P1.

(a) $x = 1 \times 10^{178}$

$y = 0$

$z = 0$

(b)

P3. ~~when $x \not\approx y$~~

~~when with $x \approx y$,~~

when $x \approx y$.

when $x \approx y$, both expression will have cancellation error. But the callcellation error in (x^3-y^3) has a ~~more~~ larger magnitude, which will enlarge this error, while $(x-y)$ will has a smaller error because we loose less ~~bits~~ digits.

Scanned with CamScanner

解.

assume $F_{n\times n}$ $A_{s\times s}$ $B_{t\times t}$, $C_{t\times t}$. where $s+t=n$.

$T(Fx=b) = O(n^3)$.

$T(Ax_1=b_1) = O(s^3)$

$T(Cx_2 = b_2 - Bx_1) = O(t^3)$.

$$\frac{t_A}{t_B} = \frac{O(n^3)}{O(s^3)+O(t^3)} = \frac{O(n^3)}{\max(O(s^3),\ O(t^3))} \gg 1$$

P5.

x_i	y_i	$f[\cdot,\cdot]$	$f[\cdot,\cdot,\cdot]$	$f[\cdot,\cdot,\cdot,\cdot]$
1	1			
2	1	0		
3	1	0	0	
4	6	5	$\frac{5}{2}$	$\frac{5}{6}$

(1) $p(x) = f[x_0] + f[x_0, x_1](x - x_0)$

$= 1 + 0$ $\quad\quad \hat{T}(1.5) = 1$

$= 1.$

(2) $p(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2]$ $(x - x_1)$ $(x - x_1)$

$= 1 + 0 + \frac{5}{2}(x - \cancel{2}\ 1)(x - 2)$

$= \frac{5}{2}x^2 - 10x + 6.$ $\quad\quad \hat{T}(1.5) = -\frac{27}{8}.$

(3) $p(x) = 1 + 0 + \frac{5}{2}(x-1)(x-2) + \frac{5}{6}(x-1)(x-2)(x-3)$

$\hat{T}(1.5) = p(1.5) = -\frac{27}{8} + \frac{5}{6} \times 0.5 \times (-0.5) \times (-1.5)$

$= \frac{-147}{8}.$

Scanned with CamScanner

P6.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \times -1 \downarrow \quad \times 0 \downarrow . \qquad M_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \quad \times -1 \downarrow \qquad M_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$L = M_1^{-1} M_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$U = . \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

P7

(a) $A = LU$

~~(b)~~. use route for solving a triangle system.

solve

$Ly = b$

then

solve.

$y = Ux$.

to get P.

$\cancel{A}\, y^T \cdot A = c^T$

$A = LU$.

$y^T \cdot LU = c^T$.

~~solve $(y^T L)\, U = c^T$~~

for U.

~~then solve~~

P8.

$fl(x^2+y)$ where $|\epsilon| \leq u = \epsilon_{mach}/2$

$= fl(fl(x^2) + fl(y))$

$= (fl(x^2) + fl(y))(1+\epsilon_1)$

$= (x^2(1+\epsilon_2) + y(1+\epsilon_3))(1+\epsilon_1)$

$= x^2(1+\epsilon_1)(1+\epsilon_2) + y(1+\epsilon_1)(1+\epsilon_3).$

$= x^2(1+\epsilon_1+\epsilon_2+\epsilon_1\epsilon_2) + y(1+\epsilon_1+\epsilon_3+\epsilon_1\epsilon_3)$

$\epsilon_1\epsilon_2 \ll u$

$\epsilon_2\epsilon_3 \ll u$

$\approx x^2(1+\epsilon_1+\epsilon_2) + y(1+\epsilon_1+\epsilon_3)$

$\cancel{= (1+\epsilon_1)(x^2+y) + \epsilon_2 x^2 + \epsilon_3 y}$

denote $\delta_{12} = \epsilon_1+\epsilon_2$

denote $\delta_{13} = \epsilon_1+\epsilon_2$

$= x^2(1+\delta_{12}) + y(1+\delta_{13})$ where $\delta \leq 2u$

Scanned with CamScanner