Note Title 10/17/2008

$$x = 2x' + y'$$
$$y = x' + y'$$
$$w = 2x' + y' + w'$$

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix}$$

$$x^2 + y^2 = 1$$

$$\Rightarrow (2x' + y')^2 + (x' + y')^2 = 1$$

$$\Rightarrow 4x'^2 + y'^2 + 4x'y' + x'^2 + y'^2 + 2x'y' = 1$$

Note we are interested in $u = \frac{x'}{w'}$ & $v = \frac{y'}{w'}$, since no guarantee that $w' = 1$.

$$\therefore 1 = w^2 \quad (\because w = 1)$$

$$\therefore w^2 = 4x'^2 + y'^2 + w'^2 + 4x'y' + 4x'w' + 2y'w'$$

$$\therefore \cancel{4x'^2} + \cancel{y'^2} + \cancel{4x'y'} + x'^2 + y'^2 + 2x'y' = \cancel{4x'^2} + \cancel{y'^2} + w'^2 + \cancel{4x'y'} + 4x'w' + 2y'w'$$

or $x'^2 + y'^2 + 2x'y' - 4x'w' - 2y'w' - w'^2 = 0$

Divide by w'^2

$\therefore \left(\frac{x'}{w'}\right)^2 + \left(\frac{y'}{w'}\right)^2 + \frac{2x'}{w'} \cdot \frac{y'}{w'} - \frac{4x'}{w'} - \frac{2y'}{w'}$

$-1 = 0$

$\therefore u^2 + v^2 + 2uv - 4u - 2v - 1 = 0$

This is a parabola.

$\therefore$ finite pts. on circle has been made to infinite pts.

<u>Two parallel lines</u>

$4x + y = 5 \qquad 4x + y = 3$

$4x + y = 5w \qquad (w = 1)$

or $8x' + 4\cancel{y'} + x' + \cancel{y'} = 10x' + \cancel{5}y' + 5w'$

or $-x' = 5w'$

or $-\dfrac{x'}{w'} = 5$

$\Rightarrow \dfrac{x'}{w'} = 5$ or $u = 5$

$4x + y = 3w$

$8x' + 4y' + x' + y' = 6x' + 3y' + 3w'$

or $3x' + 2y' = 3w'$

$\Rightarrow 3\dfrac{x'}{w'} + \dfrac{2y'}{w'} = 3$

or $3u + 2v = 3$

$\therefore$ These two lines intersect.

Lets take the general case

$m_1 x + y = c$

$m_2 x + y = d$

$$m_1(2x' + y') + (x' + y') = c_1(2x' + y' + 2w')$$

$$w\left(2m_1 + 1 - 2c\right)\frac{x'}{w'} + \left(m_1 + 1 - c\right)\frac{y'}{w'}$$

$$= c$$

$$w\left(2m_1 + 1 - 2c\right)u + (m_1 + 1 - c)v$$
$$= c$$

Similarly

$$(2m_2 + 1 - 2d)u + (m_2 + 1 - d)v = c$$

$$\frac{2m_1 + 1 - 2c}{m_1 + 1 - c} = \frac{2m_2 + 1 - 2d}{m_2 + 1 - d}$$

Let $c = 0$

$$\frac{2m_1+1}{m_1+1} = \frac{2m_2+1-2d}{m_2+1-d}$$

$$\cancel{2m_1m_2} + 2m_1 - \cancel{2m_1d} + m_2 + \cancel{1} - d$$

$$= \cancel{2m_1m_2} + m_1 - \cancel{2dm_1} + 2m_2 + \cancel{1} - 2d$$

or $m_1 - m_2 + d = 0$

or $m_2 - m_1 = d$

$\therefore$ $y = m_1 x$

$y = m_2 x + d$

$\therefore$ Let $d = 1$.

$m_2 = 4$ $\quad$ $m_1 = 3$

$\therefore$ $y = 3x$

$y = 4x + 1$

$\therefore$ These two lines that intersect in (x, y) space will be parallel after they are projected.

With slope

$$\frac{2m_1+1}{m_1+1} = \frac{7}{4} = \frac{2m_2+1-2d}{m_2+1-d}$$

$$= \frac{8+1-2}{4+1-1} = \frac{7}{4}$$