LITTLE ANGEL SCHOOL, KICHHA

Class12th–Mathematics

Half Yearly Exam

Time: 3 Hour **Max Marks: 80**

General Instructions:

the following instructions very carefully and strictly follow them:

(i) *This Question paper contains 38 questions.* ***All*** *questions are compulsory.*

(ii) *Question paper is divided into* ***FIVE*** *Sections – Section* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A*** *– Questions Number* ***1*** *to* ***18*** *are Multiple Choice Questions (****MCQ****s) type and Questions Number* ***19*** *&* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B*** *– Questions Number* ***21*** *to* ***25****are Very Short Answer (****VSA****) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C*** *– Questions Number* ***26*** *to* ***31*** *are Short Answer (****SA****) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D*** *– Questions Number* ***32*** *to* ***35*** *are Long Answer (****LA****) type questions, carrying* ***5*** *marks each*

Section A – Multiple Choice Questions (1 Mark Each)

Q1. If A and B are two non-zero square matrices of the same order such that $(A+B)^2 = A^2 + B^2$, then:

(A) AB = O
(B) AB = – BA
(C) BA = O
(D) AB = BA

Q2. The interval in which the function f(x)=$2x^3 + 9x^2 + 12x - 1$ is decreasing, is :

(A) (-1,∞)
(B) (-2,-1)
(C) (-∞, -2)
(D) [-1,1]

Q3. The value of x for which $(x - x^2)$ is maximum, is

A).$\frac{3}{4}$
B).$\frac{1}{2}$
C).$\frac{1}{3}$
D).$\frac{1}{4}$

Q4. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

(A) 27
(B) 18
(C) 81
(D) 512

Q5.Lets A be a square matrix of order 3 × 3, then kA is equal to

$(A)k|A|$
$(B)k^2|A|$
$(C)k^3|A|$
$(D)3k|A|$

Q6. $\frac{d}{dx}\log(\tan x)$ is equals to:

(A) 2sec 2x
(B) 2 cosec(2x)
(C) sec 2x
(D) cosec 2x

Q7. The rate of change of the area of a circle with respect to its radius r when r = 6 is

(A) 10π
(B) 12π
(C) 8π
(D) 11π

Q8. The interval in which y = x^2e^{-x} is increasing is

(A) $(-\infty, \infty)$ (B) (–2, 0)
(C) $(2, \infty)$ (D) (0, 2)

Q9. $\sin(\frac{\pi}{3} - \sin^{-1}\frac{1}{2})$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) 1

Q10. If $f(x) = \begin{cases} 1+x & x \leq 2 \\ 5-x & x \leq 3 \end{cases}$ then,

(A) f is continuous at x = 2 (B) f is discontinuous at x = 2
(C) f is continuous at x = 3 (D) f is continuous at x = 3 and discontinuous at x=2

Q11. If $\begin{vmatrix} -1 & 2 & 4 \\ 1 & x & 1 \\ 0 & 3 & 3x \end{vmatrix}$ =-57, the product of possible value of x:

(A) -24 (B) -16
(C) 16 (D) 24

Q12. If f:$\mathbb{N} \to \mathbb{R}$ is define as, f(x)=$\begin{cases} \frac{n}{2} & \text{if n is even} \\ 0 & \text{if n is odd} \end{cases}$

(A) Injective only (B) Surjective Only
(C) a Bijection (D) neither injective nor surjective

Q13. If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$, then the value of $\cos^{-1}x + \cos^{-1}y$ is

(A) $\frac{\pi}{2}$ (B) π
(C) 0 (D) $\frac{2\pi}{3}$

Q14. The domain of y = $\cos^{-1}(x^2 - 4)$ is

(A) [3,5] (B) [0,π]
(C) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{5}, \sqrt{3}]$ (D) $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Q15 The area of a triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units. The value of k will be

(A) 9 (B) 3
(C) -9 (D) 6

Q16. If y =$ax^2 + b$, then $\frac{dy}{dx}$ at x = 2 is equal to

(A) 2a (B) 3a
(C) 4a (D) None of these

Q17. Given that A is a square matrix of order 3 and $|A|$= -4, then $|adjA|$ is equal to

(A) -4 (B) 4
(C) -16 (D) 16

Q18. If $x = t^2, y = t^3$, then $\frac{d^2y}{dx^2}$ =

(A) $\frac{3}{2}4$ (B) $\frac{3}{4t}$
(C) $\frac{3}{2t}$ (D) $\frac{3t}{2}$

Questions number **19** and **20** are Assertion and Reason based questions.Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below:

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion(A) and Reason(R) are true, but Reason(R) is not the correct explanation of the Assertion(A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Q19. Assertion (A): If A is $m \times n$ and B is $n \times p$, then **AB** is defined.
Reason (R): A relation R is symmetric if $aRb \Rightarrow bRa$

Q20. Assertion (A): The matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is both symmetric and skew-symmetric.
Reason (R): A matrix cannot be both symmetric and skew-symmetric unless it is a zero matrix.

Section B –Very Short Answer Questions (2 Marks Each)

Q21. Let R be the set of real numbers, f: $\mathbb{R} \to \mathbb{R}$ be the function defined by f(x) = 4x +3, x ∈ R. Show that f is invertible. Find the inverse of f.

Q22.If $x = a(\cos t + t \sin t)$,$y = a(\sin t – t \cos t)$. Find $\frac{d^2y}{dx^2}$

Q23. Determine if the relation **R**={(x,y)∈ $\mathbb{R} \times \mathbb{R} : x^2 + y^2 = 1$} is symmetric, reflexive or transitive.

Q24. Let A= $\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$,B=$\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and C=$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find a matrix D such that CD – AB = O

Q25**(a)**. Simplify $\sin^{-1}(\frac{x}{\sqrt{1+x^2}})$

OR

(b). Find the domain of $\sin^{-1}\sqrt{x-1}$

Section C- Short Answer Questions (3 Marks Each)

Q26. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right] = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$, $\frac{-1}{\sqrt{2}} \leq x \leq 1$

Q27. Prove that the determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ

Q28. Show that the function f :$\mathbb{R} \to \{x \in \mathbb{R} : -1 < x < 1\}$ defined by f(x)=$\frac{x}{1+|x|}$, $x \in \mathbb{R}$ is one-one and onto function.

Q29. Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix}$=O

Q30 **(a)**. Show that y= $\log(1+x) - \frac{2x}{2+x}, x > -1$ is an increasing function of x throughout its domain.

OR

(b). Find the local maxima and local minima of the function $f(x) = \frac{8}{3}x^3 - 12x^2 + 18x + 5$

Q31. If $y = e^{a\cos^{-1}x}, -1 \leq x \leq 1$ show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0$

Section D Long Answer Questions (5 Marks Each)

Q32. If A=$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$,find A^{-1}. Using A–1, solve the system of equations:

$$2x – 3y + 5z = 11 \quad (1)$$
$$3x + 2y – 4z = –5 \quad (2)$$
$$x + y – 2x = –3 \quad (3)$$

Q33(a).An open box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

OR

(b). Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

Q34. Show that the relation R in the set $\mathbb{R}$ of real numbers, defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Q35 (a). If f(x)=$\begin{cases} \frac{1-\cos 4x}{x^2} & \text{when} x < 0 \\ \text{a} & \text{when} x = 0 \\ \frac{\sqrt{x}}{\sqrt{16-\sqrt{x-4}}} & \text{when} x > 0 \end{cases}$ and f is continuous at x = 0, then find the value of a.

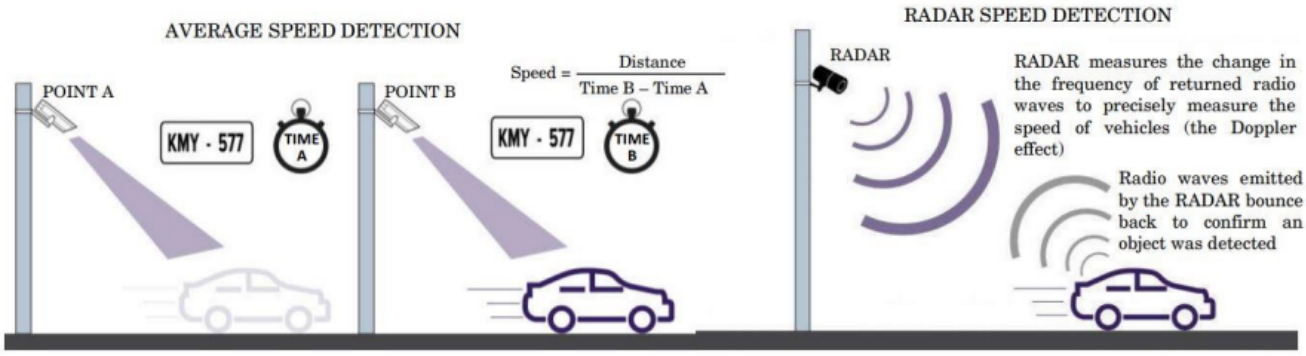
OR

(b). Find the value of k, so that the following function is continuous at x = 2.

$$f(x) = \begin{cases} \frac{x^3+x^2-16x+20}{(x-2)^2} & \text{when} x \neq 2 \\ k & \text{when} x = 2 \end{cases}$$

Section E Case Study Based Question (4 Marks for each Question)

Q36. The traffic police have installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at a height of 5 m. It detects a car travelling away from the pole at a speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ. On the basis of the above information, answer the following questions:

(i) Express θ in terms of the height of the camera installed on the pole and x.

(ii) Find $\frac{d\theta}{dx}$

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

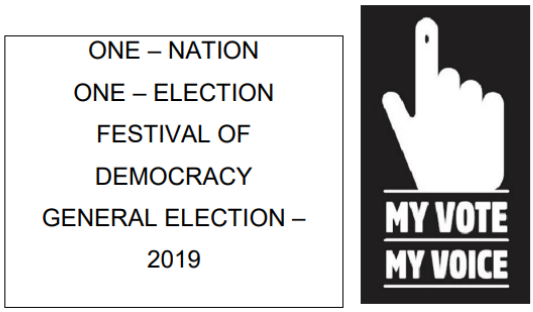
OR

(b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is 3/101 rad/s, then find the speed of the car.

Q37. A general election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever Let I be the set of all citizens of India who were eligible to exercise their voting right in general election held in 2019. A relation 'R' is defined on I as follows:

R= $\{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in the general election – 2019$\}$

(i). Two neighbours X and Y $\in I$. X exercised his voting right while Y did not cast her vote in general election –



2019. Which of the following is true?

(A) (X,Y) $\in \mathbb{R}$ (B) (Y,X) $\in \mathbb{R}$
(C) (Y,X)$\notin \mathbb{R}$ (D) (X,Y) $\notin \mathbb{R}$

(ii). Mr.'**X**' and his wife '**W**' both exercised their voting rights in the general election -2019. Which of the following is true?

(A) both (X,W) and (W,X) $\in \mathbb{R}$ (B) (X,W) $\in \mathbb{R}$ but (W,X) $\notin \mathbb{R}$
(C) both (X,W) and (W,X)$\notin \mathbb{R}$ (D) (W,X) $\in \mathbb{R}$ but (X,W) $\notin \mathbb{R}$

(iii). Three friends F_1, F_2 and F_3 exercised their voting right in the general election-2019, then which of the following is true?

(A) (F1,F2) $\in$ R, (F2,F3) $\in$ R and (F1,F3) $\in$ R (B) (F1,F2) $\in$ R, (F2,F3) $\in$ R and (F1,F3) $\notin$ R
(C) (F1,F2) $\in$ R, (F2,F2) $\in$ R but (F3,F3) $\notin$ R (D) (F1,F2) $\notin$ R, (F2,F3) $\notin$ R and (F1,F3) $\notin$ R

(iv). The above defined relation R is

(A) Symmetric and transitive but not reflexive (B) Equivalence relation
(C) Universal relation (D) Reflexive but not symmetric and transitive

Q38. Sand is pouring from a pipe at the rate of 12 cm^3/ second the falling sand forms a cone on the ground in such a way that the height of the cone is always 1/6th of the radius of the base. Based on the above information, answer the following:

(i) Write the expression for volume in terms of height only.

(ii) What is the rate of Change of height, when height is 4 cm?