Assignment 2

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Question 1(a)

The Finite Difference Method is used to solve for electrostatic potential of a region, with $V = V_0$ on the left side and $V = 0$ on the right side of the region. The top and bottom of the region are not fixed, so the problem is essential in one dimension.

```
close all
clear all

nx=150; %L
ny=100; %W
BC_left=1;
BC_right=0;
BC_top=0;
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

for a=1:nx
    for b=1:ny
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==nx
            %Right Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
            % Top
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nym= a +(b)*nx;
```

```
            G(n,n) = -3;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
            B(n)=BC_top;
        elseif b==ny
            %Bottom
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;

            G(n,n) = -3;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nyp) = 1;
            B(n)=BC_bottom;
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

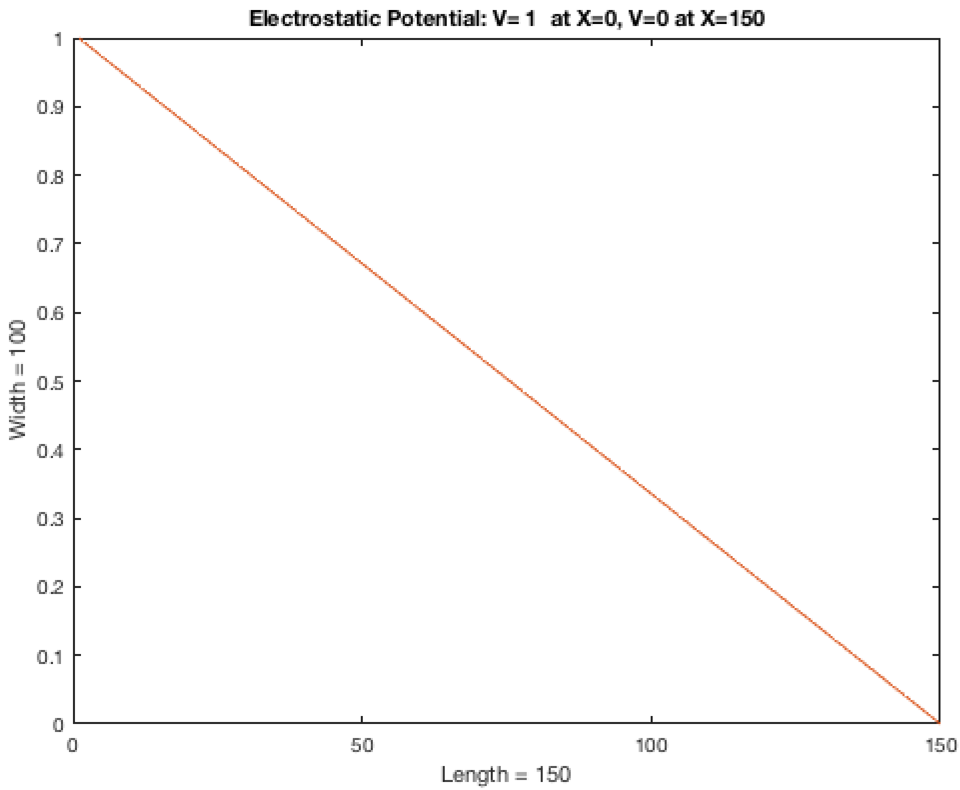
            G(n,n) = -4;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
            G(n,nyp) = 1;
        end
    end
end

% figure(1)
% grid on
% spy(G)
% title("G-matrix composition")

V=G\B';

Vmap = zeros(nx,ny);
for a=1:nx
    for b = 1:ny
      n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

figure(2)
plot(Vmap)
xlabel(sprintf('Length = %d', nx))
ylabel(sprintf('Width = %d', ny))
title(sprintf('Electrostatic Potential: V= %d  at X=0, V=%d at X=
%d',BC_left, BC_right,nx))
```



Question 1(b)

The Finite Difference Method is used to solve for electrostatic potential of a region, with $V = V_0$ on the left side and $V = 0$ on the right side of the region. The top and bottom of the region are not fixed, so the problem is essential in one dimension.

```
BC_left=1;
BC_right=1;
BC_top=0;
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

for a=1:nx
    for b=1:ny
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==nx
            %Right Side
```

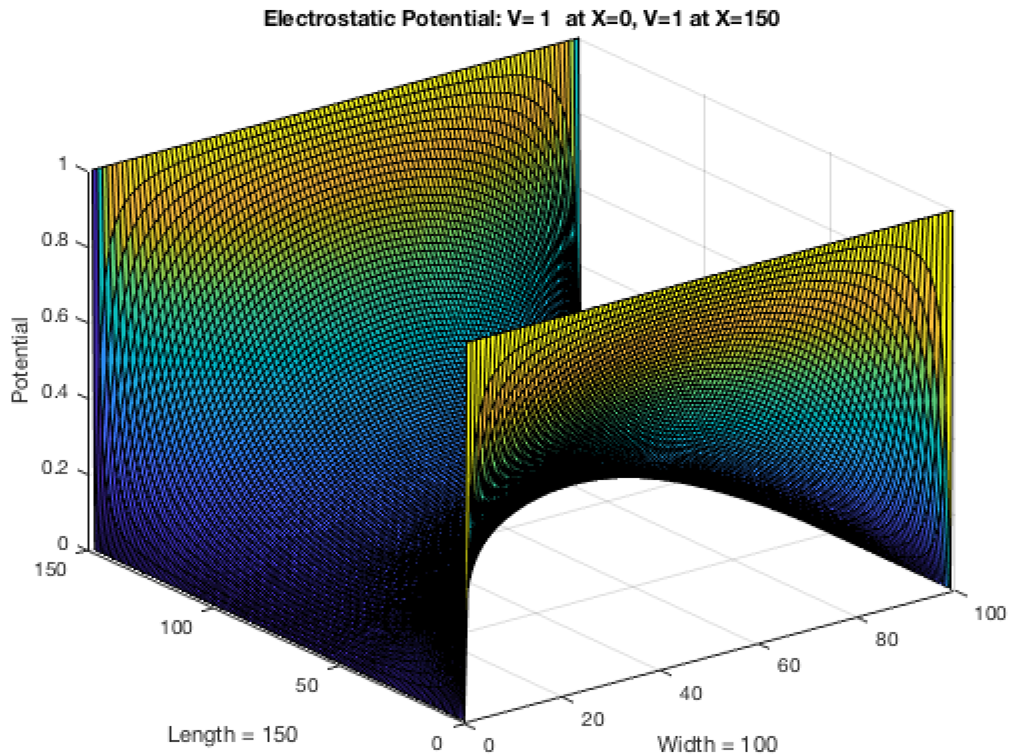
```
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
            % Top
            G(:,n) = 0;
            G(n,n) = 1;
            B(n)=BC_top;
        elseif b==ny
            %Bottom
            G(:,n) = 0;
            G(n,n) = 1;
            B(n)=BC_bottom;
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

            G(n,n) = -4;
            G(n,nxm) = 1;
            G(n,nxp) = 1;
            G(n,nym) = 1;
            G(n,nyp) = 1;
        end
    end
end

V=G\B';

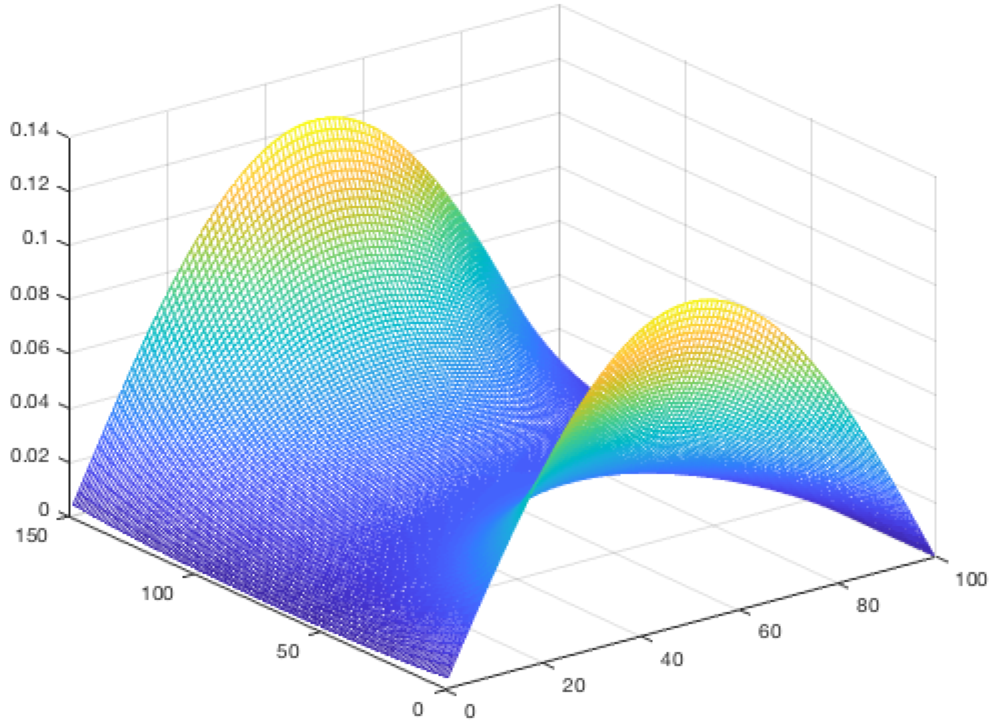
Vmap = zeros(nx,ny);
for a=1:nx
    for b = 1:ny
      n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

figure(3)
surf(Vmap)
ylabel(sprintf('Length = %d', nx))
xlabel(sprintf('Width = %d', ny))
zlabel('Potential')
title(sprintf('Electrostatic Potential: V= %d  at X=0, V=%d at X=
%d',BC_left, BC_right,nx))
```



Analytical Solution to electrostatic potential where $V_0 = 1$

```
L=nx;
W=ny;
Vx = zeros(L,W);
for n = 1:2:100
    for x = 1:L
        for y = 1:W
            Vx(x,y)=Vx(x,y)+(4/pi)*(1/n)*cosh((n*pi*(x-L/2))/W)/
cosh(n*pi*L/W).*sin(n*pi*y/W);
        end
    end
   figure(4)
   mesh(Vx)
   hold on
   pause(0.01)
end
```



Ultimately, the solution should be stopped when the solution reaches the desired accuracy, indicated by some pre-decided variance factor. In this example, I arbitrarily chose a finite number of repetitions to simulate the solution. 100 repetitions is more than sufficient to obtain the correct representation. More repetitions than this does not provide increased accuracy. The analytical and mesh solutions agree to the extenet that supports the use of the mesh solution.

Question 2

In this section we invesitgate the Electric field and current diensity of a region under the same conditions as above, but this time we will introduce a dielectric type material, with a low conductivity and see how it changes the potential of the region.

```
nx=150; %L
ny=100; %W
BC_left=1;
BC_right=0;
BC_top=0;
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

Llow = round(L/3);
Lhigh= round(L-L/3);
Wlow =round(W/3);
Whigh=round(W-W/3);
```

```
sig = ones(W,L);

% set conductivity in boxes
for a = 1:Wlow
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end
for a = Whigh:W-1
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end
figure(5)
mesh(sig)


for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==ny
            %Right Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
            % Top
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            B(n)=BC_top;
        elseif b==nx
            %Bottom
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;

            yp = (sig(a,b)+sig(a,b-1))/2;
```

```
            xp = (sig(a-1,b)+sig(a,b))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;

            G(n,n) =-(yp+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nyp) = yp;
            B(n)=BC_bottom;
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            yp = (sig(a,b)+sig(a,b-1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(yp+ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            G(n,nyp) = yp;
            B(n)=0;
        end
    end
end


V=G\B';

Vmap = zeros(ny,nx);
for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

figure(6)
mesh(Vmap)
    xlim([0 L])
    ylim([0 W])
xlabel(sprintf('Length = %d', nx))
ylabel(sprintf('Width = %d', ny))
title(sprintf('Electrostatic Potential with Conductivity: V= %d  at
 X=0, V=%d at X=%d',BC_left, BC_right,nx))

[Ex,Ey] = gradient(-Vmap);

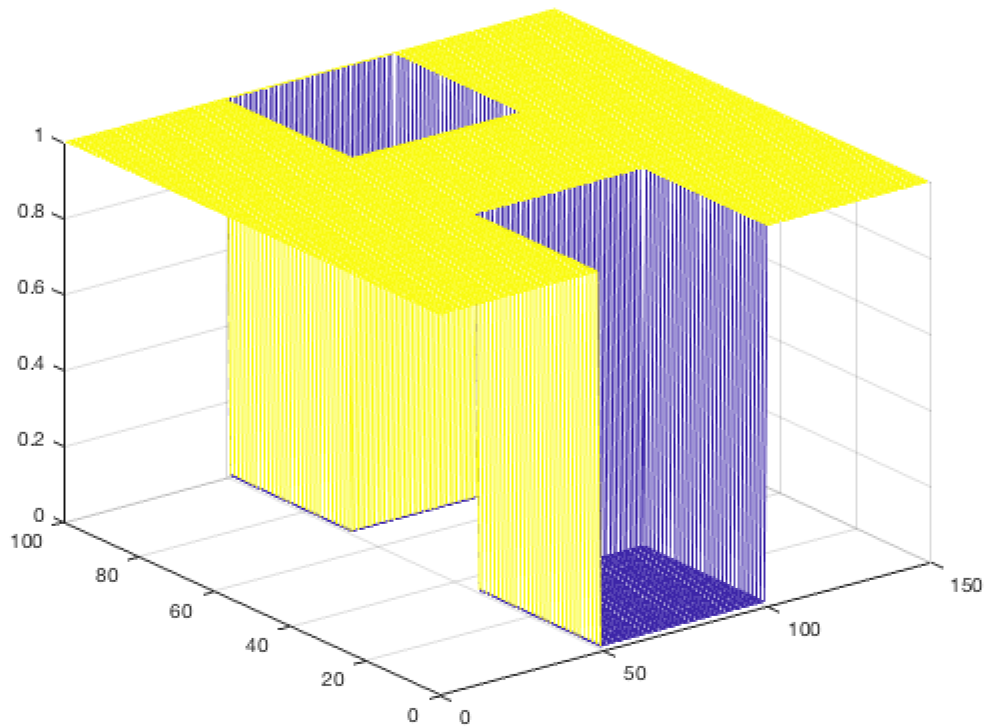
figure(7)
quiver(Ex,Ey)
```

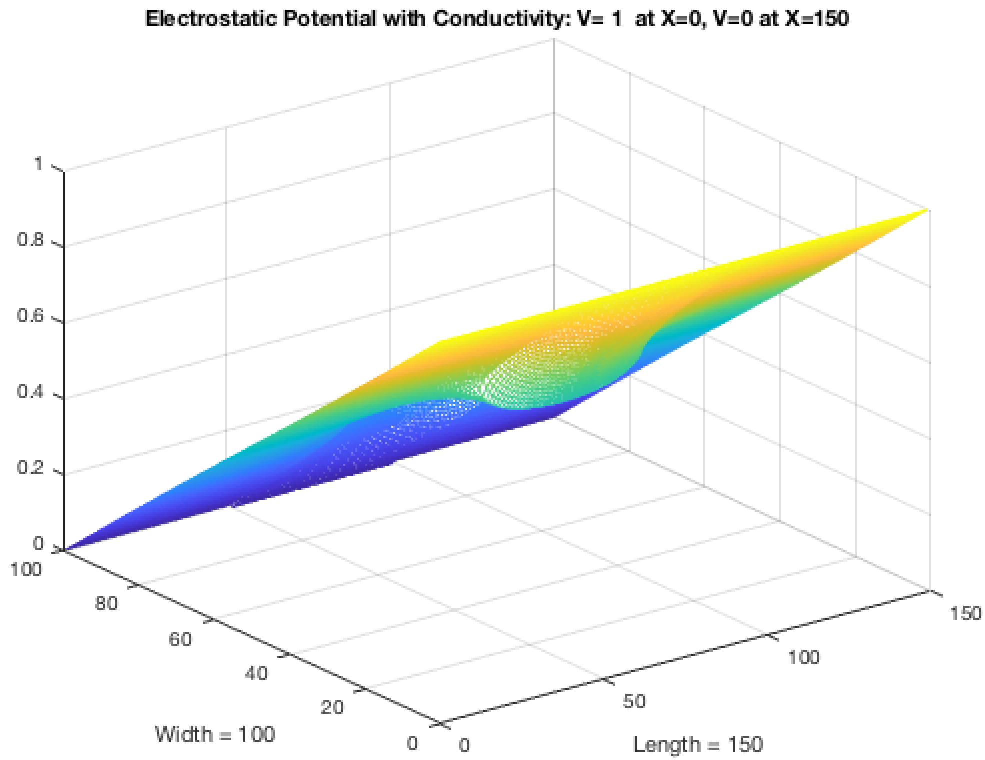
```
    xlim([0 L])
    ylim([0 W])
title('Electric Field')

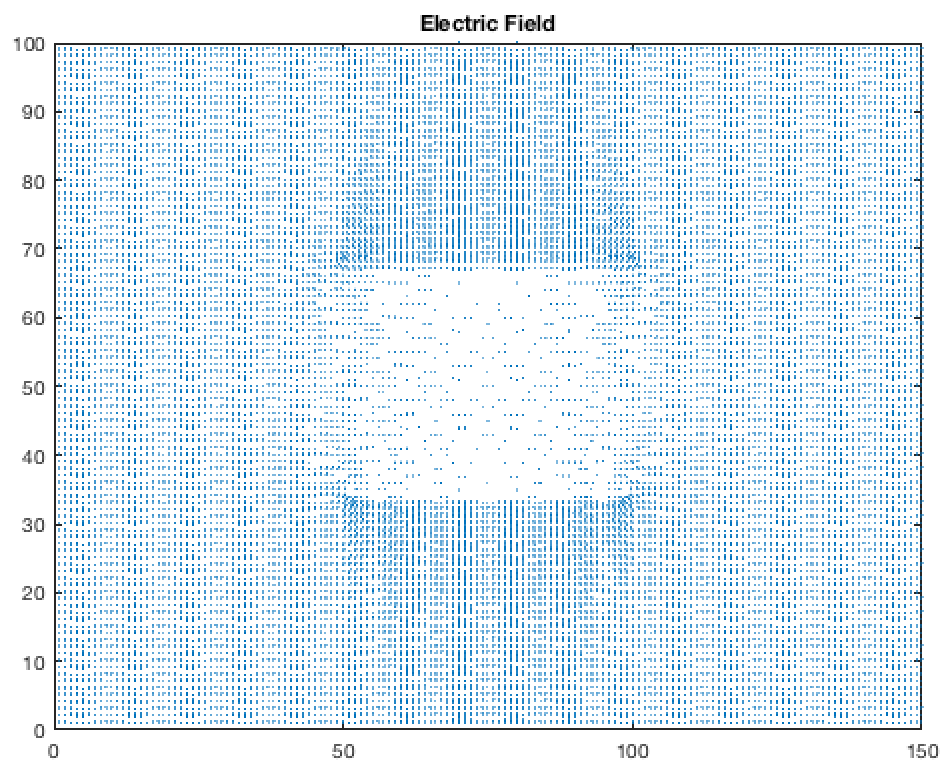
Jx=sig.*Ex;
Jy=sig.*Ey;

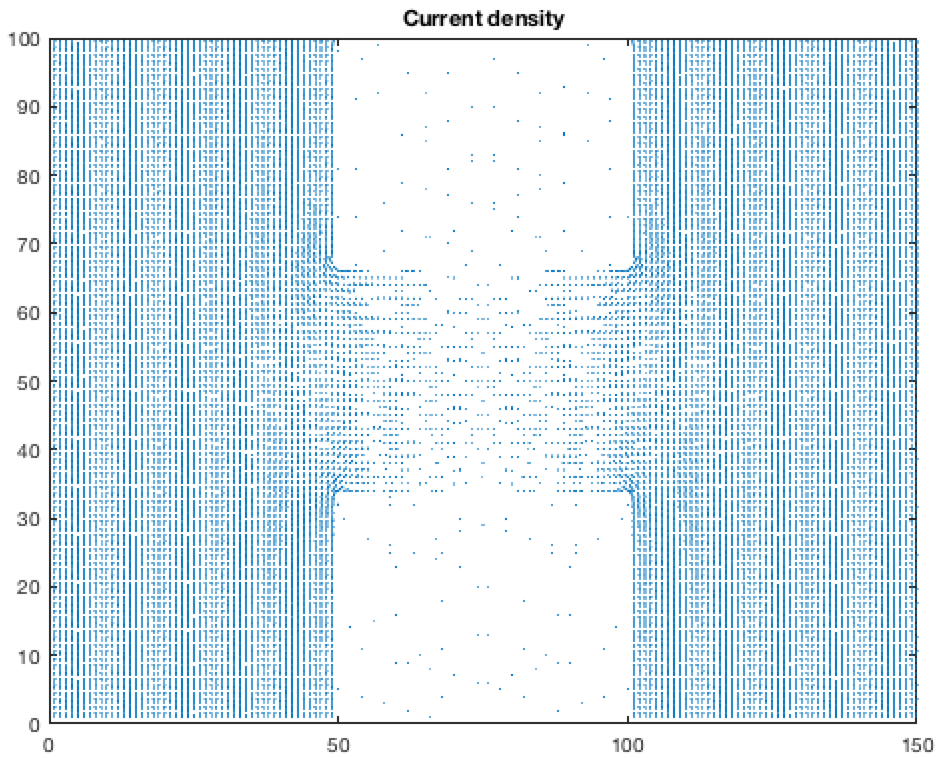
figure(8)
quiver(Jx,Jy)
    xlim([0 L])
    ylim([0 W])
title('Current density')
```

Warning: Matrix is singular to working precision.




Electrostatic Potential with Conductivity: V= 1 at X=0, V=0 at X=150
1
0.8
0.6
0.4
0.2
0
100
80
60
40
20
0
Width = 100
0
50
100
150
Length = 150


Electric Field
100
90
80
70
60
50
40
30
20
10
0
0
50
100
150



Mesh Density

In this section, we investigate how the mesh density changes the resulting current

```
nx=75; %L
for nx = 20:10:150
ny=round((2/3)*nx); %W
L=nx;
W=ny;
BC_left=1;
BC_right=0;
BC_top=0;
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

Llow = round(L/3);
Lhigh= round(L-L/3);
Wlow =round(W/3);
Whigh=round(W-W/3);

sig = ones(W,L);

% set conductivity in boxes
for a = 1:Wlow
```

```
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end
for a = Whigh:W-1
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end

for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==ny
            %Right Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
            % Top
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            B(n)=BC_top;
        elseif b==nx
            %Bottom
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;

            yp = (sig(a,b)+sig(a,b-1))/2;
            xp = (sig(a-1,b)+sig(a,b))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;

            G(n,n) =-(yp+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nyp) = yp;
            B(n)=BC_bottom;
```

```
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            yp = (sig(a,b)+sig(a,b-1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(yp+ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            G(n,nyp) = yp;
            B(n)=0;
        end
    end
end


V=G\B';

Vmap = zeros(ny,nx);
for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

[Ex,Ey] = gradient(-Vmap);

Jx=sig.*Ex;
Jy=sig.*Ey;


I=0;
for b=1:ny
    I=I+Jy(1,b)*(ny);%
end

figure(9)
plot(nx,I,'*')
hold on
ylabel('Current')
xlabel('Number of Mesh Divisions')
title('Current vs. Mesh Size')
pause(0.01);

end
```

```
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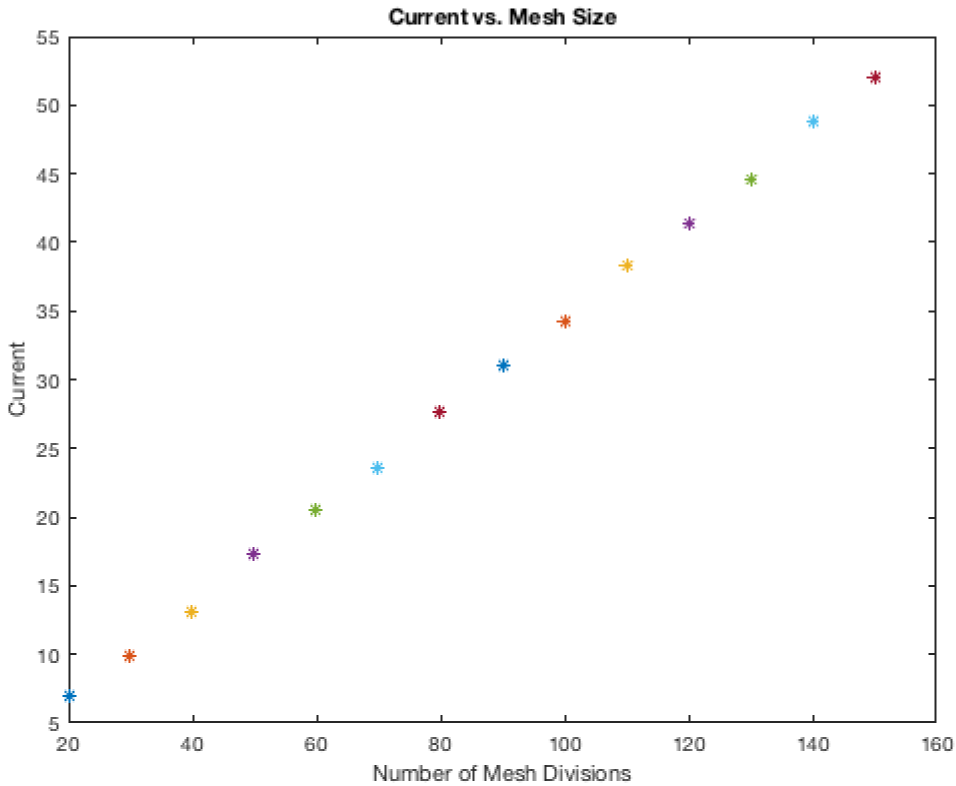
```



Bottle-Neck Size

Here we determine the effects of the size of the bottle-neck on the analysis.

```
nx=75; %L
ny=round((2/3)*nx); %W
L=nx;
W=ny;
BC_left=1;
BC_right=0;
BC_top=0;
```

```
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

for div=3:0.1:4
Llow = round(L/div);
Lhigh= round(L-L/div);
Wlow =round(W/div);
Whigh=round(W-W/div);

sig = ones(W,L);

% set conductivity in boxes
for a = 1:Wlow
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end
for a = Whigh:W-1
    for b=Llow:Lhigh
        sig(a,b) = 0.01;
    end
end

for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==ny
            %Right Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
            % Top
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            B(n)=BC_top;
```

```
        elseif b==nx
            %Bottom
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;

            yp = (sig(a,b)+sig(a,b-1))/2;
            xp = (sig(a-1,b)+sig(a,b))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;

            G(n,n) =-(yp+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nyp) = yp;
            B(n)=BC_bottom;
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            yp = (sig(a,b)+sig(a,b-1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(yp+ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            G(n,nyp) = yp;
            B(n)=0;
        end
    end
end


V=G\B';

Vmap = zeros(ny,nx);
for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

[Ex,Ey] = gradient(-Vmap);

Jx=sig.*Ex;
Jy=sig.*Ey;
```

```
I=0;
for b=1:ny
    I=I+Jy(1,b)*(ny);%
end

figure(10)
plot(div,I,'*')
hold on
ylabel('Current')
xlabel('Proportion of Region in Bottle-Neck')
title('Current vs. Bottle-Neck')
pause(0.01);

end

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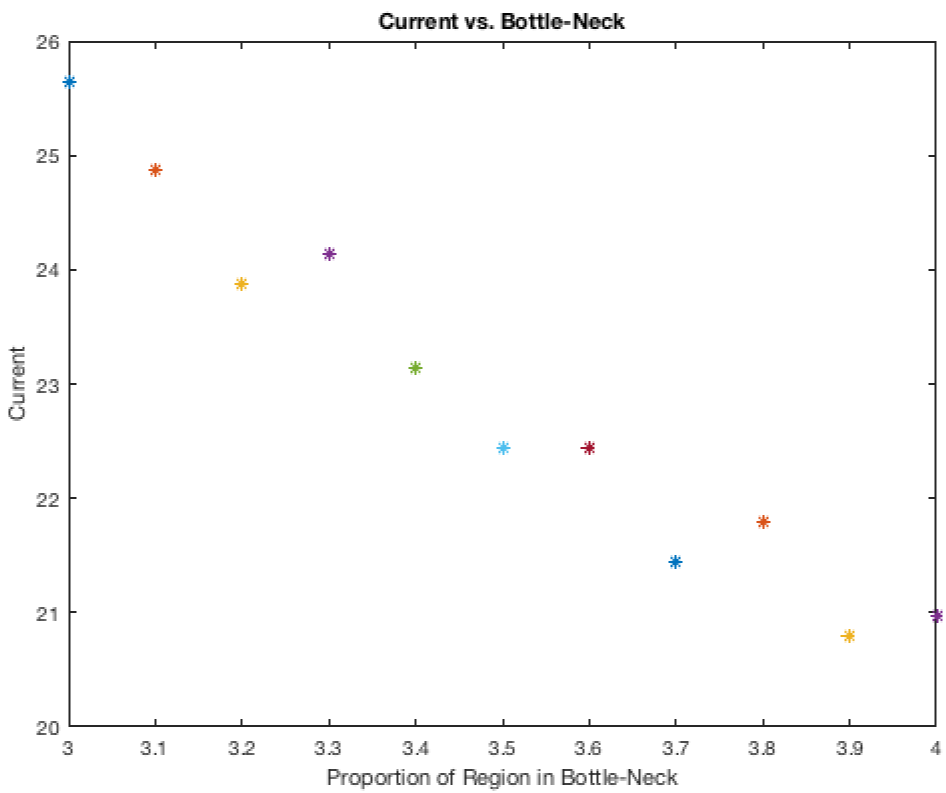
```



Varying Conductivity

In this section, we investigate how varying conductivy of materials will affect the resulting current in the region.

```
nx=75; %L
ny=round((2/3)*nx); %W
L=nx;
W=ny;
BC_left=1;
BC_right=0;
BC_top=0;
BC_bottom=0;

G=sparse(nx*ny);
B=zeros(1,nx*ny);

div = 3;
Llow = round(L/div);
Lhigh= round(L-L/div);
Wlow =round(W/div);
Whigh=round(W-W/div);



for s=0.001:0.005:2
   % set conductivity in boxes
   for a = 1:Wlow
       for b=Llow:Lhigh
           sig(a,b) = s;
       end
   end
   for a = Whigh:W-1
       for b=Llow:Lhigh
           sig(a,b) = s;
       end
   end

for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;

        if a==1
            %Left Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_left;
        elseif a==ny
            %Right Side
            G(n,:) = 0;
            G(n,n) = 1;
            B(n)=BC_right;
        elseif b==1
```

```
            % Top
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            B(n)=BC_top;
        elseif b==nx
            %Bottom
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;

            yp = (sig(a,b)+sig(a,b-1))/2;
            xp = (sig(a-1,b)+sig(a,b))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;

            G(n,n) =-(yp+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nyp) = yp;
            B(n)=BC_bottom;
        else
            %All Central Nodes
            nxm= a-1 +(b-1)*nx;
            nxp= a+1 +(b-1)*nx;
            nyp= a +(b-2)*nx;
            nym= a +(b)*nx;

            ym = (sig(a,b)+sig(a,b+1))/2;
            yp = (sig(a,b)+sig(a,b-1))/2;
            xm = (sig(a-1,b)+sig(a,b))/2;
            xp = (sig(a+1,b)+sig(a,b))/2;

            G(n,n) =-(yp+ym+xp+xm);
            G(n,nxm) = xm;
            G(n,nxp) = xp;
            G(n,nym) = ym;
            G(n,nyp) = yp;
            B(n)=0;
        end
    end
end


V=G\B';
```

```
Vmap = zeros(ny,nx);
for a=1:ny
    for b=1:nx
        n=a+(b-1)*nx;
      Vmap(a,b) = V(n);
    end
end

[Ex,Ey] = gradient(-Vmap);

Jx=sig.*Ex;
Jy=sig.*Ey;


I=0;
for b=1:ny
    I=I+Jy(1,b)*(ny);%
end

figure(10)
plot(s,I,'*')
hold on
ylabel('Current')
xlabel('Conductivity')
title('Current vs. Conductivity')
pause(0.01);

end

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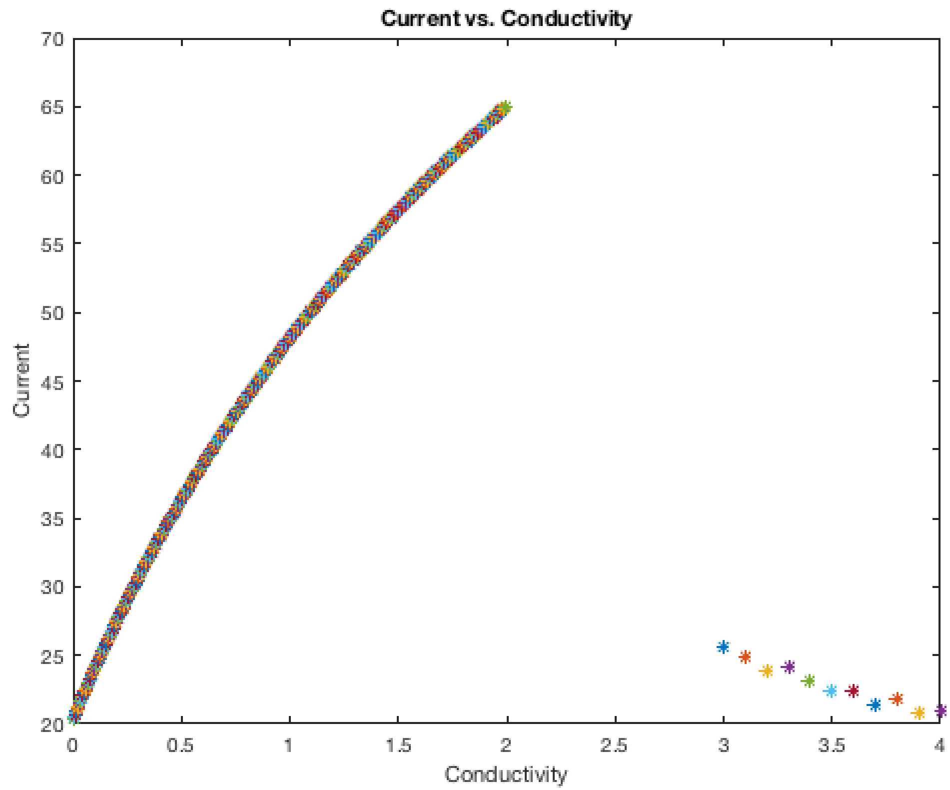

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