CBE 140 Design Game

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1 Reactor Mass Balance Calculations

1.1 Species Balance on Reactant, A

$$a = i - o + g - c$$

$$0 = q_{in}C_{0A} - q_{out}C_A - kC_AV$$

$$0 = qC_{0A} - qC_A - kC_AV$$

$$C_A = \frac{qC_{0A}}{q + kV}$$

$$C_A = \frac{C_{0A}}{1 + k\frac{V}{q}}$$

1.2 Species Balance on Product, D

$$a = i - o + g - c$$

$$0 = -q_{out}C_D + kC_AV$$

$$0 = -qC_D + kV\frac{C_{0A}}{1 + k\frac{V}{q}}$$

$$0 = -qC_D + \frac{C_{0A}}{\frac{1}{kV} + \frac{1}{q}}$$

$$V = \frac{q}{k}\left(\frac{C_D}{C_{0A} - C_D}\right)$$

2 Cost Analysis

2.1 Determining the Optimal Conversion Ratio

$$Cost = q(\Delta t)C_{0A}(Price_A) + V(Cost_{per\ V})$$

$$Cost = q(\Delta t)C_{0A}(Price_A) + \frac{q}{k}\left(\frac{C_D}{C_{0A} - C_D}\right)(Cost_{per\ V})$$

$$\frac{Cost}{qC_D} = \frac{C_{0A}(\Delta t)}{C_D}(Price_A) + \frac{1}{k}\left(\frac{1}{C_{0A} - C_D}\right)(Cost_{per\ V})$$

$$\frac{Cost}{qC_D} = \frac{\Delta t}{X}(Price_A) + \frac{1}{kC_{0A}(1 - X)}(Cost_{per\ V})$$

$$\frac{d}{dX}\left[\frac{Cost}{qC_D}\right] = -\frac{\Delta t}{X^2}(Price_A) + \frac{1}{kC_{0A}(1 - X)^2}(Cost_{per\ V})$$

$$0 = -\frac{\Delta t}{X^2}(Price_A) + \frac{1}{kC_{0A}(1 - X)^2}(Cost_{per\ V})$$

$$0 = \frac{\alpha}{X^2} + \frac{\beta}{(1 - X)^2}, \quad \alpha = -\Delta t(Price_A), \quad \beta = \frac{Cost_{per\ V}}{kC_{0A}}$$

$$0 = (\alpha + \beta)X^2 - (2\alpha)X + \alpha$$

Plugging in $\Delta t = 5.04 \times 10^5 \,\text{min}$, $Price_A = \$0.28 \,\text{mol}^{-1}$, $Cost_{per\ V} = \$47 \,\text{L}^{-1}$, $k = 0.005 \,\text{min}^{-1}$, $C_{0A} = 0.20 \,\text{mol} \,\text{L}^{-1}$:

$$\alpha = -\$1.411 \times 10^5 \,\mathrm{min \,mol}^{-1} \quad \beta = \$4.70 \times 10^4 \,\mathrm{min \,mol}^{-1}$$

$$X = \frac{2\alpha \pm \sqrt{(2\alpha)^2 - 4(\alpha + \beta)(\alpha)}}{2(\alpha + \beta)}$$

$$X = 0.634$$

3 Reactor Sizing (Work in progress)

We will size the reactor to achieve the optimal (cost minimized) conversion ratio, X, when 10% of the time-averaged projected market demand is reached.

$$\bar{D} = \frac{\Sigma[D_1 \dots D_5]}{5}$$
$$\bar{D} = 3.12 \times 10^7 \,\mathrm{mol}\,\mathrm{yr}^{-1}$$

This amounts to a production rate, qC_D , of:

$$qC_D = \frac{(0.1)D}{(\Delta t)}$$
$$qC_D = 6.19 \,\text{mol/min}$$

$$V = \frac{1}{k} \frac{qC_D}{C_{0A} - XC_{0A}}$$
$$V = \frac{qC_D}{kC_{0A}(1 - X)}$$

 $Plugging \ in \ k = 0.005 \, \mathrm{min^{-1}}, \ \ C_{0A} = 0.20 \, \mathrm{mol} \, \mathrm{L^{-1}}, \ \ qC_D = 6.19 \, \mathrm{mol/min}, \ \ X = 0.634 : 100 \, \mathrm{mol/min}$

$$V = 1.69 \times 10^4 \,\mathrm{L}$$

4 Year 1 Deliverables

4.1 Reactor Volume

$$V = 1.69 \times 10^4 \, \mathrm{L}$$

4.2 Product Concentration

$$C_D = C_{0A}X$$

 $C_D = 0.127 \,\text{mol}\,\text{L}^{-1}$

4.3 Reactor Flow Rate

$$q = \frac{qC_D}{C_D}$$