, y_i) + f(t_{i+1}, y_i + hf(t_i, y_i))]$$

for integrating the ODE $y' = f(t, y)$. What is the condition for h so this method is stable when applied to $y' = \lambda y$ with $\lambda < 0$.

Problem 11 [5 points, Bonus] Consider the following method for solving the equation $f(x) = 0$:

$$x_{n+1} = x_n - \frac{f(x_n)}{g(x_n)}, \quad \text{where} \quad g(x) = \frac{f\big(x + f(x)\big) - f(x)}{f(x)}.$$

Derive its rate of convergence.