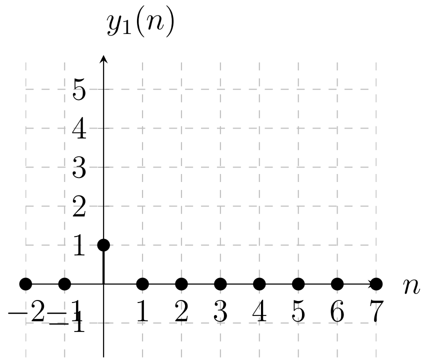
Musterlösung zur Klausur
"Digitale Signalverarbeitung"
vom 18.09.2014

Aufgabe 1: Analyse eines LTI-Systems

(14 Punkte gesamt)

a.) (2 Punkte)
$y_1(n) = \delta(n)$

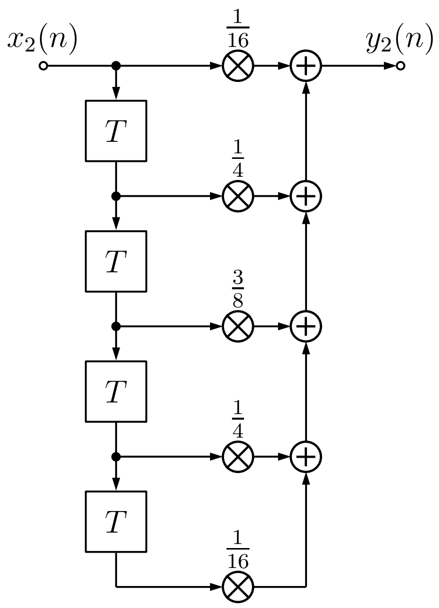


b.) (2 Punkte)

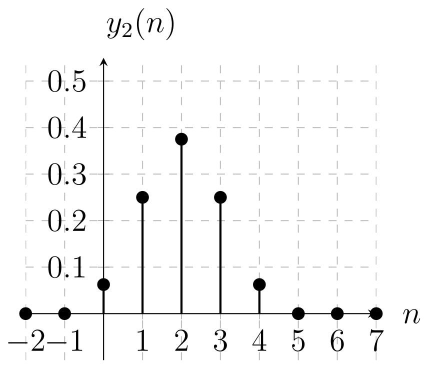
$$H_2(e^{j\Omega}) = \frac{1}{16}\left[1 + 4e^{-j\Omega} + 6e^{-j2\Omega} + 4e^{-j3\Omega} + 1e^{-j4\Omega}\right]$$
$$H_2(z) = \frac{1}{16} + \frac{1}{4}z^{-1} + \frac{3}{8}z^{-2} + \frac{1}{4}z^{-3} + \frac{1}{16}z^{-4}$$

$$\begin{aligned} \Rightarrow y_2(n) &= \frac{1}{16}\left[x_2(n) + 4x_2(n-1) + 6x_2(n-2) + 4x_2(n-3) + x_2(n-4)\right] \\ &= \frac{1}{16}x_2(n) + \frac{1}{4}x_2(n-1) + \frac{3}{8}x_2(n-2) + \frac{1}{4}x_2(n-3) + \frac{1}{16}x_2(n-4) \end{aligned}$$

c.) (2 Punkte)



d.) (2 Punkte)



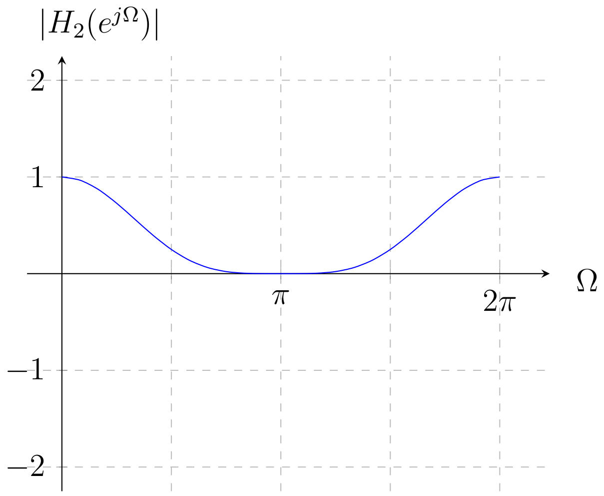
e.) (1 Punkt)
FIR, da endliche Impulsantwortlänge.
Typ I, N_b gerade, spiegelsymmetrisch.

f.) (3 Punkte)

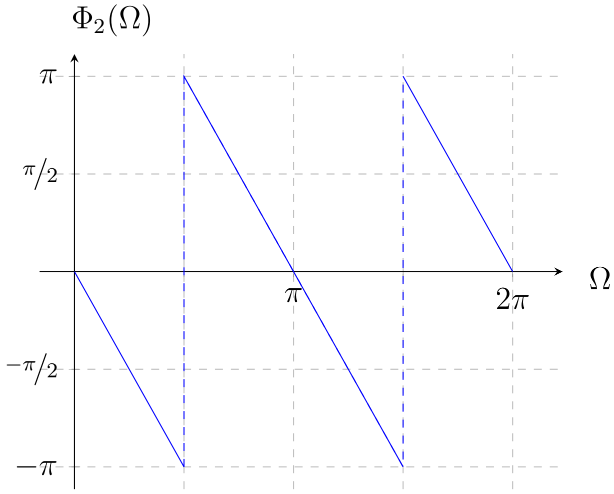
$$
\begin{aligned}
H_2(e^{j\Omega}) &= \frac{1}{16}(1+e^{-j\Omega})^4 \\
&= \left(\underbrace{\frac{1}{2}(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}})}_{\cos(\frac{\Omega}{2})} e^{-j\frac{\Omega}{2}} \right)^4 \\
&= \cos^4(\frac{\Omega}{2}) \cdot e^{-j2\Omega}
\end{aligned}
$$

$$\Rightarrow |H_2(e^{j\Omega})| = |\cos^4(\frac{\Omega}{2})|$$

$$\Rightarrow \Phi_2(\Omega) = -2\Omega$$



$|H_2(e^{j\Omega})|$
2
1
−1
−2
π
2π
Ω



$\Phi_2(\Omega)$
π
$\pi/2$
$-\pi/2$
$-\pi$
π
2π
Ω

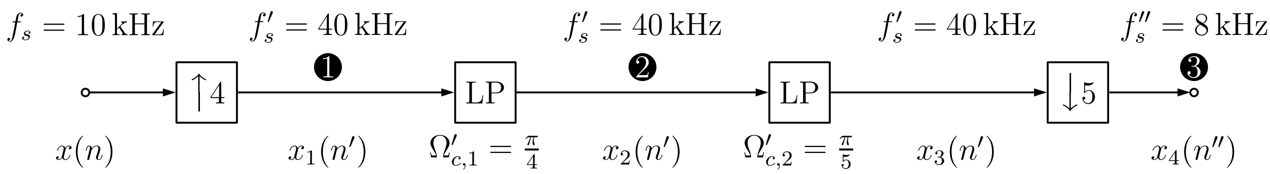
g.) (2 Punkte)
Tiefpass, siehe Amplitudengang
Linearphasig, siehe Phasengang

Aufgabe 2: Abtastratenwandlung

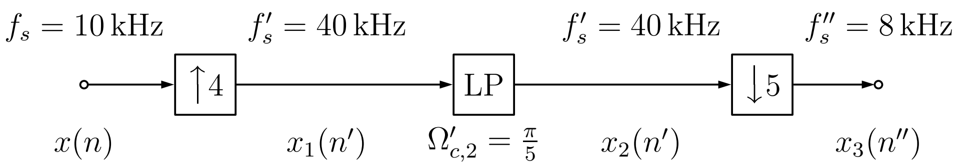
(12 Punkte gesamt)

a.) (1 Punkt)
$r = \frac{8\,\text{kHz}}{10\,\text{kHz}} = \frac{4}{5}$

b.) (2 Punkte)

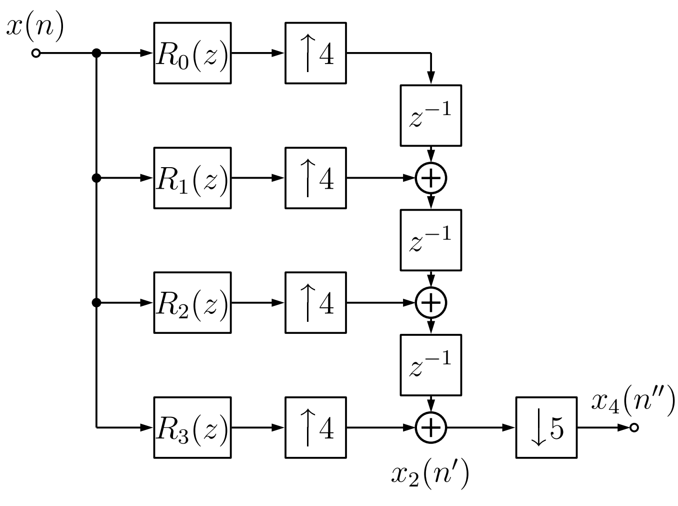


c.) (1 Punkte)

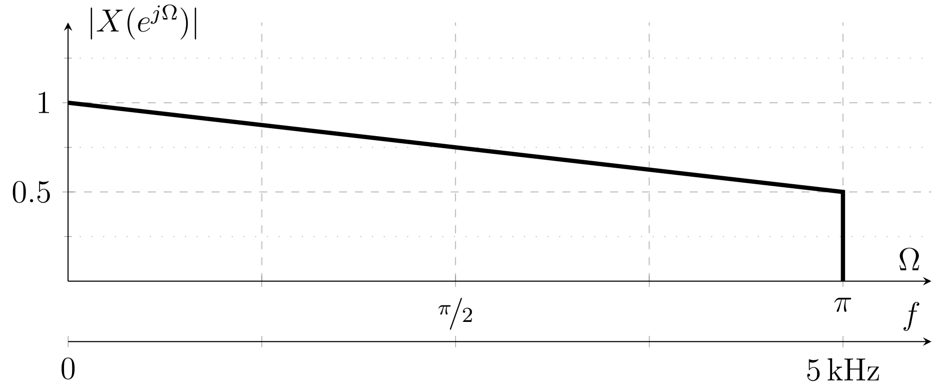


d.) (4 Punkte)
(siehe nächste Seite)

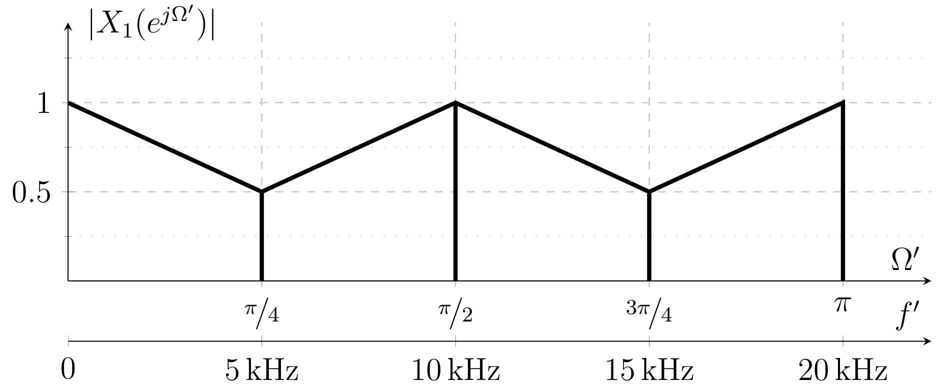
e.) (3 Punkte)



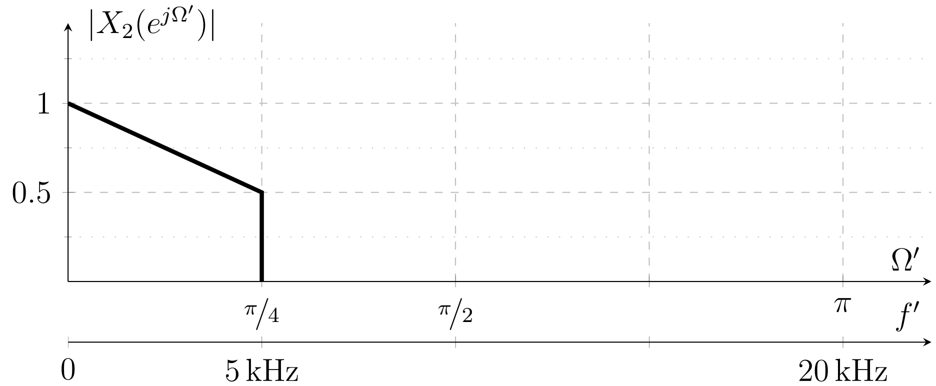
f.) (1 Punkte)
Reduktion der MAC's durch Polyphasendarstellung auf ca. $1/p = 1/4 = 25\%$, d.h. Reduktion um ca. 75%.


$|X(e^{j\Omega})|$
1
0.5
Ω
$\pi/2$
π
f
0
5 kHz

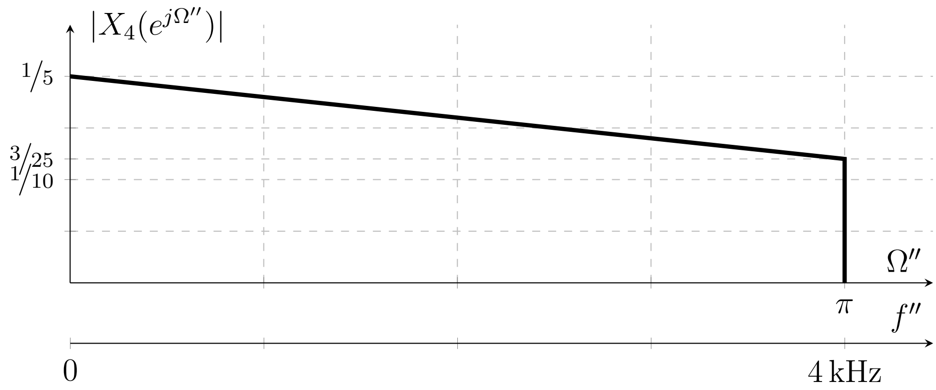
❶


$|X_1(e^{j\Omega'})|$
1
0.5
Ω'
$\pi/4$
$\pi/2$
$3\pi/4$
π
f'
0
5 kHz
10 kHz
15 kHz
20 kHz

❷


$|X_2(e^{j\Omega'})|$
1
0.5
Ω'
$\pi/4$
$\pi/2$
π
f'
0
5 kHz
20 kHz

❸


$|X_4(e^{j\Omega''})|$
$1/5$
$3/25$
$1/10$
Ω''
π
f''
0
4 kHz

Aufgabe 3: Filterentwurf

(11 Punkte gesamt)

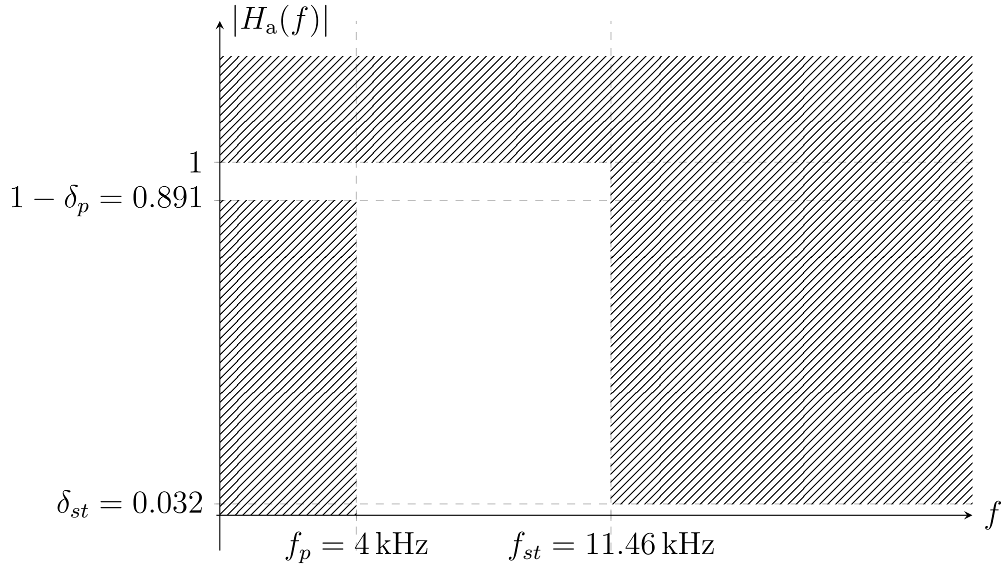
a.) (1 Punkt)
zeitkontinuierliches Butterworth IIR-Tiefpassfilter (rekursiv, zeitkontinuierlich, Unterdrückung hoher Frequenzen, flaches Amplitudenspektrum bei $\omega = 0$, monoton fallendes Amplitudenspektrum)

b.) (3 Punkte)

$$\begin{aligned} R_p = 1\,\mathrm{dB} \Rightarrow \delta_p &= 1 - 10^{-1\,\mathrm{dB}/20\,\mathrm{dB}} = 0{,}109 \\ \Rightarrow 1 - \delta_p &= \underline{0{,}891} \\ d_{st} = 30\,\mathrm{dB} \Rightarrow \delta_{st} &= 10^{-d_{st}/20\,\mathrm{dB}} = \underline{0{,}032} \\ \omega_p = \Omega_p \cdot f_S \Rightarrow f_p &= \frac{\Omega_p}{2\pi} \cdot f_s = \frac{\pi/6}{2\pi} \cdot 48\,\mathrm{kHz} = \underline{4\,\mathrm{kHz}} \end{aligned}$$

Bilineare Transformation: $\Omega' = \Omega_p$

$$\begin{aligned} v &= \frac{\omega'}{\tan(\frac{\Omega'}{2})} = \frac{\Omega_p \cdot f_s}{\tan(\frac{\Omega_p}{2})} = \frac{\pi/6 \cdot 48\,\mathrm{kHz}}{\tan(\frac{\pi}{12})} = \underline{93797\,\mathrm{s}^{-1}} \\ \omega_{st} = v\tan(\frac{\Omega_{st}}{2}) \Rightarrow f_{st} &= \frac{v \cdot \tan(\frac{\pi/6+\pi/4}{2})}{2\pi} = 11455\,\mathrm{Hz} = \underline{11{,}46\,\mathrm{kHz}} \end{aligned}$$



c.) (2 Punkte)

$$\left| \begin{aligned} |H_a(j\omega_p)|^2 &= \frac{1}{1 + \left(\frac{\omega_p}{\omega_c}\right)^{2N}} = (1 - \delta_p)^2 = 0{,}79433 \\ |H_a(j\omega_{st})|^2 &= \frac{1}{1 + \left(\frac{\omega_{st}}{\omega_c}\right)^{2N}} = (\delta_{st})^2 = 1{,}024 \cdot 10^{-3} \end{aligned} \right|$$

$$\left| \begin{array}{l} \left(\frac{\omega_p}{\omega_c}\right)^N = 0{,}50885 \\ \left(\frac{\omega_{st}}{\omega_c}\right)^N = 31{,}2340 \end{array} \right|$$

$\Rightarrow \left(\frac{\omega_{st}}{\omega_p}\right)^N \geq \frac{31{,}2340}{0{,}50885}$

$\Rightarrow N \cdot \log\left(\frac{11{,}46}{4}\right) \geq \log(61{,}3815)$

$\Rightarrow N \geq 3{,}9115$

$\Rightarrow \underline{N = 4}$

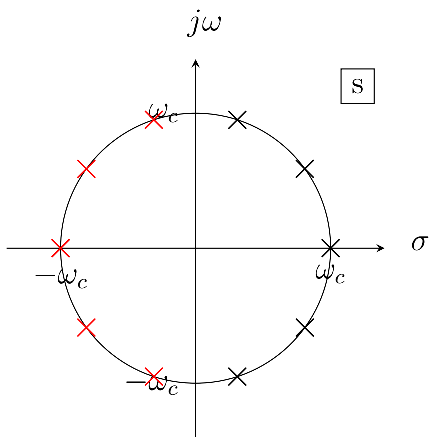
d.) (2 Punkte)

$$\left(\frac{\omega_p}{\omega_c}\right)^N = 0{,}50885 \Rightarrow \omega_c = 28768{,}81039\,\mathrm{s}^{-1} \Rightarrow f_c = \frac{\omega_c}{2\pi} = \underline{4578{,}70\,\mathrm{Hz}}$$

Die Dämpfung beträgt ca. 3 dB.

e.) (3 Punkte)

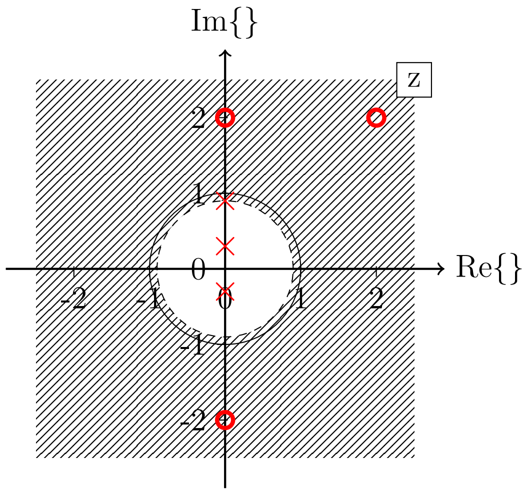
$$\begin{aligned}
s_{\infty,\nu} &= \omega_c \cdot e^{j(\pi/2N + \pi/2 + \nu \cdot \pi/N)}, \text{ mit } \nu = 0,1,\ldots,4 \\
s_{\infty,0} &= \omega_c \cdot e^{j\frac{6\pi}{10}} = 28769\,\mathrm{s}^{-1} \cdot e^{j\frac{3\pi}{5}} \\
s_{\infty,1} &= \omega_c \cdot e^{j\frac{8\pi}{10}} = 28769\,\mathrm{s}^{-1} \cdot e^{j\frac{4\pi}{5}} \\
s_{\infty,2} &= \omega_c \cdot e^{j\frac{10\pi}{10}} = 28769\,\mathrm{s}^{-1} \cdot e^{j\pi} \\
s_{\infty,3} &= \omega_c \cdot e^{j\frac{2\pi}{10}} = 28769\,\mathrm{s}^{-1} \cdot e^{j\frac{\pi}{5}} \\
s_{\infty,4} &= \omega_c \cdot e^{j\frac{4\pi}{10}} = 28769\,\mathrm{s}^{-1} \cdot e^{j\frac{2\pi}{5}}
\end{aligned}$$



Aufgabe 4: Zerlegung eines LTI-Systems

(13 Punkte gesamt)

a.) (3 Punkte)



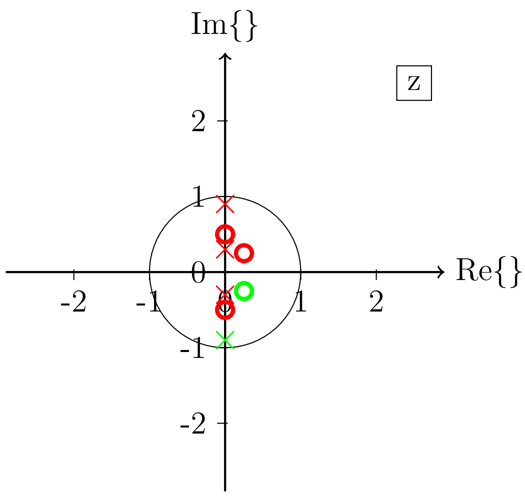
$$z_{0,1} = 2j \qquad z_{\infty,1} = 0{,}3j$$
$$z_{0,2} = -2j \qquad z_{\infty,2} = -0{,}3j \qquad ROC: |z| > 0{,}9$$
$$z_{0,3} = 2+2j \qquad z_{\infty,3} = 0{,}9j$$

b.) (2 Punkte)
ja, da alle Polstellen innerhalb des Einheitskreises!

c.) (3 Punkte)

$$H_{\min}(z) = \frac{(1-0{,}5jz^{-1})(1+0{,}5jz^{-1})(1-(1/4+1/4j)z^{-1})}{(1-0{,}3jz^{-1})(1+0{,}3jz^{-1})(1-0{,}9jz^{-1})} \cdot b_0$$
$$H_{\mathrm{AP}}(z) = \frac{(1-2jz^{-1})(1+2jz^{-1})(1-(2+2j)z^{-1})}{(1-0{,}5jz^{-1})(1+0{,}5jz^{-1})(1-(1/4+1/4j)z^{-1})} \cdot 1/b_0$$

d.) (3 Punkte)



$$
\begin{aligned}
z_{0,1} &= 0{,}5j & z_{\infty,1} &= 0{,}3j \\
z_{0,2} &= -0{,}5j & z_{\infty,2} &= -0{,}3j \\
z_{0,3} &= 0{,}25 + 0{,}25j & z_{\infty,3} &= 0{,}9j \\
z_{0,4} &= 0{,}25 - 0{,}25j & z_{\infty,4} &= -0{,}9j
\end{aligned}
$$

e.) (2 Punkte)

$$
\begin{aligned}
1) ROC: &\quad 0{,}3 < |z| < 0{,}9 \\
2) ROC: &\quad |z| < 0{,}3
\end{aligned}
$$