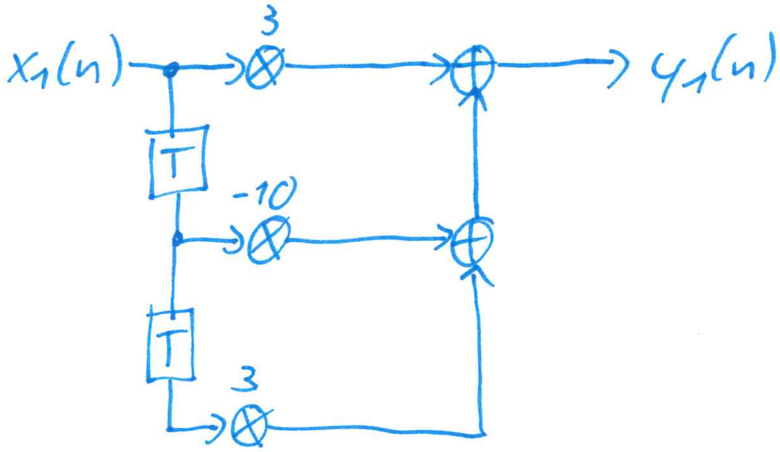
Aufgabe 1 (16P)

a) FIR, da nicht rekursiv (1P)

b) (1P)



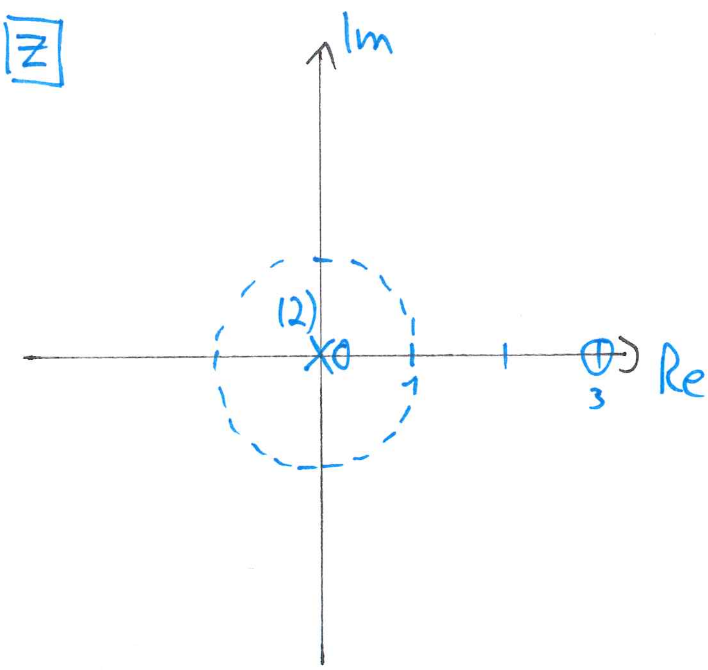
c) $y_1(n) = 3x_1(n-2) - 10\,x_1(n-1) + 3x_1(n)$

(1P) $Y_1(z) = 3 \cdot X_1(z)\,[\,z^{-2} - \frac{10}{3} z^{-1} + 1\,]$

$$H_1(z) = \frac{Y_1(z)}{X_1(z)} = 3 \cdot [\,z^{-2} - \frac{10}{3} z^{-1} + 1\,]$$

d) $z_{0,1} = \{ \frac{1}{3}, 3 \}$; $z_{\infty_{0,1}} = 0$

(4P)

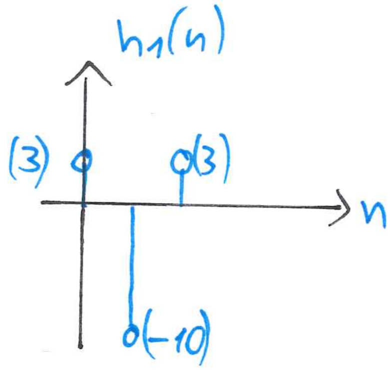


e) 1P $|z| > 0$, rechtsseitige Impulsantwort

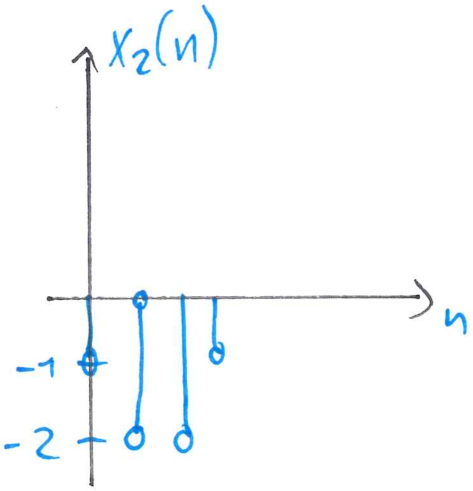
f) 3P
- **kausal**, da zur Berechnung von $y_1(n)$ keine zukünftigen Werte von $\{x_1\}$ benötigt werden
- **stabil**, da alle Pole innerhalb des EHK
- **nicht minimalphasig**, da nicht alle NST innerhalb des EHK liegen

g) $h_1(n) = 3 \cdot \delta(n) - 10 \cdot \delta(n-1) + \delta(n-2)$

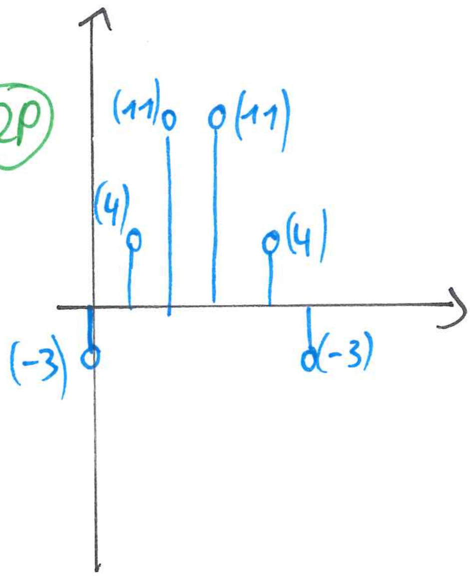
1P



h) 1P



i) 2P



j) TP, da FIR Typ II

1P

Aufgabe 2 (7P)

a) $y(n) = -x(n+2) + x(n-2)$

1P

b) $Y(e^{j\Omega}) = X(e^{j\Omega}) \cdot \left(e^{-j2\Omega} - e^{j2\Omega}\right)$

1P

c) $H(e^{j\Omega}) = \dfrac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \left(e^{-j2\Omega} - e^{j2\Omega}\right) \cdot \overbrace{\left(\dfrac{-2j}{-2j}\right)}^{=1}$

3P

$$= -\frac{1}{2j}\left(e^{-j2\Omega} - e^{j2\Omega}\right) \cdot (-2j) \qquad \boxed{-j = e^{-j\frac{\pi}{2}}}$$

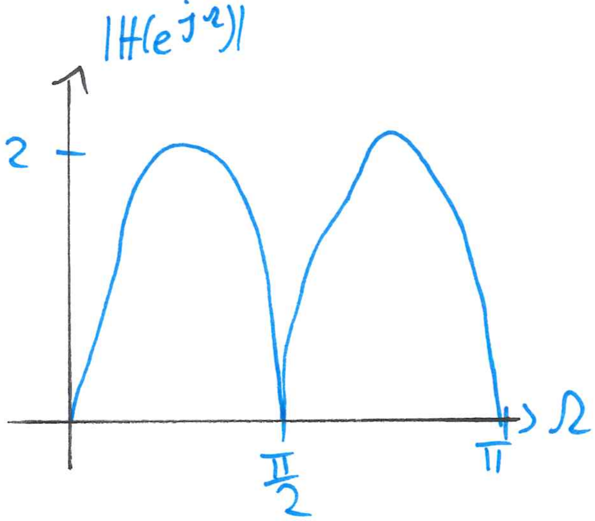
$$= \frac{1}{2j}\left(e^{j2\Omega} - e^{-j2\Omega}\right) \cdot 2 \cdot e^{-j\frac{\pi}{2}}$$

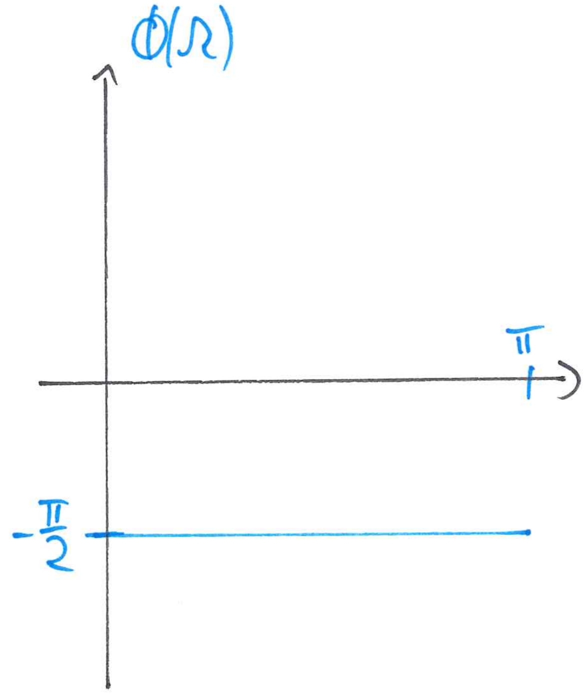
$$= 2 \cdot \sin(2\Omega) \cdot e^{-j\frac{\pi}{2}}$$

$|H(e^{j\Omega})| = |2 \cdot \sin(2\Omega)|$ $\qquad \phi(\Omega) = -\frac{\pi}{2}$

d)

2P





Aufgabe 3 (10P)

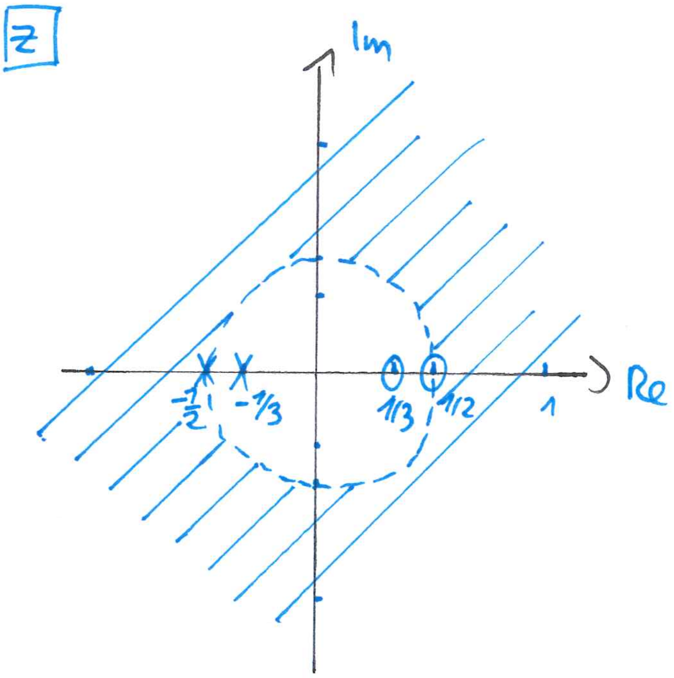
a) (1,5P) $H(z) = \frac{(z-1/2)(z-1/3)}{(z+1/2)(z+1/3)}$

b) (1,5P) ROC_1 : $|z| > 1/2$ (rechtsseitige Impulsantwort)

ROC_2 : $|z| < 1/3$ (Linksseitige Impulsantwort)

ROC_3 : $1/3 < |z| < 1/2$ (beidseitige Impulsantwort)

c) (2P)



d) (5P) $R_0 = H(0) = 1$

$$R_{1,1} = \lim_{z \to -\frac{1}{2}} \left\{ \frac{z^2 - 5/6\, z + 1/6}{z^2 + 1/3\, z} \right\} = 10$$

$$R_{2,1} = \lim_{z \to -\frac{1}{3}} \left\{ \frac{z^2 - 5/6\, z + 1/6}{z^2 + 1/2\, z} \right\} = -10$$

$$H(z) = 1 + 10 \cdot \frac{z}{z+1/2} - 10 \frac{z}{z+1/3}$$

$$h(n) = \delta(n) + 10 \cdot \left(-\tfrac{1}{2}\right)^n \cdot \varepsilon(n) + (-10) \cdot \left(-\tfrac{1}{3}\right)^n \cdot \varepsilon(n)$$

Aufgabe 4 (17P)

a) (1P) $f_s = 96\,\mathrm{kHz}$ $\quad f_s''' = 72\,\mathrm{kHz}$

b) (1P) $r = \frac{3}{4}$

c) (1P) $\Omega_{c,p} = \frac{\pi}{3}$ $\quad$ d) (1P) $\Omega_{c,q} = \frac{\pi}{4}$ $\quad$ e) (1P) $\Omega_{c,pq} = \frac{\pi}{4}$

f) (2P)

$f_s = 96\,\mathrm{kHz}$ $\quad f_s' = 288\,\mathrm{kHz}$ $\quad f_s'' = 288\,\mathrm{kHz}$ $\quad f_s''' = 72\,\mathrm{kHz}$

$$x(n) \longrightarrow \boxed{\uparrow 3} \xrightarrow{x_1(n')} \boxed{H_{pq}(z)} \xrightarrow{x_2(n'')} \boxed{\downarrow 4} \longrightarrow x_3(n''')$$

$$\Omega_{c,pq} = \frac{\pi}{4}$$

g) (6P) s. nächste Seite

h) (1P)
$$h(n) = \frac{1}{4} \cdot si\left(\frac{\pi}{4}(n-1440)\right) \cdot w(n)$$
$$w(n) = \begin{cases} 1 & : n = 0, \ldots, N_b = 2880 \\ 0 & : \text{sonst} \end{cases}$$

i) (1P) $\tau = \frac{N_b}{2} = 1440$ Samples

Verzögerung: $\frac{\tau}{f_s'} = 5\,\mathrm{ms}$

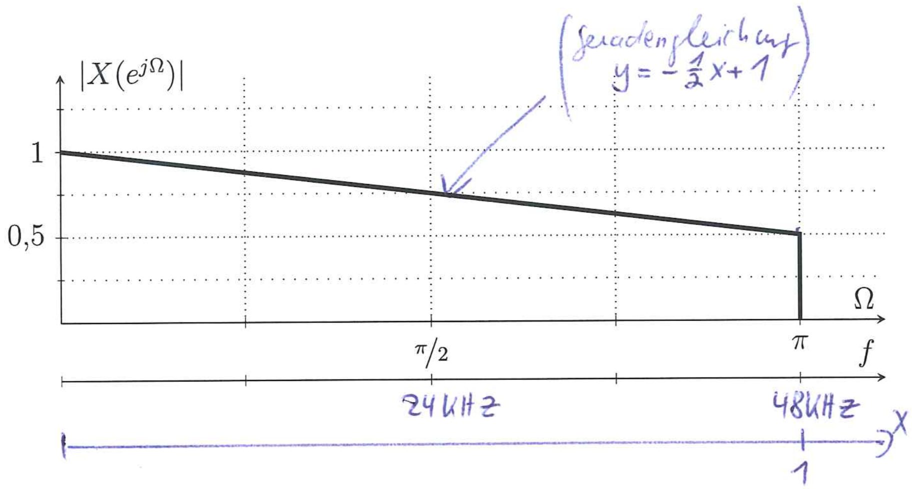
j) (1P) $N_b = 2880$ Verzögerungsglieder

$\Rightarrow N_b \cdot 16\,\mathrm{Bit} = 46080\,\mathrm{Bit}$

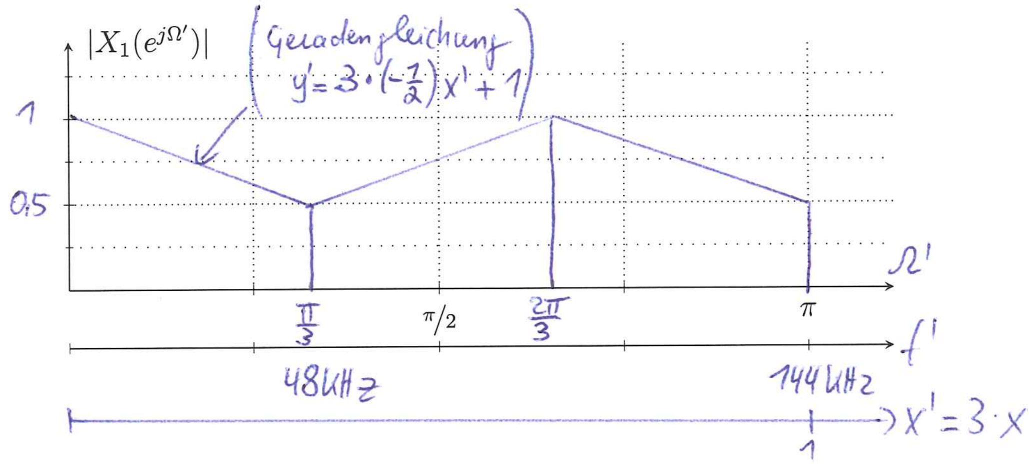
k) (1P)
$$\frac{4\pi}{N} \stackrel{!}{=} \frac{12\pi}{N_b^*} = \frac{12\pi}{N^* - 1}$$
$$\Rightarrow N^* - 1 = 12\pi \cdot \frac{N}{4\pi} = 3 \cdot N = 8643$$
$$\Rightarrow N^* = 3N + 1 = 8644$$

9)

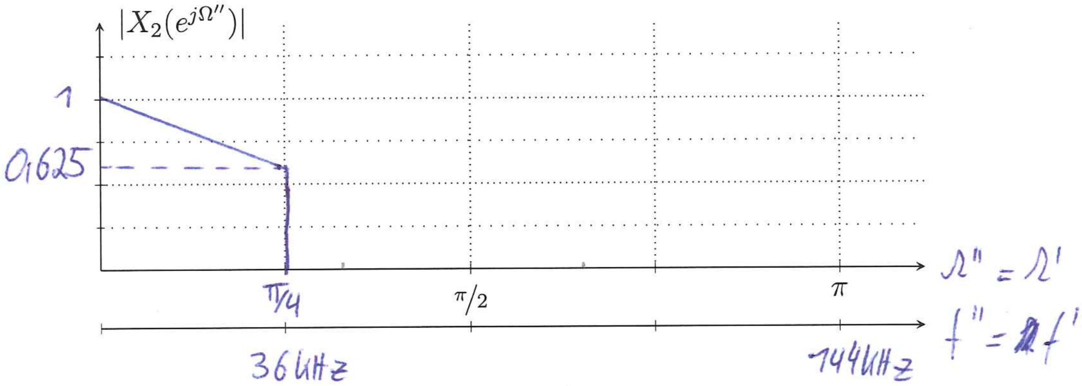
6P


$|X(e^{j\Omega})|$
(Geradengleichung $y = -\frac{1}{2}x + 1$)
1
0,5
Ω
$\pi/2$
π
f
24 kHz
48 kHz
x
1

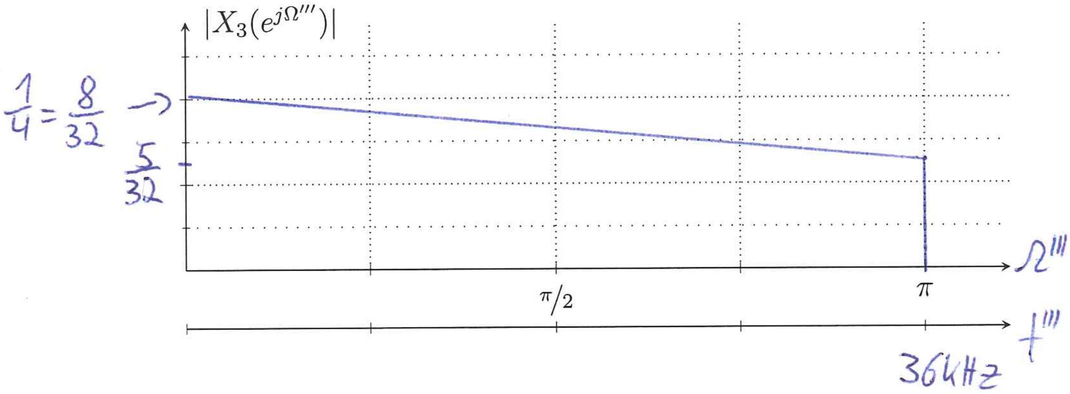
❶


$|X_1(e^{j\Omega'})|$
(Geradengleichung $y' = 3 \cdot (-\frac{1}{2}) x' + 1$)
1
0,5
Ω'
$\frac{\pi}{3}$
$\pi/2$
$\frac{2\pi}{3}$
π
f'
48 kHz
144 kHz
$x' = 3 \cdot x$
1

❷


$|X_2(e^{j\Omega''})|$
1
0,625
$\pi/4$
$\pi/2$
π
$\Omega'' = \Omega'$
$f'' = f'$
36 kHz
144 kHz

❸


$|X_3(e^{j\Omega'''})|$
$\frac{1}{4} = \frac{8}{32}$
$\frac{5}{32}$
Ω'''
$\pi/2$
π
f'''
36 kHz