Sensor Placement Project

Aurimas Racas, Matthew Shaw, Hayden Dessommes

Problem Background

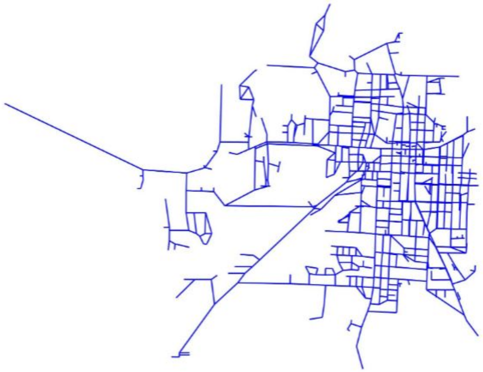


Figure 1: Water distribution network in Kentucky.

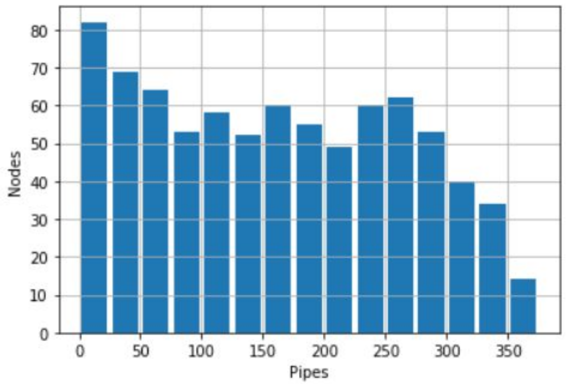
A water utility company in Kentucky is interested in allocating pressure sensors to detect pipe bursts in their water distribution network.

With 1123 pipes and 811 nodes where sensors can be placed, we would like to optimize where and how many sensors should be placed in order to detect bursts.

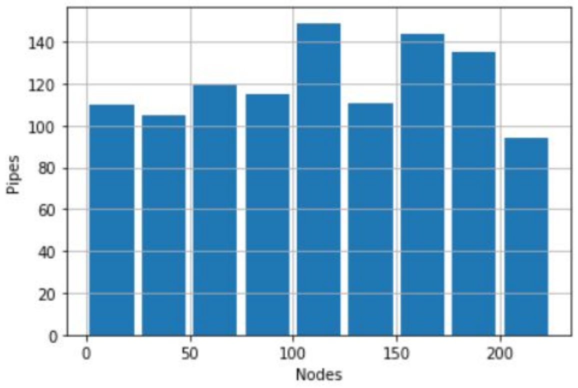
In the data given, each pipe is a row, and each node is a column. The corresponding value for each pipe/node combination is either a 1 (if a sensor placed and node *i* can detect a burst at pipe *j*) or a 0 (if a sensor cannot detect a burst).

Data Background

Distribution of how many pipe leaks can be detected by each sensor (node), i.e ~80 sensors detect between 0-25 pipes.



Distribution of how many sensors (nodes) can sense a leak for each pipe, i.e ~110 pipes are detected by between 0-25 nodes.



IP #1 - Pipe Detection

Determine the location of *b* sensors that maximizes that maximizes the expected number of pipe bursts that are detected (given that each pipe has the same probability of bursting of 0.1).

N_i - a binary variable that represents whether node i has a sensor installed.

P_j - a binary variable that represents whether pipe j will be detected.

Objective Function:

$$Max \quad 0.1 \sum_{j=1}^{1123} P_j$$

IP #1 - Constraints

$$P_j \geq N_i \quad \forall (i,j) | I(N_i, P_j) = 1$$

If node *i* has a sensor installed and can detect pipe *j*:
a burst in pipe *j* will be detected. (Pipe Detection)

$$P_j \leq \sum_{i | I(i,j)=1} N_i \quad \forall j$$

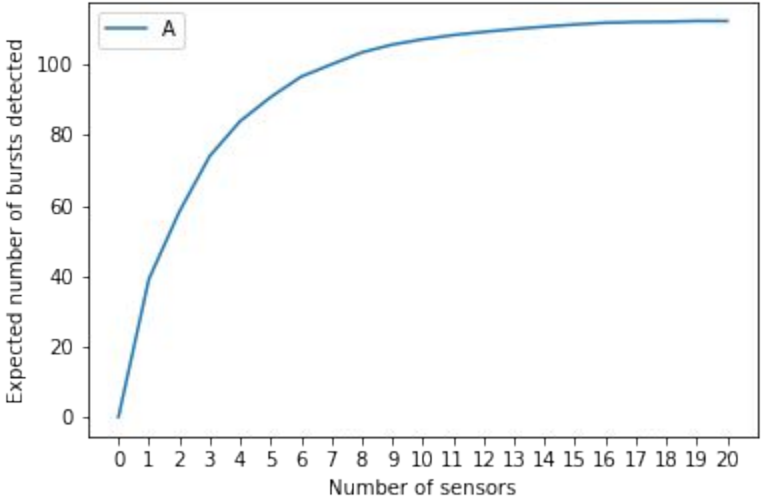
If no nodes capable of detecting pipe *j* have a sensor:
a burst in pipe *j* will be undetected. (Pipe Non-Detection)

$$\sum_{i=1}^{811} N_i \leq b$$

Limits the number of sensors placed to *b*. (Total Placement)

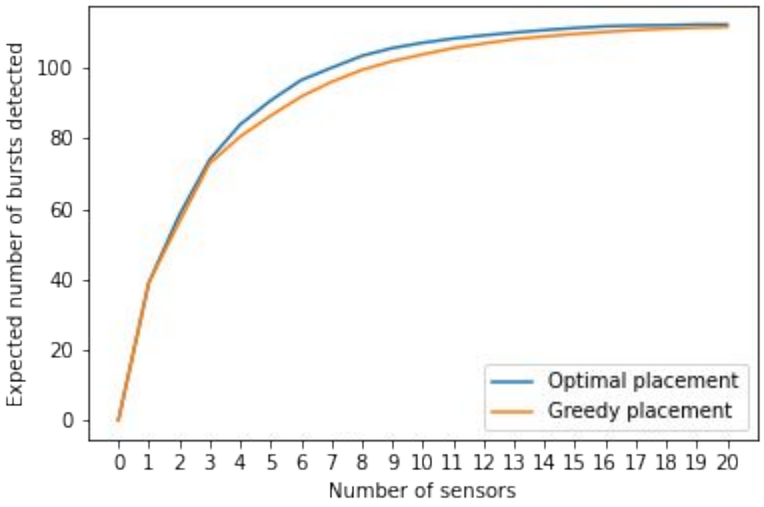
IP #1 - Results

Detection	Optimal
50%	2 Sensors
90%	8 Sensors
95%	10 Sensors
99%	15 Sensors
100%	19 Sensors



IP #1 - Greedy Solution

Detection	Optimal	Greedy
50%	2 Sensors	2 Sensors
90%	8 Sensors	9 Sensors
95%	10 Sensors	12 Sensors
99%	15 Sensors	19 Sensors
100%	19 Sensors	24 Sensors



IP #2 - Pipe Criticality

- Each pipe is assigned a criticality number (between 0 and 1). The higher the number the more critical the pipe is.

- Our new goal is to minimize the highest criticality of a pipe that is not detected by any sensor.

Formulation

Min-Max problem: $$\min \max_{j \in [1;1123]} (1 - P_j) w_j$$

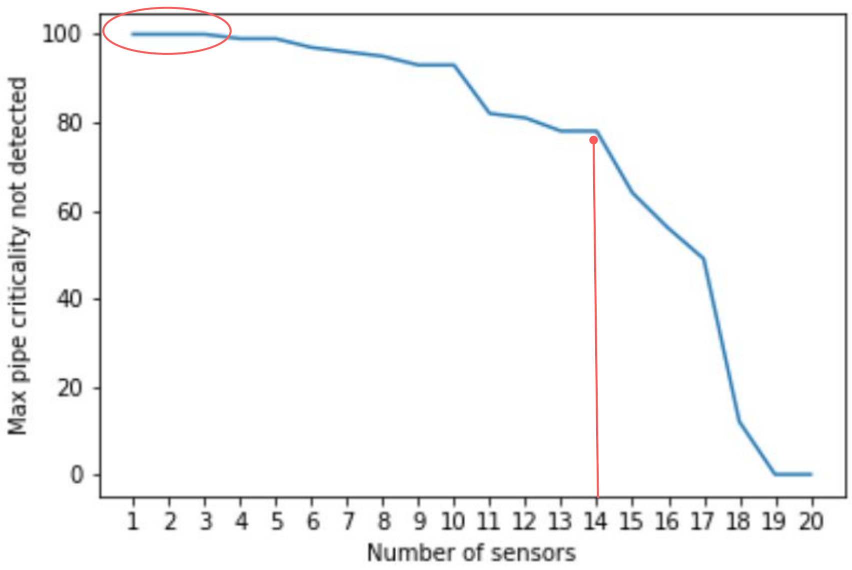
A linear formulation

$$\min z$$

$$z \geq (1 - P_J) w_j \quad \forall j$$

\+ all constraints from previous formulation

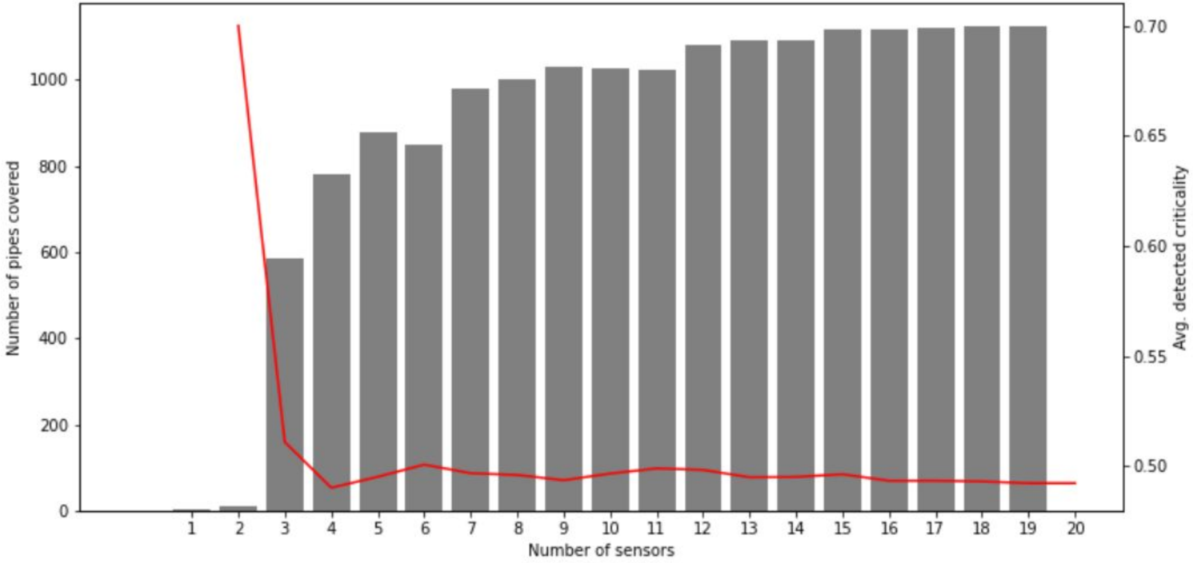
IP #2 - Results



- No impact to objective function with less than 3 sensors!
- Highly critical pipes not detected until at least 14 sensors are placed
- As previously, 19 sensors cover the entire network

IP #2 - A practical issue with limited sensors

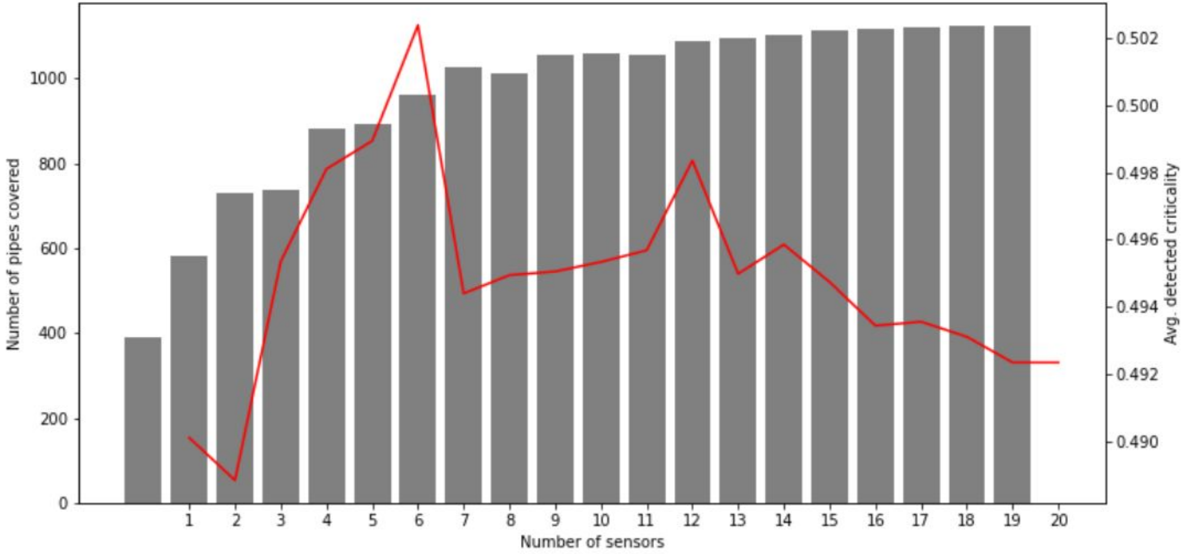
- Mathematically correct, but practically suboptimal solver choices for <4 sensors



IP #3 - Modified objective function to obtain better results

- Idea: introduce average detected pipe criticality into the objective function without affecting its primary objective

$$\min z - \frac{1}{1123} P_j w_j$$



Q&A

Thank you!