Assignment 4 - Circuit modelling

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Question 1

The differential equations for the ciruit in the time-domain are as follows:

1) $V1 = Vin$

2) $V2 - L(d/dt) * I3 - V3 = 0$

3) $V3 - I3 * R3 = 0$

4) $I3 - V1/R1 - V2 * (R2 + R1)/(R1 * R2) - C(V1 - V2) = 0$

5) $V4 - a * I3 = 0$

6) $V5 * (R0 + R4) - R0 * V4 = 0$

The same equations in the frequency domain are:

1) $V1 = Vin$

2) $V2 - j\omega L * I3 - V3 = 0$

3) $V3 - I3 * R3 = 0$

4) $I3 - V1/R1 - V2 * (R2 + R1)/(R1 * R2) - j\omega C * (V1 - V2) = 0$

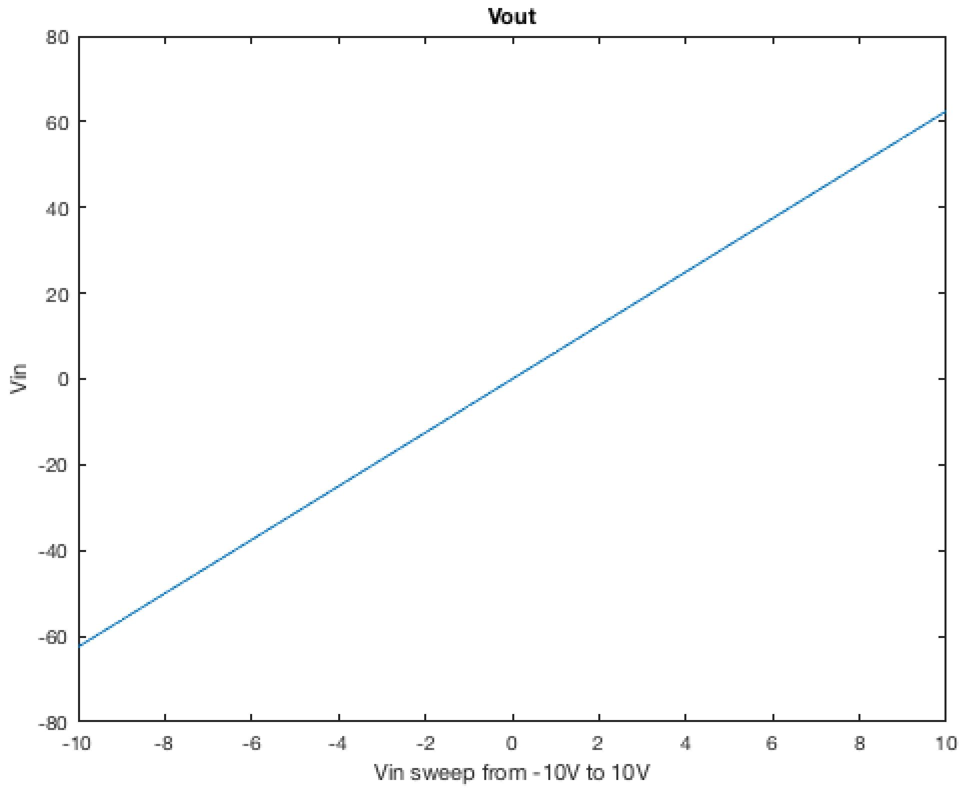
5) $V4 - a * I3 = 0$

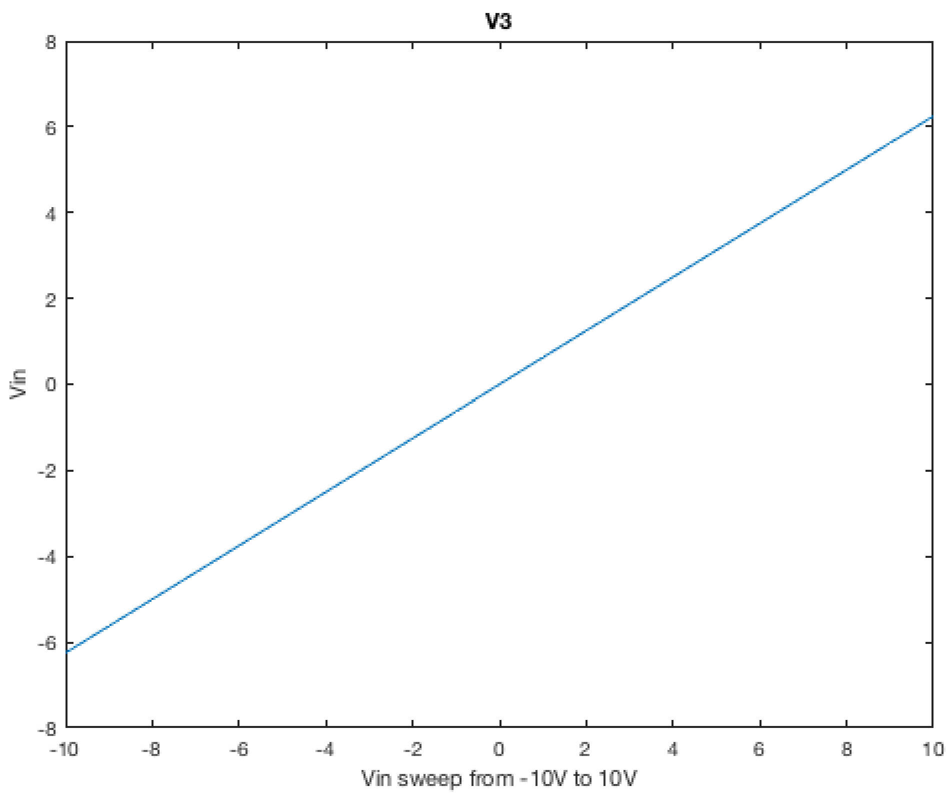
6) $V5 * (R0 + R4) - R0 * V4 = 0$

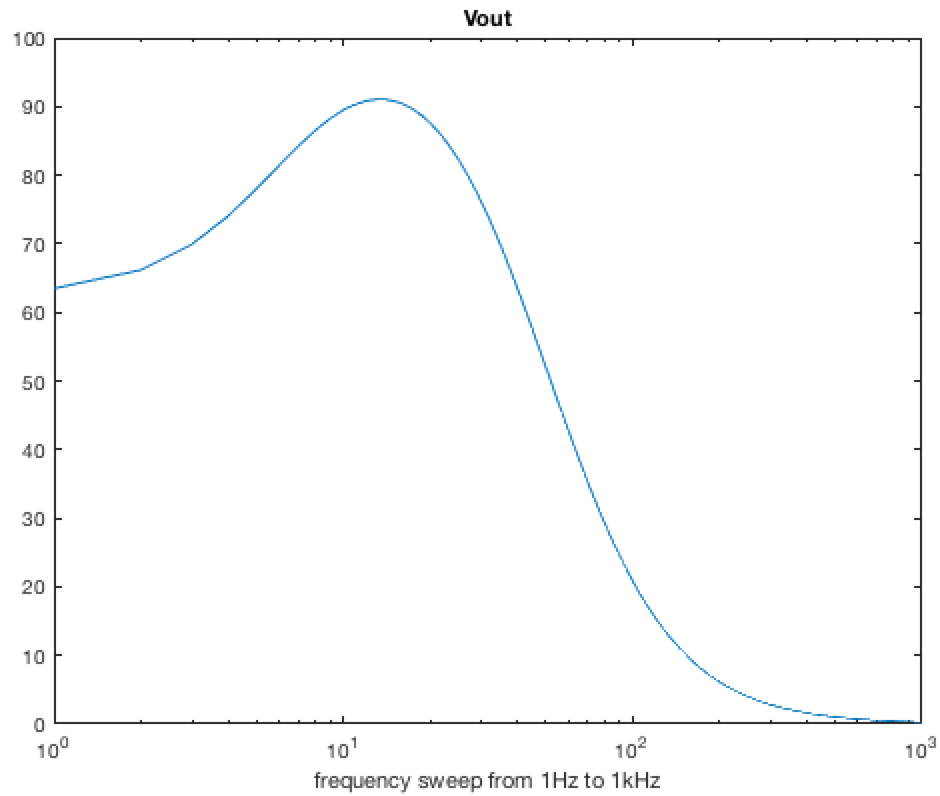
```
close all
clear all

R1=1;
c=0.25;
R2=2;
L=0.2;
R3=10;
a=100;
R4=0.1;
R0=1000;
```

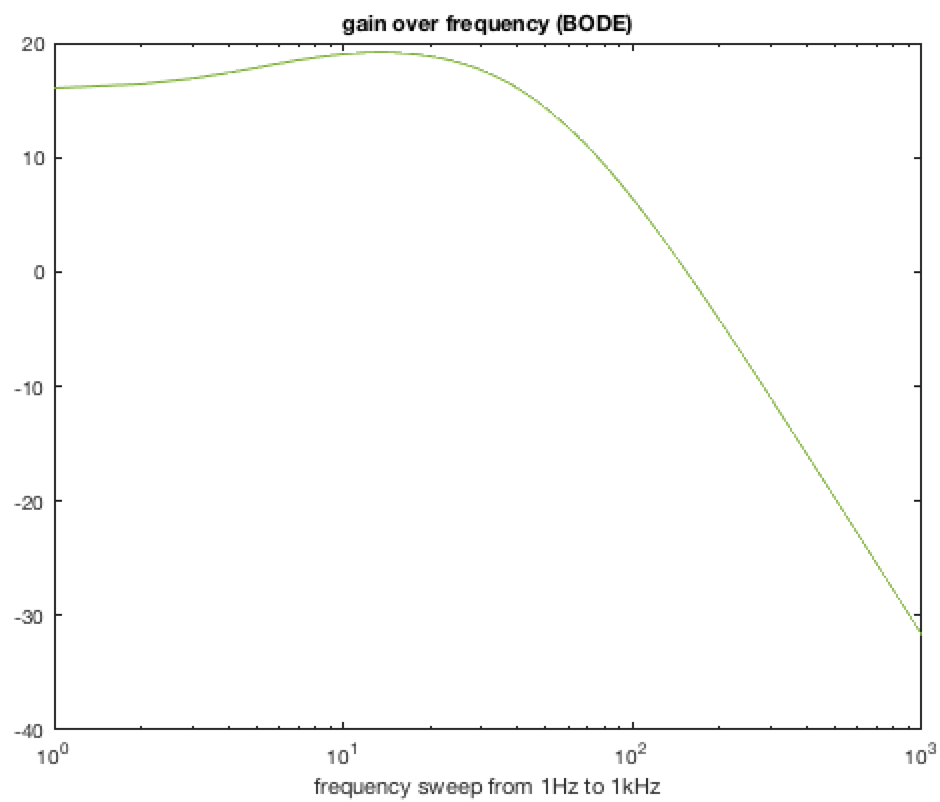
```
Vin=1;

%V=[V1 V2 V3 I3 V4 V5]

F = [Vin 0 0 0 0 0];
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 0 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];
G=[1 0 0 0 0 0;
    0 1 -1 0 0 0;
    0 0 1 -R3 0 0;
    -1/R1 ((R1+R2)/(R1*R2)) 0 1 0 0;
    0 0 0 -a 1 0;
    0 0 0 0 -R0 (R0+R4)];

% DC case
out= zeros(21,6);
in= zeros(1,21);
b=0;
for vin= -10:1:10
    b=b+1;
    F(1)=vin;
    V=G\F';

    out(b,:)=V;
    in(b) = vin;
end

figure(1)
plot(in,out(:,6))%plot vin vs. V5 (Vout)
title('Vout')
xlabel('Vin sweep from -10V to 10V')
ylabel('Vin')

figure(2)
plot(in,out(:,3))
title('V3')
xlabel('Vin sweep from -10V to 10V')
ylabel('Vin')

%AC case - frequency sweep
out= zeros(1001,6);
in= zeros(1,1001);
b=0;
for w= 1:1:1000
    b=b+1;

    V=(G+1j*w*C)\F';

    out(b,:)=V;
    in(b) = w;
```

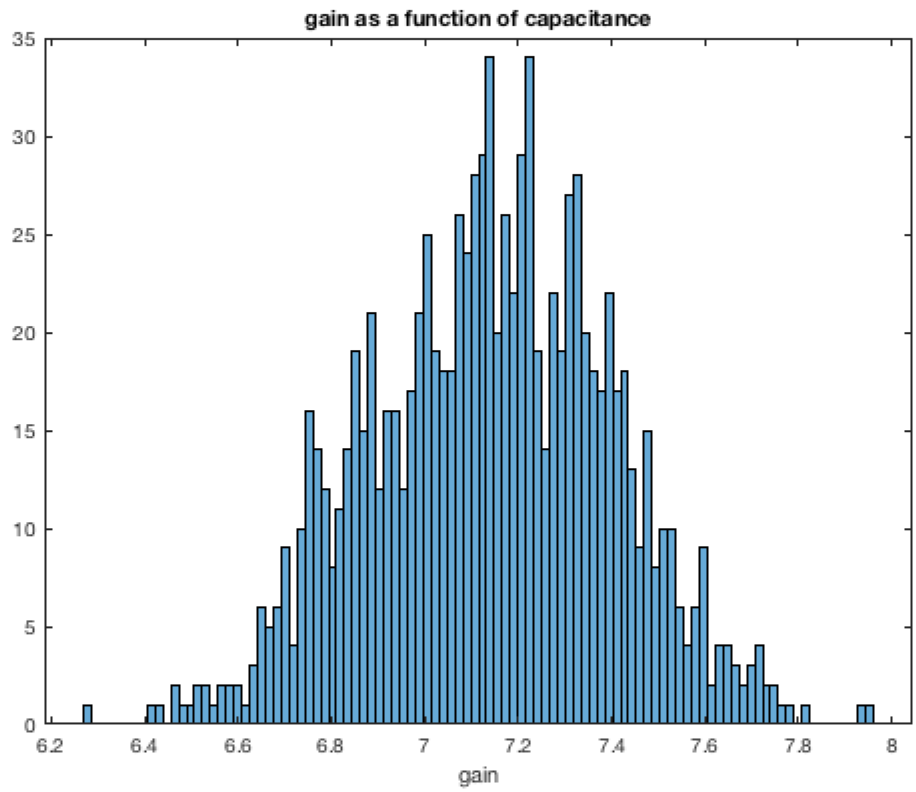
```
end

figure(3)
semilogx(in,real(out(:,6)))
title('Vout')
xlabel('frequency sweep from 1Hz to 1kHz')


figure(4)
semilogx(in,real(20*log10(real(out(:,6)/out(:,1)))))
title('gain over frequency (BODE)')
xlabel('frequency sweep from 1Hz to 1kHz')

%AC case - capacitance sweep (cs)
n=1000;
out= zeros(n,6);
gain= zeros(1,n);
b=0;
w=pi;
 cs=c+0.05*randn(1,n); %std=0.05; c=0.25;
for a=1:n
   c=cs(a);
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 0 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];

    V=(G+1j*w*C)\F';
     out(a,:)=V;
    gain(a) = (abs(V(6))/abs(V(1)));

end


figure(5)
histogram(gain,100)
title('gain as a function of capacitance')
xlabel('gain')
```


Vout
Vin
80
60
40
20
0
-20
-40
-60
-80
-10
-8
-6
-4
-2
0
2
4
6
8
10
Vin sweep from -10V to 10V


V3
Vin
8
6
4
2
0
-2
-4
-6
-8
-10
-8
-6
-4
-2
0
2
4
6
8
10
Vin sweep from -10V to 10V


Vout
100
90
80
70
60
50
40
30
20
10
0
10^0
10^1
10^2
10^3
frequency sweep from 1Hz to 1kHz


gain over frequency (BODE)
20
10
0
-10
-20
-30
-40
10^0
10^1
10^2
10^3
frequency sweep from 1Hz to 1kHz



Question 2- Transient Circuit Simulation

Using the above formulation, we can do a transient analysis of the circuit. a) This circuit functions as a low pass filter.

b) For a low pass filter, signals at frequencies below some cut-off frequency are sent to the output whereas signals at frequencies above the cut-off frequency are attenuated.

The finite difference equation used for the numerical solution of transient analysis is $V_i = (F + C * V_{(i-1)}/dt)(C/dt + G)^{(-1)}$

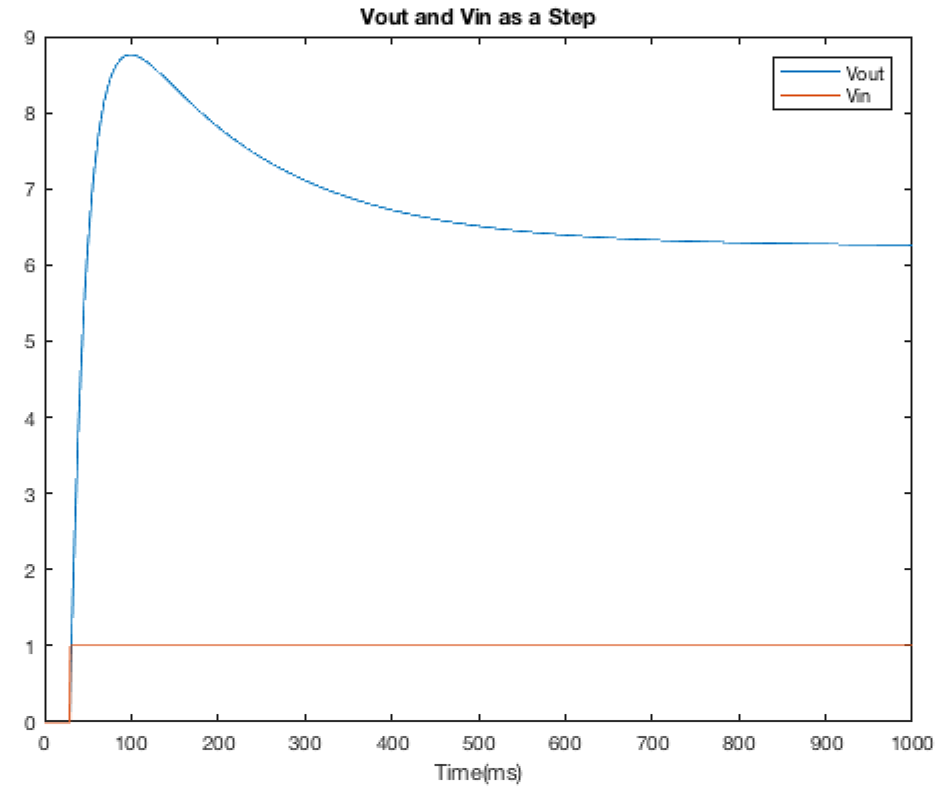
```
% AC case - transient sweep
% Step
n=1000;
out= zeros(n,6);
in= zeros(1,n);
Vin=0;

F = [Vin 0 0 0 0 0];
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 0 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];
```

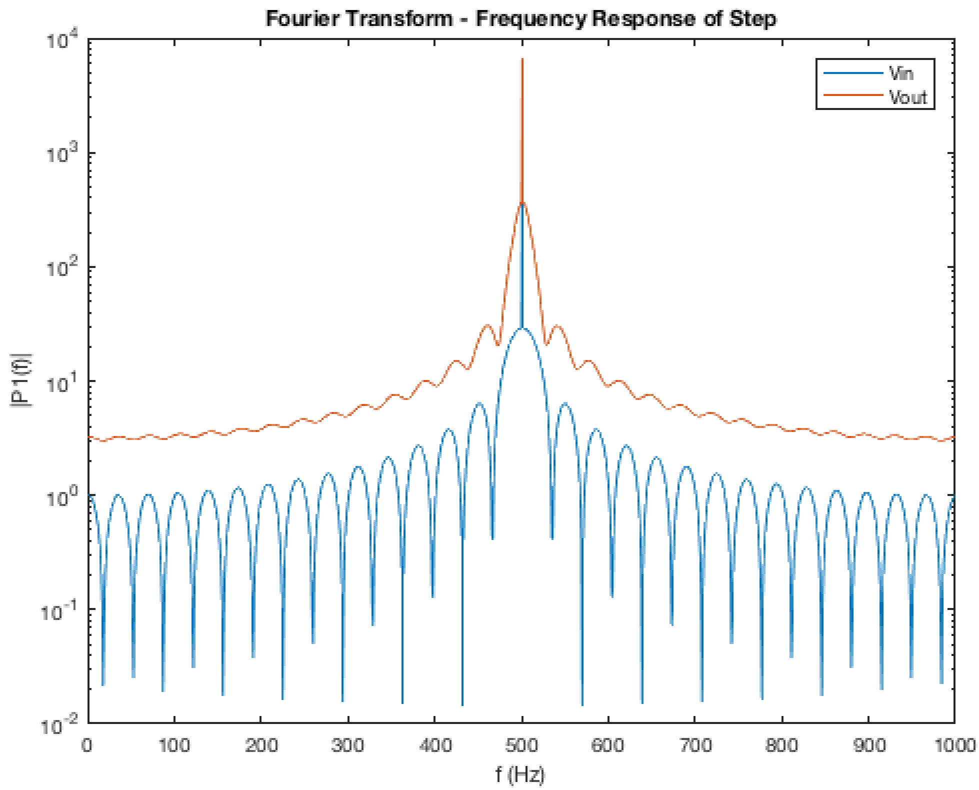
```
dt = 1e-3;

Vt=zeros(n,6); %present voltage
Vt(1,:) = G\F' ; %V(t=0)

% C = C*0;
H = (C/dt + G);
Hi = inv(H);

t(1) = 0;

for k= 2:1:n %ms
    t(k) = (k-1)*1e-3;

    if k<30
        Vin=0;
        F = [Vin 0 0 0 0 0];
        Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    elseif k>=30
        Vin=1;
        F = [Vin 0 0 0 0 0];
        Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    end
    out(k,:)=Vt(k,:);
    %plot(k,out(k,1),'+');hold on; plot(k,out(k,6),'*');
    in(k) = k; %ms
end

Fs=1./t;

figure(6)
% Vin and Vout for step
plot(in,out(:,6))
hold on;
plot(in,out(:,1))
title('Vout and Vin as a Step')
xlabel('Time(ms)')
hold off
legend('Vout','Vin')

Fin = fft(out(:,1));
Fout = fft(out(:,6));


figure(7)
semilogy(abs(fftshift(Fin)))
hold on
 semilogy(abs(fftshift(Fout)))
title('Fourier Transform - Frequency Response of Step')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')
```

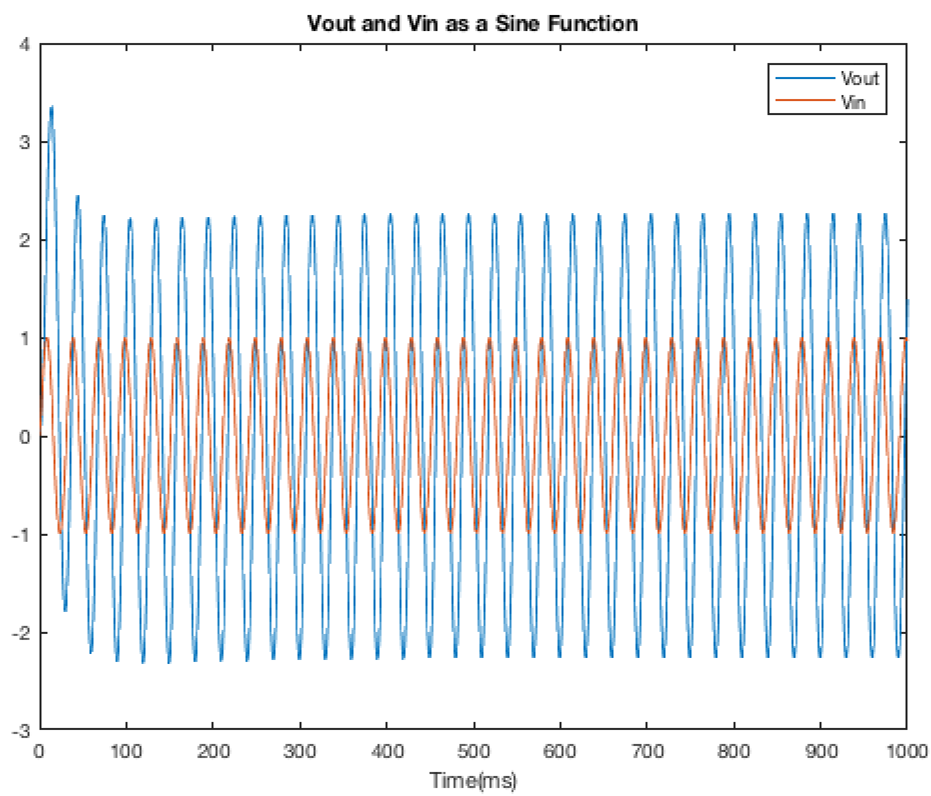
```
%-----------------
%Sine Function
n=1000;
out= zeros(n,6);
in= zeros(1,n);

dt = 1e-3;

Vt=zeros(n,6); %present voltage
f=1/0.03; %Hz
t(1) = 0;

Vin=sin(2*pi*f*t(1));

F = [Vin 0 0 0 0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*(F' );

t(1) = 0;

for  k= 2:1:n %ms
    t(k) = (k-1)*1e-3;

    Vin=sin(2*pi*f*t(k));
    F = [Vin 0 0 0 0 0];

        Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');

    out(k,:)=Vt(k,:);
    in(k) = k; %ms

end
Fs=1./t;


figure(8)
plot(in,out(:,6))
hold on;
plot(in,out(:,1))
title('Vout and Vin as a Sine Function')
xlabel('Time(ms)')
hold off
legend('Vout','Vin')

Fin = fft(out(:,1));
Fout = fft(out(:,6));


figure(9)
semilogy(abs(fftshift(Fin)))
hold on
 semilogy(abs(fftshift(Fout)))
```

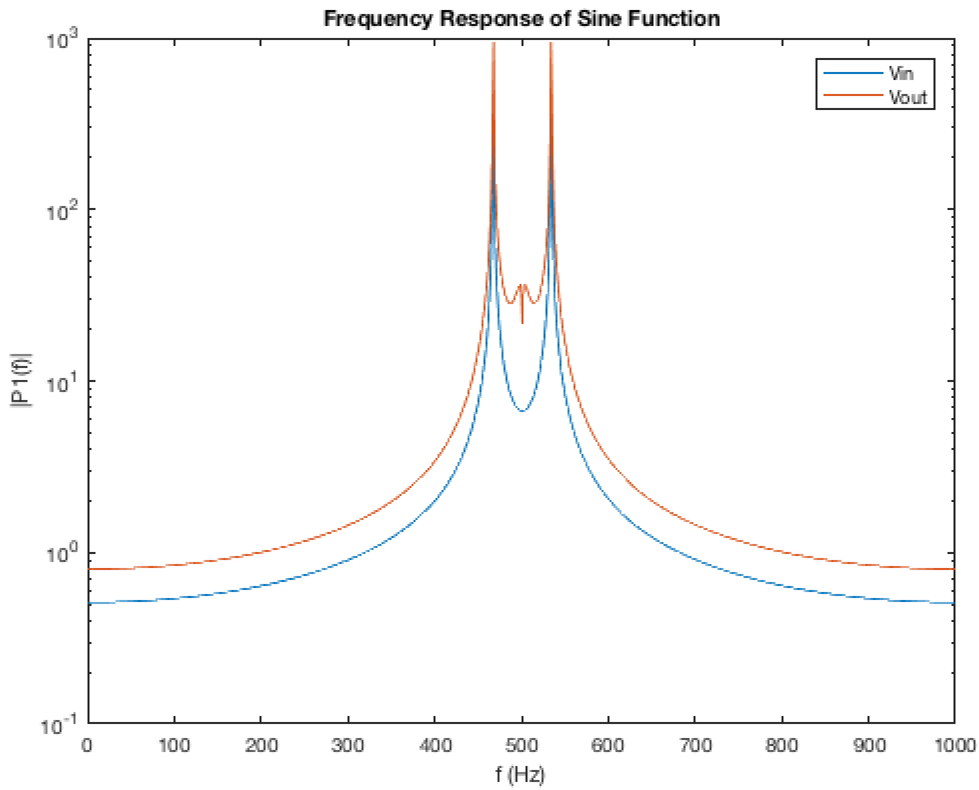
```
title('Frequency Response of Sine Function')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')


%-----------------
% Gaussian Pulse
n=1000;
out= zeros(n,6);
in= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;

Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 0 0 0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*(F' );

for  k= 1:1:n %ms
    t(k) = (k-1)*1e-3;

    Vin=exp(-((t(k)-delay)/sd) *((t(k)-delay)/sd)/2);
    F = [Vin 0 0 0 0 0];
    if k>1
       Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    end
    out(k,:)=Vt(k,:);
    in(k) = k; %ms

end

Fs=1./t;

figure(10)
plot(t,out(:,6))
hold on;
plot(t,out(:,1))
title('Vout and Vin as a Gaussian Pulse')
xlabel('Time(ms)')
hold off
legend('Vout','Vin')

Fin = fft(out(:,1));
Fout = fft(out(:,6));
```

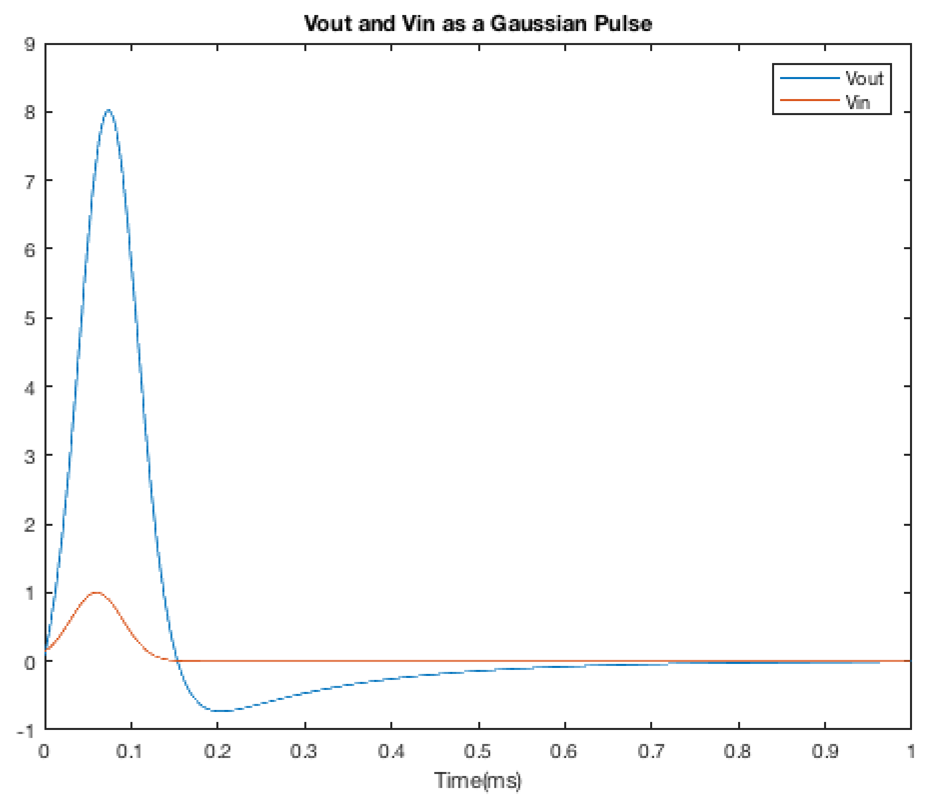
```
figure(11)
semilogy(abs(fftshift(Fin)))
hold on
semilogy(abs(fftshift(Fout)))
title('Frequency Response of Gaussian Pulse')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')
```

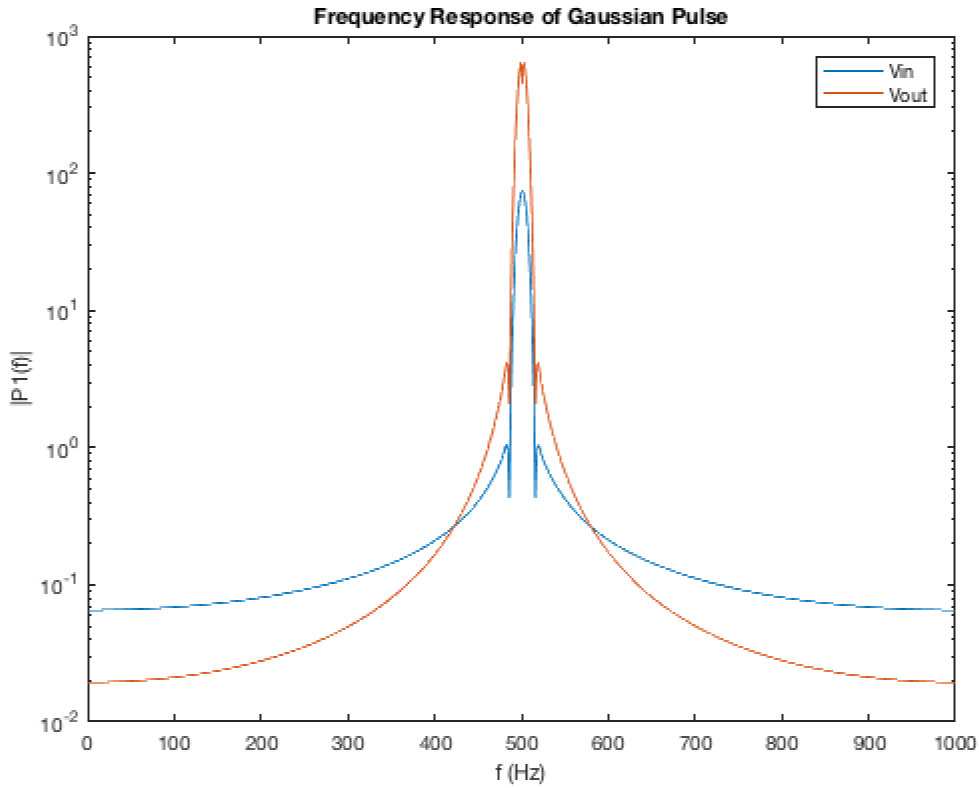



Fourier Transform - Frequency Response of Step
Vin
Vout
|P1(f)|
f (Hz)
10^4
10^3
10^2
10^1
10^0
10^-1
10^-2
0
100
200
300
400
500
600
700
800
900
1000


Vout and Vin as a Sine Function
Vout
Vin
Time(ms)
4
3
2
1
0
-1
-2
-3
0
100
200
300
400
500
600
700
800
900
1000


Frequency Response of Sine Function
Vin
Vout
|P1(f)|
f (Hz)


Vout and Vin as a Gaussian Pulse
Vout
Vin
Time(ms)

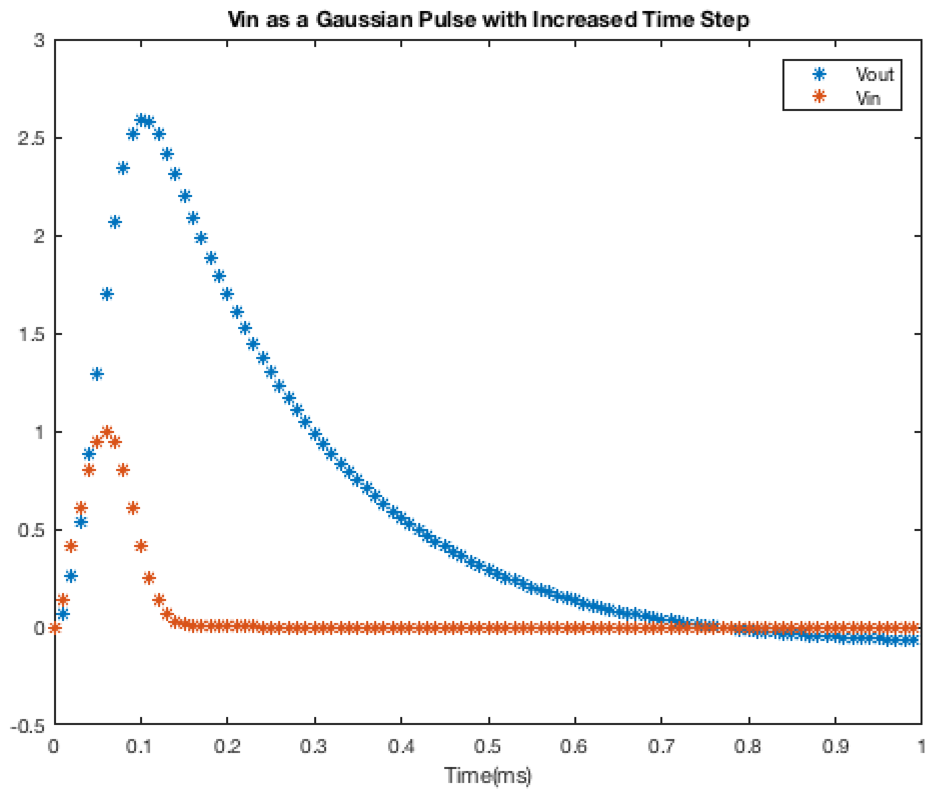


Increased Time Step

With the increased time step, we can see that the circuit changes from being an underdamped system to an overdamped system. The output is more sluggish and not as responsive to changes in the input.

```
n=100;
out= zeros(n,6);
in= zeros(1,n);
t= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;

Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 0 0 0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*(F' );
b=0;
for  k= 11:10:(n*10-1) %ms
```

```
    b=b+1;
    t(b) = (k-1)*1e-3;

    Vin=exp(-((t(b)-delay)/sd) *((t(b)-delay)/sd)/2);
    F = [Vin 0 0 0 0 0];
    if k>11
       Vt(b,:) = Hi*(F' + C/dt*Vt(b-1,:)');
    end
    out(b,:)=Vt(b,:);
    in(b) = k; %ms

end

Fs=1./t;

figure(12)
plot(t,out(:,6),'*')
hold on;
plot(t,out(:,1),'*')
title('Vin as a Gaussian Pulse with Increased Time Step')
xlabel('Time(ms)')
hold off
legend('Vout','Vin')
```



Question 3

Circuit with Noise The differential equations for the ciruit in the time-domain are as follows: 1) $V1 = Vin$

2) $V2 - L(d/dt) * I3 - V3 = 0$

3) $I3 - V3/R3 - Cn * (d/dt) * V3 = In$

4) $I3 - V1/R1 + V2 * (R2 + R1)/(R1 * R2) - C * (d/dt) * (V1 - V2) = 0$

5) $V4 - a * I3 = 0$

6) $V5 * (R0 + R4) - R0 * V4 = 0$

The same equations in the frequency domain are:

1) $V1 = Vin$

2) $V2 - j\omega L * I3 - V3 = 0$

3) $I3 - V3/R3 - j\omega Cn * (d/dt) * V3 = In$

4) $I3 - V1/R1 + V2 * (R2 + R1)/(R1 * R2) - j\omega C * (d/dt) * (V1 - V2) = 0$

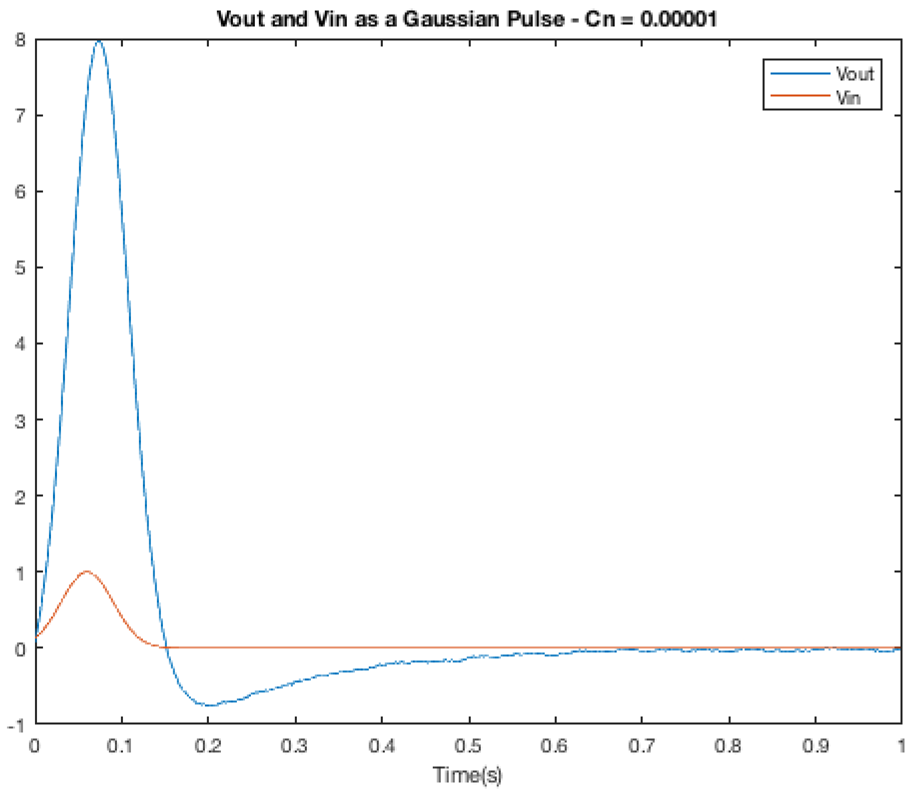
5) $V4 - a * I3 = 0$

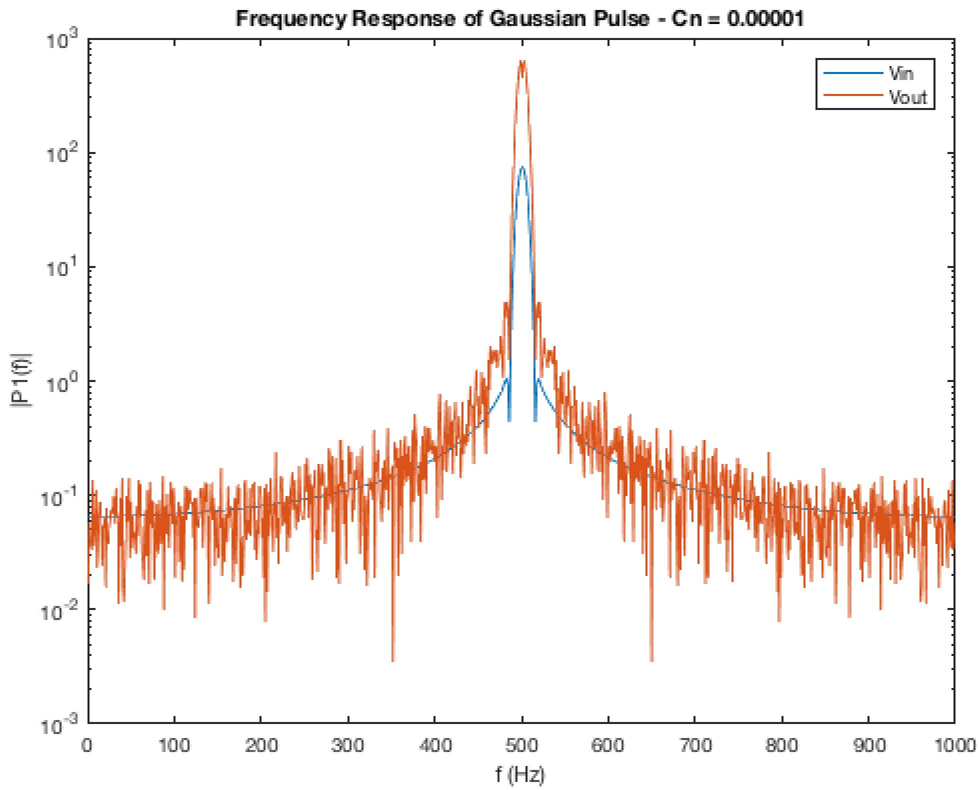
6) $V5 * (R0 + R4) - R0 * V4 = 0$

```
clear all

R1=1;
c=0.25;
R2=2;
L=0.2;
R3=10;
a=100;
R4=0.1;
R0=1000;

Cn=0.00001;
In=0.001*randn();

%V=[V1 V2 V3 I3 V4 V5]
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 -Cn 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];
G=[1 0 0 0 0 0;
```

```
    0 1 -1 0 0 0;
    0 0 -1/R3 1 0 0;
    -1/R1 ((R1+R2)/(R1*R2)) 0 1 0 0;
    0 0 0 -a 1 0;
    0 0 0 0 -R0 (R0+R4)];


% Gaussian Pulse
n=1000;
out= zeros(n,6);
in= zeros(1,n);
noise= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;

Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 In 0  0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*F';

for  k= 1:1:n %ms
    t(k) = (k-1)*1e-3;

    In=0.001*randn();
    Vin=exp(-((t(k)-delay)/sd) *((t(k)-delay)/sd)/2);
    F = [Vin 0 In 0  0 0];
    if k>1
       Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    end
    out(k,:)=Vt(k,:);
    in(k) = k; %ms
    noise(k) = In;

end

Fs=1./t;

figure(13)
plot(t,out(:,6))
hold on;
plot(t,out(:,1))
title('Vout and Vin as a Gaussian Pulse - Cn = 0.00001')
xlabel('Time(s)')
hold off
legend('Vout','Vin')
```

```
Fin = fft(out(:,1));
Fout = fft(out(:,6));


figure(14)
semilogy(abs(fftshift(Fin)))
hold on
 semilogy(abs(fftshift(Fout)))
title('Frequency Response of Gaussian Pulse - Cn = 0.00001')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')
hold off

% figure (15)
% hist(noise,100)
% title('Distribution of Noise')
```





Different Capacitance Values

By increasing the value of Cn, the noise is proportionally overwhelmed and the signal is smoothed out. The bankdwidth of the signal does not appear to be affected.

```
Cn=0.0001;
In=0.001*randn();

%V=[V1 V2 V3 I3 V4 V5]
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 -Cn 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];


n=1000;
out= zeros(n,6);
in= zeros(1,n);
noise= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;
```

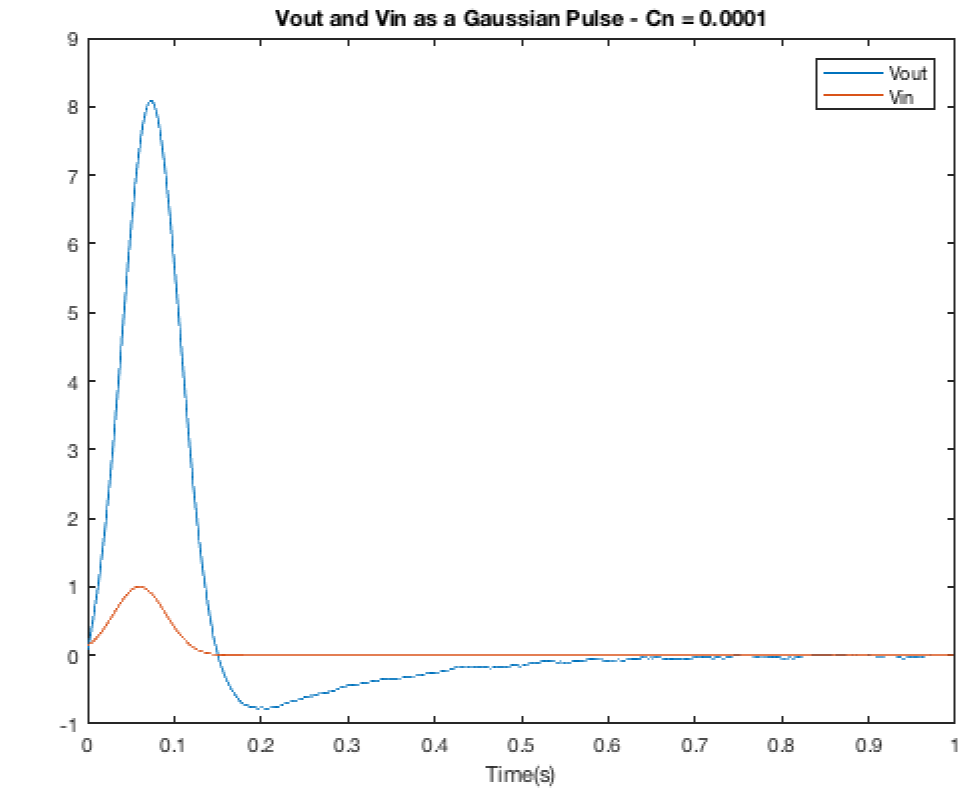
```
Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 In 0  0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*F';

for  k= 1:1:n %ms
    t(k) = (k-1)*1e-3;

    In=0.001*randn();
    Vin=exp(-((t(k)-delay)/sd) *((t(k)-delay)/sd)/2);
    F = [Vin 0 In 0  0 0];
    if k>1
       Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    end
    out(k,:)=Vt(k,:);
    in(k) = k; %ms
    noise(k) = In;

end

Fs=1./t;

figure(16)
plot(t,out(:,6))
hold on;
plot(t,out(:,1))
title('Vout and Vin as a Gaussian Pulse - Cn = 0.0001')
xlabel('Time(s)')
hold off
legend('Vout','Vin')

Fin = fft(out(:,1));
Fout = fft(out(:,6));


figure(17)
semilogy(abs(fftshift(Fin)))
hold on
 semilogy(abs(fftshift(Fout)))
title('Frequency Response of Gaussian Pulse - Cn = 0.0001')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')
hold off
%-------

Cn=0.001;
In=0.001*randn();
```

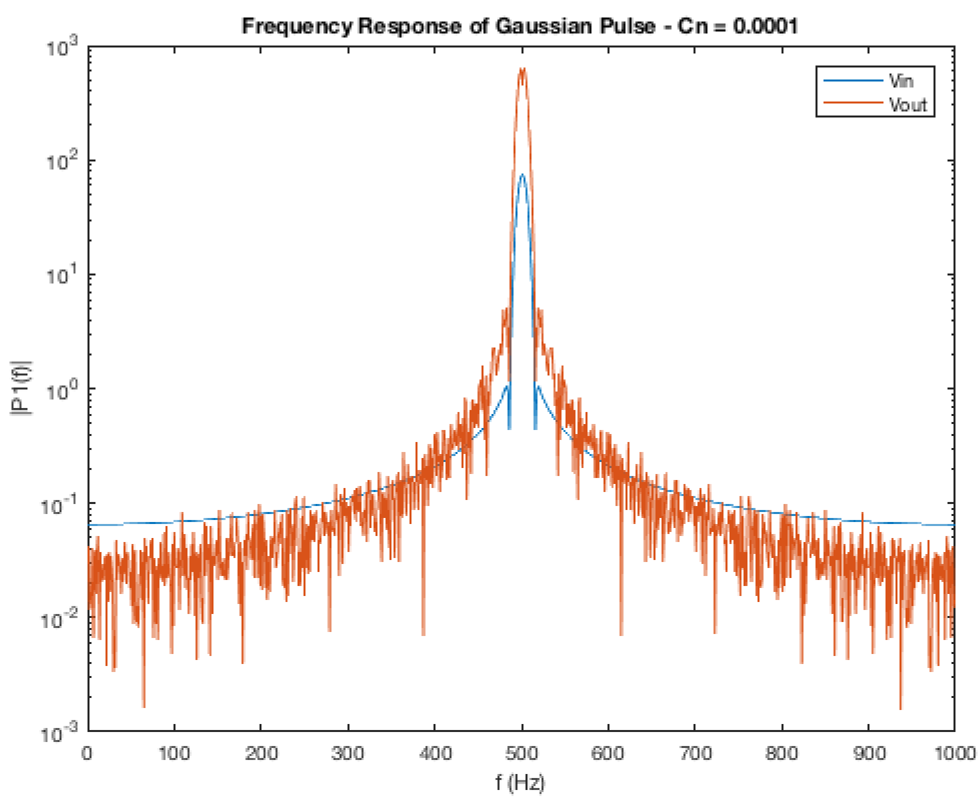
```
%V=[V1 V2 V3 I3 V4 V5]
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 -Cn 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];

n=1000;
out= zeros(n,6);
in= zeros(1,n);
noise= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;

Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 In 0  0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*F';

for  k= 1:1:n %ms
    t(k) = (k-1)*1e-3;

    In=0.001*randn();
    Vin=exp(-((t(k)-delay)/sd) *((t(k)-delay)/sd)/2);
    F = [Vin 0 In 0  0 0];
    if k>1
       Vt(k,:) = Hi*(F' + C/dt*Vt(k-1,:)');
    end
    out(k,:)=Vt(k,:);
    in(k) = k; %ms
    noise(k) = In;

end

Fs=1./t;

figure(18)
% Vin and Vout for gaussian pulse
plot(t,out(:,6))
hold on;
plot(t,out(:,1))
title('Vout and Vin as a Gaussian Pulse - Cn = 0.001')
xlabel('Time(s)')
hold off
```

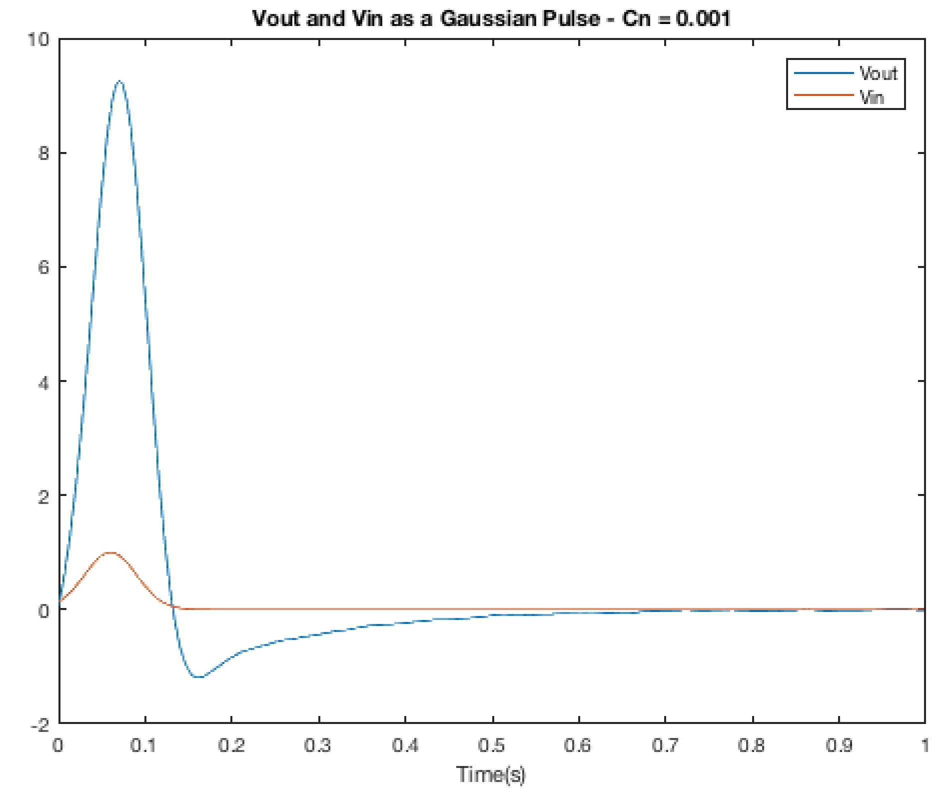
```
legend('Vout','Vin')

Fin = fft(out(:,1));
Fout = fft(out(:,6));


figure(19)
semilogy(abs(fftshift(Fin)))
hold on
 semilogy(abs(fftshift(Fout)))
title('Frequency Response of Gaussian Pulse - Cn =0.001')
 xlabel('f (Hz)')
 ylabel('|P1(f)|')
legend('Vin','Vout')
hold off

% Increased Time Step
%
% Using the original capacitance with an increased time step, the same
% effect occurs where the output becomes overdamped, but in this case,
 it
% is affected by some noise. The effect of the noise is diminished
 with the
% increased time step, but the relationship between Vin and Vout is
 also
% significantly altered.

Cn=0.00001;
In=0.001*randn();

%V=[V1 V2 V3 I3 V4 V5]
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 -Cn 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];


n=100;
out= zeros(n,6);
in= zeros(1,n);
t= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;

Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 0 0 0 0];
```

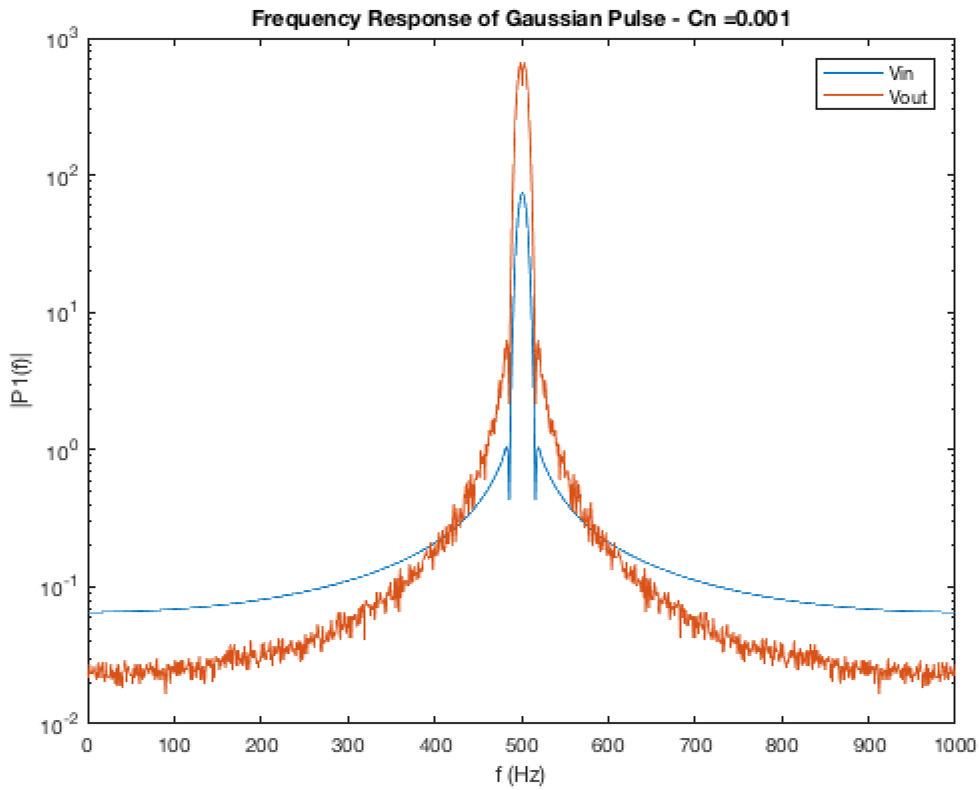
```
H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*(F' );
b=0;
for  k= 11:10:(n*10-1) %ms
    b=b+1;
    t(b) = (k-1)*1e-3;

    In=0.001*randn();
    Vin=exp(-((t(b)-delay)/sd) *((t(b)-delay)/sd)/2);
    F = [Vin 0 In 0  0 0];
    if b>1
       Vt(b,:) = Hi*(F' + C/dt*Vt(b-1,:)');
    end
    out(b,:)=Vt(b,:);
    in(b) = k; %ms
    noise(b) = In;

end


Fs=1./t;

figure(20)
plot(t,out(:,6),'*')
hold on;
plot(t,out(:,1),'*')
title('Vin as a Gaussian Pulse with Increased Time Step (x10)')
xlabel('Time(s)')
hold off
legend('Vout','Vin')

Cn=0.00001;
In=0.001*randn();

%V=[V1 V2 V3 I3 V4 V5]
C=[0 0 0 0 0 0;
    0 0 0 -L 0 0;
    0 0 -Cn 0 0 0;
    -c c 0 0 0 0;
    0 0 0 0 0 0;
    0 0 0 0 0 0];


n=100;
out= zeros(n,6);
in= zeros(1,n);
t= zeros(1,n);

delay = 0.06;%s
sd = 0.03; %s
dt = 1e-3;
```

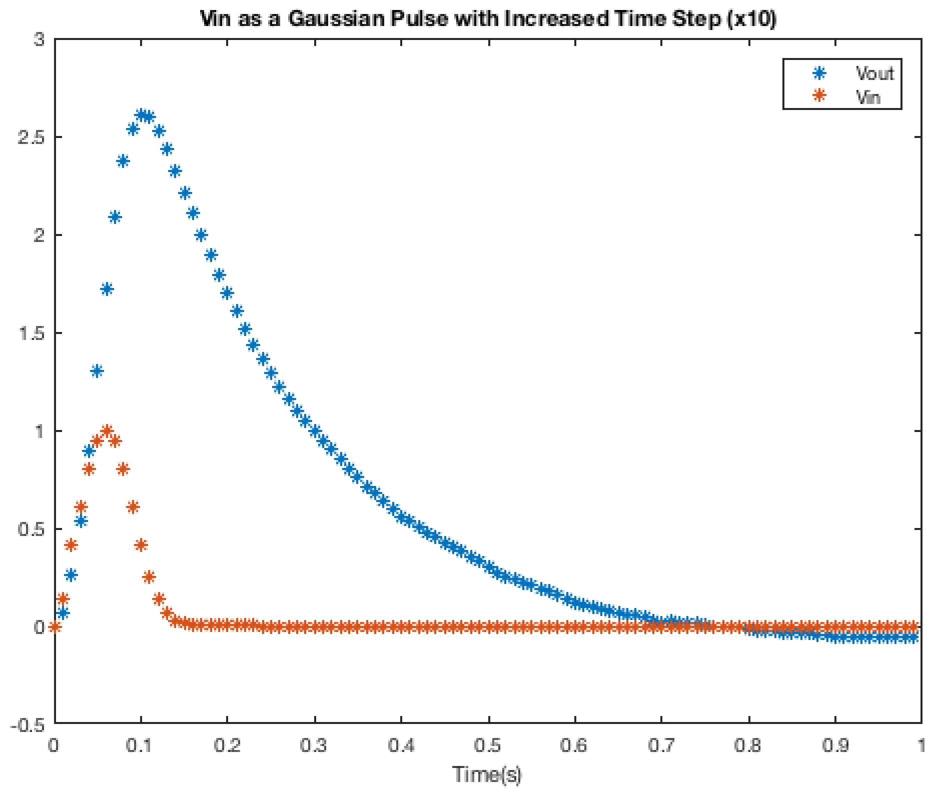
```
Vt=zeros(n,6); %present voltage
t(1) = 0;

Vin=exp(-((t(1)-delay)/sd) *((t(1)-delay)/sd)/2);

F = [Vin 0 0 0 0 0];

H = (C/dt + G);
Hi = inv(H);
Vt(1,:) = Hi*(F' );
b=0;
for  k= 11:20:(n*10-1) %ms
    b=b+1;
    t(b) = (k-1)*1e-3;

    In=0.001*randn();
    Vin=exp(-((t(b)-delay)/sd) *((t(b)-delay)/sd)/2);
    F = [Vin 0 In 0  0 0];
    if b>1
       Vt(b,:) = Hi*(F' + C/dt*Vt(b-1,:)');
    end
    out(b,:)=Vt(b,:);
    in(b) = k; %ms
    noise(b) = In;

end


Fs=1./t;

figure(21)
plot(t,out(:,6),'*')
hold on;
plot(t,out(:,1),'*')
title('Vin as a Gaussian Pulse with Increased Time Step (x20)')
xlabel('Time(s)')
hold off
legend('Vout','Vin')
```

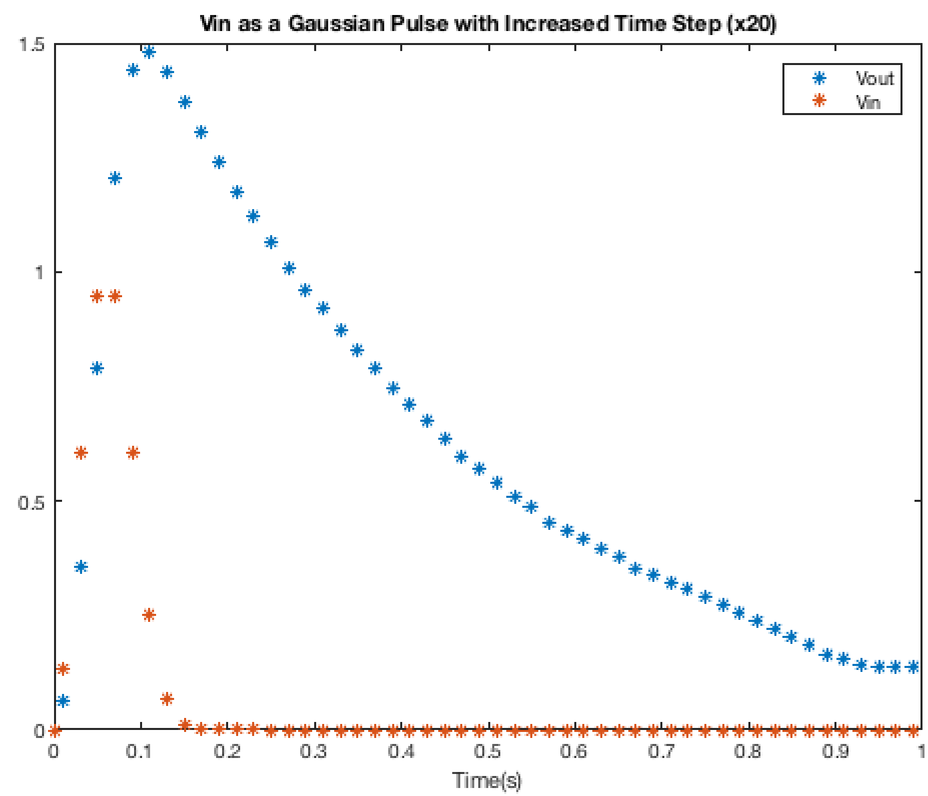

Vout and Vin as a Gaussian Pulse - Cn = 0.0001
Vout
Vin
Time(s)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-1
0
1
2
3
4
5
6
7
8
9


Frequency Response of Gaussian Pulse - Cn = 0.0001
Vin
Vout
|P1(f)|
f (Hz)
0
100
200
300
400
500
600
700
800
900
1000
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


Vout and Vin as a Gaussian Pulse - Cn = 0.001
Vout
Vin
Time(s)


Frequency Response of Gaussian Pulse - Cn =0.001
Vin
Vout
|P1(f)|
f (Hz)


Vin as a Gaussian Pulse with Increased Time Step (x10)
Vout
Vin
Time(s)


Vin as a Gaussian Pulse with Increased Time Step (x20)
Vout
Vin
Time(s)

Question 4 - Non-Linearity

There are likely several ways to achieve modeling a parameter with polynomial-form dependence. One method would require the use of an additional, or several additional matrices with another source value. A matrix of this type would have many zeros in it, and strategically placed coefficients to create the I^2 and I^3 factors. These would be multiplied by the G matrix to create a new composite matrix that would fit in the equation $C * dV/dt + GV = F$. Another method would be the implementation of a B matrix $C * dV/dt + GV + B(V) = F$ which is typically used for non-linear situations. Lastly, it may be possible to do something with the wonderful polynomial functions that Matlab has at its disposal, along the lines of polyfit, but any solution of this type would almost certainly also require the use of a matrix to be applicable to the type of analysis in the rest of this simulation.

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