SECTION 3 Time — 25 minutes 9 Questions

Directions: For each question in this section, calculate the correct answer from the choices given.

Example:

Fill in the blanks to make the following equation valid.

- (A) $\times, +, -, +, 1$
- (B) $+, \div, \div, \times, 2$
- (C) $\times, -, \div, \times, 3$
- (D) $+, \times, -, +, 4$
- (E) $+,+,-,\div,5$

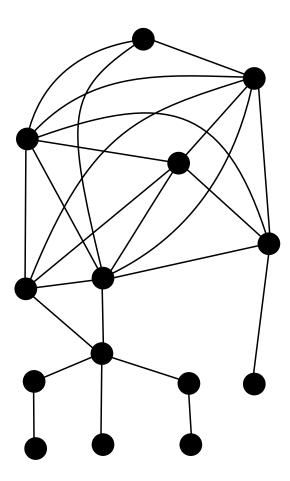
- 1. 12 of the playing cards are trying to decide which of their 6 prisoners to execute. They each have 3 prisoners who they either like or dislike.
 - The 2 of Hearts dislikes Bill the Lizard, dislikes the Dodo, and likes the Eaglet.
 - The 2 of Spades likes Bill the Lizard, dislikes the Cheshire Cat, and dislikes the Frog Footman.
 - The 3 of Diamonds dislikes Alice, likes Bill the Lizard, and likes the Frog Footman.
 - The 4 of Clubs dislikes Alice, dislikes Bill the Lizard, and dislikes the Eaglet.
 - The 4 of Spades likes Alice, dislikes the Cheshire Cat, and dislikes the Eaglet.
 - The 5 of Hearts likes Bill the Lizard, likes the Dodo, and dislikes the Frog Footman.
 - The 6 of Clubs dislikes Bill the Lizard, likes the Dodo, and likes the Frog Footman.
 - The 7 of Diamonds dislikes Bill the Lizard, likes the Dodo, and dislikes the Frog Footman.
 - The 7 of Spades likes Alice, dislikes the Dodo, and dislikes the Eaglet.
 - The 8 of Clubs dislikes Bill the Lizard, likes the Cheshire Cat, and likes the Frog Footman.
 - The 8 of Spades likes Alice, likes Bill the Lizard, and likes the Frog Footman.
 - The 9 of Hearts likes Bill the Lizard, likes the Cheshire Cat, and dislikes the Dodo.

Each of the cards will be satisfied if one of the prisoners they like is spared or if one of the prisoners they dislike is killed. What is the maximum possible number of cards that can be satisfied?

- (A) 1
- (B) 5
- (C) 2
- (D) 13
- (E) 12

Questions 2-3 refer to the graph below.

Each vertex represents a position in the Red Queen's castle. Edges connect positions that are within sight of each other.



- 2. The Red Queen is putting guards in her castle. She has one condition: every position in her castle (every vertex in the given graph) must be either occupied by a guard or within sight of a guard (directly connected to a vertex occupied by a guard). What is the smallest number of guards she would need in order to meet this condition?
 - (A) 18
 - (B) 11
 - (C) 16
 - (D) 6
 - (E) -4
- 3. The Red Queen has changed her mind and would now like a guard to be stationed at every position in her castle. But she knows that every guard is a member of a particular brigade, and if two guards from the same brigade are within sight of one another, they may be tempted to chat while on duty and neglect their surroundings. So her condition is that no two guards within sight of one another may be from the same brigade. What is the smallest number of brigades she can enlist to guard her castle while meeting this condition?
 - (A) 0
 - (B) 22
 - (C) 1
 - (D) 6
 - (E) 24

4. Alice wants to pick the prettiest mushrooms from the mushroom forest. However, she can only carry 20 kg in her knapsack. She has assigned each mushroom a prettiness score from 1 to 10. The table below lists the prettiness and weight of each type of mushroom.

Mushroom	Prettiness	Weight (kg)
Black morel	8	12
Candy cap	2	4
Chanterelle	3	12
Enokitake	4	4
Giant puffball	1	2
Maitake	6	12
Matsutake	4	13
Porcino	2	2
Portobello	9	13
Scotch bonnet	1	5
Shaggy parasol	7	7
Shimeji	10	14

For each type of mushroom, there is exactly one instance that Alice can reach and pick. What is the highest possible total prettiness for a set of mushrooms that she can fit in her knapsack?

- (A) 0
- (B) 10
- (C) 3
- (D) 11
- (E) 7

5. The Wonderland Croquet Club is trying to order croquet clubs for the upcoming season. Only one game can be played at a time, and all the people who play in a game must be friends with each other. Friendship is a symmetric and irreflexive, but not transitive, relation. The chart below specifies for every pair of members whether or not they are friends (• = yes, o = no).

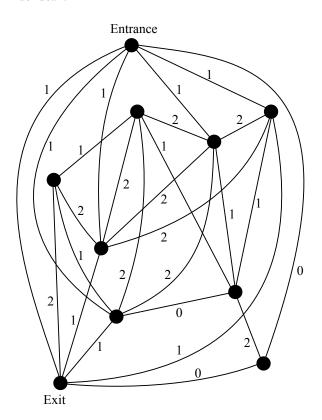
	Tortoise	Puppy	Mouse	Lory	Hatter	Gryphon	Eaglet	Duck	Duchess
Dodo	•	0	•	•	•	•	•	0	•
Duchess	•	0	•	•	•	0	•	0	
Duck	•	0	0	•	0	•	•		
Eaglet	•	•	•	•	•	0			
Gryphon	•	•	•	0	0				
Hatter	•	•	•	•					
Lory	•	•	•						
Mouse	•	0							
Puppy	0								

Assuming every player in a game needs exactly one club, how many clubs must they order so that the largest group of friends can all play?

- (A) 3
- (B) 0
- (C) 10
- (D) 2
- (E) 13

Questions 6-7 refer to the graph below.

Each vertex represents a location in the garden of the Queen of Hearts. Edges represent direct paths between locations, labeled with the distance between the locations. This graph is not to scale, nor is it even Euclidean.



- **6.** The Queen of Hearts is taking a leisurely stroll through her garden. What is the maximum possible length for a route she can take from the entrance to the exit without visiting any location twice?
 - (A) 7
 - (B) 2
 - (C) 4
 - (D) 6
 - (E) 18
- 7. Meanwhile, the White Rabbit is trying to find the Queen of Hearts! He knows she is somewhere in her garden, but he is not sure where. What is the minimum possible length for a route he can take to visit each of the locations exactly once and return to wherever he started?
 - (A) 5
 - (B) 8
 - (C) 4
 - (D) 2
 - (E) 11

- **8.** It's time for a tea party! The Hatter wants to stock up on all 16 varieties of tea on sale at the tea shop. But the shop only sells combo packs, as follows:
 - · Combo A: Genmaicha, Lapsang Souchong
 - Combo B: Hōjicha, Lu'An Melon Seed
 - Combo C: Darjeeling, Genmaicha
 - · Combo D: Hōjicha, Oolong
 - · Combo E: Earl Grey, Genmaicha, Hōjicha
 - Combo F: Hōjicha, Pu-erh
 - · Combo G: Hōjicha, Keemun
 - Combo H: Hōjicha, Russian Caravan
 - Combo I: Genmaicha, Jasmine
 - Combo J: Genmaicha, Gyokuro
 - · Combo K: Genmaicha, Hōjicha, Nilgiri
 - Combo L: Genmaicha, Lu'An Melon Seed
 - Combo M: Genmaicha, Irish Breakfast
 - Combo N: Genmaicha, Hōjicha, Oolong
 - Combo O: Genmaicha, Pu-erh
 - Combo P: Hōjicha, Lapsang Souchong
 - · Combo Q: Gunpowder, Hōjicha
 - Combo R: Genmaicha, Hōjicha
 - Combo S: Genmaicha, Hōjicha, Konacha
 - · Combo T: Darjeeling, Hōjicha

What is the minimum number of combo packs the Hatter must purchase in order to get all 16 varieties?

- (A) 8
- (B) 2
- (C) 9
- (D) 12
- (E) 24

9. The Duchess's cook is trying to fulfill the following orders. There is exactly one of each dish to make, and each dish takes some number of minutes to prepare and has a specified deadline.

Dish	Prep (min)	Deadline
Acharuli khachapuri	17	6:40
Bún bò Huế chay	17	7:40
Curry spaghetti	23	7:05
Kik alicha wot	25	7:25
Kimchi bokkeumbap	20	7:25
Monjayaki	6	7:10
Pannkakor med lingon	21	7:55
Patlıcan salatası	15	6:50
Poutine végétalienne	17	6:35
Tortilla española	19	7:10

It is now 6:00 and the cook can begin immediately. She can only work on one dish at a time, but may prepare dishes in any order. After finishing a dish, she can instantaneously begin preparing the next dish.

A dish is considered on time if the cook finishes preparing it at exactly the time of the deadline or earlier. What is the maximum number of dishes she can prepare on time?

- (A) 10
- (B) 5
- (C) 13
- (D) 20
- (E) 4