CS/SE 4X03 Final Examination

DAY CLASS — Dr. N. Nedialkov
DURATION OF EXAMINATION: 2 hours
MCMASTER UNIVERSITY FINAL EXAMINATION — 14 December, 2021, 16:00–18:00

Special Instructions:

1. You must not communicate with anybody during this exam.
2. You must not use the Internet.
3. Matlab and similar programs are not allowed. You can use a calculator.
4. Textbooks are allowed.
5. Write your solutions in the space provided in this PDF, or on separate paper.
6. Sign the next page and you must submit it. Electronic signature, or signing a printed copy and scanning it, or signing the PDF using a tablet is fine.

Submission. You will have 10 minutes extra time for submission. This time is not for writing the exam. There will be 3 drop boxes.

Dropbox 1 will be open from 16:00 to 18:09:59

Dropbox 2 will be open from 18:10 to 18:24:59. A submission in it will have a 20% penalty.

Dropbox 3 will be open from 18:25 to 18:40. A submission in it will have a 40% penalty.

Only exams on Avenue will be marked.

If you have a SAS accommodation, please email me your exam within the time indicated in your SAS letter + 10min.

COURSE: CS/SE4X03

EXAM DATE: 14 December, 2021

McMaster University Statement on Academic Integrity:
You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: "Grade of F assigned for academic dishonesty") and/or suspension of expulsion from the university.

"By signing this document I agree to follow the McMaster University Policy on Academic Integrity. My signature below confirms that the work submitted for this exam is my own and did not involve the use of unauthorized aids."

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STUDENT ID: 400316660

STUDENT SIGNATURE: Mingzhe Wang

P1 (2) would be more accurate.

Because there will be floating point error accumated in the evaluation of each x_i in (1); while for (2), we calculate every term x_i directly based on a, i, and h.

P2 a. when $x \approx 1$, there will be cancellation error both in $\sqrt{x^2-1}$ and $x - \sqrt{x^2-1}$.

b. $\log\left(\frac{1}{x+\sqrt{x^2-1}}\right)$

P3. Write $x = \frac{1}{R^2}$, $f(x) = \cancel{1-R^2 x}\ \frac{1}{x} - R^2$.

Apply Newton's method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

$$\cancel{= x_n - \frac{1-R^2 x_n}{-R^2}}$$

$$\cancel{= \frac{1}{R^2} - x_n}$$

~~Set x_0 = some initial guess, apply $x_{n+1} = \frac{1}{R^2} - x_n$ iteratively~~

$$x_{n+1} = x_n - \frac{\frac{1}{x_n} - R^2}{-\frac{1}{x_n^2}}.$$

we get

$$x_{n+1} = -R^2 x_n^2 + 2x_n \quad (*)$$

Approach:

step 1. set x_0 = some initial guess,

step 2. apply (*) until get the accurate result we want.

~~P4~~

P4. if $F'(x) = \begin{pmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{pmatrix}$ is singular, it will break down.

That is, $\begin{vmatrix} 2x_1 & -2x_2 \\ 2x_2 & 2x_1 \end{vmatrix} = 4x_1^2 + 4x_2^2 = 0$.

when $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

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P5. We will use newton's interpolation for this question.

i	x_i	$f[x_i]$.	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
0	0	1			
1	1	9	8		
2	2	23	14	3	
3	4	93	35	7	1

$$P_3(x) = f[x_0] + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1) + f[x_0, x_1, x_2, x_3](x-x_0)(x-x_1)(x-x_2)$$

$$= 1 + 8(x-0) + 3\cdot(x-0)(x-1) + 1\cdot(x-0)(x-1)(x-2)$$

$$= x^3 + 7x + 1.$$

$$f(2.5) = 2.5^3 + 7 \times 2.5 + 1 = 34.1250$$

P6. Assume we have n points for interpolation. i.e. we have $x_1, x_2, \cdots, x_n$ $y_1, y_2, \cdots, y_n$.

solve $Ax = b$ for x.

where

$$A = \begin{bmatrix} 1 & \cos(x_1) & \sin(x_1) \\ 1 & \cos(x_2) & \sin(x_2) \\ \vdots & \vdots & \vdots \\ 1 & \cos(x_n) & \sin(x_n) \end{bmatrix}.$$

$$x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

$$b = \begin{bmatrix} \cancel{f(x_1)}\ y_1 \\ \cancel{f(x_2)}\ y_2 \\ \vdots \\ \cancel{f(x_n)}\ y_n \end{bmatrix}.$$

P7. (a) Set $x = A^{-1}(B+C)d$,

then $Ax = (B+C)d$

Approach:

step 1. Calculate $y = (B+C)\cdot d$

step 2. Use Guess Elimination solve $Ax = y$ for x.

step 3. Use backward substitution for solving x in $Ax = y$.

(b). In first step,

for, $(B+C)$, there is n^2 addition.

then for $(B+C)\cdot d$, there is n^2 multiplication and $n(n-1)$ addition.

In second step,

the Guess Elimination takes $2n^3/3 - n^2 + n/6$ time.

In third step,

the backward substitution takes. n^2 time.

So the total time complexity is $O(n^3)$.

P8. If $Ax = b$ is an overdetermined system: more equations than variables. i.e. if $A \in \mathbb{R}^{m \times n}$, then $m > n$.

P9. let $n = \#$ points.

$r = n-1$

$a = 0$

$b = \pi$

$h = \frac{b-a}{r} = \frac{\pi}{n-1}$

(a) for trapezoid composite rule,

$$\text{error} = -\frac{f''(\mu)}{12}(b-a)h^2$$

$$= -\frac{-\sin(\mu)}{12} \cdot \pi \cdot \left(\frac{\pi}{n-1}\right)^2$$

let $|\text{error}_{max}| = \left|\frac{\pi^3}{12} \cdot \frac{1}{(n-1)^2}\right| \leqslant 10^{-6}$.

$n_{min} = 1609$.

(b) for Simpson's composite rule,

$$\text{error} = -\frac{f^{(4)}(\xi)}{180}(b-a)h^4.$$

$$= -\frac{\sin(\xi)}{180} \cdot \pi \cdot \left(\frac{\pi}{n-1}\right)^4$$

let $|\text{error}_{max}| = \left|\frac{\pi^5}{180} \cdot \frac{1}{(n-1)^4}\right| \leqslant 10^{-6}$.

$n_{min} = 38$

P10.

This is explicit trapezoidal method.

$$y_{i+1} = y_i + \frac{h}{2}\cdot[\lambda y_i + \lambda y_{i+1}]$$

that is,

$$(1-\frac{h\lambda}{2})y_{i+1} = (1+\frac{h\lambda}{2})y_i.$$

we want. $|y_{i+1}| \leq |y_i|$, that is.

$$\left|\frac{y_{i+1}}{y_i}\right| = \left|\frac{1+\frac{h\lambda}{2}}{1-\frac{h\lambda}{2}}\right| \leq 1.$$

because $\lambda < 0$

that is. $\frac{|2+h\lambda|}{2-h\lambda} \leq 1.$

$$h \leq 0.$$

the condition is $h \leq 0$.

P11. Based on theorem of covergence,

we have

$$|e_{n+1}| \leq c(\delta)\,|e_n|^2.$$

where

$$c(\delta) = \frac{1}{2}\cdot\frac{\max_{|r-x|\leq\delta}|f''(x)|}{\min_{|r-x|\leq\delta}|f'(x)|}.$$

$$f'(x_n) = \frac{f'(x_n)g(x_n) - f(x_n)g'(x_n)}{g^2(x_n)}$$

$$f''(x_n) = .$$

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