Stochastic Modelling of Financial Derivatives

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Total Marks: 100 **Deadline:** 13^{th} June, 2025

Week 3: Assignment 3

Instructions: All the following are computational problems. Please compile them into a single Jupyter Notebook (or Google Colab file), and provide complete solutions for each question. Ensure that the code is well-commented and clearly explained for ease of understanding.

Q1. Simulate a single path of a one-dimensional Wiener process (standard Brownian motion) over the interval $[0, T]$. Your output should be the full simulated path.

Q2. Let α and $\sigma > 0$ be constants, and define the *geometric Brownian motion*

$$S(t) = S(0)\exp\left\{\sigma W(t) + \left(\alpha - \frac{1}{2}\sigma^2\right)t\right\}.$$

Simulate 5 paths in a single plot for the above Geometric Brownian Motion.

Q3. Show that for standard Brownian motion, $\mathbb{E}[W_s W_t] = \min(s,t)$ for $s, t \geq 0$.

Q4. Let $0 \leq s < t$. Show that $W_t - W_s$ is normally distributed with mean 0 and variance $t - s$, and that increments over non-overlapping intervals are independent.

Q5. Show that for any $t \geq 0$, $\mathbb{E}[W_t|\mathcal{F}_s] = W_s$ for $0 \leq s \leq t$. Conclude that Brownian motion is a martingale.
