1 Elektrisches Feld

Punkte: 19

a) $C = \varepsilon_0 \varepsilon_r \frac{A}{d}$ (1)

$C_0 = \varepsilon_0 \frac{ab}{d_0}$ (1)

$\sum_a 2$

b) $C_1 = \varepsilon_0 \frac{ab}{d_0 - x}$ (1)

$C_2 = \varepsilon_0 \varepsilon_{rx} \frac{ab}{x}$ (1)

$C(x) = \frac{C_1 \cdot C_2}{C_1 + C_2}$ (1)

$C(x) = \frac{\varepsilon_0 \varepsilon_{rx} ab}{\varepsilon_{rx}(d_0 - x) + x}$ (1)

$\frac{C(x)}{C_0} = \frac{\varepsilon_{rx} d_0}{\varepsilon_{rx}(d_0 - x) + x}$ (1)

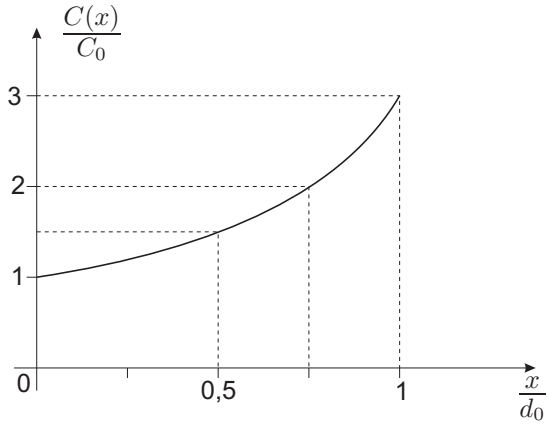
$C^* = \frac{\varepsilon_{rx}}{\varepsilon rx + (1 - \varepsilon_{rx})\frac{x}{d_0}} = \frac{1}{1 + (\frac{1}{\varepsilon_{rx}} - 1)\frac{x}{d_0}}$ (1)

$\sum_b 6$

c) $C^* = \frac{1}{1 - \frac{2}{3} \cdot \frac{x}{d_0}}$ (1)

$\frac{x}{d_0}$	0	$\frac{1}{2}$	$\frac{3}{4}$	1
C^*	1	1,5	2	3

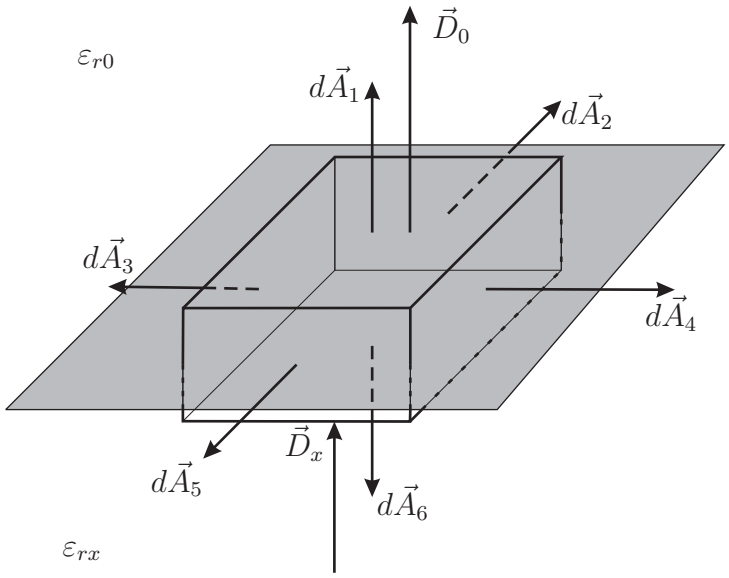
(1)



Skizze (2)

$\sum_c 4$

d)



(1)

$\oint_A \vec{D} d\vec{A} = Q$ (1)

Fl 1 : $d\vec{A}_1 \parallel \vec{D}_0,\ \angle(d\vec{A}_1, D_0) = 0 \quad \Rightarrow \quad \vec{D}_0 \cdot d\vec{A}_1 = D_0 dA_1$ (0,5)

Fl 2 − 5 : $d\vec{A}_i \perp \vec{D}_i \quad \Rightarrow \quad \vec{D}_i \cdot d\vec{A}_i = 0$ (0,5)

Fl 6 : $d\vec{A}_6 \parallel \vec{D}_x,\ \angle(d\vec{A}_1, D_0) = \pi \quad \Rightarrow \quad \vec{D}_x \cdot d\vec{A}_6 = -D_x dA_6$ (0,5)

$\oint_A \vec{D} d\vec{A} = 0$ (0,5)

$\int_{A1} \vec{D}_0 d\vec{A}_1 + \int_{A6} \vec{D}_x d\vec{A}_6 = D_0 A_1 - D_x A_6 = (D_0 - D_x) A_{1|6} = 0$ (0,5)

$D_0 = D_x$ (0,5)

$\vec{D} = \varepsilon \vec{E}$ (1)

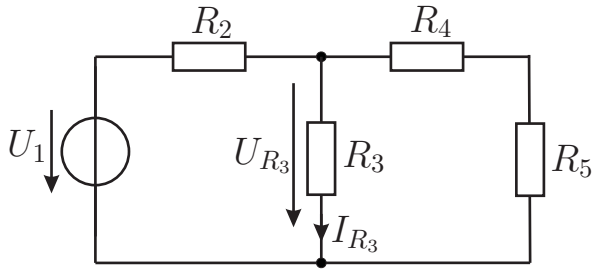
$\frac{E_x}{E_0} = \frac{\varepsilon_0 \varepsilon_{r0}}{\varepsilon_0 \varepsilon_{rx}} = \frac{1}{\varepsilon_{rx}}$ (1)

$\sum_d 7$

2 Gleichstromnetzwerk

Punkte: 11

a) I) Wirkung der Spannungsquelle U_1 betrachten. Stromquellen I_1 und I_2 passivieren.



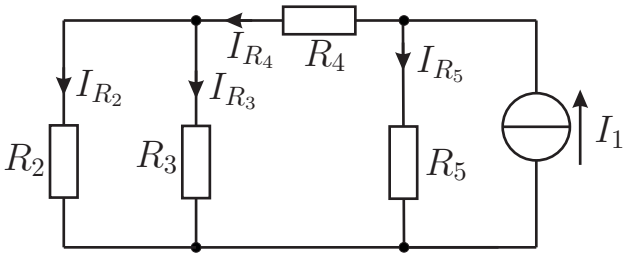
Skizze oder Ansatz (1)

$$R_{345} = \frac{R_3(R_4+R_5)}{R_3+R_4+R_5} \text{ (1)}$$

$$U_{R_3} = \frac{R_{345}}{R_{345}+R_2}U_1 = \frac{R_3(R_4+R_5)}{R_3(R_4+R_5)+R_2(R_3+R_4+R_5)}U_1 \text{ (1)}$$

$$I_{R_{3_I}} = \frac{U_{R_3}}{R_3} = \frac{R_4+R_5}{R_2R_3+(R_2+R_3)(R_4+R_5)}U_1 \text{ (1)}$$

II) Wirkung der Stromquelle I_1 betrachten. Spannungsquelle U_1 und Stromquelle I_2 passivieren.



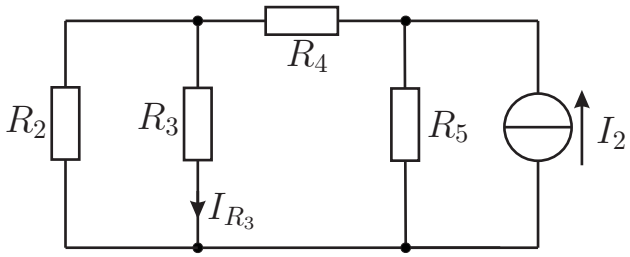
Skizze oder Ansatz (1)

$$R_{234} = R_4 + \frac{R_2R_3}{R_2+R_3} \text{ (1)}$$

$$I_{R_4} = \frac{R_5}{R_5+R_{234}}I_1 \text{ (1)}$$

$$I_{R_{3_{II}}} = \frac{R_2}{R_2+R_3}I_{R_4} = \frac{R_2R_5}{R_2R_3+(R_2+R_3)(R_4+R_5)}I_1 \text{ (1)}$$

III) Wirkung der Stromquelle I_2 betrachten. Spannungsquelle U_1 und Stromquelle I_1 passivieren.



Skizze oder Ansatz (1)

Anordnung identisch wie die bei Punkt II)

$$I_{R_{3_{III}}} = \frac{R_2 R_5}{R_2 R_3 + (R_2 + R_3)(R_4 + R_5)} I_2 \quad (1)$$

Superposition

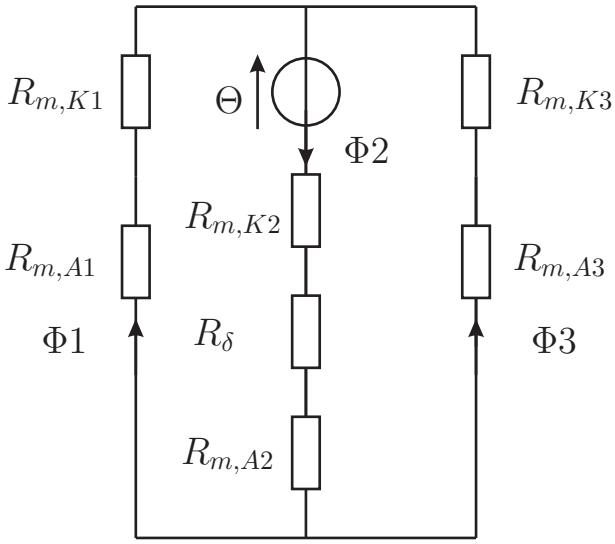
$$I_{R_3} = \frac{(R_4 + R_5)U_1 + R_2 R_5 (I_1 + I_2)}{R_2 R_3 + (R_2 + R_3)(R_4 + R_5)} \quad (1)$$

$\sum_a 11$

3 Magnetischer Kreis

Punkte: 20

a)



Skizze 1 Punkt

$$
\begin{aligned}
R_m &= \frac{l}{\mu A} \\
R_{m,A1} &= R_{m,A3} = \frac{l + h/2}{\mu_{r,gg}\mu_0 h^2} \\
R_{m,\delta} &= \frac{\delta}{\mu_0 h^2} \\
R_{m,K1} &= R_{m,K3} = \frac{2l}{\mu_{r,dyn}\mu_0 h^2} \\
R_{m,K2} &= \frac{l - \delta}{\mu_{r,dyn}\mu_0 h^2} \\
R_{m,A2} &= \frac{h/2}{\mu_{r,gg}\mu_0 h^2}
\end{aligned}
$$

Je Zeile 1 Punkt = 6 Punkte
Symmetrie 1 Punkt

$\sum_a 8$

b)

$$\left[\frac{2h^2}{\mu_0}\right]\left(\frac{Vs}{m^2}\right)^2 = \frac{m^2}{\frac{Vs}{Am}}\cdot\left(\frac{Vs}{m^2}\right)^2 = \frac{Am^3}{Vs}\cdot\frac{V^2s^2}{m^4} =$$
$$= \frac{AVs}{m} = \frac{Ws}{m} = \frac{Nm}{m} = N$$

Je Zeile 1 Punkt = 2 Punkte

$\sum_b 2$

c) gegeben:

$$F_L = \frac{B^2A}{2\mu_0}$$

$$B = \sqrt{\frac{2\mu_0 F_L}{A}}$$
$$\Phi_1 = \frac{1}{2}\Phi_2$$
$$\text{mit } B = \frac{\Phi}{A}$$
$$\text{Phi\_1} = \frac{1}{2}\sqrt{2\mu_0 F_L A} = \frac{1}{2}\sqrt{2\mu_0\frac{2h^2}{\mu_0}\left(\frac{Vs}{m^2}\right)^2 h^2} = h^2\frac{Wb}{m^2}$$

Je Zeile 1 Punkt = 4 Punkte

$\sum_c 4$

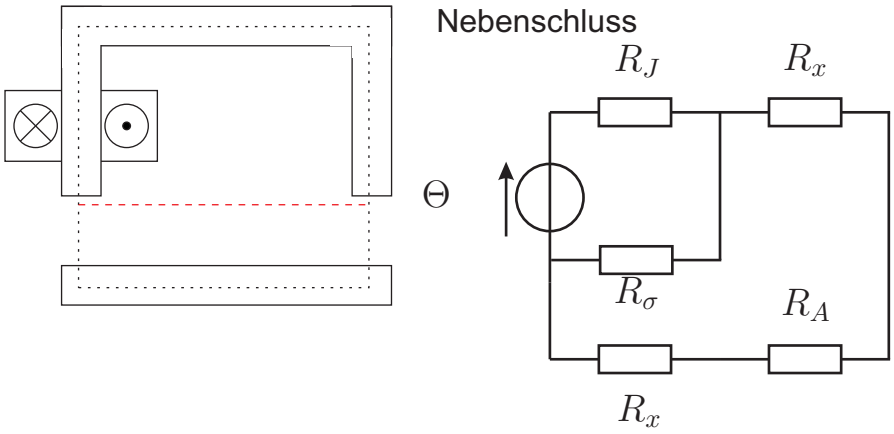
d)

$$V_m = \Phi_2 \cdot R_{m,\delta}$$
$$= 2\Phi_1 \cdot \frac{\delta}{\mu_0 \cdot h^2} = \frac{2h^2\delta}{\mu_0 h^2}\frac{Wb}{m^2} = \frac{2\delta}{\mu_0}\frac{Wb}{m^2}$$

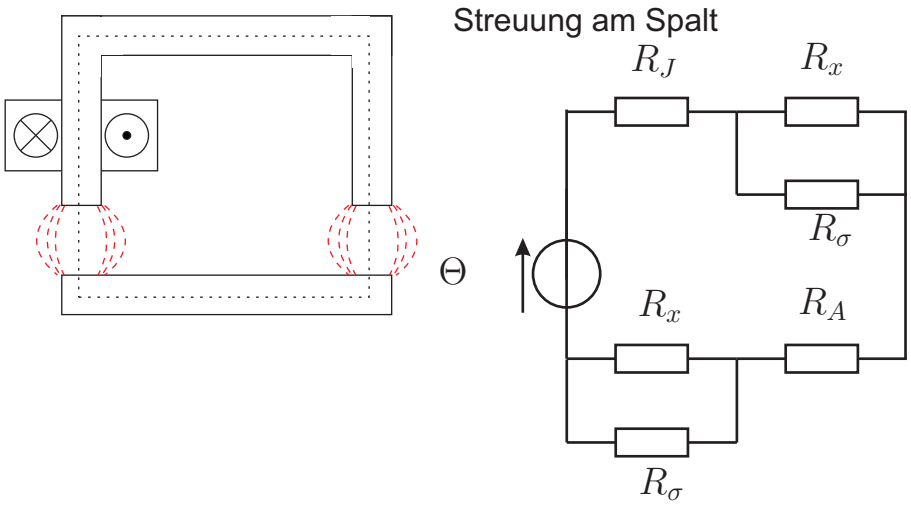
Je Zeile 1 Punkt = 2 Punkte

$\sum_d 2$

e)



und



Skizze und richtige Modellierung je 1 Punkt

$\sum_e 4$

4 Komplexe Wechselstromrechnung

Punkte: 30

a) $\omega = 0 \quad \Rightarrow \quad$ Spule - Kurzschluss, Kondensator - Leerlauf (1)

$\omega \Rightarrow \infty \quad \Rightarrow \quad$ Spule - Leerlauf, Kondensator - Kurzschluss (1)

$$R = \frac{U}{I} (1)$$

$$R_1 = \left.\frac{|\underline{U}_0|}{|\underline{I}_0|}\right|_{\omega=0} = \frac{30\,\mathrm{V}}{6\,\mathrm{A}} = 5\,\Omega \ (0,5)$$

$$R_2 = \left.\frac{|\underline{U}_0|}{|\underline{I}_0|}\right|_{\omega\to\infty} = \frac{30\,\mathrm{V}}{2\,\mathrm{A}} = 15\,\Omega \ (0,5)$$

$\sum_a 4$

b) $$\underline{Z}_{R_1L} = R_1 + j\omega L \ (1)$$

$$\underline{Z}_{R_2C} = R_2 - \frac{j}{\omega C} \ (1)$$

$$\underline{Z} = \frac{\underline{Z}_{R_1L} \cdot \underline{Z}_{R_2C}}{\underline{Z}_{R_1L} + \underline{Z}_{R_2C}} \ (0,5) = \frac{(R_1 + j\omega L)(R_2 - \frac{j}{\omega C})}{\underline{Z}_{R_1L} + R_2 - \frac{j}{\omega C}} \ (0,5)$$

$$\underline{Z} = \frac{R_1R_2 + \frac{L}{C} + j(\omega R_2 L - \frac{R_1}{\omega C})}{R_1 + R_2 + j(\omega L - \frac{1}{\omega C})} = \frac{\omega R_1 R_2 C + \omega L + j(\omega^2 R_2 LC - R_1)}{\omega(R_1 + R_2)C + j(\omega^2 LC - 1)} \ (1)$$

$\sum_b 4$

c) $$\underline{I}_2 = \frac{\underline{U}_{R_2}}{R_2} = \frac{45\,\mathrm{V}}{15\,\Omega} = 3\,\mathrm{A} \ (1)$$

$$\underline{U}_C = -j\frac{1}{\omega C}\underline{I}_2 = -j10\,\Omega \cdot (3\,\mathrm{A}) = -j30\,\mathrm{V} \ (1)$$

$$\underline{U}_0 = \underline{U}_C + \underline{U}_{R_2} = (-j30)\mathrm{V} + 45\,\mathrm{V} = (45 - j30)\,\mathrm{V} \ (1)$$

$$\underline{Z}_{R_1L} = R_1 + j\,(\omega L) = R_1 + j\left(250\,\frac{1}{\mathrm{s}} \cdot 20\,\mathrm{mH}\right) = 5 + j5\,\Omega \ (0{,}5)$$

$$\underline{I}_1 = \frac{\underline{U}_0}{\underline{Z}_{R_1L}} = \frac{(45 - j30)\,\mathrm{V}}{(5 + j5)\,\Omega} = 3\frac{(5 - j2)\,\mathrm{V}}{(1 + j)\,\Omega} = 3\frac{(3 - j2)(1 - j)}{(1 + j)(1 - j)}\,\mathrm{A}$$

$$= 3\frac{(1 - j5)}{2}\,\mathrm{A} = (1,5 - j7,5)\,\mathrm{A} \ (1{,}5)$$

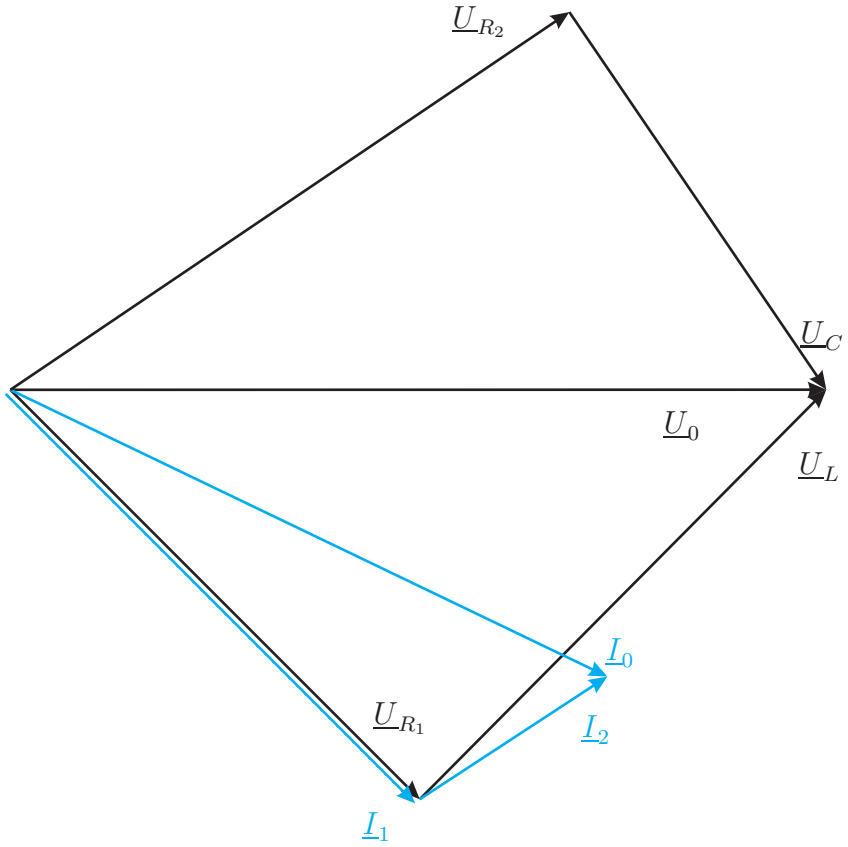
$$\underline{U}_L = j\omega L\underline{I}_1 = j5\,\Omega \cdot (1,5 - j7,5\,\mathrm{A}) = (37,5 + j7,5)\,\mathrm{V} \ (1)$$

$$\underline{U}_{R_1} = R_1\underline{I}_0 = 5\,\Omega \cdot (1,5 - j7,5)\mathrm{A} = (7,5 - j37,5)\mathrm{V} \ (1)$$

$$\underline{I}_0 = \underline{I}_1 + \underline{I}_2 = (4,5 - j7,5)\mathrm{A} \ (1)$$

$\sum_c 8$

d)



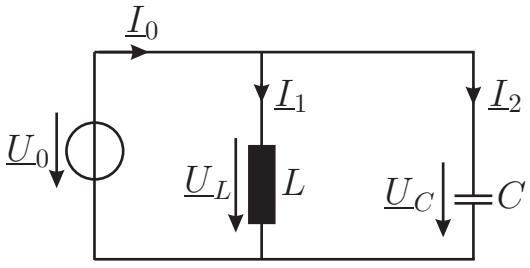
je Richtiger Zeiger 0.5 Punkte

$\sum_d 4$

e) $\varphi \approx 26^\circ$ (1)
induktives Verhalten (Spannung vor Strom) (1)

$\sum_e 2$

f)



$\sum_f 1$

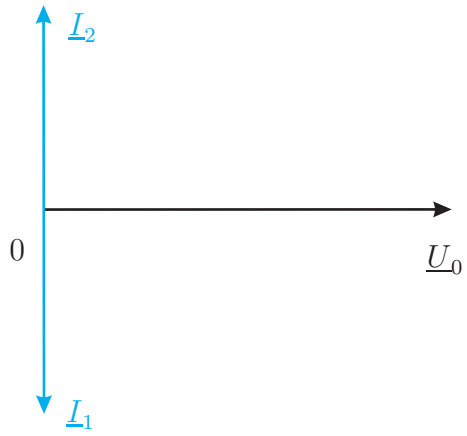
g) $\omega_0 = \dfrac{1}{\sqrt{LC}}$ (1)

$$L = \frac{1}{\omega_0^2 \cdot C} = \frac{1}{250 \cdot 250\frac{1}{\mathrm{s}^2} \cdot 400 \cdot 10^{-6}\,\mathrm{F}} = 40\,\mathrm{mH}\ (1)$$

$\sum_g 2$

h) $\underline{I}_1 = \dfrac{\underline{U}_0}{j\omega L} = \dfrac{54\,\mathrm{V}}{j250\frac{1}{\mathrm{s}}40\,\mathrm{mH}} = -j5,4\,\mathrm{A}$ (1)

$$\underline{I}_2 = \underline{U}_0 \cdot j\omega C = 54\,\mathrm{V} j250\frac{1}{\mathrm{s}}400\,\mu\mathrm{F} = j5,4\,\mathrm{A}\ (1)$$



Skizze 1 Punkt

$\sum_h 3$

i) $\underline{I}_0 = \underline{I}_1 + \underline{I}_2 = -j5,4\,\mathrm{A} + j5,4\,\mathrm{A} = 0$ (1)

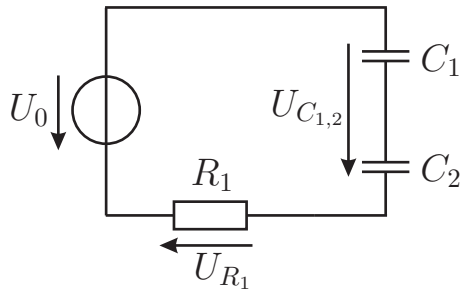
$$\underline{Z}_{LC} = \frac{\underline{U}_0}{\underline{I}_0} = \frac{54\,\mathrm{V}}{0\,\mathrm{A}} \to \infty\ (1)$$

$\sum_i 2$

5 Kondensatornetzwerk

Punkte: 20

a)



$\sum_a 1$

b)

$$U_{C_{1,2}} = U_0 - U_{R_1} \ (1) = 15\,\text{V} - 10\,\text{V} = 5\,\text{V} \ (0,5)$$

$$C_{GES} = \frac{C_1 \cdot C_2}{C_1 + C_2} \ (1) \ = \frac{6\,\mu\text{F} \cdot 3\,\mu\text{F}}{6\,\mu\text{F} + 3\,\mu\text{F}} = 2\,\mu\text{F} \ (0,5)$$

$$Q_{GES} = C_{GES} \cdot U_{C_{1,2}} \ (1) \ = 2\,\mu\text{F} \cdot 5\,\text{V} = 10\,\mu\text{C} \ (0,5)$$

$$W_{GES} = \frac{1}{2} Q_{GES} \cdot U_{C_{1,2}} \ (1) \ = \frac{1}{2} \cdot 10\,\mu\text{C} \cdot 5\,\text{V} = 25\,\mu\text{Ws} \ (0,5)$$

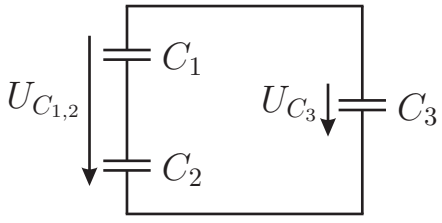
$$Q_{C_1} = Q_{C_2} = Q_{GES} = 10\,\mu\text{C} \ (1)$$

$$U_{C_1} = \frac{Q_{C_1}}{C_1} = \frac{10\,\mu\text{C}}{6\,\mu\text{F}} = \frac{5}{3}\,\text{V} \ (0,5)$$

$$U_{C_2} = \frac{Q_{C_2}}{C_2} = \frac{10\,\mu\text{C}}{3\,\mu\text{F}} = \frac{10}{3}\,\text{V} \ (0,5)$$

$\sum_b 8$

c)



$\sum_c 1$

d) Ladungserhaltung. Umverteilung der Ladung auf alle Kondensatoren.

$\sum_d 1$

e)

$$C^*_{GES} = C_{GES} + C_3 = 2\,\mu\text{F} + 3\,\mu\text{F} = 5\,\mu\text{F}\ (1)$$

$\sum_e 1$

f)

$$Q_{GES} = Q^*_{GES}\ \text{(Ladungserhaltung)}$$

$$U^*_{C_3} = \frac{Q^*_{GES}}{C^*_{GES}} = \frac{10\,\mu\text{C}}{5\,\mu\text{F}} = 2\,\text{V}\ (1)$$

$$U^*_{C_1} = \frac{C_2}{C_1 + C_2} U^*_{C_{1,2}}\ (1)\ = \frac{C_2}{C_1 + C_2} U^*_{C_3}\ (0,5)\ = \frac{3\,\mu\text{F}}{6\,\mu\text{F} + 3\,\mu\text{F}} \cdot 2\,\text{V} = \frac{2}{3}\,\text{V}\ (0,5)$$

$$U^*_{C_2} = \frac{C_1}{C_1 + C_2} U^*_{C_3}\ (0,5)\ = \frac{6\,\mu\text{F}}{6\,\mu\text{F} + 3\,\mu\text{F}} \cdot 2\,\text{V} = \frac{4}{3}\,\text{V}\ (0,5)$$

$\sum_f 4$

g)

$$W^*_{GES} = \frac{1}{2} Q^*_{GES} \cdot U^*_{C_3} = \frac{1}{2} \cdot 10\,\mu\text{C} \cdot 2\,\text{V} = 10\,\mu\text{Ws}\ (1)$$

Energieverlust durch Hochfrequenz-Strahlung beim Schließen des Schalters S_2 (1)

$\sum_g 2$

h)

$$C^*_{GES} = \frac{Q^*_{GES}}{U^*_{C_3}} = \frac{10\,\mu\text{C}}{1\,\text{V}} = 10\,\mu\text{F}\ (1)$$

$$C_3 = C^*_{GES} - C_{GES} = 10\,\mu\text{F} - 2\,\mu\text{F} = 8\,\mu\text{F}\ (1)$$

$\sum_h 2$