CGS 698C , Assignment 1

Part 1

1.1 (a) $\Omega = \{HH, HT, TH, TT\}$

(b) number of possible events $= 2^4 = \underline{\underline{16}}$

$\mathcal{F} = \{\phi, \{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\}, \{TH, TT\}, \{HH, HT, TH\}, \{HH, TH, TT\}, \{HH, HT, TT\}, \{HT, TH, TT\}, \Omega\}$

(c) (i) $P(\{HH\}) = P(HT) = P(TH) = P(TT) = \underline{\underline{1/4}}$

(ii) P (atleast one head) $= P(HH) + P(HT) + P(TH)$

$= \underline{\underline{3/4}}$

(iii) $P(\text{exactly one head}) = P(HT) + P(TH)$

$$= \frac{1}{2}$$

Part 2

2.1) $f(45, 50, 0.9) = \frac{50!}{45! \cdot 5!} (0.9)^{45} (0.1)^{5}$

$$\boxed{f \approx 0.1849}$$

2.2) (a) $f(0, 10) = \frac{10^0 e^{-10}}{0!} = 0.000045$

(b) $f(8,10) + f(9,10) = \frac{10^8 e^{-10}}{8!} + \frac{10^9 \cdot e^{-10}}{9!} = 0.2139$

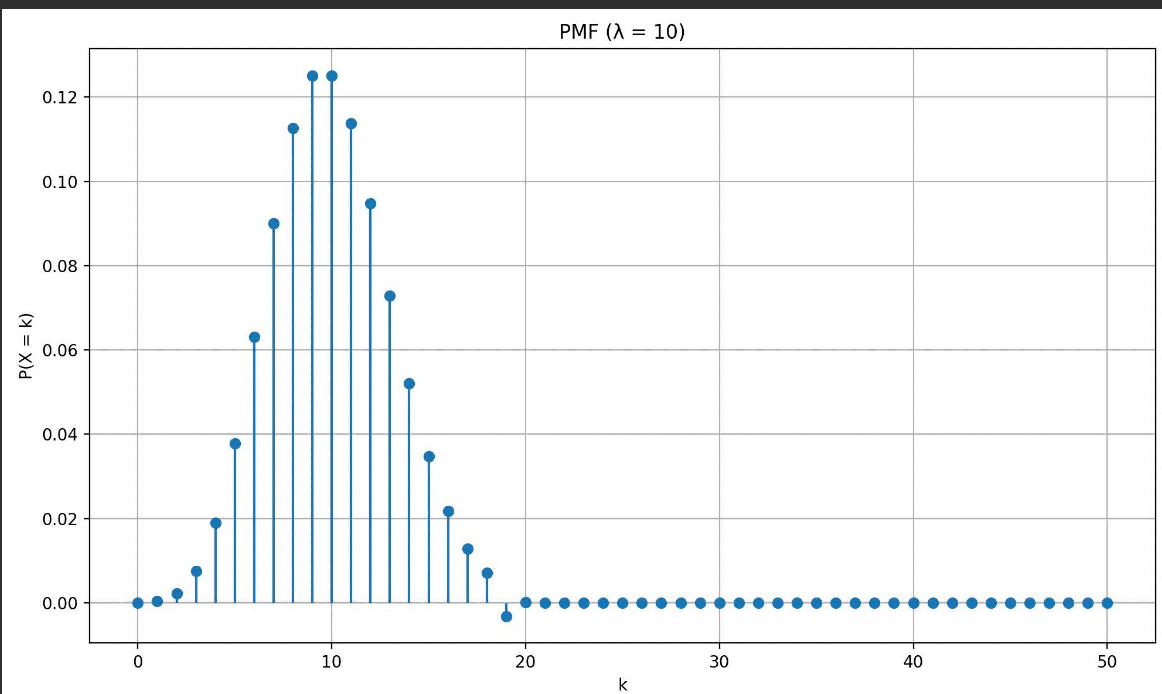
$$= \frac{10^8}{8!} e^{-10} \left(1 + \frac{10}{9} \right)$$

$$= \frac{10^8}{8!} e^{-10} \times \frac{19}{10}$$

(c)

2_2.py

```python
import math
import numpy as np
import matplotlib.pyplot as plt


lambda_val = 10


def pmf(k, lam):
    return (lam ** k) * math.exp(-lam) / math.factorial(k)

# Generate X values from 0 to 50
x = np.arange(0, 51)
# Compute PMF values
pmf = [pmf(k, lambda_val) for k in x]

# Plotting
plt.figure(figsize=(10, 6))
plt.stem(x, pmf, basefmt=" ")
plt.title(f'PMF (λ = {lambda_val})')
plt.xlabel('k')
plt.ylabel('P(X = k)')
plt.grid(True)
plt.tight_layout()
plt.show()

```



Part 3

3.1) $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(a) $$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(-1)^2}{2}} = \frac{e^{-1/2}}{\sqrt{2\pi}} = 0.2419$$

(b) $$f(1) = \frac{1}{\sqrt{2\pi}} e^{-1/2} = 0.2419$$

(c) $$P(x_1 \le X \le x_2) = \int_0^{x_2} f\,dx - \int_0^{x_1} f\,dx = 0.3 \quad \text{— (1)}$$

$$P(x_3 \le X \le x_2) = \int_0^{x_3} f\,dx - \int_0^{x_2} f\,dx = 0.45 \quad \text{— (2)}$$

(2) − (1) $\Rightarrow$ $$\int_0^{x_3} f\,dx - \int_0^{x_2} f\,dx = 0.15$$

$$P(x_3 \le X \le x_2) = \underline{\underline{0.15}}$$

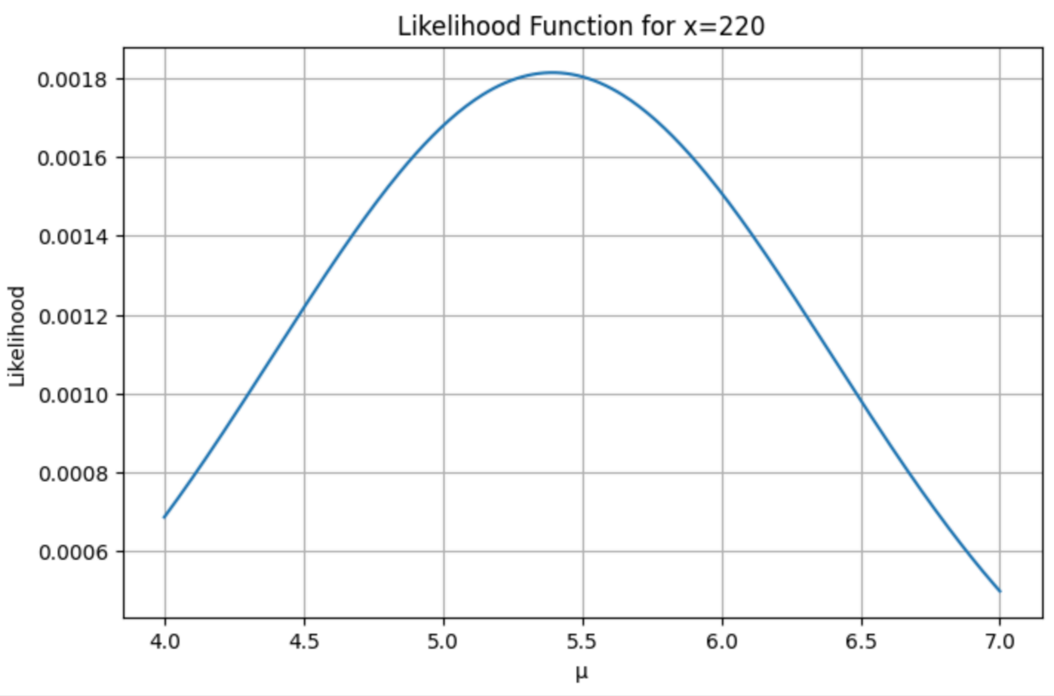
Made with Goodnotes

Part 4

4.1 (a)

```python
import numpy as np
import matplotlib.pyplot as plt

x_fixed = 220
def likelihood_function(x, mu):
    return (1 / (x * np.sqrt(2 * np.pi))) * np.exp(-((np.log(x) - mu)**2)/2)

mu_values = np.linspace(4, 7, 300)
likelihood_values = likelihood_function(x_fixed, mu_values)

plt.figure(figsize=(8,5))
plt.plot(mu_values, likelihood_values)
plt.xlabel('μ')
plt.ylabel('Likelihood')
plt.title('Likelihood Function for x=220')
plt.grid(True)
plt.show()
```

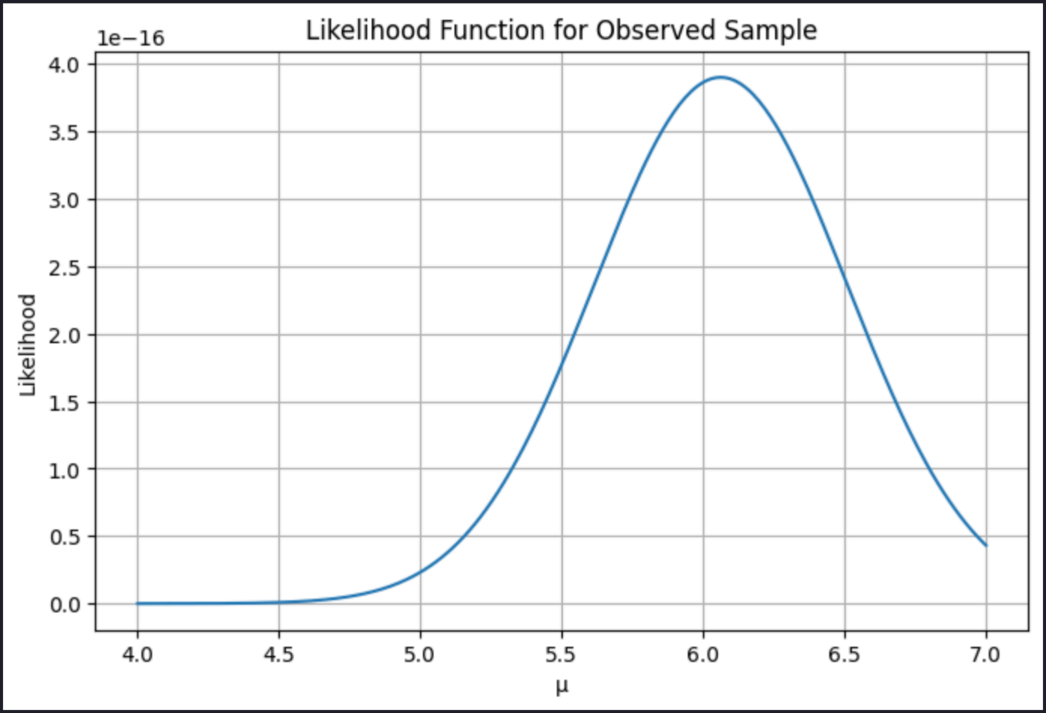
✓ 0.6s



4.1 (b)

```
x_sample = np.array([303, 443, 220, 560, 880])

def likelihood_function_sample(mu, x):
    n = len(x)
    product_x = np.prod(x)
    term1 = 1 / (product_x * (np.sqrt(2 * np.pi)**n))
    term2 = np.exp(-np.sum((np.log(x) - mu)**2)/2)
    return term1 * term2

mu_values = np.linspace(4, 7, 300)
likelihood_values_sample = [likelihood_function_sample(mu, x_sample) for mu in mu_values]

plt.figure(figsize=(8,5))
plt.plot(mu_values, likelihood_values_sample)
plt.xlabel('μ')
plt.ylabel('Likelihood')
plt.title('Likelihood Function for Observed Sample')
plt.grid(True)
plt.show()
```

✓ 0.0s



4.1 (c)

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} \log x_i$$

$$\hat{\mu} = \frac{\ln 303 + \ln 443 + \ln 560 + \ln 880 + \ln 220}{5}$$

$$\boxed{\hat{\mu} \approx 6.06}$$