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Case study for lecture “Production Systems and Supply Chains”

Tutorial II: Assessment

Summer term 2023



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Tutorial II: Assessment

Exercise 4: Dominance and efficiency

The brewers have been experimenting with several temperatures for boiling. This resulted in different electricity consumptions and labor demand, which is illustrated in the following three activities of technology $T = \{z_1, z_2, z_3\}$:

$$z_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix} \quad z_2 = \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix} \quad z_3 = \begin{pmatrix} -4 \\ -4 \\ 1 \end{pmatrix}$$

- a) Examine each of the three activities in pairs for dominance. Input and output object types are desirable!

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Solution:

$$z_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix} \quad \begin{matrix} = \\ < + \\ = \end{matrix} \quad \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix} = z_2 \quad \rightarrow \text{Dominance: } z_2 \text{ dominates } z_1$$

$$z_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix} \quad \begin{matrix} + > \\ < + \\ = \end{matrix} \quad \begin{pmatrix} -4 \\ -4 \\ 1 \end{pmatrix} = z_3 \quad \rightarrow \text{No dominance relation}$$

$$z_2 = \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix} \quad \begin{matrix} + > \\ + > \\ = \end{matrix} \quad \begin{pmatrix} -4 \\ -4 \\ 1 \end{pmatrix} = z_3 \quad \rightarrow \text{Dominance relation: } z_2 \text{ dominates } z_3$$

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- b) Draw the three activities in a **two-dimensional diagram** and mark the **efficient frontier of technology**. Input and output object types are desirable!

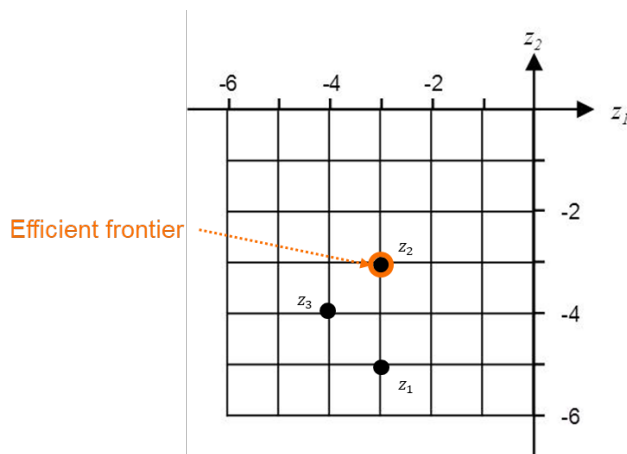
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Solution:

$$z_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}$$

$$z_2 = \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix}$$

$$z_3 = \begin{pmatrix} -4 \\ -4 \\ 1 \end{pmatrix}$$



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c) How does the efficient frontier change when both **input object types** are **undesirable**? The output object type remains **desirable**.

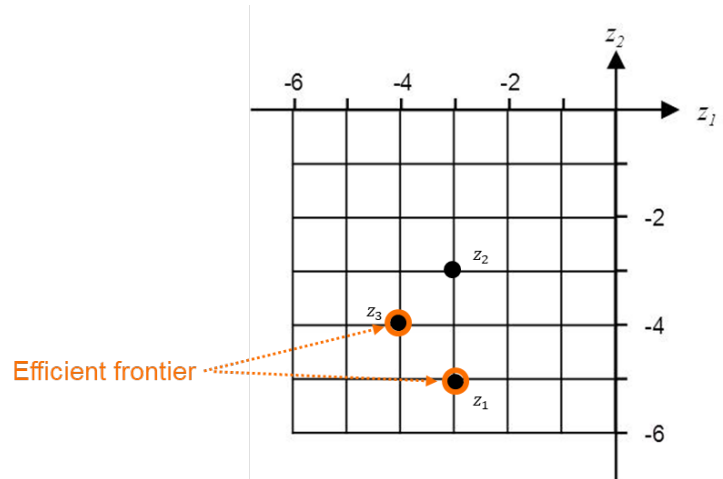
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Solution:

$$z_1 = \begin{pmatrix} -3 \\ -5 \\ 1 \end{pmatrix}$$

$$z_2 = \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix}$$

$$z_3 = \begin{pmatrix} -4 \\ -4 \\ 1 \end{pmatrix}$$



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Exercise 5: Economic assessment (Non-linear cost functions, learning curves)

One of the brewers (brewer 1) tells you that when he started working at the nanobrewery, it took him a lot of time to get to know all the characteristics of all the machines and the most efficient sequences of all the production steps. Nowadays, he has more routine and has been able to optimize certain steps and completes the processes faster. This phenomenon is called learning effects.

Due to learning effects, the processing time for a bottle (unit) of beer can be reduced over time. Therefore, the unit cost depends on the cumulative quantity of beer bottles produced so far according to the following equation:

$$c_1(y) = 100 \cdot y^{-0.5} \quad (\text{Unit cost function})$$

- a) What is the unit cost reduction (in %) when the cumulative product quantity is doubled?

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Solution:

Given unit cost function $c_1(y) = 100 \cdot y^{-0.5}$
with $\alpha = 100$; $\beta = 0.5$

Doubling of cumulative product quantity

$$\frac{c_1(2y)}{c_1(y)} = \frac{100 \cdot (2y)^{-0.5}}{100 \cdot y^{-0.5}} = \frac{2^{-0.5} \cdot y^{-0.5}}{y^{-0.5}} = 2^{-0.5} = \frac{1}{\sqrt{2}} \approx 0.71$$

$$\text{unit cost reduction in \%} = 100\% - 0.71 \cdot 100\% = 29\%$$

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- b) We assume that brewer 1 has produced 1,000 units of beer (= bottles) after some time. How many additional units does he need to produce to halve the unit cost?

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Solution:

$$c_1(y) = 100 \cdot y^{-0.5} \stackrel{!}{=} 0.5 \cdot 100 \cdot 1000^{-0.5} \approx 1.58$$

$a^{-n} = \frac{1}{a^n} \quad (a^n)^m = a^{n \cdot m}$

$$y^{-0.5} = 0.5 \cdot 1000^{-0.5}$$

$$y^{0.5} = 2 \cdot 1,000^{0.5}$$

$$\Rightarrow y = (2 \cdot 1,000^{0.5})^2 = 4 \cdot 1,000 = 4,000$$

Unit costs are cut in **half**, when output **quadruplicates** → for this, additional 3,000 units must be produced (4,000 units – 1,000 units)

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- c) A colleague of brewer 1, brewer 2, has already produced 55,760 units of beer since he started working at the brewery. So far, each doubling of the cumulative quantity has yielded a 19% reduction in cost. For this reason, the unit cost of the beer he produces is now only 1.5 MU per unit. Determine brewer 2's unit cost function $c_2(y)$ as a function of the cumulative quantity y .

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Solution:

Given unit cost function $c_1(y) = 100 \cdot y^{-0.5}$ **Objective: Determine α and β**

“So far, each doubling of the cumulative quantity has yielded a **19% reduction** in cost.”

Double cumulative quantity

Reduce unit costs by 19 %
(1 – 0.19 = 0.81)

$$\alpha \cdot (2 \cdot y)^{-\beta} = 0.81 \cdot \alpha \cdot (y)^{-\beta}$$

$$\Leftrightarrow \cancel{\alpha} \cdot (2)^{-\beta} \cdot \cancel{(y)^{-\beta}} = 0.81 \cdot \cancel{\alpha} \cdot \cancel{(y)^{-\beta}}$$

$$\Leftrightarrow 2^{-\beta} = 0.81$$

$$\Leftrightarrow -\beta \cdot \ln 2 = \ln 0.81$$

$$\ln c^d = d \cdot \ln c$$

$$\Leftrightarrow \beta = -\frac{\ln 0.81}{\ln 2} \approx 0.304$$

“Brewer 2 has already produced **55,760 units** of beer since he started working. For this reason, the unit cost of the beer he produces is now only 1.5 MU per unit.”

$$1.5 \text{ MU/unit} = \alpha \cdot 55,760^{-0.3}$$

Cost reduction potential β
(calculated previously, insert here)

$$\Leftrightarrow \alpha = \frac{1.5}{55,760^{-0.3}} = 39.809$$

55,760 units produced

Unit cost function $c_2(y)$: $\Rightarrow c_2(y) = 39.81 \cdot y^{-0.3}$

Exercise 6: Ecological assessment

In the lecture, you learned about a method to assess ecological sustainability, the Life Cycle Assessment (LCA). This method includes both midpoint and endpoint indicators. With these, it is possible to attribute certain impacts more specifically and compare different “aspects” of sustainability.

You share your newly acquired knowledge with the brewers, and you enter an intense discussion. The brewers realize that although they care a lot about sustainable production activities, there is at least one input factor which they depend on and which they cannot change or influence: electricity.

In order to assess the impact of the electricity generation, they ask you to compare the electricity mix of Braunschweig (BS) and Magdeburg (MD) (which will be considered as an optional new brewery location, later on). Electricity mix, also known as power generation mix, refers to the combination of fuels utilized to generate the electricity in a specific geographical area.

[We assume that the two cities have different electricity mixes, although this is difficult to measure and is mostly compared among countries. Numbers are fictitious]

- a) Please explain the difference between midpoint and endpoint indicators for life cycle assessments and give an example for each indicator type.

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Solution:

Midpoint Indicators:

- Describe the environmental impacts linked to the analyzed system
- Characterize the environmental impacts through effects which can be described in scientific terms (release of emissions, resource use, etc.)
- Examples: particulate matter, photochemical oxidant formation (human health), ionizing radiation, stratospheric ozone depletion, human toxicity (carcinogenic), human toxicity (non-carcinogenic), global warming, water use, freshwater ecotoxicity, freshwater eutrophication, photochemical oxidant formation (eco system), terrestrial ecotoxicity, terrestrial acidification, land use/transformation, marine ecotoxicity, depletion of mineral resources, depletion of fossil resources

Endpoint Indicators:

- Describe the environmental impacts associated with an investigated system on different protected areas.
- Environmental impact of an **endpoint** indicator results from the environmental impacts of **several midpoint** indicators
- Examples: damage to human health, damage to ecosystems, damage to resource availability

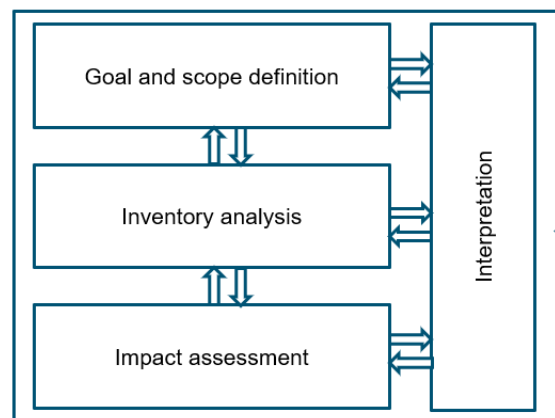
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- b) The brewers are curious and want to know more about Life Cycle Assessment (LCA). Please name the four phases of an LCA and sketch the interactions among them.

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Solution:

- 1) Definition of goal and scope
- 2) Inventory analysis
- 3) Impact assessment
- 4) Interpretation



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- c) Now we want to take a closer look at the two midpoint indicators **Global Warming (GW)** and **Photochemical oxidant formation (POF)**; released e.g. from the burning of hard coal) and the corresponding units in which they are measured. Two tables with several numbers are given; some are missing. Please complete the tables. The functional unit is 1 kWh electricity.

Table 3: Ecological impacts of the generation of 1 kWh electricity in different cities

	GW	POF
Electricity BS	?	21.2
Electricity MD	17.3	?

Table 4: Life Cycle Inventory results for electricity generation in different cities

	Electricity BS	Electricity MD
CO_2	25.1	?
CH_4	?	45
NMVOC	20.1	4.8
CO	178.7	37.2

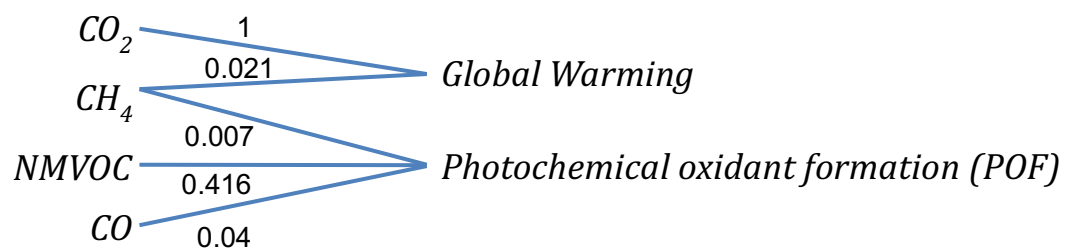


Figure 1: Allocation of Life Cycle Inventory results to ecological impact category

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Solution:

Electricity BS:

$$POF = 0.007 CH_4 + 0.416 NMVOC + 0.04 CO$$

$$21.2 = 0.007 CH_4 + 0.416 * 20.1 + 0.04 * 178.7 = 0.007 CH_4 + 15.51$$

$$0.007 CH_4 = 5.69 \rightarrow CH_4 = 812.9$$

$$GW = 1 CO_2 + 0.021 CH_4$$

$$GW = 25.1 + 0.021 * 812.9 = 25.1 + 17.1 \rightarrow GW = 42.2$$

Electricity MD

$$POF = 0.007 CH_4 + 0.416 NMVOC + 0.04 CO$$

$$POF = 0.007 * 45 + 0.416 * 4.8 + 0.04 * 37.2$$

$$POF = 3.8$$

$$GW = 1 CO_2 + 0.021 CH_4$$

$$17.3 = CO_2 + 0.021 * 45 = CO_2 + 0.95$$

$$CO_2 = 16.35$$

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- d) Determine the system vector of the brewing process (Figure 2) using the activity analytical model. Then use this system vector and the characterization factor matrix (Figure 3) to calculate the impact categories global warming in [CO₂ eq] and noise [dB].

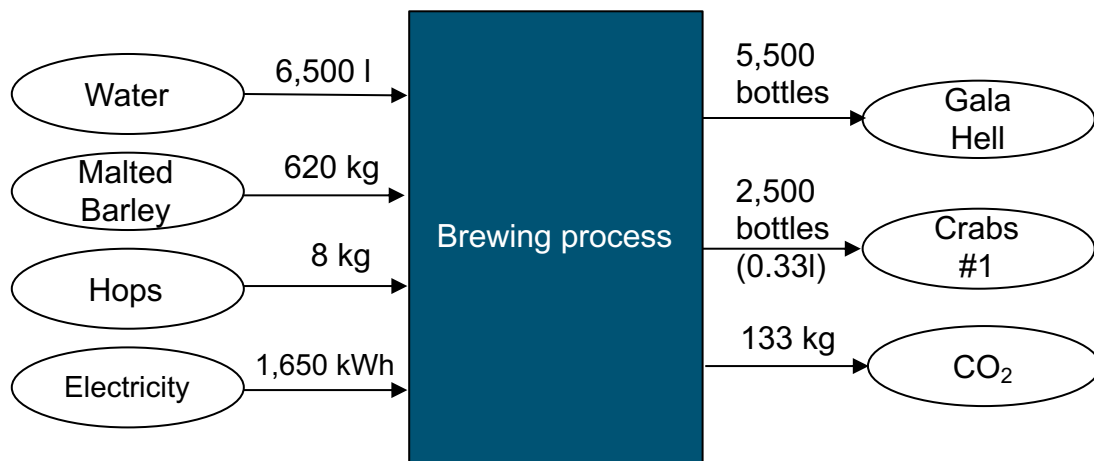


Figure 2: Brewing process

$$Q = \begin{pmatrix} \text{Water} & \text{Malted Barley} & \text{Hops} & \text{Electricity} & \text{Gala Hell} & \text{Crabs \#1} & \text{CO}_2 \\ \begin{pmatrix} 0 & 0 & 0 & -0.4 & 0 & 0 & 1 \\ -0.05 & 0 & 0 & 0 & 0.1 & 0.1 & 0 \end{pmatrix} & \begin{matrix} \text{Global warming} \\ \text{Noise} \end{matrix} \end{pmatrix}$$

Figure 3: Characterization factor matrix

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Solution:

$$\mathbf{z} = \begin{pmatrix} -6.500 \\ -620 \\ -8 \\ -1.650 \\ 5.500 \\ 2.500 \\ 133 \end{pmatrix}$$

Global warming: $w_1(\mathbf{z}) = 0 + 0 + 0 + (-0,4 * -1.650) + 0 + 0 + 1 * 133 =$
793 [kg CO₂-eq]

Noise: $w_1(\mathbf{z}) = (-0,05 * -6.500) + 0 + 0 + 0 + 0,1 * 5.500 + 0,1 * 2.500 + 0 =$
1.125 [dB]