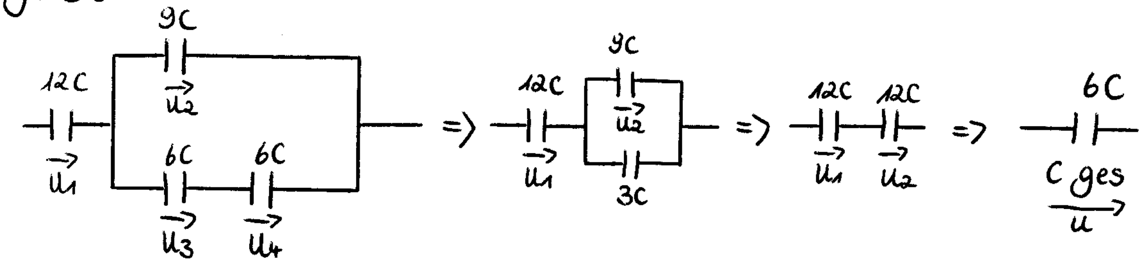
Aufgabe 1

a)



b) $u_3 = u_4 = \frac{u_2}{2}$; Widerstandsbetrachtung $u_1 = u_2 = \frac{u}{2}$

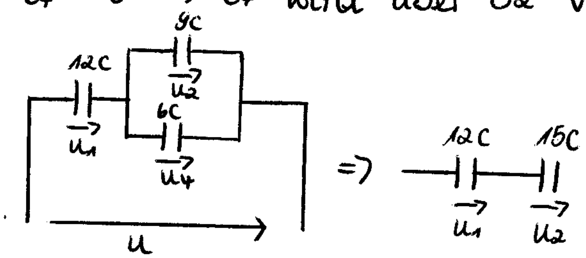
$u_3 = u_4 = \frac{u_2}{2} = \frac{u_1}{2} = \frac{u}{4}$; u_1 und u_2 sind die größten Spannungen im Netzwerk => $u = u_{max} = 2u_1 = 2u_2 = 2u_N$
$= 2 \cdot 50\ V = 100\ V$

c) $W_1 = \frac{1}{2} C_{ges} u^2 = \frac{1}{2} \cdot 6C \cdot u^2 = 3 \cdot 1\,uF \cdot 10^4\ V = 3 \cdot 10^{-2}\ Ws$

d) Ladung bleibt erhalten !

Vor Schließen von S2 : $Q = u_1 C_1 = \frac{u_{max}}{2} \cdot 12C = 6\,u_{max} C$

$C_4 = 0V$ => C_4 wird über S2 vollständig entladen !



Ladung auf C_1 => Spannung bleibt erhalten

$u_1 = \frac{u_{max}}{2} = 50\,V$

$\underline{\underline{u_2 = u_4}}$

$\frac{u_2}{u_1} = \frac{\frac{1}{15C}}{\frac{1}{12C}} = \frac{12}{15} = \frac{4}{5}$ => $u_2 = \frac{4}{5} \cdot u_1 = \frac{\cancel{4}^{2}}{5} \cdot \frac{u_{max}}{2} = 40\,V$

$u = 90\,V$

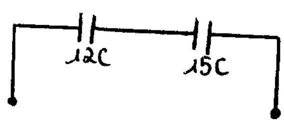
$u = u_1 + u_2 = 50 + 40 = 90\,V$

$u_1 = 50\,V$

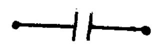
$u_2 = u_4 = 40\,V$

$u_3 = 0\,V$

e) $W = \frac{Q^2}{C_{ges}} = \frac{C_{ges}\,u^2}{2}$



=>



$C_{ges} = \frac{180}{27}\ C$

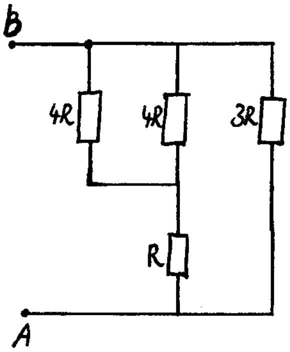
$\frac{20}{3}\ C$

$W_2 = \frac{1}{2} \cdot \frac{180}{27} C \cdot 90^2\,V = 0{,}027\ Ws$

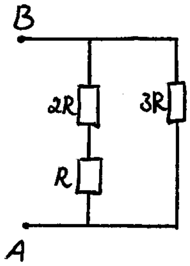
Energieabgabe bei Umschalten durch HF - Strahlung

Aufgabe 2

a)

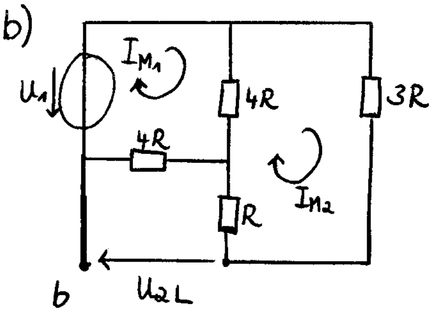


$\Rightarrow$



$$\Rightarrow R_i = \frac{9}{6} R = \frac{3}{2} R$$

b)



$= I_{M2} \cdot R + I_{M1} \cdot 4R$

$$I_{M1}(4R + 4R) - U_1 - I_{M2} 4R = 0$$

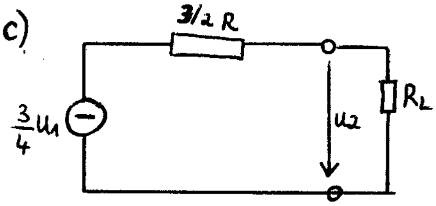
$$I_{M2}(3R + R + 4R) - I_{M1} 4R = 0 \rightarrow I_{M2} = \frac{I_{M1} \cancel{4R}}{{}_2\cancel{8R}} = \frac{1}{2} = \frac{I_{M1}}{2}$$

$$I_{M1} 8R - U_1 - \frac{I_{M1}}{\cancel{2}} \cancel{4}^{2} R = 0$$

$$6 I_{M1} R = U_1$$

$$U_{2L} = \frac{U_1}{12\cancel{R}} \cancel{R} + \frac{U_1}{{}_3\cancel{6}\cancel{R}} \cancel{4}^{2}\cancel{R} = \frac{U_1}{12} + \frac{2}{3} U_1 = \frac{U_1 + 8U_1}{12} = \frac{9}{12} U_1 = \underline{\underline{\frac{3}{4} U_1}}$$

c)

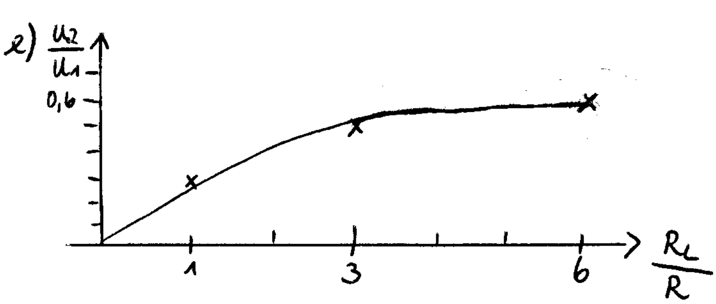


$$\frac{U_2}{3/4\, U_1} = \frac{R_L}{3/2\, R + R_L} \Rightarrow \frac{U_2}{U_1} = \frac{3/4\, R_L}{3/2\, R + R_L} = \underline{\underline{\frac{3/4\; R_L/R}{3/2 + \frac{R_L}{R}}}}$$

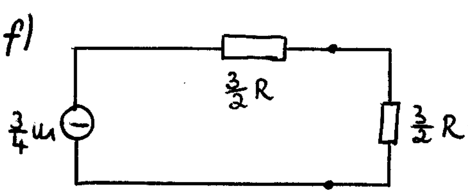
d) $$\frac{R_L}{R} = \frac{3\, U_2/U_1}{3/2 - 2\, U_2/U_1}$$

R_L/R	1	3	6
U_2/U_1	0,3	0,5	0,6

e)



f)

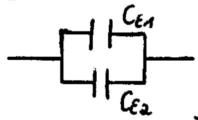


Leistungsanpassung

$R_L = R_i$

$$P = \frac{\left(\frac{3}{4} U_1\right)^2}{\frac{3}{2} R + \frac{3}{2} R} = \frac{\frac{9}{16} U_1^2}{3R} = \frac{3}{16} \frac{U_1^2}{R} = 2\,W$$

Aufgabe 3

a) ESB 

b) $Q = \oint |\vec{D}_i| \cdot d\vec{A}$

$= D_1 A_1 + D_2 A_2$

für $\underline{r_1 < r < r_2}$:

$A = 4\pi r^2$

$\underline{A_1} = \frac{3}{4} \cdot 4\pi r^2 = \underline{3\pi r^2}$

$\underline{A_2} = \frac{1}{4} \cdot 4\pi r^2 = \underline{\pi r^2}$

$\Rightarrow Q = 3 D_1 \cdot \pi r^2 + D_2 \cdot \pi r^2$

$Q = \pi r^2 (3 D_1 + D_2)$

c) $E = \frac{1}{\varepsilon} D$; $D_1 = \varepsilon_0 \varepsilon_{r1} \cdot E$

$D_2 = \varepsilon_0 \varepsilon_{r2} \cdot E$

$\rightarrow Q = \pi r^2 \cdot E \cdot (3 \varepsilon_1 + \varepsilon_2)$

$\Rightarrow E(r) = \frac{Q}{\pi r^2 \cdot (3\varepsilon_1 + \varepsilon_2)} = \frac{Q}{\pi r^2 \varepsilon_0 (3\varepsilon_{r1} + \varepsilon_{r2})} = \frac{Q}{10 \cdot \varepsilon_0 \pi r^2}$

d) $U = \int \vec{E} \cdot d\vec{r}$

$\varphi_{r1} - \varphi_{r2} = U_{r1\,r2} = \int_{r_1}^{r_2} E(r) \cdot dr = \int_{r_1}^{r_2} \frac{Q}{\pi r^2 \cdot \varepsilon_0 \cdot (3\varepsilon_{r1} + \varepsilon_{r2})} \cdot dr$

$= \frac{Q}{\pi \cdot \varepsilon_0 (3\varepsilon_{r1} + \varepsilon_{r2})} \int_{r_1}^{r_2} \frac{dr}{r^2} = \frac{Q}{\pi \varepsilon_0 \cdot (3\varepsilon_{r1} + \varepsilon_{r2})} \left(-\frac{1}{r} \Big|_{r_1}^{r_2} \right)$

$= \frac{Q}{\pi \varepsilon_0 (3\varepsilon_{r1} + \varepsilon_{r2})} \left[-\frac{1}{r_2} + \frac{1}{r_1} \right] = \frac{Q}{\pi \varepsilon_0 (3\varepsilon_r + \varepsilon_{r2})} \left(\frac{r_2 - r_1}{r_1 r_2} \right)$

$= \frac{10^{-9} As}{\pi \cdot \frac{10^{-9}}{36\pi} \frac{As}{Vm} \cdot 10} \cdot \frac{4 \cdot 10^{-2} m}{12 \cdot 10^{-4} m} = \underline{\underline{120\ V}}$

e) $C = \frac{Q}{U}$

$C_{Kugel} = \frac{Q}{U_{r1\,r2}} = \frac{Q}{\frac{Q}{\pi \varepsilon_0 (3\varepsilon_{r1} + \varepsilon_{r2})} \left(\frac{r_2 - r_1}{r_1 r_2} \right)} = \pi \varepsilon_0 (3\varepsilon_{r1} + \varepsilon_{r2}) \frac{r_2 \cdot r_1}{r_1 - r_2}$

$= 10 \pi \varepsilon_0 \cdot \frac{r_2 \cdot r_1}{r_2 - r_1}$

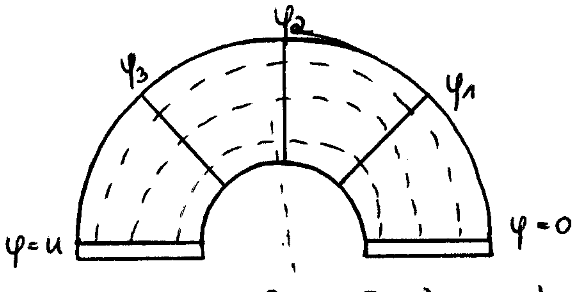
f) $C = \frac{Q}{U} = \frac{10^{-9} As}{120\ V} = \frac{1}{120}\ nF = \underline{\underline{8{,}33\ pF}}$

$\underline{\text{oder:}}$ $C = 10\cancel{\pi} \cdot \frac{10^{-9}}{{}_3\cancel{36}\cancel{\pi}} \frac{As}{Vm} \cdot \frac{\cancel{12} \cdot 10^{-4} m^2}{4 \cdot 10^{-2} m}$

$= \frac{1}{12} \cdot 10^{-10} \frac{As}{V} = \underline{\underline{8{,}33\ pF}}$

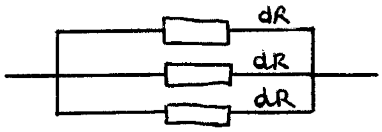
Aufgabe 4

a)



$S(r)$, $E(r) =$ const

b)



Widerstandselemente dR müssen auf Fläche mit konstanter Stromdichte liegen!

Allgemein $R = \rho \frac{l}{A}$ hier $l = \pi r$, $dA = b\,dr$

$$dR = \rho \frac{\pi r}{b\,dr}$$

c) Stromdichte und Feldstärke sind auf Mantelflächen des Hohlzylindersegments konstant -> radiale Abhängigkeit $(\sim \frac{1}{r})$

Aus $I = \int \vec{S}\, d\vec{A}$ => $S(r) = \frac{dI}{dA} = \frac{U}{dR\, dA} = \frac{U}{\rho \pi r}$

$E(r) = \rho \cdot S(r) = \frac{U}{\pi r}$

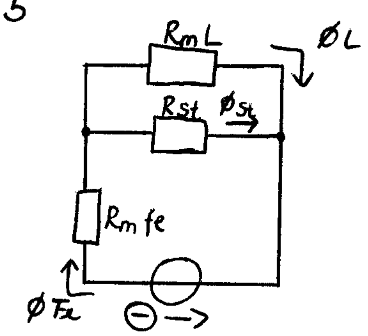
d) $R = \frac{U}{I} = \frac{\int_0^{\pi} \frac{U}{\pi \cdot r} \cdot r\,d\alpha}{\int_{r_i}^{r_a} \frac{U}{\rho \pi r} \cdot b\,dr} = \rho \frac{\pi}{b \ln \frac{r_a}{r_i}}$

$U = \int E\,ds$

$ds = r\,d\alpha$

Aufgabe 5

a) ESB



b) $\phi_L = 0{,}8 \cdot \phi_{fe}$

$\phi_{SL} = 0{,}2 \; \phi_{fe}$ $\qquad \phi_{St} + \phi_L = \phi_{fe}$

$\phi_{fe} = \frac{\phi_L}{0{,}8}$

$\phi_L = B_L \cdot A = 1\,T \cdot \pi R^2 = 1\,T \cdot \pi \left(\frac{2 \cdot 10^{-2} m}{2}\right)^2$

$\underline{\underline{\phi_L = 3{,}14 \cdot 10^{-4} \; Tm^2}}$ $\qquad Tm^2 = V$

$\underline{\underline{\phi_{fe} = 3{,}927 \cdot 10^{-4} \; Vs}}$

$\underline{\underline{A}} = \pi R^2 = \pi \left(\frac{2 \cdot 10^{-2} m}{2}\right)^2 = \underline{\underline{3{,}14 \cdot 10^{-4} \, m^2}}$

$\underline{\underline{\phi_{St} = 0{,}7854 \cdot 10^{-4} \; Vs}}$

$\underline{\underline{B_{fe}}} = \frac{\phi_{fe}}{A} = \underline{\underline{1{,}25 \; T}}$

$R_{mfe} = \frac{l_{fe}}{\mu_0 \mu_{rfe} \cdot A}$ $\qquad l_{fe} = \pi \cdot D - \delta = \pi \cdot 0{,}2\,m - 5 \cdot 10^{-3}\,m$

$\underline{\underline{l_{fe} = 0{,}623\,m}}$

$\underline{\underline{R_{mfe} = 157{,}96 \cdot 10^3 \; \frac{1}{H}}}$

$\underline{\underline{R_\delta}} = \frac{\delta}{\mu_0 A} = \frac{5 \cdot 10^{-3}\,m}{1{,}256 \cdot 10^{-6} \frac{H}{m} \cdot 3{,}14 \cdot 10^{-4}\,m^2} = \underline{\underline{1{,}2678 \cdot 10^7 \; \frac{1}{H}}}$

$\underline{\underline{H_L}} = \frac{B_L}{\mu_0} = \frac{1\,T}{1{,}256 \cdot 10^{-6} \frac{H}{m}} = \underline{\underline{0{,}796 \cdot 10^6 \; \frac{A}{m}}}$

$\underline{\underline{H_{fe}}} = \frac{B_{fe}}{\mu_0 \mu_{rfe}} = \frac{1{,}25\,T}{1{,}256 \cdot 10^{-6} \frac{H}{m} \cdot 10^4} = \underline{\underline{99{,}5 \; \frac{A}{m}}}$

$V_{fe} = \phi_{fe} \cdot R_{mfe} = \phi_{fe} \cdot \frac{l_{fe}}{\mu_0 \cdot \mu_{rfe} \cdot A} = \phi_{fe} \cdot \frac{\pi \cdot D - \delta}{\mu_0 \cdot \mu_{rfe} \cdot A}$

$\underline{\underline{V_{fe} = 62{,}03\,A}}$ oder $\underline{\underline{V_{fe}}} = H_{fe} \cdot l_{fe} = \underline{\underline{61{,}98\,A}}$

$\underline{\underline{V_L}} = \phi_L \cdot R_\delta = H_L \cdot \delta = \underline{\underline{3{,}98 \cdot 10^3\,A}}$

$\underline{\underline{\Theta}} = V_{fe} + V_L = H_{fe} \cdot l_{fe} + H_L \cdot \delta = \underline{\underline{4041{,}98\,A}}$

c) $V_q = \frac{\Theta}{N} (R_i + R_m) = \frac{4041{,}98A}{2000} (0{,}5\,\Omega + R_m)$

$R_m = \frac{S \cdot l_m \cdot N}{A_m} = \frac{1{,}78\,\Omega m \cdot 8 \cdot 10^{-2}\,m \cdot 2000}{0{,}785 \cdot 10^{-6}\,m^2} =$ $\qquad A_m = \pi R^2 = \pi \left(\frac{d}{2}\right)^2$

$d = 0{,}001\,m$ Drahtdurchmesser

$\underline{\underline{A_m = 0{,}785 \cdot 10^{-6}\,m^2}}$

$\underline{\underline{R_m = 362{,}8 \cdot 10^6\,\Omega}}$

$\underline{\underline{V_q = 733{,}2 \cdot 10^6\,V}}$

d) Durchflutungsgesetz

$$B^* = \mu_0 \cdot H$$

$$\underline{\underline{H^*}} = \frac{\Theta}{l_{sp}} = \frac{\Theta}{2\pi R} = \frac{\Theta}{2\pi \cdot \frac{D}{2}} = \frac{4041{,}98\,A}{2\pi \cdot \frac{0{,}2\,m}{2}} = \underline{\underline{6433\,\frac{A}{m}}}$$

$$\underline{\underline{B^*}} = 1{,}256 \cdot 10^{-6}\,\frac{H}{m} \cdot 6433\,\frac{A}{m} = \underline{\underline{8{,}08 \cdot 10^{-3}\,\frac{Vs}{m^2}}}$$

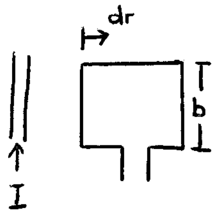
Aufgabe 6

a) Leiter 1 und 2

(I) $U_i = -N \cdot \frac{d\Phi}{dt}$ (II) $\Phi = \int \vec{B} \, d\vec{A}$ (III) $B = \mu_0 \mu_r \cdot H \overset{!}{=} \mu_0 H \quad (\mu_r = 1)$

$= \mu_0 \cdot \frac{I}{2\pi r}$

(IV) $dA = b \cdot dr$



$$\Rightarrow \Phi = \int_a^{a+b} \mu_0 \cdot \frac{I}{2\pi r} \cdot b \, dr = \frac{\mu_0 I b}{2\pi} \int_a^{a+b} \frac{1}{r} \, dr = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+b}{a}\right) \quad \text{(V)}$$

V in I:

$$U_i = -N \cdot d\left(\frac{\mu_0 b}{2\pi} \ln\left(\frac{a+b}{a}\right) \cdot \hat{I} \cdot \sin(wt)\right) dt$$

$$= -N \cdot \frac{\mu_0 b}{2\pi} \ln\left(\frac{a+b}{a}\right) \cdot \hat{I} \cdot \left[\sin(wt) \frac{d}{dt}\right]$$

$$= -N \cdot \frac{\mu_0 b}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right) \cdot \hat{I} \cdot w \cdot \cos(wt)$$

Aufgrund der Geometrie: $U_{iges}(t) \overset{= 2 U_1(t)}{=} U_{i1}(t) + U_{i2}(t)$ mit $i_1(t) = i_2(t)$

$$U_{iges} = \cancel{2} \cdot \left(-N \frac{\mu_0 b}{\cancel{2}\pi} \cdot \ln\left(\frac{a+b}{a}\right) \cdot \hat{I} \cdot w \cdot \cos(wt)\right)$$

$$= -N\mu_0 \cdot \frac{b}{\pi} \cdot \ln\left(\frac{a+b}{a}\right) \cdot \hat{I} \cdot w \cdot \cos(wt)$$

$$[= -10 \cdot 4\cancel{\pi} \cdot 10^{-7} \frac{Vs}{A\cancel{m}} \cdot \frac{20 \cdot 10^{-3} \cancel{m}}{\cancel{\pi}} \cdot \hat{I} \cdot w \cdot \cos(wt)$$

$$= 8 \cdot 10^{-8} \cdot I \cdot w \cdot \cos(wt) \frac{Vs}{A} \cdot \underbrace{\ln\left(\frac{a+b}{a}\right)}_{1,1}]$$

b) Leiter 3

Da sich an der Geometrie prinzipiell nichts ändert: wie U_{i1} bzw. U_{i2}

$\Rightarrow$ wenn Formulierung oder $\Phi = \mu_0 \frac{I_3 b}{2\pi} \ln\left(\frac{a+b}{a}\right)$

oder

$$\Phi = \frac{\mu_0 I_3 b}{2\pi} \int_a^{a+b} \frac{1}{r} \, dr$$

$$U_{i3} = -N \cdot \mu_0 \cdot \frac{b}{2\pi} \cdot \ln\left(\frac{a+b}{a}\right) \cdot \hat{I}_3 \cdot w \cdot \cos(wt)$$

c) Gesamt induzierte Spannung
- ist max, wenn $\cos(wt) = 1$
- $U_{ges} = U_{12max} - U_{3max}$ (aufgrund der Richtungen)

$U_{ges\,max} = -2{,}071 \cdot 10^{-4}\ V = -0{,}207\ mV$

$U_{eff} = \frac{U_{max}}{\sqrt{2}} = -0{,}146\ mV$

d) $U_{ges\,max} = 0$, wenn $\hat{I}_3 - 2\hat{I} = 0$

$\Rightarrow \hat{I}_3 \stackrel{!}{=} 2\hat{I}$

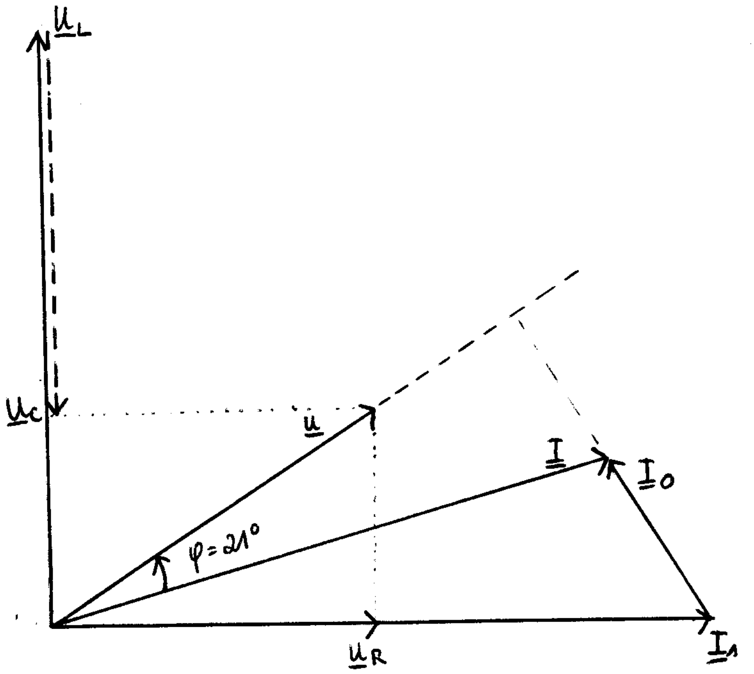
Aufgabe 7

a) $\underline{U}_L = j\omega L \, \underline{I}_1 \rightarrow |\underline{I}_1| = \dfrac{|\underline{U}_L|}{\omega L} = \dfrac{10\,V}{10^6\,s^{-1} \cdot 10^{-3}\,VsA^{-1}} = 10^{-2}\,A = 10\,mA$

$|\underline{U}_R| = |\underline{I}_1| \cdot R = 10^{-2}\,A \cdot 0{,}4 \cdot 10^3\,\Omega = 4\,V$

$|\underline{U}_C| = \dfrac{|\underline{I}_1|}{\omega C} = \dfrac{10^{-2}\,A}{10^6\,s^{-1} \cdot 1428 \cdot 10^{-12}\,As\,V^{-1}} = \dfrac{10^4}{1428}\,V = 7\,V$

b)



aus ZD:

$|\underline{U}| = 5\,V$

$\underline{U}_{C2} = \underline{U} = \dfrac{1}{j\omega C_2} \cdot \underline{I}_2$

$|\underline{I}_2| = |\underline{U}_{C2}| \, \omega C_2 = 5\,V \cdot 10^6\,s^{-1} \cdot 600 \cdot 10^{-12}\,\dfrac{As}{V}$

$= 3 \cdot 10^{-3}\,A = 3\,mA$

$\underline{I} = \underline{I}_1 + \underline{I}_2 \quad \underline{I}_2 \perp \underline{U}$

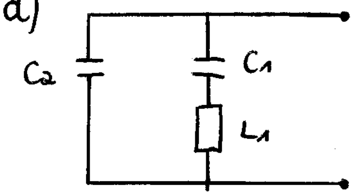
aus ZD $\underline{I} = 8{,}6 \cdot mA$

c) $P = |\underline{U}| \cdot |\underline{I}| \cdot \cos\varphi = 5\,V \cdot 8{,}6 \cdot 10^{-3}\,A \cdot \cos 21° = 40{,}14\,mW$

oder $P = (\underline{I}_1)^2 \cdot R = (10\,mA)^2 \cdot 400\,\Omega = 40\,mW$

$Q = |\underline{U}| \cdot |\underline{I}| \cdot \sin\varphi = 5\,V \cdot 8{,}6 \cdot 10^{-3}\,A \; \sin 21° = 15{,}4\,mW$

d)



$\underline{Z} = \left(j\omega L + \dfrac{1}{j\omega C_1}\right) \,\|\, \dfrac{1}{j\omega C_2}$

$\underline{Z} = \dfrac{\left(j\omega L + \dfrac{1}{j\omega C_1}\right) \cdot \dfrac{1}{j\omega C_2}}{j\omega L + \dfrac{1}{j\omega C_1} + \dfrac{1}{j\omega C_2}} \quad \Big| \cdot j\omega C_2$

$= \dfrac{j\omega L + \dfrac{1}{j\omega C_1}}{1 + \dfrac{C_2}{C_1} - \omega^2 C_2 L} \quad \Big| \cdot \omega L_1 \quad = j \dfrac{\omega L \cdot \omega C_1 - 1}{\omega C_1 + \omega C_2 - \omega^3 C_1 C_2 L}$

$\underline{Z} = j \dfrac{\omega^2 L C_1 - 1}{\omega (C_1 + C_2) - \omega^3 C_1 C_2 L} = j \dfrac{\omega L - \dfrac{1}{\omega C_1}}{1 - \omega^2 L C_2 + \dfrac{C_2}{C_1}}$

e) $\omega_{SR}^2 \, L C_1 - 1 = 0$

$$\omega_{SR}^2 = \frac{1}{LC_1} \qquad f_{SR} = \frac{1}{2\pi} \sqrt{\frac{1}{LC}}$$

$$\omega_{PR} \, (C_1 + C_2) - \omega_{PR}^3 \, C_1 C_2 L = 0$$

$$\omega_{PR}^2 = \frac{C_1 + C_2}{C_1 C_2 L}$$

$$f_{PR} = \frac{1}{2\pi} \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}} = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC_1}} \sqrt{1 + \frac{C_1}{C_2}}$$

$$f_{PR} = f_{SR} \sqrt{1 + \frac{C}{C_0}}$$

Aufgabe 8

a)
$$\frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} + R = Z$$

$$\frac{j\omega L \cdot \frac{1}{j\omega C}}{\frac{1-\omega^2 LC}{j\omega C}} + R = j\omega L \cdot \frac{1}{j\omega C} \cdot \frac{j\omega C}{1-\omega^2 LC} + R$$

$$= R - j\frac{\omega L}{\omega^2 LC - 1} = R + \frac{j\omega L}{1-\omega^2 LC} = Z$$

b) Parallelschwingkreis

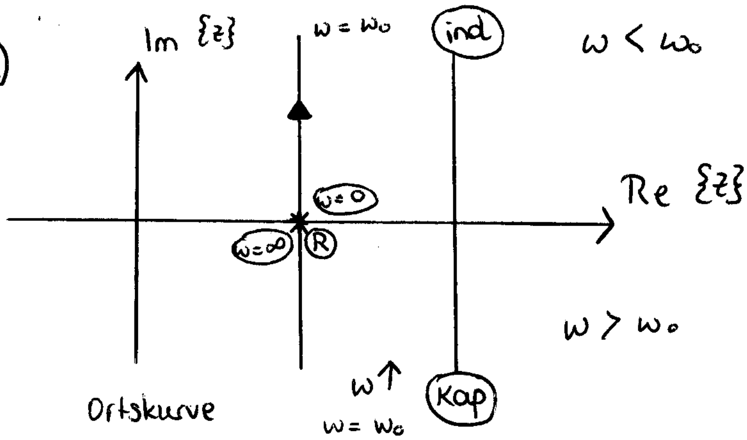
Resonanzbedingung $\underline{\underline{Im\{Z\} = \infty}}$ oder $\underline{\underline{1-\omega^2 LC \overset{!}{=} 0}}$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

c)

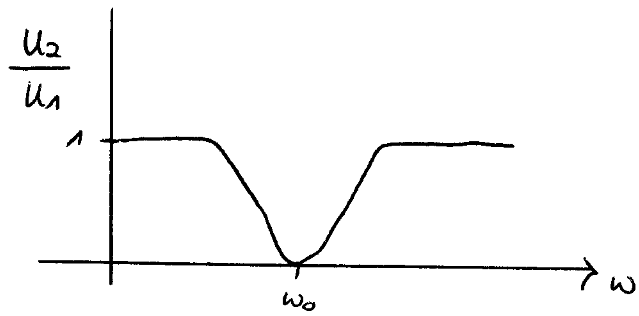
	$\omega = 0$	$\omega = \omega_0$	$\omega = \infty$
$\underline{Z}$	R	∞	R

d)



e)
$$\underline{\underline{\frac{U_2}{U_1}}} = \frac{R}{\underline{Z}} = \frac{R}{\frac{j\omega L}{1-\omega^2 LC} + R} = \frac{R - \omega^2 LCR}{j\omega L - \omega^2 LCR + R} = \frac{R - \omega^2 LCR}{R + \omega(jL - \omega LCR)}$$

$\frac{U_2}{U_1}$	1	0	1
$\underline{Z}$	R	∞	R
ω	0	ω_0	∞



f) $Q = \infty$ Parallelschwingreis ohne Re

$d = 0$

$$d = \frac{1}{Q}$$