SENSOR PLACEMENT PROJECT, REPORT DUE: DECEMBER 5, 2021

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Project Rules:

1. Each team should hand in one report on Canvas.

2. You may *not* interact with fellow students outside of your team when conducting your project.

3. The solutions and Python codes should be submitted electronically on Canvas prior to the deadline. Any handwritten solutions should be scanned so that they are easily read by the TA, and then submitted electronically. Typed solutions are preferred.

4. When formulating optimization problems, clearly state the parameters, decision variables, constraints, and objective function of your formulation. If you are using compact notations, i.e., $\sum, \prod, \forall$, state the ranges of indices for each of them.

5. Show your work. Answers requiring a justification will not receive full grades if the justification is not provided. Similarly, answers requiring intermediate computations will not receive full grades if those computations are not shown in your solutions.

Problem Background

Ensuring the security of critical infrastructures such as water distribution networks is crucial for the welfare and prosperity of our society. Such infrastructure networks may span huge geographical areas, and are inherently vulnerable to disruptions. In recent years, numerous incidents have been reported that highlight the inherent vulnerability of infrastructure networks (website). Operations Research can provide new solutions to help the water utilities to proactively recognize the security risks to their infrastructure, and develop specific capabilities to reduce them.

In this project, a water utility company, and more specifically a network operator, is interested in allocating pressure sensors in order to detect pipe bursts in its water distribution network. The network is located in Kentucky, and is composed of 1123 pipes and 811 nodes. It provides 2.09 million gallons of water per day to its 5,010 customers. The layout of the water distribution network is given in Figure 1.

The network operator possesses b pressure sensors that they wish to position on nodes of the network in order to detect pipe bursts. We denote $\mathcal{V}$ the set of 811 nodes/sensor locations, and $\mathcal{E}$ the set of 1123 network pipes. Each node can receive at most one sensor. You will be given a detection matrix, denoted $\boldsymbol{F} \in \{0,1\}^{|\mathcal{E}| \times |\mathcal{V}|}$, that represents the sensing capabilities of the pressure sensors. $\boldsymbol{F}$ is a binary matrix of size $|\mathcal{E}| \times |\mathcal{V}| = 1123 \times 811$, and is such that $f_{e,v} = 1$ if a sensor placed at location $v \in \mathcal{V}$ can detect a burst of pipe $e \in \mathcal{E}$.

The goal of this project is to determine the optimal location of the b sensors to protect the network as best as possible (we will formalize this notion below).
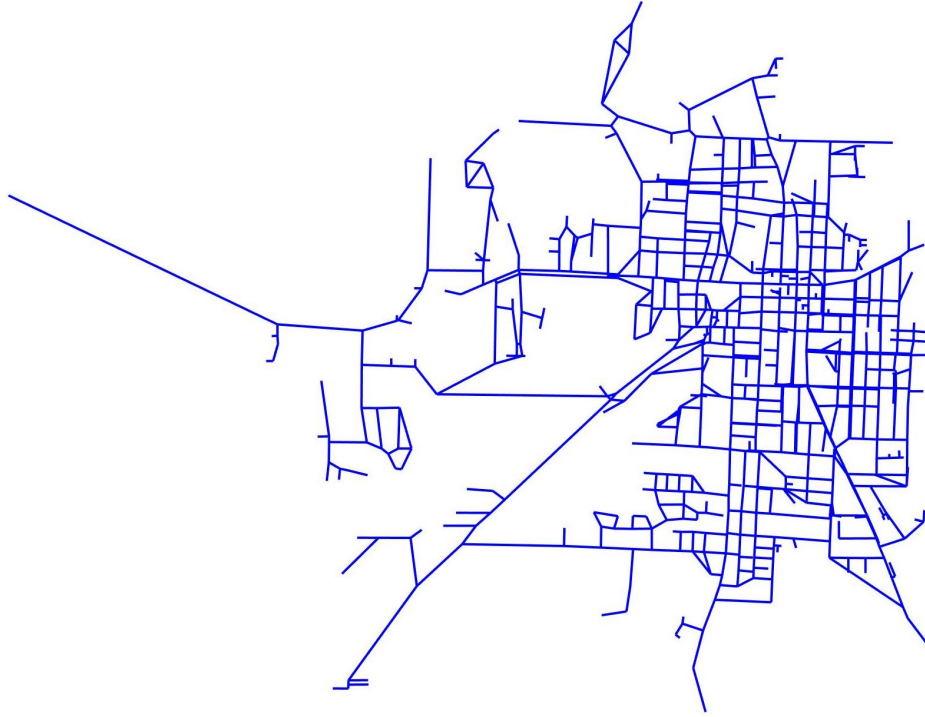
Figure 1: Water distribution network in Kentucky.

Questions:

(a) (15 points) In this first question, it is assumed that each pipe independently bursts with probability 0.1. Formulate an integer program that determines the location of the b sensors that maximizes the expected number of pipe bursts that are detected.

(b) (15 points) Code and solve the optimization model formulated in part (a) for $b \in [\![0, 20]\!]$ using Gurobi Solver. Plot and analyze the optimal value of the problem as a function of the number of sensors b. What is the minimum number of sensors to detect all pipe bursts?

(c) (20 points) Instead, the operator wants to investigate a faster solution approach, where sensor locations are sequentially (and myopically) selected. Specifically, the operator takes their first sensor, and places it at the node that detects the maximum expected number of pipe bursts. Then, they take the second sensor, and places it at the node that detects the maximum expected number of pipe bursts that were not previously detected, and so on until all sensors are positioned.

Code this iterative algorithm, and plot the resulting expected number of pipe bursts that are detected by that solution, as a function of the number of sensors available, $b \in [\![0, 20]\!]$. Compare this plot with the one obtained in part (b).

(d) (15 points) It turns out that some pipes are more critical than others, depending on their locations. For example, a pipe burst next to a hospital will be more problematic than a pipe

burst in a remote area. Each pipe $e \in \mathcal{E}$ is assigned a criticality level $w_e \in [0, 1]$. The higher w_e is, the more critical pipe e is. The goal of the network operator is to position their b sensors as to minimize the highest criticality of a pipe that is not detected by any sensor. Formulate this problem as an IP.

(e) (10 points) Code and solve the optimization model formulated in part (d) for $b \in [\![0, 20]\!]$ using Gurobi Solver. Plot and analyze the optimal value of the problem as a function of the number of sensors b.

(f) (25 points) Present your project. Your presentation should introduce the problem (for the students who have not worked on this project), and describe your results in a clean and organized manner.