

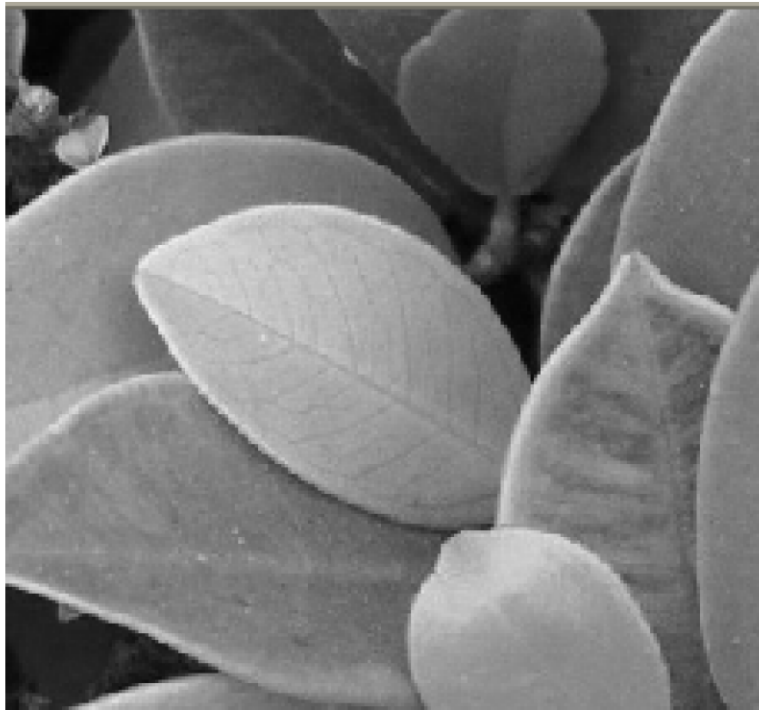
Extending DFT to 2D

Axes

- Frequency
 - Only positive
- Orientation
 - 0 to 180
- Repeats in negative frequency
 - Just as in 1D

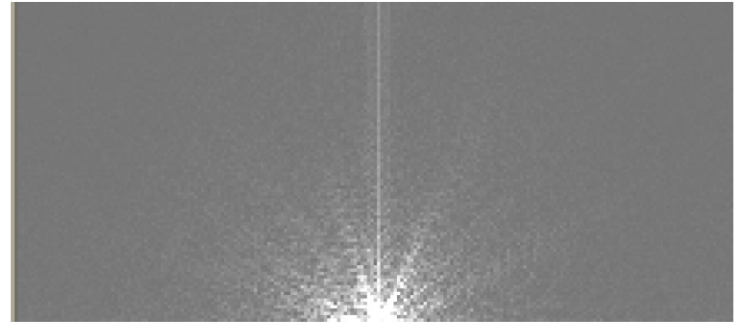
Example

Spatial Domain

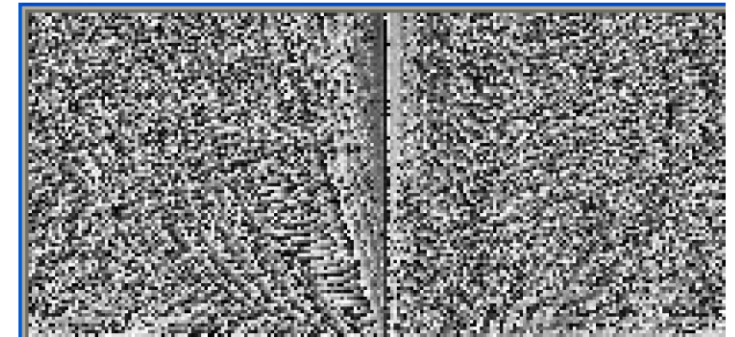


Frequency Domain

Amplitude

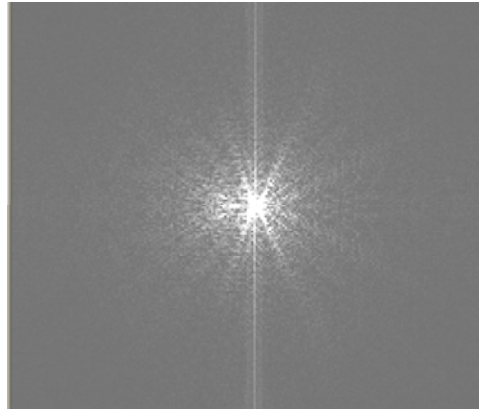
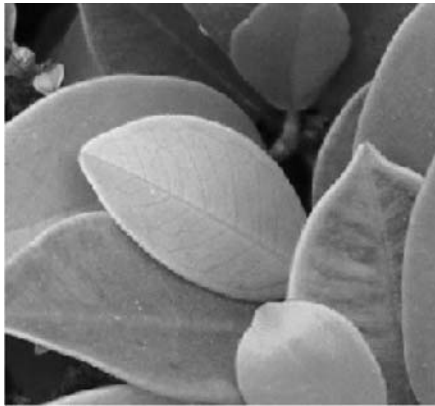


Phase



How it repeats?

- Just like in 1D
 - Even function for amplitude
 - Odd function for phase
- For amplitude
 - Flipped on the bottom



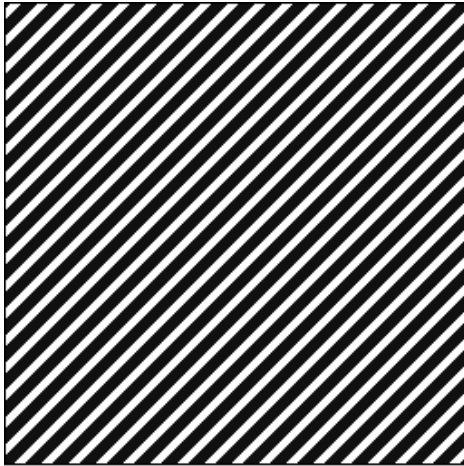
Why all the noise?

- Values much bigger than 255
- DC is often 1000 times more than the highest frequencies
- Difficult to show all in only 255 gray values

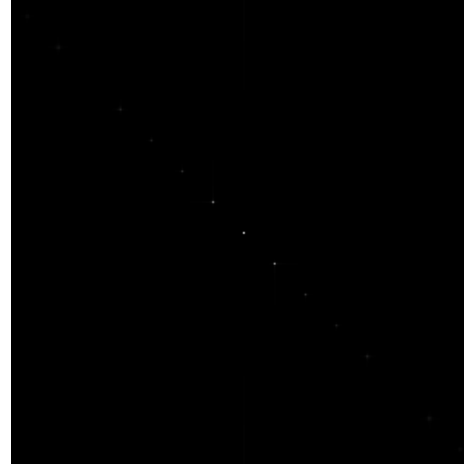
Mapping

- Numerical value = i
- Gray value = g
- Linear Mapping is $g = ki$
- Logarithmic mapping is $g = k \log(i)$
 - Compresses the range
 - Reduces noise
 - May still need thresholding to remove noise

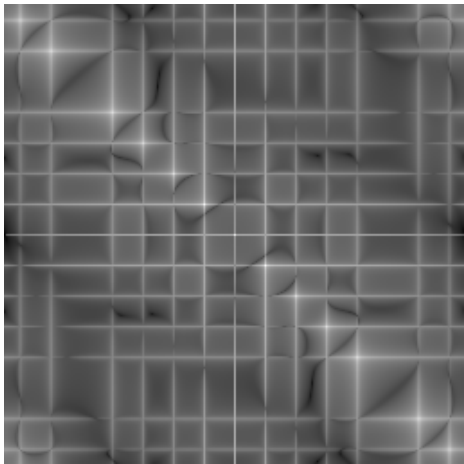
Example



Original



DFT Magnitude

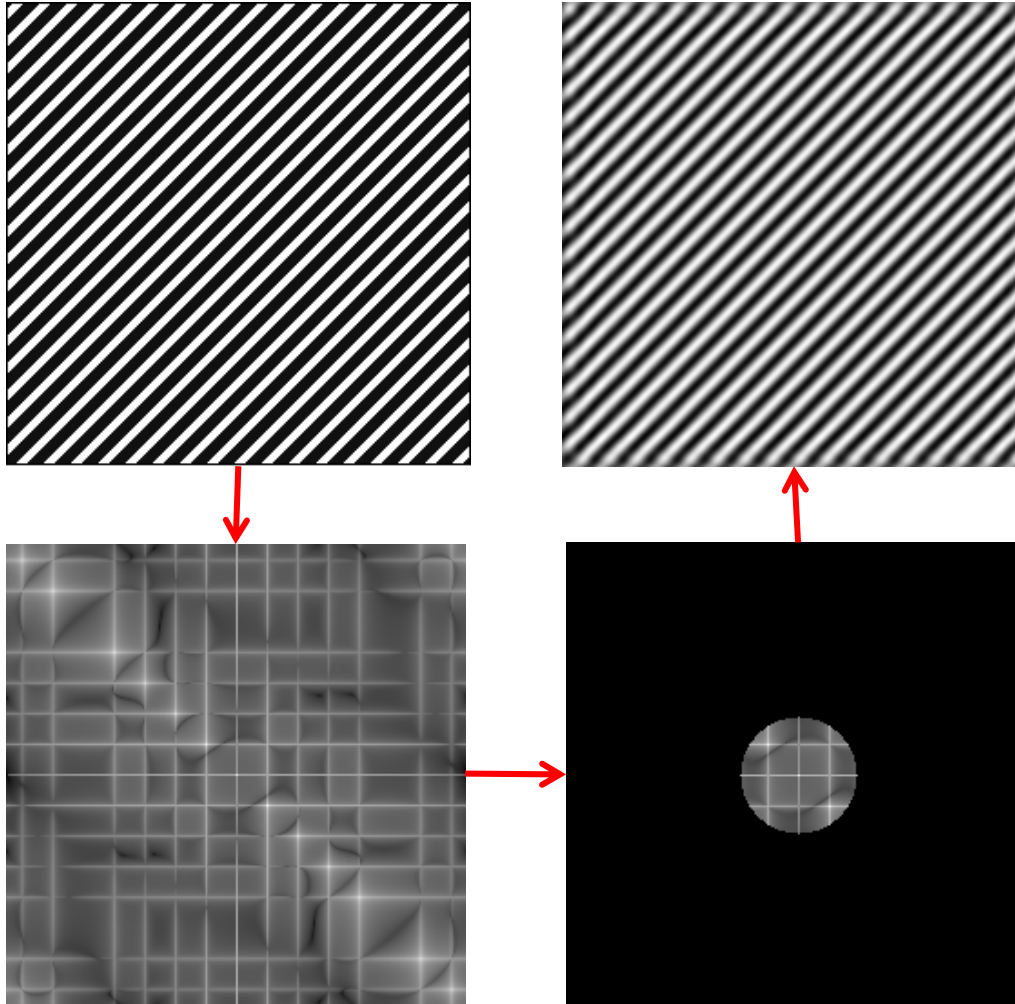


In Log scale

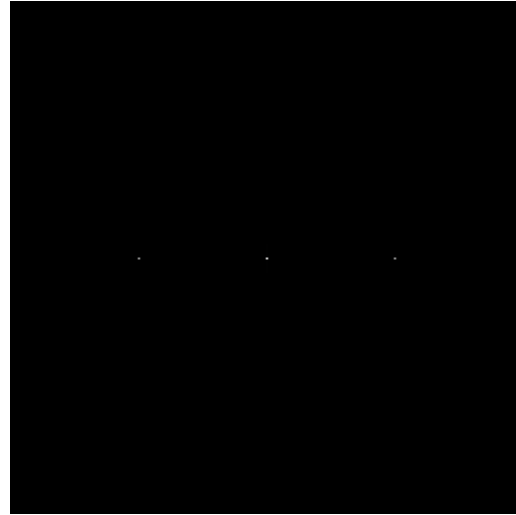
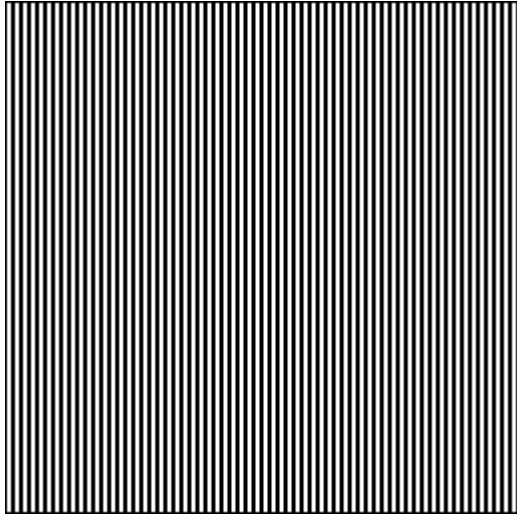


Post Thresholding

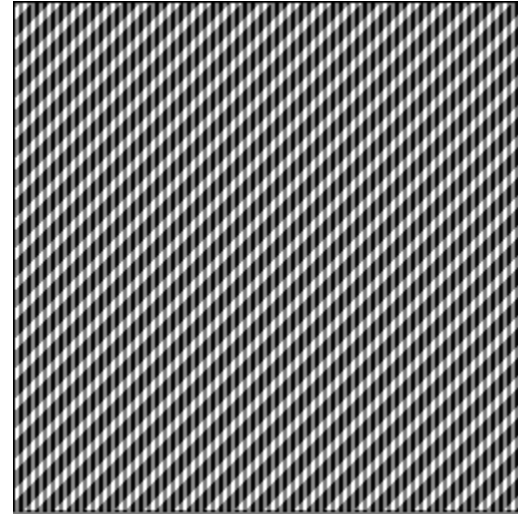
Low Pass Filter Example



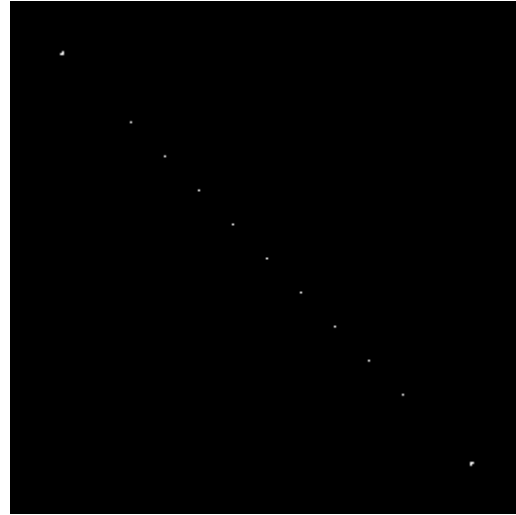
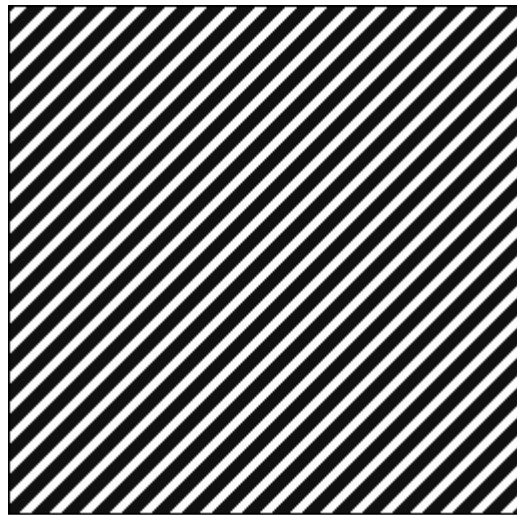
Additivity



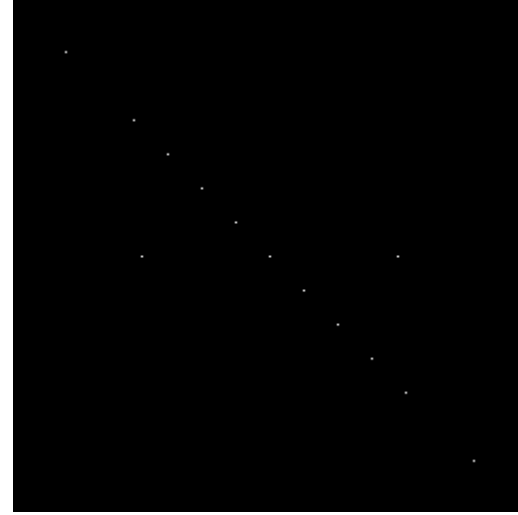
+



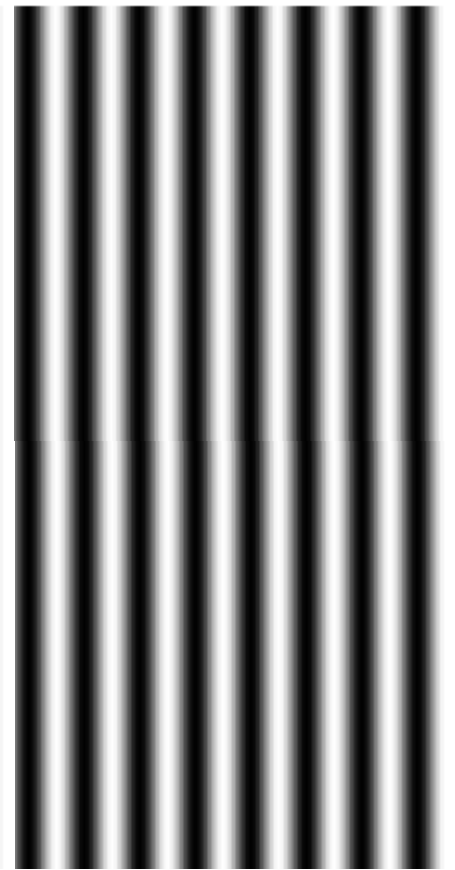
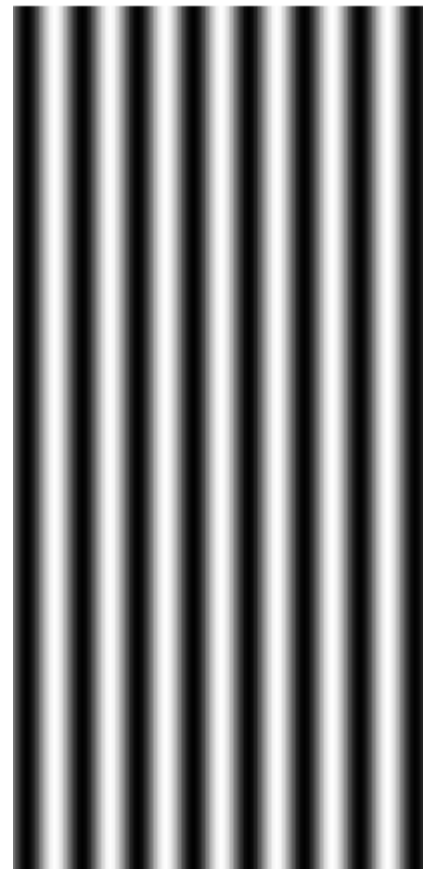
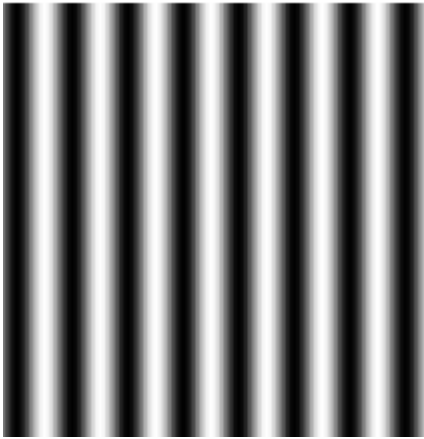
Inverse DFT

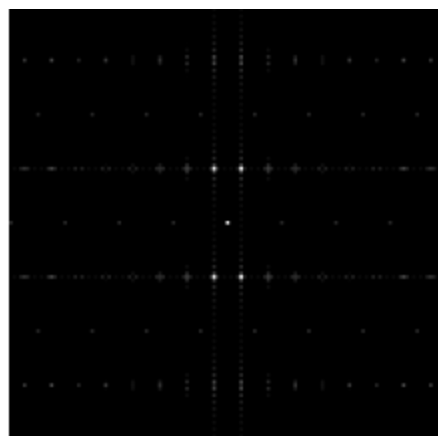
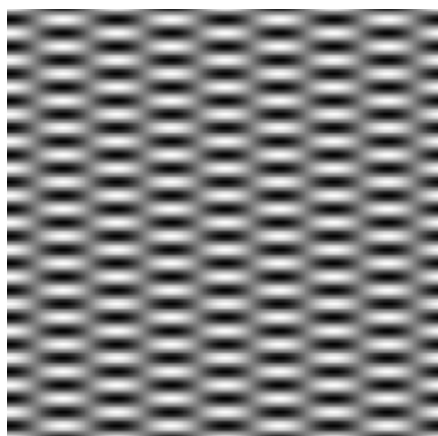
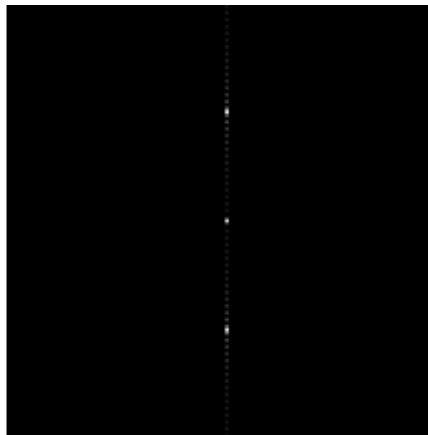
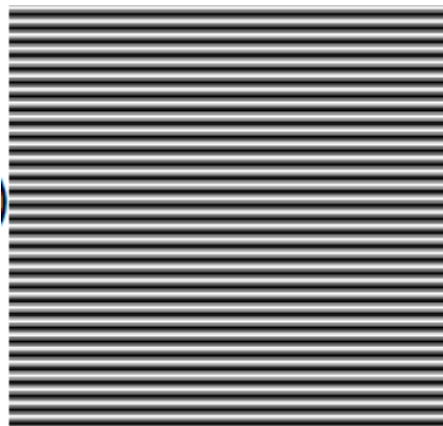


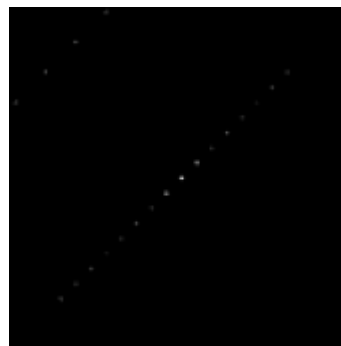
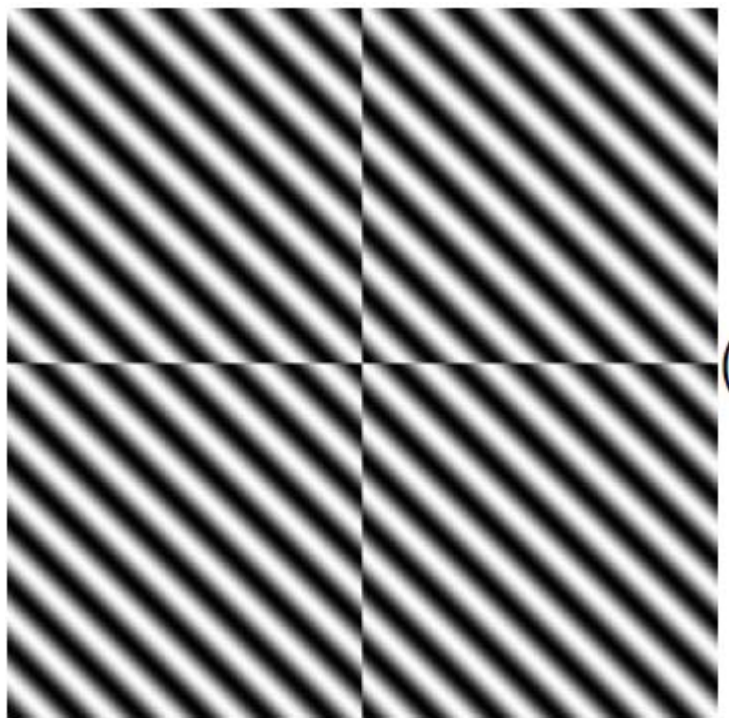
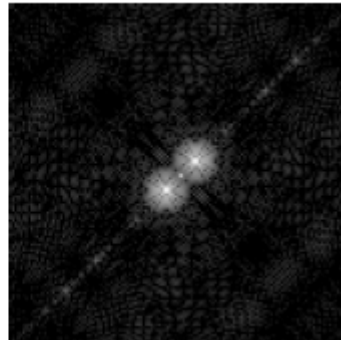
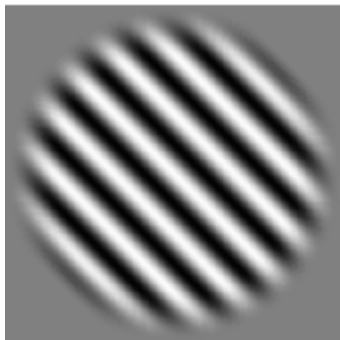
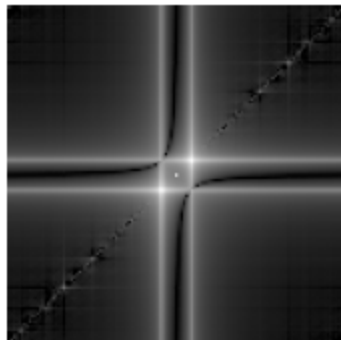
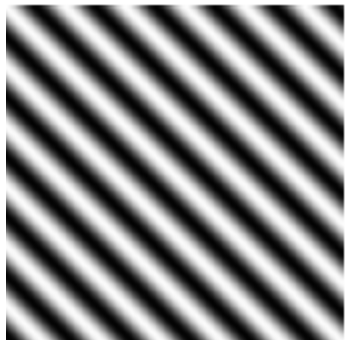
=



Nuances





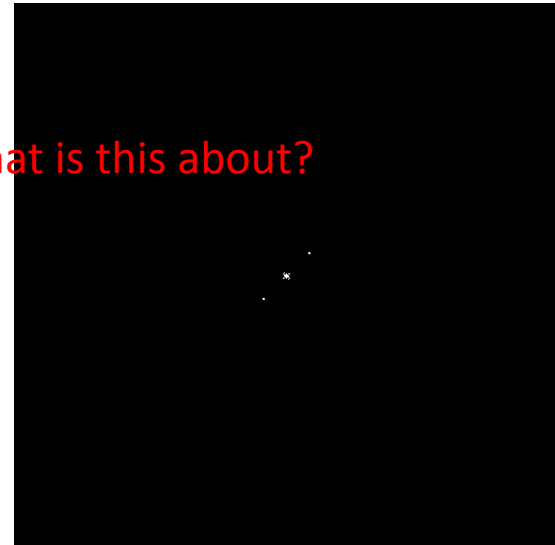
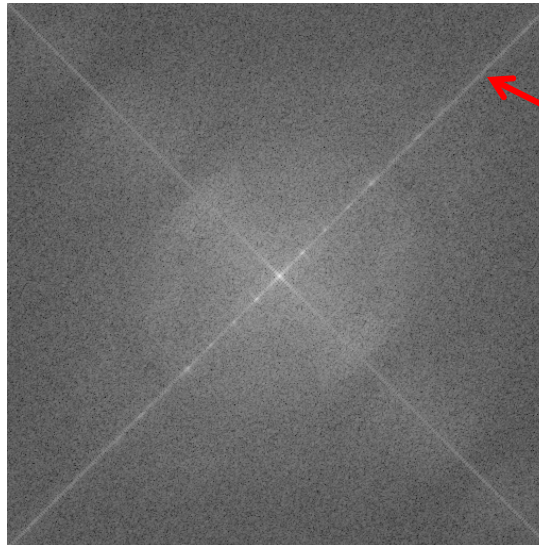
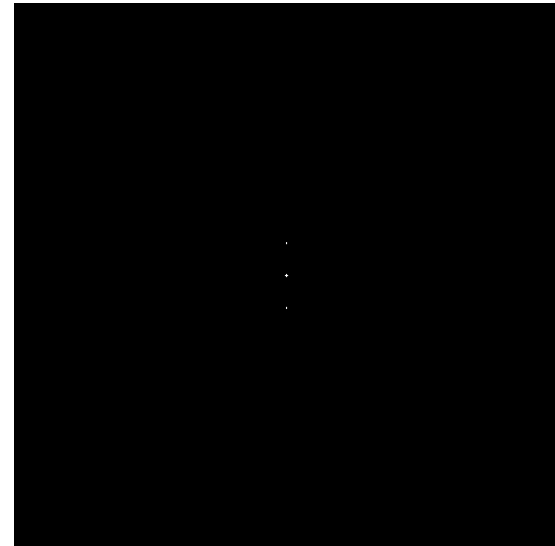
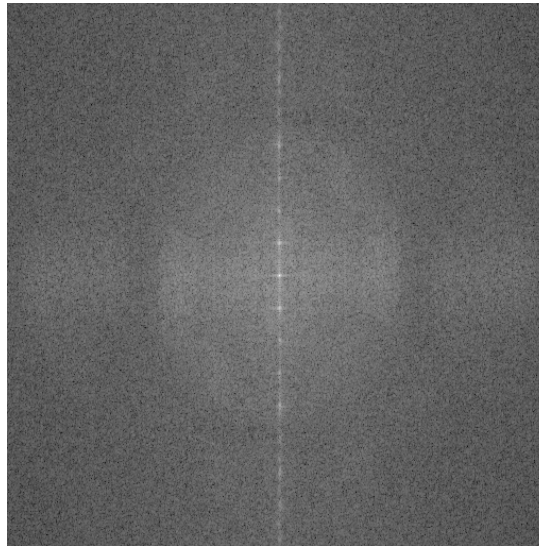


Rotation

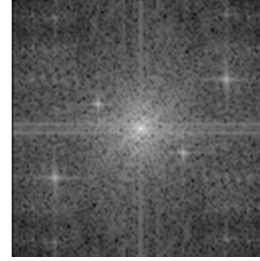
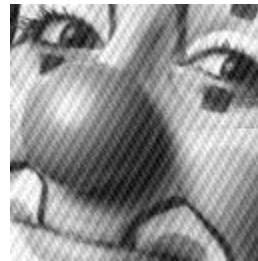
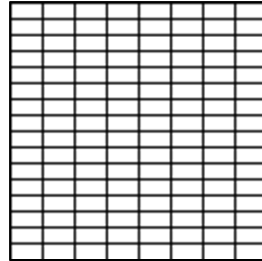
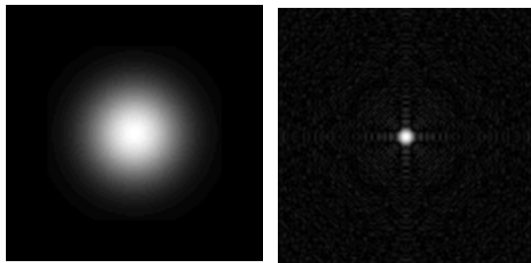
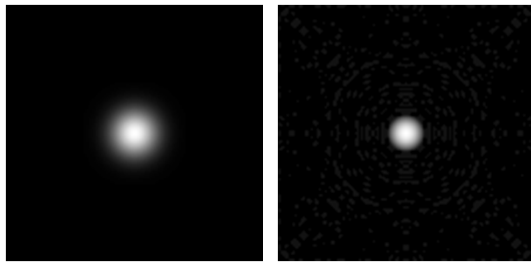
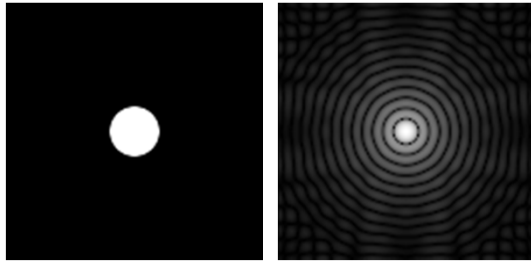
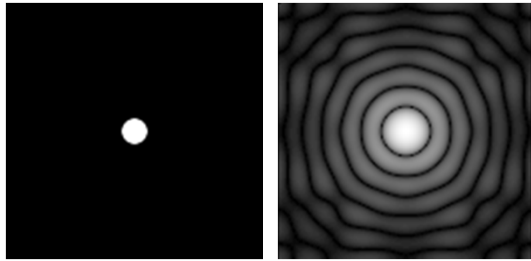
Sonnet for Lena

O dear Lena, your beauty is no vast
It is hard sometimes to describe it fast.
I thought the entire world I would impress
If only your portrait I could compress.
Alas! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
Hard to match with sums of discrete cosines.
And for your lips, sensual and tactual
Thirteen Crays found not the proper fractal.
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when Stern took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

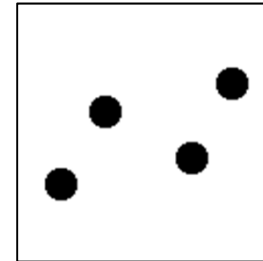
Thomas Colthart

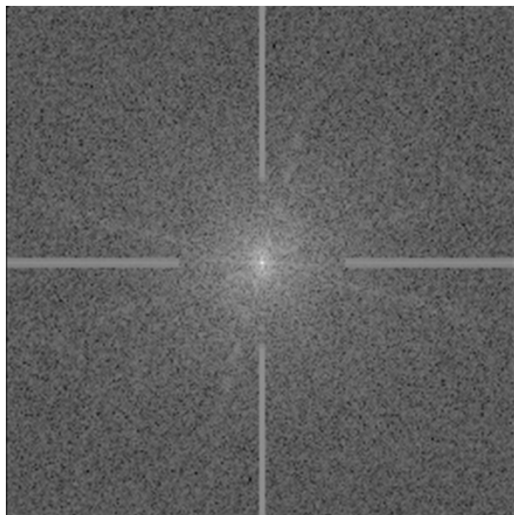


What is this about?

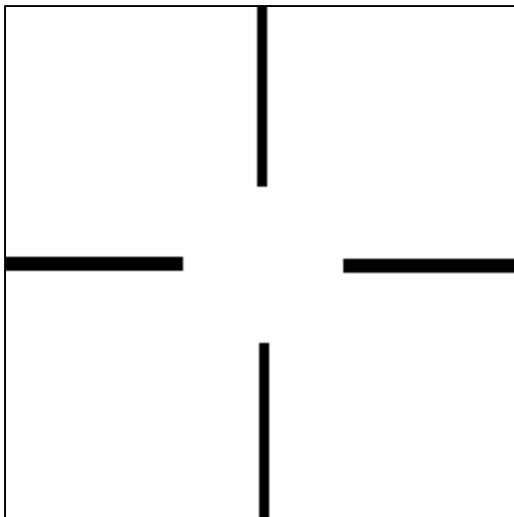


X

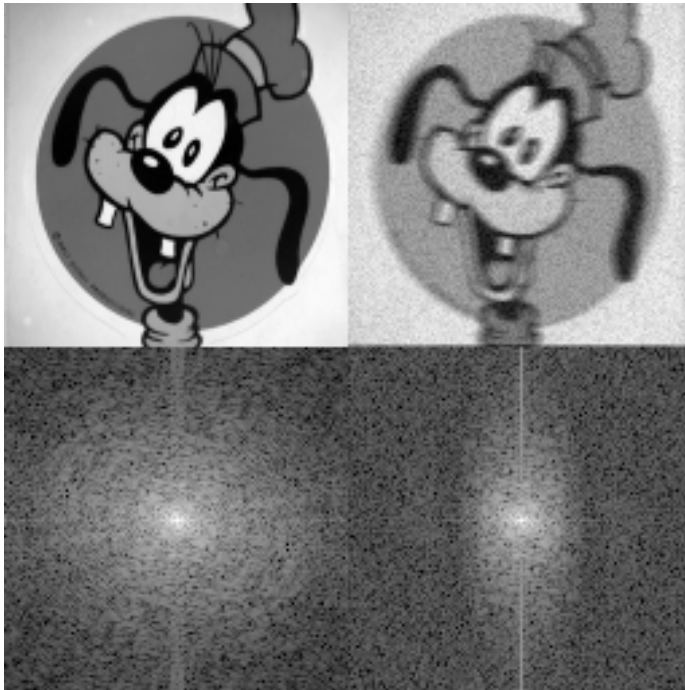




X

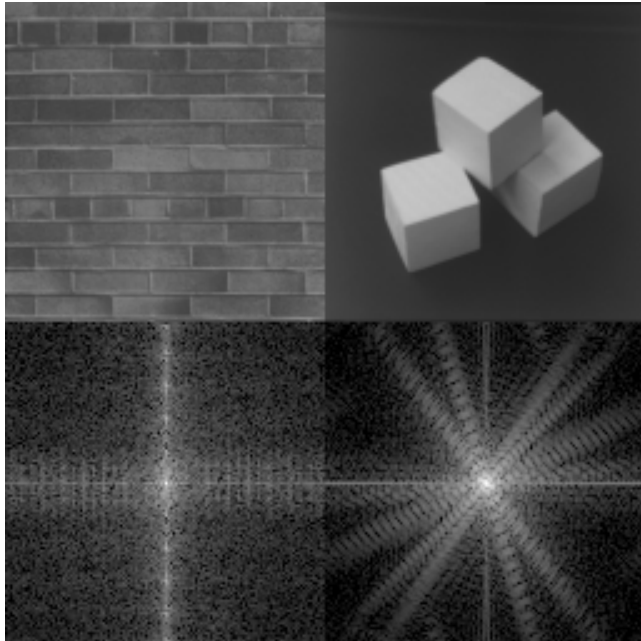


More examples: Blurring



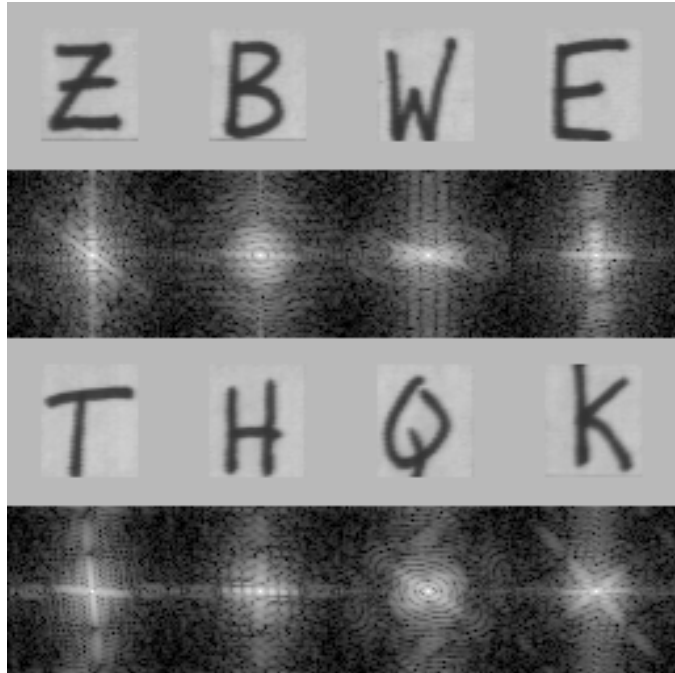
- Note energy reduced at higher frequencies
- What is direction of blur?
 - Horizontal
- Noise also added
 - DFT more noisy

More examples: Edges



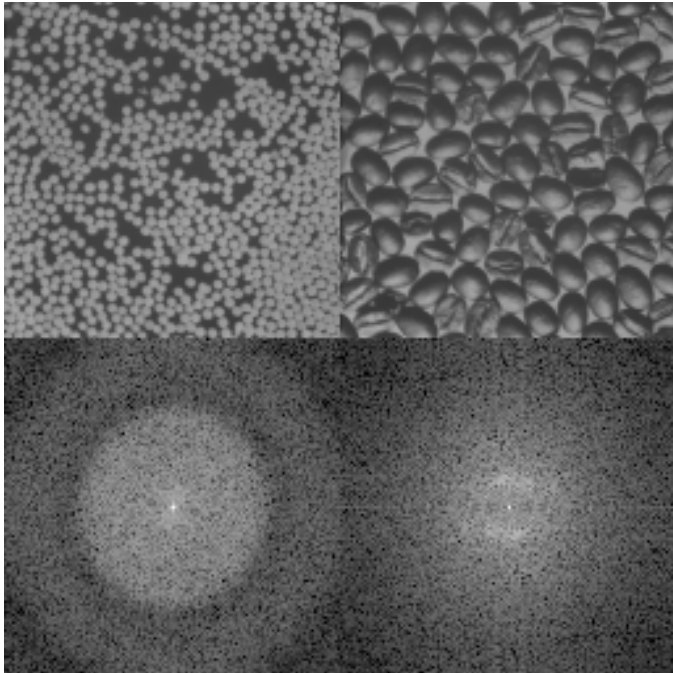
- Two direction edges on left image
 - Energy concentrated in two directions in DFT
- Multi-direction edges
 - Note how energy concentration synchronizes with edge direction

More examples: Letters



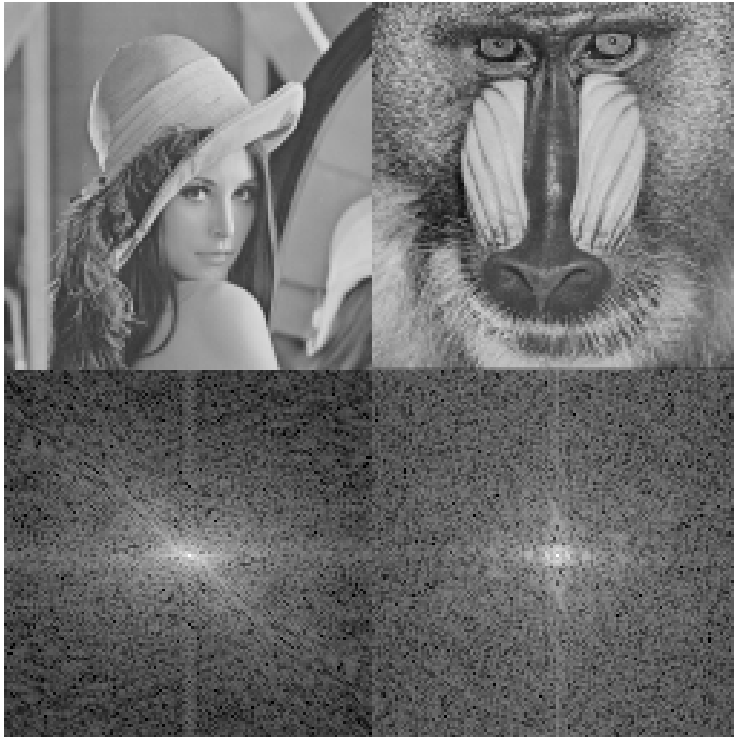
- DFTs quite different
 - Specially at low frequencies
- Bright lines perpendicular to edges
- Circular segments have circular shapes in DFT

More examples: Collections



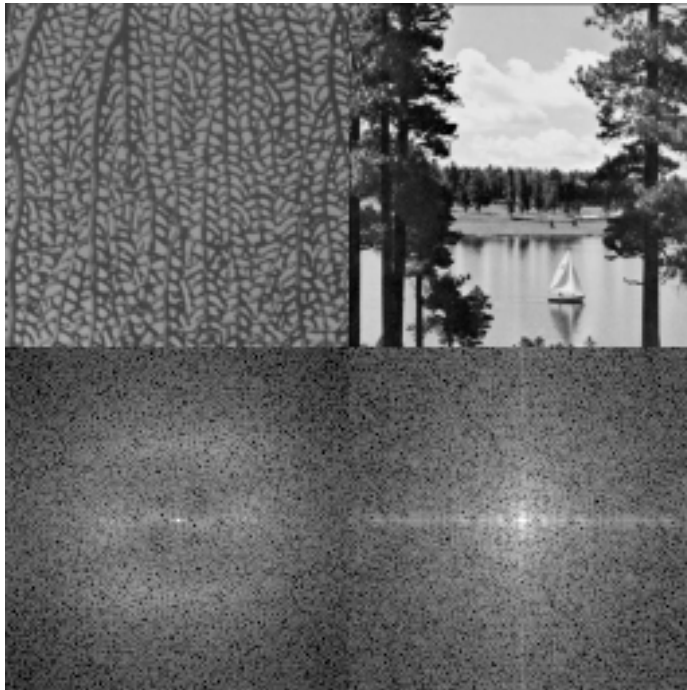
- Concentric circle
 - Due to pallets symmetric shape
 - DFT of one pallet
 - Similar
- Coffee beans have no symmetry
 - Why the halo?
 - Illumination

More examples: Natural Images



- Natural Images
- Why the diagonal line in Lena?
 - Strongest edge between hair and hat
- Why higher energy in higher frequencies in Mandril?
 - Hairs

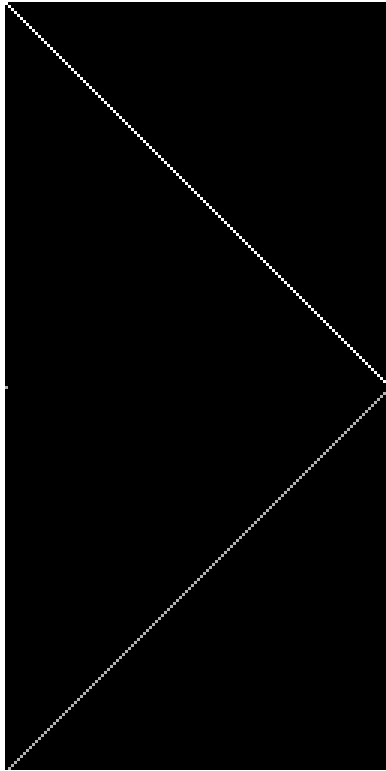
More examples: Natural Images



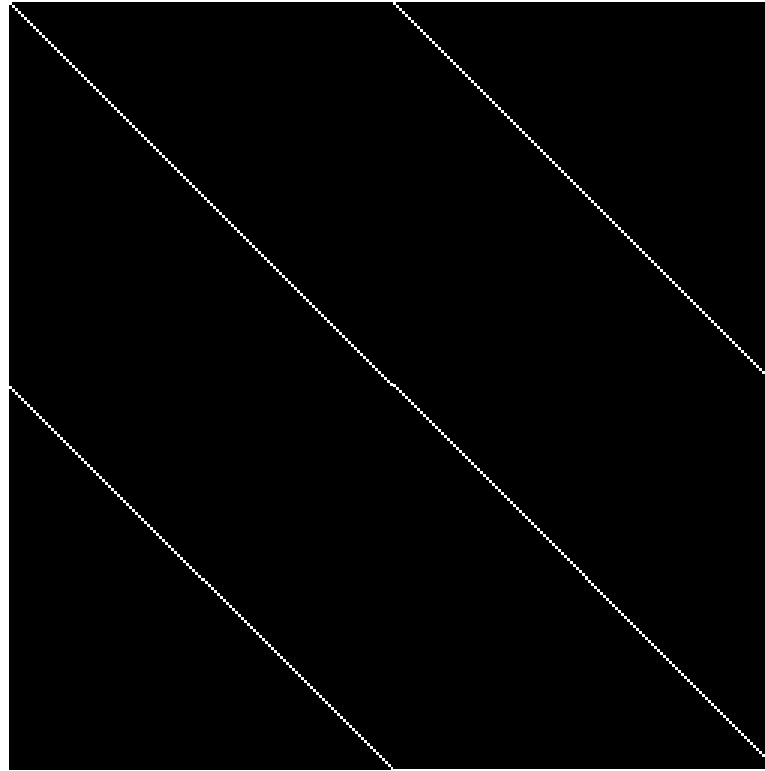
- Why high energy half way out towards maximum?
 - 2 pixels wide holes
 - Frequency half way to maximum
- Why horizontal energy concentration?
 - Tree Trunks
- Why a little lower vertical energy concentration?
 - Boundary between lake and land

More examples

Spatial



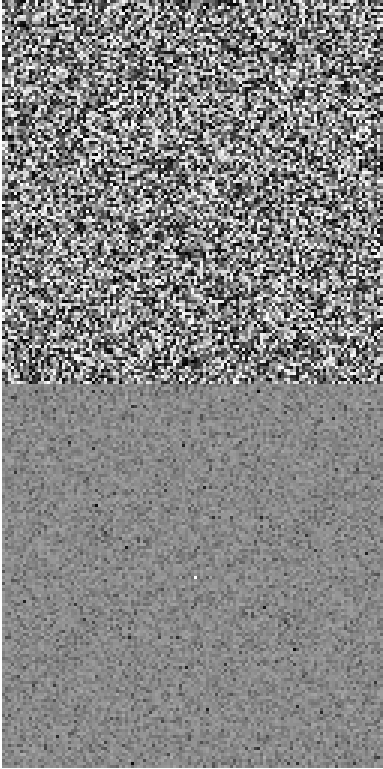
Frequency



- Repeatability makes perfect periodic signal
- Therefore perfect result perpendicular to it

More examples

Spatial



Frequency

- Just a gray telling all frequencies
- Why the bright white spot in the center?