Proposals for Parameter Optimization in dAMMs (WIP)

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Abstract

We provide some approaches for optimally parameterizing a vAMM (or an entirely configurable vAMM, also known as a dAMM). Our first approach treats the dAMM as a reinforcement learning problem, from which we can determine a temporal-based optimal policy for parameterizing the dAMM based on trading activity. Our second approach uses machine and deep learning techniques on historical data collected from the dAMM to obtain an alternative policy for setting the peg multiplier/k.

1 dAMM Background

The aim of the dAMM is to automate the virtual market-making process through the constant product formula $x \cdot y = k$, where x can be considered as the "base asset amount", y can be considered as the "quote asset amount", and k is an arbitrary constant considered to be the "liquidity depth" that we wish to maintain.

Assets in a base-quote pair are priced as follows:

$$\text{price}_x = \frac{y}{x} \cdot \text{peg multiplier}$$

where the "peg multiplier" can be considered as an arbitrary constant designed to bring market prices closer to the oracle price, thus creating a healthier perpetuals market.

For simplicity, we will first consider a cross margin scenario (i.e. a protocol with only one quote asset, USDC). An example of a cross margin protocol currently implementing the dAMM is Drift Protocol.

We will also primarily consider the problem of finding an optimal "peg multiplier" and "k" for the dAMM; other parameters, such as traunch limits for the fee pool, may exist within the dAMM, however these seem to be more arbitrary in nature and thus have less overall impact on trading activity.

2 Literature Review

The idea of using deep/reinforcement learning techniques in traditional finance isn't necessarily a new one; for instance, Mani et al. [2019] introduced a framework for automated market-making using risk-sensitive reinforcement learning. In particular, they established an agent (market maker)-environment (trading activity) model by defining it as a finite Markov decision process in discrete time with a linear reward function, and solving it using temporal difference learning algorithms.

Moreover, Zhang et al. [2019] have proposed using deep reinforcement learning to design trading strategies for continuous futures contracts, considering both discrete and continuous action spaces, and found that their algorithms typically outperformed baseline models, such as classical time series momentum strategies, despite large transaction costs.

Silva et al. [2014] have also proposed using artificial neural networks to support the market making process in high frequency trading, using multilayer perceptrons to effectively predict short-term oscillations in particular.

3 Q-Learning/Reinforcement Learning Approach

3.1 Proposed Model

3.1.1 Typical Setup

In a typical portfolio-maximization problem, we might expect an agent to choose the sequence of actions that will maximize the expected utility of wealth at the end of finite time horizon T:

$$E[U(W_T)] = E\Big[U\Big(W_0 + \sum_{t=1}^{T} \Delta W_t\Big)\Big]$$

where W_0 is the initial wealth and ΔW_t represents the change in wealth over the t-th time interval. The performance of the policy essentially depends on sequences of interdependent actions where optimal trading decisions affect both immediate returns and subsequent future returns.

We aim to take a similar approach in our model by defining the concept of expected "wealth" (i.e. immediate reward) in the dAMM setting.

3.1.2 Notation

We aim to define the dAMM as a finite stationary Markov Decision Process (MDP), described by the tuple $M = \langle S, A, R, P \rangle$, where:

- S is a finite collection of states; in this case, the long-short imbalance at time t.

$$S_t = IMB_t = \text{long open interest}_t - \text{short open interest}_t$$

- A is a finite collection of actions; in this case, the agents' (collective) choice to long or short at time t.

$$A_t = \begin{cases} +1 & \text{for a long position} \\ -1 & \text{for a short position} \end{cases}$$

- R is the set of possible values of immediate rewards; in this case we consider "reward" to mean the fee pool (in Drift Protocol, this is 0.1% of trading volume) capped by the traunch limit (in Drift Protocol, this is 50%) at time t.

$$R_t = \text{trading volume}_t \cdot 0.001 \cdot 0.5$$

  Since Drift's trading volume (measured in USDC) can be treated as a function of both price and liquidity depth, we can initially run a linear regression model to obtain a numerical approximation for trading volume in the form:

$$\text{trading volume}_t = \beta_0 + \beta_1 \cdot \text{price}_t + \beta_2 \cdot k_t$$

  where $\beta \in \mathbb{R}^3$ are the regression coefficients. Depending on the relative magnitudes of typical prices and liquidity depths, it might make sense to scale either variable by some further transformation, such as square-rooting k or the price. Nonetheless, the "reward" can essentially be written in terms of both k and the peg multiplier by:

$$\begin{aligned} R_t &= (\beta_0 + \beta_1 \cdot \text{price}_t + \beta_2 \cdot k_t) \cdot 0.001 \cdot 0.5 \\ &= (\beta_0 + \beta_1 \cdot \frac{y_t}{x_t} \cdot \text{peg multiplier}_t + \beta_2 \cdot k_t) \cdot 0.001 \cdot 0.5 \\ &= (\beta_0 + \beta_1 \cdot \frac{k_t}{x_t^2} \cdot \text{peg multiplier}_t + \beta_2 \cdot k_t) \cdot 0.001 \cdot 0.5 \end{aligned}$$

- P describes the transition probabilities given some previous state and action i.e. $P(s', r \mid s, a) = P(S_{t+1} = s', R_{t+1} = r \mid S_t = s, A_t = a)$ for $s, s' \in S, a \in A, r \in R$. We expect to approximate the transition matrix numerically from Drift Protocol's trading data.

Furthermore, we assume that at each time step:

- Informed traders enter the market with probability λ_i, take a long position if $mark < oracle$ and a short position if $mark > oracle$.
- Uninformed traders enter the market with probability λ_u and have an even chance of taking a long or short position. In other words, the uninformed traders essentially have no strategy and trade as if in a random walk without drift.

We assume that these are mutually exclusive events such that $\lambda_i + \lambda_u = 1$.

The agent-environment dynamics is as follows: at time t, the aggregate market maker observes a state S_t and takes an action A_t according to a Markovian policy, from which a numerical reward R_{t+1} is provided at the next time step, leading to the next state S_{t+1}.

At any time t, the goal is to maximize the expected return, denoted as:

$$G_t = \sum_{k=t+1}^{T} \gamma^{k-t-1}(\lambda_i R_k + \lambda_u R_k)$$

where $\gamma \in (0, 1)$ is a discount factor representing the agent's uncertainty about the time horizon remaining on the contract. Note that the exponent for γ is configured so that we only start discounting from the $(t+2)$-th interval onwards i.e. we start becoming more "uncertain" as the time horizon extends past the first time step after t. We also weight the rewards accordingly by the probability of observing a group of informed or uninformed traders, as we expect the immediate rewards to differ based on strategic vs. naive trading activity.

Assuming that the utility of final wealth is linear over time, we can see that optimizing $E[G]$ is also equivalent to optimizing our expected wealth.

3.1.3 Optimal Policy

Define the state-action value function $Q_\pi : S \times A \to \mathbb{R}$ for some fixed policy π as follows:

$$\begin{aligned} Q_\pi(s,a) &= E_\pi\Big[\sum_{k=t+1}^{T} \gamma^{k-t-1}(\lambda_i R_k + \lambda_u R_k) |\ S_t = s, A_t = a\Big] \\ &= E_\pi\Big[R_{t+1} + \gamma \sum_{a'} Q_\pi(S_{t+1}, a')\pi(a' \mid S_{t+1}) |\ S_t = s, A_t = a\Big] \end{aligned}$$

By using the DP algorithm, we see that the optimal policy is determined by:

$$\begin{aligned} Q_*(s,a) &= \max_\pi Q_\pi(s,a) \\ &= E[R_{t+1} + \gamma \max_{a'} Q_*(S_{t+1}, a') \mid S_t = s, A_t = a] \\ \implies \pi_* &= \text{argmax}_a Q_*(s,a) \end{aligned}$$

Similar to the model that Mani et al. [2019] proposed, we would need to rely on approximation techniques such as temporal difference methods in order solve for the optimal policy, since the transition probabilities aren't known explicitly. One example could include the SARSA algorithm with appropriate learning rate.

4 Supervised Machine/Deep Learning Approach

Another way to approach the optimization problem could be supervised machine learning. In some sense, applying supervised machine learning algorithms could be useful here as it would allow us to establish relationships between data variables, from which we could infer the optimal direction or magnitude for a new peg multiplier or k, given similar market dynamics observed historically.

4.1 Classification of Repeg vs k Adjustment

One could first use Drift's historical data to classify previous instances of "success" or "failure" during previous repegs or k adjustments. Using the Drift Flat Data as an example, we could define two subproblems:

- Find previous repegs by filtering for instances where peg_multiplier_before <> peg_multiplier_after.
- Find previous k adjustments found by filtering for instances where sqrt_k_before <> sqrt_k_after.

Then we could arbitrarily classify instances as "success" or "failure" post parameter changes based on thresholds defined by the following attributes:

- market_index or base_asset_amount; Certain base assets might naturally have more liquidity depth due to popularity or time (e.g. BTC over AVAX).
- open_interest; Does changing the price or liquidity incentivize more trading volume?
- adjustment_cost as a proportion of total_fee_minus_distributions; Is the relative cost of performing the change fairly low in comparison to the fees collected post change?

Examples of appropriate classification techniques could include nearest neighbor, Naive Bayes, decision trees or logistic regression; robustness could be built into the model by taking an ensemble approach.

4.1.1 Game Theoretic Approach

Alternatively, we could define a payoff matrix fixed in time where one player is the repeg and another is the k adjustment. Always apply the dominant strategy or Pareto optimal strategy.

Mixed strategy Nash equilibrium - an equilibrium where players make a choice based on some unpredictability (like an if-else)?

4.2 Magnitude of Repeg/k Adjustment

Conditional on whether we wish to to repeg or adjust k, we could then construct a LSTM recurrent neural network to predict the appropriate change for these values. This would be preferred over an artificial neural network as the recurrent connection on the hidden state would ensure that sequential information can be captured in the input data (i.e. we aim to consider the dAMM as a time series, not a single instance in time). See `https://machinelearningmastery.com/time-series-forecasting-long-short-term-memory-network-python/`

4.3 Potential Caveats

Drift Protocol does not currently have enough historical data to ensure a robust model, especially since the team has not repegged or changed k frequently. However it'd still be interesting to train (and continuously retrain) a machine learning model in conjunction with reinforcement learning in an attempt to not only verify the optimal policy of the reinforcement learning problem, but also potentially take an ensemble approach to determining the dAMM's overall optimal policy.

Deep recurrent neural networks might also be susceptible to the "exploding and vanishing gradient" problem associated with backpropagation, however remedies such as gradient clipping can also be implemented.

5 References

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