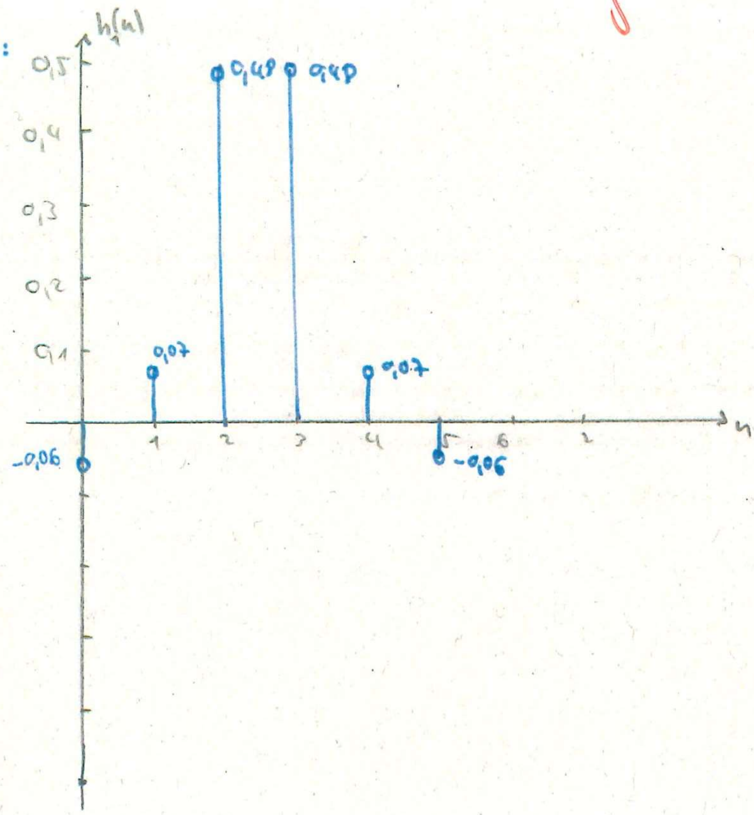
Musterlösung

Aufgabe 1:

a)



1

b) N_b odd, $h(n) = h(N_b - n)$ even $\longrightarrow$ Typ $\underline{\text{II}}$ 1

c) $N_{x_1} = 3 \quad N_h = 6 \qquad N_{y_1} = 6 + 3 - 1 = 8$ 1

d) 6 MACS + 8000 1/s = 48.000 MACS/s

e)

$\omega_{sL} = 2\pi \cdot f_{sL} = 21991{,}15\ s^{-1}$ 1

$\Omega_{sL} = \frac{7}{8}\pi$

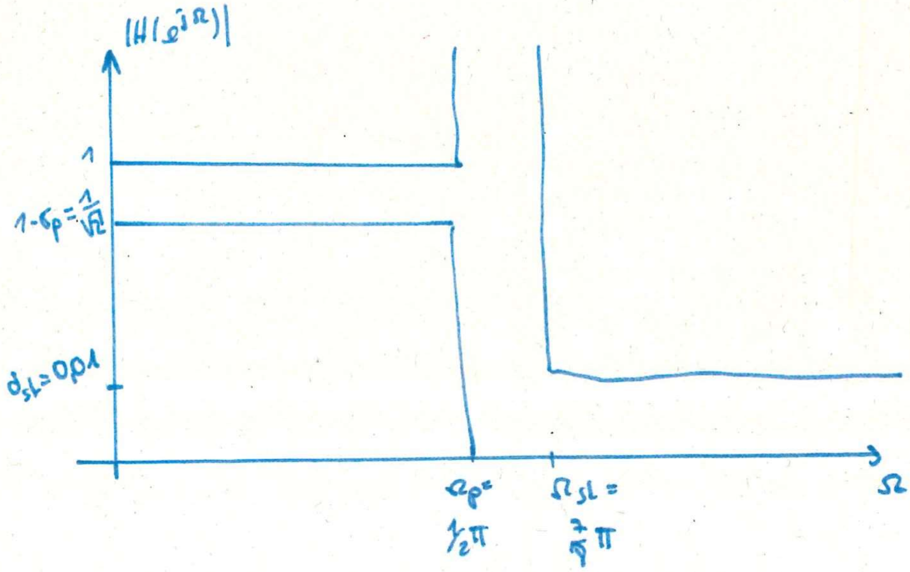
$\Omega_p = \Omega_{sL} - \Delta\Omega = \frac{7}{8}\pi - \frac{3}{8}\pi = \frac{1}{2}\pi$

$v = \frac{\omega'}{\tan\left(\frac{\Omega'}{2}\right)} = \frac{\omega_{sL}}{\tan\left(\frac{\Omega_{sL}}{2}\right)} = 4374{,}31\ s^{-1}$

$\omega_p = v \cdot \tan\left(\frac{\Omega_p}{2}\right) = 4374{,}31\ s^{-1} \cdot \tan(0{,}25\pi)$

$= 4374{,}31\ s^{-1}$ 1,5

f)



2

f) $20 \cdot \log(|H_a(j\omega_{st})| \leq -d_{st}$ $\;-40dB$

$\Leftrightarrow N \geq \dfrac{\log(10^{\frac{d_{st}}{10}} - 1)}{\log\left(\left(\frac{\omega_{st}}{\omega_p}\right)^2\right)} \approx 2{,}85$ 1,5

$N = 3$ 1

~~g) IIR: kleinere Gruppenlaufzeit ⊕, dafür frequenzabhängig ⊖~~

~~FIR: größere –"– ⊖, dafür frequenzunabhängig ⊕~~

~~IIR Filter benötigen eine geringere Filterordnung um die gleiche Spec zu erfüllen wie FIR Filter.~~

h) 7 MACS + 8000 1/s = 56.000 MACS/s

g) $|H_a(j\omega)|^2 = \dfrac{1}{1 + \left(\frac{\omega_{st}}{\omega_p}\right)^{2N}} = d_{st}^2$

$\Rightarrow 1 + \left(\frac{\omega_{st}}{\omega_p}\right)^{2N} = \dfrac{1}{d_{st}^2} = 10.000$

$\left(\frac{\omega_{st}}{\omega_p}\right)^{2N} = 9999 \quad |\sqrt{\ }\ |\log$

$N \cdot \log\left(\frac{\omega_{st}}{\omega_p}\right) = \log(99{,}99)$

$N \geq \dfrac{\log(99{,}99)}{\log\left(\frac{2{,}1991}{1{,}3617}\right)} = 2{,}85 \rightarrow N = 3$

Aufgabe 2) a)

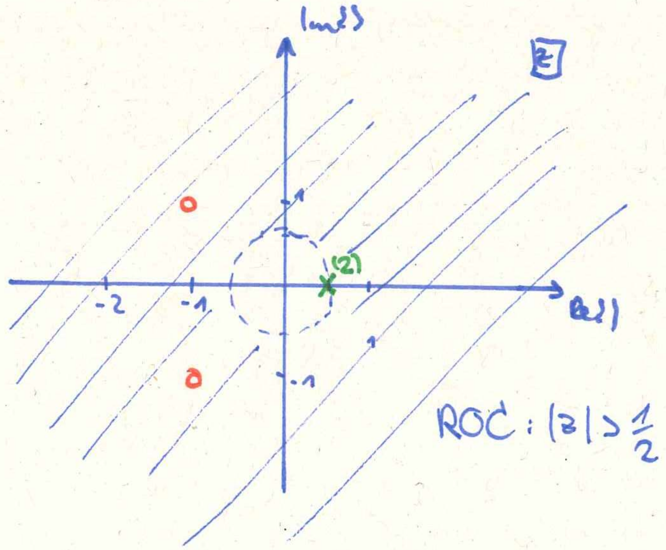
$$h_1(n) \circ\!\!-\!\!\bullet\, H_1(z) = \sum_{n=0}^{\infty} 8\,\delta(n)\, z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n \cdot (-7+13n)\, z^{-n}$$

$$= 8 - 7\,\frac{z}{z-\frac{1}{2}} + 13\,\frac{1/2 \cdot z}{(z-\frac{1}{2})^2} = \frac{z^2+2z+2}{(z-\frac{1}{2})^2}$$

NSt: p-q Formel $z_{1,2} = -1 \pm j$

Polst: $(z-\frac{1}{2})^2 \overset{!}{=} 0 \rightarrow z_{\infty,1,2} = 0{,}5$

b)



c)

d) Ja, da EHK im ROC liegt.

e) IIR-Filter. Wir haben eine unendliche Impulsantwort. Nenner von $H_1(z) \neq 1$.

f) Es liegen Nullstellen außerhalb des EHK, also ist das System nicht minimalphasig.

g) System stabil, kann also zerlegt werden.

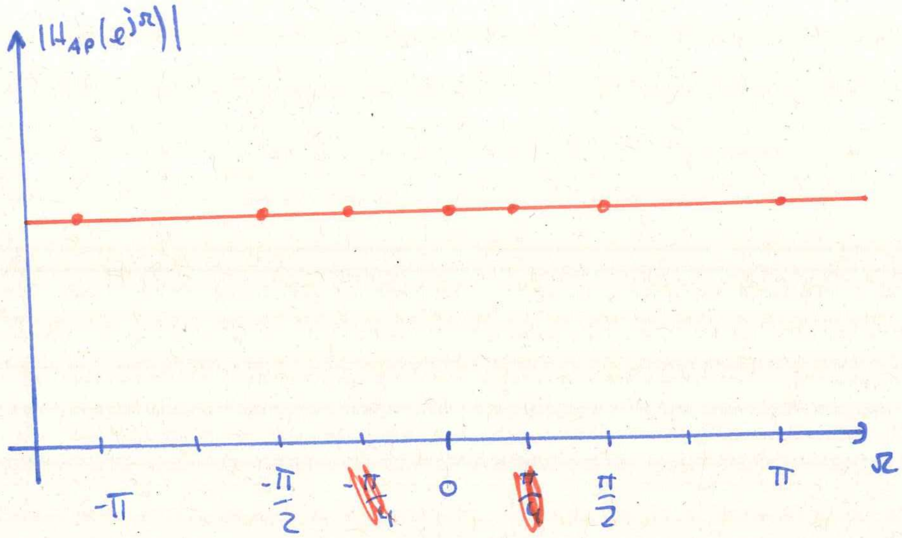
$$H_2(z) = H_{min}(z) \cdot H_{Ap}(z)$$

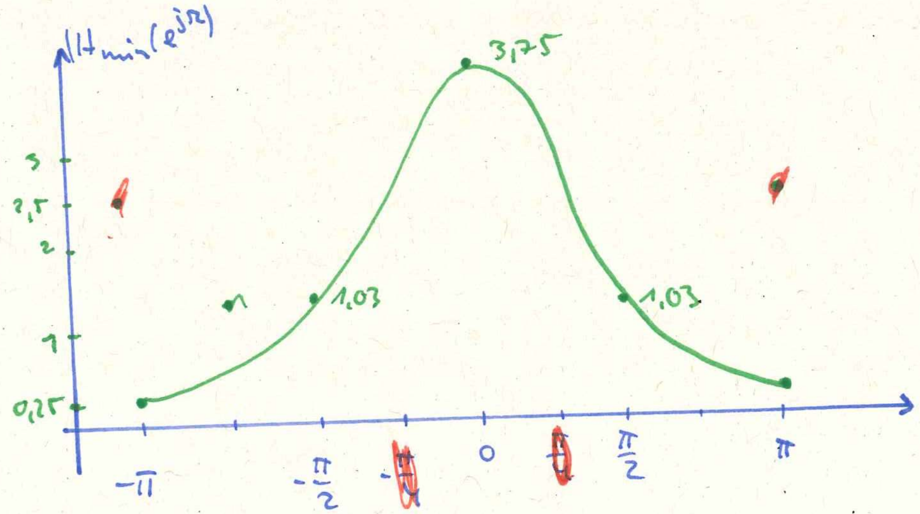
$$H_{min} = \frac{(z+1/4)(z+0{,}5)\cancel{(z+2/3)}}{z(z-0{,}5)} \cdot \frac{1}{b_0} \qquad H_{Ap}(z) = \frac{(z+4)\cancel{(z+1{,}5)}}{(z+1/4)\cancel{(z+2/3)}} \cdot b_0$$

$$\cancel{b \Rightarrow 1 \overset{!}{=} \frac{(1+4)(1+1{,}5)}{(1+\frac{1}{4})(1+\frac{2}{3})} \cdot b_0 = \frac{5 \cdot (1+0{,}5)}{1{,}25 \cdot 5/3} = 6 \cdot b_0 \Rightarrow b_0 = \frac{1}{6}}$$

$$b_0 \cdot \frac{5}{\frac{5}{4}} \overset{!}{=} 1 \Rightarrow b_0 = \frac{1}{4}$$

h)





$$|H_{min}(e^{j0})| = \frac{(1+\frac{1}{4})(1+\frac{1}{2})}{(1-\frac{1}{2})} = \frac{15/8}{1/2} = 3{,}75$$

$$|H_{min}(e^{j\pi})| = \frac{(-1+\frac{1}{4})(-1+\frac{1}{2})}{-1(-1-\frac{1}{2})} = \frac{3/8}{3/2} = \frac{1}{4} = 0{,}25 = |H_{min}(e^{j-\pi})|$$

$$|H_{min}(e^{j\frac{\pi}{2}})| = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{19}{20}\right)^2} = 1{,}03 = |H_{min}(e^{j-\frac{\pi}{2}})|$$

$$\uparrow$$

$$\sqrt{Re^2 + Im^2}$$

3. a) $N_\omega \cdot \Omega_0 = i \cdot 2\pi \Leftrightarrow N_\omega \cdot \frac{\cancel{2\pi}}{4} \stackrel{!}{=} i \cdot \cancel{2\pi} \Leftrightarrow N_\omega \stackrel{!}{=} 4 \cdot i \;,\; i \in N$

(1)

$= 4, 8, 12, \dots$

b) Parseval: $\sum_{n=0}^{7} |x(n)|^2 = \frac{1}{K} \sum_{k=0}^{7} |X_1(k)|^2 \Leftrightarrow 4 = \frac{1}{8} \cdot 2 \cdot a^2 \Leftrightarrow 16 = a^2 \Rightarrow a = 4$

(1.5)

c) 1) $w_8 = e^{-j\frac{2\pi}{8}} = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \Rightarrow w_8^0 = 1 \qquad w_8^2 = -j$

(2)

$w_8^1 = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j \qquad w_8^3 = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} j$

2) $\mathbf{x}_2 = [1,0,0,0,0,0,0,0] \Rightarrow \mathbf{X}_2 = [1,1,1,1,1,1,1,1]$

(1.5)

d) Unterschiedliche Signallänge betrachtet, keine ganze Periode => spectral leakage

(1)

e) windowing (nicht Rechteck!), z.B. Blackman o.ä.

(1)

4. a) $\frac{800.000}{16}$ samples = 50.000 samples $\Rightarrow$ $\frac{50.000 \text{ samples}}{5s} = 10\,\text{kHz}$

1

b) $f_s' = f_s \cdot 3 = 30\,\text{kHz}$

1,5

c) $r = \frac{p}{q} = \frac{3}{3}$

1

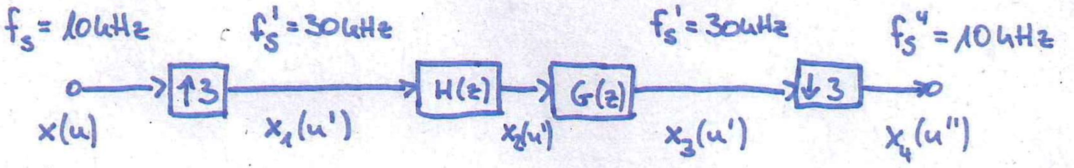
d) $H(z) = z^{-2}$

1,5

e) $f_{c,q} = 5\,\text{kHz}$

1

f)



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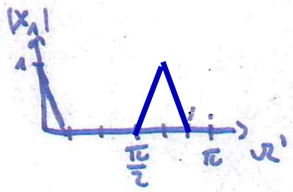
g)



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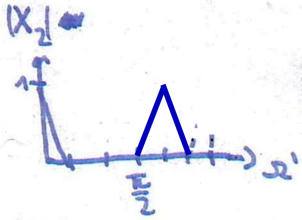

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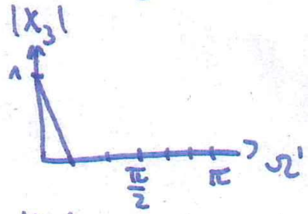

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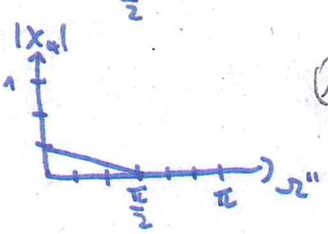

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