Aufgabe 2

a. beobachbar überprüfen

$$\underline{\underline{A}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad \underline{c} = (1 \ \ 0)$$

$$\Rightarrow \underline{\underline{Q}}_B = \begin{bmatrix} \underline{c} \\ \underline{c}\,\underline{\underline{A}} \end{bmatrix} = \begin{bmatrix} (1 \ \ 0) \\ (1\,0)\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a_{11} & a_{21} \end{bmatrix}$$

$\det \underline{\underline{Q}}_B \neq 0 \Rightarrow$ hat voll Rang $\Rightarrow$ beobachbar

b. $\underline{L}$ bestimmen

Definiere : $\underline{e} = \underline{x} - \hat{\underline{x}}$ ← geschätzt, in Simulator

reale Welt,
nicht vollständig messbar

$$\Rightarrow \dot{\underline{e}} = \dot{\underline{x}} - \dot{\hat{\underline{x}}} \qquad \left| \begin{array}{l} \dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{B}\,\underline{u} \\ \dot{\hat{\underline{x}}} = \underline{\underline{A}}\,\hat{\underline{x}} + \underline{L}(y - \hat{y}) + \underline{B}\,\underline{u} \end{array} \right.$$

$$\left| \begin{array}{l} y = \underline{c}\,\underline{x} \\ \hat{y} = \underline{c}\,\hat{\underline{x}} \end{array} \right.$$

$$\Rightarrow \dot{\underline{e}} = \underline{\underline{A}}\,\underline{x} + \underline{B}\,\underline{u} - \underline{\underline{A}}\,\hat{x} - \underline{L}(y - \hat{y}) - \underline{B}\,\underline{u}$$

$$= \underline{\underline{A}}\,\underline{x} - \underline{\underline{A}}\,\hat{\underline{x}} - \underline{L}(\underline{c}\,\underline{x} - \underline{c}\,\hat{\underline{x}})$$

$$= \underline{\underline{A}}(\underline{x} - \hat{\underline{x}}) - \underline{L}\,\underline{c}(\underline{x} - \hat{\underline{x}})$$

$$= (\underline{\underline{A}} - \underline{L}\,\underline{c})\,\underline{e} \qquad \Rightarrow \boxed{\dot{\underline{e}} = (\underline{\underline{A}} - \underline{L}\,\underline{c})\,\underline{e}}$$

Eigenwert von $(\underline{\underline{A}} - \underline{L}\,\underline{c})$ bestimmen, wie schnell und ob $\underline{e}$ auf 0 abklingt

特征值 $\Rightarrow$ $\underline{e}$ 是否 $\to 0$

Eigenwert 的求解

$$\underline{\underline{A}}\,\underline{v} = \lambda\,\underline{v}$$

↙ Eigenwert von $\underline{\underline{A}}$

$$(\underline{\underline{A}} - \lambda \mathbb{1})\,\underline{v} = 0$$

$$\underline{L}\,\underline{c} = \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}(1 \ \ 0) = \begin{pmatrix} l_1 & 0 \\ l_2 & 0 \end{pmatrix}$$

$$\Rightarrow \det\left[s\mathbb{1} - (\underline{\underline{A}} - \underline{L}\,\underline{c}) \right] = \det\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} + \begin{pmatrix} l_1 & 0 \\ l_2 & 0 \end{pmatrix} \right]$$

$$= \det \begin{bmatrix} s - a_{11} + l_1 & -a_{12} \\ -a_{21} + l_2 & s - a_{22} \end{bmatrix} = (s - a_{11} + l_1)(s - a_{22}) + a_{12}(-a_{21} + l_2)$$

$$= s^2 - s a_{22} - s a_{11} + a_{11}a_{22} + s l_1 - a_{22} l_1 - a_{12} a_{21} + a_{12} l_2$$

$$= s^2 + s(l_1 - a_{11} - a_{22}) + a_{11}a_{22} - a_{22} l_1 - a_{12}a_{21} + a_{12} l_2 \overset{!}{=} 0$$

Polvorgabe: $s_1, s_2 \Rightarrow (s-s_1)(s-s_2) = s^2 - (s_1+s_2)s + s_1 s_2 = 0$

$$\Rightarrow \begin{cases} l_1 - a_{11} - a_{12} \overset{!}{=} -(s_1+s_2) \\ a_{11}a_{22} - a_{22}l_1 - a_{12}a_{21} + a_{12}l_2 = s_1 s_2 \end{cases}$$

$$\Rightarrow \begin{cases} l_1 = a_{11} + a_{12} - s_1 - s_2 \\ l_2 = \dfrac{s_1 s_2 - a_{11}a_{22} + a_{22}l_1 + a_{12}a_{21}}{a_{12}} \end{cases}$$

其实这里和这里在求极点, s_1, s_2

Ü1, $\underline{\dot{x}} = \underline{\underline{\tilde{A}}}\,\underline{x} + \underline{\underline{\tilde{B}}}\,\underline{w} \Rightarrow \det[s\mathbb{1} - \underline{\underline{\tilde{A}}}] = 0$

Ü2, $\underline{\dot{e}} = \underline{\underline{\tilde{A}}}\,\underline{e} \Rightarrow \det[s\mathbb{1} - \underline{\underline{\tilde{A}}}] = 0$

求谁的状态, 就求它的Ble

Ü1, Regler 上的 $\Rightarrow \underline{\dot{x}}$

Ü2, $\underline{e}$ 上的

C

- Modellunsicherheit: reales System $\neq \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$

  ≙ praxis 偏差

  \+ Störung

  \+ inkonsistenter Anfangswert

- Dynamisch: sehr viel schneller als Systemdynamik