Stochastic Modelling of Financial Derivatives

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Total Marks: 75 **Deadline:** 24th June 2025

Week-4 Assignment

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1. **European Options and Real Options Framing (25 Marks)**
   A European call option on a non-dividend paying stock is trading at \$4.20. The stock currently trades at \$38, the option has a strike price of \$35, and it expires in 4 months. The risk-free interest rate is 6% per annum compounded continuously.

   (a) Compute the implied volatility of the option using the Black-Scholes Model. Use trial-and-error or interpolation. Try at least three volatility levels in the range 0.1 to 0.5. Use the Black-Scholes formula for a European call option.

   (b) Suppose the implied volatility is accepted to be 0.28 from market estimation. Now, using this volatility, calculate the price of a European put option with the same strike and maturity. Use the Black-Scholes formula for a European put option or apply put-call parity.

   (c) A firm owns a patented drug that can be commercialized within the next 4 months. The launch involves a fixed regulatory and setup cost equivalent to \$35 million. The expected net revenue from the product launch (treated as the "stock") is currently estimated at \$38 million and follows a log-normal diffusion process. There is no intermediate revenue, and the firm can only make the launch decision now. Should the firm launch the product? Justify using a real options perspective and your results from (a) and (b). Assume the launch opportunity resembles a European call option.

2. **Pen and Paper Option Pricing (25 Marks)**
   Consider a European call option with the following characteristics:

   - Current Stock Price $S_0 = 100$
   - Strike Price $K = 105$
   - Time to Maturity $T = 10$ days

- Option Type: European Call

Part A: Discrete Binomial Model

Assume the stock follows a discrete random walk: each day, the stock price either increases or decreases by exactly \$1 with equal probability.

(a) Compute the probability that the option ends in the money after 10 days.

(b) Calculate the expected payoff of the option.

(c) Hence, determine the fair value of the option (ignore discounting).

Part B: Continuous Normal Distribution Model

Now assume the stock's daily return follows a normal distribution with:

- Mean = 0
- Standard deviation σ chosen so that the expected absolute daily move is \$1. Use the relation $\mathbb{E}[|X|] = \sigma\sqrt{2/\pi}$.

(a) Determine the daily standard deviation σ, and scale it to 10 days.

(b) Write the expected payoff of the option as an integral over the terminal price distribution:

$$\mathbb{E}[\max(S_T - K, 0)] = \int_K^\infty (S - K) f_{S_T}(S)\, dS$$

where $f_{S_T}(S)$ is the density function of the terminal stock price.

(c) Evaluate the integral numerically or using software (e.g., Python, R, Excel).

Part C: Uniform Distribution Model

Assume each day's price move is from a uniform distribution, with expected absolute move still \$1.

(a) Define the support $[a, b]$ of the uniform distribution such that $\mathbb{E}[|X|] = 1$.

(b) Compare the resulting 10-day distribution of final prices with the binomial and normal models.

(c) Propose and describe a simulation method to estimate the fair value of the call option under this model.

3. **Buffon's Needle — Monte Carlo Estimation of π (25 Marks)**
Use a classic Monte Carlo experiment to estimate the value of π by simulating a needle drop on a lined surface.

Setup:

- A floor has equally spaced parallel lines, distance d units apart.
- A needle of length $\ell \leq d$ is randomly dropped.
- Randomly generate:
  - Distance x from needle's center to nearest line: $x \sim \text{Uniform}[0, d/2]$
  - Angle θ with horizontal: $\theta \sim \text{Uniform}[0, \pi/2]$

- The needle crosses a line if $\frac{\ell}{2}\sin(\theta) \geq x$

Monte Carlo Estimator:

$$\pi \approx \frac{2N}{d \times (\text{Number of crossings})}$$

Instructions:

- Simulate at least 10,000 needle drops.
- Plot convergence of the estimated π as N increases (log scale recommended).
- Compare with the true value of π, and comment on error sources such as random seed variation, sample size, or angle generation granularity.