Aufgabe 1: Zeitdiskrete Filter

a) 2P

$H_1(z)$: ja, weil: FIR, gerade Symmetrie, N_b ungerade (=5), Typ 2

$H_2(z)$: ja, weil: FIR, ungerade Symmetrie, N_b = 3, Typ 4

wenn Typ 3 genannt: eventuell richtig werten, wenn Begründung korrekt

$H_3(z)$: nein, keine Koeffizientensymmetrie

$H(z)$: ja, weil: FIR, ungerade Symmetrie, N_b = 5, Typ 4

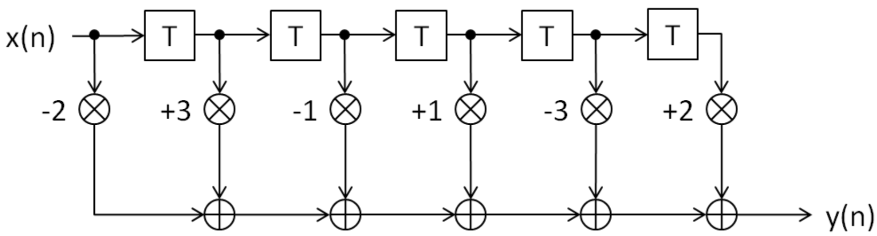
b) 1P

$$h(n) = -2\delta(n) + 3\delta(n-1) - 1\delta(n-2) + 1\delta(n-3) - 3\delta(n-4) + 2\delta(n-5)$$

c) 1P

$$y(n) = -2x(n) + 3x(n-1) - x(n-2) + x(n-3) - 3x(n-4) + 2x(n-5)$$

d) 2P



e) 4P

	0	1	2	3	4	5	6	7	8	9
x(n)	1	4	-2	3	0	0	0	0	0	0
$y_1(n)$	-2	-8	4	-6	0	-2	-8	4	-6	0
$y_2(n)$	0	3	12	-6	6	-12	6	-9	0	0

f) 2P

$y_1(n)$: $K_{min} = 6 + 4 - 1 = 9$

$y_2(n)$: $K_{min} = (5 + 4 - 1 = 8)$ besser:
$K_{min} = 4 + 4 - 1 = 7$

$y_3(n)$: $K_{min} = (6 + 4 - 1 = 9)$ besser:
$K_{min} = 4 + 4 - 1 = 7$

$y(n)$: $K_{min} = 6 + 4 - 1 = 9$

g) $\boxed{\textbf{3P}}$

	0	1	2	3	4	5	6	7
$y_1(n)$	-8	-8	4	-6	0	-2	-8	4
$y_2(n)$	0	3	12	-6	6	-12	6	-9

RECHENWEG

y_1

			-2	**0**	**0**	**0**	**0**	**-2**	**0**	**0**	
3	-2	4	1	0	0	0	0	3	-2	4	-8
	3	-2	4	1	0	0	0	0	3	-2	-8
		3	-2	4	1	0	0	0	0	3	4
			3	-2	4	1	0	0	0	0	-6
				3	-2	4	1	0	0	0	0
					3	-2	4	1	0	0	-2
						3	-2	4	1	0	-8
							3	-2	4	1	4
								3	-2	4	-6

y_2 analog zu y_1

h) $\boxed{\textbf{1P}}$

$y_1(n)$: Aliasing bei n = 0 ! $K_{min} = 9$, aber $K = 8$

$K_{min} - K = 1$ (Abweichung)

$y_2(n)$: kein Aliasing, da $K \geq K_{min}$!

Aufgabe 2: Filterentwurf

a) $\boxed{1P}$

$\delta_p = 0{,}005$

$\delta_{St} = 0{,}01$

$\Omega_p = 0{,}3\pi$

$\Omega_{st} = 0{,}6\pi$

b) $\boxed{1{,}5P}$

$d_{st} = -20\log(\delta_{st}) = 40\ dB$

$R_p = 20\log(1+\delta_p) - 20\log(1-\delta_p) = 0{,}0869\ dB$

c) $\boxed{1{,}5P}$

$N_b \geq \frac{-20\log(0{,}005) - 7{,}95}{2{,}29 \cdot 0{,}3\pi} = \frac{46{,}02 - 7{,}95}{2{,}29 \cdot 0{,}3\pi} = 17{,}64 \rightarrow N_b = 18$

d) $\boxed{1P}$

$\tilde{N}_b \geq \frac{-10\ log(\delta_p \cdot \delta_{St}) - 13}{2{,}323(\Omega_{st} - \Omega_p)} = 13{,}71 \rightarrow \tilde{N}_b = 14$

e) $\boxed{2P}$

aus b) $R_p = 0{,}0869\ dB$

$R_p = -20\log(1-\delta_p)$

→ $\delta_p' = 0{,}00995 \approx 0{,}01$

$\delta_{St}' = 0{,}01$

$\Omega_p' = 0{,}3\pi$

$\Omega_{St}' = 0{,}6\pi$

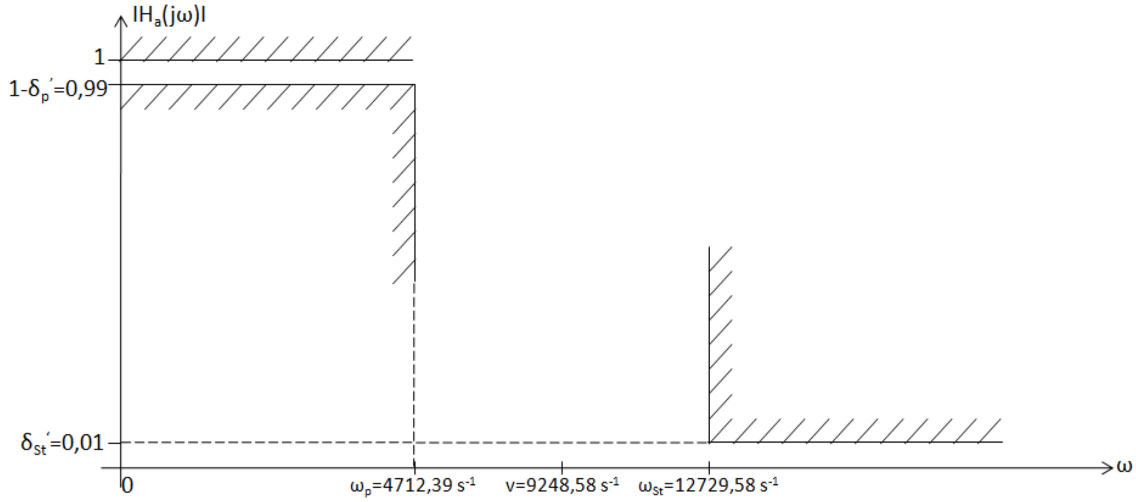
f) $\boxed{3P}$

$\omega_p = \frac{\Omega_p}{T} = 0{,}3\pi \cdot 5 \cdot 10^3 \frac{1}{s} = 4712{,}389 \frac{1}{s}$

$v = \frac{\omega_p}{\tan\left(\frac{\Omega_p'}{2}\right)} = 9248{,}584 \frac{1}{s}$

$\omega_{St} = v \cdot \tan\left(\frac{\Omega_{st}'}{2}\right) = 12729{,}584 \frac{1}{s}$

g) $\boxed{\textbf{2P}}$



h) $\boxed{\textbf{4P}}$

$$N \geq max\{N_p, N_{st}\}$$

$$|H_a(j\omega)|^2 = \frac{1}{1+(\frac{j\omega}{j\omega_c})^{2N}}$$

$$\omega_c = \nu * \tan\left(\frac{\Omega_c}{2}\right) = 7899{,}037\frac{1}{s}$$

<u>Passband:</u> $(1-\delta_p)^2 = \frac{1}{1+(\frac{\omega_p}{\omega_c})^{2N_p}}$

$$\left(\frac{\omega_p}{\omega_c}\right)^{2N_p} = \frac{1}{(1-\delta_p)^2} - 1$$

$$N_p = \frac{\log\left(\frac{1}{(1-\delta_p)^2}-1\right)}{2\log\left(\frac{\omega_p}{\omega_c}\right)} = \frac{-1{,}6924}{2\log\left(\frac{\omega_p}{\omega_c}\right)} = 3{,}77$$

<u>Stopband:</u> ${\delta_{st}}^2 = \frac{1}{1+\left(\frac{\omega_{st}}{\omega_c}\right)^{2N_{st}}}$

$$N_{st} = \frac{\log\left(\frac{1}{{\delta_{st}}^2}-1\right)}{2\log\left(\frac{\omega_{st}}{\omega_c}\right)} = 9{,}65$$

$$\Rightarrow N = 10$$

i) $\boxed{\textbf{1P}}$

FIR:	Vorteile:	Linearphasigkeit möglich, Stabilität, …
	Nachteile:	höhere Filterordnung, …
IIR:	Vorteile:	niedrigere Filterordnung, geringe algorithmische Verzögerung
	Nachteile:	Instabilität möglich, …

Aufgabe 3: Analyse eines kausalen LTI-Systems

a) $\boxed{\textbf{3P}}$

$$G(z)=\frac{[z-1{,}5]\cdot[z-(1{,}5+1{,}5j)]\cdot[z-(1{,}5-1{,}5j)]}{z[z-0{,}5j]\cdot[z+0{,}5j]}\cdot b_0$$

$$=\frac{[1-(1{,}5+1{,}5j)\cdot z^{-1}][1-(1{,}5-1{,}5j)\cdot z^{-1}][1-1{,}5z^{-1}]}{[1-0{,}5jz^{-1}]*[1+0{,}5jz^{-1}]}\cdot b_0$$

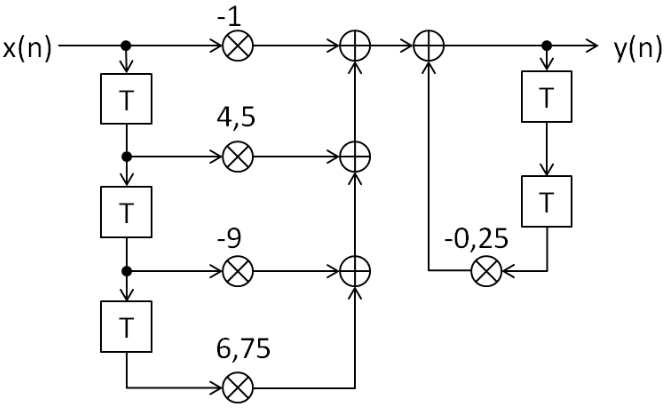
$$=\frac{1-4{,}5z^{-1}+9{,}0z^{-2}-6{,}75z^{-3}}{1+0{,}25z^{-2}}\cdot b_0 \qquad \text{mit } b_0=-1$$

b) $\boxed{\textbf{2P}}$

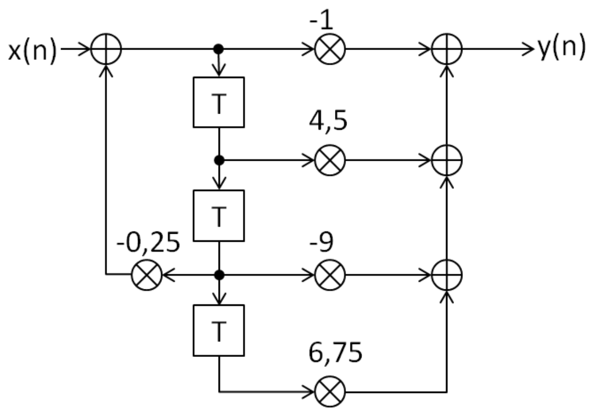
$$y(n)=-x(n)+4{,}5x(n-1)-9{,}0x(n-2)+6{,}75x(n-3)-0{,}25y(n-2)$$

c) $\boxed{\textbf{3P}}$

DF 1



DF 2



d) $\boxed{\textbf{4P}}$

$$G_{min}(z)=\frac{\left[z-\left(\frac{1}{3}+\frac{1}{3}j\right)\right]\cdot\left[z-\left(\frac{1}{3}-\frac{1}{3}j\right)\right]\cdot\left[z-\frac{2}{3}\right]}{z[z-0{,}5j]\cdot[z+0{,}5j]}\cdot b_0 \qquad \boxed{z_{0,\nu}=\frac{1}{z^*_{\infty,\nu}}}$$

$$G_{AP}(z) = \frac{[z-(1{,}5+1{,}5j)]\cdot[z-(1{,}5-1{,}5j)]\cdot[z-1{,}5]}{\left[z-\left(\frac{1}{3}+\frac{1}{3}j\right)\right]\cdot\left[z-\left(\frac{1}{3}-\frac{1}{3}j\right)\right]\cdot\left[z-\frac{2}{3}\right]}$$

e) $\boxed{\textbf{2P}}$

über z-Diagramm: IIR-Filter → nicht linearphasig !

f) $\boxed{\textbf{2P}}$

nicht stabil, da kausales LTI-System mit Polstellen außerhalb des Einheitskreises!

g) $\boxed{\textbf{1P}}$

Hochpass, für niedrige Frequenzen überwiegt NS-Einfluss,
für höhere Polstellen-Einfluss