Physics Olympiad Notes

1 Motion

1.1 Kinematics

The three main equations for kinematics are as follows.

$$v = u + at$$
$$s = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2as$$

Where v = final velocity, u = initial velocity, a = acceleration, t = time, s = displacement.

Drawing graphs may also be helpful for some questions (area/gradient).

1.2 Newton's Laws

Newton's first law: An object in motion stays in motion, an object at rest stays at rest.

Newton's second law: $F_{\text{net}} = ma$

Newton's third law: Every action has an equal and opposite reaction.

1.3 Friction

There are two types of friction: static friction and kinetic friction. Static friction occurs when the object is static (stationary) and a force is being exerted on it. Kinetic friction occurs when the object is moving.

$$f_{\text{static}} \leq \mu_s N$$
$$f_{\text{kinetic}} = \mu_k N$$

Where μ_s and μ_k is the coefficient of static and kinetic friction respectively, N is the normal force.

1.4 Vectors

1.4.1 Vector addition

Forces can be represented as vectors. To do vector addition, simply join the forces and find the resultant.

1.4.2 Multiplying a vector by a scalar

The magnitude changes, but the direction remains the same

1.4.3 Dot (scalar) product of two vectors

The scalar product of two vectors is called the dot product. The dot product of $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

Where A is the magnitude of $\vec{A}$, B is the magnitude of $\vec{B}$, θ is the angle between the two vectors.

1.4.4 Cross (vector) product of two vectors

The vector product of two vectors is called the cross product. The cross product of $\vec{A}$ and $\vec{B}$ is

$$\vec{A} \times \vec{B} = AB\sin\theta\,\hat{n}$$

Where the magnitude is $AB\sin\theta$ and the direction is the unit vector $\hat{n}$. The direction of the cross product can be determined by the right hand rule.

1.5 Work, energy and power

1.5.1 Work

SI unit: N·m (newton metres)

Work is the effort applied over a distance against a force. It is given by the equation

$$W = \vec{F} \cdot \vec{d}$$

Where $\vec{F}$ is the force and $\vec{d}$ is the displacement. Note that work is a scalar, thus dot product (section 1.4.3) is used. This also means

$$W = Fd\cos\theta.$$

Note that if force and displacement are in the same direction, $\cos\theta = 1$; if force and displacement are perpendicular to each other, $\cos\theta = 0$.

In a graph of force against distance, work is the area under graph.

1.5.2 Energy

SI unit: J (joules)

Energy is a scalar quantity that exists in several forms.

Gravitational Potential Energy

The gravitational potential energy U_g is given by the equation

$$U_g = mgh$$

where m = mass, g = gravity, h = height.

Kinetic Energy

The kinetic energy K of an object is

$$K = \frac{1}{2}mv^2$$

where m = mass, v = velocity.

Conservation of Energy is used to solve a question when there is no external work done.

1.5.3 Power

SI unit: W (watts), scalar.

Power is the rate at which energy is applied or work is done. It is calculated by

$$P = \frac{W}{t} \qquad \text{or} \qquad P = \frac{E}{t}$$

where W = work, E = energy, t = time.

Power can also be calculated as

$$P = F \cdot v$$

where F = force, v = velocity.

1.6 Momentum

SI unit: N·s (newton seconds), vector.

Momentum p is the amount of "oomph" going in a direction. It is given by

$$p = mv$$

where m = mass, v = velocity.

The net force on an object can also be calculated by

$$F_{\text{net}} = \frac{\Delta p}{\Delta t}.$$

Looking at the equations carefully, we may also see that

$$K = \frac{p^2}{2m}$$

where K = kinetic energy, p = momentum, m = mass.

Conservation of momentum is frequently used to solve collision problems (there is no external force on the system).

1.7 Rotational motion

1.7.1 Arc length

The arc length s can be calculated as

$$s = r\theta$$

where r is the radius of the circle, θ is the angle travelled (in radians).

1.7.2 Angular velocity

The angular velocity ω is the rotational velocity of an object as it travels around a circle (i.e. change in angle per unit of time). It is calculated by

$$\omega = \frac{\Delta\theta}{\Delta t}$$

where θ is the angle travelled, t = time.

Also note that

$$\omega = 2\pi f$$

where f is frequency.

1.7.3 Tangential velocity

The tangential velocity v_T is the linear velocity of a point on a rigid rotating body. It is given by

$$v_T = r\omega$$

where r = radius of circle, ω = angular velocity.

1.7.4 Angular acceleration

Angular acceleration α is the change in angular velocity with respect to time.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

where ω = angular velocity, t = time.

1.7.5 Tangential acceleration

The tangential acceleration α_T is the linear acceleration of a point on a rigid, rotating body. It is

$$\alpha_T = r\alpha$$

where r = radius of circle, α = angular acceleration.

1.7.6 Centripetal acceleration

Centripetal acceleration a_c is the constant acceleration of an object towards the center of rotation (there must be a force pulling you to the center, otherwise you will just go in a straight line).

$$a_c = \frac{v^2}{r} = r\omega^2$$

where v = velocity, r = radius, ω = angular velocity.

1.7.7 Centripetal force

Centripetal force F_c is the inward force that keeps an object moving in a circle.

$$F_c = \frac{mv^2}{r} = mr\omega^2$$

where m = mass, v = velocity, r = radius, ω = angular velocity.

1.7.8 Center of mass

The center of mass can be calculated as follows.

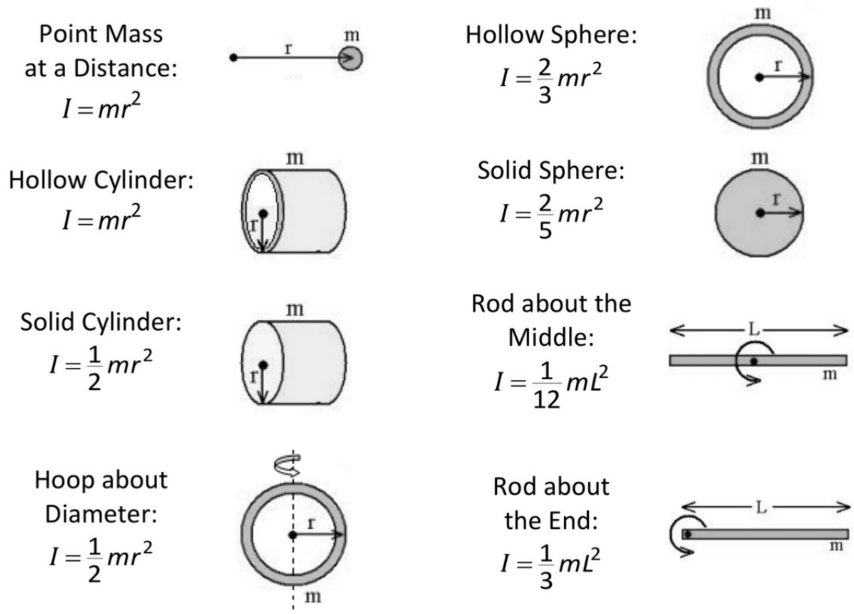
$$\text{Distance from center of mass} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

1.7.9 Moment of inertia

Inertia is the tendency for an object to continue to do what it is doing.

Similarly, rotational inertia is the tendency for a rotating object to continue rotating.

Moment of inertia I is a qualitative measure of the rotational inertia of an object.



The above figure shows the formula for I for objects of various shapes.

Parallel axis theorem

If the rotational inertia about the center of mass is I_{CM}, then the rotational inertia about a point distance x away is

$$I_{\text{shifted}} = I_{\text{CM}} + mx^2$$

where m is the mass of the object.

Perpendicular axis theorem

If the rotational inertia of a 2D object about the CM is I_x and I_y, then the rotational inertia about the z-axis is $I_z = I_x + I_y$.

1.7.10 Torque

Unit: N·m (newton meters), vector.

Torque τ measures the effectiveness of a force in causing rotation. It is calculated by

$$\tau_{\text{net}} = I\alpha$$

where I = rotational inertia, α = angular acceleration.

Torque can also be calculated by

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad \text{which gives} \qquad \text{magnitude of } \tau = rF\sin\theta$$

where r = distance from center of circle, F is the force applied, θ = angle between lever arm and applied force.

1.7.11 Rotational work

Rotational work W can be calculated as

$$W = \tau \Delta\theta$$

where τ = torque, θ = angle (in radians).

1.7.12 Rotational kinetic energy

Rotational kinetic energy K_r can be calculated as

$$K_r = \frac{1}{2} I \omega^2$$

where I = moment of inertia, ω = angular velocity.

The total kinetic energy is the sum of the rotational and translational kinetic energy (e.g. a ball rolling).

1.7.13 Angular momentum

Angular momentum L is the momentum of a rotating object in the direction of rotation. It is a vector quantity and can be calculated as

$$L = I\omega$$

where I = moment of inertia, ω = angular velocity.

Angular momentum can also be calculated as

$$\vec{L} = \vec{r} \times \vec{p} = rp \sin\theta$$

where r = radius of circle, p = linear momentum.

Conservation of angular momentum is used when there is no external torque.

1.8 Gravitation

1.8.1 Gravitational force

The gravitational force between two objects is

$$F = \frac{Gm_1m_2}{r^2}$$

where G is the gravitational constant 6.67×10^{-11}, m_1 is the mass of the first object, m_2 is the mass of the second object, r is the distance between the two objects.

Since $F = ma$, dividing by the mass of the second object gives us the gravity

$$g = \frac{Gm_1}{r^2}$$

For example, gravity on Earth can be calculated as $\frac{6.67 \times 10^{-11} \times 5.972 \times 10^{24}}{(6.371 \times 10^6)^2} = 9.81\mathrm{m/s}^2$.

1.8.2 Escape velocity

If you want to send a rocket into outer space, it needs enough kinetic energy to escape from the force of Earth's gravity. The velocity required is known as the escape velocity. It is given by

$$v_e = \sqrt{\frac{2Gm}{r}}$$

where G is the gravitational constant 6.67×10^{-11}, m is the mass of the planet, r is the radius of the planet.

Exercise: derive this equation by equating the GPE of the Earth m_1gh to the kinetic energy of the rocket $\frac{1}{2}m_2v_e^2$.

1.8.3 Kepler's third law

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

where T is the orbital period, r is the average radius of the planet from the sun, G is the gravitational constant 6.67×10^{-11}, M is the mass of the sun.

1.9 Fluid dynamics

1.9.1 Solid pressure

SI unit: Pa (pascal), scalar.

Pressure is the exertion of force upon a surface by an object. It is defined as

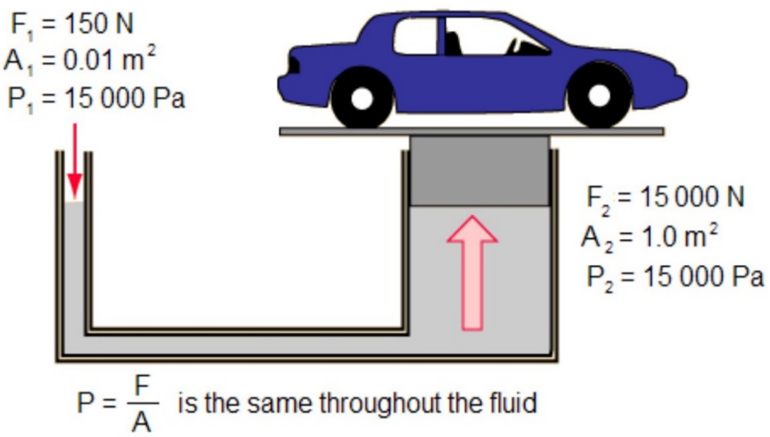
$$P = \frac{F}{A}$$

where F = force, A = area.

1.9.2 Pascal's principle

Any pressure applied to a fluid is transmitted uniformly throughout the fluid.

Because $P = \frac{F}{A}$, if pressure is the same everywhere in the fluid, then $\frac{F}{A}$ must be the same everywhere in the fluid.



If you have two pistons whose cylinders are connected, $\frac{F_1}{A_1} = \frac{F_2}{A_2}$. This principle is called hydraulics as shown in the diagram above.

1.9.3 Fluid pressure

The pressure in a fluid is

$$P = \rho g h$$

where ρ is the density of the fluid, g is gravity, h is the height of the fluid.

1.9.4 Buoyant force

Buoyant force occurs because the pressure increases as you go deeper down a fluid.

The equation for buoyant force is

$$F_B = \rho g V_f$$

where ρ = density of fluid, g = gravity, V_f = volume of fluid displaced.

Upon closer inspection, the equation can also be written as

$$F_B = m_f g$$

where m_f is the mass of fluid displaced, g is gravity.

This is known as Archimedes principle: the buoyant force on an object is equal to the weight of fluid displaced.

1.9.5 Bernoulli's equation

The pressures in a moving fluid are caused by:

- The pressure exerted by the fluid, P
- The fluid pressure, $\rho g h$
- The dynamic pressure, $\frac{1}{2}\rho v^2$, which results from the force that the moving particles exert on other fluid particles around them

A change in any of theses pressures affects the others, thus the sum of these pressures is constant, giving the equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

This is known as Bernoulli's equation.

2 Waves

2.1 Springs and pendulums

2.1.1 Spring force

The equation for spring force is given by Hooke's Law:

$$F_s = -kx$$

where x is the displacement of the end of the spring, k is the spring constant which is unique for every spring.

The negative sign is because the force is always in the opposite direction from the displacement.

2.1.2 Potential energy

The potential energy stored in a spring is given by the equation

$$U = \frac{1}{2}kx^2$$

where k is the spring constant, x is the displacement.

2.1.3 Period of a spring

The period is the time it takes for a spring to move from its maximum displacement in one direction to its maximum displacement in the opposite direction and back again. The period of a spring is given by the following equation:

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

where m is the mass of the spring, k is the spring constant.

2.1.4 Period of a pendulum

For small angles ($\theta < 15^\circ$), the period of a pendulum is given by the equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of pendulum, g is gravity.

2.1.5 Frequency

Frequency is the number of times something occurs in a given amount of time. It is measured in hertz (Hz). One hertz is the inverse of one second.

Frequency and period are reciprocals of each other:

$$T = \frac{1}{f} \qquad \text{and} \qquad f = \frac{1}{T}$$

2.2 Kinematics of simple harmonic motion

Simple harmonic motion is motion consisting of regular, periodic back-and-fourth oscillation. Examples include springs, pendulums and waves.

From calculus, the general equations of simple harmonic motion are:

$$x = A\cos(\omega t + \phi)$$
$$v = -A\omega\sin(\omega t + \phi)$$
$$a = -A\omega^2\cos(\omega t + \phi)$$

where x = displacement from equilibrium point, A = amplitude, ω = angular frequency, t = time, ϕ = phase angle, v = velocity, a = acceleration.

The phase angle is position where the cycle starts, relative to the equilibrium point. Usually it is 0.

2.3 Waves

The velocity of a wave can be calculated by

$$v = f\lambda$$

where f is the frequency of the wave, λ is the wavelength of the wave.

2.3.1 Reflection of waves

Reflection is when a wave hits a fixed point and "bounces" back.

Note that when the end of the rope is fixed, the reflected wave is inverted.

2.3.2 Superposition of waves

Constructive interference

Constructive interference is when waves add in a way that the amplitude of the resulting wave is larger than the amplitudes of the component waves.

Destructive interference

Destructive interference is when waves add in a way that the amplitude of the resulting wave is smaller than the amplitudes of the component waves.

2.3.3 Standing waves

When the wavelength is an exact fraction of the length of a medium that is vibrating, the wave reflects back and the reflected wave interferes constructively with itself. This causes the wave to appear stationary.

Points along the wave that are not moving are called nodes, points of maximum displacement are called antinodes.

2.4 Lens

This is the lens equation:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

where u is the distance to the object, v is the distance to the image, f is the focal length of the lens.

It can be derived from this equation that $u + v \geq 4f$.

2.4.1 Magnification

Magnification can be calculated as

$$M = -\frac{v}{u}$$

where v is the distance to image, u is the distance to object.

2.4.2 Snell's Law

When light crosses from one medium to another, the difference in velocity of the wave causes it to bend. This can be calculated by snell's law:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n_1, n_2 are the two refractive indexes, θ_1, θ_2 are the angle of incidence and refraction.

2.4.3 Refractive index

The refractive index n of a medium can be calculated by

$$n = \frac{c}{v}$$

where c is the speed of light in a vacuum 3.0×10^8, v is the speed of light in the medium.

2.4.4 Critical angle

Critical angle is the angle beyond which total internal reflection occurs.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the refractive index of the current medium, n_2 is the refractive index of the next medium.

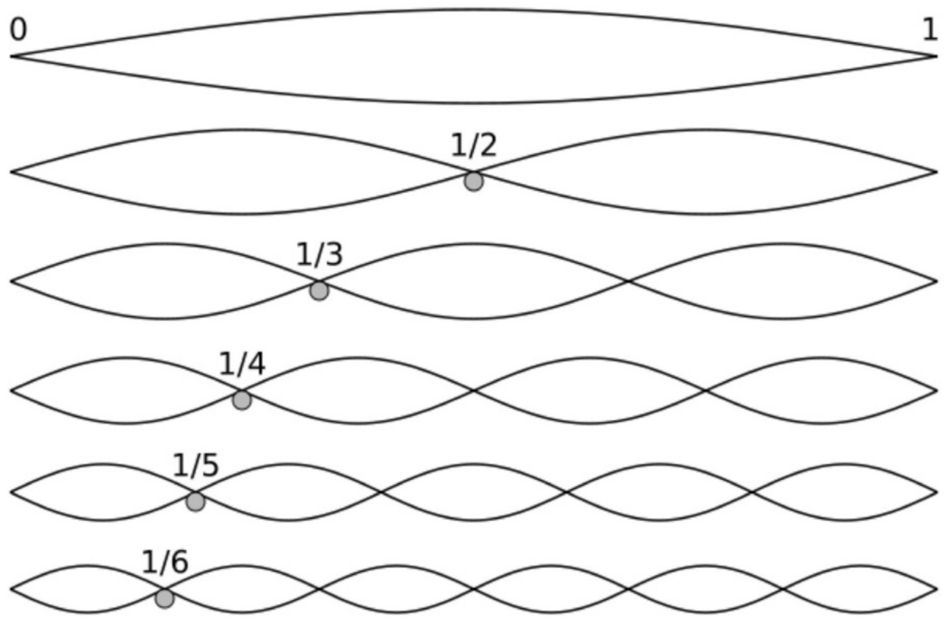
2.4.5 Malus's Law

Light can be polarized by passing it through a polarizing filter or other polarizing material. The intensity I of polarised light after passing through a polarising filter is

$$I = I_0 \cos^2 \theta$$

where I_0 is the original intensity, θ is the angle between the direction of polarisation and the axis of filter.

2.5 Harmonic series



This is for a drum (both ends are closed). For closed pipes, one end is open; for open pipes, both ends are open.

3 Thermodynamics

3.1 Heat

3.1.1 Heat transfer by conduction

Heat transfer by conduction can be calculated using Fourier's Law of Heat Conduction:

$$\frac{Q}{t} = -kA\frac{\Delta T}{L}$$

where Q = heat transferred, t = time, k = coefficient of thermal conductivity, A = cross-sectional area, ΔT = temperature difference, L = length.

3.1.2 Heating

The amount of heat gained or lost when an object changes temperature is given by

$$Q = mc\Delta T$$

where m = mass, c = specific heat capacity, ΔT = temperature change.

3.1.3 Heat when melting/boiling

The amount of heat required to melt/boil an object is

$$Q = mL$$

where m = mass, L = heat of fusion (melting) / heat of vapourisation (boiling).

3.1.4 Thermal expansion of solids and liquids

The changes in length and volume when heating are given by the equations:

$$\frac{\Delta L}{L} = \alpha \Delta T$$
$$\frac{\Delta V}{V} = \beta \Delta T$$

where ΔL = change in length, L = initial length, ΔT = temperature change, ΔV = change in volume, V = initial volume, α = linear coefficient of thermal expansion, β = volumetric coefficient of thermal expansion.

3.1.5 Heat from radiation

The rate of heat transfer by emitted radiation is determined by the Stefan-Boltzmann law of radiation:

$$\frac{Q}{t} = \epsilon \sigma A T^4$$

where t = time, A = surface area of object, T = temperature, ϵ = emissivity of object, σ = Stefan-Boltzmann constant 5.67×10^{-8}.

3.2 Thermodynamic laws

First Law: Energy is conserved.
Second Law: States tend towards thermal equilibrium (that's when entropy is maximum).
Third Law: Entropy at 0˚K is 0.

3.3 Ideal gas

3.3.1 Ideal gas law

$$PV = nRT$$

where P = pressure, V = volume, n = number of moles, R = universal gas constant 8.314, T = temperature.

3.3.2 PV graphs

Isobaric: constant pressure

Isochoric: constant volume

Isothermal: constant temperature

Adiabatic: no heat exchange

Work done *on* the system is the area under a PV graph going counter-clockwise.

To know whether a part of the graph has positive, zero, or negative internal energy, heat or work, use the following equation and draw a table:

$$\Delta U = \Delta Q + \Delta W$$

where U = internal energy, Q = heat added *to* the system, W = work done *on* the system.

4 Electricity and Magnetism

4.1 Terms and definitions

4.1.1 Electric current (current)

The flow of charged particles from one place to another, caused by a difference in electric potential. It can be calculated by

$$q = I \cdot t$$

where q = charge, I = electric current, t = time.

4.1.2 Voltage

The difference in electric potential energy between two locations.

4.1.3 Resistance

Resistance is the amount of electromotive force needed to force a given amount of current through an object. It is given by Ohm's Law:

$$V = IR$$

where V = voltage, I = electric current, R = resistance.

4.1.4 Resistivity

The innate ability of a substance to offer electrical resistance.

$$R = \frac{\rho L}{A}$$

where R = resistance, ρ = resistivity, L = length of object, A = cross-sectional area of object.

4.1.5 Power

In electric circuits,

$$P = IV = \frac{V^2}{R} = I^2 R$$

where P = power, I = electric current, V = voltage, R = resistance.

4.2 Kirchhoff's rules

4.2.1 Kirchhoff's junction rule

The total current coming into any junction must equal the total current coming out of the junction.

This is because electric charge cannot be created or destroyed.

4.2.2 Kirchhoff's loop rule

The sum of voltages around any closed loop must add up to zero.

This is because voltage is the difference in electric potential between two locations. If you come back to the same point in the circuit, the difference in electric potential must be zero.

4.3 Series circuits

4.3.1 Current

Since there is only one path, all of the current flows through every component. This means the current is the same through every component in the circuit.

$$I_{\text{total}} = I_1 = I_2 = I_3 = ...$$

4.3.2 Voltage

In a series circuit, if there are multiple voltage sources (e.g. batteries), the voltages add.

$$V_{\text{total}} = V_1 + V_2 + V_3 + ...$$

4.3.3 Resistance

If there are multiple resistors, each one contributes to the total resistances and they add.

$$R_{\text{total}} = R_1 + R_2 + R_3 + ...$$

4.3.4 Power

In all circuits, any component that has resistance dissipates power whenever current passes through it. The total power consumed by the circuit is the sum of power dissipated by each component.

$$P_{\text{total}} = P_1 + P_2 + P_3 + ...$$

4.4 Parallel circuits

4.4.1 Current

The current divides at each junction. This means the current through each path must add up to the total current.

$$I_{\text{total}} = I_1 + I_2 + I_3 + ...$$

4.4.2 Voltage

In a parallel circuit, the potential difference across the battery is always the same. This means the voltage is the same along each path.

$$V_{\text{total}} = V_1 = V_2 = V_3 = ...$$

4.4.3 Power

In a parallel circuit, any component that has resistance dissipates power whenever current passes through it. The total power is the sum of power dissipated by each component.

$$P_{\text{total}} = P_1 + P_2 + P_3 + ...$$

4.4.4 Resistance

If there are multiple resistors, the effective resistance of each path becomes less and less as there are more paths for current to flow through.

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

4.5 Electric field

The force between two charges in an electric field is

$$F_e = \frac{kq_1q_2}{r^2}$$

where k is the electrostatic constant 8.99×10^9, q_1, q_2 are the charges, r is the distance between the two charges.

Note that k can also be written as

$$k = \frac{1}{4\pi\epsilon_0}$$

where ϵ_0 is the electric constant 8.85×10^{-12}.

The force on a charge in an electric field can also be calculated by

$$F = qE$$

where q is the charge of the particle, E is the electric field force.

4.6 Capacitance

4.6.1 Capacitance

A capacitor is an electrical component that stores electric charge but does not allow current to flow through.

Capacitance is a measure of the ability of a capacitor to store charge. It is measured in farads (F). It is calculated by

$$C = \frac{Q}{V}$$

where Q is charge and V is voltage.

The capacitance of a parallel plate capacitor is also given by

$$C = \frac{\epsilon A}{d}$$

where ϵ = electrical permittivity, A = cross-sectional area, d = distance between the two plates.

4.6.2 Energy stored in a capacitor

The energy stored in a capacitor is

$$U_c = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

where C is capacitance, V is voltage, Q is charge.

4.6.3 Capacitors in parallel

When capacitors are arranged in parallel, the total capacitance is the sum of capacitances of the individual capacitors.

$$C_{\text{total}} = C_1 + C_2 + C_3 + ...$$

4.6.4 Capacitors in series

When capacitors are arranged in series, the total capacitance is

$$\frac{1}{C_{\text{total}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

4.6.5 RC circuits

An RC circuit is a circuit involving combinations of resistors and capacitors.

Charging a capacitor

The equations for charge and current as a function of time as a capacitor is charging are:

$$I = I_0 e^{\frac{-t}{RC}}$$

$$Q = CV(1 - e^{\frac{-t}{RC}})$$

where I = current, I_0 = initial current, Q = charge, C = capacitance, V = voltage, e = Euler's number. t = time, R = resistance.

Discharging a capacitor

The equation for discharging a capacitor is

$$\frac{V}{V_0} = \frac{Q}{Q_0} = \frac{I}{I_0} = e^{\frac{-t}{RC}}$$

where V = voltage, V_0 = initial voltage, Q = charge, Q_0 = initial charge, I = current, I_0 = initial current, e = Euler's number, t = time, R = resistance, C = capacitance.

4.7 Magnetism

The force on a charge moving through a magnetic field is given by the equation

$$F = qvB\sin\theta$$

where F is the force, q is the charge, v is the velocity of the charge, B is the magnetic field, θ is the angle between the direction of the velocity and the magnetic field.

The force can also be calculated as

$$F = LIB\sin\theta$$

where F is the force, L is the length of the wire passing through the magnetic field, I is the current, B is the magnetic field, θ is the angle between the current and the magnetic field.

The direction of the force can be determined by the right hand rule or by the left hand rule (FBI).

4.7.1 Magnetic field produced by electric current within a solenoid

A solenoid is a coil made of fine wire. When a current is passed through the wire, it produces a magnetic field through the center of the coil.

The magnetic field is given by

$$B = \mu_0 n I$$

where B is the magnetic field, μ_0 is the magnetic permeability of free space, n is the number of coils per meter, I is the current.

4.7.2 Magnetic field produced by electric current through a wire

The magnetic field produced by a steady current flowing in a very long straight wire encircles the wire. The magnetic field can be calculated as

$$B = \frac{\mu_0 I}{2\pi r}$$

where B is the magnetic field, μ_0 is the magnetic permeability of free space, I is the current, r is the distance from the wire.

4.7.3 Magnetic field produced by electric current in a circular loop

The magnetic field at the center of a circular loop is

$$B = \frac{\mu_0 I}{2r}$$

where B is the magnetic field, μ_0 is the magnetic permeability of free space, I is the current, r is the radius of the circular loop.