# Musterlösung zur Klausur „Digitale Signalverarbeitung“ 16.10.2007

## Aufgabe 1

a.) $y(n) = \alpha \cdot x(n) - a_1 \cdot y(n-1) = \alpha \cdot x(n) - \frac{1}{2} \cdot y(n-1)$

b.) $Y(z) = \alpha \cdot X(z) - \alpha_1 \cdot z^{-1} \cdot Y(z)$

$Y(z) = \alpha \cdot X(z) - \frac{1}{2} \cdot z^{-1} \cdot Y(z)$

$H(z) = \frac{Y(z)}{X(z)} = \frac{\alpha}{1+\frac{1}{2}z^{-1}}$

c.) $H(e^{j\frac{\pi}{2}}) = \frac{\alpha}{1+\frac{1}{2}\cdot e^{-j\frac{\pi}{2}}} = \frac{\alpha}{1+\frac{1}{2}\cdot(-j)} = \frac{\alpha(1+\frac{j}{2})}{(1-\frac{j}{2})(1+\frac{j}{2})} = \frac{\alpha(1+\frac{j}{2})}{\frac{5}{4}} = \frac{4}{5} \cdot \alpha \cdot (1 + \frac{j}{2})$

$\frac{4}{5} \cdot \alpha \cdot (1 + \frac{j}{2}) = \frac{2}{5} + \frac{1}{5} \cdot j \qquad \Rightarrow \alpha = \frac{1}{2}$

d.) nein, da IIR

e.) $|H(e^{j0})| = \frac{1}{3}$

$|H(e^{j\frac{\pi}{2}})| = \frac{1}{\sqrt{5}}$

$|H(e^{j\pi})| = 1$

f.) Hochpasscharakter ( $\rightarrow$ siehe e.) )

g.) $z \rightarrow -z \qquad z^{-1} \rightarrow -z^{-1}$

$H_2(z) = \frac{\alpha}{1-\frac{1}{2}z^{-1}} = \frac{\frac{1}{2}}{1-\frac{1}{2}z^{-1}}$

h.) $|a_1| < 1 \qquad$ ( $\rightarrow$ Pol muss innerhalb EK liegen)

# Aufgabe 2

a.) Toleranzschema vgl. Skript S. 127.

b.) $d_{\mathrm{st}} = -20\log(\delta_{\mathrm{st}}) = -20\log(0.3) = 10.458$ [dB]

$R_{\mathrm{p}} = -20\log(1-\delta_{\mathrm{p}}) = -20\log(0.85) = 1.412$ [dB]

c.) Bilineare Transformation mit $\Omega' = \Omega_{\mathrm{p}}$ und $\omega' = \frac{\Omega_{\mathrm{p}}}{T}$

$\Omega' = \Omega_{\mathrm{p}} = 0.2\pi$

$\omega' = \frac{\Omega_{\mathrm{p}}}{T} = 0.2\pi \cdot 1\ \mathrm{kHz} = 0.2\pi \cdot 10^3\ \frac{1}{\mathrm{s}}$

$v = \frac{\omega'}{\tan(\frac{\Omega'}{2})} = \frac{0.2\pi \cdot 10^3 \cdot \frac{1}{s}}{\tan(0.1\pi)} = 1933.766\ \mathrm{s}^{-1}$

$\omega_{\mathrm{st}} = v \cdot \tan(\frac{\Omega_{\mathrm{st}}}{2}) = 2661.601\ \mathrm{s}^{-1}$

$\omega_{\mathrm{p}} = v \cdot \tan(\frac{\Omega_{\mathrm{p}}}{2}) = v \cdot \tan(0.1\pi) = 628.318\ \mathrm{s}^{-1}$

d.) $|H_{\mathrm{a}}(j\omega)|^2 = \frac{1}{1+(\frac{j\omega}{j\omega_{\mathrm{c}}})^{2N}}$

$\rightarrow$ muss für Pass- und Stopband erfüllt sein $\Rightarrow N = 2$ („härteres Kriterium“!)

$\omega_{\mathrm{c}} = v \cdot \tan(\frac{\Omega_{\mathrm{c}}}{2}) = v \cdot \tan(0.2\pi) = 1404.963\ \mathrm{s}^{-1}$

e.) Polstellen im analogen Bereich

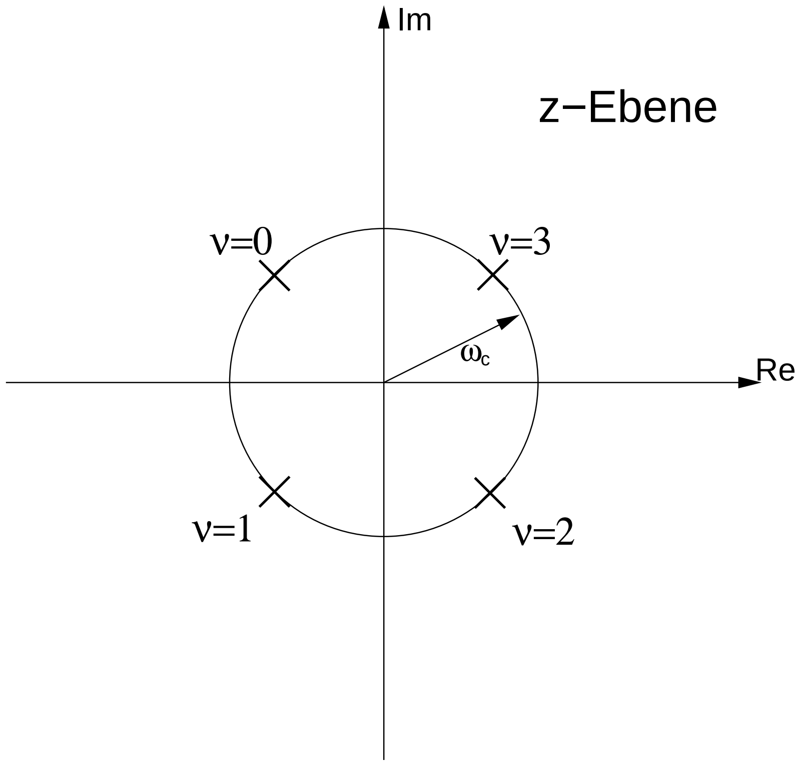
allgemein: $s_{\infty,\nu} = \omega_{\mathrm{c}} \cdot e^{j(\frac{\pi}{2N}+\frac{\pi}{2}+\nu\cdot\frac{\pi}{N})}$

$s_{\infty,0} = \omega_{\mathrm{c}} \cdot e^{j\frac{3\pi}{4}}$

$s_{\infty,1} = \omega_{\mathrm{c}} \cdot e^{j\frac{5\pi}{4}}$

$s_{\infty,2} = \omega_{\mathrm{c}} \cdot e^{j\frac{7\pi}{4}}$

$s_{\infty,3} = \omega_{\mathrm{c}} \cdot e^{j\frac{\pi}{4}}$

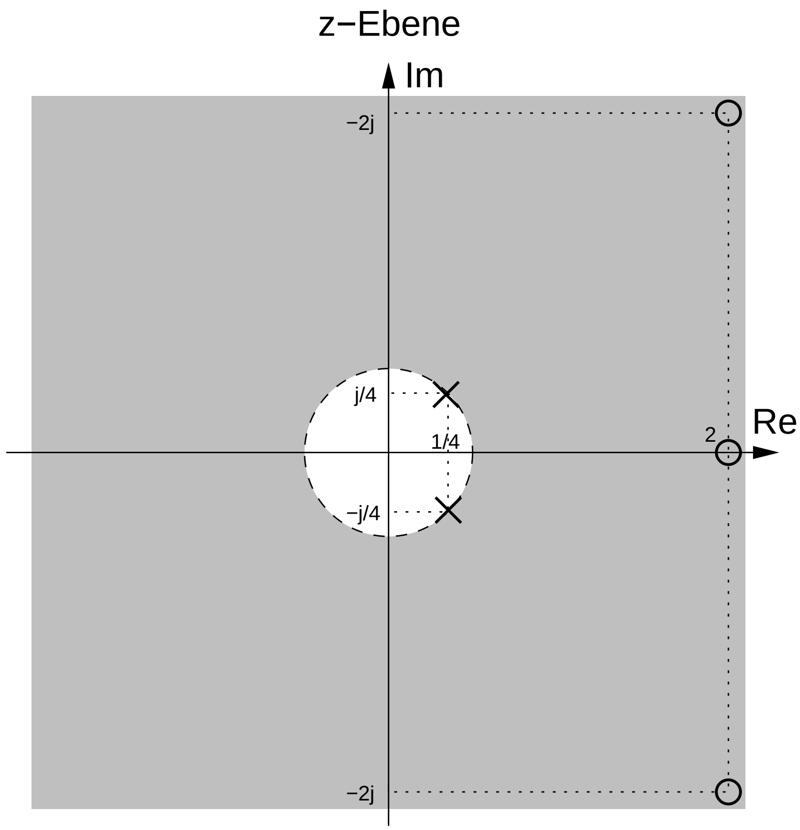



f.) $z_{\infty,i} = \frac{v+s_{\infty,i}}{v-s_{\infty,i}}$ $\quad i = 1,2$ (nur die Pole der linken $s$-Halbebene)

2-fache Nullstelle bei $z = -1$

# Aufgabe 3

a.)



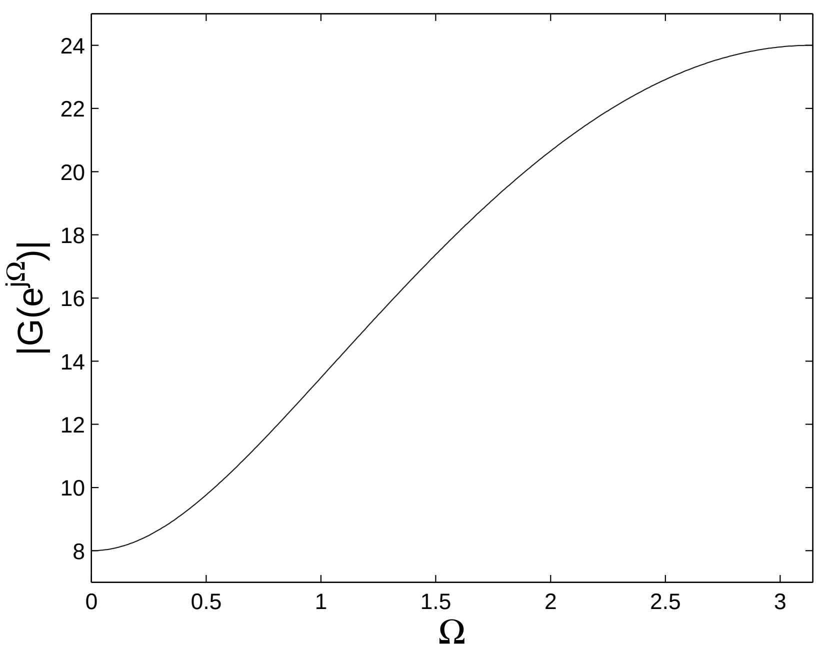

$z_{1,2}^{(0)} = 2 \pm 2j \qquad z_3^{(0)} = 2$

ROC: $|z| > \frac{1}{2\sqrt{2}}$

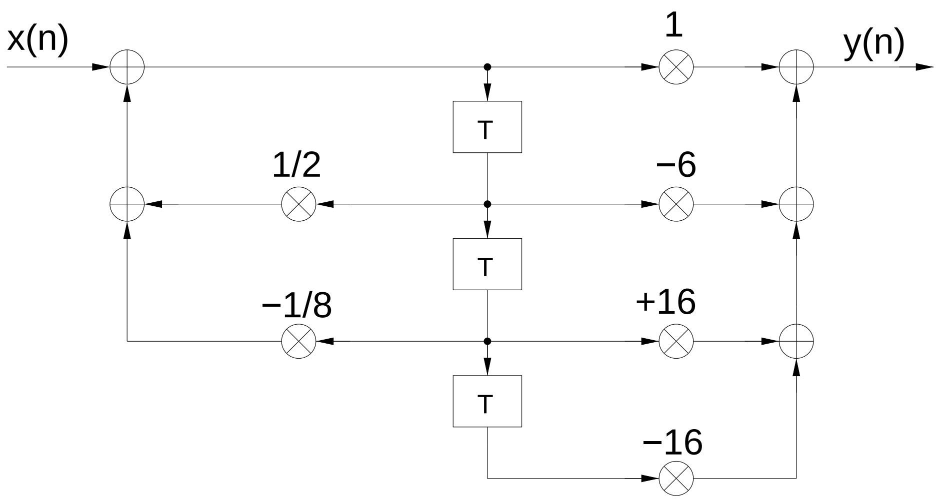
$z_{1,2}^{\infty} = \frac{1}{4} \pm \frac{1}{4}j$

b.) Ja, da alle Polstellen im Einheitskreis.

c.)




d.) $y(n) = x(n) - 6x(n-1) + 16x(n-2) - 16x(n-3) + \frac{1}{2}y(n-1) - \frac{1}{8}y(n-2)$



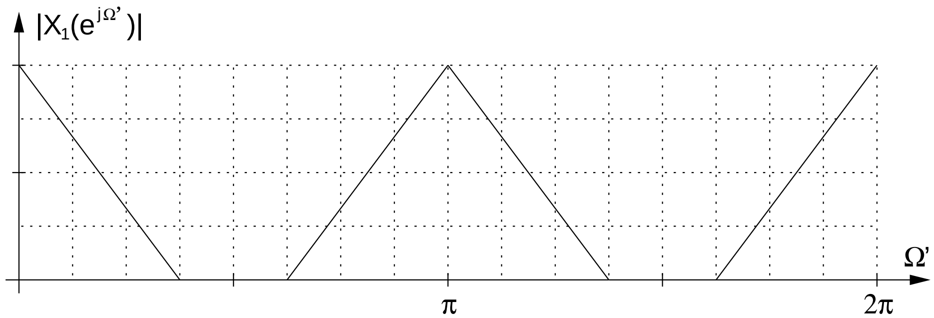

e.) Nein, da Polstellen außerhalb Einheitskreis

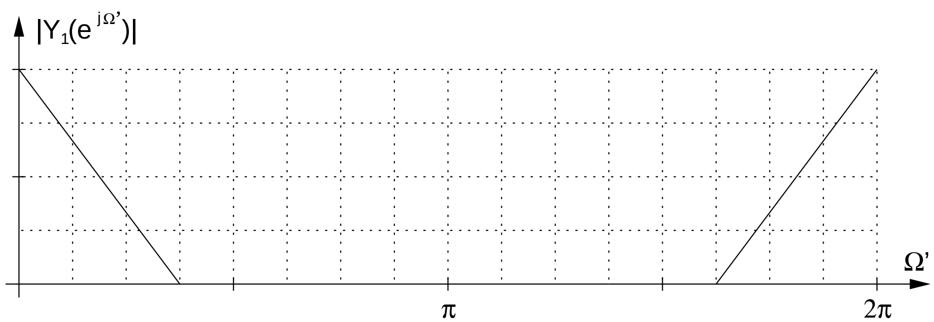
f.) $G_{\min}(z) = (1 - \frac{1}{2}z^{-1})$

## Aufgabe 4

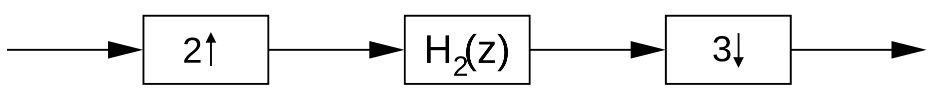
a.) $L = 2 \qquad \Omega'_\mathrm{c} = \frac{\pi}{2} \quad$ (Tiefpass)

b.)




c.)




$\Omega'_\mathrm{c} = \frac{\pi}{3} \qquad f'_\mathrm{s} = 96$ kHz

d.)