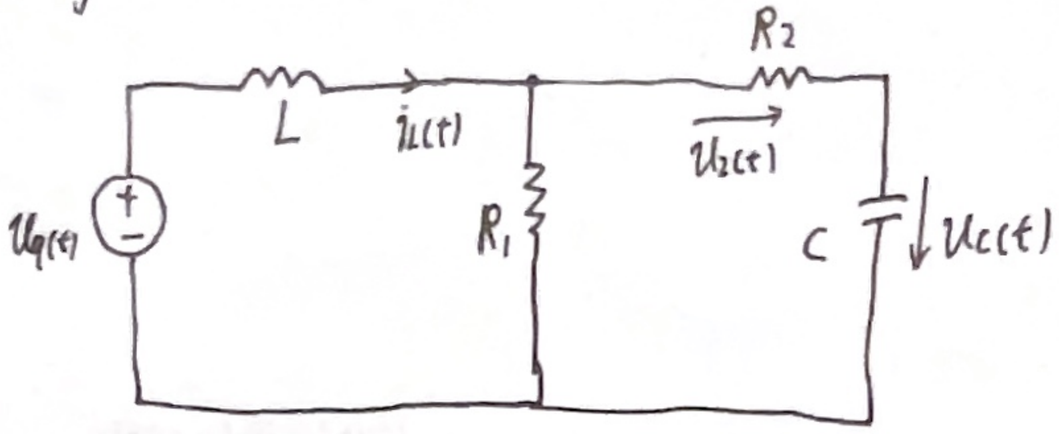
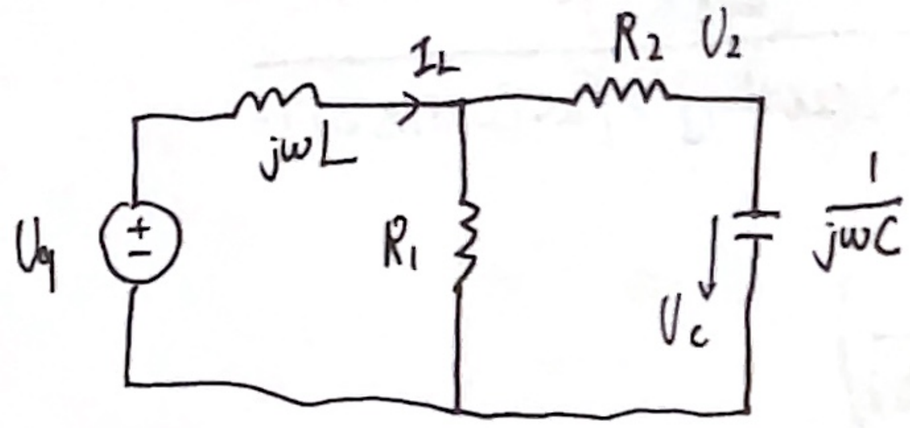
Aufgabe 5



$u_q(t) = V_0 \cos(\omega_0 t + \varphi_1)$, $L>0$, $C>0$, $(2\omega_0)^2 LC((R_1+R_2) < R_1 < (3\omega_0)^2 LC((R_1+R_2)$

i.e. Netzwerk ist eingeschwungen

a. Zeichnen Frequenzbereich



b. $\Rightarrow$ Frequenzgang $H(j\omega) = \frac{U_2}{U_q}$

$$Z_{2C} = R_2 + \frac{1}{j\omega C} \quad , \quad Z_{12C} = \frac{Z_{2C} R_1}{Z_{2C} + R_1} = \frac{(\frac{1}{j\omega C} + R_2) R_1}{\frac{1}{j\omega C} + R_2 + R_1}$$

$$Z_{ges} = j\omega L + Z_{12C}$$

$$I_L = \frac{U_q}{Z_{ges}} \quad , \quad U_{12C} = U_q \frac{Z_{12C}}{Z_{ges}} \quad , \quad U_2 = U_{12C} \frac{R_2}{Z_{2C}}$$

$$\Rightarrow U_2 = U_q \frac{Z_{12C}}{Z_{ges}} \frac{R_2}{Z_{2C}} = U_q \frac{R_2}{\frac{1}{j\omega C} + R_2} \frac{\frac{(\frac{1}{j\omega C} + R_2) R_1}{\frac{1}{j\omega C} + R_2 + R_1}}{\frac{(\frac{1}{j\omega C} + R_2) R_1}{\frac{1}{j\omega C} + R_2 + R_1} + j\omega L}$$

$$= U_q \frac{j\omega R_2 C}{1 + j\omega R_2 C} \frac{(\frac{1}{j\omega C} + R_2) R_1}{(\frac{1}{j\omega C} + R_2) R_1 + j\omega L (\frac{1}{j\omega C} + R_2 + R_1)} \frac{R_2}{\frac{1}{j\omega C} + R_2}$$

$$= U_q \frac{j\omega R_1 R_2 C}{(1 + j\omega R_2 C) R_1 + j\omega L [1 + j\omega C (R_2 + R_1)]}$$

$$= U_q \frac{j\omega R_1 R_2 C}{R_1 + j\omega R_1 R_2 C + j\omega L + (j\omega)^2 LC(R_1 + R_2)}$$

$$\Leftrightarrow H(j\omega) = \frac{U_2}{U_q} = \frac{j\omega R_1 R_2 C}{(j\omega)^2 LC(R_1 + R_2) + j\omega (L + R_1 R_2 C) + R_1}$$

c. $u_q(t) = V_0 \cos(\omega_0 t)$

求 $u_2(t)$, $-\frac{\pi}{2} < \arctan < \frac{\pi}{2}$

$$u_q(t) = \mathrm{Re}\{V_0 e^{j0} e^{j\omega_0 t}\} = \mathrm{Re}\{U_q e^{j\omega_0 t}\}$$

$$u_2(t) = \mathrm{Re}\{H(j\omega_0) U_q e^{j\omega_0 t}\} = \mathrm{Re}\{|H(j\omega_0)| e^{j\varphi_H(\omega_0)} V_0 e^{j\omega_0 t}\} = \mathrm{Re}\{|H(j\omega_0)| V_0 e^{j(\omega_0 t + \varphi_H(\omega_0))}\}$$

$$= |H(j\omega_0)| V_0 \cos(\omega_0 t + \varphi_H(\omega_0))$$

$$|H(j\omega_0)| = \left| \frac{j\omega_0 R_1 R_2 C}{R_1 - \omega_0^2 LC(R_1+R_2) + j\omega_0(L+R_1R_2C)} \right| = \sqrt{\frac{(\omega_0 R_1 R_2 C)^2}{[R_1 - \omega_0^2 LC(R_1+R_2)]^2 + [\omega_0(L+R_1R_2C)]^2}}$$

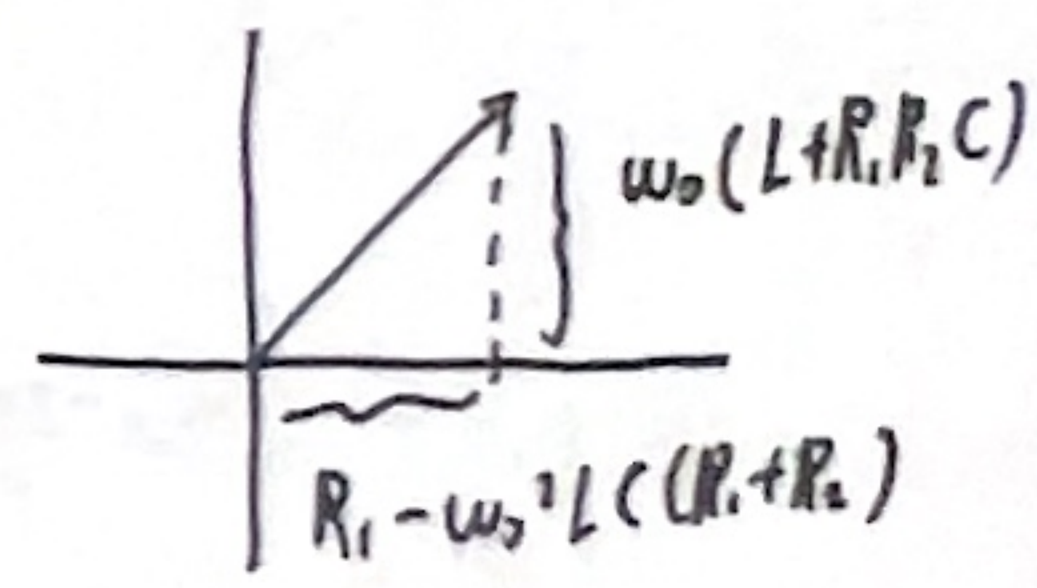
$$\varphi_H(\omega_0) = \arctan(\omega_0 R_1 R_2 C) - \arctan\left(\frac{\omega_0(L+R_1R_2C)}{R_1 - \omega_0^2 LC(R_1+R_2)}\right)$$

$$= \frac{\pi}{2} - \arctan\left(\frac{\omega_0(L+R_1R_2C)}{R_1 - \omega_0^2 LC(R_1+R_2)}\right)$$ 虚 / 实

① 实部 >0 ⇒ 保持不变

$R_1 > (2\omega_0)^2 LC(R_1+R_2)$

$\Rightarrow R_1 - \omega_0^2 LC(R_1+R_2) > 0$



d. $u_q(t) = V_0 \cos(3\omega_0 t)$

求 $u_2(t)$, $-\frac{\pi}{2} < \arctan < \frac{\pi}{2}$

$u_q(t) = \mathrm{Re}\{V_1 e^{j3\omega_0 t}\}$ 系统频率随外加而变化

$u_2(t) = |H(j3\omega_0)| V_1 \cos(3\omega_0 t + \varphi_H(3\omega_0))$

$$|H(j3\omega_0)| = \frac{3\omega_0 R_1 R_2 C}{\sqrt{[R_1 - (3\omega_0)^2 LC(R_1+R_2)]^2 + [3\omega_0(L+R_1R_2C)]^2}}$$

$3\omega_0(L+R_1R_2C)$ ↓

$R_1 - (3\omega_0)^2 LC(R_1+R_2) < 0$ & 虚 > 0

$$\varphi_H(3\omega_0) = \frac{\pi}{2} - \arctan\left(\frac{3\omega_0(L+R_1R_2C)}{R_1 - (3\omega_0)^2 LC(R_1+R_2)}\right) - \pi$$

⇒ $+\pi$

设 $\varphi = -30°$

$\Rightarrow \varphi' = \varphi + \pi = 150°$