Bsp. $\mu = 4 \quad \sigma = 9$: $\quad X \sim N(4, 9^2) \qquad Z = \frac{X-\mu}{\sigma} = \frac{X-4}{9}$

$$P\{X \le 1\} = P\{Z \le \tfrac{1-4}{9}\} = P\{Z \le -\tfrac{1}{3}\}$$

$\approx -0.33\underline{3}$

$$\Phi(-0.33) = 1 - \Phi(0.33) = 1 - 0.6293 = 0.3707$$

Beweis: 1) $F_{\underline{Z}}(z) \underset{\text{Def.}}{=} P\{Z \le z\} \underset{\text{Def}}{=} P\{\underbrace{\tfrac{X-\mu}{\sigma}}_{=:Z} \le z\} \qquad | \cdot \sigma > 0 \;| + \mu$

$$= P\{X \le \sigma \cdot z + \mu\} = F_X(\sigma \cdot z + \mu) = \Phi\left(\frac{\sigma \cdot z + \mu - \mu}{\sigma}\right) = \underline{\underline{\Phi(z)}} \quad ∎$$

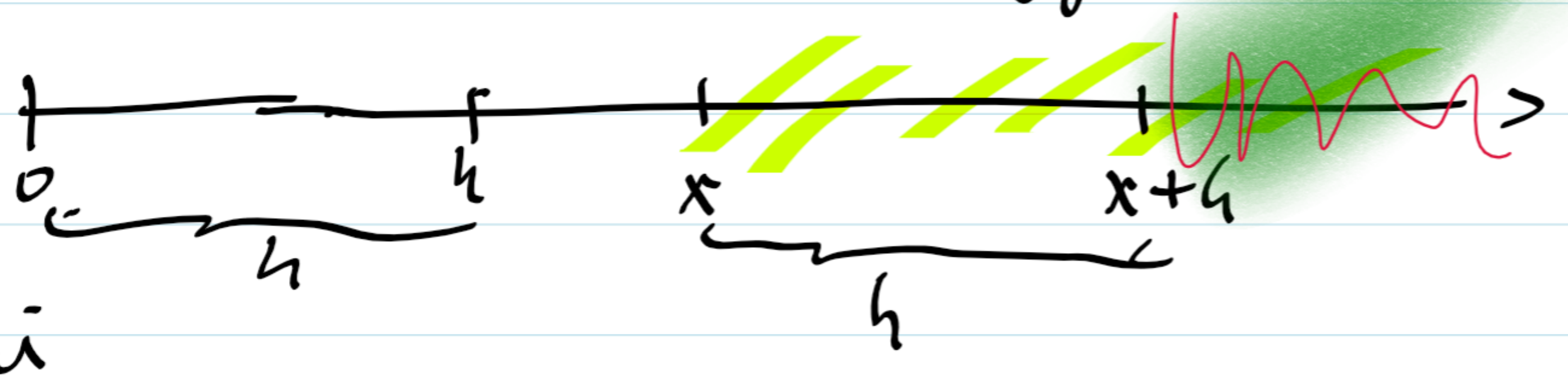
2) $F_{\underline{X}}(x) \underset{\text{Def.}}{=} P\{X \le x\} = P\{\underbrace{\sigma \cdot Z + \mu}_{=:X} \le x\} \qquad | - \mu \;| : \sigma > 0$

$$= P\{\underline{Z} \le \frac{x-\mu}{\sigma}\} \underset{\text{Def.}}{=} F_{\underline{Z}}\left(\frac{x-\mu}{\sigma}\right) = \underline{\underline{\Phi\left(\frac{x-\mu}{\sigma}\right)}} \quad ∎$$

Eigenschaft der Exponentialvert. (VF $F(x) = \begin{cases} 1 - e^{-\lambda x}, & \text{für } x \ge 0 \\ 0 & \text{für } x < 0 \end{cases}$)

Es gilt: $P\{X \ge x+h \mid X \ge x\} = P\{X \ge \underline{h}\}$

↑ gegeben, dass $\qquad$ unabhängig von x



Beweis

$$\underline{P\{X > x\}} = 1 - P\{X \le x\} = 1 - F(x) = 1 - (1 - e^{-\lambda x}) = \underline{\underline{e^{-\lambda x}}}$$

Wdh. $\quad P(A \mid B) \underset{\text{Def.}}{=} \frac{P(A \cap B)}{P(B)} \qquad A = \{X > x+h\}$

$B = \{X > x\}$

$$A \cap B = \{X > x+h\} \cap \{X > x\} = \{X > x+h\}$$

$$P\{X > x+h \mid X > x\} = \frac{P(\{X > x+h\} \cap \{X > x\})}{P\{X > x\}} = \frac{P\{X > x+h\}}{P\{X > x\}}$$

$$= \frac{e^{-\lambda(x+h)}}{e^{-\lambda x}} = e^{-\lambda h} = \underline{P\{X > h\}} \quad ∎$$

Alterungs- bzw. Gedächtnislosigkeit der Exponentialvert.