Notes

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Central Limit Theorem

- The Uniform Probability Distribution

Refers to a type of probability distribution in which all outcomes are equally likely.
The following is the probability density function for a uniform variable:

$$f(x) = \{ \begin{matrix} \frac{1}{(b-a)}; a <= x <= b \\ 0; \text{otherwise} \end{matrix} \}$$

The function is equal to zero for values outside the range between a and b.
The area under the line (1/b(b-a)) is 1 since it is a PDF.

Example, if a = 0 and b = 40, then for the range from 0 to 40 the function is $f(x) = \frac{1}{40}$. Plotting this function, it would have the form of a rectangle with base 40 and hight 1/40, so the are will be 1.

The expected value of this x random variable is given by:

$$E(x) = \frac{a+b}{2}$$

(Actually the mid-point of the range from a to b).
If a = 0 and b = 40, the expected value for x will be
$f(x) = \frac{40}{2} = 20$

The expected value of a random variable is the *theorical* mean of the random variable according to its probability distribution.
The variance of a variable with the uniform distribution is:

$$Var(x) = \frac{(b-a)^2}{12}$$

- Monte Carlo simulation

Monte Carlos simulation: creating random numbers that follow specific probability distributions.

The Central Limit Theorem Definition

The sample mean will have the following characteristics:

1) The distribution of the sample means will be close to normal distribution when you take many groups (the size of the groups should be each equal or bigger than 25).

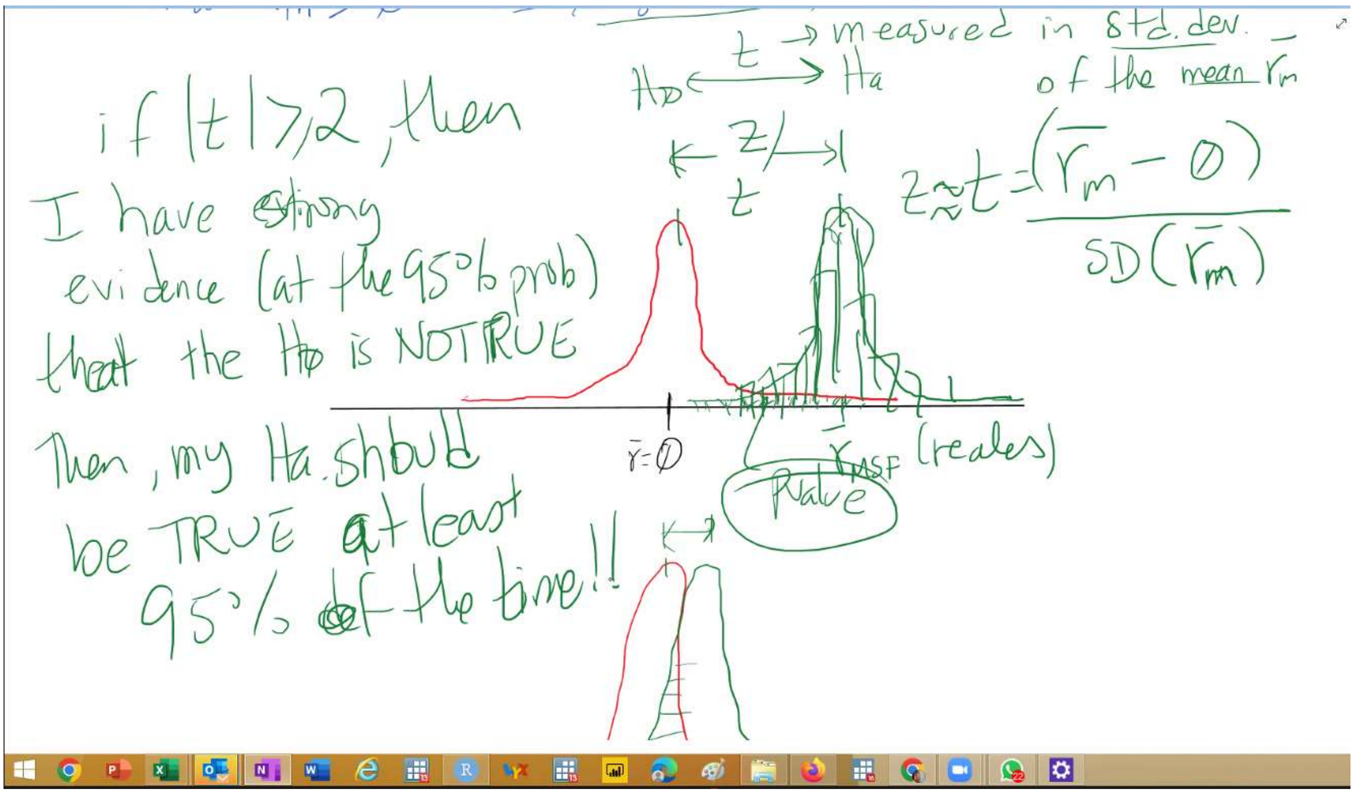
2) The standard deviation of the sample means will be much less than the standard deviation of the individuals. Being more specifically, the standard deviation of the sample mean will shrink with a factor of $\frac{1}{\sqrt{N}}$.

No matter the original probability distribution of any random variable, if we take groups of this variable, a) the means of these groups will have a probability distribution close to the normal distribution, and b) the standard deviation of the *mean will shrink according to the number of elements of each group.*

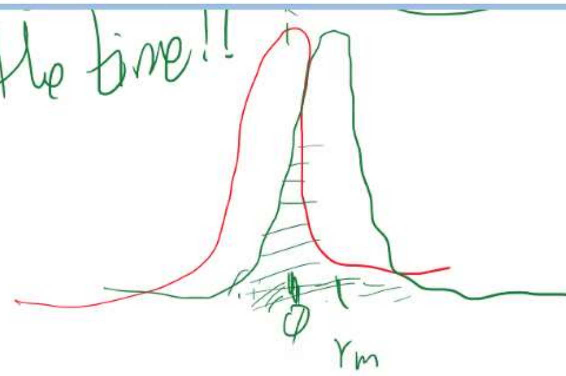
The standard deviation of a grouped sample mean is equal to the standard deviation of a non-grouped sample divided by the root of groups.

$$SD(\bar{X}) = \frac{SD(X)}{\sqrt{N}}$$

Hypothesis testing



we ...
95% of the time!!



$$SD(\bar{r}_m) = ? = \frac{SD(r_m)}{\sqrt{N}}$$

$$t = \frac{(\bar{r}_m - 0)}{\left(\frac{SD(r_m)}{\sqrt{N}}\right)} = =$$

Imagine that the mean ret of MST = 0.015 ...
for 36 months, and $SD(r_m) = 0.020$ (2%)

$$SD(\bar{r}_m) = \frac{SD(r_m)}{\sqrt{36}} = \frac{0.02}{6} \approx 0.0033 \approx 0.33\%$$

95% of the time, the mean returns of MSFT will be around: [

$$\left.\begin{array}{l} \min(95\%\,C.I.) \approx \bar{r}_m - 2(SD(\bar{r}_m)) \\ \max(95\%\,C.I.) \approx \bar{r}_m + 2 \cdot SD(\bar{r}_m) \end{array}\right\} \text{Then!}$$