

Week - 2 Assignment

		1	2	3	4	(next state)
1.						
a)	1	0.5	0.5	0	0	
	2	0.25	0.75	0	0	
	3	0	0	0.25	0.75	
	4	0	0	0.75	0.25	

(current state)

b) States $\{1, 2\}$ \rightarrow closed & finite
 \therefore Recurrent

States $\{3, 4\}$ \rightarrow closed & finite
 \therefore Recurrent

c) for stationary distribution $\pi_j = \pi$ & $\sum \pi_j = 1$
for states $\{1, 2\}$

$$\pi_1 + \pi_2 = 1$$

$$0.5 \pi_1 + 0.25 \pi_2 = \pi_1 \Rightarrow \pi_2 = 2\pi_1$$

$$0.5 \pi_1 + 0.75 \pi_2 = \pi_2$$

$$\pi_1 + 2\pi_1 = 1 \Rightarrow \pi_1 = \frac{1}{3}, \pi_2 = \frac{2}{3}$$

$$\pi = \left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$$

for state $\{3, 4\}$, $\pi = \{0, 0, \pi_3, \pi_4\}$
 $\pi_3 + \pi_4 = 1$

$$0.25 \pi_3 + 0.75 \pi_4 = \pi_3 \Rightarrow \pi_3 = \pi_4$$

$$2\pi_3 = 1 \Rightarrow \pi_3 = 0.5$$

$$\therefore \pi = (0, 0, 0.5, 0.5)$$

$$2) \quad P = \begin{matrix} & \begin{matrix} W & L \end{matrix} \\ \begin{matrix} W \\ L \end{matrix} & \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} \end{matrix}$$

$$P(D|W) = 0.7 \quad ; \quad P(D|L) = 0.2$$

a) let π denote stationary distribution
 $\pi = (\pi_W, \pi_L) \quad \pi P = \pi^T, \quad \sum_i \pi_i = 1$

$$\pi_W 0.8 + 0.3 \pi_L = \pi_W$$

$$\Rightarrow 0.3 \pi_L = 0.2 \pi_W \quad \therefore \pi_W = \frac{3}{2} \pi_L$$

$$\pi_L + \pi_W = 1$$

$$\left(\frac{3}{2} + 1\right) \pi_L = 1 \quad \Rightarrow \quad \pi_L = \frac{2}{5} \quad \pi_W = \frac{3}{5}$$

$$\pi = (0.6, 0.4)$$

\therefore Team wins 60% of games

$$b) \quad P(D) = \pi_W 0.7 + \pi_L \times 0.2 = 0.6 \times 0.7 + 0.4 \times 0.2 = 0.5$$

\therefore 50% of games result in dinner.

$$c) \text{ Expected no. of games } = \frac{1}{0.5} = 2$$

$$3) \text{ Cat } \begin{matrix} R1 & R2 \\ P_C = \text{Room 1} & \begin{bmatrix} 0.2 & 0.8 \end{bmatrix} \\ \text{Room 2} & \begin{bmatrix} 0.8 & 0.2 \end{bmatrix} \end{matrix}$$

$$\text{mouse } \begin{matrix} 1 & 2 \\ P_M = 1 & \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \\ 2 & \begin{bmatrix} 0.6 & 0.4 \end{bmatrix} \end{matrix}$$

a)

Cat chain

$$\pi_C = (\pi_1, \pi_2) \quad \pi_1 + \pi_2 = 1$$

$$\pi_1 \cdot 0.2 + 0.8 \pi_2 = \pi_1$$

$$0.8 \pi_2 = 0.8 \pi_1$$

$$\pi_1 = \pi_2$$

$$\therefore \pi_1 = \pi_2 = 0.5$$

mouse chain

$$\pi_M = (\pi_1, \pi_2)$$

$$0.7 \pi_1 + 0.6 \pi_2 = \pi_1$$

$$0.6 \pi_2 = 0.3 \pi_1$$

$$\pi_1 = 2 \pi_2$$

$$\pi_2 + 2 \pi_2 = 1$$

$$\pi_2 = \frac{1}{3}$$

$$\pi_M = \left(\frac{2}{3}, \frac{1}{3} \right)$$

b) for Z_n , state space $\{(1,1), (1,2), (2,1), (2,2)\}$

		(1,1)	(1,2)	(2,1)	(2,2)
T.P. (Cat, Mouse)	(1,1)	$\begin{pmatrix} 0.2 & 0.7 \\ 0.14 & 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.2 \\ 0.06 & 0.06 \end{pmatrix}$	$\begin{pmatrix} 0.7 & 0.3 \\ 0.56 & 0.24 \end{pmatrix}$	$\begin{pmatrix} 0.3 & 0.3 \\ 0.24 & 0.06 \end{pmatrix}$
	(1,2)	0.12	0.08	0.48	0.32
	(2,1)	0.56	0.24	0.14	0.06
	(2,2)	0.48	0.32	0.12	0.08

Here, Row Sum = 1

$\forall i, p_{ij} \leq 1 \quad \therefore$ It follows a Markov chain

Cat & Mouse follow M.C. Independently, so their joint follows Markov chain.

Q4) There are 3 types of squares = corners, edges & Interior.

Type	No.	No. of moves
Corners	4	3
Edge	$6 \times 4 = 24$	5
Interior	$64 - 28 = 36$	8

[Row 2-7, Col 2-7]

Total degree of freedom

$$\text{for corners} = 4 \times 3 = 12$$

$$\text{Edge} = 5 \times 24 = 120$$

$$\text{Interior} = 36 \times 8 = 288$$

$$\text{Total} = 420$$

Now, Stationary Probability for

$$(P_c) \text{ each corner} = \frac{3}{420} = \frac{1}{140} \quad \left[\frac{\text{No. of moves}}{\text{Total d.o.f.}} \right]$$

$$(P_e) \text{ each edge} = \frac{5}{420} = \frac{1}{84}$$

$$(P_i) \text{ each Interior} = \frac{8}{420} = \frac{2}{105}$$

$$\therefore 4P_c + 24P_e + 36P_i = 1$$

$$\& \quad \frac{P_c}{3} = \frac{P_e}{5} = \frac{P_i}{8} \quad \therefore P_c = \frac{1}{140}$$

$$P_e = \frac{1}{84}$$

$$P_i = \frac{2}{105}$$

Q5) Price opens at Rs 120 at 10:00 am

$$\begin{aligned}\text{Total open time} &= 5 \text{ hours } [10:00 \text{ am} \rightarrow 3:00 \text{ pm}] \\ &= 5 \times 3600 = 18000 \text{ seconds}\end{aligned}$$

Transition occurs at every 5 Secs

$$\text{So, total} = \frac{18,000}{5} = 3,600$$

$$P(\text{up 1 tick}) = 0.1$$

$$P(\text{stay}) = 0.25, \quad P(\text{down}) = 0.05$$

$$\begin{aligned}\text{Expected change in Price} &= 0.1 \times 1 + 0.25 \times 0 + 0.05 \times (-1) \\ &= 0.05 \text{ ticks} \\ &= 0.05 \times 0.01 = \text{Rs } +0.0005\end{aligned}$$

a)

\therefore Stock prices are unbounded

\rightarrow The Stock price is transient.

b) No stationary distribution exists

c) Strike $K = 125$

$$\text{To get Pay off} = 130 - 125 = 5$$

$$\text{Time} = 1:00 \text{ pm} = 3 \text{ hours}$$

$$= 10,800 \text{ s} = 2160 \text{ steps}$$

$$E[\text{Increase in price}] = 0.0005 \times 2160 = \text{Rs } 1.08$$

$$\therefore \text{Expected price} = 121.08 \quad [\text{confirmed using Monte-Carlo simulation}]$$