Autgabe 1

a. sicherstellen, steuerbar

steuerbar $\leftrightarrow$ $\underline{\underline{Q}}_s$ voll rang $\Leftrightarrow$ $\det \underline{\underline{Q}}_s \neq 0$ $\Leftrightarrow$ invertierbar

题中已给 2.Ordnung / 只有一个 x(t) $\Rightarrow$ $\begin{cases} \dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,u \\ y = \underline{c} \cdot \underline{x} \end{cases}$

$\Rightarrow n=2$, $\underline{\underline{Q}}_s = [\underline{\underline{B}} \quad \underline{\underline{A}}\,\underline{\underline{B}}]$

$$= \left[\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \quad \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \right] = \begin{bmatrix} b_1 & a_{11}b_1 + a_{12}b_2 \\ b_2 & a_{21}b_1 + a_{22}b_2 \end{bmatrix}$$

$$\det \underline{\underline{Q}}_s = b_1(a_{21}b_1 + a_{22}b_2) - b_2(a_{11}b_1 + a_{12}b_2)$$

根据给的 Querführungsmodell $\begin{pmatrix} 0 & 0 \\ v & 0 \end{pmatrix}$ $\Rightarrow$ 代入, $b_1 = \frac{v}{a}$

$$\Rightarrow \det \underline{\underline{Q}}_s = b_1(v b_1 + 0) - b_2(0+0) = b_1^2 v = \frac{v^2}{a^2} \cdot v = \frac{v^3}{a^2}$$

$\Rightarrow \det \underline{\underline{Q}}_s = \frac{v^3}{a^2} \neq 0$, wenn $v \neq 0$

b. Zustandsregler 上的 Systemdynamik 描述

$\Rightarrow$ 描述 x(t)

Zustandsrückführung

$$\left.\begin{aligned} \dot{x}(t) &= A\,x(t) + B\,u(t) \\ u(t) &= -F\,x(t) + M\,w(t) \end{aligned}\right\} \Rightarrow \dot{x}(t) = A\,x(t) - BF\,x(t) + BM\,w(t)$$

$$\Rightarrow \dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} - \underline{\underline{B}}\,\underline{\underline{F}}\,\underline{x} + \underline{\underline{B}}\,\underline{\underline{M}}\,\underline{w}$$

$$= \underbrace{(\underline{\underline{A}} - \underline{\underline{B}}\,\underline{\underline{F}})}_{:=\tilde{\underline{\underline{A}}}}\,\underline{x} + \underbrace{(\underline{\underline{B}}\,\underline{\underline{M}})}_{:=\tilde{\underline{\underline{B}}}}\,\underline{w}$$

$\circ\!-\!\bullet$

$$sX = \tilde{\underline{\underline{A}}}\,\underline{X} + \tilde{\underline{\underline{B}}}\,\underline{W} \Leftrightarrow (s\mathbb{1} - \tilde{\underline{\underline{A}}})\,\underline{X} - \tilde{\underline{\underline{B}}}\,\underline{W} = 0$$

Pole: $\det(s\mathbb{1} - \tilde{\underline{\underline{A}}}) = \det(s\mathbb{1} - \underline{\underline{A}} + \underline{\underline{B}}\,\underline{\underline{F}}) = 0$

wählbar

C.

沿中线行驶 & Querablage = y ⇒ 无需外部控制，可内部自动控制

$y=0 \Rightarrow \underline{w}=0$ Eingangsvektor

$\Rightarrow \underline{u} = -\underline{\underline{F}}\,\underline{x} \quad \Rightarrow \underline{x}$ ist $\begin{pmatrix} \text{Querwinkelfehler} \\ \text{Querablage} \end{pmatrix} \Rightarrow$ soll 0 sein

D.

Pole 是 s_1 与 s_2，确定 Rückführungsmatrix $\underline{\underline{F}} = (k_1 \quad k_2) \Rightarrow$ 求出 k_1, k_2

aus B) $\det\left[s\mathbf{1} - \underline{\underline{A}} + \underline{\underline{B}}\,\underline{\underline{F}}\right] = 0$

$$\det\left[s\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} + \begin{pmatrix}b_1\\ b_2\end{pmatrix}(k_1 \quad k_2)\right] = 0$$

$$\det\left[\begin{pmatrix}s & 0\\ 0 & s\end{pmatrix} - \begin{pmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{pmatrix} + \begin{pmatrix}b_1k_1 & b_1k_2\\ b_2k_1 & b_2k_2\end{pmatrix}\right] = 0$$

$$\det\begin{bmatrix} s-a_{11}+b_1k_1 & -a_{12}+b_1k_2 \\ -a_{21}+b_2k_1 & s-a_{22}+b_2k_2 \end{bmatrix} = 0$$

$$(s-a_{11}+b_1k_1)(s-a_{22}+b_2k_2) - (-a_{12}+b_1k_2)(-a_{21}+b_2k_1) = 0$$

außer a_{21}, alle $=0$

$$\Rightarrow (s+b_1k_1)(s+b_2k_2) - (b_1k_2)(-a_{21}+b_2k_1) = 0$$

$$s^2 + s(b_1k_1+b_2k_2) + b_1b_2k_1k_2 + a_{21}b_1k_2 - b_1b_2k_1k_2 = 0$$

$$s^2 + s(b_1k_1+b_2k_2) + \underline{\underline{a_{21}b_1k_2}} = 0$$

Pole 为 $s_1, s_2 \Rightarrow (s-s_1)(s-s_2) = 0 \Leftrightarrow s^2 - (s_1+s_2)s + s_1s_2 = 0$

$$\Rightarrow \begin{cases} b_1k_1+b_2k_2 = -(s_1+s_2) & ① \\ a_{21}b_1k_2 = s_1s_2 & ② \end{cases} \Rightarrow \begin{cases} k_2 = \dfrac{-(s_1+s_2)-b_1k_1}{b_2} & ① \\ k_2 = \dfrac{s_1s_2}{a_{21}b_1} & ② \end{cases} \Leftrightarrow \begin{cases} -k_2 = \dfrac{s_1+s_2+b_1k_1}{b_2} & ① \\ k_2 = \dfrac{s_1s_2}{a_{21}b_1} & ② \end{cases}$$

$$①+② \Rightarrow -\frac{s_1+s_2+b_1k_1}{b_2} = \frac{s_1s_2}{a_{21}b_1} \Leftrightarrow s_1+s_2+b_1k_1 = -\frac{s_1s_2b_2}{a_{21}b_1}$$

$$\Rightarrow \begin{cases} k_1 = \dfrac{b_2s_1s_2+(s_1+s_2)a_{21}b_1}{a_{21}b_1^2} \\ k_2 = \dfrac{s_1s_2}{a_{21}b_1} \end{cases}$$