Week- 2 Assignment

(next State)

1.

a)

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0.5 & 0.5 & 0 & 0 \\ 2 & 0.25 & 0.75 & 0 & 0 \\ 3 & 0 & 0 & 0.25 & 0.75 \\ 4 & 0 & 0 & 0.75 & 0.25 \end{array}$$

(current state)

b) States $\{1,2\}$ $\rightarrow$ Closed & finite

$\therefore$ Recurrent

States $\{3,4\}$ $\rightarrow$ closed & finite

$\therefore$ Recurrent

c) for stationary distribution $\pi Q = \pi$ & $\Sigma \pi_i = 1$

for states $\{1,2\}$

$$\pi_1 + \pi_2 = 1$$
$$0.5\,\pi_1 + 0.25\,\pi_2 = \pi_1 \Rightarrow \pi_2 = 2\pi_1$$
$$0.5\,\pi_1 + 0.75\,\pi_2 = \pi_2$$

$$\pi_1 + 2\pi_1 = 1 \Rightarrow \pi_1 = \frac{1}{3},\ \pi_2 = \frac{2}{3}$$

$$\pi = \left(\frac{1}{3}, \frac{2}{3}, 0, 0\right)$$

for state $\{3,4\}$, $\pi = \{0, 0, \pi_3, \pi_4\}$

$$\pi_3 + \pi_4 = 1$$
$$0.25\,\pi_3 + 0.75\,\pi_4 = \pi_3 \Rightarrow \pi_3 = \pi_4$$
$$2\pi_3 = 1 \Rightarrow \pi_3 = 0.5$$
$$\therefore \pi = (0, 0, 0.5, 0.5)$$

2)

$$P = \begin{array}{c|cc} & W & L \\ \hline W & 0.8 & 0.2 \\ L & 0.3 & 0.7 \end{array}$$

$P(D|W) = 0.7$; $P(D|L) = 0.2$

a) Let π denote stationary distribution

$\pi = (\pi_W, \pi_L)$ $\pi P = \pi$, $\sum_i \pi_i = 1$

$\pi_W \, 0.8 + 0.3\pi_L = \pi_W$

$\Rightarrow 0.3\pi_L = 0.2\pi_W$ $\therefore \pi_W = \frac{3}{2}\pi_L$

$\pi_L + \pi_W = 1$

$\left(\frac{3}{2} + 1\right)\pi_L = 1 \Rightarrow \pi_L = \frac{2}{5}$ $\pi_W = \frac{3}{5}$

$\pi = (0.6, 0.4)$

$\therefore$ Team wins 60% of games

b)

$P(D) = \pi_W \, 0.7 + \pi_L \times 0.2 = 0.6 \times 0.7 + 0.4 \times 0.2$

$= 0.5$

$\therefore$ 50% of games result in dinner.

c) Expected no. of games $= \frac{1}{0.5} = 2$

3) Cat

$$P_C = \begin{array}{c|cc} & R1 & R2 \\ \hline \text{Room1} & 0.2 & 0.8 \\ \text{Room2} & 0.8 & 0.2 \end{array}$$

mouse

$$P_M = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 0.7 & 0.3 \\ 2 & 0.6 & 0.4 \end{array}$$

a)

Cat chain

$\pi_c = (\pi_1, \pi_2)$ $\pi_1 + \pi_2 = 1$

$\pi_1 0.2 + 0.8\pi_2 = \pi_1$

$0.8\pi_2 = 0.8\pi_1$

$\pi_1 = \pi_2$

$\therefore \pi_1 = \pi_2 = 0.5$

Mouse chain

$\pi_m = (\pi_1, \pi_2)$

$0.7\pi_1 + 0.6\pi_2 = \pi_1$

$0.6\pi_2 = 0.3\pi_1$

$\pi_1 = 2\pi_2$

$\pi_2 + 2\pi_2 = 1$

$\pi_2 = \frac{1}{3}$

$\pi_m = \left(\frac{2}{3}, \frac{1}{3}\right)$

b) for Z_n, state space $\{(1,1), (1,2), (2,1), (2,2)\}$

T. P.

(Cat, Mouse)

$$\begin{array}{c|cccc} & (1,1) & (1,2) & (2,1) & (2,2) \\ & (0.2\times0.7) & (0.3\times0.2) & (0.7\times0.8) & (0.8\times0.3) \\ \hline (1,1) & 0.14 & 0.06 & 0.56 & 0.24 \\ (1,2) & 0.12 & 0.08 & 0.48 & 0.32 \\ (2,1) & 0.56 & 0.24 & 0.14 & 0.06 \\ (2,2) & 0.48 & 0.32 & 0.12 & 0.08 \end{array}$$

Here, Row sum $= 1$

$0 \le p_{ij} \le 1$ $\therefore$ It follow a Markov chain

Cat & Mouse follow M. C. Independently, so their joint follows Markov chain.

Q4) There are 3 types of squares = corners, edges & Interior.

Type	No.	No. of Moves
Corners	4	3
Edge	6×4=24	5
Interior [Row 2-7, col 2-7]	64-28=36	8

Total degree of freedom

for corners = $4 \times 3 = 12$

Edge = $5 \times 24 = 120$

Interior = $36 \times 8 = \underline{288}$

Total = 420

Now, Stationary Probability for

(P_c) each corner = $\frac{3}{420} = \frac{1}{140}$ $\left[\frac{\text{No. of moves}}{\text{Total d.o.f.}}\right]$

(P_e) each Edge = $\frac{5}{420} = \frac{1}{84}$

(P_i) each Interior = $\frac{8}{420} = \frac{2}{105}$

$\because 4p_c + 24p_e + 36p_i = 1$

& $\frac{P_c}{3} = \frac{P_e}{5} = \frac{P_i}{8}$ $\therefore$ $P_c = \frac{1}{140}$

$P_e = \frac{1}{84}$

$P_i = \frac{2}{105}$

Q5) Price opens at Rs 120 at 10:00 am

Total Open time = 5 hours [10:00am → 3:00 pm]

$= 5 \times 3600 = 18000$ seconds

Transition occurs at every 5 secs

so, total $= \frac{18,000}{5} = 3,600$

$P(\text{up 1 tick}) = 0.1$

$P(\text{Stay}) = 0.85$, $P(\text{down}) = 0.05$

Expected change in Price $= 0.1 \times 1 + 0.85 \times 0 + 0.05 \times (-1)$

$= 0.05$ ticks

$= 0.05 \times 0.01 =$ Rs $+0.0005$

a)

∵ Stock prices are unbounded

→ The Stock price is transient.

b) No Stationary distribution exists

c) Strike $K = 125$

To get Pay off $= 130 - 125 = 5$

Time = 1:00 pm = 3 hours

$= 10,800$s $= 2160$ steps

E[Increase in price] $= 0.0005 \times 2160 =$ Rs 1.08

∴ Expected Price = 121.08 [Confirmed using Monte-Carlo Simulation]

Estimated Probability $= 0.0134$

$\approx 0.01\%$