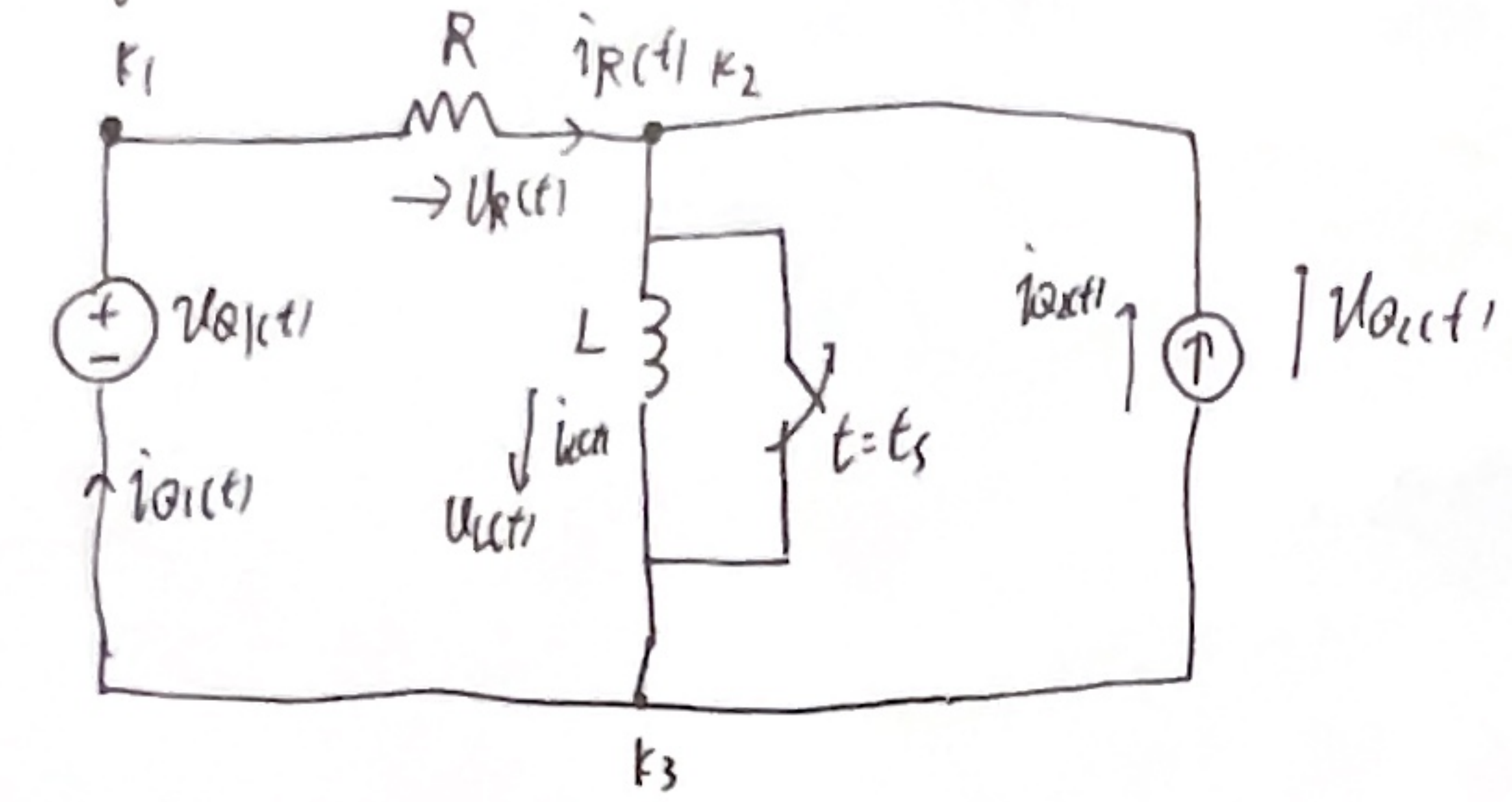
Aufgabe 4



$u_{Q1}(t)$、$i_{Q2}(t)$ 是直流

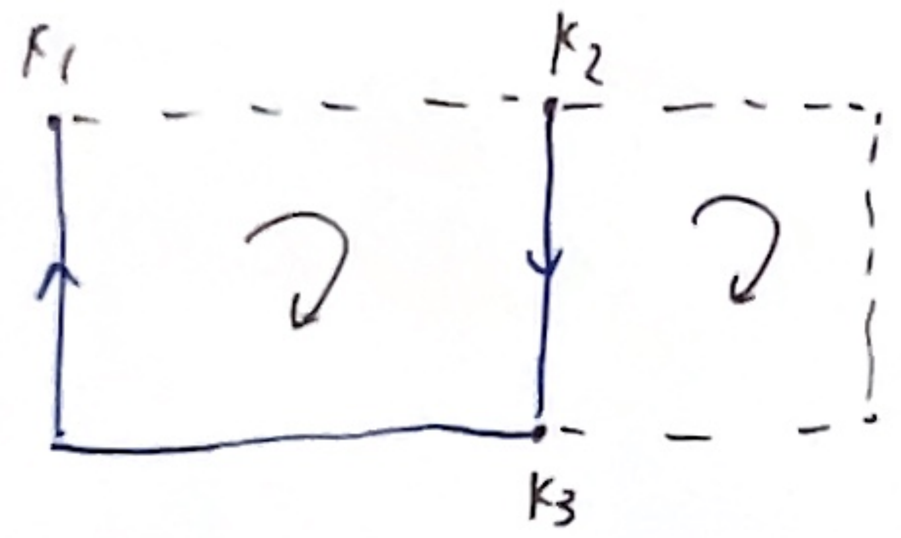
$t=t_s=0$ 时，开关打开

$i_L(0^-)=i_L(0^+)=i_L(0)$

求：$i_L(t)$，$t>0$

此题可用~~叠加定理~~ 但要逐次引入回论

在此域不行 ⇒ 会有两个 homogene Lösung



$2BZ$, $2VZ$, $3K$

$\Rightarrow m = z-k+1 = 4-3+1=2$

$p = k-1 = 2$

KVL: $\overset{\text{已知}}{u_{Q1}(t)} = u_R(t) + u_L(t)$

$u_{Q2}(t) = u_L(t)$ 未知

KCL: $i_{Q2}(t) + i_R(t) = i_L(t)$

电感特性

$u_L(t) = L\dfrac{di_L(t)}{dt}$

$$\Rightarrow u_{Q1}(t) = u_R(t) + u_L(t) = R\cdot i_R(t) + L\frac{di_L(t)}{dt} = R\left[i_L(t) - i_{Q2}(t)\right] + L\frac{di_L(t)}{dt}$$

$$\Leftrightarrow \frac{di_L(t)}{dt} = \frac{1}{L}u_{Q1}(t) - \frac{R}{L}\left[i_L(t) - i_{Q2}(t)\right] = \frac{1}{L}u_{Q1}(t) + \frac{R}{L}i_{Q2}(t) - \frac{R}{L}i_L(t)$$

DGL: $$\frac{di_L(t)}{dt} = \frac{1}{L}u_{Q1}(t) + \frac{R}{L}i_{Q2}(t) - \frac{R}{L}i_L(t)$$

Lösg: $i_L(t) = i_{L,h}(t) + i_{L,p}(t)$

für homogen

$u_{Q1}(t) = i_{Q2}(t) = 0$ 无外部激励

$$\frac{di_L(t)}{dt} = -\frac{R}{L}i_L(t)$$

$$\frac{1}{i_L(t)}di_L(t) = -\frac{R}{L}dt$$

$$\int_{i_L(0^+)}^{i_L(t)} \frac{1}{i_L(t')}di_L(t') = \int_{0^+}^{t} -\frac{R}{L}dt$$

$$\ln\frac{i_L(t)}{i_L(0^+)} = -\frac{R}{L}t$$

$$\Rightarrow i_{L,h}(t) = i_L(0)e^{-\frac{R}{L}t}$$

für inhomogen

根据通解 $i_L(0)e^{-\frac{R}{L}t}$

$$\Rightarrow i_{L,p}(t) = K_p(t)e^{-\frac{R}{L}t}$$

$$\frac{d}{dt}i_{L,p}(t) = e^{-\frac{R}{L}t}\frac{d}{dt}K_p(t) - \frac{R}{L}K_p(t)e^{-\frac{R}{L}t}$$

$$= \frac{1}{L}u_{Q1}(t) + \frac{R}{L}i_{Q2}(t) - \frac{R}{L}K_p(t)e^{-\frac{R}{L}t}$$

$$\Rightarrow e^{-\frac{R}{L}t}\frac{d}{dt}K_p(t) = \frac{1}{L}u_{Q1}(t) + \frac{R}{L}i_{Q2}(t)$$

$$dK_p(t) = \left[\frac{1}{L}u_{Q1}(t) + \frac{R}{L}i_{Q2}(t)\right]e^{\frac{R}{L}t}dt$$

$$\int_{K_p(0^+)}^{K_p(t)} dK_p(t') = \int_0^t \left[\frac{1}{L}u_{Q1}(t') + \frac{R}{L}i_{Q2}(t')\right]e^{\frac{R}{L}t'}dt'$$

$$K_{p}(t) - K_{p}(0) = \int_0^t \frac{1}{L} u_{Q1}(t') e^{\frac{R}{L}t'} dt' + \int_0^t \frac{R}{L} i_{Q2}(t') e^{\frac{R}{L}t'} dt'$$

Anfangswert $i_{L,p}(0^+) = 0 \Rightarrow K_p(0) = 0$

$$\Rightarrow i_{L,p}(t) = K_p(t) e^{-\frac{R}{L}t}$$

$$= \int_0^t \frac{1}{L} u_{Q1}(t') e^{\frac{R}{L}t'-t} dt' + \int_0^t \frac{R}{L} i_{Q2}(t') e^{\frac{R}{L}t'-t} dt'$$

$$= \frac{1}{L} u_{Q1}(t') \frac{L}{R} \left[e^{\frac{R}{L}t'-t} \right]_0^t + \frac{R}{L} i_{Q2}(t') \frac{L}{R} \left[e^{\frac{R}{L}t'-t} \right]_0^t$$

$$= \frac{u_{Q1}(t')}{R} \left(1 - e^{-\frac{R}{L}t}\right) + i_{Q2}(t') \left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow i_L(t) = i_{L,h}(t) + i_{L,p}(t) = i_L(0) e^{-\frac{R}{L}t} + \left[\frac{u_{Q1}(t')}{R} + i_{Q2}(t')\right]\left[1 - e^{-\frac{R}{L}t}\right]$$

$u_{Q1}(t)$ & $i_{Q2}(t)$ sind feste Quelle

ungesteuert, 和 Gleichstrom

$$\Rightarrow i_L(t) = i_L(0) e^{-\frac{R}{L}t} + \int_0^t \frac{1}{L} u_{Q1}(t') e^{-\frac{R}{L}t-t'} dt' + \int_0^t \frac{R}{L} i_{Q2}(t') e^{-\frac{R}{L}t-t'} dt'$$