Musterlösung zur Klausur „Digitale Signalverarbeitung“ 14.07.2009

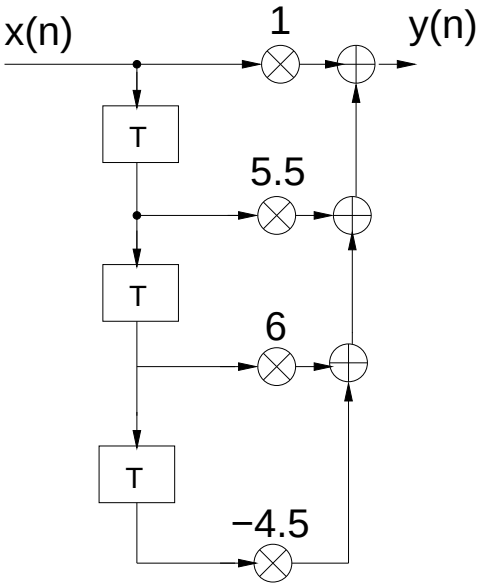
Aufgabe 1

a.) FIR, da Nenner = 1 (kein Bruch)

b.) $1 + 6z^{-1} + 9z^{-2} - 0.5z^{-1} - 3z^{-2} - 4.5z^{-3} = H(z)$

$\Rightarrow y(n) = x(n) + 5.5x(n-1) + 6x(n-2) - 4.5x(n-3)$

c.)



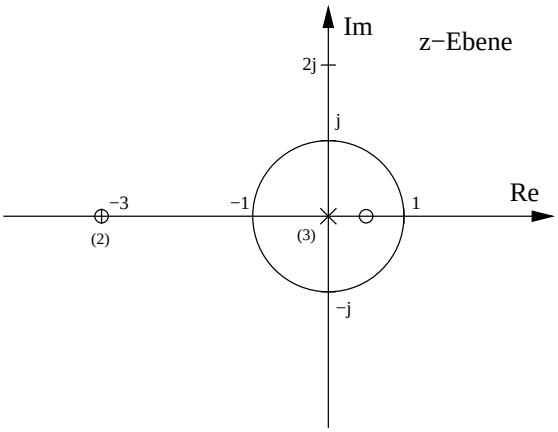
d.) $z_{0,1} = 0.5$

$1 + 6z^{-1} + 9z^{-2} = 0$

$z^2 + 6z + 9 = 0$

$z_{0,2/3} = -3 \pm \sqrt{9-9} \Rightarrow z_{0,2} = -3 \ , \ z_{0,3} = -3$

$z_{\infty,1} = z_{\infty,2} = z_{\infty,3} = 0$



e.) $H(z) = (1 - 0.5z^{-1})(1 + 3z^{-1})(1 + 3z^{-1})$

$H_{min}(z) = (1 - 0.5z^{-1})(1 + \frac{1}{3}z^{-1})(1 + \frac{1}{3}z^{-1})$

$H_{AP}(z) = \frac{(1+3z^{-1})(1+3z^{-1})}{(1+\frac{1}{3}z^{-1})(1+\frac{1}{3}z^{-1})}$

Aufgabe 2

a.) siehe Skript

b.) d_{st} = -20 log(δ_{st}) = -20 log(0.05) = 26.0206 dB

R_p = 20 log(1 + δ_p) - 20 log(1 - δ_p) = 20 log(1.08) - 20 log(0.92) = 1.3927 dB

c.) $N_b \geq \frac{26.0206-7.95}{2.29 \cdot 0.4\pi} = 6.2795$

$\Rightarrow N_b = 7$

d.) d_{st} = 26.0206 dB

R_p = -20 log(1 - δ_p) = 0.7242 dB

e.) $\delta_p = 0.05$
$\Omega' = \Omega_p = 0.2\pi$
$\omega' = \frac{\Omega_p}{T} = 1000\pi \cdot \frac{1}{s}$
$\Omega_c = 0.4\pi$

$\omega_{st} = v \cdot \tan(\frac{\Omega_{st}}{2}) = 9668.8\frac{1}{s} \cdot \tan(0.3\pi) = 13308\frac{1}{s}$

$v = \frac{\omega'}{\tan(\frac{\Omega'}{2})} = \frac{\pi \cdot 1000 \cdot \frac{1}{s}}{\tan(0.1\pi)} = 9668.8\frac{1}{s}$

$\omega_c = 7024.8\frac{1}{s}$

$\omega_p = 3141.6\frac{1}{s}$

f.) siehe Skript

g.) Passband: $0.92^2 = 0.8464$

$|H_a(j\omega)|^2 = \frac{1}{1+(\frac{j\omega_p}{j\omega_c})^{2N}}$

$\mathrm{N} = 2 \Rightarrow \frac{1}{1+(\frac{j\omega_p}{j\omega_c})^{2N}} = 0.96154 \rightarrow$ erfüllt

Stopband: $0.05^2 = 0.0025$

$|H_a(j\omega)|^2 = \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}}$

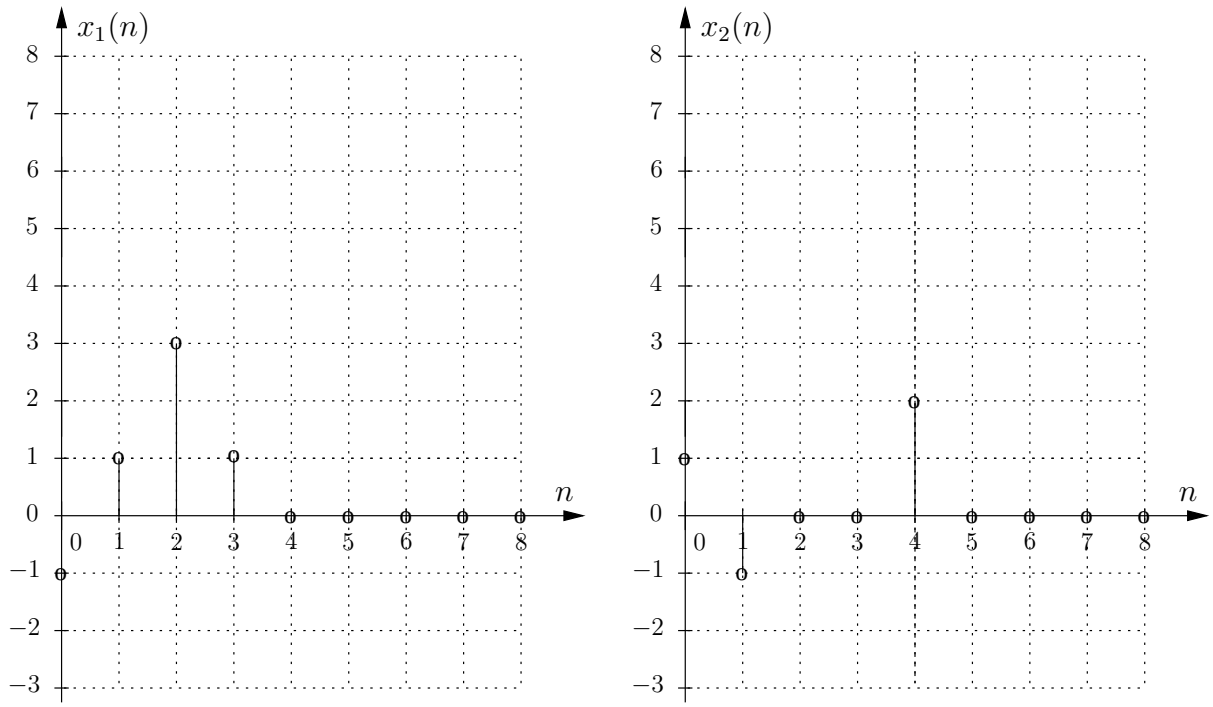
$\mathrm{N} = 1 \Rightarrow \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}} = 0.2179 \rightarrow$ nicht erfüllt
$\mathrm{N} = 2 \Rightarrow \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}} = 0.072 \rightarrow$ nicht erfüllt
$\mathrm{N} = 3 \Rightarrow \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}} = 0.0212 \rightarrow$ nicht erfüllt
$\mathrm{N} = 4 \Rightarrow \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}} = 0.0060 \rightarrow$ nicht erfüllt
$\mathrm{N} = 5 \Rightarrow \frac{1}{1+(\frac{jw_{st}}{j\omega_c})^{2N}} = 0.0017 \rightarrow$ erfüllt

h.) IIR, da hier weniger Koeffizienten.

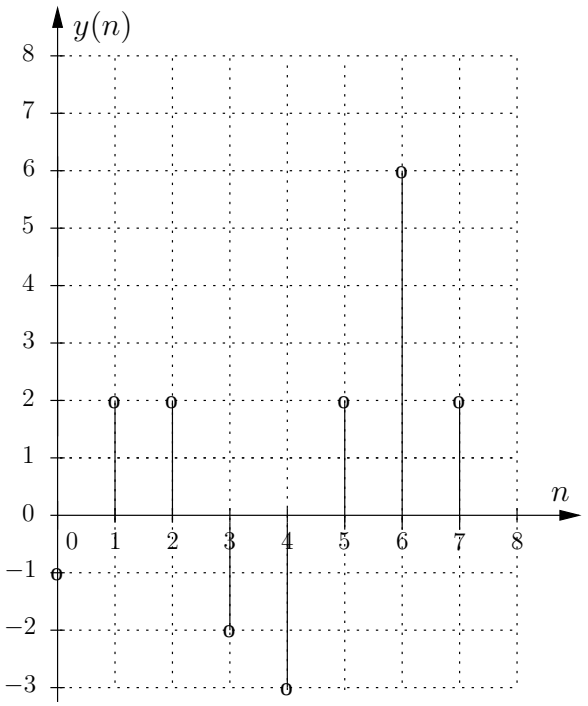
i.) FIR, da lin. Phase möglich.

Aufgabe 3

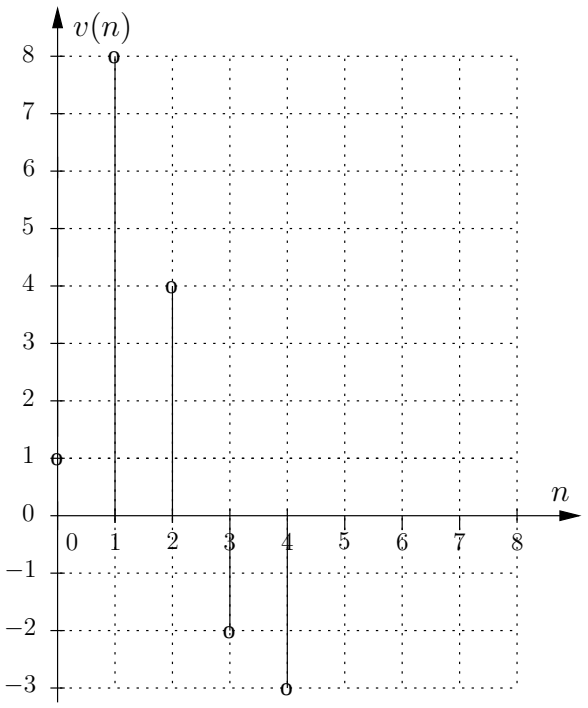
a.)



b.)



c.)



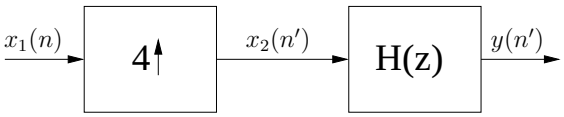
d.) b) lineare Faltung

c) zyklische Faltung

$K_{min} = 4 + 5 - 1 = 8$

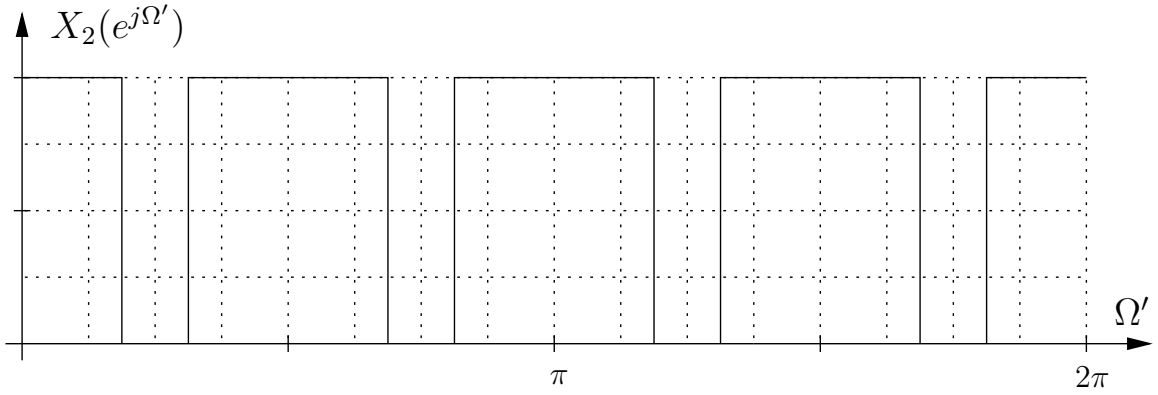
Aufgabe 4

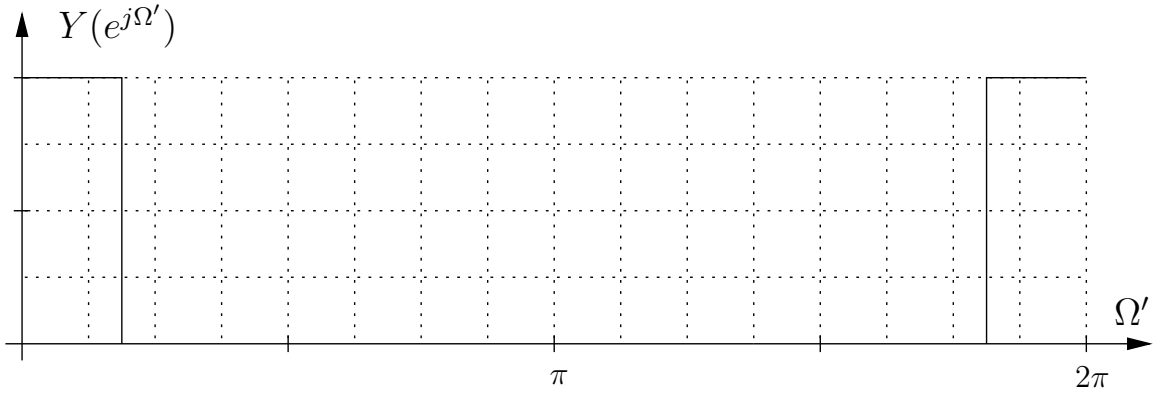
a.)



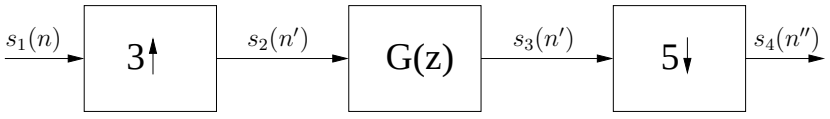
b.) $\Omega'_g = \frac{\pi}{4}$

c.)





d.)



e.) geringere Rechenkomplexität