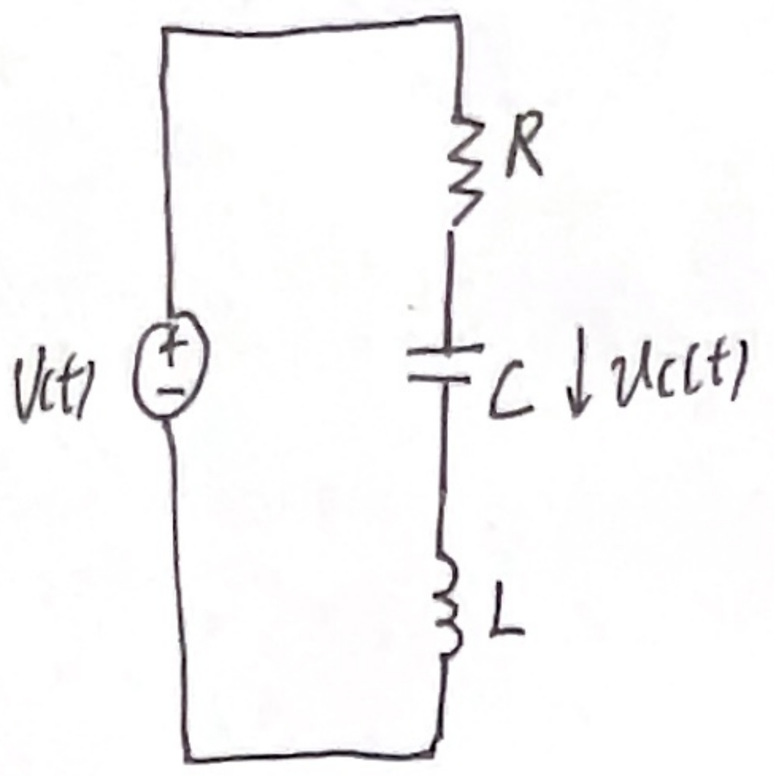
Frequenzbereich



$v(t) = V_0 \cos(\omega_1 t)$

eingeschwungen

1. $H(j\omega)$

输入：$V(j\omega)$，输出 $\begin{cases} R \to R \\ C \to \frac{1}{j\omega C} \\ L \to j\omega L \end{cases}$ $\dfrac{\frac{1}{j\omega C}}{\left(\frac{1}{j\omega C} + j\omega L + R\right)} V(j\omega) = U_c(j\omega)$

$$H(j\omega) = \frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L + R} = \frac{1}{(j\omega)^2 LC + j\omega RC + 1}$$

2. Polstellen des Frequenzgangs

$$\text{Nenner} \overset{!}{=} 0 = (j\omega)^2 LC + j\omega RC + 1$$

$$= (j\omega)^2 + j\omega \frac{R}{L} + \frac{1}{LC}$$

$s_{1,2} = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$, Pole werden komplex und es kommt zur Resonanz,

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0 \Leftrightarrow \frac{R^2}{4L^2} < \frac{1}{LC} \Leftrightarrow R^2 < 4\frac{L}{C} \Rightarrow R < 2\sqrt{\frac{L}{C}}$$

为了使一个网络能够产生共振，该网络在无激励的情况下必须也要振荡

DGL 的 homogene Lösungen

⇒ 频率响应必须具有一对共轭复数极点

3. $u_c(t)$

$$U_c = H(j\omega) \cdot V = \frac{1}{(j\omega)^2 LC + j\omega RC + 1} V_0 \cos(\omega_1 t)$$

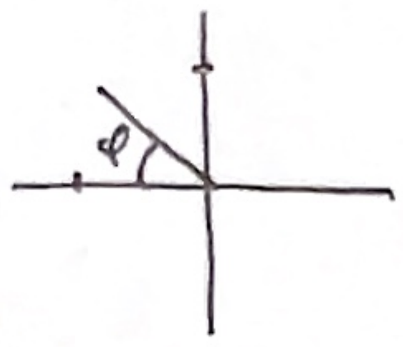
$$u_c(t) = \mathrm{Re}\{e^{j0} e^{j\omega_1 t} U_c\} = \mathrm{Re}\{H(j\omega_1) V e^{j\omega_1 t}\} = \mathrm{Re}\{|H(j\omega_1)| e^{j\varphi} V_0 e^{j\omega_1 t}\}$$

$$= |H(j\omega_1)| V_0 \cos(\omega_1 t + \varphi)$$

$$|H(j\omega)| = \left| \frac{1}{(j\omega)^2 LC + j\omega RC + 1} \right| = \sqrt{\frac{1}{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}} = \frac{1}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$

$$\varphi_{H(\omega)} = \arctan\left(\frac{0}{1}\right) - \arctan\left(\frac{\omega RC}{1-\omega^2 LC}\right) = -\arctan\left(\frac{\omega RC}{1-\omega^2 LC}\right)$$

① $1-\omega^2 LC < 0$



假设 $\varphi = -45^\circ$ $\Rightarrow 1-\omega^2 LC < 0$

$\Rightarrow \varphi + \pi$ $\quad -\left[\arctan\left(\frac{\omega RC}{1-\omega^2 LC}\right) + \pi\right] = -\arctan\left(\frac{\omega RC}{1-\omega^2 LC}\right) - \pi$

② $1-\omega^2 LC > 0$

$\Rightarrow -\arctan\left(\frac{\omega RC}{1-\omega^2 LC}\right)$

③ $1-\omega^2 LC = 0$

$\Rightarrow -\frac{\pi}{2}$

4. Resonanzfrequenz ω_0

$$\frac{d}{d\omega}|H(j\omega)| = \frac{d}{d\omega}\left[(1-\omega^2 LC)^2 + (\omega RC)^2\right]^{-\frac{1}{2}} = -\frac{1}{2}\left[(1-\omega^2 LC)^2 + (\omega RC)^2\right]^{-\frac{3}{2}} \underbrace{\left[2(1-\omega^2 LC)(-2\omega LC) + 2\omega R^2 C^2\right]}_{=0} = 0$$

$$\Rightarrow (1-\omega^2 LC)(-2\omega LC) + \omega R^2 C^2 = 0$$

$$-2\omega LC + 2\omega^3 L^2 C^2 + \omega R^2 C^2 = 0$$

$$2\omega^2 L^2 C - 2L + R^2 C = 0$$

$$2\omega^2 L^2 C = 2L - R^2 C$$

$$\omega^2 = \frac{2L - R^2 C}{2L^2 C} = \frac{2L}{2L^2 C} - \frac{R^2 C}{2L^2 C} = \frac{1}{LC} - \frac{R^2}{2L^2}$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R}{2L^2}}$$