# Assignment 1 -APG3013F

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# 1 Chapter 1 - Introduction

## 1.1 About the report

Investigate the general law of error propagation with respect to non-linear functional models for uncorrelated observations. The report also investigates the use of error ellipses and the interpretation of these structures.

## 1.2 Structure of report

The aim of the report, problem and steps to solve the problem will be presented in this chapter. The setting, explanation and analysis of findings will follow up in the next chapters. Conclusions and recommendations will then be drawn based on the findings.

## 1.3 Aim of report

The aim of investigation is to simulate a sequence of polars within the network of points P1, P2, ...P9. Assuming the observations are uncorrelated, a variance matrix must be created and the co-variance matrix computed at all the points. Once the matrices have been made assume numerical variances, distances, directions and specify coordinates for the known point P1. Substitute these values to evaluate the final co-variance matrix and compute the error ellipses at each point. Plot the network of points as well as the ellipses using a module in python.

## 1.4 The problem

The implementation of calculating variances and co-variance matrices are straight forward. The only concerning matter for this assignment is working with huge matrices and keeping track of all the data to be used.

## 1.5 Steps to solving the problem

The steps are as follows:

1. Verify all the knowns, observations and constants.
2. Assess the given variables and decide on the functional model ( a polar in this case)
3. Compute symbolically the polars from P1 to P9
4. Find the Jacobian by differentiating all the polars in a single column matrix with respect to the observations.
5. Populate the variance matrix symbolically with variances of distances and directions between points
6. Calculate the co-variance matrix by using the law of error propagation
7. Substitute numerical values in the covariance matrix and calculate the orientation, semi-major and semi-minor axis of the error ellipses using the error ellipse formula.
8. Plot the network of points and error ellipses

# 2 Chapter 2 - Input

## 2.1 Input data for the problem

The data provided include a network of points P1 to P9. The specific structure of nodes and edges is relevant to the task although distances and directions weren't initially given. After the symbolic data has been manipulated to represent the matrices we can provide numerical data which is both assumed and somewhat adherent to the structure. The figures below displays a network of nodes followed by the input data for the polars and covariance matrix:

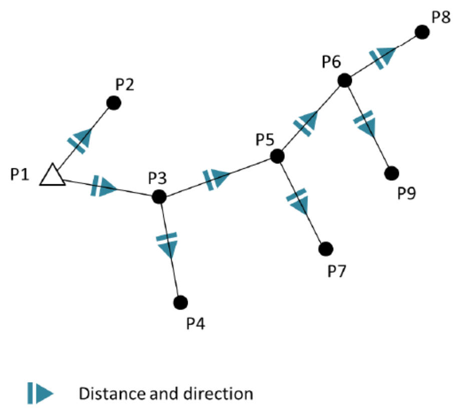


Figure 1: Network of nodes

| point | Y | X |
|---|---|---|
| P1 | 21233.32 | 3729000.12 |

Figure 2: Coordinates of P1

| from | to | distance | direction | distvar | dirvar |
|---|---|---|---|---|---|
| P1 | P2 | 350 | 315.202 | 0.005 | 0.016 |
| P1 | P3 | 299.8 | 25.005 | 0.004 | 0.012 |
| P3 | P4 | 150 | 146 | 0.003 | 0.011 |
| P3 | P5 | 210.2 | 173.285 | 0.005 | 0.014 |
| P5 | P6 | 310.3 | 55.4806 | 0.002 | 0.013 |
| P5 | P7 | 180.2 | 245.3402 | 0.003 | 0.010 |
| P6 | P8 | 233 | 127 | 0.005 | 0.015 |
| P6 | P9 | 89.5 | 304.4525 | 0.002 | 0.012 |

Figure 3: Table of values

# 3 Chapter 3 - Calculating points by polar method

## 3.1 Theory

The polar formula calculates a second points coordinates from the initial point's known coordinates as well as the observed direction and distance from the known to the unknown point. The formula below describes how a known point 'a' can be used to find point b's x and y coordinate:

$$Xb = Xa + Scos(\alpha)$$

$$Yb = Ya + Ssin(\alpha)$$

where S is the distance and alpha is the direction from a to b.

## 3.2 Calculating the points by polar from point P1

The figure below represents the numerical values of P2 to P9 using the input data of P1 and observations between points.

| | X | Y |
|---|---|---|
| P2 | 372917.75 | 21263.552 |
| P3 | 372929.67 | 21229.523 |
| P4 | 372930.93 | 21244.470 |
| P5 | 372911.04 | 21219.408 |
| P6 | 372926.22 | 21191.805 |
| P7 | 372928.83 | 21224.833 |
| P8 | 372931.63 | 21214.467 |
| P9 | 372840.25 | 21216.710 |

Figure 4: Coordinates of points

# 4 Chapter 4 - Calculating the covariance

## 4.1 Theory

The law is usually referred to the law of propagation of variances and covariances. For some non-linear function y = f(x) the law aims to determine how the variances and covariances of the observations in x will be propagated in the variable y. If the system is non-linear the system of equations given y = f(x) must be linearized to give :

$$dy = Jdx$$

where J is the Jacobian of the functional model differentiated with respect to the observations x and dx is correction to x. In other words

$$x = xo + dx$$

where

$$xo = (xo1, xo2, xo3, ..., xon);$$

and the vector of initial values $dx = x - xo$. Now $xo$ is a set of constants so by covariance law

$$\Sigma x = \Sigma dx$$

Where $\Sigma x$ is the covariance matrix of $dx$ Applying the same argument to y we get

$$\Sigma y = \Sigma dy$$

Where $\Sigma dy$ is the covariance matrix of $dy$. Now applying the covariance law to the linearized system $dy = Jdx$ we get

$$\Sigma dy = J\Sigma dx J^T$$

But $\Sigma y = \Sigma dy$ and $\Sigma x = \Sigma dx$ therefore

$$\Sigma y = J\Sigma x J^T$$

## 4.2 Setting up the Jacobian

The design matrix of the problem is filled with the polar functions from P1 to P9. These functions are symbolically inserted into the matrix in order to partially differentiate it with respect to the observations; distance and direction between points. This partially derived matrix is known as the Jacobian and must be done for any non-linear functions. The matrix below is the Jacobian of the problem. There are 9 nodes, however P1 is known hence there are 8 nodes which has 8 directions and 8 distances. Therefore the

Jacobian is a 16X16 matrix as follows:

$$
\begin{bmatrix}
\frac{\partial Xp3}{\partial Sp1p3} & \frac{\partial Xp3}{\partial \alpha p1p3} & \frac{\partial Xp3}{\partial Sp1p2} & \frac{\partial Xp3}{\partial \alpha p1p2} & \frac{\partial Xp3}{\partial Sp3p4} & \frac{\partial Xp3}{\partial \alpha p3p4} & \frac{\partial Xp3}{\partial Sp3p5} & \frac{\partial Xp3}{\partial \alpha p3p5} & \frac{\partial Xp3}{\partial Sp5p6} & \frac{\partial Xp3}{\partial \alpha p5p6} & \frac{\partial Xp3}{\partial Sp5p7} & \frac{\partial Xp3}{\partial \alpha p5p7} & \frac{\partial Xp3}{\partial Sp6p8} & \frac{\partial Xp3}{\partial \alpha p6p8} & \frac{\partial Xp3}{\partial Sp6p9} & \frac{\partial Xp3}{\partial \alpha p6p9} \\
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\end{bmatrix}
$$

## 4.3 Covariance matrix

The variance matrix below is a diagonal matrix containing symbolic representations of the distance and direction variances between the points. Since the observations are uncorrelated this matrix contains zero in place of the covariances usually present in the off diagonals.

$$
\begin{bmatrix}
\sigma Sp1p3^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sigma \alpha p1p3^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma Sp1p2^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \sigma \alpha p1p2^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & \sigma Sp3p4^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma \alpha p3p4^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma Sp3p5^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma \alpha p3p5^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma Sp5p6^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma \alpha p5p6^2 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma Sp5p7^2 & 0 & 0 & 0 & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma \alpha p5p7^2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma Sp6p8^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma \alpha p6p8^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma Sp6p9^2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma \alpha p6p9^2
\end{bmatrix}
$$

Finally the covariance matrix of all points can be formed using the law of variances and covariance. The formula is as follows:

$$\Sigma p3p2p4p5p6p7p8p9 = J \Sigma s\alpha J^T$$

Once the final covariance matrix is formed we can sub in numerical values to evaluate the matrix.

# 5 Chapter 6 - Error ellipse

## 5.1 Theory

Error ellipse define an area of spatial confidence in two or more dimensions. For a point p in a 2-dimensional graph the ellipse about p will represent the spatial distribution of errors, i.e, it is the area of confidence for a bivariate distribution. To construct an error ellipse we need the size of the semi-major and semi-minor axis. We also require the orientation of the ellipse with respect to the x, y system. The covariance matrix of p is used to determine these quantities. Assuming the covariance matrix of the position x, y is given by:

$$\Sigma xy = \begin{bmatrix} \sigma x^2 & \sigma xy^2 \\ \sigma y^2 & \sigma xy^2 \end{bmatrix}$$

To determine $\sigma u^2$ and $\sigma v^2$ , the semi-major and semi- minor axis respectively we use the formula:

$$\sigma u^2 = \frac{\sigma x^2 + \sigma y^2 + \sqrt{(\sigma x^2 + \sigma y^2)^2 + 4\sigma xy2}}{2}$$

$$\sigma v^2 = \frac{\sigma x^2 + \sigma y^2 - \sqrt{(\sigma x^2 + \sigma y^2)^2 + 4\sigma xy2}}{2}$$

And the orientation $\alpha$ can be found by :

$$tan(2\alpha) = \frac{-2\sigma xy}{\sigma x^2 - \sigma y2}$$

# 6 Chapter 7 - Visual analysis

The figure below is a visualization of the network of polars. Using a python module matplotlib we were able to network all the nodes visually. For all graphs the y-axis is vertical and x-axis is horizontal.

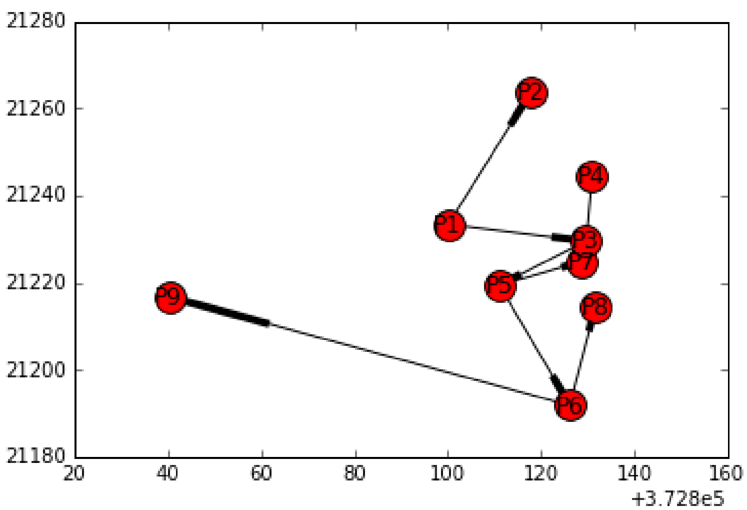


Figure 5: Coordinate specified network

The ellipse calculated for this problem set were relatively small. However they have been scaled up by a factor of 5 for better visualization.

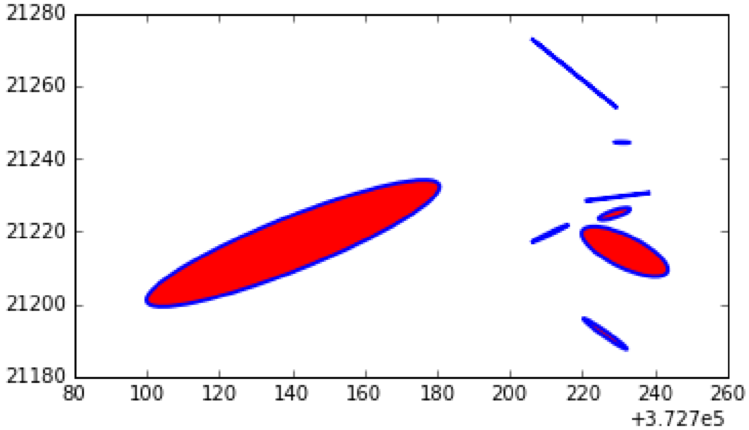


Figure 6: Error ellipse diagram

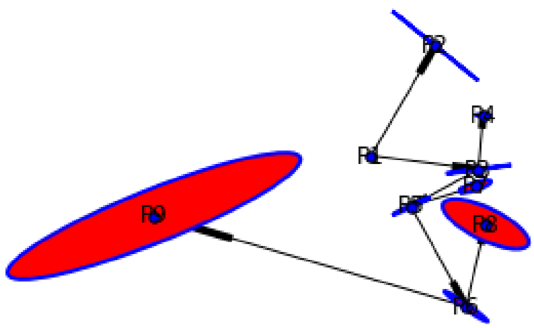

Figure 7: Error ellipse on network

# 7 Conclusion

As was speculated in the introduction, many of the problems revolved around manipulating huge data sets. The efficiency was reduced when a number of lists had to be used as every time a search operation is conducted a linear search method was done. The procedure of producing the covariance matrix was consistent with the law of variances, covariances and propagation. The size of the Jacobian and variance matrices are relevant to the number of observations made and resulted in a 16x16 large matrix. The final covariance relating all the elements was evaluated for a number of different variances, distances and directions. It was noticed that over longer distances the propagation was significantly bigger. For the purpose of simulation, smaller distances were used. It was also evident that larger distances influenced the error ellipses much more than direction. This is a direct result from variances in the covariances matrix being large. The outcome of semi-major and semi-minor axis were really big. In other words, the distance had the most effect on the confidence of the error ellipse. However for simulation and efficiency purposes the distances were reduced. The orientation of the ellipses is suggestive that the major axes of the relative ellipses are perpendicular to the fixed points and minor points are towards the fixed points. This is indicative that a weakness in orientation is present and that more azimuth control is required.