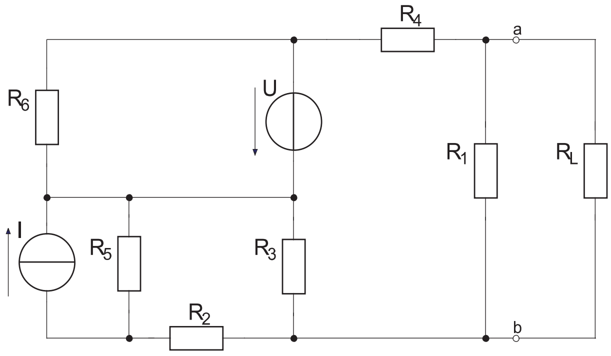
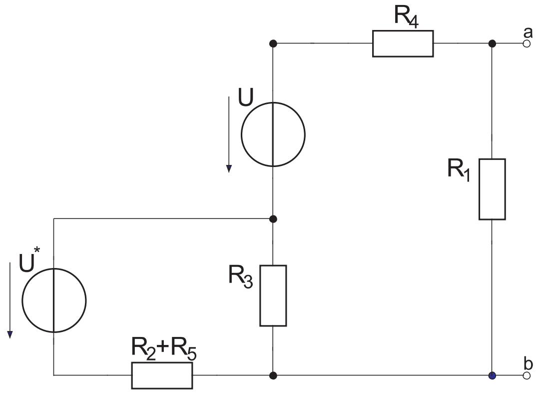
Aufgabe 1

a)

Spannungsquelle in Stromquelle umwandeln R_6 fällt weg, da parallel zur Spannungsquelle U. Aus der Stromquelle I wird die Spannungsquelle $U^* = IR_5$.





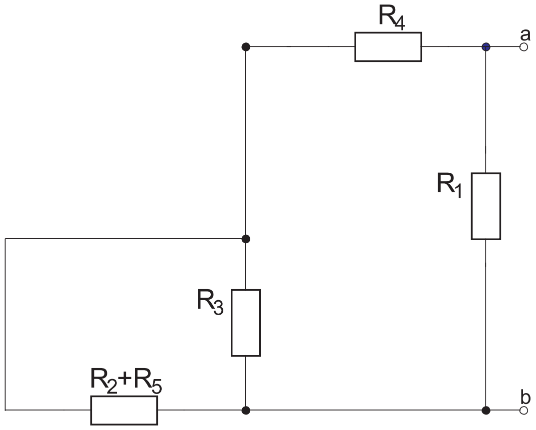
Innenwiderstand bestimmmen Für den Innenwiderstand R_i des Netzwerks bezüglich der Klemmen a und b gilt:

$$R_i = [(R_3 \parallel (R_2 + R_5)) + R_4] \parallel R_1$$

$$R_i = \left[\frac{(R_2 + R_5)\, R_3}{R_2 + R_3 + R_5} + R_4\right] \parallel R_1$$

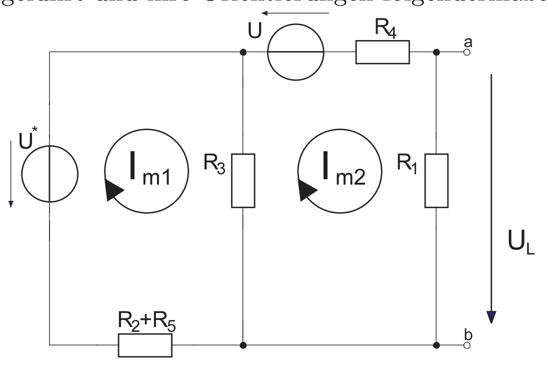
$$R_i = \frac{R_2R_3 + R_3R_5 + R_2R_4 + R_3R_4 + R_4R_5}{R_2 + R_3 + R_5} \parallel R_1$$

$$R_i = \frac{\frac{R_2R_3+R_3R_5+R_2R_4+R_3R_4+R_4R_5}{R_2+R_3+R_5} R_1}{\frac{R_2R_3+R_3R_5+R_2R_4+R_3R_4+R_4R_5}{R_2+R_3+R_5} + R_1}$$



$$R_i = \underline{\underline{\frac{(R_2R_3 + R_3R_5 + R_2R_4 + R_3R_4 + R_4R_5)\, R_1}{R_2R_3 + R_3R_5 + R_2R_4 + R_3R_4 + R_4R_5 + R_1R_2 + R_1R_3 + R_1R_5}}}$$

U_L berechnen mit Maschenstromanalyse Dazu werden die zwei Maschenströme I_{m1} und I_{m2} eingeführt und ihre Orientierungen folgendermaßen definiert:



$$\underbrace{\begin{pmatrix} R_2+R_3+R_5 & -R_3 \\ -R_3 & R_1+R_3+R_4 \end{pmatrix}}_{\underline{R}} \begin{pmatrix} I_{m1} \\ I_{m2} \end{pmatrix} = \begin{pmatrix} U^* \\ U \end{pmatrix}$$

$$U_L = I_{m2} R_1$$

Anwenden der *Cramer´schen Regel*:

$$I_{m2} = \frac{\det \underline{R}^*}{\det \underline{R}}$$

$$\det \underline{R}^* = \begin{vmatrix} R_2+R_3+R_5 & U^* \\ -R_3 & U \end{vmatrix} = U\left(R_2+R_3+R_5\right) + U^* R_3$$

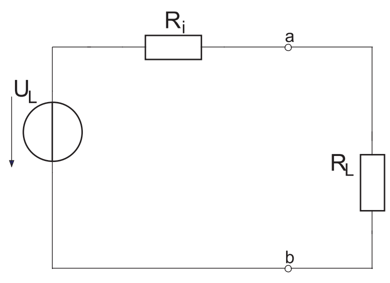
$$\det \underline{R} = \begin{vmatrix} R_2+R_3+R_5 & -R_3 \\ -R_3 & R_1+R_3+R_4 \end{vmatrix} = \left(R_2+R_3+R_5\right)\left(R_1+R_3+R_4\right) - R_3^2$$

$$I_{m2} = \frac{\det \underline{R}^*}{\det \underline{R}} = \frac{U\left(R_2+R_3+R_5\right)+U^*R_3}{\left(R_2+R_3+R_5\right)\left(R_1+R_3+R_4\right)-R_3^2}$$

$$U_L = I_{m2}R_1 = \underline{\underline{\frac{U\left(R_2+R_3+R_5\right)+U^*R_3}{\left(R_2+R_3+R_5\right)\left(R_1+R_3+R_4\right)-R_3^2}R_1}}$$

b) Beim Kurzschluss der Klemmen a und b wird R_1 überbrückt, deswegen besteht keine Abhängigkeit des Kurzschlussstromes I_K von dem Widerstand R_1.

c) Bei Leistungsanpassung muss gelten: $R_i = R_L$
Wegen $R_{1,2,4,5,6} = R$ gilt:



$$R_i = \frac{\left(RR_3 + R_3R + RR + R_3R + RR\right)R}{RR_3 + R_3R + RR + R_3R + RR + RR + RR_3 + RR}$$

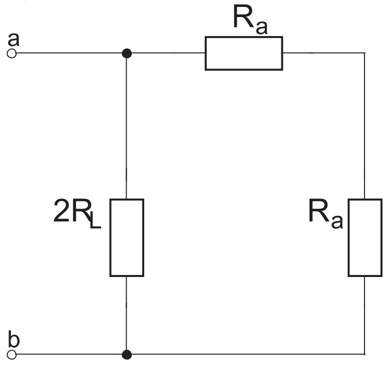
$$R_i = \frac{3R_3R + 2R^2}{4R_3 + 4R}$$

$$\Rightarrow R_L\left(4R_3 + 4R\right) = 3RR_3 + 2R^2$$

$$R_3\left(4R_L - 3R\right) = 2R^2 - 4RR_L$$

$$R_3 = \underline{\underline{2R\left(\frac{R-2R_L}{4R_L-3R}\right)}}$$

d) Für $n = 1$ gilt:



$$R_{ers} = 2R_a \parallel 2R_L$$

$$R_{ers} = \frac{2R_a R_L}{R_a + R_L}$$

für $R_a = R_L$ gilt:

$$R_{ers} = \frac{2R_L^2}{2R_L} = \underline{\underline{R_L}}$$

Damit $R_{ers} = R_L$ ist, muß $R_a = R_L$ sein.

e) Damit R_{ers} unabhängig wird von n, muss für $k = n$ und alle vorhergenden Maschen $(n-1,\, n-2,\, \ldots)$ der Ersatzwiderstand gleich R_a sein.
Für $n = \mathit{beliebig}$ gilt:

$$R_a + R_a = 2R_a$$

$$2R_L \parallel 2R_a = \frac{2R_L R_a}{R_L + R_a} \overset{!!}{=} R_a$$

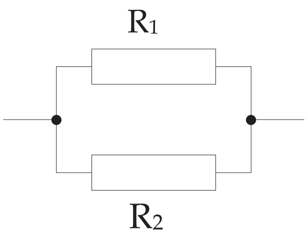
$$\Rightarrow 2R_L R_a = R_a R_L + R_a^2$$

$$R_a^2 - R_a R_L = 0 \Rightarrow R_a\,(R_a - R_L) = 0$$

$$\Rightarrow R_a = 0 \vee R_a = R_L$$

Aufgabe 2

a) Das Ersatzschaltbild hat folgende Form:



b) $dR_{\kappa_1} = \frac{dx}{\kappa_1 A_1(x)}$

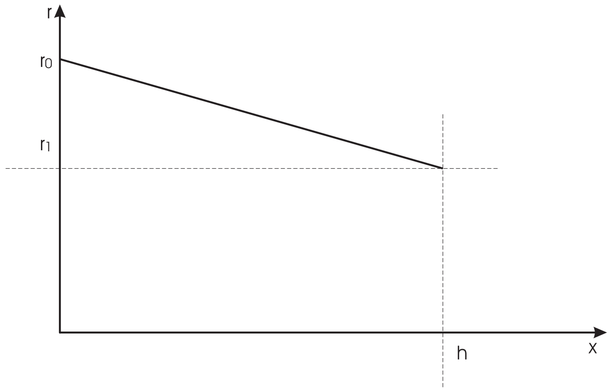
$$r(x) = ax + b$$

$$r(0) = b = r_0$$

$$r(h) = ah + r_0$$

$$\Rightarrow a = \frac{r_1 - r_0}{h}$$

$$\Rightarrow r(x) = \underline{\underline{\frac{r_1 - r_0}{h} x + r_0}}$$



Bedingung für idealisierten Feldverlauf:

$$h >> r_0, r_1$$

Bestimmen von $A(x)$:

$$A_{\kappa_1} = A_{\kappa_2} = \frac{\pi r^2}{2} = \underline{\underline{\frac{\pi}{2}\left[\left(\frac{r_1 - r_0}{h}\right)x + r_0\right]^2}}$$

Bestimmen von R_1 und R_2:

$$R_{\kappa_1} = \int_0^h \frac{2}{\pi\kappa_1\left[\left(\frac{r_1-r_0}{h}\right)x + r_0\right]^2}dx$$

$$\left(Hinweis: \int \frac{dx}{(ax+b)^2} = -\frac{1}{a}\frac{1}{ax+b}\right)$$

$$R_{\kappa_1} = \frac{2h}{(r_1 - r_0)\kappa_1\pi}\left(\frac{1}{r_0} - \frac{1}{r_1}\right)$$

$$R_{\kappa_1} = \frac{2h}{\underline{\underline{\kappa_1\pi r_0 r_1}}}$$

R_{κ_2} analog:

$$R_{\kappa_2} = \frac{2h}{\underline{\underline{\kappa_2\pi r_0 r_1}}}$$

c) Gesamtwiderstand ergibt sich aus Parallelschaltung von R_{κ_1} und R_{κ_2}:

$$R_{ges} = \frac{R_{\kappa_1}R_{\kappa_2}}{R_{\kappa_1} + R_{\kappa_2}}$$

$$R_{ges} = \frac{\frac{1}{\kappa_1}\frac{1}{\kappa_2}}{\frac{1}{\kappa_1} + \frac{1}{\kappa_2}}\frac{2h}{\pi r_0 r_1}$$

$$R_{ges} = \frac{2h}{\underline{\underline{(\kappa_1 + \kappa_2)\pi r_0 r_1}}}$$

d) Bei Gesamtstrom $I = I_0$ verteilt sich der Strom:

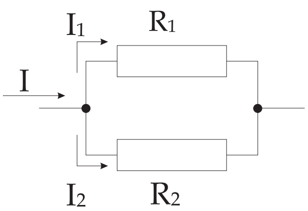
Dabei gilt:

$$U = U_1 = U_2$$

$$IR_{ges} = I_1R_1 = I_2R_2$$

$$\frac{I_1}{I} = \frac{\frac{1}{\kappa_1+\kappa_2}}{\frac{1}{\kappa 1}}$$

$$\Rightarrow I_1 = I\frac{\kappa_1}{\kappa_1 + \kappa_2} = I_0\frac{\kappa_1}{\kappa_1 + \kappa_2}$$



$$S(x) = E(x)\,\kappa$$

$$S_{\kappa_1}(x) = \frac{I_1}{A_1(x)}$$

$$S_{\kappa_1}(x) = I\frac{\kappa_1}{\kappa_1+\kappa_2}\frac{2}{\pi\left[\left(\frac{r_1-r_0}{h}\right)x+r_0\right]^2}$$

$$E_{\kappa_1}(x) = E_{\kappa_2}(x) = E(x) = \frac{I}{\kappa_1+\kappa_2}\frac{2}{\pi\left[\left(\frac{r_1-r_0}{h}\right)x+r_0\right]^2}$$

$$\varphi(x) = -\int E(x)\,dx + C,\ C\in\Re,\ \varphi(h) \overset{def.}{=} 0V$$

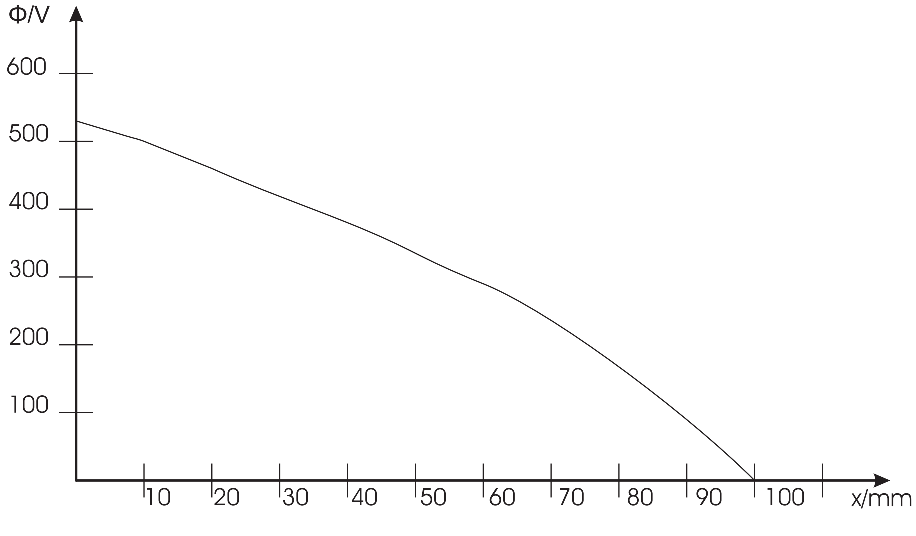
$$\varphi(x) = \frac{2hI}{(\kappa_1+\kappa_2)\,\pi\,(r_1-r_0)\left[\frac{r_1-r_0}{h}x+r_0\right]} + C$$

$$\varphi(h) = 0V \Rightarrow C = \frac{2hI}{(\kappa_1+\kappa_2)\,\pi\,(r_1-r_0)\,r_1}$$

$$\underline{\underline{\varphi(x) = \frac{2hI}{(\kappa_1+\kappa_2)\,\pi\,(r_1-r_0)}\left(\frac{1}{\frac{r_1-r_0}{h}x+r_0}-\frac{1}{r_1}\right)}}$$

e)

$\frac{x}{mm}$	0	10	20	40	60	80	100
$\frac{\varphi(x)}{V}$	530,82	496,1	458,82	374,48	273,82	151,58	0



Aufgabe 3

a) Die Platten ziehen sich an.

b) $F_{el} = \frac{dW_{el}}{dx}$, $W_{el} = \frac{1}{2} C(x) U^2$, $C(x) = \frac{\varepsilon_0 \varepsilon_r A}{d-x}$

$$F_{el} = -\frac{\varepsilon_0 \varepsilon_r A U^2 (-1)}{2\,(d-x)^2} = \underline{\underline{\frac{\varepsilon_0 \varepsilon_r A U^2}{2(d-x)^2}}}$$

c)

$$\underbrace{F_{el}}_{\text{in } x\text{-Richtg}} + \underbrace{(-F_{Feder})}_{\text{gegen } x\text{-Richtg}} = 0 \quad \Rightarrow \quad \frac{\varepsilon_0 \varepsilon_r A U^2}{2(d-x)^2} = c \cdot x$$

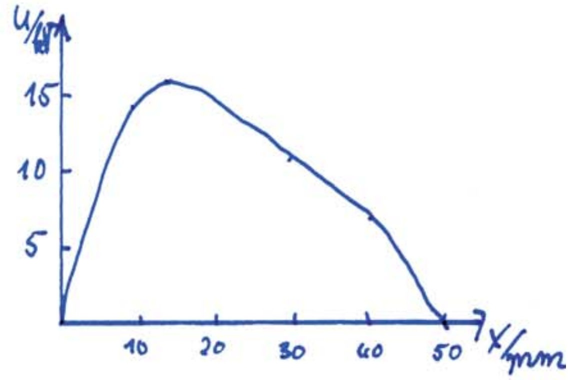
$$\Rightarrow A = \frac{2c(d-x)^2 x}{\varepsilon_0 \varepsilon_r U^2} = \frac{2 \cdot 18 \cdot 10^{-6} N \cdot (40mm)^2 \cdot 10mm\ 36\pi\ Vm}{mm\ 10^{-9} As \cdot 3 \cdot 14{,}4^2 V^2}$$

$$= \frac{25 \cdot 4 \cdot \pi \cdot 10^{-6} m^2}{3} = \underline{\underline{\frac{\pi}{3} cm^2}} \mathrel{\hat{=}} 1{,}047 cm^2$$

d)

$$U = \sqrt{\frac{2c}{\varepsilon_0 \varepsilon_r A}} \cdot (d-x) \cdot \sqrt{x} = \underline{\underline{36V \cdot \sqrt{10} \cdot \frac{d-x}{mm} \cdot \frac{\sqrt{x}}{\sqrt{mm}}}}$$

x/mm	0	10	20	30	40	50
U/kV	0	14,4	15,27	12,47	7,2	0



e)

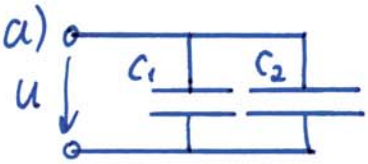
$$\frac{dU}{dx} = \frac{36V\sqrt{10}}{mm\sqrt{mm}} \left(\frac{d}{2\sqrt{x}} - \frac{3\sqrt{x}}{2} \right)$$

$$\frac{dU}{dx} \stackrel{!}{=} 0 \Rightarrow d = 3x \Rightarrow \underline{x = \frac{d}{3}}$$

$$U\left(x = \frac{d}{3}\right) = 36V\sqrt{10}\,\frac{\left(\frac{2}{3}d\right)}{mm} \cdot \sqrt{\frac{1}{3}\frac{d}{mm}} = \underline{\underline{15{,}49\ kV}}$$

Aufgabe 4

a)



b) $Q_1 = \int_A \vec{D}_1 \, d\vec{A}_1 = \int_0^{\varphi_1} D_1 \cdot l \cdot r \, d\varphi = \underline{D_1 l \cdot r \cdot \varphi_1}$

oder $Q_1 = D_1 \cdot l \cdot 2\pi r \cdot \frac{\varphi_1}{2\pi} = \underline{D_1 \cdot l \cdot r \cdot \varphi_1}$

$$Q = Q_1 + Q_2 = l \cdot r(\varphi_1 D_1 + \varphi_2 D_2) = lr\left(\frac{3}{4}\pi D_1 + \frac{5}{4}\pi D_2\right) = \underline{\underline{\frac{lr\pi}{4}(3D_1 + 5D_2)}}$$

c) $D_1 = \varepsilon_0 \varepsilon_1 E_1$, $D_2 = \varepsilon_0 \varepsilon_2 E_2$, $E = E_1 = E_2$!

$$Q = \frac{l\pi r \varepsilon_0}{4}(3\varepsilon_1 E + 5\varepsilon_2 E) \Rightarrow E = \frac{4Q}{l\pi r \varepsilon_0 (3\varepsilon_1 + 5\varepsilon_2)}$$

$$U = \int_{r_1}^{r_2} E \, dr = \underline{\underline{\frac{4Q}{l\pi \, \varepsilon_0 (3\varepsilon_1 + 5\varepsilon_2)} \cdot \ln \frac{r_2}{r_1}}}$$

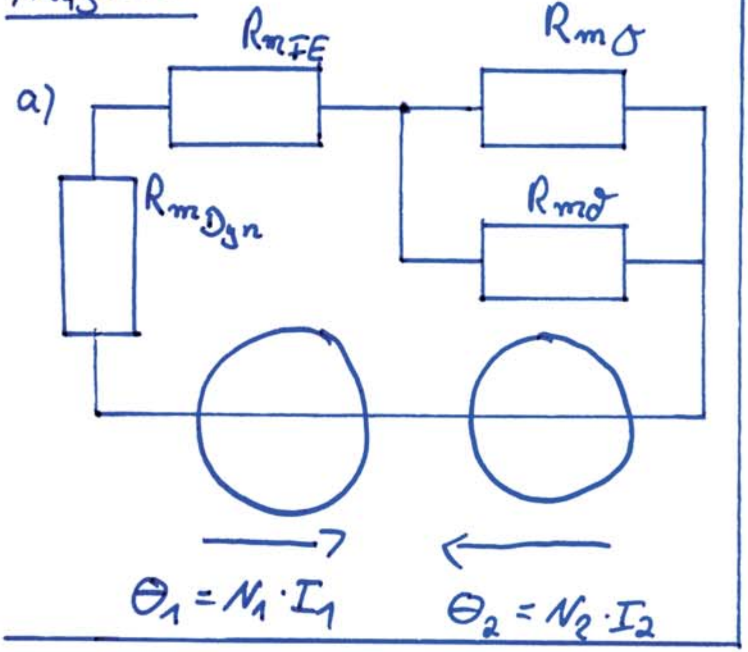
$$U = \frac{4 \cdot 10^{-9} As \cdot \ln 2 \; 36\pi Vm}{1m \cdot \pi \cdot 10^{-9} As \, (26)} = \frac{4 \cdot 36}{26} \cdot \ln 2 \, V = \underline{\underline{3{,}84 V}}$$

d) $$C' = \frac{Q}{U} = \underline{\underline{\frac{l\pi \cdot \varepsilon_0 (3\varepsilon_1 + 5\varepsilon_2)}{4 \ln \frac{r_2}{r_1}}}}$$

e) $$C' = \frac{Q}{U} = \frac{10^{-9} As}{3{,}84 V} = \underline{\underline{260 pF}}$$

Aufgabe 5

a)



$$A = \frac{(r_2 - r_1)^2 \pi}{4}, \quad R_m = \frac{l}{\mu_0 \mu_r A}$$

b)

$$R_{mDyn} = \frac{\frac{3}{2}\pi \cdot 0{,}3m \cdot 4}{\mu_0 \cdot \mu_{rDyn} (r_2 - r_1)^2 \pi}$$

$$R_{mFE} = \frac{\frac{1}{2}\pi \cdot 0{,}3m \cdot 4}{\mu_0 \mu_{rFE} (r_2 - r_1)^2 \pi}$$

$$R_{m\vartheta} = \frac{\vartheta \cdot 4}{\mu_0 (r_2 - r_1)^2 \pi}$$

$$\Phi_\vartheta \cdot R_{m\vartheta} = \Phi_\sigma \cdot R_{m\sigma}$$

$$\Phi_\vartheta = (1-\sigma) \cdot \Phi_{ges}, \quad \Phi_\sigma = \sigma \cdot \Phi_{ges}$$

$$\Rightarrow R_\sigma = \frac{1-\sigma}{\sigma} \cdot R_{m\vartheta}$$

$$\mu_{rFE} \to \infty \Rightarrow R_{mFE} \to 0$$

c)

$$R_{mges} = R_{mDyn} + R_{m\vartheta} = \frac{(6\pi \cdot 0{,}3m + 12.000\vartheta) Am}{3000 \cdot 4\pi \cdot 10^{-7} Vs \cdot 4\pi \cdot 10^{-4} m^2}$$

$$R_{mges} = 7{,}53 \, 10^6 \frac{1}{Vs}$$

d)

$$L_1 = N_1^2 \frac{1}{R_m} = 132{,}8 \, mH \qquad L_2 = N_2^2 \frac{1}{R_m} = 0{,}332 \, mH$$

e) $k = 1$

$$M = k \sqrt{L_1 \cdot L_2} = 6{,}64 \, mH$$

f)

$$\max\left\{\frac{di_1(t)}{dt}\right\} = 0{,}2A \cdot \sqrt{2} \cdot \frac{50}{\pi} Hz \cdot 2\pi \Rightarrow u_{2max} = M \cdot \frac{di_{max}}{dt} = 0{,}18V$$

g) Blindleistung! $P_{Blind} = I_1^2 \cdot X$ $\quad |P|_{Blind} = I_1^2 \omega L_1 = 0{,}2^2 A^2 \cdot 100Hz \cdot 132{,}8mH$

$$P = 531{,}2 \, mW$$

h)

$$\Phi_{ges} = 0 \Rightarrow \frac{\Theta_1}{R_m} + \frac{-\Theta_2}{R_m} = 0 \Rightarrow \Theta_1 \overset{!}{=} \Theta_2 \quad N_1 I_1 = N_2 I_2 \Rightarrow I_2 = 4A$$

$$(-U_1) = -L_1 \frac{di_1(t)}{dt} + M \frac{di_2(t)}{dt} \Rightarrow U_1(t) = (133{,}32 \cdot 0{,}2 - 6{,}64 \cdot 4) \cdot mH \cdot \sqrt{2} \cdot 100Hz \cdot A = 14{,}7 mV$$

Anmerkung Markus Steimle: Aufgabenstellung ist nicht ideal formuliert; bei Bereich 1 könnte man genauso gut x=a*cos(alpha) hinzunehmen und in Bereich 2 weglassen (würde sogar besser mit der Aufgabenstellung übereinstimmen)

Aufgabe 6

a) Bereich 1: $0 \le x < a\cdot\cos\alpha$ $\qquad \Phi = \int \vec{B}\, d\vec{A} \qquad dA = b\cdot dx$

$\vec{B} \parallel d\vec{A}$

$$\Phi = \int_0^x kx'\cdot b\,dx' = \underline{\underline{\frac{kbx^2}{2}}}$$

Bereich 2: $a\cdot\cos\alpha \le x$

$$\Phi = \int_{x-a\cos\alpha}^{x} kx'\cdot b\,dx' = \frac{k\cdot b}{2}\left(x^2-(x^2-2ax\cos\alpha+a^2\cos^2\alpha)\right)$$

$$= \frac{k\cdot b}{2}\left(2ax\cos\alpha - a^2\cos\alpha\right) = \underline{\underline{\frac{k\cdot a\cdot b\cdot\cos\alpha}{2}\left(2x - a\cos\alpha\right)}}$$

b) Bereich 1: $0 \le t \le \frac{a\cdot\cos\alpha}{v}$ $\qquad x = v\cdot t\,;\ dx = v\cdot dt$

$U_i = -\left(-N\frac{d\Phi}{dt}\right)$ wegen Lenz'scher Regel $\quad , N=1$

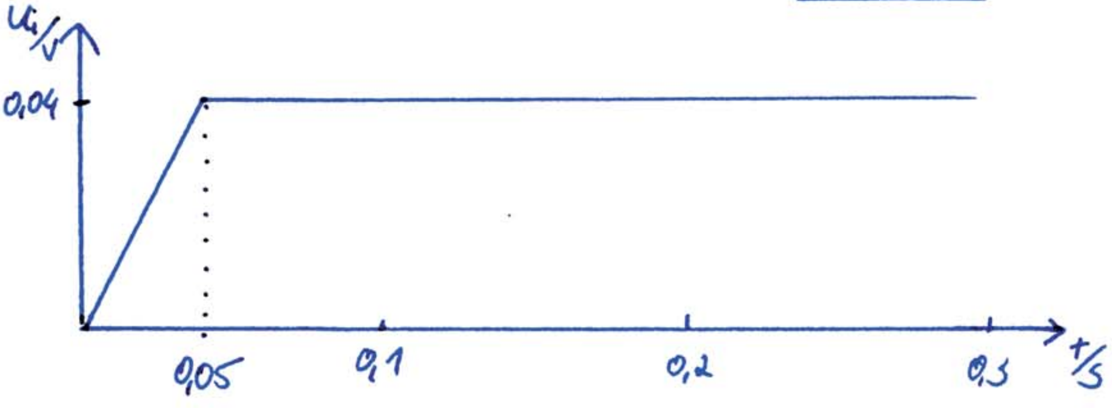
$$U_i = \frac{d}{dt}\left(\frac{kbv^2t^2}{2}\right) = \frac{k\cdot bv^2}{2}\cdot 2t = \underline{\underline{kbv^2t}}$$

Bereich 2: $\frac{a\cos\alpha}{v} \le t$

$$U_i = \frac{d}{dt}\left(\frac{kab\cos\alpha}{2}\left(2vt - a\cos\alpha\right)\right) = \underline{\underline{kabv\cos\alpha}} = \text{const.}$$

c) $\frac{a\cdot\cos\alpha}{v} \hat{=} \frac{0{,}1\,m\cdot\frac{1}{\sqrt{2}}}{\sqrt{2}\,\frac{m}{s}} = \underline{0{,}05\,s} \Rightarrow U_i = \begin{cases} U_{i1}, & t < 0{,}05\,s \\ U_{i2}, & t \geq 0{,}05\,s \end{cases}$

$U_{i1} = \frac{1\,T}{m}\cdot 0{,}4\,m\cdot 0{,}1\,m\cdot\left(\sqrt{2}\,\frac{m}{s}\right)^2\cdot t = \underline{\underline{0{,}8\,V\,\frac{t}{s}}} \qquad U_{i2} = \underline{\underline{0{,}04\,V}}$



Aufgabe 7

a) $f = 100\,Hz$, $X_L = 28\,\Omega = \omega L \Rightarrow L = \frac{28\,\Omega}{2\pi f} = \underline{\underline{44{,}6\,mH}}$

$|\underline{U}_L| = |\underline{I}_L| \cdot X_L = 0{,}25\,A \cdot 28\,\Omega = \underline{\underline{7\,V}}$

$|\underline{I}_C| = |\underline{U}_C| \cdot \omega C = 5{,}5\,V \cdot 2\pi \cdot 100\,Hz \cdot 248 \cdot 10^{-6} \frac{As}{V} = \underline{\underline{0{,}85\,A}}$

b) $|\underline{U}_{R2}| = |\underline{I}_L| R_2 = 12\,V$

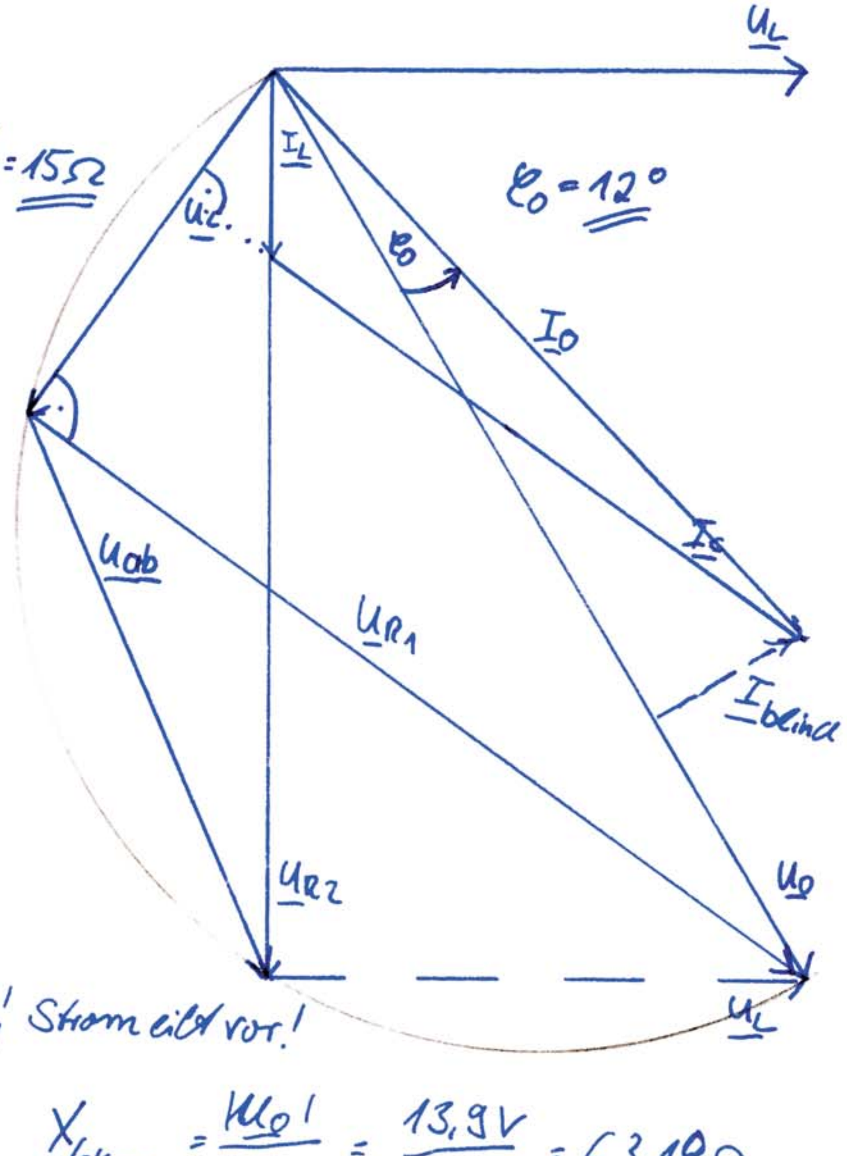
$|\underline{U}_0| = \underline{\underline{13{,}9\,V}}$

$|\underline{U}_{R1}| = 12{,}7\,V \Rightarrow R_1 = \frac{12{,}7\,V}{0{,}85\,A} = \underline{\underline{15\,\Omega}}$

$|\underline{U}_{ab}| = \underline{\underline{8{,}3\,V}}$

$|\underline{I}_0| = \underline{\underline{1{,}02\,A}}$

$\varphi_0 = \underline{\underline{12°}}$



c) Kapazitive Belastung! Strom eilt vor!

d) $\underline{I}_{blind} = 0{,}22\,A$ $\quad X_{L\,komp} = \frac{|\underline{U}_0|}{|\underline{I}_{blind}|} = \frac{13{,}9\,V}{0{,}22\,A} = \underline{63{,}18\,\Omega}$

$\Rightarrow L_{komp} = \frac{63{,}18\,\Omega}{\omega} = \frac{63{,}18\,\Omega}{2\pi f} = \underline{\underline{100\,mH}}$

e) Abgleich: $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$

$\frac{j\omega L^*}{R_2} = \frac{R_1}{\frac{1}{j\omega C}} \Rightarrow \underline{\underline{L^* = R_1 R_2 C}} = \underline{\underline{179\,mH}}$

Aufgabe 8

a) $$\underline{Y} = \frac{1}{R_i} + \frac{1}{R} + \frac{1}{j\omega L + \frac{1}{j\omega C'}} = \underline{\underline{\frac{R_i + R}{R_i \cdot R} + \frac{j\omega C'}{1-\omega^2 LC'}}}$$

b) $$\underline{Y}_{t0} = \frac{1}{R_i} + \frac{1}{j\omega L} + j\omega C' = \frac{1}{R_i} + j\left(\frac{1-\omega^2 LC'}{-\omega L}\right)$$

c) Parallelschwingkreis! Resonanz: $\mathrm{Im}\{\underline{Y}_{t0}\} = 0$

$$\Rightarrow \frac{1-\omega^2 LC'}{-\omega L} = 0 \Rightarrow \underline{\underline{\omega = \frac{1}{\sqrt{LC'}}}}$$

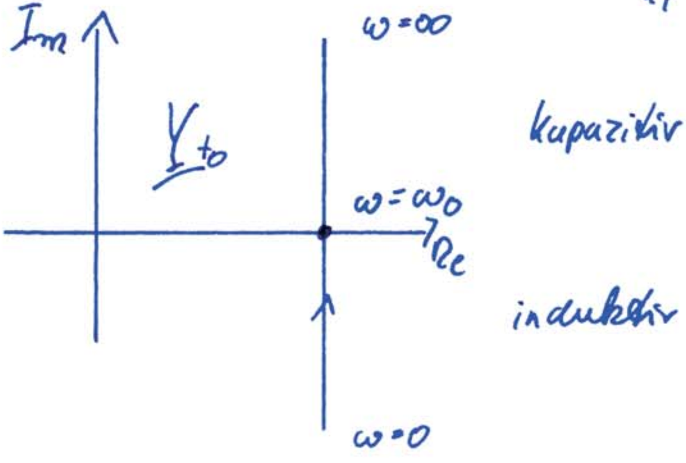
d)

ω	$\lvert\underline{Y}_{t0}\rvert$
0	$\to \infty$
ω_0	$\frac{1}{R_i}$
∞	$\to \infty$

"$\frac{1}{R_i} - j\infty$"

"$\frac{1}{R_i} + j\infty$"

e)



f) $$\omega_0 = \frac{1}{\sqrt{LC'}} = \frac{1}{\sqrt{100 \cdot 10^{-3}\,\frac{Vs}{A} \cdot 900 \cdot 10^{-9}\,\frac{As}{V}}} = \underline{\underline{3{,}3\,kHz}}$$

$$Q = \frac{R_i}{\omega_0 L} = R_i\,\omega_0 C' = \underline{\underline{1{,}2}}$$