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STANDARD NINE

TERM - III

VOLUME 2

MATHEMATICS

Untouchability is Inhuman and a Crime

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SYMBOLS

$=$	equal to
$\neq$	not equal to
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\approx$	equivalent to
$\cup$	union
$\cap$	intersection
$\mathbb{U}$	universal Set
$\in$	belongs to
$\notin$	does not belong to
$\subset$	proper subset of
$\subseteq$	subset of or is contained in
$\not\subset$	not a proper subset of
$\not\subseteq$	not a subset of or is not contained in
A' (or) A^c	complement of A
$\varnothing$ (or) $\{\ \}$	empty set or null set or void set
$n(A)$	number of elements in the set A
$P(A)$	power set of A
$\sum$	summation
$\|\|^{ly}$	similarly
Δ	symmetric difference
$\mathbb{N}$	natural numbers
$\mathbb{W}$	whole numbers
$\mathbb{Z}$	integers
$\mathbb{R}$	real numbers
$\triangle$	triangle
$\angle$	angle
$\perp$	perpendicular to
$\|$	parallel to
$\Longrightarrow$	implies
$\therefore$	therefore
$\because$	since (or) because
$\lvert\ \rvert$	absolute value
$\simeq$	approximately equal to
$\mid$ (or) :	such that
$\equiv$ (or) $\cong$	congruent
$\equiv$	identically equal to
π	pi
$\pm$	plus or minus

CONTENTS

FYIDKS

E - book

FYNLYI

Assessment

B566_9_MAT_EM_T3

DIGI links

QR GUIDE

Lets use the QR code in the text books ! How ?

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- Open the QR code scanner application
- Once the scanner button in the application is clicked, camera opens and then bring it closer to the QR code in the text book.
- Once the camera detects the QR code, a url appears in the screen.Click the url and goto the content page.

Captions used in this Textbook

எண்ணென்ப ஏனை எழுத்தென்ப இவ்விரண்டும்
கண்ணென்ப வாழும் உயிர்க்கு – குறள் 392

Numbers and letters, they are known as
eyes to humans. - Kural 392

Learning Outcomes

To transform the classroom processes into learning centric with a set of bench marks

Note

To provide additional inputs for students in the content

Activity / Project

To encourage students to involve in activities to learn mathematics

ICT Corner

To encourage learner's understanding of content through application of technology

Thinking Corner

To kindle the inquisitiveness of students in learning mathematics. To make the students to have a diverse thinking

Points to Remember

To recall the points learnt in the topic

Multiple Choice Questions

To provide additional assessment items on the content

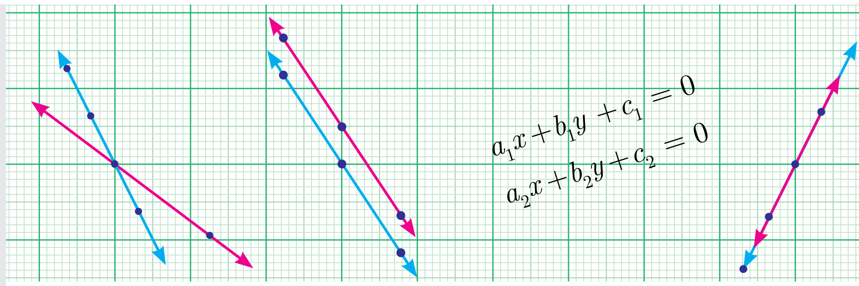
Progress Check

Self evaluation of the learner's progress

Exercise

To evaluate the learners' in understanding the content

"The essence of mathematics is not to make simple things complicated but to make complicated things simple" -S. Gudder



1 ALGEBRA

In real life, I assure you, there is no such things as Algebra.
- Fran Lebowitz

Diophantus

Diophantus of Alexandria an Alexandrian Hellenistic mathematician who lived for about 84 years, was born between A.D(C.E) 201 and A.D(C.E) 215. Diophantus was the author of a series of books called Arithmetica. His texts deal with solving algebraic equations. He is also called as "the father of algebra".

Learning Outcomes

- To recall linear equation in one variable.
- To identify and understand linear equation in two variables.
- To know the slope of a line and its intercepts.
- Able to draw graph for a given linear equation.
- To solve simultaneous linear equations in two variables by
  - Graphical method
  - Algebraic method
    - ❖ Substitution method
    - ❖ Elimination method
    - ❖ Cross multiplication method
- To understand consistency and inconsistency of linear equations in two variables.

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1.1 Introduction

Mathematics may be thought of as a systematic study of relationships. Here we are set to learn what are known as linear relationships. These relationships can be neatly organized in three different ways: preparing a table, forming an equation and sketching a graph.

Study the following information:

A call-taxi company charges a boarding fee of ₹20 (just to get into a cab) and then ₹2 for each kilometre travelled. Here is a table to show the trend of the charges levied when a person engages the cab.

Representing data with a table

Distance travelled (km)	0	1	2	3	4	5	…	10	…	20	…
Cost (₹)	20	22	24	26	28	30	…	?	…	?	…

Do you see how the charges are levied? Can you guess the charges when one travels 10 km and 20 km in this cab?

This kind of tabulation is one way of expressing the relationship between the distance and the cost.

Representing data with an equation

Let us try to put these details in the form of an equation for the cost ₹y, corresponding to a travel distance of x km.

Boarding charges (at a distance of 0 km) = ₹ 20

Travel cost for 1 km = ₹ 22 = ₹ 20 + 2×1

Travel cost for 2 km = ₹ 24 = ₹ 20 + 2×2

Travel cost for 3 km = ₹ 26 = ₹ 20 + 2×3

⋮ ⋮

Travel cost for x km (denoted by ₹y) = ? = ₹ 20 + $2\times x$

$$y = 2x + 20$$

Thus $y = 2x + 20$ represents the relationship between the distance travelled and the cost involved.

You can verify if the equation is a correct representation, by going through a backward 'check'.

If the distance travelled is 4 km, then $x = 4$ and when you substitute in the equation $y = 2x + 20$, you get the cost to be $2(4) + 20 = 28$ which is true as found in the initial table. (Verify a few more entries).

Representing data with a graph

A graph, as a visual aid, helps to understand the things clearly. In the case of our problem we can easily draw a graph using the ordered pairs listed with the help of the above mentioned table.

Notice how the axes are labelled. x represent the number of kilometres travelled and y represent the corresponding cost in rupees. Different scales are used on the two axes. Notice that the points plotted in this figure all lie on a straight line as shown in Fig. 1.1

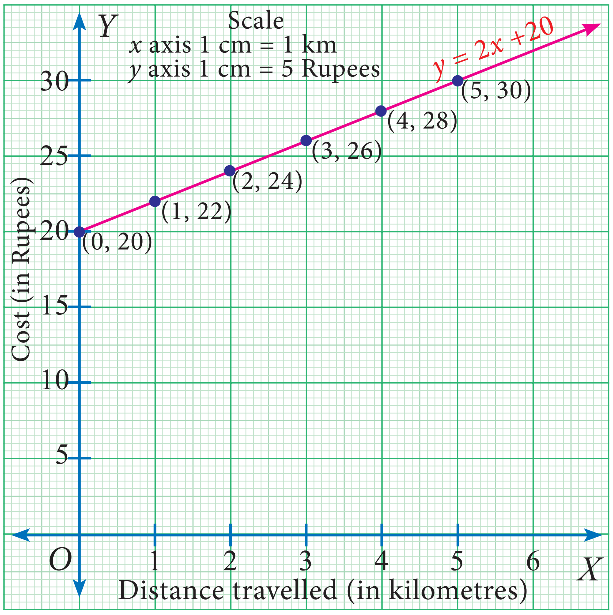


Fig. 1.1

Note that each of these methods represents the same relationship. The table gives us data numerically arranged, the equation generalized the way things behave and the graph help us to visualize the same.

Progress Check

Use the given situation to prepare a table, form an equation and sketch a graph. Kavya hires a cab to go somewhere, call taxi company charges ₹5 for each kilometres travelled. Find the charge for 10 kilometres.

1.2 Linear Equation in One Variable

Equations
$4x - 5 = 3$,
$2y + 1 = -7$,
$5a = 19$,
$x + 3 = -x$

What is an equation?

It is a pair of expressions set equal to each other.

A few samples of equations are here in the box.

What is special about these given equations?

(i) There is only one unknown (called variable) like x, y, a, etc.

(ii) The exponent on each variable is 1.

Thus an equation of the form $ax+b=0$ (where a,b are real numbers and $a\neq0$) is called a linear equation in one variable x.

For example $x = 2$, $3y - 5 = 0$, $2m = -3$ and $5k - 2 = 2k + 2$

The above equations are called 'first degree' / 'linear' equations. Why are they said to be 'linear' ? We shall see now.

Consider the equation $4x - 5 = 3$, for example. We will simplify it, keeping the variable x on LHS and taking the numerical values on the RHS.

$$\text{Given} \qquad 4x - 5 = 3$$

$$\text{Adding } +5 \text{ on both sides} \quad 4x - 5 + 5 = 3 + 5$$

$$4x + 0 = 8$$

$$4x = 8$$

$$\text{Divide both sides by } 4 \qquad \frac{4x}{4} = \frac{8}{4}$$

This simplified form is shown in the graph grid below (Fig. 1.2). From the graph, one can infer that, whatever may be the value of y, the x value is 2. That is, to draw the graph, some of the ordered pairs we would be interested are (2, −3), (2, −2), (2,−1), (2,0), (2,1), (2,2), (2,3) etc. When we plot them and join them, we find the graph to be a line which is parallel to Y axis. This is a linear graph representing the given equation. Hence the equation itself is said to be linear.

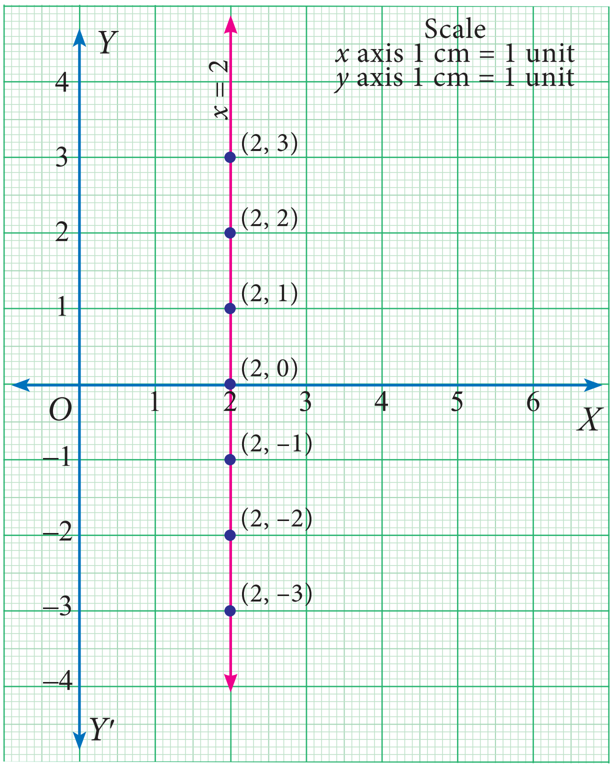


Fig. 1.2

Now similarly investigate the graphs of other equations given in the box above and find out if they are eligible to be called 'linear'. Can you think of an equation that is not linear?

Solution of an equation

Consider the equation $\frac{1}{2}x + 3 = -x$. Suppose we put $x = -2$, then

$$\text{LHS} = \frac{1}{2}\times(-2)+3=-1+3=+2$$

and also RHS $= -x = -(-2) = +2$.

Thus the value −2 satisfies the given equation and hence is a solution. (Does this equation have any other solution?)

> The solution of an equation is the set of all values that, when substituted for unknowns, make the equation true.

Progress Check

Verify whether $x = \frac{4}{3}$ is a solution of the equation $1\frac{1}{4}x + \frac{1}{2} = 2x - \frac{1}{2}$.

Exercise 1.1

1. Give any two examples for linear equations in one variable.

2. Check whether $\frac{1}{4}$ is a solution of the equation $3(x + 1) = 3(5-x) - 2(5 + x)$.

3. Solve the following linear equations

   (i) $\frac{2(x+1)}{3} = \frac{3(x-2)}{5}$ (ii) $\frac{2}{x+1} = 4 - \frac{x}{x+1}$, $(x \neq -1)$

4. Draw the graph for the following linear equations

   (i) $y = 4$ (ii) $x = -2$ (iii) $2x - 4 = 0$

   (iv) $6 + 2y = 0$ (v) $9 - 3x = 0$

1.3 Linear Equation in Two Variables

A linear equation in two variables is of the form $ax + by + c = 0$ where a, b and c are real numbers, both a and b are not zero (The two variables are denoted here by x and y and c is a constant).

Examples

Linear equation in two variables	Not a linear equation in two variables.
$2x + y = 4$	$xy + 2x = 5$ (Why?)
$-5x + \frac{1}{2} = y$	$\sqrt{x} + \sqrt{y} = 25$ (Why?)
$5x = 35y$	$x(x+1) = y$ (Why?)

If an equation has two variables each of which is in first degree such that the variables are not multiplied with each other, then it is a linear equation in two variables (If the degree of an equation in two variables is 1, then it is called a linear equation in two variables).

An understanding of linear equation in two variables will be easy if it is done along with a geometrical visualization (through graphs). We will make use of this resource.

Why do we classify, for example, the equation $2x + y = 4$ as a linear equation? You are right; because its graph will be a line. Shall we check it up?

We try to draw its graph. To draw the graph of $2x + y = 4$, we need some points on the line so that we can join them. (These are the ordered pairs satisfying the equation).

To prepare table giving ordered pairs for $2x + y = 4$. It is better, to take it as $y = 4 - 2x$. (Why? How?)

When $x = -4$, $y = 4 - 2(-4) = 4 + 8 = 12$

When $x = -2$, $y = 4 - 2(-2) = 4 + 4 = 8$

When $x = 0$, $y = 4 - 2(0) = 4 + 0 = 4$

When $x = +1$, $y = 4 - 2(+1) = 4 - 2 = 2$

When $x = +3$, $y = 4 - 2(+3) = 4 - 6 = -2$

Thus the values are tabulated as follows:

x-value	-4	-2	0	1	3
y-value	12	8	4	2	-2

(To fix a line, do we need so many points? It is enough if we have two and probably one more for verification.)

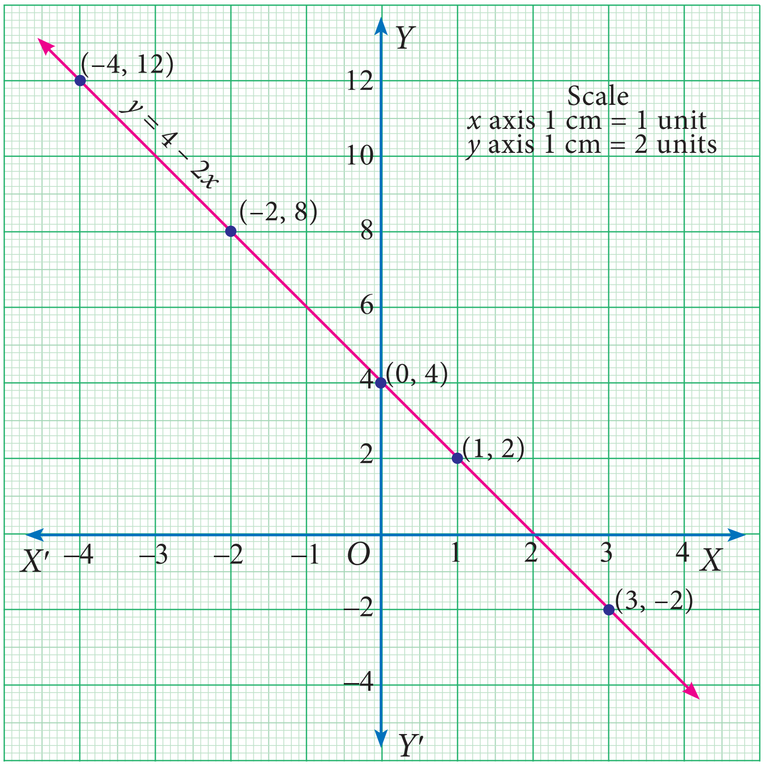


Fig. 1.3

When you plot the points $(-4,12)$, $(-2,8)$, $(0,4)$, $(1,2)$ and $(3,-2)$, you find that they all lie on a line.

This clearly shows that the equation $2x + y = 4$ represents a line (and hence said to be linear).

All the points on the line satisfy this equation and hence the ordered pairs of all the points on the line are the solutions of the equation.

1.4 Slope of a Line

We saw that every first degree equation in two variables is a line. So we need to know more about the line that represents a linear equation. In particular we need o know about an important concept known as slope of the line. The slope of a line (also called the gradient of a line) is a number that describes how "steep" it is. It is usually denoted by the letter m.

The slope of a non-vertical line is defined as follows:

$$\text{slope } (m) = \frac{rise}{run} = \frac{vertical\ change}{horizontal\ change}$$

In the figure,

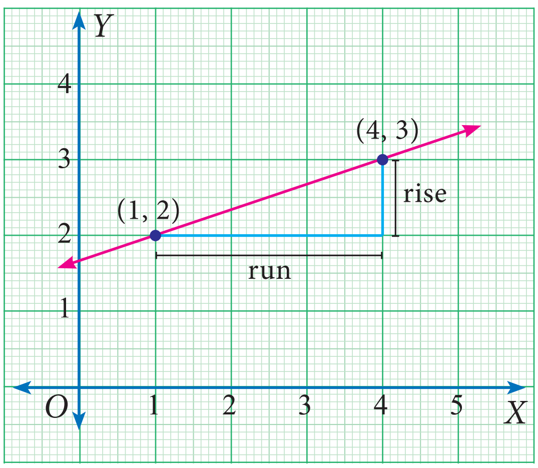


Fig. 1.4

$$\text{Slope } (m) = \frac{rise}{run} = \frac{3-2}{4-1} = \frac{1}{3}$$

In general,

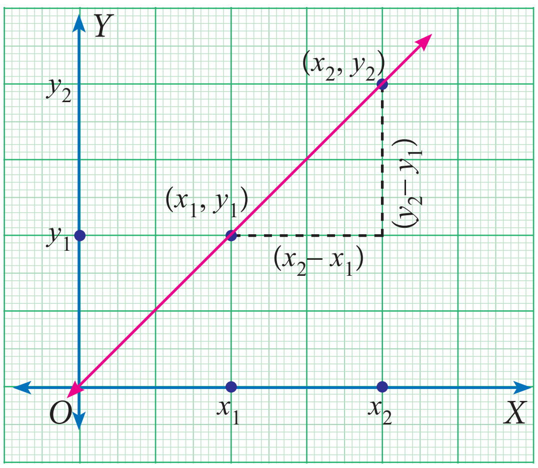


Fig. 1.5

The slope m of (a non-vertical line) passing through the points (x_1,y_1) and (x_2,y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{change\ in\ y}{change\ in\ x} \quad (x_2 \neq x_1)$$

Example 1.1 Find the slopes of all the lines from the adjacent figure,

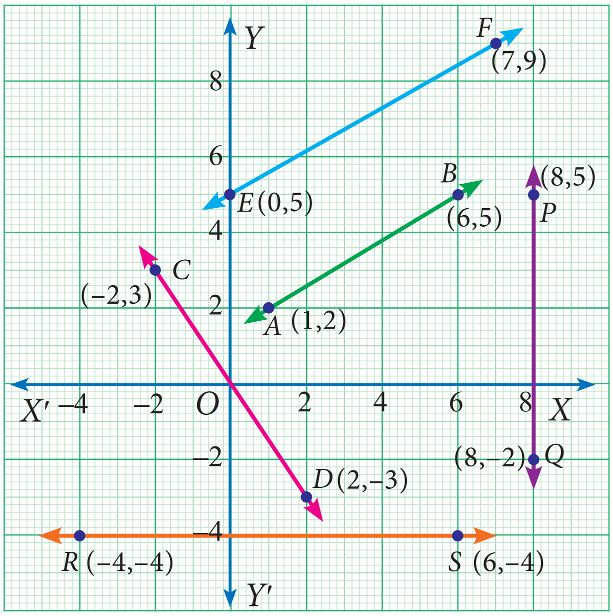


Fig. 1.6

Slope of AB $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{3}{5}$

Slope of CD $= \frac{change\ in\ y}{change\ in\ x} = -\frac{3}{2}$

Slope of EF $= \frac{4}{7}$

Slope of PQ $=$ Undefined

Slope of RS $= 0$.

1.5 Intercept of a Line

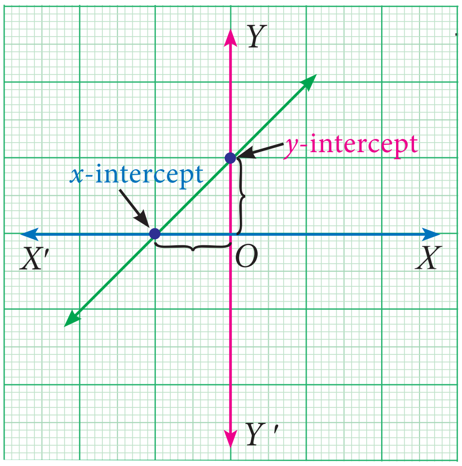


Fig. 1.7

The intercept of a line is the distance from the origin to the point at which it crosses either x or y axis (If no axis is specifically indicated, the y-axis is assumed). The y- intercept is commonly labelled by the letter c.

If 'c' is where the line crosses the y-axis (that is, if c were the y-intercept), and m is the slope of the line, then it can be given as an equation $y = mx + c$ (This is formally proved in the higher classes).

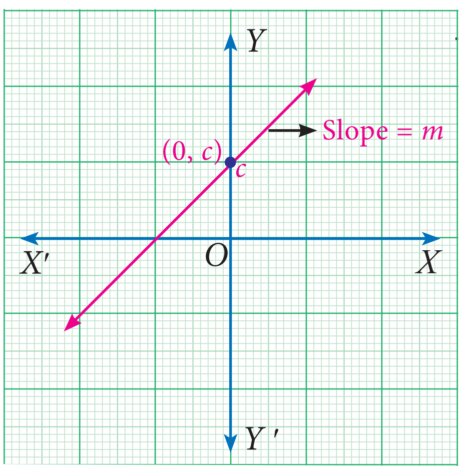


Fig. 1.8

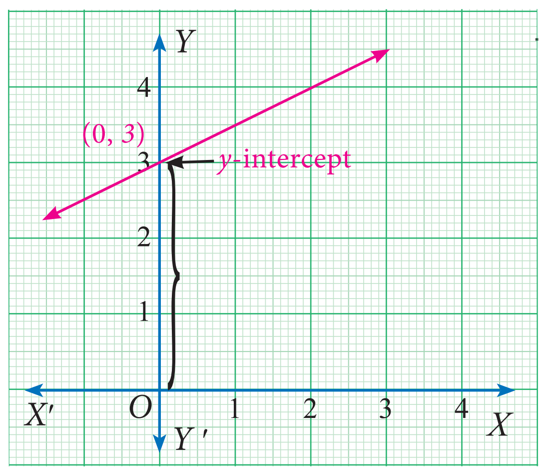


Fig. 1.9

Thus, for example, for the line in the Fig. 1.9 on the right side, the slope m is $\frac{1}{2}$ (how?) and the y-intercept c is 3; therefore the equation of the line is $y = \frac{1}{2}x + 3$.

Example 1.2

(Computing slope made easier!) Find the slope and y-intercept of the line given by the equation $2y - 3x = 12$.

Solution

The given equation is $2y - 3x = 12$

$$\Rightarrow \quad 2y = +3x + 12$$

$$\Rightarrow \quad \frac{2y}{2} = \frac{3x+12}{2}$$

$$\Rightarrow \quad y = \frac{3x}{2} + \frac{12}{2}$$

$$\Rightarrow \quad y = \frac{3}{2}x + 6$$

Compare with, $y = mx + c$

Slope $m = \frac{3}{2}$, y-intercept $c = 6$

Example 1.3 (Graphing made easier!) Draw the graph of the line given by the equation $y = 4x - 3$.

Solution

> **Note**
> y-intercept (c) may be obtained by comparing $y = 4x - 3$ with $y = mx + c$.

We have already come across one method: forming a table of values, listing and plotting ordered pairs and joining the points.

But, to fix a line, after all, how many points do we need? Just two! These can easily be obtained when a line is given in the form y = $mx + c$.

The given line $y = 4x - 3$

put $x = 0$ to get y-intercept

$$y = 4(0) - 3$$

$$y = -3$$

point is $(0, -3)$ and

$$y\text{-intercept} = -3$$

put $y = 0$ to get x-intercept

$$0 = 4x - 3$$

$$3 = 4x$$

$$\frac{3}{4} = x$$

point is $\left(\frac{3}{4}, 0\right)$ and

$$x\text{-intercept} = \frac{3}{4}$$

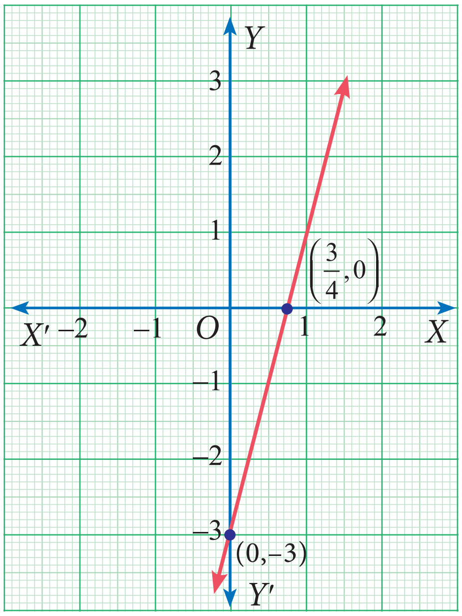


Fig. 1.10

The graph may be drawn through two points $(0, -3)$ and $\left(\frac{3}{4}, 0\right)$.

Progress Check

1. Find the slope of the lines (i) $3x + 5y + 4 = 0$, (ii) $x + y = 0$, (iii) $y = 7$.
2. Draw the graph of $2x + 3y = 6$, by transforming it to the form $y = mx + c$.
3. Find x-intercept and y-intercept of : (i) $y = 5x - 15$ (ii) $y = 7x$ (iii) $x = 5$

Example 1.4

Draw the graph for the following

(i) $y = 3x - 1$ (ii) $y = \left(\frac{2}{3}\right)x + 3$

Solution

(i) Let us prepare a table to find the ordered pairs of points for the line $y = 3x - 1$.

We shall assume any value for x, for our convenience let us take -1, 0 and 1.

When $x = -1$, $y = 3(-1) - 1 = -4$

When $x = 0$, $y = 3(0) - 1 = -1$

When $x = 1$, $y = 3(1) - 1 = 2$

x	-1	0	1
y	-4	-1	2

The points (x,y) to be plotted :

$(-1, -4)$, $(0, -1)$ and $(1, 2)$.

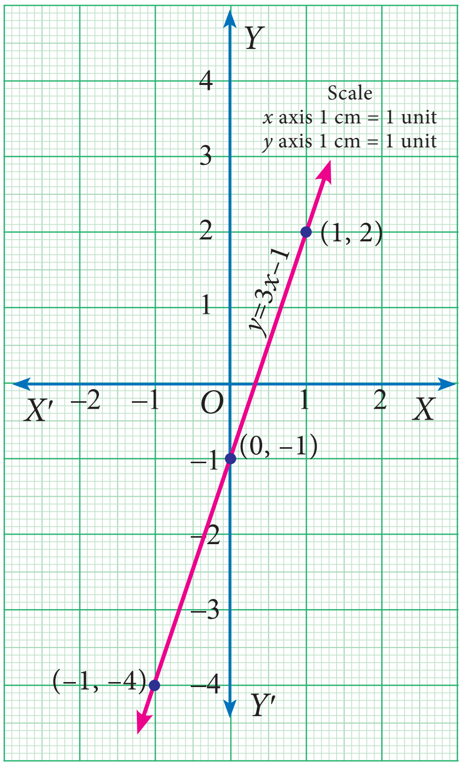


Fig. 1.11

(ii) Let us prepare a table to find the ordered pairs of points for the line $y = \left(\frac{2}{3}\right)x + 3$.

Let us assume -3, 0, 3 as x values. (why?)

When $x = -3$, $y = \frac{2}{3}(-3) + 3 = 1$

When $x = 0$, $y = \frac{2}{3}(0) + 3 = 3$

When $x = 3$, $y = \frac{2}{3}(3) + 3 = 5$

x	-3	0	3
y	1	3	5

The points (x,y) to be plotted :

$(-3,1)$, $(0,3)$ and $(3,5)$.

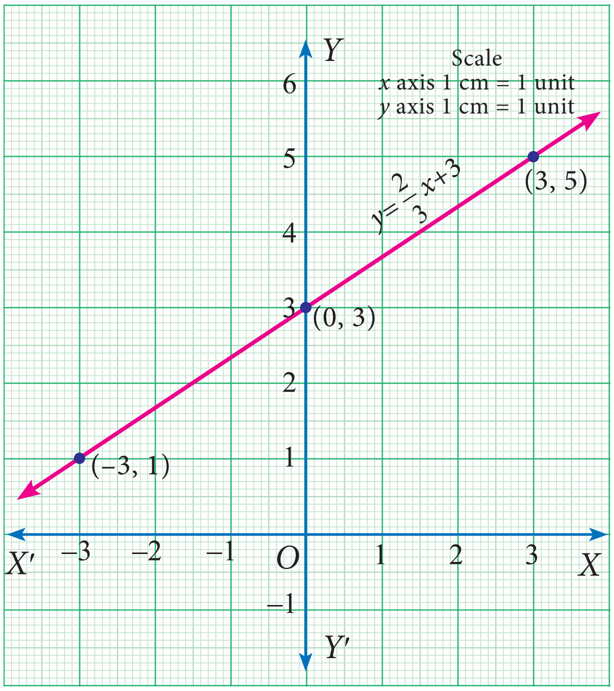


Fig. 1.12

Exercise 1.2

1. Draw the graph for the following

   (i) $y = 2x$ (ii) $y = 4x - 1$ (iii) $y = \left(\frac{3}{2}\right)x + 3$ (iv) $3x + 2y = 14$

1.6 Simultaneous Linear Equations

With sufficient background of graphing an equation, now we are set to study about system of equations, particularly pairs of simultaneous equations.

What are simultaneous linear equations? These consists of two or more linear equations with the same variables.

Why do we need them? A single equation like $2x+y=10$ has an unlimited number of solutions. The points (1,8), (2,6), (3,4) and many more lie on the graph of the equation, which means these are some of its endless list of solutions. To be able to solve an equation like this, another equation needs to be used alongside it; then it is possible to find a single ordered pair that solves both equations at the same time.

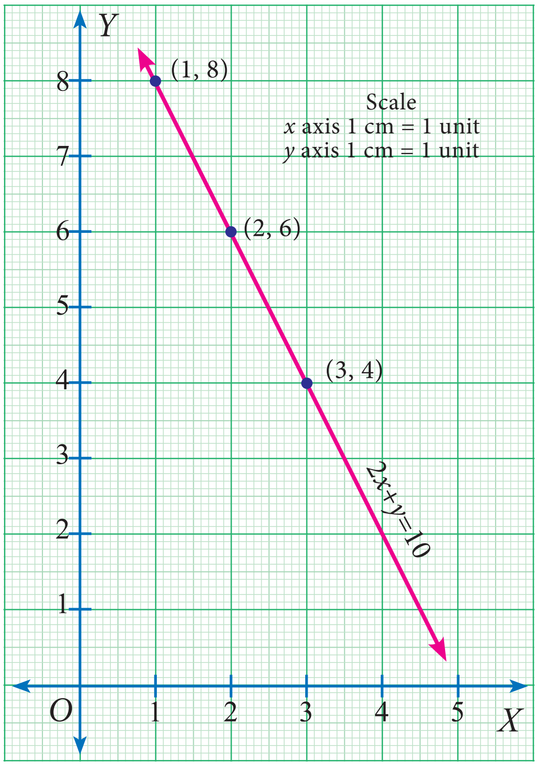


Fig. 1.13

The equations we consider together in such settings make a meaningful situation and are known as simultaneous linear equations.

Real life Situation to understand the simultaneous linear equations

Consider the situation, Anitha bought two erasers and a pencil for ₹10. She does not know the individual cost of each. We shall form an equation by considering the cost of eraser as 'x' and that of pencil as 'y'.

That is $2x + y = 10$ ... (1)

Now, Anitha wants to know the individual cost of an eraser and a pencil. She tries to solve the first equation, assuming various values of x and y.

$2(1)+8=10$

$2(1.5)+7=10$

$2(2)+6=10$

$2(2.5)+5=10$

$2(3)+4=10$

$\vdots \quad \vdots$

G2T8JA

Points to be plotted :

x	1	1.5	2	2.5	3	...
y	8	7	6	5	4	...

She gets infinite number of answers. So she tries to find the cost with the second equation.

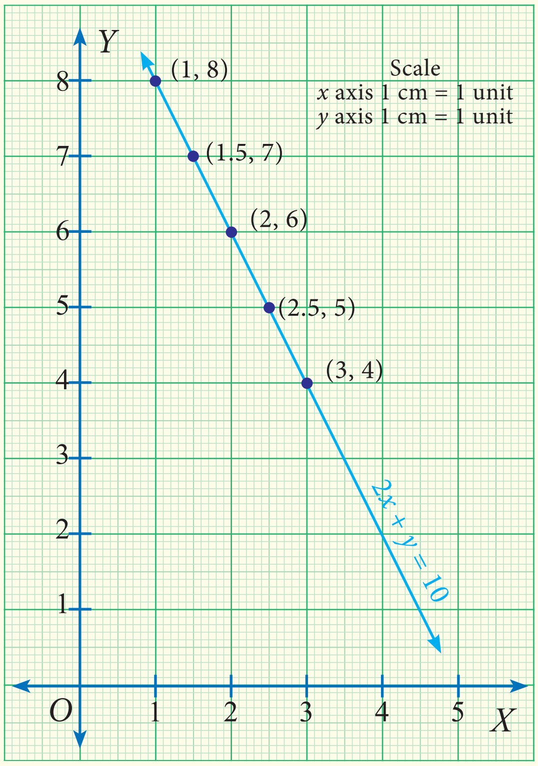


Fig. 1.14

Again, Anitha needs some more pencils and erasers. This time, she bought 3 erasers and 4 pencils and the shopkeeper received ₹30 as the total cost from her. We shall form an equation like the previous one.

The equation is $3x + 4y = 30$ ...(2)

Even then she arrives at an infinite number of answers.

3 × cost of eraser + 4 × cost of pencil = 30

$3(2)+4(6) = 30$

$3(4)+4(4.5) = 30$

$3(6)+4(3) = 30$

$3(8)+4(1.5) = 30$

⋮ ⋮

Points to be plotted :

x	2	4	6	8	...
y	6	4.5	3	1.5	...

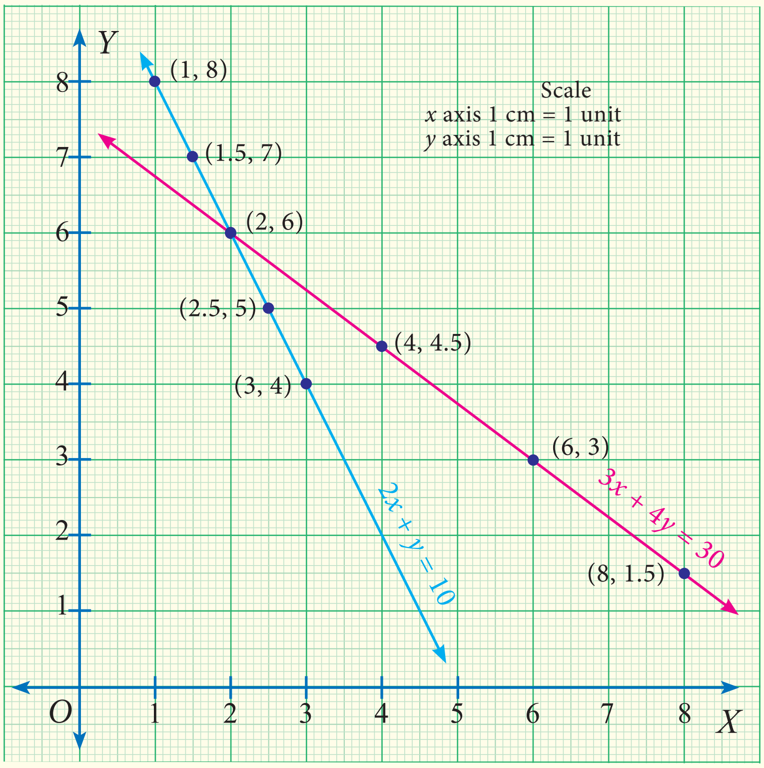


Fig. 1.15

While discussing this with her teacher, the teacher suggested that she can get a unique answer if she solves both the equations together.

By solving equations (1) and (2) we have the cost of an eraser as ₹2 and cost of a pencil as ₹6. It can be visualised in the graph.

The equations we consider together in such settings make a meaningful situation and are known as simultaneous linear equations.

Thus a system of linear equations consists of two or more linear equations with the same variables. Then such equations are called Simultaneous linear equations or System of linear equations or a Pair of linear equations.

Example 1.5 Check whether (5, −1) is a solution of the simultaneous equations $x - 2y = 7$ and $2x + 3y = 7$.

Solution

Given $x - 2y = 7$ ...(1)

$2x + 3y = 7$ ...(2)

When $x = 5,\ y = -1$ we get

From (1) $x - 2y = 5 - 2(-1) = 5 + 2 = 7$ which is RHS of (1)

From (2) $2x + 3y = 2(5) + 3(-1) = 10 - 3 = 7$ which is RHS of (2)

Thus the values $x = 5,\ y = -1$ satisfy both (1) and (2) simultaneously. Therefore (5,−1) is a solution of the given equations.

Progress Check

Examine if (3,3) will be a solution for the simultaneous linear equations $2x - 5y - 2 = 0$ and $x + y - 6 = 0$ by drawing a graph.

1.6.1 Methods of solving simultaneous linear equations

There are different methods to find the solution of a pair of simultaneous linear equations. It can be broadly classified as geometric way and algebraic ways.

Geometric way	Algebraic ways
1. Graphical method	1. Substitution method
	2. Elimination method
	3. Cross multiplication method

Solving by Graphical Method

Already we have seen graphical representation of linear equation in two variables. Here we shall learn, how we are graphically representing a pair of linear equations in two variables and find the solution of simultaneous linear equations.

Example 1.6 Use graphical method to solve the following system of equations $x + y = 5;\ 2x - y = 4$.

Solution

Given $\quad x + y = 5 \qquad \ldots(1)$

$\qquad 2x - y = 4 \qquad \ldots(2)$

To draw the graph (1) is very easy. We can find the x and y intercepts and thus two of the points on the line (1).

When $x = 0$, (1) gives $y = 5$.

Thus $A(0,5)$ is a point on the line.

When $y = 0$, (1) gives $x = 5$.

Thus $B(5,0)$ is another point on the line.

Plot A and B ; join them to produce the line (1).

To draw the graph of (2), we can adopt the same procedure.

When $x = 0$, (2) gives $y = -4$.

Thus P(0,–4) is a point on the line.

When $y = 0$, (2) gives $x = 2$.

Thus $Q(2,0)$ is another point on the line.

Plot P and Q ; join them to produce the line (2).

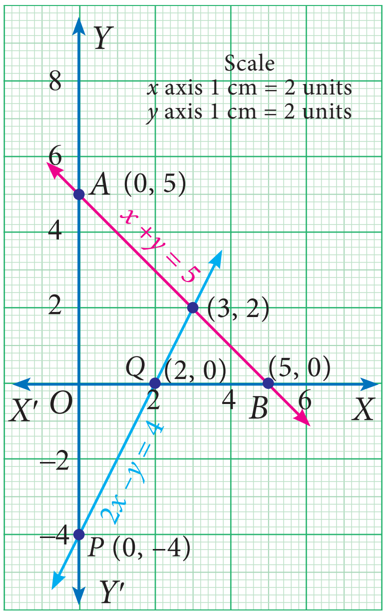


Fig. 1.16

The point of intersection (3, 2) of lines (1) and (2) is a solution.

The solution is the point that is common to both the lines. Here we find it to be (3,2). We can give the solution as $x = 3$ and $y = 2$.

Note

It is always good to verify if the answer obtained is correct and satisfies both the given equations.

Example 1.7 Use graphical method to solve the following system of equations $3x + 2y = 6;\ 6x + 4y = 8$

Solution

Let us form table of values for each line and then fix the ordered pairs to be plotted.

Graph of $3x + 2y = 6$

x	-2	0	2
y	6	3	0

Points to be plotted :

(–2,6), (0,3), (2,0)

Graph of $6x + 4y = 8$

x	-2	0	2
y	5	2	-1

Points to be plotted :

(–2,5), (0,2), (2,–1)

When we draw the graphs of these two equations, we find that they are parallel and they fail to meet to give a point of intersection. As a result there is no ordered pair that can be common to both the equations. In this case there is no solution to the system.

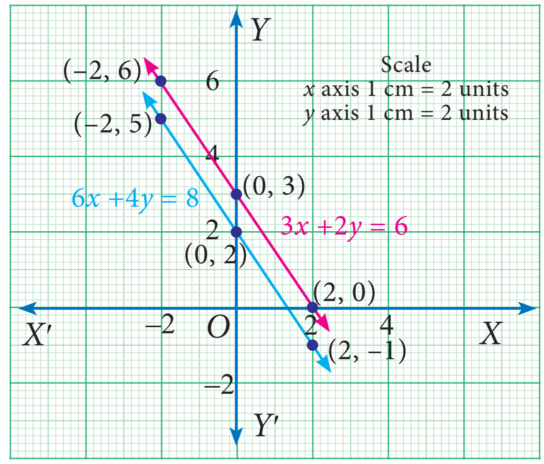


Fig. 1.17

This could have been easily guessed even without drawing the graphs. Writing the two equations in the form $y = mx + c$.

Note that the slopes are equal

Therefore the lines are parallel and will not meet at any point and hence no solution exists.

Example 1.8

Use graphical method to solve the following system of equations $y = 2x + 1$; $-4x + 2y = 2$

Solution

Let us form table of values for each line and then fix the ordered pairs to be plotted.

Graph of $y = 2x + 1$

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
1	1	1	1	1	1
$y = 2x+1$	-3	-1	1	3	5

Points to be plotted :

$(-2, -3), (-1, -1), (0,1), (1,3), (2, 5)$

Graph of $-4x + 2y = 2$

$$2y = 4x + 2$$

$$y = 2x + 1$$

x	-2	-1	0	1	2
$2x$	-4	-2	0	2	4
1	1	1	1	1	1
$y = 2x+1$	-3	-1	1	3	5

Points to be plotted :

$(-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5)$

Here both the equations are identical; they were only represented in different forms. Since they are identical, their solutions are same. All the points on one line are also on the other!

This means we have an infinite number of solutions which are the ordered pairs of all the points on the line.

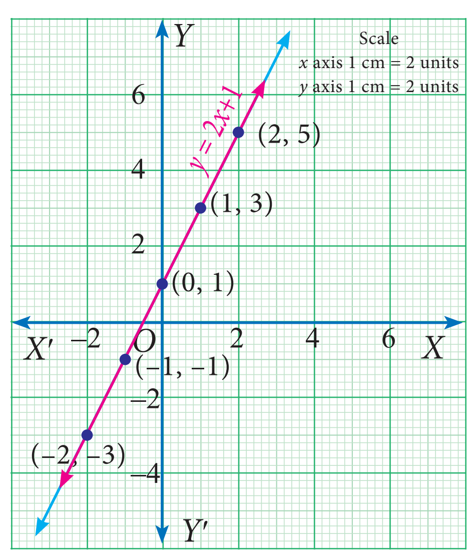


Fig. 1.18

Example 1.9 The perimeter of a rectangle is 36 metres and the length is 2 metres more than three times the width. Find the dimension of rectangle by using the method of graph.

Solution

Let us form equations for the given statement.

Let us consider l and b as the length and breadth of the rectangle respectively.

Now let us frame the equation for the first statement

Perimeter of rectangle = 36

$$2(l+b)=36$$

$$l+b=\frac{36}{2}$$

$$l=18-b \qquad \text{... (1)}$$

b	2	4	5	8
18	18	18	18	18
−b	−2	−4	−5	−8
l = 18−b	16	14	13	10

Points: (2,16), (4,14), (5,13), (8,10)

The second statement states that the length is 2 metres more than three times the width which is a straight line written as $l=3b+2$ ... (2)

Now we shall form table for the above equation (2).

b	2	4	5	8
$3b$	6	12	15	24
2	2	2	2	2
$l=3b+2$	8	14	17	26

Points: (2,8), (4,14), (5,17), (8,26)

The solution is the point that is common to both the lines. Here we find it to be (4,14). We can give the solution to be b = 4, l = 14.

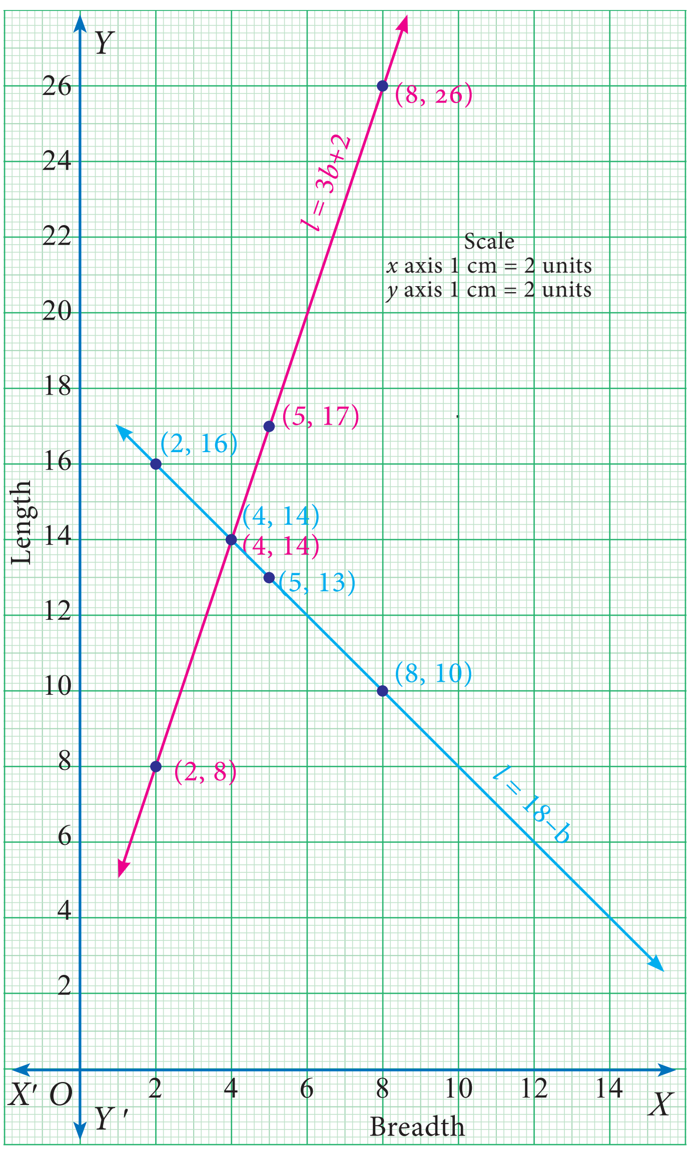


Fig. 1.19

Verification :

$$\begin{aligned} 2(l+b) &= 36 \quad ...(1) \\ 2(14+4) &= 36 \\ 2\times 18 &= 36 \\ 36 &= 36 \quad \text{true} \end{aligned} \qquad\qquad \begin{aligned} l &= 3b+2 \quad ...(2) \\ 14 &= 3(4)+2 \\ 14 &= 12+2 \\ 14 &= 14 \quad \text{true} \end{aligned}$$

Exercise 1.3

1. Solve graphically

   (i) $x+y=7;\ x-y=3$ (ii) $3x+2y=4;\ 9x+6y-12=0$

   (iii) $\frac{x}{2}+\frac{y}{4}=1;\ \frac{x}{2}+\frac{y}{4}=2$ (iv) $x-y=0;\ \ y+3=0$

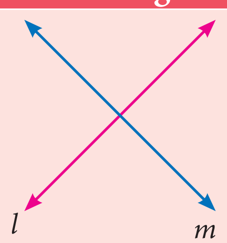
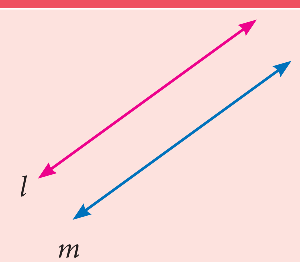
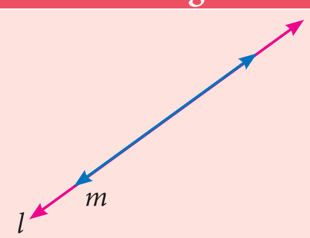
   (v) $x-2y=1;\ x-2y+5=0$ (vi) $2x+y=4;\ \ 4x+2y=8$

   (vii) $y=2x+1;\ y+3x-6=0$ (viii) x = -3; y = 3

2. Two cars are 100 miles apart. If they drive towards each other they will meet in 1 hour. If they drive in the same direction they will meet in 2 hours. Find their speed by using graphical method.

Some special terminology

We found that the graphs of equations within a system tell us how many solutions are there for that system. Here is a visual summary.

Intersecting lines	Parallel lines	Coinciding lines
		
One single solution	*No solution*	*Infinite number of solutions*

- When a system of linear equation has one solution (the graphs of the equations intersect once), the system is said to be a consistent system.

- When a system of linear equation has no solution (the graphs of the equations don't intersect at all), the system is said to be an inconsistent system.

- When a system of linear equation has infinitely many solutions, the lines are the same (the graph of lines are identical at all points), the system is consistent.

Solving by Substitution Method

In this method we substitute the value of one variable, by expressing it in terms of the other variable to reduce the given equation of two variables into equation of one variable (in order to solve the pair of linear equations). Since we are substituting the value of one variable in terms of the other variable, this method is called substitution method.

The procedure may be put shortly as follows

Step 1: From any of the given two equations, find the value of one variable in terms of the other.

Step 2: Substitute the value of the variable, obtained in step 1 in the other equation and solve it.

Step 3: Substitute the value of the variable obtained in step 2 in the result of step 1 and get the value of the remaining unknown variable.

Example 1.10

Solve the system of linear equations $x+3y=16$ and $2x-y=4$ by substitution method.

Solution

Given $x+3y=16$ ... (1)

$2x-y=4$ ... (2)

Step 1	Step 2	Step 3	Solution
From equation (2) $2x-y=4$ $-y=4-2x$ $y=2x-4$ ...(3)	Substitute (3) in (1) $x+3y=16$ $x+3(2x-4)=16$ $x+6x-12=16$ $7x=28$ $x=4$	Substitute $x=4$ in (3) $y=2x-4$ $y=2(4)-4$ $y=4$	$x=4$ and $y=4$

Progress Check

(i) In step 1, instead of taking equation (2), can we take equation (1) and express x in term of y ? Is there any other way of expressing one variable in terms of the other ? Discuss.

(ii) Verify by substitution that the solution obtained for the above problem is correct.

(iii) We got the solution (4, 4) for the above problem. Are we sure that there is no other solution? Draw the graphs for the equations given and try to verify that there is only one solution.

Example 1.11 The sum of the digits of a given two digit number is 5. If the digits are reversed, the new number is reduced by 27. Find the given number.

Solution

Let x be the digit at ten's place and y be the digit at unit place.

Given that $x+y=5$ …… (1)

	Tens	Ones	Value
Given Number	x	y	$10x+y$
New Number (after reversal)	y	x	$10y+x$

Given, Original number – reversing number $=27$

$$(10x+y)-(10y+x)=27$$

$$10x-x+y-10y=27$$

$$9x-9y=27$$

$$\Rightarrow \qquad x-y=3 \qquad \text{... (2)}$$

Also from (1), $y = 5 - x$ ... (3)

Substitute (3) in (2) to get $x-(5-x)=3$

$$x-5+x=3$$

$$2x=8$$

$$x=4$$

Substituting $x=4$ in (3), we get $y=5-x \ =5-4$

$$y=1$$

Thus, $10x+y=10\times4+1=40+1 \ = 41.$

Therefore, the given two-digit number is 41.

Verification :

sum of the digits = 5

$x + y = 5$

$4 + 1 = 5$

$5 = 5$ true

Original number – reversed number = 27

$41 - 14 = 27$

$27 = 27$ true

Exercise 1.4

1. Solve, using the method of substitution

(i) $2x-3y=7;\ 5x+y=9$

(ii) $1.5x+0.1y=6.2;\ 3x-0.4y=11.2$

(iii) $10\%\ of\ x+20\%\ of\ y=24;\ 3x-y=20$

(iv) $\sqrt{2}\,x-\sqrt{3}\,y=1;\ \sqrt{3}\,x-\sqrt{8}\,y=0$

(v) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$; $\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$ (Hint : $Put\ \frac{1}{\sqrt{x}}=a;\frac{1}{\sqrt{y}}=b$)

2. Raman's age is three times the sum of the ages of his two sons. After 5 years his age will be twice the sum of the ages of his two sons. Find the age of Raman.

3. The middle digit of a number between 100 and 1000 is zero and the sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the original number by 495. Find the number.

Solving by Elimination Method

This is another algebraic method for solving a pair of linear equations. This method is more convenient than the substitution method. Here we eliminate (i.e. remove) one of the two variables in a pair of linear equations, so as to get a linear equation in one variable which can be solved easily.

The various steps involved in the technique are given below:

Step 1: Multiply one or both of the equations by a suitable number(s) so that either the coefficients of first variable or the coefficients of second variable in both the equations become numerically equal.

Step 2: Add both the equations or subtract one equation from the other, as obtained in step 1, so that the terms with equal numerical coefficients cancel mutually.

Step 3: Solve the resulting equation to find the value of one of the unknowns.

Step 4: Substitute this value in any of the two given equations and find the value of the other unknown.

Example 1.12 Given $4a+3b=65$ and $a+2b=35$ solve by elimination method.

Solution

Given, $4a+3b=65$ .....(1)

$a+2b=35$ .....(2)

(2) × 4 gives $4a + 8b = 140$

$(-)\ (-)\ (-)$

Already (1) is $\underline{4a + 3b = 65}$

$5b = 75$ which gives $b = 15$

Put $b = 15$ in (2):

$a + 2(15) = 35$ which simplifies to $a = 5$

Thus the solution is $a = 5$, $b = 15$.

Verification :

$4a+3b = 65$ ...(1)

$4(5)+3(15) = 65$

$20 + 45 = 65$

$65 = 65$ True

$a + 2b = 35$ ...(2)

$5 + 2(15) = 35$

$5+30 = 35$

$35 = 35$ True

Example 1.13

Solve $2x+3y=14$ and $3x-4y=4$ by the method of elimination.

Solution

Given, $2x+3y=14$ ...(1)

$3x-4y=4$ ...(2)

Here our aim is to make the coefficients of y same, so that we can eliminate y. The LCM of 3 and 4=12 comes to our help!

To eliminate y:

Multiply (1) by 4, to get $8x+12y=56$

Multiply (2) by 3, to get $\underline{9x-12y=12}$

Adding, we get $17x=68$

Therefore, $x=4$

Substitute $x=4$ in (1) to get $2x+3y=14$

$$2(4)+3y=14$$

$$8+3y=14$$

$$y=2$$

Thus the solution is $x=4$, $y=2$.

Verification :

$2x+3y$	=	14	...(1)
$2(4)+3(2)$	=	14	
$8+6$	=	14	
14	=	14	True
$3x-4y$	=	4	...(2)
$3(4)-4(2)$	=	4	
$12-8$	=	4	
4	=	4	True

Example 1.14

Solve for x and y: $8x-3y=5xy$, $6x-5y=-2xy$ by the method of elimination.

Solution

The given system of equations are $8x-3y=5xy$ ...(1)

$6x-5y=-2xy$ ...(2)

Observe that the given system is not linear because of the occurrence of xy term. Also note that if $x=0$, then $y=0$ and vice versa. So, (0,0) is a solution for the system and any other solution would have both $x\neq 0$ and $y\neq 0$.

Let us take up the case where $x\neq 0$, $y\neq 0$.

Dividing both sides of each equation by xy,

$$\frac{8x}{xy}-\frac{3y}{xy}=\frac{5xy}{xy} \quad \text{we get,} \quad \frac{8}{y}-\frac{3}{x}=5 \quad \text{...(3)}$$

$$\frac{6x}{xy}-\frac{5y}{xy}=\frac{-2xy}{xy} \qquad \frac{6}{y}-\frac{5}{x}=-2 \quad \text{...(4)}$$

Let $a=\frac{1}{x}$, $b=\frac{1}{y}$.

(3)&(4) respectively become, $8b-3a=5$ ...(5)

$6b-5a=-2$ ...(6)

which are linear equations in a and b.

To eliminate a, we have, $(5)\times 5 \Rightarrow \quad 40b - 15a = 25 \quad .....(7)$

$(6)\times 3 \Rightarrow \quad 18b - 15a = -6 \quad .....(8)$

Now proceed as in the previous example to get the solution $\left(\frac{11}{23}, \frac{22}{31}\right)$.

Thus, the system have two solutions $\left(\frac{11}{23}, \frac{22}{31}\right)$ and $(0,0)$.

Exercise 1.5

1. Solve by the method of elimination

(i) $2x - y = 3; \quad 3x + y = 7$

(ii) $x - y = 5; \quad 3x + 2y = 25$

(iii) $\frac{x}{10} + \frac{y}{5} = 14; \quad \frac{x}{8} + \frac{y}{6} = 15$

(iv) $3(2x + y) = 7xy; \quad 3(x + 3y) = 11xy$

(v) $\frac{4}{x} + 5y = 7; \quad \frac{3}{x} + 4y = 5$

(vi) $\frac{3}{x+y} + \frac{2}{x-y} = 3; \quad \frac{2}{x+y} + \frac{3}{x-y} = \frac{11}{3}$

(vii) $13x + 11y = 70; \quad 11x + 13y = 74$

(viii) $37x + 29y = 45; \quad 29x + 37y = 21$

2. The monthly income of A and B are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹ 5,000 per month, find the monthly income of each.

3. Five years ago, a man was seven times as old as his son, while five year hence, the man will be four times as old as his son. Find their present age.

Solving by Cross Multiplication Method

The substitution and elimination methods involves many arithmetic operations, whereas the cross multiplication method utilize the coefficients effectively, which simplifies the procedure to get the solution. This method of cross multiplication is so called because we draw cross ways between the numbers in the denominators and cross multiply the coefficients along the arrows ahead. Now let us discuss this method as follows.

Suppose we are given a pair of linear simultaneous equations such as

$$a_1x + b_1y + c_1 = 0 \qquad ...(1)$$

$$a_2x + b_2y + c_2 = 0 \qquad ...(2)$$

such that $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. We can solve them as follows :

(1) $\times\, b_2$ – (2) $\times\, b_1$ gives $b_2(a_1x+b_1y+c_1)-b_1(a_2x+b_2y+c_2)=0$

$$\Rightarrow \quad x(a_1b_2-a_2b_1)=(b_1c_2-b_2c_1)$$

$$\Rightarrow \quad x=\frac{(b_1c_2-b_2c_1)}{(a_1b_2-a_2b_1)}$$

(1) $\times\, a_2$ – (2) $\times\, a_1$ similarly can be considered and that will simplify to

$$y=\frac{(c_1a_2-c_2a_1)}{(a_1b_2-a_2b_1)}$$

Hence the solution for the system is

$$\boxed{x=\frac{(b_1c_2-b_2c_1)}{(a_1b_2-a_2b_1)},\quad y=\frac{(c_1a_2-c_2a_1)}{(a_1b_2-a_2b_1)}}$$

This can also be written as

$$\boxed{\frac{x}{b_1c_2-b_2c_1}=\frac{y}{c_1a_2-c_2a_1}=\frac{1}{a_1b_2-a_2b_1}}$$

which can be remembered as

$$\begin{array}{ccccccc} & x & & y & & 1 & \\ b_1 & & c_1 & & a_1 & & b_1 \\ b_2 & & c_2 & & a_2 & & b_2 \end{array}$$

Example 1.15 Solve $3x-4y=10$ and $4x+3y=5$ by the method of cross multiplication.

Solution

The given system of equations are

$$3x-4y=10 \quad \Rightarrow 3x-4y-10=0 \quad .....(1)$$

$$4x+3y=5 \quad \Rightarrow 4x+3y-5=0 \quad .....(2)$$

For the cross multiplication method, we write the co-efficients as

$$\begin{array}{ccccccc} & x & & y & & 1 & \\ -4 & & -10 & & 3 & & -4 \\ 3 & & -5 & & 4 & & 3 \end{array}$$

$$\frac{x}{(-4)(-5)-(3)(-10)}=\frac{y}{(-10)(4)-(-5)(3)}=\frac{1}{(3)(3)-(4)(-4)}$$

$$\frac{x}{(20)-(-30)}=\frac{y}{(-40)-(-15)}=\frac{1}{(9)-(-16)}$$

$$\frac{x}{20+30}=\frac{y}{-40+15}=\frac{1}{9+16}$$

$$\frac{x}{50}=\frac{y}{-25}=\frac{1}{25}$$

Therefore, we get $\quad x=\frac{50}{25}; \quad y=\frac{-25}{25}$

$$x=2; \quad y=-1$$

Thus the solution is $x = 2$, $y = -1$.

Verification :

$$\begin{aligned} 3x-4y &= 10 \quad ...(1) \\ 3(2)-4(-1) &= 10 \\ 6+4 &= 10 \\ 10 &= 10 \quad \text{True} \end{aligned}$$

$$\begin{aligned} 4x+3y &= 5 \quad ...(2) \\ 4(2)+3(-1) &= 5 \\ 8-3 &= 5 \\ 5 &= 5 \quad \text{True} \end{aligned}$$

Example 1.16 Solve $2x=-7y+5;\ -3x=-8y-11$ by cross multiplication method.

Solution

The given system of equation can be written as

$$2x+7y-5=0$$

$$-3x+8y+11=0$$

For the cross multiplication method, we write the coefficients as

$$\begin{array}{cccc} & x & y & 1 \\ 7 & -5 & 2 & 7 \\ 8 & 11 & -3 & 8 \end{array}$$

$$\frac{x}{(7)(11)-(8)(-5)}=\frac{y}{(-5)(-3)-(11)(2)}=\frac{1}{(2)(8)-(-3)(7)}$$

$$\frac{x}{77+40}=\frac{y}{15-22}=\frac{1}{16+21}$$

$$\frac{x}{117}=\frac{y}{-7}=\frac{1}{37}$$

$$\frac{x}{117}=\frac{1}{37}, \qquad \frac{y}{-7}=\frac{1}{37}$$

Hence the solution is $\left(\frac{117}{37}, \frac{-7}{37}\right)$

Verification :

$$\begin{aligned} 2x+7y-5 &= 0 \quad ...(1) \\ 2\left(\frac{117}{37}\right)+7\left(\frac{-7}{37}\right)-5 &= 0 \\ \frac{234}{57}-\frac{49}{37}-5 &= 0 \\ \frac{185}{37}-5 &= 0 \\ 5-5 &= 0 \quad \text{True} \end{aligned}$$

$$\begin{aligned} 3x+8y+11 &= 0 \quad ...(2) \\ 3\left(\frac{117}{37}\right)+8\left(\frac{-7}{37}\right)+11 &= 0 \\ \frac{-351}{37}-\frac{56}{37}+11 &= 0 \\ \frac{-407}{37}+11 &= 0 \\ -11+11 &= 0 \quad \text{True} \end{aligned}$$

Example 1.17 Solve by cross multiplication method :

$$3x+5y=21; \quad -7x-6y=-49$$

Solution

The given system of equations are $3x+5y-21=0$; $-7x-6y+49=0$

Now using the coefficients for cross multiplication, we get,

$$\begin{array}{cccc} & x & y & 1 \\ 5 & -21 & 3 & 5 \\ -6 & 49 & -7 & -6 \end{array}$$

$$\Rightarrow \frac{x}{(5)(49)-(-6)(-21)} = \frac{y}{(-21)(-7)-(49)(3)} = \frac{1}{(3)(-6)-(-7)(5)}$$

$$\frac{x}{119} = \frac{y}{0} = \frac{1}{17}$$

$$\Rightarrow \quad \frac{x}{119} = \frac{1}{17}, \quad \frac{y}{0} = \frac{1}{17}$$

$$\Rightarrow \quad x = \frac{119}{17}, \quad y = \frac{0}{17}$$

$$\Rightarrow \quad x = 7, \quad y = 0$$

Verification :

$$\begin{aligned} 3x+5y &= 21 \quad ...(1) \\ 3(7)+5(0) &= 21 \\ 21+0 &= 21 \\ 21 &= 21 \quad \text{True} \end{aligned}$$

$$\begin{aligned} -7x-6y &= -49 \quad ...(2) \\ -7(7)-6(0) &= -49 \\ -49 &= -49 \\ -49 &= -49 \quad \text{True} \end{aligned}$$

Note

Here $\frac{y}{0} = \frac{1}{17}$ is to mean $y = \frac{0}{17}$. Thus, $\frac{y}{0}$ is only a notation and it is not division by zero. It is always true that division by zero is not defined.

Exercise 1.6

1. Solve by cross-multiplication method

   (i) $8x-3y=12$; $5x=2y+7$ (ii) $6x+7y-11=0$; $5x+2y=13$

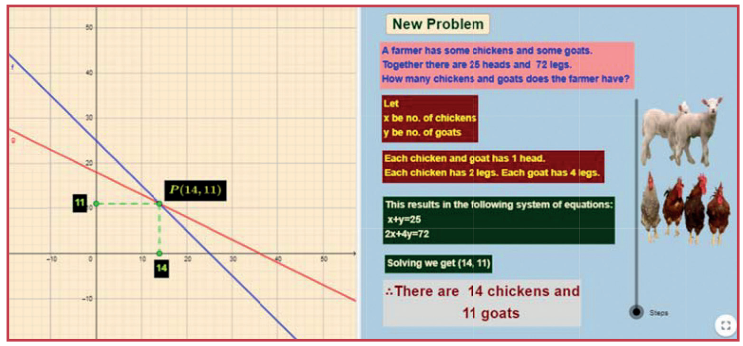
   (iii) $\frac{2}{x}+\frac{3}{y}=5$; $\frac{3}{x}-\frac{1}{y}+9=0$

2. Akshaya has 2 rupee coins and 5 rupee coins in her purse. If in all she has 80 coins totalling ₹ 220, how many coins of each kind does she have.

3. It takes 24 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 8 hours and the pipe of the smaller diameter is used for 18 hours. Only half of the pool is filled. How long would each pipe take to fill the swimming pool.

ICT Corner

Expected Result is shown in this picture



Step – 1

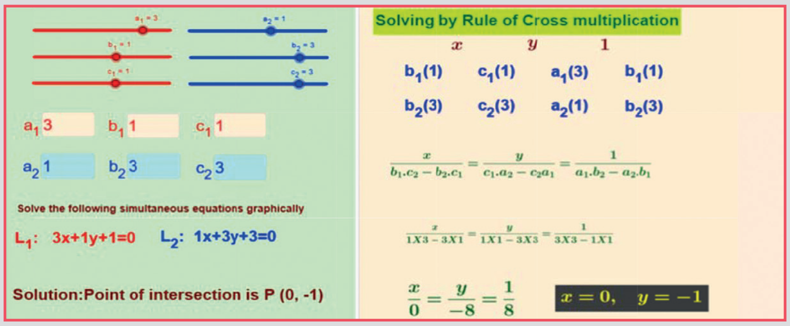
Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Algebra" will open. There are three worksheets under the title Solving by rule of cross multiplication, Graphical method and Chick-Goat puzzle.

Step - 2

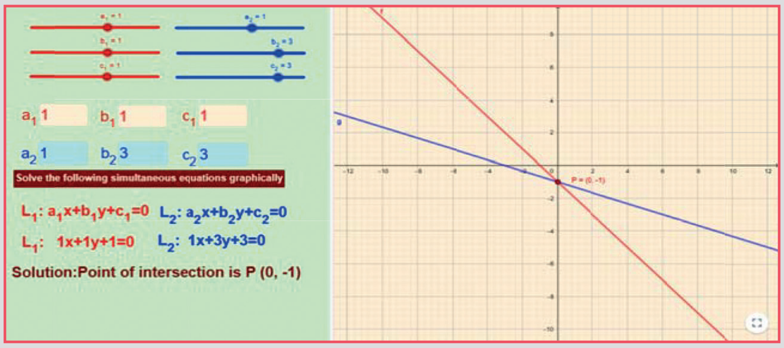
Move the sliders or type the respective values in the respective boxes to change the equations. Work out the solution and check the solutions.

In third title click on new problem and solve. Move the slider to see the steps.

Step 1



Step 2



Browse in the link

Algebra: https://ggbm.at/qampr4ta or Scan the QR Code.

B566_9_MAT_EM_T3

1.7 Consistency and Inconsistency of Linear Equations in Two Variables

Consider linear equations in two variables say

$$a_1x+b_1y+c_1=0 \quad ...(1)$$

$$a_2x+b_2y+c_2=0 \quad ...(2)$$ where a_1, a_2, b_1, b_2, c_1 and c_2 are real numbers.

Then the system has :

(i) a unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (Consistent)

(ii) an Infinite number of solutions if

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (Consistent)

(iii) no solution if $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ (Inconsistent)

Example 1.18 Check whether the following system of equation is consistent or inconsistent and say how many solutions we can have if it is consistent.

(i) $2x - 4y = 7$, $x - 3y = -2$ (ii) $4x + y = 3$, $8x + 2y = 6$ (iii) $4x + 7 = 2y$, $2x + 9 = y$

Solution

Sl. No	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
(i)	$2x-4y=7$ $x-3y=-2$	$\frac{2}{1}=2$	$\frac{-4}{-3}=\frac{4}{3}$	$\frac{7}{-2}=\frac{-7}{2}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution
(ii)	$4x+y=3$ $8x+2y=6$	$\frac{4}{8}=\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6}=\frac{1}{2}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$	Coinciding lines	Infinite many solutions
(iii)	$4x+7=2y$ $2x+9=y$	$\frac{4}{2}=2$	$\frac{2}{1}=2$	$\frac{7}{9}$	$\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

Activity - 1

Check whether the following system of equation is consistent or inconsistent and say how many solutions we can have if it is consistent.

(i) $2x - y = 3$ (ii) $3x - 4y = 12$ (iii) $2x - y = 5$

$4x - 2y = 6$ $9x - 12y = 24$ $3x + y = 10$

Example 1.19 Find the value of k for which the given system of equations $kx+2y=3;\ 2x-3y=1$ has a unique solution.

Solution

Given linear equations are

$$\begin{aligned} kx+2y&=3 \ .....(1) \\ 2x-3y&=1 \ .....(2) \end{aligned} \quad \begin{bmatrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{bmatrix}$$

Here $a_1=k,\ b_1=2,\ a_2=2,\ b_2=-3;$

For unique solution we take $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$; therefore $\frac{k}{2} \neq \frac{2}{-3}$; $k \neq \frac{4}{-3}$, that is $k \neq -\frac{4}{3}$.

Example 1.20 Find the value of k, for the following system of equation has infinitely many solutions.

$$2x-3y=7;\quad (k+2)x-(2k+1)y=3(2k-1)$$

Solution

Given two linear equations are

$$\begin{aligned} 2x-3y&=7 \\ (k+2)x-(2k+1)y&=3(2k-1) \end{aligned} \quad \begin{bmatrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{bmatrix}$$

Here $a_1=2,\ b_1=-3,\ a_2=(k+2),\ b_2=-(2k+1),\ c_1=7,\ c_2=3(2k-1)$

For infinite number of solution we consider $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$$\frac{2}{k+2}=\frac{-3}{-(2k+1)}=\frac{7}{3(2k-1)}$$

$$\frac{2}{k+2}=\frac{-3}{-(2k+1)}$$

$$2(2k+1)=3(k+2)$$

$$4k+2=3k+6$$

$$k=4$$

$$\frac{-3}{-(2k+1)}=\frac{7}{3(2k-1)}$$

$$9(2k-1)=7(2k+1)$$

$$18k-9=14k+7$$

$$4k=16$$

$$k=4$$

Example 1.21

Find the value of k for which the system of linear equations $8x+5y=9;\ kx+10y=15$ has no solution.

Solution

Given linear equations are

$$\begin{aligned} 8x+5y&=9 \\ kx+10y&=15 \end{aligned} \quad \begin{bmatrix} a_1x+b_1y+c_1=0 \\ a_2x+b_2y+c_2=0 \end{bmatrix}$$

Here $a_1=8,\ b_1=5,\ c_1=9,\ a_2=k, b_2=10, c_2=15$

For no solution, we know that $\dfrac{a_1}{a_2}=\dfrac{b_1}{b_2}\neq\dfrac{c_1}{c_2}$ and so, $\dfrac{8}{k}=\dfrac{5}{10}\neq\dfrac{9}{15}$

$$80=5k$$

$$k=16$$

Activity - 2

1. Find the value of k for the given system of linear equations satisfying the condition below:
   (i) $2x+ky=1;\ 3x-5y=7$ has a unique solution
   (ii) $kx+3y=3;\ 12x+ky=6$ has no solution
   (iii) $(k-3)x+3y=k;\ kx+ky=12$ has infinite number of solution
2. Find the value of a and b for which the given system of linear equation has infinite number of solutions $3x-(a+1)y=2b-1,\quad 5x+(1-2a)y=3b$

Activity - 3

For the given linear equations, find another linear equation satisfying each of the given condition

Given linear equation	Another linear equation		
	Unique Solution	Infinite many solutions	No solution
$2x+3y=7$	$3x+4y=8$	$4x+6y=14$	$6x+9y=15$
$3x-4y=5$			
$y-4x=2$			
$5y-2x=8$			

Exercise 1.7

Solve by any one of the methods

1. Draw the graph of the equations $x=3$, $x=5$ and $2x-y-4=0$. Also find the area of the quadrilateral formed by these lines and the x-axis.

2. The sum of a two digit number and the number formed by interchanging the digits is 110. If 10 is subtracted from the first number, the new number is 4 more than 5 times the sums of the digits of the first number. Find the first number.

3. The sum of the numerator and denominator of a fraction is 12. If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$. Find the fraction.

4. Points A and B are 70 km apart on a highway. A car starts from A and another car starts from B simultaneously. If they travel in the same direction, they meet in 7 hours, but if they travel towards each other, they meet in one hour. Find the speed of the two cars.

5. ABCD is a cyclic quadrilateral such that $\angle A = (4y + 20)^\circ$, $\angle B = (3y - 5)^\circ$, $\angle C = (4x)^\circ$ and $\angle D = (7x + 5)^\circ$. Find the four angles.

6. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gains ₹2000. But if he sells the T.V. at 10% gain and the fridge at 5% loss, he gains Rs.1500 on the transaction. Find the actual price of the T.V. and the fridge.

7. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

8. The age of Arjun is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

9. The taxi charges in a city comprise of a fixed charge together with the charge for the distance covered. For a journey of 10 km the charge paid is ₹ 75 and for a journey of 15 km the charge paid is ₹110. What will a person have to pay for travelling a distance of 25 km ? (You may also try to illustrate through a graph).

10. A railway half ticket costs half the full fare and the reservation charge is the same on half ticket as on full ticket. One reserved first class ticket from Mumbai to Ahmadabad costs ₹ 216 and one full and one half reserved first class ticket costs ₹ 327. What is the basic first class full fare and what is the reservation charge?

11. A lending library gives books on rent for reading. It takes a fixed charge for the first two days and an additional charge for each day thereafter. Latika paid ₹ 22 for a book kept for 6 days, while Anand paid ₹ 16 for the book kept for 4 days. Find the fixed charges and the charge for each extra day.

12. 4 men and 4 boys can do a piece of work in 3 days. While 2 men and 5 boys can finish it in 4 days. How long would it take for 1 boy to do it? How long would it take for 1 men to do it?

Exercise 1.8

Multiple choice questions

1. Which of the following statement is true for the equation $2x+3y=15$

   (1) the equation has unique solution (2) the equation has two solution

   (3) the equation has no solution (4) the equation has infinite solutions

2. Find the value of m from the equation $2x+3y=m$. If its one solution is $x=2$ and $y=-2$.

   (1) 2 (2) -2 (3) 10 (4) 0

3. Which of the following is a linear equation

   (1) $x+\frac{1}{x}=2$ (2) $x(x-1)=2$ (3) $3x+5=\frac{2}{3}$ (4) $x^3-x=5$

4. Which of the following is a solution of the equation $2x-y=6$

   (1) (2,4) (2) (4,2) (3) (3, -1) (4) (0,6)

5. The linear equation in one variable is

   (1) $2x+2=y$ (2) $5x-7=6-2x$ (3) $2t(5-t)=0$ (4) $7p-q=0$

6. If (2,3) is a solution of linear equation $2x+3y=k$ then, the value of k is

   (1) 12 (2) 6 (3) 0 (4) 13

7. Which condition does not satisfy the linear equation $ax+by+c=0$

   (1) $a\neq 0$, $b=0$ (2) $a=0$, $b\neq 0$ (3) $a=0$, $b=0$, $c\neq 0$ (4) $a\neq 0$, $b\neq 0$

8. Which of the following is not a linear equation in two variable

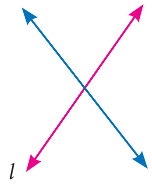
   (1) $ax+by+c=0$ (2) $0x+0y+c=0$

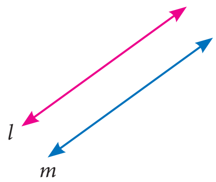
   (3) $0x+by+c=0$ (4) $ax+0y+c=0$

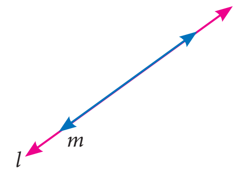
9. The value of k for which the pair of linear equations $4x+6y-1=0$ and $2x+ky-7=0$ represents parallel lines is

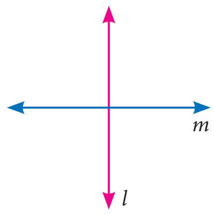
   (1) $k=3$ (2) $k=2$ (3) $k=4$ (4) $k=-3$

10. A pair of linear equations has no solution then the graphical representation is

(1) 

(2) 

(3) 

(4) 

11. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ where $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the given pair of linear equation has ___________ solution(s)

(1) no solution (2) two solutions (3) unique (4) infinite

12. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ where $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the given pair of linear equation has ___________ solution(s)

(1) no solution (2) two solutions (3) infinite (4) unique

Points to Remember

- Linear relationships can be organised/expressed by representing the data (i) by a table (ii) by a graph (iii) by an equation.
- An equation is a pair of expression set equal to each other.
- An equation with only one variable with 1 as its exponent is called a linear equation with single variable ($ax+b = 0$ where, $a \neq 0$).
- Solution of an equation is the set of all values that when substituted for unknowns make an equation true.
- An equation with two variable each with exponent as 1 and not multiplied with each other is called a linear equation with two variables.
- $y = mx + c$ is another form of linear equation in two variables.
- Slope of a line is a number that defines the steepness of the line

$$\text{Slope} = \frac{\textit{vertical change}}{\textit{horizontal change}}$$

- The intercept of a line is the distance from the origin to the point at which it crosses either x-axis or y-axis
- Linear equation in two variables has infinite number of solutions.
- The graph of a linear equation in two variables is a straight line.
- Simultaneous linear equations consists of two or more linear equations with the same variables.

2 COORDINATE GEOMETRY

Divide each difficulty into as many parts as is feasible and necessary to resolve it.
- Rene Descartes

Pierre de Fermat
A.D (C.E) 1601-1665

Pierre de Fermat was one of the leading mathematician of the first half of 17^{th} century. He made notable contribution to coordinate geometry. In particular, he is recognized for his discovery of the method of finding the greatest and the smallest ordinates of curved lines. His pioneering work in coordinate geometry was circulated in manuscript form in 1636, predating the publication of Descarte's famous 'La geometrie'.

Learning Outcomes

- To understand the mid-point formula and use it in problem solving.
- To derive the section formula and apply this in problem solving.
- To understand the centroid formula and to know its applications.

FZ25P3

2.1 Introduction

We have already seen plotting points on the plane, using ordered pairs of coordinates. This led us to find the distance between any two points.

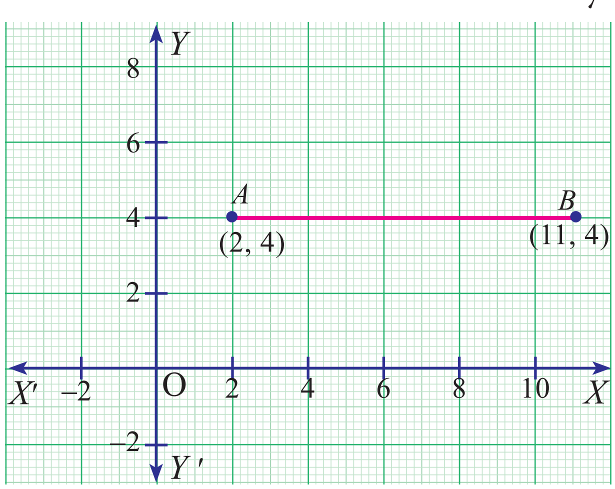


Fig. 2.1

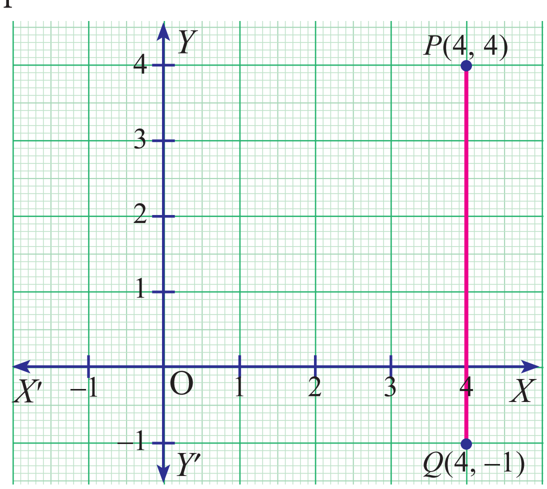


Fig. 2.2

The line segment AB is parallel to X axis AB = difference of x coordinates $= \|11 - 2\| = 9$	The line segment PQ is parallel to Y axis PQ = difference of y coordinates $= \|(-1) - 4\| = 5$

It is not necessary that always the line segment joining the points are parallel to the axis.

For example, in the Fig. 2.3, distance d between $A(2,2)$ and $B(5,6)$ is shown. This is the general case of finding the distance between end points of a segment which is neither horizontal nor vertical.

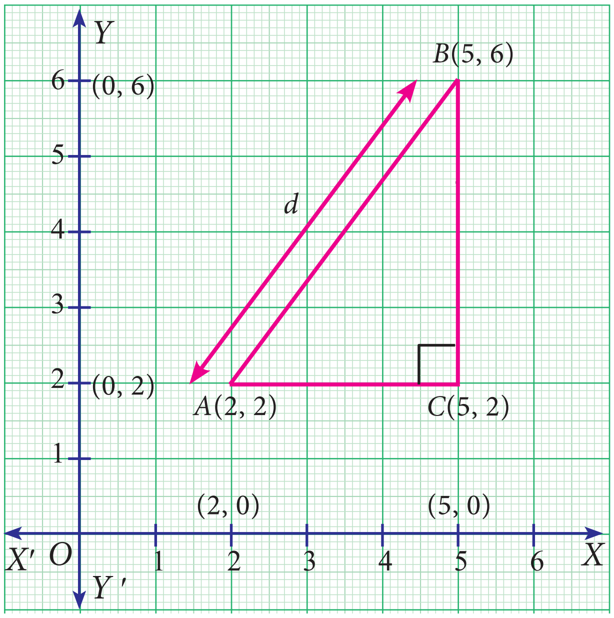


Fig. 2.3

Here, $AC = 5 - 2 = 3$, that is here AC is parallel to x-axis. So its length is nothing but the difference in the x-coordinates and $BC = 6 - 2 = 4$ (since BC is parallel to y-axis and so its length can be calculated accordingly).

ΔABC is right angled at C and by Pythagoras theorem it follows that,

$$AB = \sqrt{(AC)^2 + (BC)^2}$$

$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

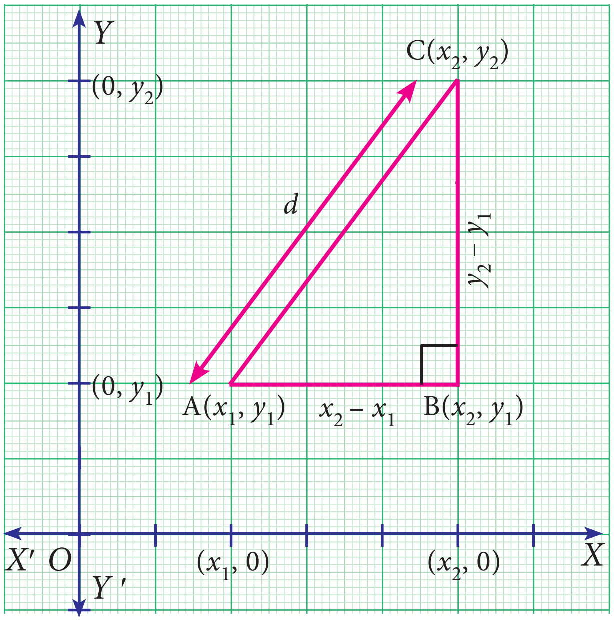


Fig. 2.4

This can be generalized (observe Fig. 2.4) and we get the distance formula to find the distance between two points (not necessarily parallel to either axis). $A(x_1,y_1)$ and $C(x_2,y_2)$ as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

2.2 The Mid-point of a Line Segment

Imagine a person riding his two-wheeler on a straight road towards East from his college to village A and then to village B. At some point in between A and B, he suddenly realises that there is not enough petrol for the journey. On the way there is no petrol bunk in between these two places. Should he travel back to A or just try his luck moving towards B? Which would be the shorter distance? There is a dilemma. He has to know whether he crossed the half way mid-point or not.


Fig. 2.5

Fig. 2.6

The above Fig. 2.6 illustrates the situation. Imagine college as origin O from which the distances of village A and village B are respectively x_1 and x_2 $(x_1 < x_2)$. Let M be the mid-point of AB then x can be obtained as follows.

$AM = MB$ and so, $\qquad x - x_1 = x_2 - x$

and this is simplified to $\qquad x = \dfrac{x_1 + x_2}{2}$

Now it is easy to discuss the general case. If $A(x_1, y_1)$, $B(x_2, y_2)$ are any two points and $M(x, y)$ is the mid-point of the line segment AB, then M' is the mid-point of AC (in the Fig. 2.7). In a right triangle the perpendicular bisectors of the sides intersect at the mid-point of the hypotenuse. (Also, this property is due to similarity among the two coloured triangles shown; In such triangles, the corresponding sides will be proportional).

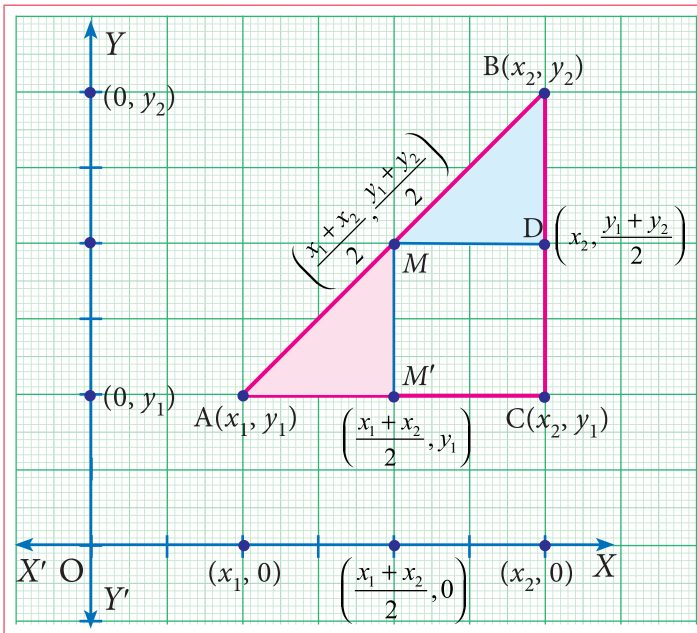


Fig. 2.7

Another way of solving

(Using similarity property)

Let us take the point M as $M(x,y)$

Now, $\Delta AMM'$ and ΔMBD are similar. Therefore,

$$\frac{AM'}{MD} = \frac{MM'}{BD} = \frac{AM}{MB}$$

$$\frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y} = \frac{1}{1} \quad (AM = MB)$$

Consider, $\dfrac{x - x_1}{x_2 - x} = 1$

$2x = x_2 + x_1 \Rightarrow x = \dfrac{x_2 + x_1}{2}$

Similarly, $y = \dfrac{y_2 + y_1}{2}$

The x-coordinate of M = the average of the x-coordinates of A and C = $\dfrac{x_1 + x_2}{2}$ and

similarly, the y-coordinate of M = the average of the y-coordinates of B and C = $\dfrac{y_1 + y_2}{2}$

The mid-point M of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Thinking Corner

If D is the mid-point of AC and C is the mid-point of AB, then find the length of AB if $AD = 4cm$.

For example, The mid-point of the line segment joining the points $(-8,-10)$ and $(4,-2)$ is given by $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ where $x_1=-8$, $x_2=4$, $y_1=-10$ and $y_2=-2$.

The required mid-point is $\left(\frac{-8+4}{2},\frac{-10-2}{2}\right)$ or $(-2,-6)$.

Let us now see the application of mid-point formula in our real life situation, consider the longitude and latitude of the following cities.

Name of the city	Longitude	Latitude
Chennai (Besant Nagar)	80.27° E	13.00° N
Mangaluru (Kuthethoor)	74.85° E	13.00° N
Bengaluru (Rajaji Nagar)	77.56° E	13.00° N

Fig. 2.8

Let us take the longitude and latitude of Chennai (80.27° E, 13.00° N) and Mangaluru (74.85° E, 13.00° N) as pairs. Since Bengaluru is located in the middle of Chennai and Mangaluru, we have to find the average of the coordinates, that is $\left(\frac{80.27+74.85}{2},\frac{13.00+13.00}{2}\right)$. This gives (77.56°E, 13.00°N) which is the longitude and latitude of Bengaluru. In all the above examples, the point exactly in the middle is the mid-point and that point divides the other two points in the same ratio.

Example 2.1

The point $(3,-4)$ is the centre of a circle. If AB is a diameter of the circle and B is $(5,-6)$, find the coordinates of A.

Solution

Let the coordinates of A be (x_1, y_1) and the given point is $B(5,-6)$. Since the centre is the mid-point of the diameter AB, we have

$$\frac{x_1+x_2}{2}=3 \qquad\qquad \frac{y_1+y_2}{2}=-4$$
$$x_1+5=6 \qquad\qquad y_1-6=-8$$
$$x_1=6-5 \qquad\qquad y_1=-8+6$$
$$x_1=1 \qquad\qquad y_1=-2$$

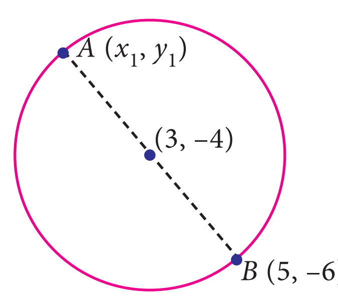


Fig. 2.9

Therefore, the coordinates of A is $(1,-2)$.

Example 2.2 Use the mid-point formula to show that the mid-point of the hypotenuse of a right angled triangle is equidistant from the vertices (with suitable points).

Solution

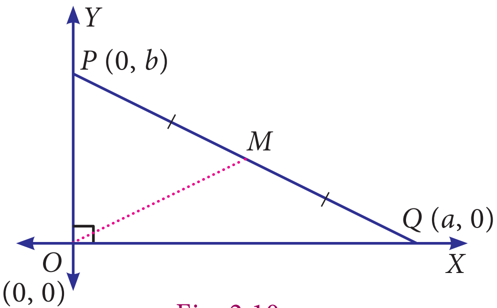


Fig. 2.10

Let POQ be the right angled triangle and O be placed at the origin. Let $OQ = a$ units and OP be b units. Let us name the coordinates of P as $(0,b)$ and Q as $(a,0)$.

By mid-point formula, if M is the mid-point of the hypotenuse PQ [$PM=MQ$], then M is

$$\left(\frac{a+0}{2},\frac{b+0}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right).$$

We now use the distance formula and find that

$OM=\sqrt{\left(\frac{a}{2}-0\right)^2+\left(\frac{b}{2}-0\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$ which is the same value as

$QM=\sqrt{\left(a-\frac{a}{2}\right)^2+\left(0-\frac{b}{2}\right)^2}=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$ and similarly $PM=\sqrt{\frac{a^2}{4}+\frac{b^2}{4}}$

This shows $OM = QM = PM$, which we desired to prove.

Progress Check

(i) Let X be the mid-point of the line segment joining $A(3,0)$ and $B(-5,4)$ and Y be the mid-point of the line segment joining $P(-11,-8)$ and $Q(8,-2)$. Find the mid-point of the line segment XY.

(ii) If $(3,x)$ is the mid-point of the line segment joining the points $A(8,-5)$ and $B(-2,11)$, then find the value of 'x'.

Example 2.3 If $(x,3)$, $(6,y)$, $(8,2)$ and $(9,4)$ are the vertices of a parallelogram taken in order, then find the value of x and y.

Solution

Let $A(x,3)$, $B(6,y)$, $C(8,2)$ and $D(9,4)$ be the vertices of the parallelogram $ABCD$.

By definition, diagonals AC and BD bisect each other.

Mid-point of AC = Mid-point of BD

$$\left(\frac{x+8}{2},\frac{3+2}{2}\right)=\left(\frac{6+9}{2},\frac{y+4}{2}\right)$$

equating the coordinates on both sides, we get

$$\frac{x+8}{2}=\frac{15}{2} \quad \text{and} \quad \frac{5}{2}=\frac{y+4}{2}$$
$$x+8=15 \qquad 5=y+4$$
$$x=7 \qquad y=1$$

Hence, $x = 7$ and $y = 1$.

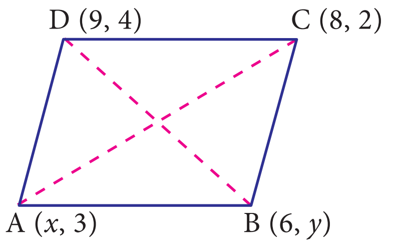


Fig. 2.11

FZ7E2T

Thinking Corner

$A(6,1)$, $B(8,2)$ and $C(9,4)$ are three vertices of a parallelogram $ABCD$ taken in order. Find the fourth vertex D. If (x_1,y_1), (x_2,y_2), (x_3,y_3) and (x_4,y_4) are the four vertices of the parallelogram, then using the given points, find the value of $(x_1+x_3-x_2, y_1+y_3-y_2)$ and state the reason for your result.

Example 2.4 Find the points which divide the line segment joining $A(-11,4)$ and $B(9,8)$ into four equal parts.

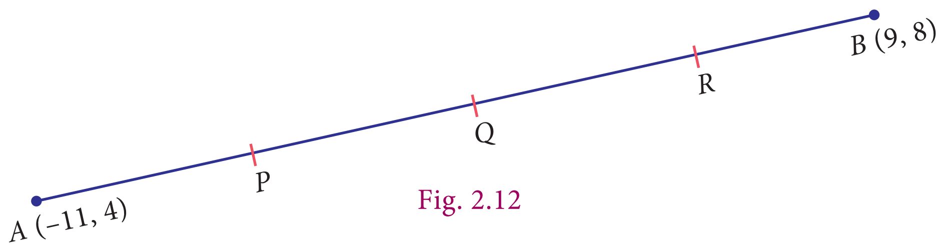


Fig. 2.12

Solution

Let P, Q, R be the points on the line segment joining $A(-11,4)$ and $B(9,8)$ such that $AP=PQ=QR=RB$.

Here Q is the mid-point of AB, P is the mid-point of AQ and R is the mid-point of QB.

$$Q \text{ is the mid-point of } AB=\left(\frac{-11+9}{2},\frac{4+8}{2}\right)=\left(\frac{-2}{2},\frac{12}{2}\right)=(-1,6)$$

$$P \text{ is the mid-point of } AQ=\left(\frac{-11-1}{2},\frac{4+6}{2}\right)=\left(\frac{-12}{2},\frac{10}{2}\right)=(-6,5)$$

$$R \text{ is the mid-point of } QB=\left(\frac{-1+9}{2},\frac{6+8}{2}\right)=\left(\frac{8}{2},\frac{14}{2}\right)=(4,7)$$

Hence the points which divides AB into four equal parts are $P(-6, 5)$, $Q(-1, 6)$ and $R(4, 7)$.

Activity - 1

1. $A(1,-2)$, $B(7,2)$, $C(5,8)$ and $D(-1,4)$ are the vertices of the parallellogram taken in order. Prove that the line joining the mid-points of the sides of the parallelogram $ABCD$ forms another parallelogram.

2. The points $A(7,10)$, $B(-2,5)$ and $C(3,-4)$ are the vertices of a right angled triangle with $\angle B = 90^0$. Consider another triangle congruent to ΔABC. Join these two triangles by taking a suitable side, common for both triangles and D as the third vertex, form the following polygons. Find the coordinates of D when,

   (i) $ABCD$ is a rectangle. (ii) $ADBC$ is a parallelogram.

   (iii) ACD is a isoceles triangle where AB is the altitude.

   (iv) ACD is a isoceles triangle where BC is the altitude.

Example 2.5 The mid-points of the sides of a triangle are $(5,1)$, $(3,-5)$ and $(-5,-1)$. Find the coordinates of the vertices of the triangle.

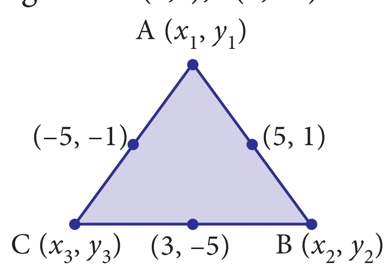


Fig. 2.13

Solution

Let the vertices of the ΔABC be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ and the given mid-points of the sides AB, BC and CA are $(5,1)$, $(3,-5)$ and $(-5,-1)$ respectively. Therefore,

$$\frac{x_1+x_2}{2} = 5 \Rightarrow x_1 + x_2 = 10 \quad \ldots(1)$$

$$\frac{x_2+x_3}{2} = 3 \Rightarrow x_2 + x_3 = 6 \quad \ldots(2)$$

$$\frac{x_3+x_1}{2} = -5 \Rightarrow x_3 + x_1 = -10 \quad \ldots(3)$$

Adding (1), (2) and (3)

$$2x_1 + 2x_2 + 2x_3 = 6$$

$$x_1 + x_2 + x_3 = 3 \quad \ldots(4)$$

$(4) - (2) \Rightarrow x_1 = 3 - 6 = -3$

$(4) - (3) \Rightarrow x_2 = 3 + 10 = 13$

$(4) - (1) \Rightarrow x_3 = 3 - 10 = -7$

$$\frac{y_1+y_2}{2} = 1 \Rightarrow y_1 + y_2 = 2 \quad \ldots(5)$$

$$\frac{y_2+y_3}{2} = -5 \Rightarrow y_2 + y_3 = -10 \quad \ldots(6)$$

$$\frac{y_3+y_1}{2} = -1 \Rightarrow y_3 + y_1 = -2 \quad \ldots(7)$$

Adding (5), (6) and (7),

$$2y_1 + 2y_2 + 2y_3 = -10$$

$$y_1 + y_2 + y_3 = -5 \quad \ldots(8)$$

$(8) - (6) \Rightarrow y_1 = -5 + 10 = 5$

$(8) - (7) \Rightarrow y_2 = -5 + 2 = -3$

$(8) - (5) \Rightarrow y_3 = -5 - 2 = -7$

Therefore the vertices of the triangles are $A(-3,5)$, $B(13,-3)$ and $C(-7,-7)$.

Thinking Corner

If (a_1,b_1), (a_2,b_2) and (a_3,b_3) are the mid-points of the sides of a triangle, using the mid-points given in example 2.5 find the value of $(a_1+a_3-a_2,\ b_1+b_3-b_2)$, $(a_1+a_2-a_3,\ b_1+b_2-b_3)$ and $(a_2+a_3-a_1,\ b_2+b_3-b_1)$. Compare the results. What do you observe? Give reason for your result?

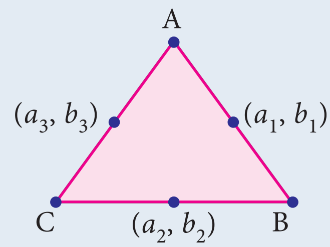


Fig. 2.14

Exercise 2.1

1. Find the mid-points of the line segment joining the points

   (i) $(-2,3)$ and $(-6,-5)$ (ii) $(8,-2)$ and $(-8,0)$

   (iii) (a,b) and (a+2b,2a−b) (iv) $\left(\frac{1}{2},-\frac{3}{7}\right)$ and $\left(\frac{3}{2},\frac{-11}{7}\right)$

2. The centre of a circle is $(-4,2)$. If one end of the diameter of the circle is $(-3,7)$ then find the other end.

3. If the mid-point (x,y) of the line joining $(3,4)$ and $(p,7)$ lies on $2x+2y+1=0$, then what will be the value of p?

4. The mid-point of the sides of a triangle are $(2,4)$, $(-2,3)$ and $(5,2)$. Find the coordinates of the vertices of the triangle.

5. $O(0,0)$ is the centre of a circle whose one chord is AB, where the points A and B are $(8,6)$ and $(10,0)$ respectively. OD is the perpendicular from the centre to the chord AB. Find the coordinates of the mid-point of OD.

6. The points $A(-5,4)$, $B(-1,-2)$ and $C(5,2)$ are the vertices of an isosceles right-angled triangle where the right angle is at B. Find the coordinates of D so that $ABCD$ is a square.

7. The points $A(-3,6)$, $B(0,7)$ and $C(1,9)$ are the mid-points of the sides DE, EF and FD of a triangle DEF. Show that the quadrilateral $ABCD$ is a parallellogram.

8. $A(-3,2)$, $B(3,2)$ and $C(-3,-2)$ are the vertices of the right triangle, right angled at A. Show that the mid-point of the hypotenuse is equidistant from the vertices.

9. Show that the line segment joining the mid-points of two sides of a triangle is half of the third side. (Hint: Place triangle ABC in a clever way such that A is $(0,0)$, B is $(2a,0)$ and C to be $(2b,2c)$. Now consider the line segment joining the mid-points of AC and BC. This will make calculations simpler).

10. Prove that the diagonals of the parallellogram bisect each other. [Hint: Take scale on both axes as $1cm = a$ units]

2.3 Points of Trisection of a Line Segment

The mid-point of a line segment is the point of bisection, which means dividing into two parts of equal length. Suppose we want to divide a line segment into three parts of equal length, we have to locate points suitably to effect a trisection of the segment.

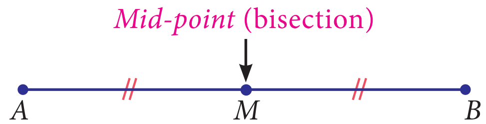


Fig. 2.15

$AM = MB$

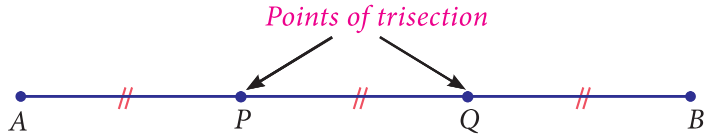


Fig. 2.16

$AP = PQ = QB$

For a given line segment, there are two points of trisection. The method of obtaining this is similar to that of what we did in the case of locating the point of bisection (i.e., the mid-point). Observe the given Fig. 2.17. Here P and Q are the points of trisection of the line segment AB where A is (x_1, y_1) and B is (x_2, y_2). Clearly we know that, P is the mid-point of AQ and Q is the mid-point of PB. Now consider the ΔACQ and ΔPDB (Also, can be verified using similarity property of triangles which will be dealt in detail in higher classes).

$$A'P' = P'Q' = Q'B'$$

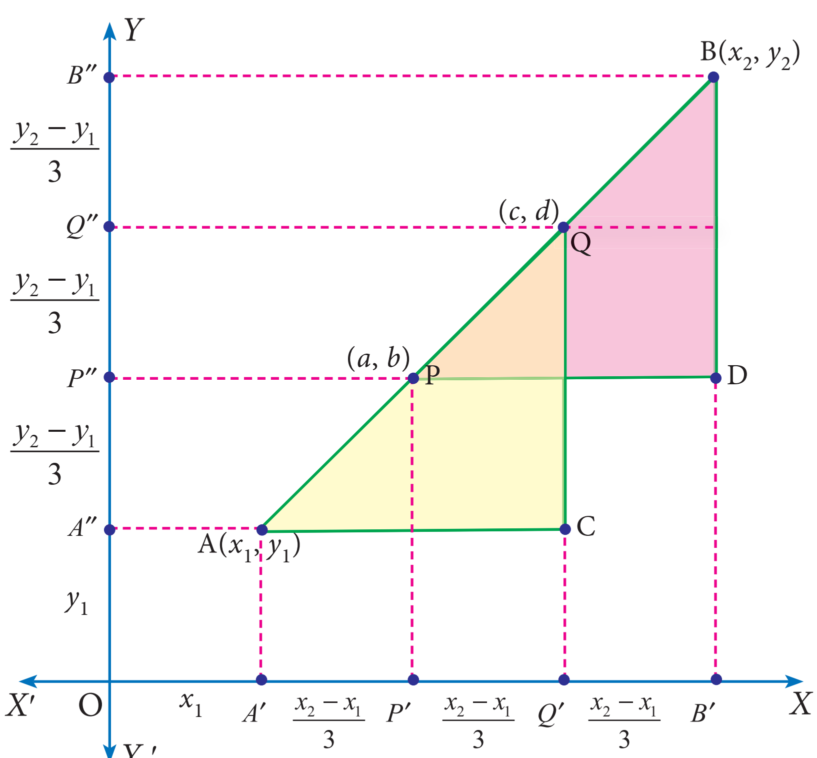


Fig. 2.17

Note that when we divide the segment into 3 equal parts, we are also dividing the horizontal and vertical legs into three equal parts.

If P is (a, b), then

$$a = OP' = OA' + A'P' = x_1 + \frac{x_2 - x_1}{3} = \frac{x_2 + 2x_1}{3};$$

$$b = PP' = OA'' + A''P'' = y_1 + \frac{y_2 - y_1}{3} = \frac{y_2 + 2y_1}{3}$$

Thus we get the point P as $\left(\frac{x_2 + 2x_1}{3}, \frac{y_2 + 2y_1}{3}\right)$

If Q is (c, d), then

$$c = OQ' = OB' - Q'B' = x_2 - \left(\frac{x_2 - x_1}{3}\right) = \frac{2x_2 + x_1}{3};$$

$$d = OQ'' = OB'' - Q''B'' = y_2 - \left(\frac{y_2 - y_1}{3}\right) = \frac{2y_2 + y_1}{3}$$

Thus the required point Q is $\left(\frac{2x_2 + x_1}{3}, \frac{2y_2 + y_1}{3}\right)$

Example 2.6 Find the points of trisection of the line segment joining $(-2,-1)$ and $(4,8)$.

Solution

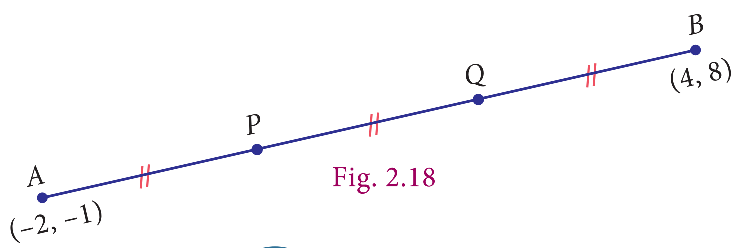


Fig. 2.18

Let $A(-2,-1)$ and $B(4,8)$ are the given points.

Let $P(a,b)$ and $Q(c,d)$ be the points of trisection of AB, so that $AP = PQ = QB$.

By the formula proved above,

P is the point

$$\left(\frac{x_2 + 2x_1}{3}, \frac{y_2 + 2y_1}{3}\right) = \left(\frac{4 + 2(-2)}{3}, \frac{8 + 2(-1)}{3}\right)$$

$$= \left(\frac{4-4}{3}, \frac{8-2}{3}\right) = (0, 2)$$

Q is the point

$$\left(\frac{2x_2 + x_1}{3}, \frac{2y_2 + y_1}{3}\right) = \left(\frac{2(4) - 2}{3}, \frac{2(8) - 1}{3}\right)$$

$$= \left(\frac{8-2}{3}, \frac{16-1}{3}\right) = (2, 5)$$

Progress Check

(i) Find the coordinates of the points of trisection of the line segment joining $(4,-1)$ and $(-2,-3)$.

(ii) Find the coordinates of points of trisection of the line segment joining the point $(6,-9)$ and the origin.

2.4 Section Formula

We studied bisection and trisection of a given line segment. These are only particular cases of the general problem of dividing a line segment joining two points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$.

Given a segment AB and a positive real number r.

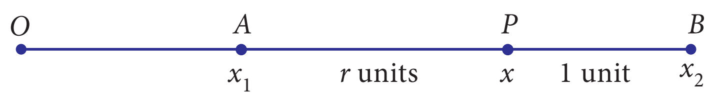


Fig. 2.19

We wish to find the coordinate of point P which divides AB in the ratio r :1.

This means $\frac{AP}{PB} = \frac{r}{1}$ or $AP = r(PB)$.

This means that $x - x_1 = r(x_2 - x)$

Solving this, $$x = \frac{rx_2 + x_1}{r+1} \quad ..... (1)$$

We can use this result for points on a line to the general case as follows.

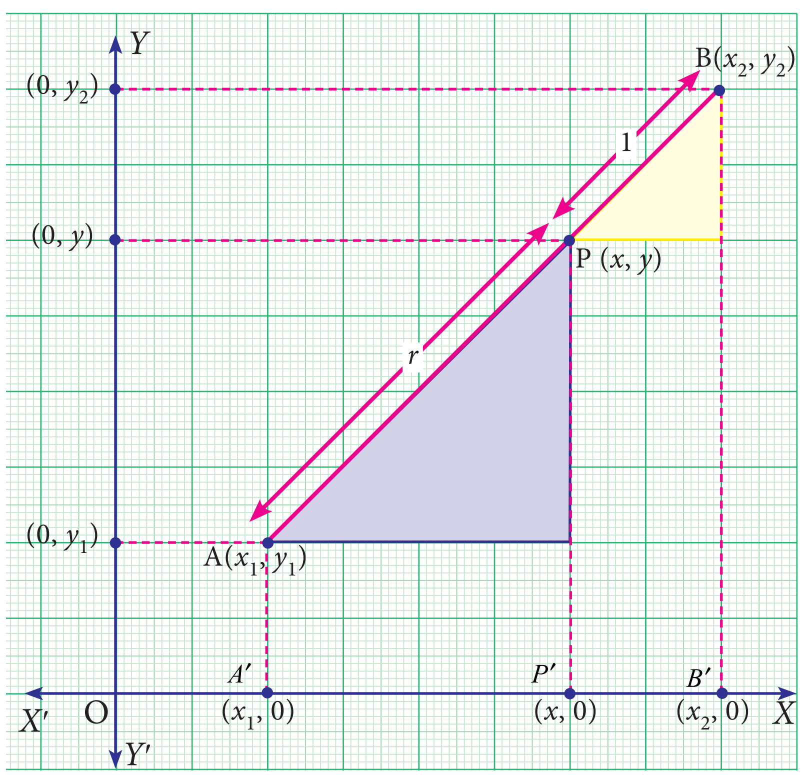


Fig. 2.20

Taking $AP : PB = r : 1$, we get $A'P' : P'B' = r : 1$.

Therefore $\quad A'P' = r(P'B')$

Thus, $(x - x_1) = r(x_2 - x)$

which gives $x = \dfrac{rx_2 + x_1}{r+1}$ … [see (1)]

Precisely in the same way we can have $y = \dfrac{ry_2 + y_1}{r+1}$

If P is between A and B, and $\dfrac{AP}{PB} = r$, then we have the formula,

$$P \text{ is } \left(\frac{rx_2 + x_1}{r+1}, \frac{ry_2 + y_1}{r+1}\right).$$

If r is taken as $\dfrac{m}{n}$, then the section formula is $\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$, which is the standard form.

Thinking Corner

(i) What happens when $m = n = 1$? Can you identify it with a result already proved?

(ii) $AP : PB = 1 : 2$ and $AQ : QB = 2:1$. What is $AP : AB$? What is $AQ : AB$?

Activity - 2

1. Draw a line segment by joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ on the graph sheet.

2. Draw a line AL from the point A and parallel to x-axis.

3. To divide the line segment AB in the ratio 2:1 (Here, $m = 2$ and $n = 1$). Locate $2+1 = 3$ $(m+n)$ points on AL which are at equal distance from each other. That is $AC_1 = C_1C_2 = C_2C_3$

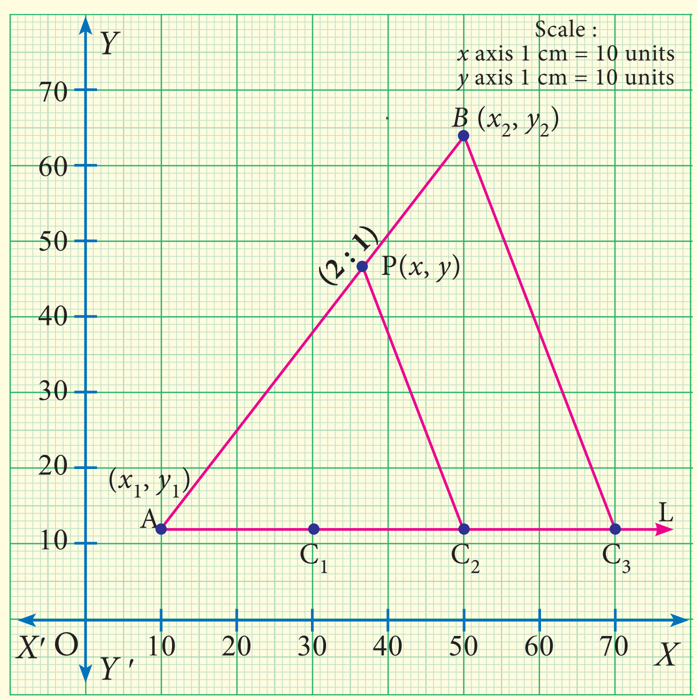


Fig. 2.21

4. Join BC_3 and draw a line parallel to BC_3 through C_2. This line touches AB at $P(x,y)$

5. Now P divides AB internally in the ratio 2:1. (that is m:n)

Observation

(i) In the diagram the coordinates of $A(x_1, y_1) =$ ____________,
$B(x_2, y_2) =$ ____________ and $P(x, y) =$ ____________.

(ii) The point P which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio 2:1 (using section formula) is __________.

Note

(i) The line joining the points (x_1, y_1) and (x_2, y_2) is divided by x-axis in the ratio $\frac{-y_1}{y_2}$ and by y-axis in the ratio $\frac{-x_1}{x_2}$.

(ii) If three points are collinear, then one of the points divide the line segment joining the other two points in the ratio $r : 1$.

(iii) Remember that the section formula can be used only when the given three points are collinear.

(iv) This formula is helpful to find the centroid, incenter and excenters of a triangle. It has applications in physics too; it helps to find the center of mass of systems, equilibrium points and many more.

Example 2.7

Find the coordinates of the point which divides the line segment joining the points (3,5) and (8,–10) internally in the ratio 3:2.

Solution

Let $A(3,5)$, $B(8,-10)$ be the given points and let the point $P(x,y)$ divides the line segment AB internally in the ratio 3:2.

By section formula,

$$P(x, y) = P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Here $x_1 = 3$, $y_1 = 5$, $x_2 = 8$, $y_2 = -10$ and $m = 3$, $n = 2$

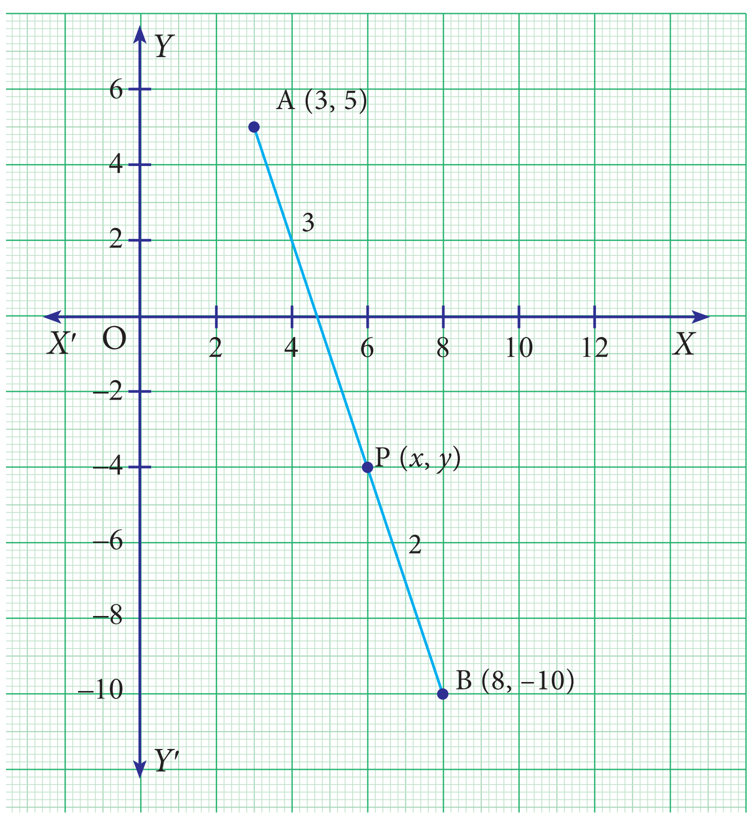
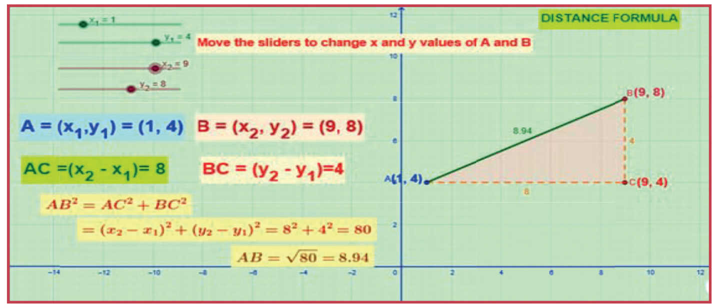


Fig. 2.22

$$\text{Therefore } P(x, y) = P\left(\frac{3(8) + 2(3)}{3+2}, \frac{3(-10) + 2(5)}{3+2}\right) = P\left(\frac{24+6}{5}, \frac{-30+10}{5}\right) = P(6, -4)$$

ICT Corner

Expected Result is shown in this picture



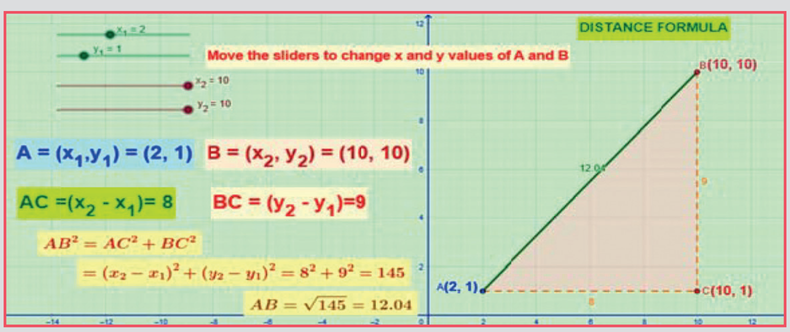
Step – 1

Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Co-ordinate Geometry" will open. There are two worksheets under the title Distance Formula and Section Formula.

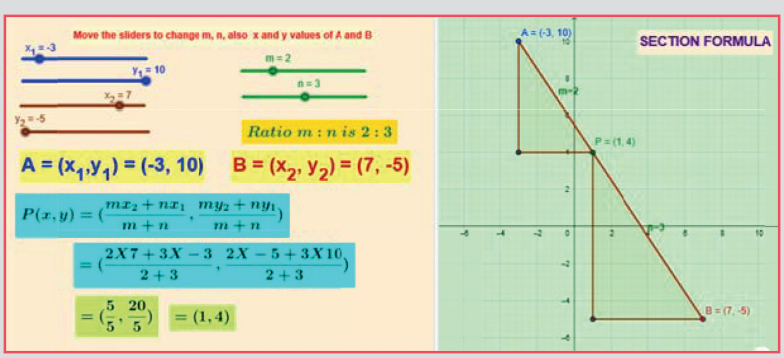
Step - 2

Move the sliders of the respective values to change the points and ratio. Work out the solution and check and click on the respective check box and check the answer.

Step 1



Step 2



Browse in the link

Co-Ordinate Geometry: https://ggbm.at/sfszfe24 or Scan the QR Code.

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Example 2.8

In what ratio does the point $P(-2, 4)$ divide the line segment joining the points $A(-3, 6)$ and $B(1, -2)$ internally?

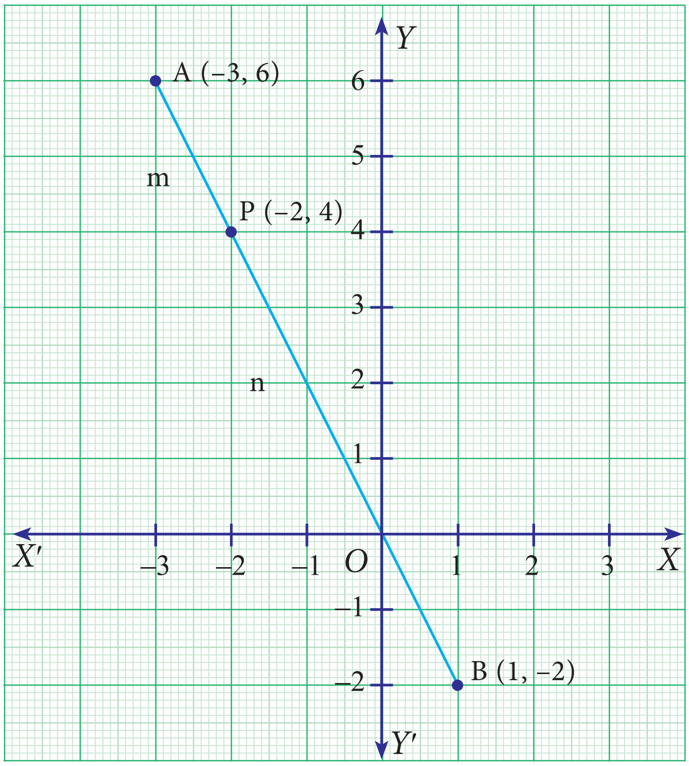


Fig. 2.23

Solution

Given points are $A(-3, 6)$ and $B(1, -2)$. $P(-2, 4)$ divide AB internally in the ratio $m : n$.

By section formula,

$$P(x, y) = P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= P(-2, 4) \quad .....(1)$$

Here $x_1 = -3,\ y_1 = 6,\ x_2 = 1,\ y_2 = -2$

$$(1) \Rightarrow \left(\frac{m(1) + n(-3)}{m+n}, \frac{m(-2) + n(6)}{m+n}\right) = P(-2, 4)$$

Equating x-coordinates, we get

$$\frac{m - 3n}{m+n} = -2 \text{ or } m - 3n = -2m - 2n$$

$$3m = n$$

$$\frac{m}{n} = \frac{1}{3}$$

$$m : n = 1 : 3$$

Note

We may arrive at the same result by also equating the y-coordinates.

Try it.

Hence P divides AB internally in the ratio 1:3.

Example 2.9

What are the coordinates of B if point $P(-2,3)$ divides the line segment joining $A(-3,5)$ and B internally in the ratio 1:6?

Solution

Let $A(-3,5)$ and $B(x_2, y_2)$ be the given two points.

Given $P(-2,3)$ divides AB internally in the ratio 1:6.

By section formula, $P\left(\dfrac{mx_2+nx_1}{m+n}, \dfrac{my_2+ny_1}{m+n}\right) = P(-2,3)$

$$P\left(\frac{1(x_2)+6(-3)}{1+6}, \frac{1(y_2)+6(5)}{1+6}\right) = P(-2,3)$$

Equating the coordinates

$\dfrac{x_2-18}{7} = -2$	$\dfrac{y_2+30}{7} = 3$
$x_2 - 18 = -14$	$y_2 + 30 = 21$
$x_2 = 4$	$y_2 = -9$

Therefore, the coordinate of B is $(4,-9)$

Exercise 2.2

1. Find the coordinates of the point which divides the line segment joining the points $A(4,-3)$ and $B(9,7)$ in the ratio 3:2.

2. Find the coordinates of the point which divides the line segment joining $A(-5,11)$ and $B(4,-7)$ in the ratio 7:2.

3. In what ratio does the point $P(2,-5)$ divide the line segment joining $A(-3,5)$ and $B(4,-9)$.

4. Find the coordinates of a point P on the line segment joining $A(1,2)$ and $B(6,7)$ in such a way that $AP = \dfrac{2}{5}AB$.

5. Find the coordinates of the points of trisection of the line segment joining the points $A(-5,6)$ and $B(4,-3)$.

6. The line segment joining $A(6,3)$ and $B(-1, -4)$ is doubled in length by adding half of AB to each end. Find the coordinates of the new end points.

7. Using section formula, show that the points $A(7, -5)$, $B(9, -3)$ and $C(13,1)$ are collinear.

8. A line segment AB is increased along its length by 25% by producing it to C on the side of B. If A and B have the coordinates $(-2,-3)$ and $(2,1)$ respectively, then find the coordinates of C.

9. A car travels at an uniform speed. At 2 *pm* it is at a distance of 180 *km* and at 6*pm* it is at 360 *km*. Using section formula, find at what distance it will reach 12 midnight.

2.5 The Coordinates of the Centroid

Consider a ΔABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Let AD, BE and CF be the medians of the ΔABC.

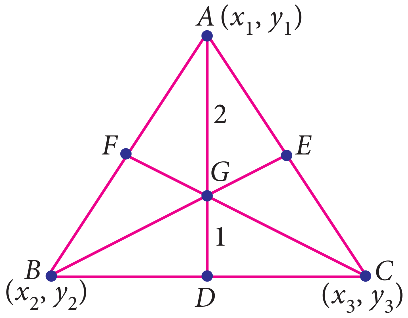


Fig. 2.24

The mid-point of BC is $D\left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$

The centroid G divides the median AD internally in the ratio 2:1 and therefore by section formula, the centroid

$$G(x,y) \text{ is } \left(\frac{\frac{2(x_2+x_3)}{2}+1(x_1)}{2+1}, \frac{\frac{2(y_2+y_3)}{2}+1(y_1)}{2+1}\right) = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

> The centroid G of the triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is
>
> $$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Activity - 3

1. Draw a ΔABC with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ on the graph sheet.
2. Draw medians and locate the centroid of ΔABC

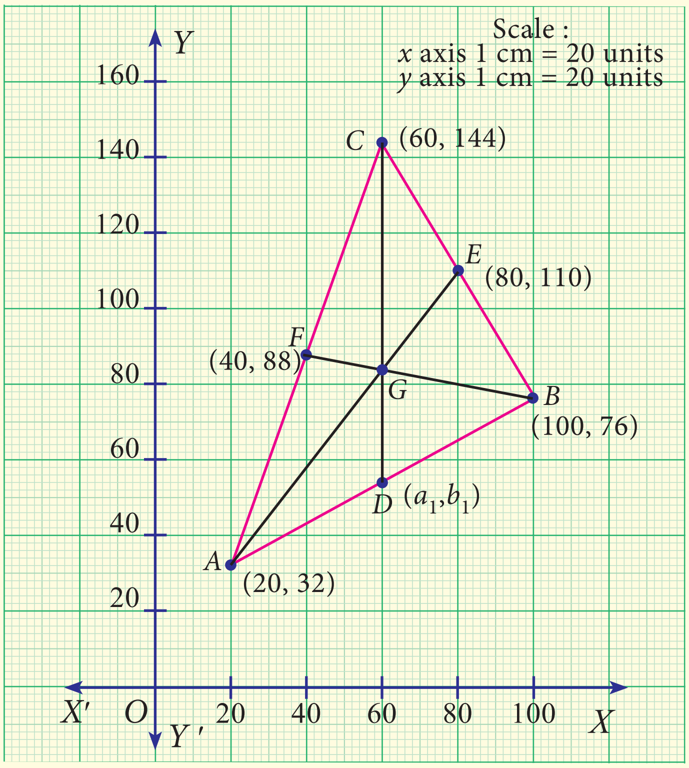


Fig. 2.25

Observation

(i) The coordinates of the vertices of ΔABC where $A(x_1, y_1) =$ ____________, $B(x_2, y_2) =$ ____________ and $C(x_3, y_3) =$ _________

(ii) The coordinates of the centroid G= ___________

(iii) Use the formula to locate the centroid, whose coordinates are = __________.

(iv) Mid-point of AB is ___________.

(v) Find the point which divides the line segment joining (x_3, y_3) and the mid-point of AB internally in the ratio 2:1 is ___________.

Note

1. The medians of a triangle are concurrent and the point of concurrence, the centroid G, is one-third of the distance from the opposite side to the vertex along the median.

2. The centroid of the triangle obtained by joining the mid-points of the sides of a triangle is the same as the centroid of the original triangle.

3. If (a_1, b_1), (a_2, b_2) and (a_3, b_3) are the mid-points of the sides of a triangle ABC then its centroid G is given by

$$G\left(\frac{a_1+a_2+a_3}{3}, \frac{b_1+b_2+b_3}{3}\right)$$

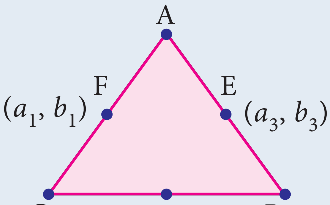


Fig. 2.26

Example 2.10

Find the centroid of the triangle whose veritices are $A(6,-1)$, $B(8,3)$ and $C(10,-5)$.

FZB1QQ

Solution

The centroid $G(x, y)$ of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$G(x, y) = G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

We have $(x_1, y_1) = (6,-1)$; $(x_2, y_2) = (8,3)$;

$(x_3, y_3) = (10,-5)$

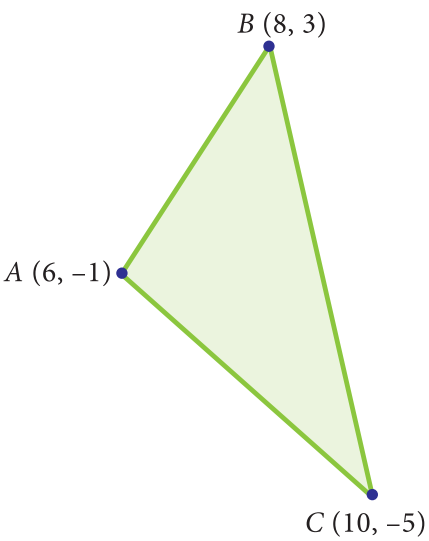


Fig. 2.27

The centroid of the triangle

$$G(x, y) = G\left(\frac{6+8+10}{3}, \frac{-1+3-5}{3}\right)$$

$$= G\left(\frac{24}{3}, \frac{-3}{3}\right) = G(8,-1)$$

Note

1. The Euler line of a triangle is the line that passes through the orthocenter (H), centroid (G) and the circumcenter (S). G divides H and S in the ratio 2:1 from the orthocenter. That is centroid divides orthocenter and circumcenter internally in the ratio 2:1.

2. In an equilateral triangle, orthocentre, incentre, centroid and circumcentre are all the same.

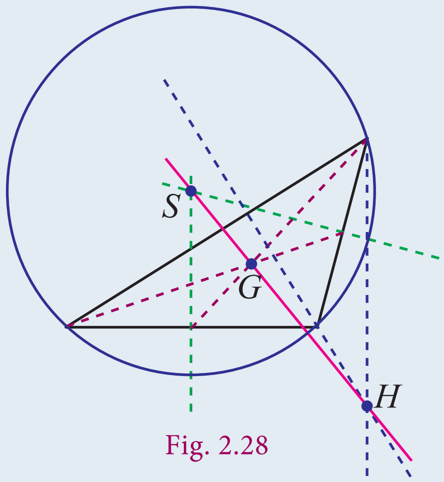


Fig. 2.28

Example 2.11 If the centroid of a triangle is at $(-2,1)$ and two of its vertices are $(1,-6)$ and $(-5,2)$, then find the third vertex of the triangle.

Solution

Let the vertices of a triangle be

$A(1,-6),\ B(-5,2)$ and $C(x_3, y_3)$

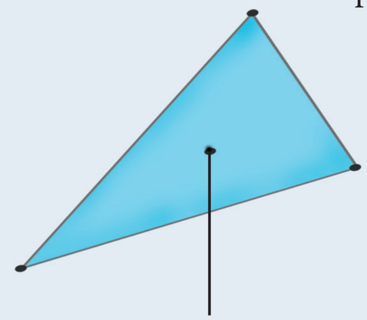
Given the centroid of a triangle as $(-2,1)$ we get,

$\frac{x_1+x_2+x_3}{3}=-2$	$\frac{y_1+y_2+y_3}{3}=1$
$\frac{1-5+x_3}{3}=-2$	$\frac{-6+2+y_3}{3}=1$
$-4+x_3=-6$	$-4+y_3=3$
$x_3=-2$	$y_3=7$

Therefore, third vertex is (–2,7).

Thinking Corner

Master gave a trianglular plate with vertices A(5, 8), B(2, 4), C(8, 3) and a stick to a student. He wants to balance the plate on the stick. Can you help the boy to locate that point which can balance the plate.



Exercise 2.3

1. Find the centroid of the triangle whose vertices are

   (i) (2,–4), (–3,–7) and (7,2) (ii) (–5,–5), (1,–4) and (–4,–2)

2. If the centroid of a triangle is at (4,–2) and two of its vertices are (3,–2) and (5,2) then find the third vertex of the triangle.

3. Find the length of median through A of a triangle whose vertices are $A(-1,3)$, $B(1,-1)$ and $C(5,1)$.

4. The vertices of a triangle are $(1,2)$, $(h,-3)$ and $(-4,k)$. If the centroid of the triangle is at the point $(5,-1)$ then find the value of $\sqrt{(h+k)^2+(h+3k)^2}$.

5. Orthocentre and centroid of a triangle are $A(-3,5)$ and $B(3,3)$ respectively. If C is the circumcentre and AC is the diameter of this cicle, then find the radius of the circle.

6. ABC is a triangle whose vertices are $A(3,4)$, $B(-2,-1)$ and $C(5,3)$. If G is the centroid and $BDCG$ is a parallelogram then find the coordinates of the vertex D.

7. If $\left(\frac{3}{2},5\right)$, $\left(7,\frac{-9}{2}\right)$ and $\left(\frac{13}{2},\frac{-13}{2}\right)$ are mid-points of the sides of a triangle, then find the centroid of the triangle.

FZGA4G

Exercise 2.4

Multiple choice questions

1. The coordinates of the point C dividing the line segment joining the points $P(2,4)$ and $Q(5,7)$ internally in the ratio 2:1 is

(1) $\left(\frac{7}{2},\frac{11}{2}\right)$ (2) $(3,5)$ (3) $(4,4)$ (4) $(4,6)$

2. If $P\left(\frac{a}{3},\frac{b}{2}\right)$ is the mid-point of the line segment joining $A(-4,3)$ and $B(-2,4)$ then (a,b) is

(1) $(-9,7)$ (2) $\left(-3,\frac{7}{2}\right)$ (3) $(9,-7)$ (4) $\left(3,-\frac{7}{2}\right)$

3. In what ratio does the point $Q(1,6)$ divide the line segment joining the points $P(2,7)$ and $R(-2,3)$

(1) 1:2 (2) 2:1 (3) 1:3 (4) 3:1

4. If the coordinates of one end of a diameter of a circle is (3,4) and the coordinates of its centre is (−3,2), then the coordinate of the other end of the diameter is

(1) $(0,-3)$ (2) $(0,9)$ (3) $(3,0)$ (4) $(-9,0)$

5. The ratio in which the x-axis divides the line segment joining the points $A(a_1,b_1)$ and $B(a_2,b_2)$ is

(1) $b_1:b_2$ (2) $-b_1:b_2$ (3) $a_1:a_2$ (4) $-a_1:a_2$

6. The ratio in which the x-axis divides the line segment joining the points (6,4) and (1, −7) is

(1) 2:3 (2) 3:4 (3) 4:7 (4) 4:3

7. If the coordinates of the mid-points of the sides AB, BC and CA of a triangle are (3,4), (1,1) and (2,−3) respectively, then the vertices A and B of the triangle are

(1) (3,2), (2,4) (2) (4,0), (2,8)
(3) (3,4), (2,0) (4) (4,3), (2,4)

8. The mid-point of the line joining $(-a,2b)$ and $(-3a,-4b)$ is

(1) $(2a,3b)$ (2) $(-2a,-b)$ (3) $(2a,b)$ (4) $(-2a,-3b)$

9. In what ratio does the y-axis divides the line joining the points (−5,1) and (2,3) internally

(1) 1:3 (2) 2:5 (3) 3:1 (4) 5:2

10. If (1,−2), (3,6), $(x,10)$ and (3,2) are the vertices of the parallelogram taken in order, then the value of x is

(1) 6 (2) 5 (3) 4 (4) 3

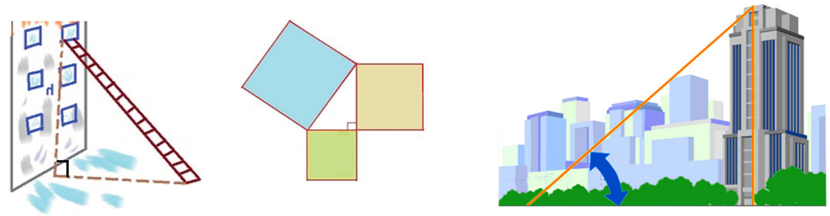
11. The centroid of the triangle with vertices (−1,−6), (−2,12) and (9,3) is

(1) (3,2) (2) (2,3) (3) (4,3) (4) (3,4)

Points to Remember

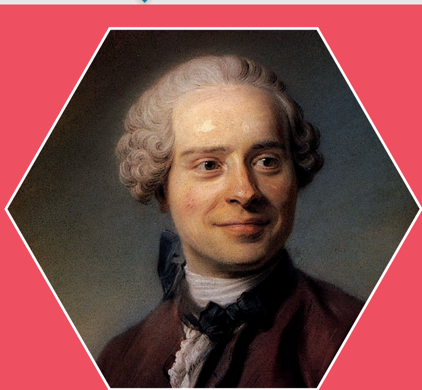
- The mid-point M of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- The point P which divides the line segment joining the two points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m:n$ is $P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
- The centroid G of the triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is $G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
- The centroid of the triangle obtained by joining the mid-points of the sides of a triangle is the same as the centroid of the original triangle.

3



TRIGONOMETRY

There is perhaps nothing which so occupies the middle position of mathematics as Trigonometry.- J. F. Herbart



Leonhard Euler
A.D (C.E) 1707 - 1783

Euler, like Newton, was the greatest mathematician of his generation. He studied all areas of mathematics and continued to work hard after he had gone blind. Euler made discoveries in many areas of mathematics, especially calculus and Trigonometry. He was the first to prove several theorems in geometry.

FZJHSD

Learning Outcomes

- To learn the trigonometric ratios.
- To understand the relationship among various trigonometric ratios.
- To recognize the values of trigonometric ratios and their reciprocals.
- To use the concept of complementary angles.
- To understand the usage of trigonometric tables.

3.1 Introduction

Let us recall Pythagoras theorem, since we will be frequently making use of right angled triangles.

In a right angled triangle, the side opposite to the right angle is the hypotenuse, the other two sides are called the legs. In the figure 3.1 , a and b are the length of the legs and c is the length of the hypotenuse. Then by using Pythagoras theorem we get $a^2 + b^2 = c^2$.

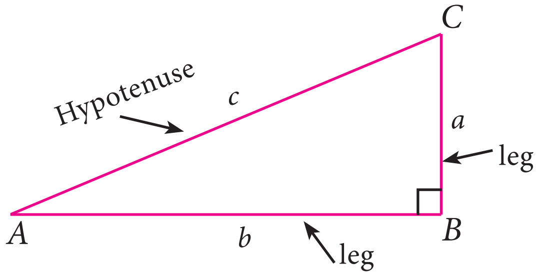


Fig. 3.1

Thus, one can guess that a triangle with side lengths 3, 4 and 5 units will be right angled, since $3^2 + 4^2 = 5^2$. (the hypotenuse will be of length 5 units. Why?) Can a triangle of sides 5, 12 and 13 be right-angled? How about a triangle with side lengths 8, 10 and 12?

Trigonometry (which comes from Greek word trigonon means triangle and metron means measure) is the branch of mathematics that studies the relationships involving lengths of sides and measures of angles of triangles. It is a useful tool for engineers, scientists, and surveyors and is applied even in seismology and navigation.

Observe the three given right angled triangles; in particular scrutinize their measures. The corresponding angles shown in the three triangles are of the same size. Draw your attention to the lengths of "opposite" sides (meaning the side opposite to the given angle) and the "adjacent" sides (which is the side adjacent to the given angle) of the triangle.

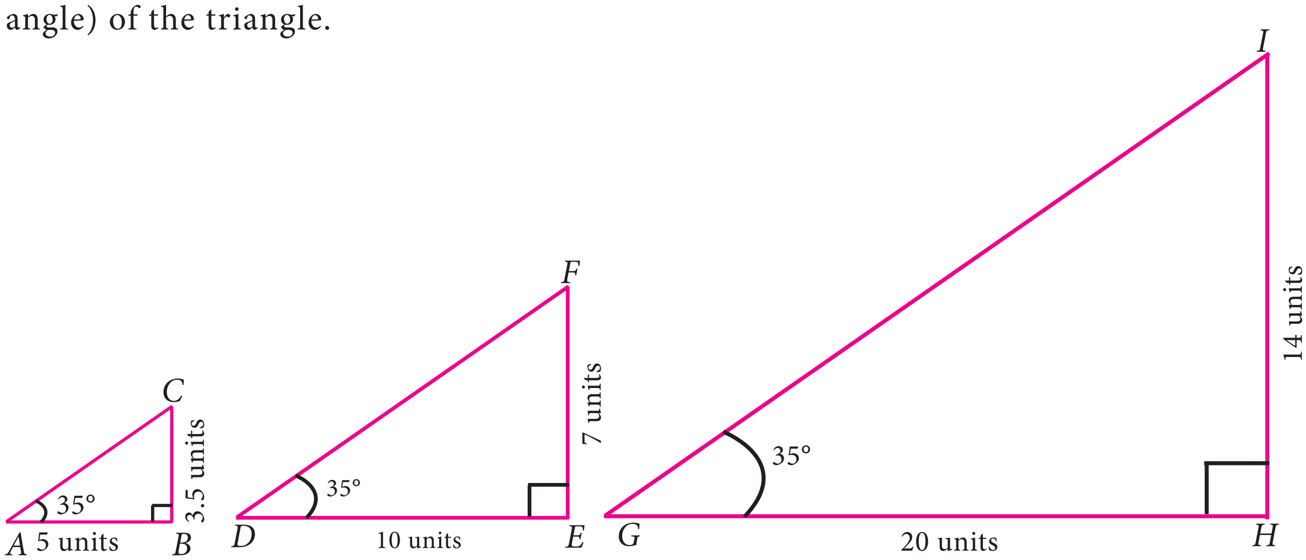


Fig. 3.2

What can you say about the ratio $\frac{\textit{opposite side}}{\textit{adjacent side}}$ in each case? Every right angled triangle given here has the same ratio 0.7 ; based on this finding, now what could be the length of the side marked 'x' in the adjacent figure? Is it 15?

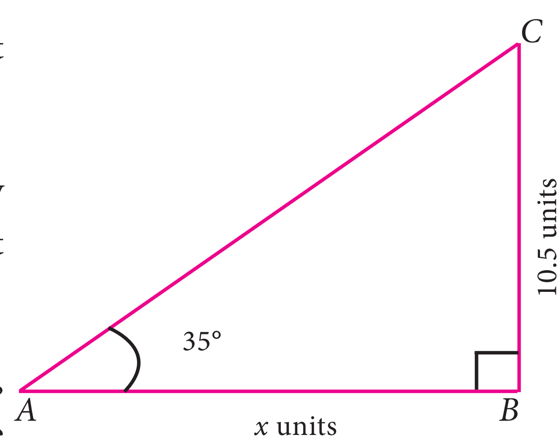
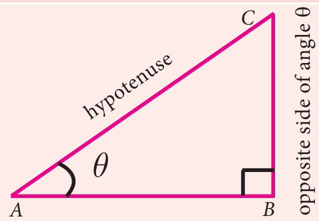
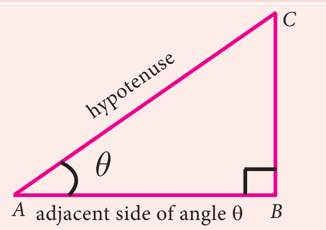
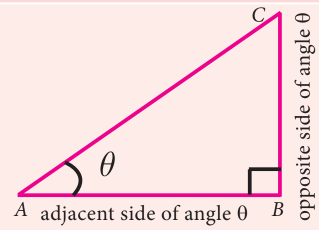


Fig. 3.3

Such remarkable ratios stunned early mathematicians and paved the way for the subject of trigonometry.

There are three basic ratios in trigonometry, each of which is one side of a right-angled triangle divided by another.

The three ratios are:

Name of the angle	sine	cosine	tangent
Short form	sin	cos	tan
Related measurements			
Relationship	$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$	$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$	$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$

Example 3.1 For the measures in the figure, compute sine, cosine and tangent ratios of the angle θ.

Solution

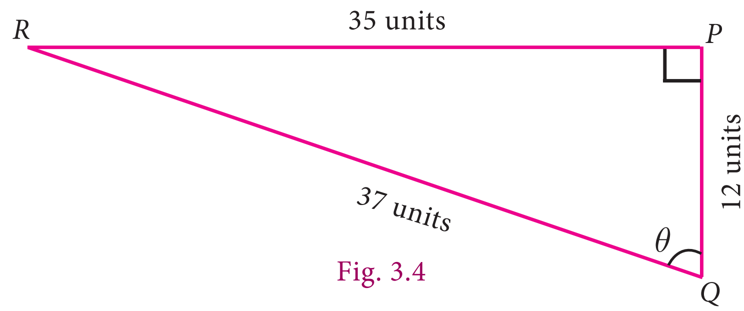


Fig. 3.4

In the given right angled triangle, note that for the given angle θ, PR is the 'opposite' side and PQ is the 'adjacent' side.

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{PR}{QR} = \frac{35}{37}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{PQ}{QR} = \frac{12}{37}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{PR}{PQ} = \frac{35}{12}$$

FZQ664

It is enough to leave the ratios as fractions. In case, if you want to simplify each ratio neatly in a terminating decimal form, you may opt for it, but that is not obligatory.

Note

Since trigonometric ratios are defined in terms of ratios of sides, they are unitless numbers.

Ratios like $\sin\theta$, $\cos\theta$, $\tan\theta$ are not to be treated like $(\sin)\times(\theta)$, $(\cos)\times(\theta)$, $(\tan)\times(\theta)$.

Thinking Corner

The given triangles ABC, DEF and GHI have measures 3-4-5, 6-8-10 and 12-16-20.

Are they all right triangles?

How do you know?

The angles at the vertices B, E and H are of equal size (each angle is equal to θ).

With these available details, fill up the following table and comment on the ratios that you get.

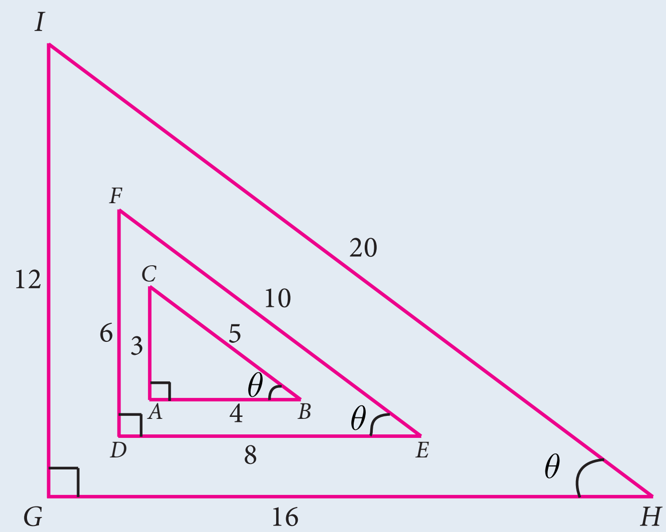


Fig. 3.5

In ΔABC	In ΔDEF	In ΔGHI
$\sin\theta = \frac{3}{5}$	$\sin\theta = \frac{6}{10} = ?$	$\sin\theta = \frac{12}{20} = ?$
$\cos\theta = ?$	$\cos\theta = ?$	$\cos\theta = ?$
$\tan\theta = \frac{3}{4}$	$\tan\theta = ?$	$\tan\theta = ?$

Reciprocal ratios

We defined three basic trigonometric ratios namely, sine, cosine and tangent. The reciprocals of these ratios are also often useful during calculations. We define them as follows:

Basic Trigonometric Ratios	Its reciprocal	Short form
$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$	$\text{cosecant}\ \theta = \frac{\text{hypotenuse}}{\text{opposite side}}$	$\text{cosec}\ \theta = \frac{\text{hypotenuse}}{\text{opposite side}}$
$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$	$\text{secant}\ \theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$	$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$
$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$	$\text{cotangent}\ \theta = \frac{\text{adjacent side}}{\text{opposite side}}$	$\cot\theta = \frac{\text{adjacent side}}{\text{opposite side}}$

From the above ratios we can observe easily the following relations:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

$$\sin\theta = \frac{1}{\operatorname{cosec}\theta} \qquad \cos\theta = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

$(\sin\theta)\times(\operatorname{cosec}\theta) = 1$. We usually write this as $\sin\theta\cdot\operatorname{cosec}\theta = 1$.

$(\cos\theta)\times(\sec\theta) = 1$. We usually write this as $\cos\theta.\sec\theta = 1$.

$(\tan\theta)\times(\cot\theta) = 1$. We usually write this as $\tan\theta\ .\ \cot\theta = 1$.

Example 3.2 Find the six trigonometric ratios of the angle θ using the given diagram.

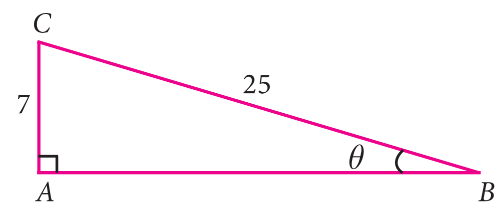


Fig. 3.6

Solution

By Pythagoras theorem,

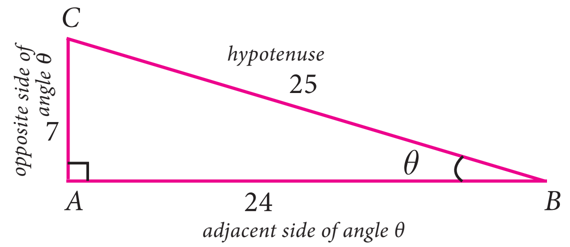


Fig. 3.7

$$AB = \sqrt{BC^2 - AC^2}$$
$$= \sqrt{(25)^2 - 7^2}$$
$$= \sqrt{625 - 49} = \sqrt{576} = 24$$

The six trignometric ratios are

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{7}{25} \qquad \cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{24}{25}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{7}{24} \qquad \operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{25}{7}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{25}{24} \qquad \cot\theta = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{24}{7}$$

Example 3.3 If $\tan A = \frac{2}{3}$, then find all the other trigonometric ratios.

Solution

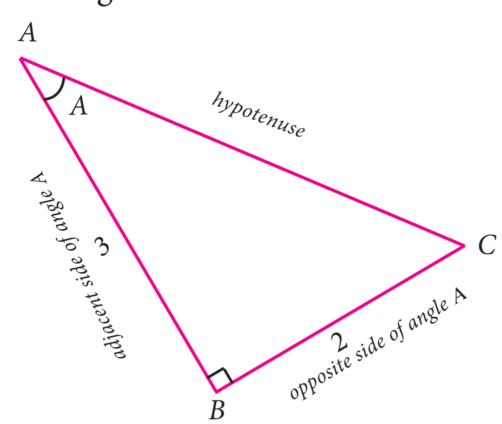


Fig. 3.8

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2}{3}$$

By Pythagoras theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$AC = \sqrt{13}$$

$$\sin A = \frac{\textit{opposite side}}{\textit{hypotenuse}} = \frac{2}{\sqrt{13}} \qquad \cos A = \frac{\textit{adjacent side}}{\textit{hypotenuse}} = \frac{3}{\sqrt{13}}$$

$$\operatorname{cosec} A = \frac{\textit{hypotenuse}}{\textit{opposite side}} = \frac{\sqrt{13}}{2} \qquad \sec A = \frac{\textit{hypotenuse}}{\textit{adjacent side}} = \frac{\sqrt{13}}{3}$$

$$\cot A = \frac{\textit{adjacent side}}{\textit{opposite side}} = \frac{3}{2}$$

Example 3.4 If $\sec\theta = \frac{13}{5}$, then show that $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

Solution:

Let $BC = 13$ and $AB = 5$

$$\sec\theta = \frac{\textit{hypotenuse}}{\textit{adjacent side}} = \frac{BC}{AB} = \frac{13}{5}$$

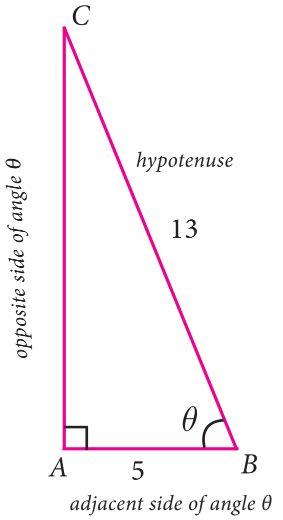


Fig. 3.9

By the Pythagoras theorem,

$$AC = \sqrt{BC^2 - AB^2}$$

$$= \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} = \sqrt{144} = 12$$

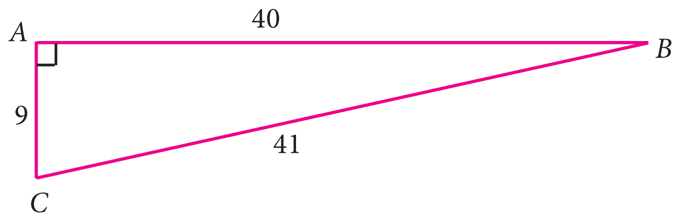
Therefore, $\sin\theta = \frac{AC}{BC} = \frac{12}{13}$; $\cos\theta = \frac{AB}{BC} = \frac{5}{13}$

$$LHS = \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} - 9\times\frac{5}{13}} = \frac{\frac{24-15}{13}}{\frac{48-45}{13}} = \frac{9}{3} = 3 = RHS$$

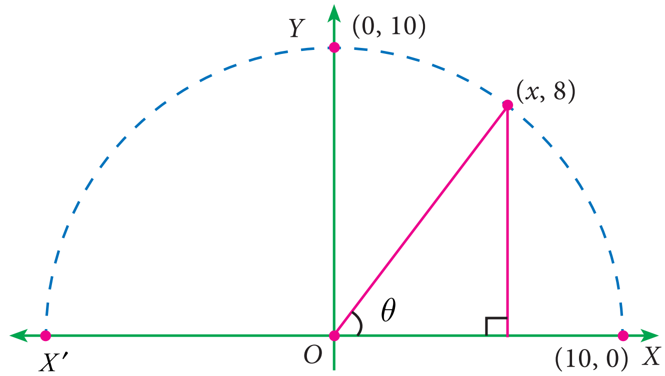
Note: We can also take the angle 'θ' at the vertex 'C' and proceed in the same way.

Exercise 3.1

1. From the given figure find all the trigonometric ratios of angle B.



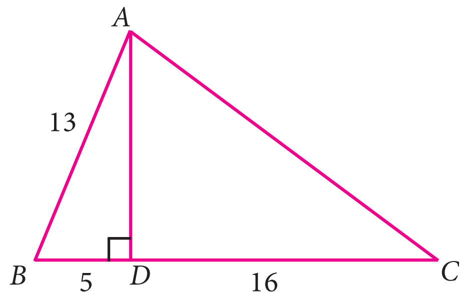
2. From the given figure, find all the trigonometric ratios of angle θ.



3. From the given figure, find the values of

(i) $\sin B$ (ii) $\sec B$ (iii) $\cot B$

(iv) $\cos C$ (v) $\tan C$ (vi) $\operatorname{cosec} C$



4. If $2\cos\theta = \sqrt{3}$, then find all the trigonometric ratios of angle θ.

5. If $\cos A = \dfrac{3}{5}$, then find the value of $\dfrac{\sin A - \cos A}{2\tan A}$.

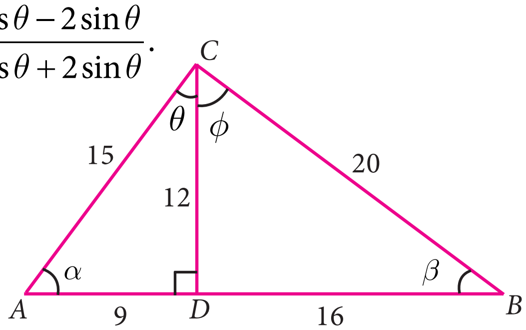
6. If $\cos A = \dfrac{2x}{1+x^2}$, then find the values of $\sin A$ and $\tan A$ in terms of x.

7. If $\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$, then show that $b\sin\theta = a\cos\theta$.

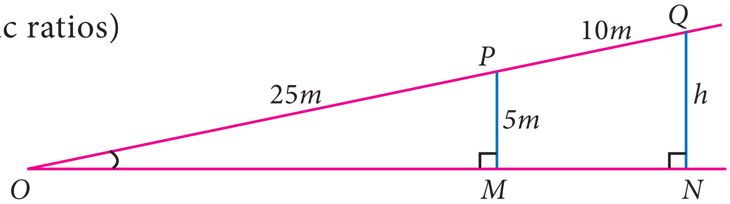
8. If $3\cot A = 2$, then find the value of $\dfrac{4\sin A - 3\cos A}{2\sin A + 3\cos A}$.

9. If $\cos\theta : \sin\theta = 1:2$, then find the value of $\dfrac{8\cos\theta - 2\sin\theta}{4\cos\theta + 2\sin\theta}$.

10. From the given figure, prove that $\theta + \phi = 90°$. Also prove that there are two other right angled triangles. Find $\sin\alpha$, $\cos\beta$ and $\tan\phi$.



11. A boy standing at a point O finds his kite flying at a point P with distance $OP=25m$. It is at a height of $5m$ from the ground. When the thread is extended by $10m$ from P, it reaches a point Q. What will be the height QN of the kite from the ground? (use trigonometric ratios)



3.2 Trigonometric Ratios of Some Special Angles

The values of trigonometric ratios of certain angles can be obtained geometrically. Two special triangles come to our help here.

3.2.1 Trigonometric ratios of 45°

Consider a triangle *ABC* with angles $45°$, $45°$ and $90°$ as shown in the figure 3.10.

It is the shape of half a square, cut along the square's diagonal. Note that it is also an isosceles triangle (both legs have the same length, *a* units).

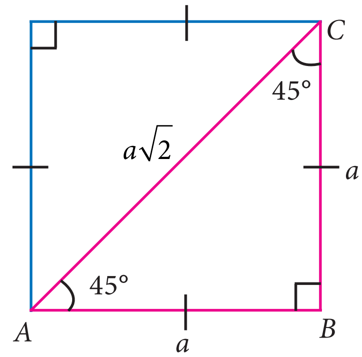


Fig. 3.10

Use Pythagoras theorem to check if the diagonal is of length $a\sqrt{2}$.

Now, from the right-angled triangle *ABC*,

$$\sin 45^o = \frac{opposite\,side}{hypotenuse} = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos 45^o = \frac{adjacent\ side}{hypotenuse} = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan 45^o = \frac{opposite\ side}{adjacent\ side} = \frac{BC}{AB} = \frac{a}{a} = 1$$

from these you can easily write

$\text{cosec}\, 45° = \sqrt{2}$;

$\sec 45° = \sqrt{2}$ and

$\cot 45° = 1$

3.2.2 Trigonometric Ratios of 30° and 60°

Consider an equilateral triangle *PQR* of side length 2 units.

Draw a bisector of $\angle P$. Let it meet *QR* at *M*.

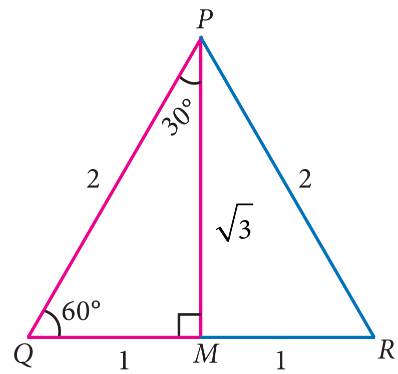


Fig. 3.11

$PQ = QR = RP = 2$ units.

$QM = MR = 1$ unit (Why?)

Knowing *PQ* and *QM*, we can find *PM*, using Pythagoras theorem,

we find that $PM = \sqrt{3}$ units.

Now, from the right-angled triangle *PQM*,

$$\sin 30^o = \frac{opposite\,side}{hypotenuse} = \frac{QM}{PQ} = \frac{1}{2}$$

from these you can easily write

$\text{cosec}\,30° = 2, \sec 30° = \frac{2}{\sqrt{3}}$

and $\cot 30° = \sqrt{3}$

$$\cos 30^o = \frac{adjacent\ side}{hypotenuse} = \frac{PM}{PQ} = \frac{\sqrt{3}}{2}$$

$$\tan 30^o = \frac{opposite\ side}{adjacent\ side} = \frac{QM}{PM} = \frac{1}{\sqrt{3}}$$

We will use the same triangle but the other angle of measure 60° now.

$$\sin 60^o = \frac{opposite\ side}{hypotenuse} = \frac{PM}{PQ} = \frac{\sqrt{3}}{2}$$

$$\cos 60^o = \frac{adjacent\ side}{hypotenuse} = \frac{QM}{PQ} = \frac{1}{2}$$

$$\tan 60^o = \frac{opposite\ side}{adjacent\ side} = \frac{PM}{QM} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

from these you can easily write

$\text{cosec}60° = \frac{2}{\sqrt{3}}$; $\sec 60° = 2$

and $\cot 60° = \frac{1}{\sqrt{3}}$

Activity - 1

With the values obtained above, fill in the blanks of the given table.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°			
45°			
60°			

3.2.3 Trigonometric ratios of 0° and 90°

To find the trigonometric ratios of 0° and 90°, we take the help of what is known as a unit circle.

A unit circle is a circle of unit radius (that is of radius 1 unit), centred at the origin.

Why make a circle where the radius is 1unit?

This means that every reference triangle that we create here has a hypotenuse of 1unit, which makes it so much easier to compare angles and ratios.

We will be interested only in the positive values since we consider 'lengths' and it is hence enough to concentrate on the first quadrant.

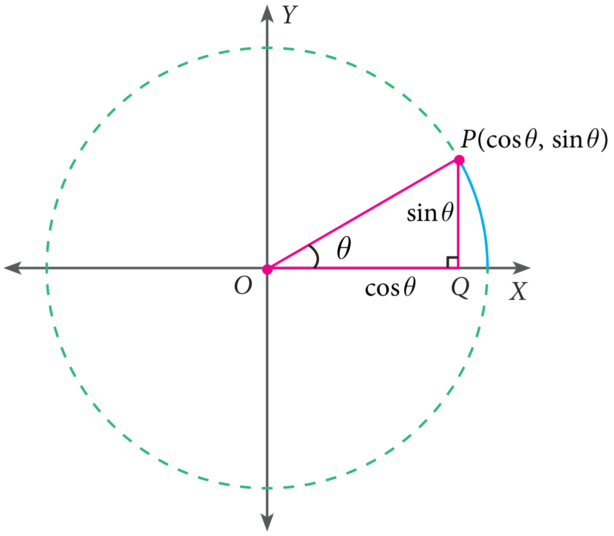


Fig. 3.12

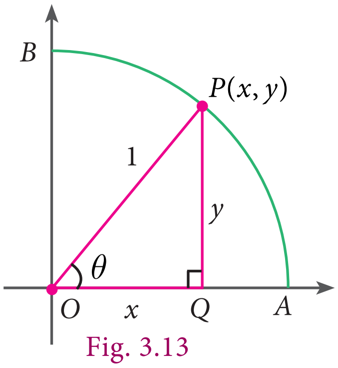


Fig. 3.13

We can see that if $P(x,y)$ be any point on the unit circle in the first quadrant and $\angle POQ = \theta$

$$\sin\theta = \frac{PQ}{OP} = \frac{y}{1} = y \;;\quad \cos\theta = \frac{OQ}{OP} = \frac{x}{1} = x \;;\quad \tan\theta = \frac{PQ}{OQ} = \frac{y}{x}$$

When $\theta = 0°$, OP coincides with OA, where A is $(1,0)$ giving $x = 1,\ y = 0$.

We get thereby,

$$\sin 0° = 0 \;;\quad \text{cosec}\, 0° = \text{not defined (why?)}$$

$$\cos 0° = 1 \;;\quad \sec 0° = 1$$

$$\tan 0° = \frac{0}{1} = 0 \;;\quad \cot 0° = \text{not defined (why?)}$$

When $\theta = 90°$, OP coincides with OB, where B is $(0,1)$ giving $x = 0,\ y = 1$.

Hence,

$$\sin 90° = 1 \;;\quad \text{cosec}\, 90° = 1$$

$$\cos 90° = 0 \;;\quad \sec 90° = \text{not defined}$$

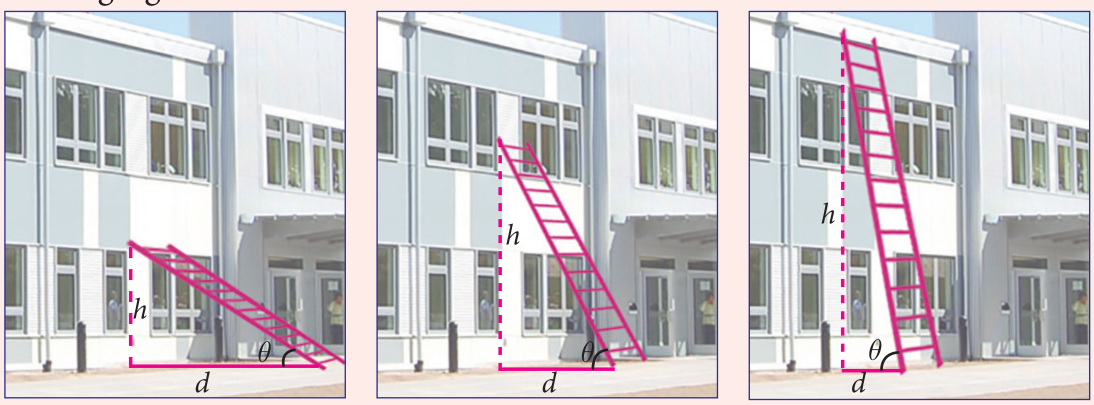
$$\tan 90° = \frac{1}{0} = \text{not defined} \;;\quad \cot 90° = 0$$

Let us summarise all the results in the table given below:

Trigonometric ratio \ θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined
$\text{cosec}\,\theta$	not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	not defined
$\cot\theta$	not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Thinking Corner

The various positions of a ladder used by a painter in painting a wall are given in the following figures.



i ii iii

Fig. 3.14

Observe the three right angled triangles formed by the ladder with the wall. Discuss the change in the values of (i) d (ii) h (iii) θ (iv) hypotenuse (v) stability of the painter while painting.

Example 3.5

Evaluate: (i) $\sin 30° + \cos 30°$ (ii) $\tan 60° . \cot 60°$

(iii) $\dfrac{\tan 45°}{\tan 30° + \tan 60°}$

(iv) $\sin^2 45° + \cos^2 45°$

Note

(i) $(\sin\theta)^2$ is written as $\sin^2\theta = (\sin\theta)\times(\sin\theta)$

(ii) $(\sin\theta)^2$ is not written as $\sin\theta^2$, because it may mean as $\sin(\theta\times\theta)$.

Solution

(i) $\sin 30° + \cos 30° = \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} = \dfrac{1+\sqrt{3}}{2}$

(ii) $\tan 60° . \cot 60° = \sqrt{3} \times \dfrac{1}{\sqrt{3}} = 1$

(iii) $$\frac{\tan 45°}{\tan 30° + \tan 60°} = \frac{1}{\frac{1}{\sqrt{3}} + \frac{\sqrt{3}}{1}} = \frac{1}{\frac{1+\left(\sqrt{3}\right)^2}{\sqrt{3}}} = \frac{1}{\frac{1+3}{\sqrt{3}}} = \frac{\sqrt{3}}{4}$$

(iv) $$\sin^2 45° + \cos^2 45° = \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1^2}{\left(\sqrt{2}\right)^2} + \frac{1^2}{\left(\sqrt{2}\right)^2} = \frac{1}{2} + \frac{1}{2} = 1$$

Thinking Corner

The following sets of three numbers are called as Pythagorean triplets as they form the sides of a right angled triangle:

(i) 3, 4, 5 (ii) 5, 12, 13 (iii) 7, 24, 25

Progress Check

Multiply each number in any of the above Pythagorean triplet by a non-zero constant. Verify whether each of the resultant set so obtained is also a Pythagorean triplet or not.

Example 3.6 Find the values of the following:

(i) $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

(ii) $\tan^2 60° - 2\tan^2 45° - \cot^2 30° + 2\sin^2 30° + \frac{3}{4}\operatorname{cosec}^2 45°$

Solution

(i) $(\cos 0° + \sin 45° + \sin 30°)(\sin 90° - \cos 45° + \cos 60°)$

$$= \left[1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right]\left[1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right]$$

$$= \left[\frac{2\sqrt{2} + 2 + \sqrt{2}}{2\sqrt{2}}\right]\left[\frac{2\sqrt{2} - 2 + \sqrt{2}}{2\sqrt{2}}\right] = \left[\frac{3\sqrt{2}+2}{2\sqrt{2}}\right]\left[\frac{3\sqrt{2}-2}{2\sqrt{2}}\right]$$

$$= \frac{18-4}{4\left(\sqrt{2}\right)^2} = \frac{14}{4\times 2} = \frac{7}{4}$$

(ii) $\tan^2 60° - 2\tan^2 45° - \cot^2 30° + 2\sin^2 30° + \frac{3}{4}\operatorname{cosec}^2 45°$

$$= \left(\sqrt{3}\right)^2 - 2(1)^2 - \left(\sqrt{3}\right)^2 + 2\left(\frac{1}{2}\right)^2 + \frac{3}{4}\left(\sqrt{2}\right)^2$$

$$= 3 - 2 - 3 + \frac{1}{2} + \frac{3}{2}$$

$$= -2 + \frac{4}{2} = -2 + 2 = 0$$

Activity - 2

In a graph sheet draw the triangle OBA with the following measurements, shown in the figure 3.15.

- Observe that the sides are in the ratio $5:5:5\sqrt{2}$ that is $1:1:\sqrt{2}$
- Measure the angles $\angle A$, $\angle B$ and $\angle O$
- We get $\angle A = 45°, \angle B = 45°, \angle O = 90°$
- Therefore, $\angle A : \angle B : \angle O$ is $45° : 45° : 90°$

Draw different triangles with sides in the ratio $1:1:\sqrt{2}$. Measure the angles each time and record it. What do you observe?

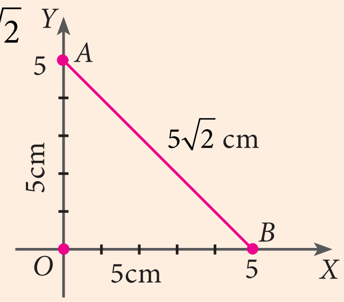


Fig. 3.15

Note

(i) In a right angled triangle, if the angles are in the ratio $45° : 45° : 90°$, then the sides are in the ratio $1:1:\sqrt{2}$.

(ii) Similarly, if the angles are in the ratio $30° : 60° : 90°$, then the sides are in the ratio $1:\sqrt{3}:2$.

(The two set squares in your geometry box is one of the best example for the above two types of triangles).

Exercise 3.2

1. Verify the following equalities:

   (i) $\sin^2 60° + \cos^2 60° = 1$

   (ii) $1 + \tan^2 30° = \sec^2 30°$

   (iii) $\cos 90° = 1 - 2\sin^2 45° = 2\cos^2 45° - 1$

   (iv) $\sin 30° \cos 60° + \cos 30° \sin 60° = \sin 90°$

2. Find the value of the following:

   (i) $\dfrac{\tan 45°}{\text{cosec} 30°} + \dfrac{\sec 60°}{\cot 45°} - \dfrac{5\sin 90°}{2\cos 0°}$

   (ii) $(\sin 90° + \cos 60° + \cos 45°) \times (\sin 30° - \cos 0° + \cos 45°)$

   (iii) $\sin^2 30° - 2\cos^3 60° + 3\tan^4 45°$

3. Verify $\cos 3A = 4\cos^3 A - 3\cos A$, when $A = 30°$

4. Find the value of $8\sin 2x.\cos 4x.\sin 6x$, when $x = 15°$

3.3 Trigonometric Ratios for Complementary Angles

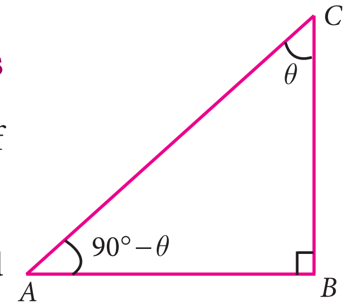


Fig. 3.16

Recall that two acute angles are said to be complementary if the sum of their measures is equal to $90°$.

What can we say about the acute angles of a right-angled triangle?

In a right angled triangle the sum of the two acute angles is equal to 90°. So, the two acute angles of a right angled triangle are always complementary to each other.

In the above figure 3.16, the triangle is right-angled at B. Therefore, if $\angle C$ is θ, then $\angle A = 90° - \theta$.

We find that

$$\left.\begin{aligned} \sin\theta &= \frac{AB}{AC} & \operatorname{cosec}\theta &= \frac{AC}{AB} \\ \cos\theta &= \frac{BC}{AC} & \sec\theta &= \frac{AC}{BC} \\ \tan\theta &= \frac{AB}{BC} & \cot\theta &= \frac{BC}{AB} \end{aligned}\right\} \text{....(1)}$$

Similarly for the angle (90° – θ), We have

$$\left.\begin{aligned} \sin(90°-\theta) &= \frac{BC}{AC} & \operatorname{cosec}(90°-\theta) &= \frac{AC}{BC} \\ \cos(90°-\theta) &= \frac{AB}{AC} & \sec(90°-\theta) &= \frac{AC}{AB} \\ \tan(90°-\theta) &= \frac{BC}{AB} & \cot(90°-\theta) &= \frac{AB}{BC} \end{aligned}\right\} \text{.....(2)}$$

Comparing (1) and (2), we get

$$\begin{aligned} \sin\theta &= \cos(90°-\theta) & \operatorname{cosec}\theta &= \sec(90°-\theta) \\ \cos\theta &= \sin(90°-\theta) & \sec\theta &= \operatorname{cosec}(90°-\theta) \\ \tan\theta &= \cot(90°-\theta) & \cot\theta &= \tan(90°-\theta) \end{aligned}$$

Example 3.7

Express (i) $\sin 74°$ in terms of cosine (ii) $\tan 12°$ in terms of cotangent (iii) $\operatorname{cosec} 39°$ in terms of secant

Solution

(i) $\sin 74° = \sin(90° - 16°)$ (since, $90° - 16° = 74°$)

RHS is of the form $\sin(90° - \theta) = \cos\theta$

Therefore $\sin 74° = \cos 16°$

(ii) $\tan 12^\circ = \tan(90^\circ - 78^\circ)$ (since, $12^\circ = 90^\circ - 78^\circ$)

RHS is of the form $\tan(90^\circ - \theta) = \cot\theta$

Therefore $\tan 12^\circ = \cot 78^\circ$

(iii) $\operatorname{cosec} 39^\circ = \operatorname{cosec}(90^\circ - 51^\circ)$ (since, $39^\circ = 90^\circ - 51^\circ$)

RHS is of the form $\operatorname{cosec}(90^\circ - \theta) = \sec\theta$

Therefore $\operatorname{cosec} 39^\circ = \sec 51^\circ$

Example 3.8 Evaluate: (i) $\dfrac{\sin 49^\circ}{\cos 41^\circ}$ (ii) $\dfrac{\sec 63^\circ}{\operatorname{cosec} 27^\circ}$

Solution

(i) $\dfrac{\sin 49^\circ}{\cos 41^\circ}$

$\sin 49^\circ = \sin(90^\circ - 41^\circ) = \cos 41^\circ$, since $49^\circ + 41^\circ = 90^\circ$ (complementary),

Hence on substituting $\sin 49^\circ = \cos 41^\circ$ we get, $\dfrac{\cos 41^\circ}{\cos 41^\circ} = 1$

(ii) $\dfrac{\sec 63^\circ}{\operatorname{cosec} 27^\circ}$

$\sec 63^\circ = \sec(90^\circ - 27^\circ) = \operatorname{cosec} 27^\circ$, here, 63° and 27° are complementary angles.

we have $\dfrac{\sec 63^\circ}{\operatorname{cosec} 27^\circ} = \dfrac{\operatorname{cosec} 27^\circ}{\operatorname{cosec} 27^\circ} = 1$

Example 3.9 Find the values of (i) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

(ii) $\dfrac{\cos 35^\circ}{\sin 55^\circ} + \dfrac{\sin 12^\circ}{\cos 78^\circ} - \dfrac{\cos 18^\circ}{\sin 72^\circ}$

Solution

(i) $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$

$= \tan 7^\circ \tan 83^\circ \tan 23^\circ \tan 67^\circ \tan 60^\circ$ (Grouping complementary angles)

$= \tan 7^\circ \tan(90^\circ - 7^\circ) \tan 23^\circ \tan(90^\circ - 23^\circ) \tan 60^\circ$

$= (\tan 7^\circ . \cot 7^\circ)(\tan 23^\circ . \cot 23^\circ) \tan 60^\circ$

$= (1) \times (1) \times \tan 60^\circ$

$= \tan 60^\circ \quad = \sqrt{3}$

(ii) $\dfrac{\cos 35°}{\sin 55°}+\dfrac{\sin 12°}{\cos 78°}-\dfrac{\cos 18°}{\sin 72°}$

$$=\frac{\cos(90°-55°)}{\sin 55°}+\frac{\sin(90°-78°)}{\cos 78°}-\frac{\cos(90°-72°)}{\sin 72°}\quad\left[\begin{array}{l}\text{since}\\ \cos 35°=\cos(90°-55°)\\ \sin 12°=\sin(90°-78°)\\ \cos 18°=\cos(90°-72°)\end{array}\right]$$

$$=\frac{\sin 55°}{\sin 55°}+\frac{\cos 78°}{\cos 78°}-\frac{\sin 72°}{\sin 72°}$$

$$=1+1-1=1$$

Example 3.10 (i) If $\operatorname{cosec} A=\sec 34°$, find A (ii) If $\tan B=\cot 47°$, find B.

Solution

(i) We know that $\operatorname{cosec} A=\sec(90°-A)$

$$\sec(90°-A)=\sec(34°)$$

$$90°-A=34°$$

We get $A=90°-34°$

$$A=56°$$

(ii) We know that $\tan B=\cot(90°-B)$

$$\cot(90°-B)=\cot 47°$$

$$90°-B=47°$$

We get $B=90°-47°$

$$B=43°$$

Exercise 3.3

Find the value of the following:

(i) $\left(\dfrac{\cos 47°}{\sin 43°}\right)^2+\left(\dfrac{\sin 72°}{\cos 18°}\right)^2-2\cos^2 45°$

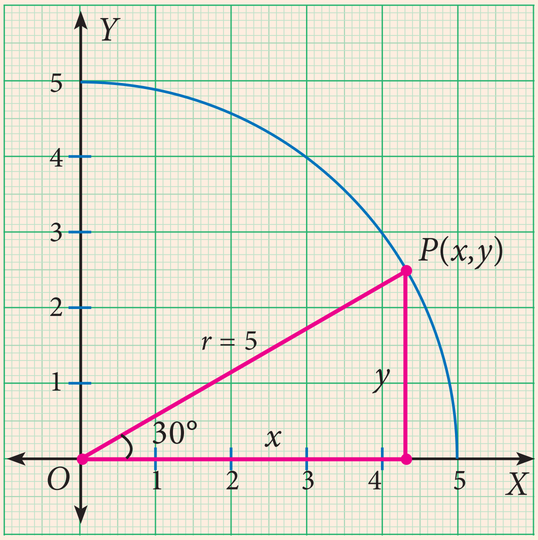
(ii) $\dfrac{\cos 70°}{\sin 20°}+\dfrac{\cos 59°}{\sin 31°}+\dfrac{\cos\theta}{\sin(90°-\theta)}-8\cos^2 60°$

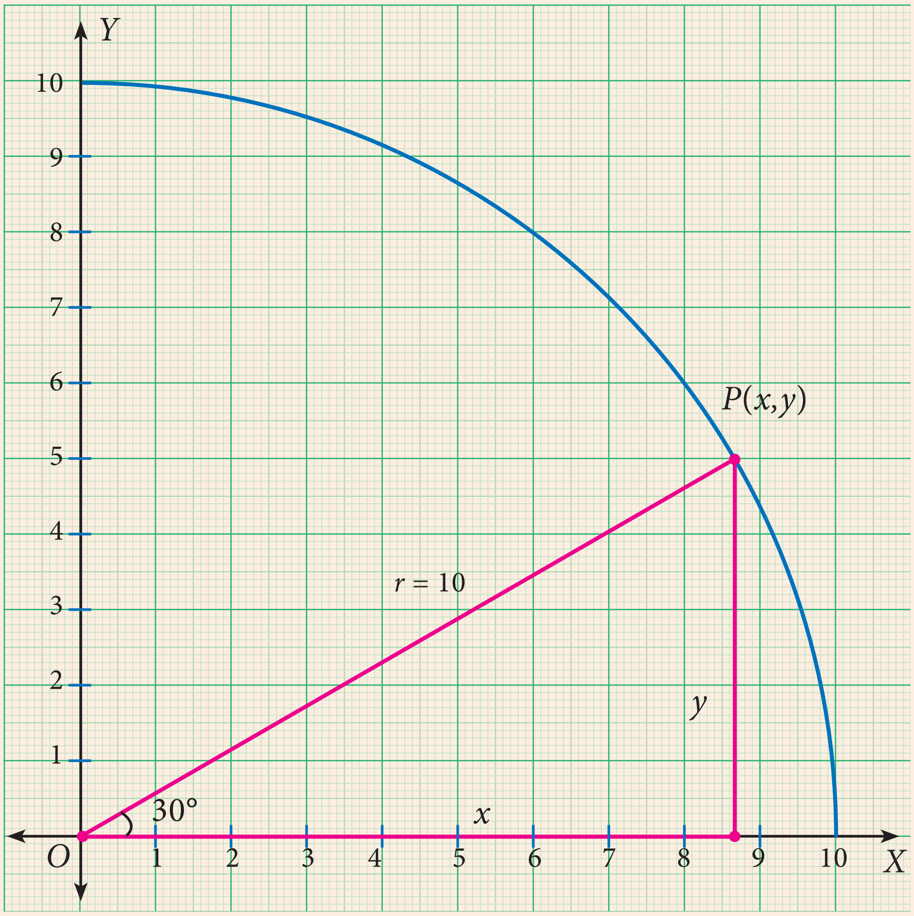
(iii) $\tan 15°\ \tan 30°\ \tan 45°\ \tan 60°\ \tan 75°$

(iv) $\dfrac{\cot\theta}{\tan(90°-\theta)}+\dfrac{\cos(90°-\theta)\tan\theta\sec(90°-\theta)}{\sin(90°-\theta)\cot(90°-\theta)\operatorname{cosec}(90°-\theta)}$

Activity - 3

In a graph sheet, draw two arcs of radius 5 *cm* and 10 *cm* in the first quadrant as shown:





Draw OP such that $\theta = \angle XOP = 30°$ and find the point $P(x,y)$.

Repeat the same steps for $\theta = 45°, 60°, 90°, 0°$ and tabulate all the readings for various values of θ as follows.

Trigonometric Ratio	r=5cm (sector of a circle)					r=10cm (sector of a circle)				
	30°	45°	60°	90°	0°	30°	45°	60°	90°	0°
$\sin\theta = \frac{y}{r}$										
$\cos\theta = \frac{x}{r}$										
$\tan\theta = \frac{y}{x}$										

1. What do you observe from the above table?
2. What do you infer by comparing this table with the trigonometric ratios table given at the end of this chapter?

Thinking Corner

(i) What is the minimum and maximum values of $\sin\theta$?

(ii) What is the minimum and maximum values of $\cos\theta$?

3.4 Method of using Trigonometric Table

We have learnt to calculate the trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°. But during certain situations we need to calculate the trigonometric ratios of all the other acute angles. Hence we need to know the method of using trigonometric tables.

One degree (1°) is divided into 60 minutes ($60'$) and one minute ($1'$) is divided into 60 seconds ($60''$). Thus, $1° = 60'$ and $1' = 60''$.

The trigonometric tables give the values, correct to four places of decimals for the angles from 0° to 90° spaced at intervals of $60'$. A trigonometric table consists of three parts.

A column on the extreme left which contains degrees from 0° to 90°, followed by ten columns headed by $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$.

Five columns under the head mean difference has values from 1,2,3,4 and 5.

For angles containing other measures of minutes (that is other than $0'$, $6'$, $12'$, $18'$, $24'$, $30'$, $36'$, $42'$, $48'$ and $54'$), the appropriate adjustment is obtained from the mean difference columns.

The mean difference is to be added in the case of sine and tangent while it is to be subtracted in the case of cosine.

Now let us understand the calculation of values of trigonometric angle from the following examples.

Example 3.11 Find the value of $\sin 64°34'$.

Solution

	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
64°						0.9026								5	

Write $64°34' = 64°30' + 4'$

From the table we have, $\sin 64°30' = 0.9026$

Mean difference for $4' = \quad 5$(Mean difference to be added for sine)

$$\sin 64°34' = 0.9031$$

Example 3.12 Find the value of $\cos 19°59'$

Solution

	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
19°										0.9403					5

Write $19°59' = 19°54' + 5'$

From the table we have,

$$\cos 19°54' = 0.9403$$

Mean difference for $5' = \quad 5$ (Mean difference to be subtracted for cosine)

$$\cos 19°59' = 0.9398$$

Example 3.13 Find the value of $\tan 70°13'$

Solution

	$0'$	$6'$	$12'$	$18'$	$24'$	$30'$	$36'$	$42'$	$48'$	$54'$	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
70°			2.7776								26				

Write $70°13' = 70°12' + 1'$

From the table we have, $\tan 70°12' = 2.7776$

Mean difference for $1' = 26$ (Mean difference to be added for tan)

$\tan 70°13' = 2.7802$

Example 3.14 Find the value of

(i) $\sin 38°36' + \tan 12°12'$ (ii) $\tan 60°25' - \cos 49°20'$

Solution

(i) $\sin 38°36' + \tan 12°12'$

$\sin 38°36' = 0.6239$

$\tan 12°12' = 0.2162$

$\sin 38°36' + \tan 12°12' = 0.8401$

(ii) $\tan 60°25' - \cos 49°20'$

$\tan 60°25' = 1.7603 + 0.0012 = 1.7615$

$\cos 49°20' = 0.6521 - 0.0004 = 0.6517$

$\tan 60°25' - \cos 49°20' = 1.1098$

Example 3.15 Find the value of θ if

(i) $\sin\theta = 0.9858$ (ii) $\tan\theta = 0.5902$ (iii) $\cos\theta = 0.7656$

Solution

(i) $\sin\theta = 0.9858 = 0.9857 + 0.0001$

From the natural sines table $0.9857 = 80°18'$

Mean difference $1 = 2'$ (add the mean difference value)

$0.9858 = 80°20'$

$\sin\theta = 0.9858 = \sin 80°20'$

$\theta = 80°20'$

(ii) $\tan\theta = 0.5902 = 0.5890 + 0.0012$

From the natural tangent table,

$$0.5890 = 30°30'$$

Mean difference $\quad 12 = \quad 3'$ (add the mean difference value)

$$0.5902 = 30°33'$$

$$\tan\theta = 0.5902 \quad = \tan 30°33'$$

$$\theta = 30°33'$$

(iii) $\cos\theta = 0.7656 = 0.7660 - 0.0004$ From the natural cosine table

$$0.7660 = 40°0'$$

Mean difference $\quad 4 = \quad 2'$ (subtract the mean difference value)

$$0.7656 = 40°2'$$

$$\cos\theta = 0.7656 \quad = \cos 40°2'$$

$$\theta = 40°2'$$

Example 3.16 Find the area of the right angled triangle with hypotenuse $5cm$ and one of the acute angle is $48°30'$

Solution

From the figure,

$$\sin\theta = \frac{AB}{AC}$$

$$\sin 48°30' = \frac{AB}{5}$$

$$0.7490 = \frac{AB}{5}$$

$$5 \times 0.7490 = AB$$

$$AB = 3.7450\ cm$$

$$\cos\theta = \frac{BC}{AC}$$

$$\cos 48°30' = \frac{BC}{5}$$

$$0.6626 = \frac{BC}{5}$$

$$0.6626 \times 5 = BC$$

$$BC = 3.313\ cm$$

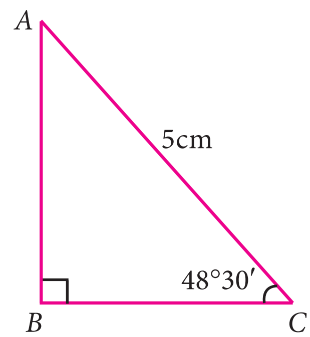


Fig. 3.17

$$\text{Area of right triangle} = \frac{1}{2}bh$$

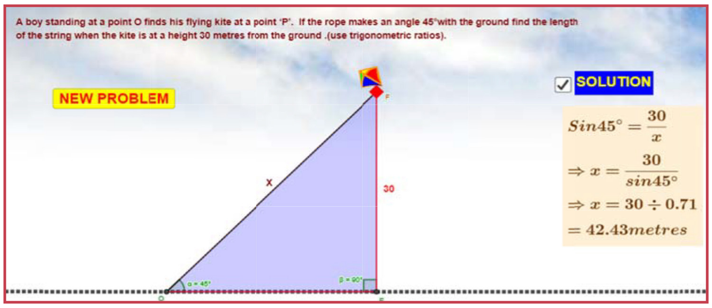
$$= \frac{1}{2} \times BC \times AB$$

$$= \frac{1}{2} \times 3.3130 \times 3.7450$$

$$= 1.6565 \times 3.7450 \quad = 6.2035925\ cm^2$$

ICT Corner

Expected Result is shown in this picture



Step – 1

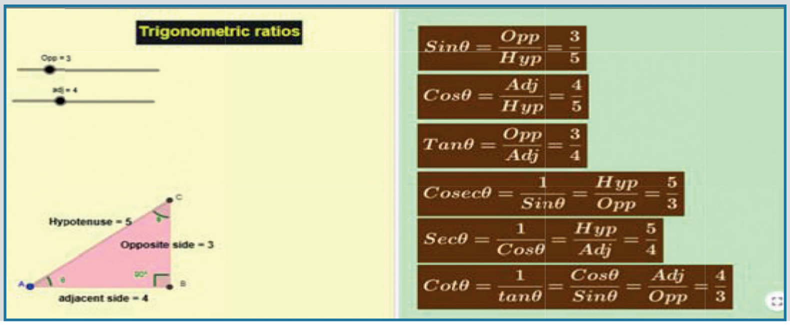
Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Trigonometry" will open. There are three worksheets under the title Trigonometric ratios and Complimentary angles and kite problem.

Step - 2

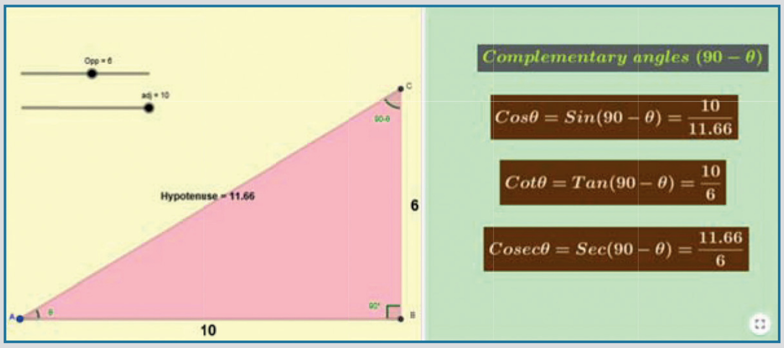
Move the sliders of the respective values to change the points and ratio. Work out the solution and check.

For the kite problem click on "NEW PROBLEM" to change the question and work it out. Click the check box for solution to check your answer.

Step 1



Step 2



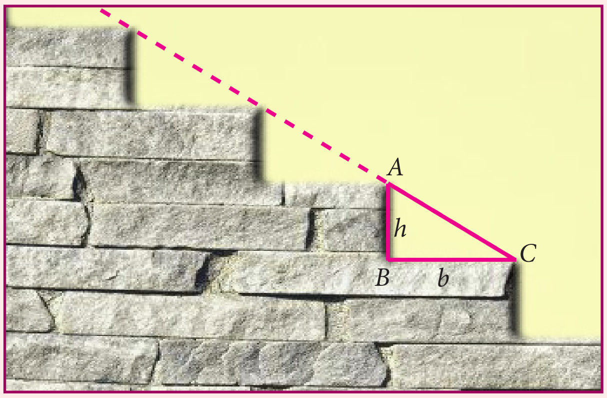
Browse in the link

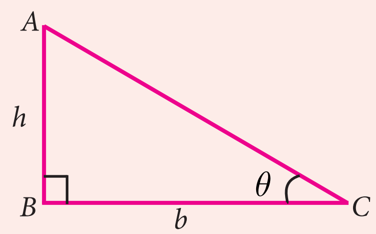
Trigonometry: https://ggbm.at/hkwnccr6 or Scan the QR Code.

B566_9_MAT_EM_T3

Activity - 4

Observe the steps in your home. Measure the breadth and the height of one step. Enter it in the following picture and measure the angle (of elevation) of that step.





(i) Compare the angles (of elevation) of different steps of same height and same breadth and discuss your observation.

(ii) Sometimes few steps may not be of same height. Compare the angles (of elevation) of different steps of those different heights and same breadth and dicuss your observation.

Exercise 3.4

1. Find the value of the following:

   (i) $\sin 49°$ (ii) $\cos 74°39'$ (iii) $\tan 54°26'$ (iv) $\sin 21°21'$ (v) $\cos 33°53'$ (vi) $\tan 70°17'$

2. Find the value of θ if

   (i) $\sin\theta = 0.9975$ (ii) $\cos\theta = 0.6763$ (iii) $\tan\theta = 0.0720$

   (iv) $\cos\theta = 0.0410$ (v) $\tan\theta = 7.5958$

3. Find the value of the following:

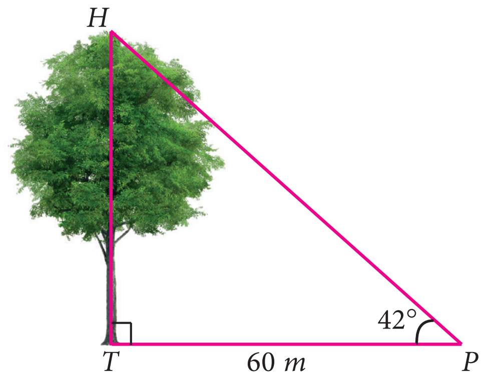
   (i) $\sin 65°39' + \cos 24°57' + \tan 10°10'$

   (ii) $\tan 70°58' + \cos 15°26' - \sin 84°59'$

4. Find the area of a right triangle whose hypotenuse is $10cm$ and one of the acute angle is $24°24'$

5. Find the angle made by a ladder of length $5m$ with the ground, if one of its end is $4m$ away from the wall and the other end is on the wall.

6. In the given figure, HT shows the height of a tree standing vertically. From a point P, the angle of elevation of the top of the tree (that is $\angle P$) measures $42°$ and the distance to the tree is 60 metres. Find the height of the tree.



Exercise 3.5

Multiple choice questions

1. If $\sin 30° = x$ and $\cos 60° = y$, then $x^2 + y^2$ is

(1) $\frac{1}{2}$ (2) 0 (3) $\sin 90°$ (4) $\cos 90°$

2. If $\tan\theta = \cot 37°$, then the value of θ is

(1) 37° (2) 53° (3) 90° (4) 1°

3. The value of $\tan 72°.\tan 18°$ is

(1) 0 (2) 1 (3) 18° (4) 72°

4. The value of $\frac{\tan 15°}{\cot 75°}$ is

(1) $\cos 90°$ (2) $\sin 30°$ (3) $\tan 45°$ (4) $\cos 30°$

5. The value of $\frac{2\tan 30°}{1-\tan^2 30°}$ is equal to

(1) $\cos 60°$ (2) $\sin 60°$ (3) $\tan 60°$ (4) $\sin 30°$

6. If $\sin\alpha = \frac{1}{2}$ and α is acute, then $(3\cos\alpha - 4\cos^3\alpha)$ is equal to

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{1}{6}$ (4) -1

7. If $2\sin 2\theta = \sqrt{3}$, then the value of θ is

(1) 90° (2) 30° (3) 45° (4) 60°

8. The value of $3\sin 70° \sec 20° + 2\sin 49° \sec 51°$ is

(1) 2 (2) 3 (3) 5 (4) 6

9. The value of $2\tan 30° \tan 60°$ is

(1) 1 (2) 2 (3) $2\sqrt{3}$ (4) 6

10. The value of $\dfrac{1-\tan^2 45°}{1+\tan^2 45°}$ is

(1) 2 (2) 1 (3) 0 (4) $\dfrac{1}{2}$

11. If $\cos A = \dfrac{3}{5}$, then the value of $\tan A$ is

(1) $\dfrac{4}{5}$ (2) $\dfrac{3}{4}$ (3) $\dfrac{5}{3}$ (4) $\dfrac{4}{3}$

12. The value of $\text{cosec}(70° + \theta) - \sec(20° - \theta) + \tan(65° + \theta) - \cot(25° - \theta)$ is

(1) 0 (2) 1 (3) 2 (4) 3

13. The value of $\tan 1°.\tan 2°.\tan 3° \ldots \tan 89°$ is

(1) 0 (2) 1 (3) 2 (4) $\dfrac{\sqrt{3}}{2}$

14. Given that $\sin\alpha = \dfrac{1}{2}$ and $\cos\beta = \dfrac{1}{2}$, then the value of $\alpha + \beta$ is

(1) 0° (2) 90° (3) 30° (4) 60°

15. The value of $\dfrac{\sin 29°31'}{\cos 60°29'}$ is

(1) 0 (2) 2 (3) 1 (4) –1

Points to Remember

- Trigonometric ratios are

$\sin\theta = \dfrac{\textit{opposite side}}{\textit{hypotenuse}}$	$\text{cosec}\,\theta = \dfrac{\textit{hypotenuse}}{\textit{opposite side}}$
$\cos\theta = \dfrac{\textit{adjacent side}}{\textit{hypotenuse}}$	$\sec\theta = \dfrac{\textit{hypotenuse}}{\textit{adjacent side}}$
$\tan\theta = \dfrac{\textit{opposite side}}{\textit{adjacent side}}$	$\cot\theta = \dfrac{\textit{adjacent side}}{\textit{opposite side}}$

- Reciprocal trigonometric ratios

$\sin\theta = \dfrac{1}{\text{cosec}\,\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

$\text{cosec}\,\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

- Complementary angles

$\sin\theta = \cos(90° - \theta)$ $\text{cosec}\,\theta = \sec(90° - \theta)$

$\cos\theta = \sin(90° - \theta)$ $\sec\theta = \text{cosec}(90° - \theta)$

$\tan\theta = \cot(90° - \theta)$ $\cot\theta = \tan(90° - \theta)$

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	15
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	12	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	12	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	12	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	11	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2233	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	14
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	14
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	10	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10

NATURAL SINES

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	3	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	2	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	2
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0

NATURAL COSINES

(Numbers in mean difference columns to be subtracted, not added)

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	2
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	2	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	3	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10

NATURAL COSINES

(Numbers in mean difference columns to be subtracted, not added)

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	14
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	14
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	12	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	12	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	15
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15

NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	18	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	29

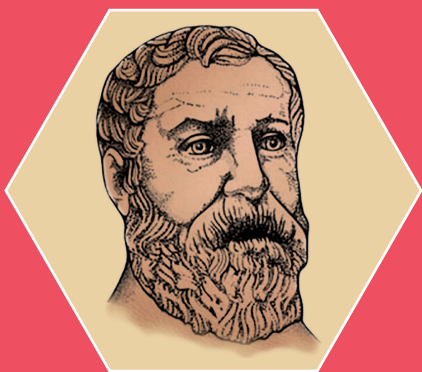
NATURAL TANGENTS

Degree	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	Mean Difference				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1	2	3	4	5
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3222	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4373	4.4737	4.5107	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.6768	9.8448	10.0187	10.1988	10.3854	10.5789	10.7797	10.9882	11.2048					
85	11.4301	11.6645	11.9087	12.1632	12.4288	12.7062	12.9962	13.2996	13.6174	13.9507					
86	14.3007	14.6685	15.0557	15.4638	15.8945	16.3499	16.8319	17.3432	17.8863	18.4645					
87	19.0811	19.7403	20.4465	21.2049	22.0217	22.9038	23.8593	24.8978	26.0307	27.2715					
88	28.6363	30.1446	31.8205	33.6935	35.8006	38.1885	40.9174	44.0661	47.7395	52.0807					
89	57.2900	63.6567	71.6151	81.8470	95.4895	114.5887	143.2371	190.9842	286.4777	572.9572					

4

MENSURATION

The most beautiful plane figure is the circle and the most beautiful solid figure is the sphere. - *Pythagoras.*



Heron
A.D (C.E) 10-75

Heron of Alexandria was a Greek mathematician. He wrote books on mathematics, mechanics and physics. His famous book 'Metrica' consists of three volumes. This book shows the way to calculate area and volume of plane and solid figures. Heron has derived the formula for the area of triangle when three sides are given.

FZZ27R

Learning Outcomes

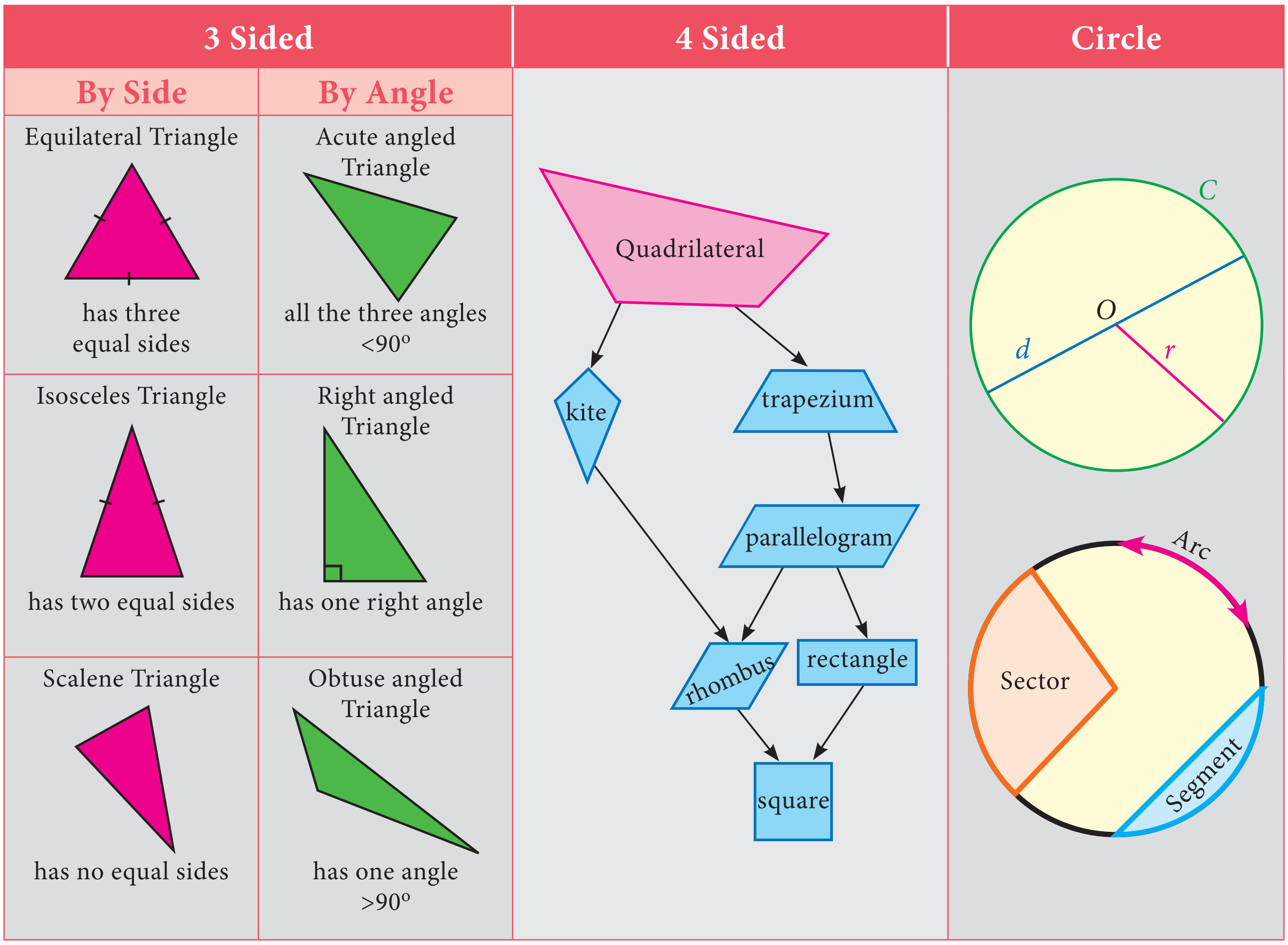
- To use Heron's formula for calculating area of triangles and quadrilaterals.
- To find Total Surface Area (TSA), Lateral Surface Area (LSA) and Volume of cuboids and cubes.

4.1 Introduction

Mensuration is the branch of mathematics which deals with the study of areas and volumes of different kinds of geometrical shapes. In the broadest sense, it is all about the process of measurement.

Mensuration is used in the field of architecture, medicine, construction, etc. It is necessary for everyone to learn formulae used to find the perimeter and area of two dimensional figures as well as the surface area and volume of three dimensional solids in day to day life. In this chapter we deal with finding the area of triangles (using Heron's formula) and surface area and volume of cuboids and cubes.

Do you remember the various shapes of plane figures we have already learnt in earlier classes? The following table may help us to recall them.



Activity - 1

Look at the trapezium given below and answer the following:

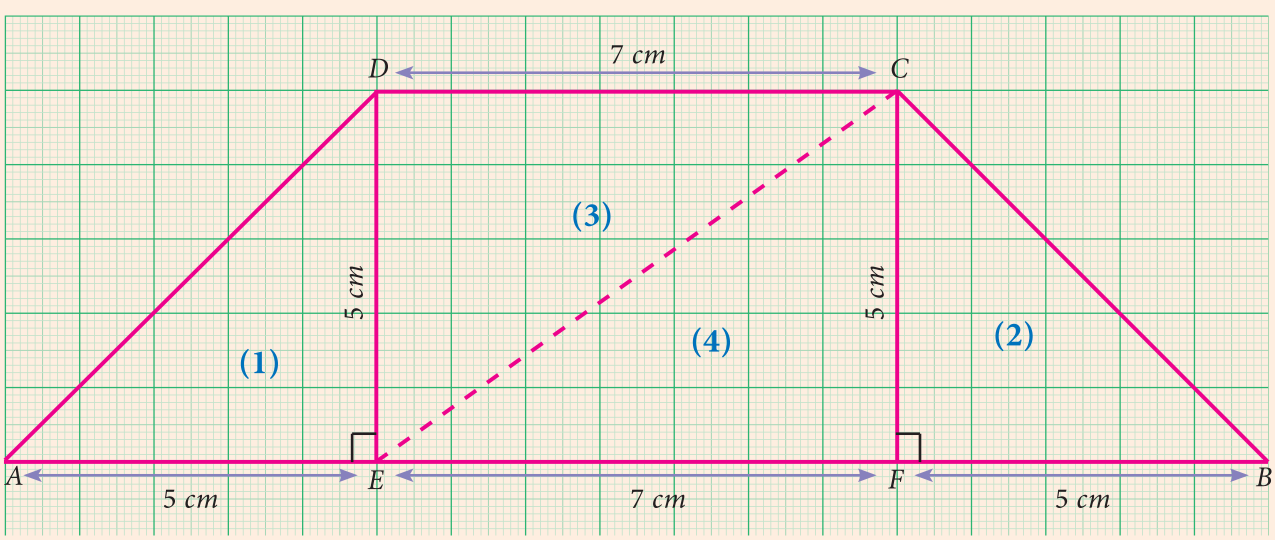


Fig. 4.1

(i) Find the area of triangles (1), (2), (3) and (4)

(ii) The diagonal EC of the rectangle *CDEF* divides it into two parts. What type of two shapes do we get? Are they equal?

(iii) Is it possible to make a square using the triangles (1) and (2).

(iv) Verify that the sum of the area of all triangles (1), (2), (3) and (4) is equal to the area of trapezium *ABCD*.

(v) Find the area of the trapezium using unit squares in the graph sheet.

Recall: For a closed plane figure (a quadrilateral or a triangle), what do we call the distance around its boundary? What is the measure of the region covered inside the boundary?

In general, the area of a triangle is calculated by the formula

$$Area = \frac{1}{2} \times Base \times Height \quad \text{sq. units}$$

That is, $A = \frac{1}{2} \times b \times h$ sq. units

where, b is base and h is height of the triangle.

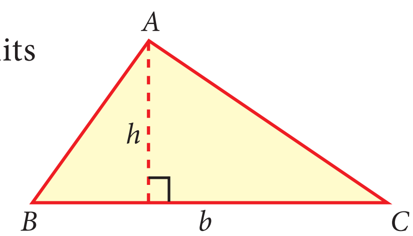


Fig. 4.2

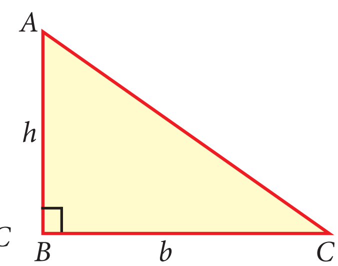


Fig. 4.3

From the above, we know how to find the area of a triangle when its 'base' and 'height' (that is altitude) are given.

G13NVN

4.2 Heron's Formula

How will you find the area of a triangle, if the height is not known but the lengths of the three sides are known?

For this, Heron has given a formula to find the area of a triangle.

If a, b and c are the sides of a triangle, then

the area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$ sq.units.

where $s = \frac{a+b+c}{2}$, 's' is the semi-perimeter (that is half of the perimeter) of the triangle.

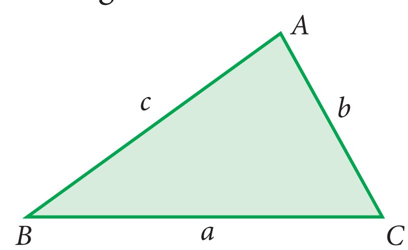


Fig. 4.4

Note

If we assume that the sides are of equal length that is $a = b = c$, then Heron's formula will be $\frac{\sqrt{3}}{4}a^2$ sq.units, which is the area of an equilateral triangle.

Example 4.1 The lengths of sides of a triangular field are 28 m, 15 m and 41 m. Calculate the area of the field. Find the cost of levelling the field at the rate of ₹ 20 per m^2.

Solution

Let $a = 28\,m,\ b = 15\,m$ and $c = 41\,m$

Then, $s = \dfrac{a+b+c}{2} = \dfrac{28+15+41}{2} = \dfrac{84}{2} = 42\ m$

$$\text{Area of triangular field} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{42(42-28)(42-15)(42-41)}$$
$$= \sqrt{42\times14\times27\times1}$$
$$= \sqrt{2\times3\times7\times7\times2\times3\times3\times3\times1}$$
$$= 2\times3\times7\times3$$
$$= 126\ m^2$$

Given the cost of levelling is ₹ 20 per m^2.

The total cost of levelling the field $= 20\times126$ = ₹ 2520.

Example 4.2 Three different triangular plots are available for sale in a locality. Each plot has a perimeter of 120 m. The side lengths are also given:

Shape of plot	Perimeter	Length of sides
Right angled triangle	120 m	30 m, 40 m, 50 m
Acute angled triangle	120 m	35 m, 40 m, 45 m
Equilateral triangle	120 m	40 m, 40 m, 40 m

Help the buyer to decide which among these will be more spacious.

Solution

For clarity, let us draw a rough figure indicating the measurements:

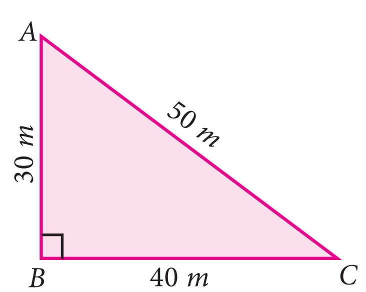


Fig. 4.5

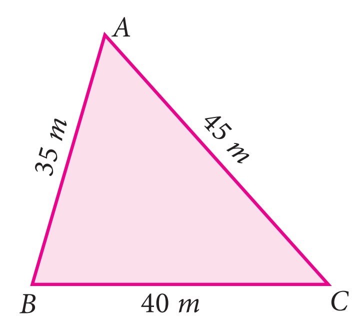


Fig. 4.6

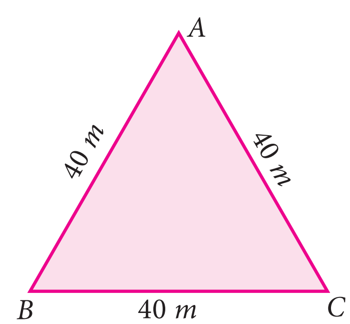


Fig. 4.7

(i) The semi-perimeter of Fig. 4.5, $s = \frac{30+40+50}{2} = 60\ m$

$$\text{Fig. 4.6, } s = \frac{35+40+45}{2} = 60\ m$$

$$\text{Fig. 4.7, } s = \frac{40+40+40}{2} = 60\ m$$

Note that all the semi-perimeters are equal.

(ii) Area of triangle using Heron's formula:

In fig.4.5, Area of triangle

$$= \sqrt{60(60-30)(60-40)(60-50)}$$
$$= \sqrt{60 \times 30 \times 20 \times 10}$$
$$= \sqrt{30 \times 2 \times 30 \times 2 \times 10 \times 10}$$
$$= 600\ m^2$$

In fig.4.6, Area of triangle

$$= \sqrt{60(60-35)(60-40)(60-45)}$$
$$= \sqrt{60 \times 25 \times 20 \times 15}$$
$$= \sqrt{20 \times 3 \times 5 \times 5 \times 20 \times 3 \times 5}$$
$$= 300\sqrt{5} \qquad (\text{ Since } \sqrt{5} = 2.236\ )$$
$$= 670.8\ m^2$$

In fig.4.7, Area of triangle

$$= \sqrt{60(60-40)(60-40)(60-40)}$$
$$= \sqrt{60 \times 20 \times 20 \times 20}$$
$$= \sqrt{3 \times 20 \times 20 \times 20 \times 20}$$
$$= 400\sqrt{3} \qquad (\text{ Since } \sqrt{3} = 1.732\ )$$
$$= 692.8\ m^2$$

We find that though the perimeters are same, the areas of the three triangular plots are different. The area of triangle in fig. 4.7 is the greatest among these; the buyer can be suggested to choose this since it is more spacious.

Note

If the perimeter of different types of triangles have the same value, among all the types of triangles, the equilateral triangle possess the greatest area. We will learn more about maximum areas in higher classes.

Example 4.3 The sides of a triangular park are in the ratio 9:10:11 and its perimeter is 300 m. Find the area of the triangular park.

Solution

Given the sides are in the ratio 9:10:11, let the sides be $9k$, $10k$, $11k$

The perimeter of the triangular park $= 300\ m$

$$9k + 10k + 11k = 300\,m$$

$$30k = 300$$

$$k = 10m$$

Therefore, the sides are $a = 90\ m$, $b = 100\ m$, $c = 110\ m$

$$s = \frac{a+b+c}{2} = \frac{90+100+110}{2} = \frac{300}{2} = 150\,m$$

Hence, Area of triangular park $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{150(150-90)(150-100)(150-110)}$$

$$= \sqrt{150 \times 60 \times 50 \times 40}$$

$$= \sqrt{3 \times 50 \times 20 \times 3 \times 50 \times 2 \times 20}$$

$$= 50 \times 20 \times 3\sqrt{2}$$

$$= 3000 \times 1.414 = 4242\ m^2$$

Exercise 4.1

1. Using Heron's formula, find the area of a triangle whose sides are

   (i) 10 cm, 24 cm, 26 cm (ii) 1.8 m, 8 m, 8.2 m

2. The sides of the triangular ground are 22 m, 120 m and 122 m. Find the area and cost of levelling the ground at the rate of ₹ 20 per m^2.

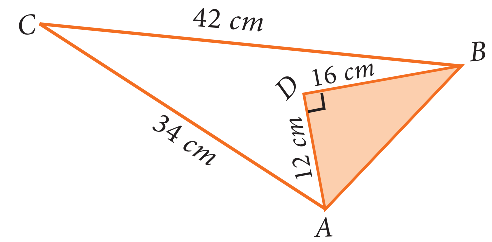
3. The perimeter of a triangular plot is 600 m. If the sides are in the ratio 5:12:13, then find the area of the plot.

4. Find the area of an equilateral triangle whose perimeter is 180 cm.

5. An advertisement board is in the form of an isosceles triangle with perimeter 36m and each of the equal sides are 13 m. Find the cost of painting it at ₹ 17.50 per square metre.

6. A triangle and a parallelogram have the same area. The sides of the triangle are 48 cm, 20 cm and 52 cm. The base of the parallelogram is 20 cm. Find (i) the area of triangle using Heron's formula. (ii) the height of the parallelogram.

7. Find the area of the unshaded region.



4.3 Application of Heron's Formula in Finding Areas of Quadrilaterals

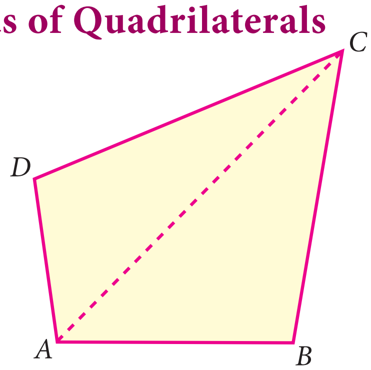


Fig. 4.8

A plane figure bounded by four line segments is called a quadrilateral.

Let $ABCD$ be a quadrilateral. To find the area of a quadrilateral, we divide the quadrilateral into two triangular parts and use Heron's formula to calculate the area of the triangular parts.

In figure 4.8,

Area of quadrilateral $ABCD$ = Area of triangle ABC + Area of triangle ACD

Example 4.4 Find the area of a quadrilateral $ABCD$ whose sides are $AB = 8\,cm$, $BC = 15\ cm$, $CD = 12\ cm$, $AD = 25\ cm$ and $\lfloor B = 90°$.

Solution

In the quadrilateral $ABCD$, join one of the diagonals, say AC.

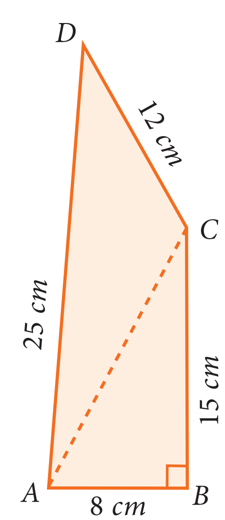


Fig. 4.9

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 8 \times 15 = 60\ cm^2$$

By Pythagoras theorem, in right angled triangle ABC,

$$AC^2 = AB^2 + BC^2$$

$$= 8^2 + 15^2 = 64 + 225 = 289\ cm$$

Therefore, $AC = \sqrt{289} = 17 cm$

Now, for $\Delta\,ACD$, let us consider a = 17 cm, b =12 cm, c =25 cm

$$\text{then, } s = \frac{a+b+c}{2} = \frac{17+12+25}{2} = \frac{54}{2} = 27\ cm$$

$$\text{Area of } \Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{27(27-17)(27-12)(27-25)}$$

$$= \sqrt{27 \times 10 \times 15 \times 2}$$

$$= \sqrt{3 \times 3 \times 3 \times 2 \times 5 \times 5 \times 3 \times 2}$$

$$= 3 \times 3 \times 2 \times 5 = 90 cm^2$$

Therefore, Area of quadrilateral ABCD

$$= \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= 60 + 90 = 150\ cm^2$$

Example 4.5

A farmer has a field in the shape of a rhombus. The perimeter of the field is 400 m and one of its diagonal is 120 m. He wants to divide the field into two equal parts to grow two different types of vegetables. Find the area of the field.

Solution

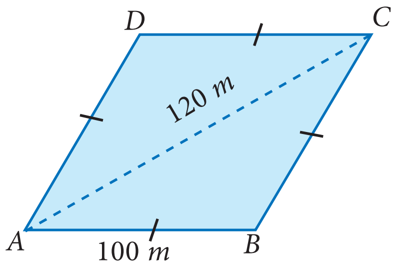


Fig. 4.10

Let $ABCD$ be the rhombus.

Its perimeter = 4 × side = 400 m

Therefore, each side of the rhombus = 100 m

Given the length of the diagonal AC = 120 m

In ΔABC, let a =100 m, b =100 m, c =120 m

$$s = \frac{a+b+c}{2} = \frac{100+100+120}{2} = 160\ m$$

$$\text{Area of } \Delta ABC = \sqrt{160(160-100)(160-100)(160-120)}$$

$$= \sqrt{160\times 60\times 60\times 40}$$

$$= \sqrt{40\times 2\times 2\times 60\times 60\times 40}$$

$$= 40\times 2\times 60 = 4800\, m^2$$

Therefore, Area of the field $ABCD = 2\times$ Area of ΔABC $= 2\times 4800 = 9600\ m^2$

Exercise 4.2

1. Find the area of a quadrilateral $ABCD$ whose sides are $AB = 13\,cm$, $BC = 12\,cm$, $CD = 9\,cm$, $AD = 14\,cm$ and diagonal $BD = 15\,cm$.
2. A park is in the shape of a quadrilateral. The sides of the park are 15 m, 20 m, 26 m and 17 m and the angle between the first two sides is a right angle. Find the area of the park.
3. A land is in the shape of rhombus. The perimeter of the land is 160 m and one of the diagonal is 48 m. Find the area of the land.
4. The adjacent sides of a parallelogram measures 34 m, 20 m and the measure of the diagonal is 42 m. Find the area of parallelogram.
5. The parallel sides of a trapezium are 15 m and 10 m long and its non-parallel sides are 8 m and 7 m long. Find the area of the trapezium.

4.4 Surface Area of Cuboid and Cube

We have learnt in the earlier classes about 3D structures. The 3D shapes are those which do not lie completely in a plane. Any 3D shape has dimensions namely length, breadth and height.

G18X9E

Some of the three dimensional (3D) shapes are given below.

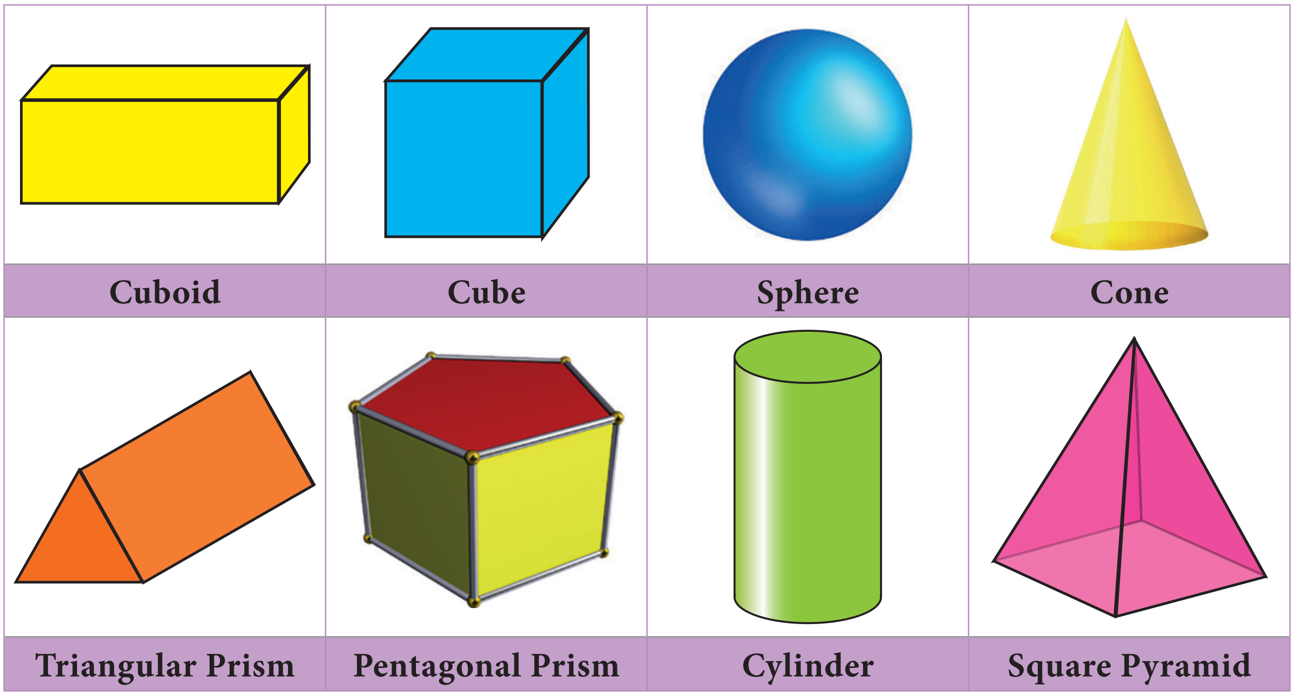


Fig. 4.11

Shown below are some examples of different kinds of solids that we use in the daily life such as brick, cube, gas cylinder, cone ice cream and ball.

Fig. 4.12

Now we turn our attention to find the surface area and volume of two of these solid shapes namely the cuboid and the cube.

Thinking Corner

Can you find some more 3D shapes?

4.4.1 Cuboid and its Surface Area

Cuboid : A cuboid is a closed solid figure bounded by six rectangular plane regions. Here are some examples:

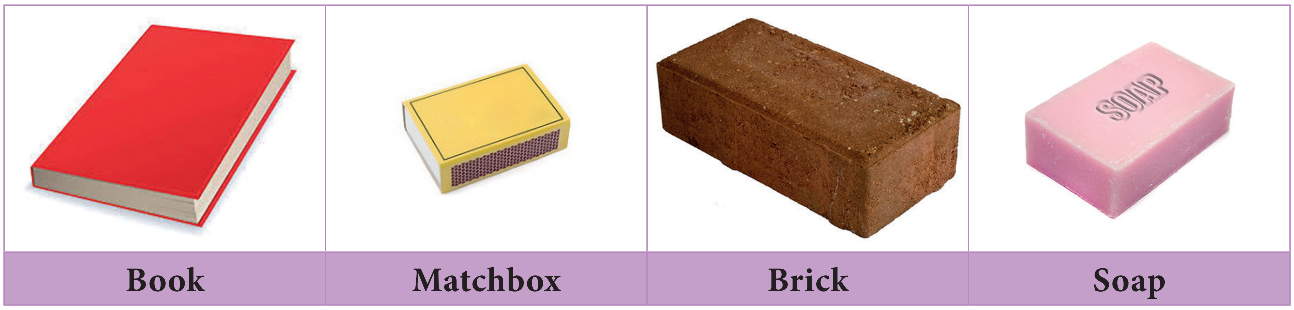


Fig. 4.13

Activity - 2

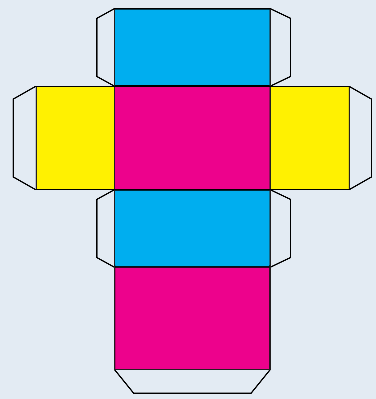


Fig. 4.14

Here is a suggestive template to make a Cuboid. Try to use a thick sheet paper / chart and prepare a net yourself. Colour it, cut it out, fold it and glue it together. Investigate about its area, (ignoring flaps). What precautions will you need to take to fold the net correctly into a cuboid?

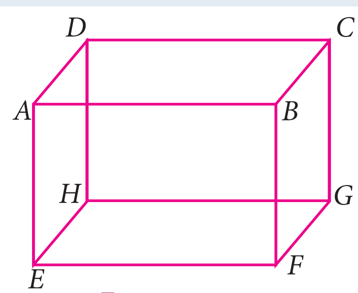


Fig. 4.15

Face: Let us consider the cuboid shown in the figure 4.15. It is made of six rectangular plane regions *ABCD*, *EFGH*, *AEHD*, *BFGC*, *AEFB* and *CDHG*. These plane regions are the faces of the cuboid.

Edge: A line segment where any two adjacent faces of a cuboid meet.

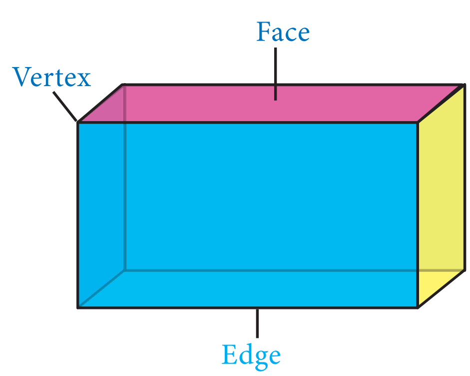


Fig. 4.16

Vertex: The point of intersection of three edges of a cuboid. (Plural : Vertices).

A cuboid has 6 faces, 12 edges and 8 vertices. Ultimately, a cuboid has the shape of a rectangular box.

Total Surface Area (TSA) of a cuboid is the sum of the areas of all the faces that enclose the cuboid. If we leave out the areas of the top and bottom of the cuboid we get what is known as its Lateral Surface Area (LSA).

Illustration

A closed box in the form of a cuboid is 7 cm length (l), 5 cm breadth (b) and 3 cm height (h). Find its total surface area and the lateral surface area.

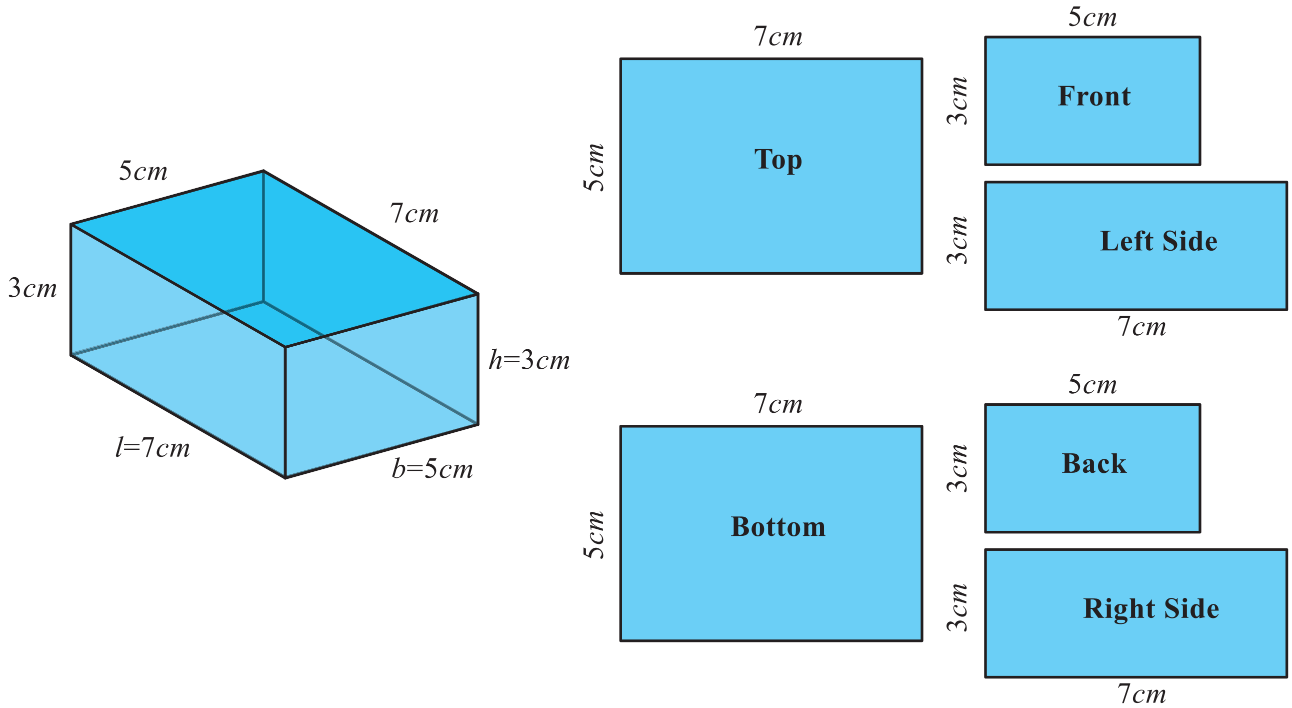


Fig. 4.17

Total surface area $= 2(7)(5) + 2(5)(3) + 2(7)(3)$

$= 70+30+42$

$= 142\ cm^2$.

This is same as $2[(7)(5) + (5)(3) + (7)(3)]\ cm^2$

Lateral surface area $= 2(7)(3) + 2(5)(3)$

$= 42+30$

$= 72\ cm^2$

This is same as $2(7+ 5)3\ cm^2$

Now we are ready to derive a formula for the Total Surface Area and Lateral Surface Area of a cuboid.

In the figure 4.18, l, b and h represents length, breadth and height respectively.

(i) Total Surface Area (TSA) of a cuboid

Top and bottom	$2 \times lb$
Front and back	$2 \times bh$
Left and Right sides	$2 \times lh$

$= 2\ (lb + bh\ +\ lh\ )$ sq. units.

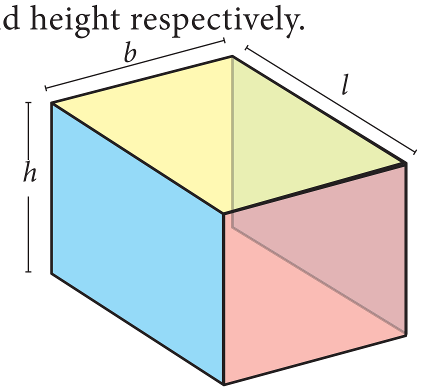


Fig. 4.18

(iii) Lateral Surface Area (LSA) of a cuboid

Front and back	$2 \times bh$
Left and Right sides	$2 \times lh$

$= 2\,(l+b)h$ sq. units.

We are using the concept of Lateral Surface Area (LSA) and Total Surface Area (TSA) in real life situations. For instance a room can be cuboidal in shape that has different length, breadth and height. If we require to find areas of only the walls of a room, avoiding floor and ceiling then we can use LSA. However if we want to find the surface area of the whole room then we have to calculate the TSA.

If the length, breadth and height of a cuboid are l, b and h respectively. Then

(i) Total Surface Area $= 2\,(lb + bh + lh)$ sq.units.

(ii) Lateral Surface Area $= 2\,(l+b)h$ sq.units.

Note

(i) The top and bottom area in a cuboid is independent of height. The total area of top and bottom is $2lb$. Hence LSA is obtained by removing $2lb$ from $2(lb+bh+lh)$.

(ii) The units of length, breadth and height should be same while calculating surface area of the cuboid.

Example 4.6 Find the TSA and LSA of a cuboid whose length, breadth and height are 7.5 m, 3 m and 5 m respectively.

Solution

Given the dimensions of the cuboid;

that is length $(l) = 7.5\ m$, breadth $(b) = 3\ m$ and height $(h) = 5\ m$.

$$\begin{aligned}
\text{TSA} &= 2(lb + bh + lh) \\
&= 2\left[(7.5\times 3) + (3\times 5) + (7.5\times 5)\right] \\
&= 2(22.5 + 15 + 37.5) \\
&= 2\times 75 \\
&= 150\ m^2 \\
\text{LSA} &= 2(l+b)\times h \\
&= 2(7.5+3)\times 5 \\
&= 2\times 10.5\times 5 \\
&= 105\ m^2
\end{aligned}$$

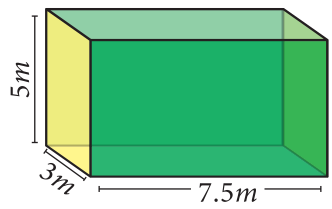


Fig. 4.19

Example 4.7 A closed wooden box is in the form of a cuboid. Its length, breadth and height are 6 m, 1.5 m and 300 cm respectively. Find the total surface area and cost of painting its entire outer surface at the rate of ₹ 50 per m^2.

Solution

Here, length (l) = 6 m, breadth (b) = 1.5 m, height (h) = $\frac{300}{100}$ m = 3 m

The wooden box is in the shape of cuboid.

The painting area of the wooden box = Total Surface Area of cuboid

$$= 2(lb + bh + lh)$$
$$= 2(6 \times 1.5 + 1.5 \times 3 + 6 \times 3)$$
$$= 2(9 + 4.5 + 18) \;\; = 2 \times 31.5$$
$$= 63\ m^2$$

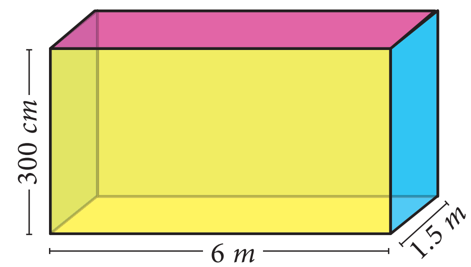


Fig. 4.20

Given that cost of painting of 1 m^2 is ₹ 50

The cost of painting area for 63 m^2 $= 50 \times 63$ = ₹ 3150.

Example 4.8 The length, breadth and height of a hall are 25 m, 15 m and 5 m respectively. Find the cost of renovating its floor and four walls at the rate of ₹80 per m^2.

Solution

Here, length (l) = 25 m, breadth (b) =15 m, height (h) = 5 m.

Area of four walls = LSA of cuboid

$$= 2(l + b) \times h$$
$$= 2(25 + 15) \times 5$$
$$= 80 \times 5 = 400\ m^2$$

Area of the floor $= l \times b$

$$= 25 \times 15$$
$$= 375\ m^2$$

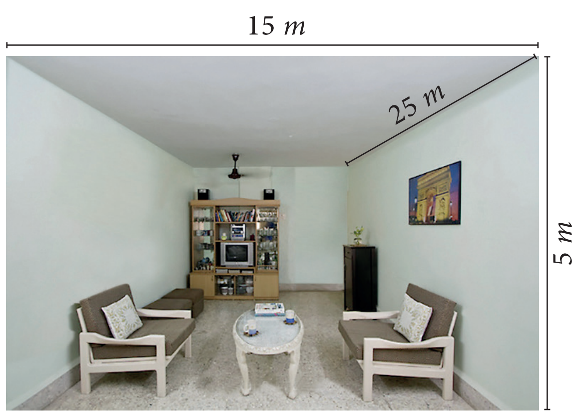


Fig. 4.21

Total renovating area of the hall

= (Area of four walls + Area of the floor)

$$= (400 + 375)\ m^2 \;\; = 775\ m^2$$

Therefore, cost of renovating at the rate of ₹80 per m^2 $= 80 \times 775$

= ₹ 62,000

4.4.2 Cube and its Surface Area

Cube: A cuboid whose length, breadth and height are all equal is called as a cube. That is a cube is a solid having six square faces. Here are some real-life examples.

Dice

Ice cubes

Sugar cubes

Fig. 4.22

A cube being a cuboid has 6 faces, 12 edges and 8 vertices.

Consider a cube whose sides are 'a' units as shown in the figure 4.23. Now,

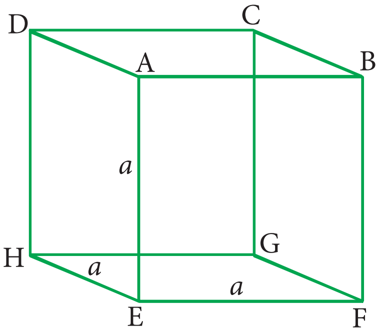


Fig. 4.23

(i) Total Surface Area of the cube

= sum of area of the faces ($ABCD+EFGH+AEHD+BFGC+ABFE+CDHG$)

$= (a^2 + a^2 + a^2 + a^2 + a^2 + a^2)$

$= 6a^2$ sq. units

(ii) Lateral Surface Area of the cube

= sum of area of the faces ($AEHD+BFGC+ABFE+CDHG$)

$= (a^2 + a^2 + a^2 + a^2)$

$= 4a^2$ sq. units

If the side of a cube is a units, then,

(i) The Total Surface Area = $6a^2$ sq.units

(ii) The Lateral Surface Area = $4a^2$ sq.units

Thinking Corner

Can you get these formulae from the corresponding formula of Cuboid?

Example 4.9 Find the Total Surface Area and Lateral Surface Area of the cube, whose side is 5 *cm*.

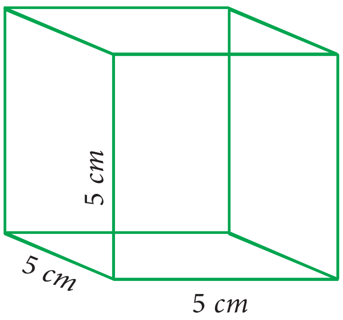


Fig. 4.24

Solution

The side of the cube (a) = 5 *cm*

Total Surface Area $= 6a^2 = 6(5^2) = 150 \ sq.\ cm$

Lateral Surface Area $= 4a^2 = 4(5^2) = 100 \ sq.\ cm$

Example 4.10 A cube has the total surface area of 486 cm^2. Find its lateral surface area.

Solution

Here, total surface area of the cube = 486 cm^2

$6a^2 = 486 \Rightarrow a^2 = \frac{486}{6}$ and so, $a^2 = 81$. This gives $a = 9$.

The side of the cube $= 9\ cm$

Lateral Surface Area $= 4a^2 = 4 \times 9^2 = 4 \times 81 = 324 \ cm^2$

Example 4.11 Two identical cubes of side 7 *cm* are joined end to end. Find the total surface area and lateral surface area of the new resulting cuboid.

Solution

Side of a cube = 7 *cm*

Now length of the resulting cuboid (l) = 7+7 =14 *cm*

Breadth (b) = 7 *cm*, Height (h) = 7 *cm*

So, Total surface area $= 2(lb + bh + lh)$

$= 2[(14 \times 7) + (7 \times 7) + (14 \times 7)]$

$= 2(98 + 49 + 98)$

$= 2 \times 245$

$= 490 \ cm^2$

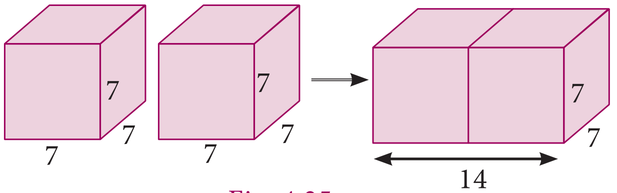


Fig. 4.25

Lateral surface area $= 2(l + b) \times h$

$= 2(14 + 7) \times 7 = 2 \times 21 \times 7$

$= 294 \ cm^2$

Thinking Corner

Which solid has greater Total Surface Area? Why?

(i)

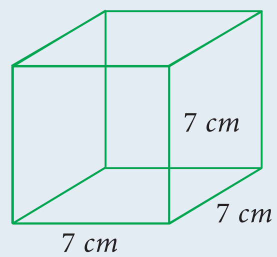


Fig. 4.26

(ii)

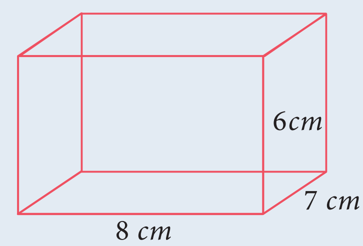


Fig. 4.27

Exercise 4.3

1. Find the Total Surface Area and the Lateral Surface Area of a cuboid whose dimensions are
   (i) length = 20 *cm*, breadth = 15 *cm*, height = 8 *cm*
   (ii) length = 16 *m*, breadth = 12 *m*, height = 8.5 *m*
2. The dimensions of a cuboidal box are 6 *m* × 400 *cm* × 1.5 *m*. Find the cost of painting its entire outer surface at the rate of ₹22 per cm^2.
3. The dimensions of a hall is 10 *m* × 9 *m* × 8 *m*. Find the cost of white washing the walls and ceiling at the rate of ₹8.50 per m^2.
4. Find the TSA and LSA of the cube whose side is (i) 8 *m* (ii) 21 *cm* (iii) 7.5 *cm*
5. (i) If the total surface area of a cube is 2400 cm^2 then, find its lateral surface area.
   (ii) The perimeter of one face of a cube is 36 *cm*. Find its total surface area.
6. A cubical container of side 6.5 *m* is to be painted on the entire outer surface. Find the area to be painted and the total cost of painting it at the rate of ₹24 per m^2.
7. Three identical cubes of side 4 *cm* are joined end to end. Find the total surface area and lateral surface area of the new resulting cuboid.

4.5 Volume of Cuboid and Cube

G1CJXB

All of us have tasted 50 *ml* and 100 *ml* of ice cream. Take one such 100 *ml* ice cream cup. This cup can contain 100 *ml* of water, which means that the capacity or volume of that cup is 100 *ml*. Take a 100 *ml* cup and find out how many such cups of water can fill a jug. If 10 such 100 *ml* cups can fill a jug then the capacity or volume of the jug is 1 litre $\left(10 \times 100\,ml = 1000\,ml = 1l\right)$. Further check how many such jug of water can fill a bucket. That is the capacity or volume of the bucket. Likewise we can calculate the volume or capacity of any such things.

Note

Unit Cube :

A cube with side 1 unit.

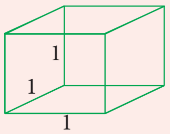


Fig. 4.28

Volume is the measure of the amount of space occupied by a three dimensional solid. Cubic centimetres (cm^3) , cubic metres (m^3) are some cubic units to measure volume.

Consider the two hollow solids shown (Fig.4.29 and Fig.4.30). The volume of an object is obtained by the number of unit cubes that can be put together to fill the entire space within the object. Without such a measure it may be difficult to judge the actual volume of an object.

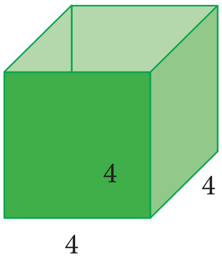


Fig. 4.29

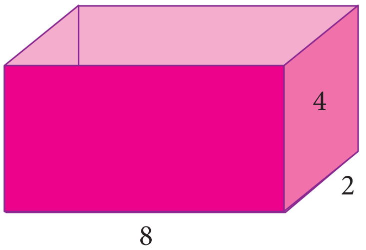


Fig. 4.30

By just observing, which one do you think has more volume? If you investigate, you will find that the same number of unit cubes would be required to completely 'fill' them up! Each one requires 64 unit cubes and the volume of each of them is 64 cubic units!

How will you find the volume of this cuboid?

It is not possible to 'fill' this with small cubes of volume 1 cm^3 and then count how many have been used. Alternately you can 'slice' the solid and then do such a counting of centimetre cubes. (See figure4.31).

Can you do this without slicing? Yes, you can visualize the situation shown in the figure and then calculate the number of 'centimetre cubes' needed to fill it up completely.

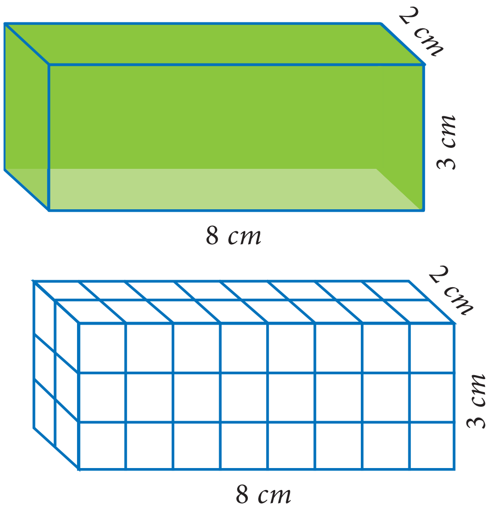


Fig. 4.31

The number of centimetre cubes needed would be $8 \times 3 \times 2 = 48$. This means the volume of the cuboid is 48 cm^3.

We find that this counting of the unit cubes has something to do with the 'base area' of the solid. In the solid cuboid discussed above,the base area is (8×2) cm^2. We multiplied this by the 'height' 3 cm to get the volume. Thus volume of the solid is the product of 'base area' and 'height'.

This can easily be understood from a practical situation. You might have seen the bundles of A4 size paper. Each paper is rectangular in shape and has an area $(=lb)$. When you pile them up, it becomes a bundle in the form of a cuboid; h times lb make the cuboid.

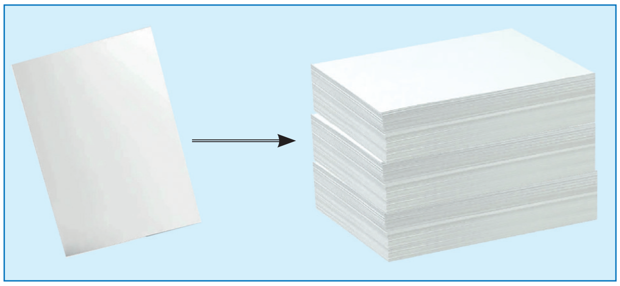


Fig. 4.32

4.5.1 Volume of a Cuboid

Let the length, breadth and height of a cuboid be l, b and h respectively.

Then, volume of the cuboid

$$V = (\text{cuboid's base area}) \times \text{height}$$
$$= (l \times b) \times h = lbh \text{ cubic units}$$

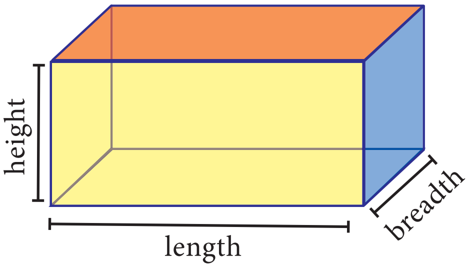


Fig. 4.33

Note

The units of length, breadth and height should be same while calculating the volume of a cuboid.

4.5.2 Volume of a Cube

It is easy to get the volume of a cube whose side is a units. Simply put $l = b = h = a$ in the formula for the volume of a cuboid. We get volume of cube to be a^3 cubic units.

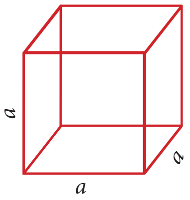


Fig. 4.34

If the side of a cube is 'a' units then the Volume of the cube $(V) = a^3$ cubic units.

Note

For any two cubes, the following results are true.

(i) Ratio of surface areas = (Ratio of sides)^2

(ii) Ratio of volumes = (Ratio of sides)^3

(iii) (Ratio of surface areas)^3 = (Ratio of volumes)^2

Example 4.12 The length, breadth and height of a cuboid is 120 *mm*, 10 *cm* and 8 *cm* respectively. Find the volume of 10 such cuboids.

Solution

Since both breadth and height are given in *cm*, it is necessary to convert the length also in *cm*.

So we get, $l = 120\ mm = \frac{120}{10} = 12\ cm$ and take $b = 10\ cm$, $h = 8\ cm$ as such.

Volume of a cuboid $= l \times b \times h$

$= 12 \times 10 \times 8$

$= 960\ cm^3$

Volume of 10 such cuboids $= 10 \times 960$

$= 9600\ cm^3$

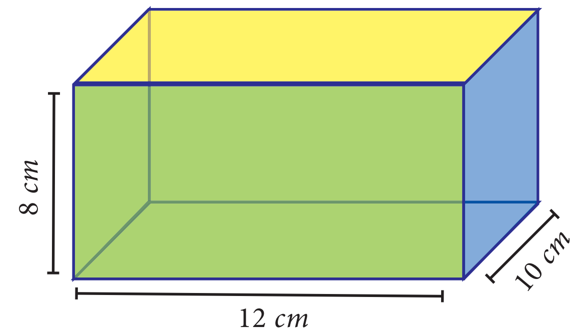


Fig. 4.35

Example 4.13 The length, breadth and height of a cuboid are in the ratio 7:5:2. Its volume is 35840 cm^3. Find its dimensions.

Solution

Let the dimensions of the cuboid be

$l = 7x,\ b = 5x$ and $h = 2x$.

Given that volume of cuboid = 35840 cm^3

$$l \times b \times h = 35840$$

$$(7x)(5x)(2x) = 35840$$

$$70x^3 = 35840$$

$$x^3 = \frac{35840}{70}$$

$$x^3 = 512$$

$$x = \sqrt[3]{8 \times 8 \times 8}$$

$$x = 8\ cm$$

Length of cuboid $= 7x = 7 \times 8 = 56\ cm$

Breadth of cuboid $= 5x = 5 \times 8 = 40\ cm$

Height of cuboid $= 2x = 2 \times 8 = 16\ cm$

THINKING CORNER

Each cuboid given below has the same volume 120 cm^3. Can you find the missing dimensions?

(i)

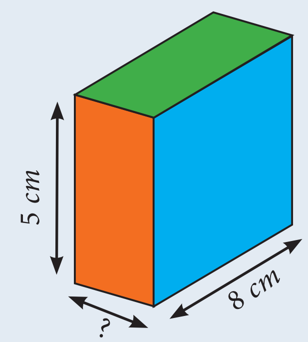


Fig. 4.36

(ii)

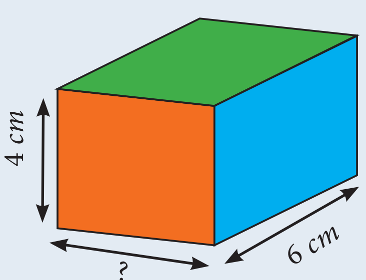


Fig. 4.37

Example 4.14 The dimensions of a fish tank are 3.8 m × 2.5 m × 1.6 m. How many litres of water it can hold?

Fig. 4.38

Solution

Length of the fish tank $l = 3.8\ m$

Breadth of the fish tank b $= 2.5\ m$, Height of the fish tank h $= 1.6\ m$

Volume of the fish tank $= l \times b \times h$

$= 3.8 \times 2.5 \times 1.6$

$= 15.2\ m^3$

$= 15.2 \times 1000\ litres$

$= 15200\ litres$

Note

A few important conversions

$1\ cm^3 = 1\ ml$, $1000\ cm^3 = 1\ litre$, $1 m^3 = 1000\ litres$

Example 4.15 The dimensions of a sweet box are 22 cm × 18 cm × 10 cm. How many such boxes can be packed in a carton of dimensions 1 m × 88 cm × 63 cm?

Solution

Here, the dimensions of a sweet box are Length (l) = 22cm, breadth (b) = 18cm, height (h) = 10 cm.

Volume of a sweet box = $l \times b \times h$

$= 22 \times 18 \times 10\ cm^3$

The dimensions of a carton are

Length (l) = 1m= 100 cm, breadth (b) = 88 cm, height (h) = 63 cm.

Volume of the carton = $l \times b \times h$

$= 100 \times 88 \times 63\ cm^3$

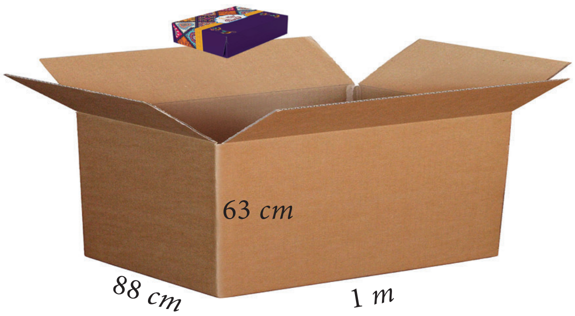


Fig. 4.39

The number of sweet boxes packed $= \dfrac{\text{volume of the carton}}{\text{volume of a sweet box}}$

$= \dfrac{100 \times 88 \times 63}{22 \times 18 \times 10}$

$= 140$ boxes

Example 4.16 Find the volume of cube whose side is 10 cm.

Solution

Given that side (a) = 10 cm

volume of the cube $= a^3$

$= 10 \times 10 \times 10$

$= 1000\ cm^3$

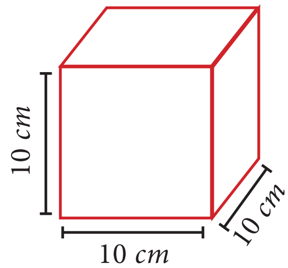


Fig. 4.40

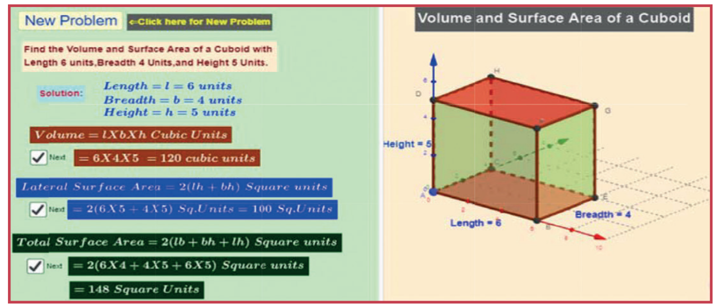
Note

Relation between side and volume of a cube

Side of a cube (in units)	1	2	3	4	5	6	7	8	9	10	Number
Volume of a cube (in cubic units)	1	8	27	64	125	216	343	512	729	1000	Its cube

ICT Corner

Expected Result is shown in this picture



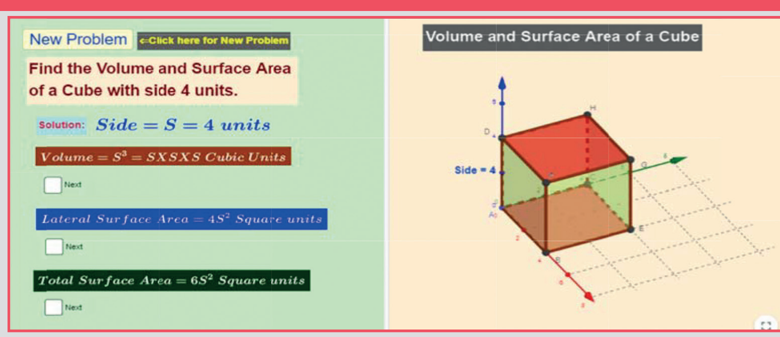
Step – 1

Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Mensuration" will open. There are two worksheets under the title CUBE and CUBOID.

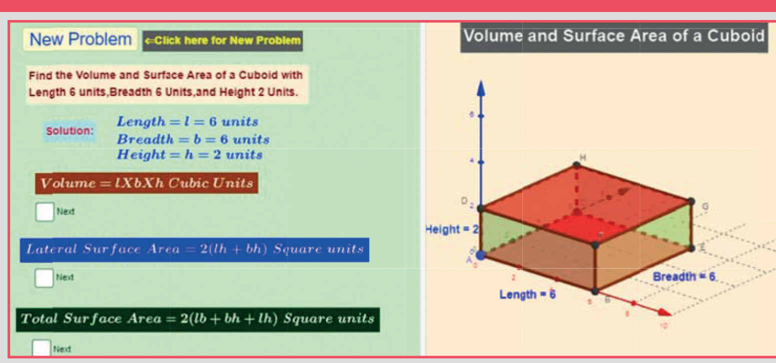
Step - 2

Click on "New Problem". Volume, Lateral surface and Total surface area are asked. Work out the solution, and click on the respective check box and check the answer.

Step 1



Step 2



Browse in the link

Mensuration: https://ggbm.at/czsby7ym or Scan the QR Code.

B566_9_MAT_EM_T3

Example 4.17 The total surface area of a cube is 864 cm^2. Find its volume.

Solution

Let 'a' be the side of the cube.

Given that, total surface area = 864 cm^2

$$6a^2 = 864$$

$$a^2 = \frac{864}{6}$$

$$a^2 = 144$$

Therefore, side (a) = 12 cm

Now, volume of the cube $= a^3$

$$= 12^3 = 12 \times 12 \times 12 = 1728 \ cm^3$$

THINKING CORNER

If [cube] =1cubic unit, then what will be the volume of given solid?

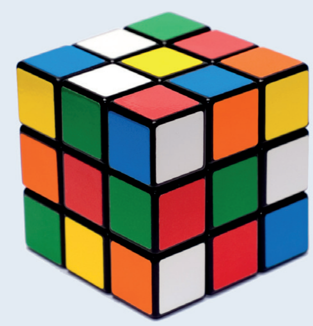


Fig. 4.41

Example 4.18 A cubical tank can hold 64,000 litres of water. Find the length of its side in metres.

Solution

Let 'a' be the side of cubical tank.

Here, volume of the tank $= 64{,}000$ litres

$$\text{i.e., } a^3 = 64{,}000 = \frac{64000}{1000} \quad [\text{since}, 1000 \text{ litres} = 1m^3]$$

$$a^3 = 64 \ m^3$$

$$a = \sqrt[3]{64} \quad a = 4 \ m$$

Therefore, length of the side of the tank is 4 metres.

Example 4.19 The side of a metallic cube is 12 cm. It is melted and formed into a cuboid whose length and breadth are 18 cm and 16 cm respectively. Find the height of the cuboid.

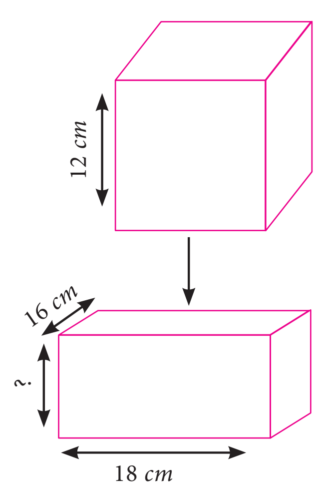


Fig. 4.42

Solution

Cube	**Cuboid**
Side $(a) = 12\,cm$	length (l) = 18cm
	breadth (b) = 16cm
	height (h) = ?

Here, Volume of the Cuboid $=$ Volume of the Cube

$$l \times b \times h = a^3$$

$$18 \times 16 \times h = 12 \times 12 \times 12$$

$$h = \frac{12 \times 12 \times 12}{18 \times 16}$$

$$h = 6\ cm$$

Therefore, the height of the cuboid is 6 *cm*.

Activity - 3

Take some square sheets of paper / chart paper of given dimension 18 *cm* × 18 *cm*. Remove the squares of same sizes from each corner of the given square paper and fold up the flaps to make a cuboidal box. Then tabulate the dimensions of each of the cuboidal boxes made. Also find the volume each time and complete the table. The side measures of corner squares that are to be removed is given in the table below.

Sl. No	Side of the corner square	Dimensions of boxes			Volume
		l	b	h	V
1.	1 *cm*				
2.	2 *cm*				
3.	3 *cm*				
4.	4 *cm*				
5.	5 *cm*				

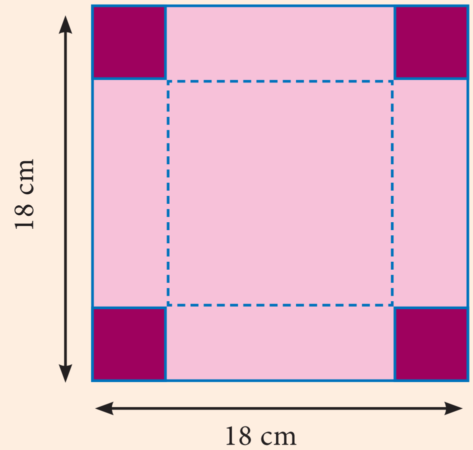


Fig. 4.43

Observe the above table and answer the following:

(i) What is the greatest possible volume?

(ii) What is the side of the square that when removed produces the greatest volume?

Exercise 4.4

1. Find the volume of a cuboid whose dimensions are
   (i) length = 12 *cm*, breadth = 8 *cm*, height = 6 *cm*
   (ii) length = 60 *m*, breadth = 25 *m*, height = 1.5 *m*
   (iii) length = 2 *m* , breadth = 60 *cm*, height = 72 *cm*

2. The dimensions of a match box are 6 *cm* × 3.5 *cm* × 2.5 *cm*. Find the volume of a packet containing 12 such match boxes.

3. The length, breadth and height of a chocolate box are in the ratio 5:4:3. If its volume is 7500 cm^3, then find its dimensions.

4. The length, breadth and depth of a pond are 20.5 m, 16 m and 8 m respectively. Find the capacity of the pond in litres.

5. The dimensions of a brick are 24 $cm \times 12\ cm \times 8\ cm$. How many such bricks will be required to build a wall of 20 m length, 48 cm breadth and 6 m height?

6. The volume of a cuboid is 1800 cm^3. If its length is 15 cm and height 12 cm, then find the breadth of the cuboid.

7. The volume of a container is 1440 m^3. The length and breadth of the container are 15 m and 8 m respectively. Find its height.

8. Find the volume of a cube each of whose side is (i) 5 cm (ii) 3.5 m (iii) 21 cm

9. If the total surface area of a cube is 726 cm^2, then find its volume.

10. A cubical milk tank holds 125000 litres of milk. Find the length of its side in metres.

11. A metallic cube with side 15 cm is melted and formed into a cuboid. If the length and height of the cuboid is 25 cm and 9 cm respectively then find the breadth of the cuboid.

12. The dimensions of a water tank are $12\ m \times 10\ m \times 8\ m$. If it is filled with water up to the level of 5 m, how much more water would be needed to fill the tank completely?

13. External dimensions of a closed wooden cuboidal box are $30\ cm \times 25\ cm \times 20\ cm$. If the thickness of the wood is 2 cm all around, find the volume of the wood contained in the cuboidal box formed.

Exercise 4.5

Multiple choice questions

1. The area of a triangle whose sides are a, b and c is

(1) $\sqrt{(s-a)(s-b)(s-c)}$ sq. units (2) $\sqrt{s(s+a)(s+b)(s+c)}$ sq. units

(3) $\sqrt{s(s\times a)(s\times b)(s\times c)}$ sq. units (4) $\sqrt{s(s-a)(s-b)(s-c)}$ sq. units

2. The semi-perimeter of a triangle having sides 15 cm, 20 cm and 25 cm is

(1) 60 cm (2) 45 cm (3) 30 cm (4) 15 cm

3. If the sides of a triangle are 3 cm, 4 cm and 5 cm, then the area is

(1) 3 cm^2 (2) 6 cm^2 (3) 9 cm^2 (4) 12 cm^2

4. The perimeter of an equilateral triangle is 30 cm. The area is

(1) $10\sqrt{3}\ cm^2$ (2) $12\sqrt{3}\ cm^2$ (3) $15\sqrt{3}\ cm^2$ (4) $25\sqrt{3}\ cm^2$

5. The total surface area of a cuboid is

(1) $4a^2$ sq. units (2) $6a^2$ sq. units

(3) $2(l+b)h$ sq. units (4) $2(lb+bh+lh)$ sq. units

6. The lateral surface area of a cube of side 12 cm is

(1) 144 cm^2 (2) 196 cm^2 (3) 576 cm^2 (4) 664 cm^2

G1HTB2

7. If the lateral surface area of a cube is 600 cm^2, then the total surface area is

(1) 150 cm^2 (2) 400 cm^2 (3) 900 cm^2 (4) 1350 cm^2

8. The total surface area of a cuboid with dimension 10 $cm \times 6$ $cm \times 5$ cm is

(1) 280 cm^2 (2) 300 cm^2 (3) 360 cm^2 (4) 600 cm^2

9. If the ratio of the sides of two cubes are 2:3, then ratio of their surface areas will be

(1) 4:6 (2) 4:9 (3) 6:9 (4) 16:36

10. The volume of a cuboid is 660 cm^3 and the area of the base is 33 cm^2. Its height is

(1) 10 cm (2) 12 cm (3) 20 cm (4) 22 cm

11. The capacity of a water tank of dimensions 10 $m \times 5$ $m \times 1.5$ m is

(1) 75 litres (2) 750 litres (3) 7500 litres (4) 75000 litres

12. The number of bricks each measuring 50 $cm \times 30$ $cm \times 20$ cm that will be required to build a wall whose dimensions are 5 $m \times 3$ $m \times 2$ m is

(1) 1000 (2) 2000 (3) 3000 (4) 5000

Points to Remember

- If a, b and c are the sides of a triangle, then the area of a triangle $=\sqrt{s(s-a)(s-b)(s-c)}$ sq.units, where $s=\dfrac{a+b+c}{2}$.
- If the length, breadth and height of the cuboid are l, b and h respectively, then

  (i) Total Surface Area(TSA) $=2(lb+bh+lh)$ sq.units

  (ii) Lateral Surface Area(LSA) $=2(l+b)h$ sq.units
- If the side of a cube is 'a' units, then

  (i) Total Surface Area(TSA) $=6a^2$ sq.units

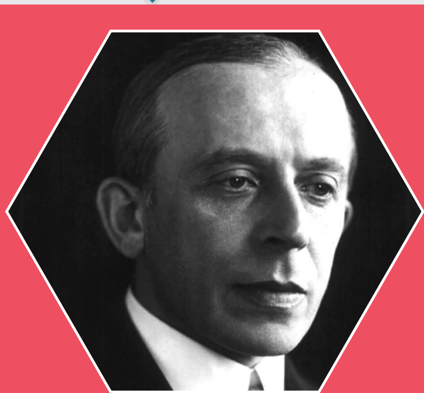
  (ii) Lateral Surface Area(LSA) $=4a^2$ sq.units
- If the length, breadth and height of the cuboid are l, b and h respectively, then the Volume of the cuboid $(V)=lbh$ cu.units
- If the side of a cube is 'a' units then, the Volume of the cube $(V)=a^3$ cu.units.

5



PROBABILITY

Probability theory is nothing more than common sense reduced to calculation. *- Pierre Simon Laplace.*



Richard Von Mises
A.D (C.E) 1883-1953

The statistical or empirical, attitude towards probability has been developed mainly by R.F.Fisher and R.Von Mises. The notion of sample space comes from R.Von Mises. This notion made it possible to build up a strictly mathematical theory of probability based on measure theory. Such an approach emerged gradually in the last century under the influence of many authors.

Learning Outcomes

- To understand the basic concepts of probability.
- To understand classical approach and empirical approach of probability.
- To familiarise the types of events in probability.

G1RPCP

5.1 Introduction

To understand the notion of probability, we look into some real life situations that involve some traits of uncertainty.

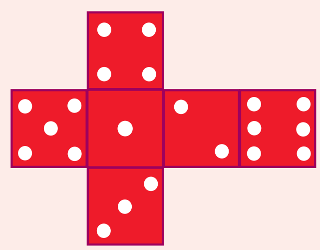
A life-saving drug is administered to a patient admitted in a hospital. The patient's relatives may like to know the probability with which the drug will work; they will be happy if the doctor tells that out of 100 patients treated with the drug, it worked well with more than 80 patients. This percentage of success is illustrative of the concept of probability; it is based on the frequency of occurrence. It helps one to arrive at a conclusion under uncertain conditions. Probability is thus a way of quantifying or measuring uncertainty.

You should be familiar with the usual complete pack of 52 playing cards. It has 4 suits(Hearts ♥, Clubs ♣, Diamonds ♦, Spades♠), each with 13 cards. Choose one of the suits or cards, say spades. Keep these 13 cards facing downwards on the table. Shuffle them well and pick up any one card. What is the chance that it will be a King? Will the chances vary if you do not want a King but an Ace? You will be quick to see that in either case, the chances are 1 in 13 (Why?). It will be the same whatever single card you choose to pick up. The word 'Probability' means precisely the same thing as 'chances' and has the same value, but instead of saying 1 in 13 we write it as a fraction $\frac{1}{13}$. (It would be easy to manipulate with fractions when we combine probabilities). It is 'the ratio of the favourable cases to the total number of possible cases'.

Have you seen a 'dice' ? (Some people use the word 'die' for a single 'dice'; we use 'dice' here, both for the singular and plural cases). A standard dice is a cube, with each side having a different number of spots on it, ranging from one to six, rolled and used in gambling and other games involving chance.

Note

In a fair die the sum of the numbers turning on the opposite sides will always be equal to 7.



If you throw a dice, what is the probability of getting a five? a two? a seven?

In all the answers you got for the questions raised above, did you notice anything special about the concept of probability? Could there be a maximum value for probability? or the least value? If you are sure of a certain occurrence what could be its probability? For a better clarity, we will try to formalize the notions in the following paragraphs.

5.2 Basic Ideas

When we carry out experiments in science repeatedly under identical conditions, we get almost the same result. Such experiments are known as *deterministic*. For example, the experiments to verify Archimedes principle or to verify Ohm's law are *deterministic*. The outcomes of the experiments can be predicted well in advance.

But, there are experiments in which the outcomes may be different even when performed under identical conditions. For example, when a fair dice is rolled, a fair coin is flipped or while selecting the balls from an urn, we cannot predict the exact outcome of these experiments; these are *random experiments*. Each performance of a random experiment is called a trial and the result of each trial is called an outcome. (*Note: Many statisticians use the words 'experiment' and 'trial' synonymously.*)

Now let us see some of the important terms related to probability.

Trial : Rolling a dice and flipping a coin are trials. A *trial* is an action which results in one or several outcomes.

Outcome : While flipping a coin we get Head or Tail . Head and Tail are called outcomes. The result of the trial is called an *outcome*.

Sample point : While flipping a coin, each outcome H or T are the sample points. Each outcome of a random experiment is called a *sample point*.

Sample space : In a single flip of a coin, the collection of sample points is given by $S = \{H,T\}$.

If two coins are tossed the collection of sample points $S=\{(HH),(HT),(TH),(TT)\}$.

The set of all possible outcomes (or Sample points) of a random experiment is called the *Sample space*. It is denoted by S. The number of elements in it are denoted by $n(S)$.

Event : If a dice is rolled, it shows 4 which is called an outcome (since, it is a result of a single trial). In the same experiment the event of getting an even number is {2,4,6}. So any subset of a sample space is called an *event*. Hence an event can be one or more than one outcome.

For example

Random experiment : Flipping a coin

Possible outcomes : Head(H) or Tail(T)

Sample space : $S = \{H,T\}$

Subset of S : $A=\{H\}$ or $A=\{T\}$

Thus, in this example A is an event.

Similarly when we roll a single dice the collection of all sample points is $S = \{1,2,3,4,5,6\}$. When we select a day in a week the collection of sample points is S = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}.

Activity - 1

Perform the experiment of tossing two coins at a time. List out the following in the above experiment.	Perform the experiment of throwing two dice at a time. List out the following in this experiment also.
Random experiment : Possible outcomes : Sample space : Any three subsets of S : (or any 3 events)	Random experiment : Possible outcomes : Sample space : Any three subsets of S : (or any 3 events)

Activity - 2

Each student is asked to flip a coin 10 times and tabulate the number of heads and tails obtained in the following table.

Number of tosses	Number of times head comes up	Number of times tail comes up
⋮	⋮	⋮

(i) Fraction 1 : $\dfrac{\textit{Number of times head comes up}}{\textit{Total number of times the coin is tossed}}$

(ii) Fraction 2 : $\dfrac{\textit{Number of times tail comes up}}{\textit{Total number of times the coin is tossed}}$

Repeat it by tossing the coin 20, 30, 40, 50 times and find the fractions.

Activity - 3

Divide the class students into groups of pairs. In each pair, the first one tosses a coin 50 times, and the second one records the outcomes of tosses. Then we prepare a table given below.

Group	Number of times head comes up	Number of times tail comes up	$\dfrac{\text{Number of times head comes up}}{\text{Total number of times the coin is tossed}}$	$\dfrac{\text{Number of times tail comes up}}{\text{Total number of times the coin is tossed}}$
1				
2				
3				
⋮	⋮	⋮	⋮	⋮

The chance of an event happening when expressed quantitatively is probability.

5.3 Classical Approach

For example, An urn contains 4 Red balls and 6 Blue balls. You choose a ball at random from the urn. What is the probability of choosing a Red ball?

The phrase 'at random' assures you that each one of the 10 balls has the same chance (that is, probability) of getting chosen. You may be blindfolded and the balls may be mixed up for a "fair" experiment. This makes the outcomes "equally likely".

The probability that the Red Ball is chosen is $\frac{4}{10}$ (You may also give it as $\frac{2}{5}$ or 0.4).

What would be the probability for choosing a Blue ball? It is $\frac{6}{10}$ (or $\frac{3}{5}$ or 0.6).

Note that the sum of the two probabilities is 1. This means that no other outcome is possible.

The approach we adopted in the above example is classical. It is calculating a *priori probability*. (The Latin phrase a *priori* means '*without investigation or sensory experience*'). Note that the above treatment is possible only when the outcomes are equally likely.

Classical probability is so named, because it was the first type of probability studied formally by mathematicians during the 17th and 18th centuries.

Thinking Corner

If the probability of success of an experiment is 0.4, what is the probability of failure?

Let S be the set of all equally likely outcomes of a random experiment. (S is called the sample space for the experiment.)

Let E be some particular outcome or combination of outcomes of an experiment. (E is called an event.)

The probability of an event E is denoted as $P(E)$.

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}$$

G21KEC

The empirical approach (relative frequency theory) of probability holds that if an experiment is repeated for an extremely large number of times and a particular outcome occurs at a percentage of the time, then that particular percentage is close to the probability of that outcome.

5.4 Empirical Approach

For example, A manufacturer produces 10,000 electric switches every month and1,000 of them are found to be defective. What is the probability of the manufacturer producing a defective switch every month?

The required probability, according to relative frequency concept, is nearly 1000 out of 10000, which is 0.1

> **Note**
>
> The number of trials has to be large to decide this probability. The larger the number of trials, the better will be the estimate of probability.

Let us formalize the definition: "If, in a large number of trials, say n, we find r of the outcomes in an event E, then the probability of event E, denoted by $P(E)$, is given by

$$P(E) = \frac{r}{n}.$$

Is there a guarantee that this value will settle down to a constant value when the number of trials gets larger and larger? One cannot say; the concept being experimental, it is quite possible to get distinct relative frequency each time the experiment is repeated.

However, there is a security range: the value of probability can at the least take the value 0 and at the most take the value 1. We can state this mathematically as

$$0 \le P(E) \le 1.$$

> **Thinking Corner**
>
> For a question on probability the student's answer was $\frac{3}{2}$. The teacher told that the answer was wrong. Why?

Let us look at this in a little detail.

First, we know that r cannot be larger than n.

This means $\frac{r}{n} < 1$. That is $P(E) < 1$. ... (1)

Next, if $r = 0$, it means either the event cannot happen or has not occurred in a large number of trials. (Can you get a 7, when you roll a dice?).

Thus , in this case $\frac{r}{n} = \frac{0}{n} = 0$. ... (2)

Lastly, if $r = n$, the event must occur (in every trial or in a large number of trials).

In such a situation, $\frac{r}{n} = \frac{n}{n} = 1$. ... (3)

(getting any number from 1 to 6 when you roll a dice)

From (1), (2) and (3) we find $0 \le P(E) \le 1$.

Progress Check

A random experiment was conducted. Which of these cannot be considered as a probability of an outcome?

(i) 1/5 (ii) – 1/7 (iii) 0.40 (iv) – 0.52 (v) 0

(vi) 1.3 (vii) 1 (viii) 72% (ix) 107%

Example 5.1 When a dice is rolled, find the probability to get the number greater than 4?

Solution

The outcomes $S = \{1,2,3,4,5,6\}$

Let E be the event of getting a number greater than 4

$$E = \{5,6\}$$

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = 0.333...$$

G2AGFZ

Example 5.2 In an office, where 42 staff members work, 7 staff members use cars, 20 staff members use two-wheelers and the remaining 15 staff members use cycles. Find the relative frequencies.

Solution

Total number of staff members = 42.

The relative frequencies:

$$\text{Car users} = \frac{7}{42} = \frac{1}{6}$$

$$\text{Two-wheeler users} = \frac{20}{42} = \frac{10}{21}$$

$$\text{Cycle users} = \frac{15}{42} = \frac{5}{14}$$

In this example note that the total probability does not exceed 1 that is,

$$\frac{1}{6} + \frac{10}{21} + \frac{5}{14} = \frac{7}{42} + \frac{20}{42} + \frac{15}{42} = 1$$

Example 5.3 Team I and Team II play 10 cricket matches each of 20 overs. Their total scores in each match are tabulated in the table as follows:

Match numbers	1	2	3	4	5	6	7	8	9	10
Team I	200	122	111	88	156	184	99	199	121	156
Team II	143	123	156	92	164	72	100	201	98	157

What is the relative frequency of Team I winning?

Solution

In this experiment, each trial is a match where Team I faces Team II.

We are concerned about the winning status of Team I.

There are 10 trials in total; out of which Team I wins in the 1st, 6th and 9th matches.

The relative frequency of Team I winning the matches $= \dfrac{3}{10}$ or 0.3.

(**Note :** The relative frequency depends on the sequence of outcomes that we observe during the course of the experiment).

Exercise 5.1

1. You are walking along a street. If you just choose a stranger crossing you, what is the probability that his next birthday will fall on a sunday?

2. What is the probability of drawing a King or a Queen or a Jack from a deck of cards?

3. What is the probability of throwing an even number with a single standard dice of six faces?

4. There are 24 balls in a pot. If 3 of them are Red, 5 of them are Blue and the remaining are Green then, what is the probability of picking out (i) a Blue ball, (ii) a Red ball and (iii) a Green ball?

5. When two coins are tossed, what is the probability that two heads are obtained?

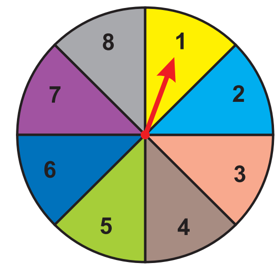
6. Two dice are rolled, find the probability that the sum is

   i) equal to 1 ii) equal to 4 iii) less than 13

7. A manufacturer tested 7000 LED lights at random and found that 25 of them were defective. If a LED light is selected at random, what is the probability that the selected LED light is a defective one.

8. In a football match, a goalkeeper of a team can stop the goal, 32 times out of 40 attempts tried by a team. Find the probability that the opponent team can convert the attempt into a goal.

9. What is the probability that the spinner will not land on a multiple of 3?



10. Frame two problems in calculating probability, based on the spinner shown here.

5.5 Types of Events

We have seen some important cases of events already.

When the likelihood of happening of two events are same they are known as equally likely events.

- If we toss a coin, getting a head or a tail are equally likely events.
- If a dice is rolled, then getting an odd number and getting an even number are equally likely events, whereas getting an even number and getting 1 are not equally likely events.

When probability is 1, the event is sure to happen. Such an event is called a sure or certain event. The other extreme case is when the probability is 0, which is known as an impossible event.

If $P(E) = 1$ then E is called Certain event or Sure event.

If $P(E) = 0$ then E is known is an Impossible event.

Consider a "coin flip". When you flip a coin, you cannot get both heads and tails simultaneously. (Of course, the coin must be fair; it should not have heads or tails on both sides!). If two events cannot occur simultaneously (at the same time), they are said to be mutually exclusive events. Are rain and sunshine mutually exclusive? What about choosing Kings and Hearts from a pack of 52 cards?

A dice is thrown. Let E be the event of getting an "even face". That is getting 2, 4 or 6. Then the event of getting an "odd face" is complementary to E and is denoted by E' or E^c. In the above sense E and E' are complementary events.

Note

(i) The events E and E' are mutually exclusive. (how?)

(ii) The probability of E + the probability of $E'=1$. Also E and E' are mutually exclusive.

(iii) Since $P(E)+P(E')=1$, if you know any one of them, you can find the other.

Progress Check

Which among the following are mutually exclusive?

Sl.No.	Trial	Event 1	Event 2
1	Roll a dice	getting a 5	getting an odd number
2	Roll a dice	getting a 5	getting an even number
3	Draw a card from a standard pack	getting a Spade Card	getting a black
4	Draw a card from a standard pack	getting a Picture Card	getting a 5
5	Draw a card from a standard pack	getting a Heart Card	getting a 7

Example 5.4 The probability that it will rain tomorrow is $\frac{91}{100}$. What is the probability that it will not rain tomorrow?

Solution

Let E be the event that it will rain tomorrow. Then E' is the event that it will not rain tomorrow.

Since $P(E) = 0.91$, we have $P(E') = 1-0.91$ (how?)

$= 0.09$

Therefore, the probability that it will not rain tomorrow

$= 0.09$

Example 5.5 In a recent year, of the 1184 centum scorers in various subjects in tenth standard public exams, 233 were in mathematics. 125 in social science and 106 in science. If one of the student is selected at random, find the probability of that selected student,

(i) is a centum scorer in Mathematics (ii) is not a centum scorer in Science

Solution

Total number of centum scorers =1184

Therefore n = 1184

(i) Let E_1 be the event of getting a centum scorer in Mathematics.

Therefore $n(E_1) = 233$, That is, $r_1 = 233$

$$P(E_1) = \frac{r_1}{n} = \frac{233}{1184}$$

(ii) Let E_2 be the event of getting a centum scorer in Science.

Therefore $n(E_2) = 106$, That is, $r_2 = 106$

$$P(E_2) = \frac{r_2}{n} = \frac{106}{1184}$$

$$P(E_2') = 1 - P(E_2)$$

$$= 1 - \frac{106}{1184}$$

$$= \frac{1078}{1184}$$

Progress Check

1. A dice is rolled once. What is the probability that the score obtained is a factor of 6?

2. You have a single standard deck of 52 cards; which of the following pairs of events are mutually exclusive?
   (i) Drawing a red card and drawing a king
   (ii) Drawing a red card and drawing a club
   (iii) Drawing a black card and drawing a spade
   (iv) Drawing a black card and drawing an ace

3. You are rolling a standard six-faced cubic dice just once. Which of the following pairs of events are not mutually exclusive?

   (i) Getting an even number and getting a multiple of 3.

   (ii) Getting an even number and getting a multiple of 5.

   (iii) Getting a prime number and getting an even number.

   (iv) Getting a non-prime and getting an odd number.

4. A number from 1 to 8 is chosen at random. What is the probability that the number chosen is not even?

Exercise 5.2

1. A company manufactures 10000 Laptops in 6 months. In that 25 of them are found to be defective. When you choose one Laptop from the manufactured, what is the probability that selected Laptop is a good one.

2. In a survey of 400 youngsters aged 16-20 years, it was found that 191 have their voter ID card. If a youngster is selected at random, find the probability that the youngster does not have their voter ID card.

3. The probability of guessing the correct answer to a certain question is $\frac{x}{3}$. If the probability of not guessing the correct answer is $\frac{x}{5}$, then find the value of x.

4. If a probability of a player winning a particular tennis match is 0.72. What is the probability of the player loosing the match?

5. 1500 families were surveyed and following data was recorded about their maids at homes

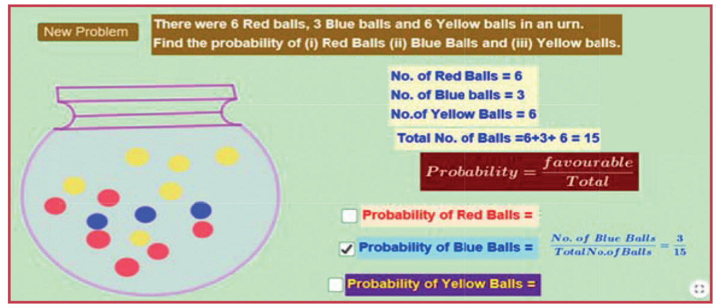
Type of maids	Only part time	Only full time	Both
Number of families	860	370	250

A family is selected at random. Find the probability that the family selected has

(i) Both types of maids (ii) Part time maids (iii) No maids

ICT Corner

Expected Result is shown in this picture



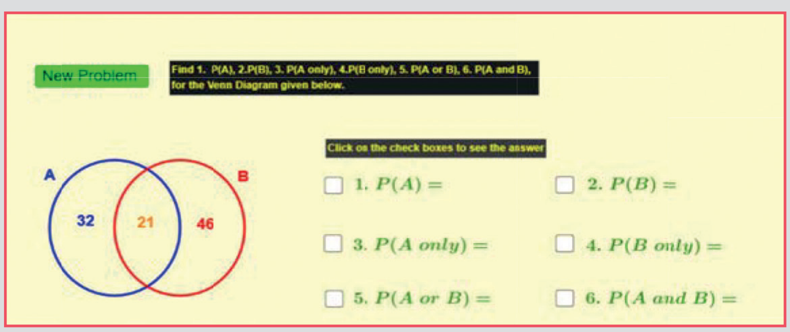
Step – 1

Open the Browser by typing the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "Probability" will open. There are two worksheets under the title Venn diagram and Basic probability.

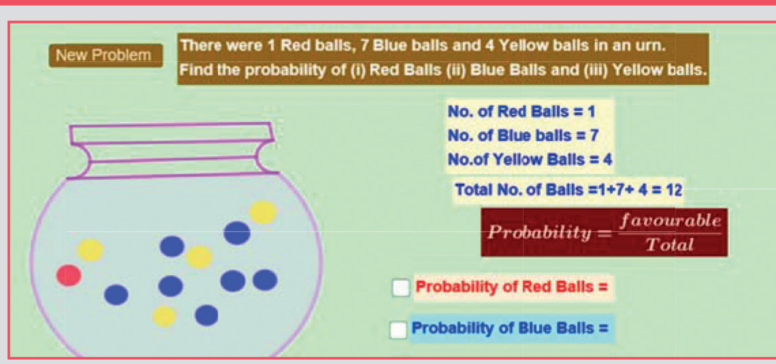
Step - 2

Click on "New Problem". Work out the solution, and click on the respective check box and check the answer.

Step 1



Step 2



Browse in the link

Probability: https://ggbm.at/mj887yua or Scan the QR Code.

B566_9_MAT_EM_T3

Exercise 5.3

Multiple choice questions

1. A number between 0 and 1 that is used to measure uncertainty is called

   (1) Random variable (2) Trial (3) Simple event (4) Probability

2. Probability lies between

   (1) -1 and $+1$ (2) 0 and 1 (3) 0 and n (4) 0 and ∞

3. The probability based on the concept of relative frequency theory is called

   (1) Empirical probability (2) Classical probability

   (3) Both (1) and (2) (4) Neither (1) nor (2)

4. The probability of an event cannot be

   (1) Equal to zero (2) Greater than zero (3) Equal to one (4) Less than zero

5. A random experiment contains

   (1) Atleast one outcome (2) Atleast two outcomes

   (3) Atmost one outcome (4) Atmost two outcomes

6. The probability of all possible outcomes of a random experiment is always equal to

   (1) One (2) Zero (3) Infinity (4) All of the above

7. If A is any event in S then its complement $P(A')$ is equal to

   (1) 1 (2) 0 (3) $1-A$ (4) $1-P(A)$

8. Which of the following cannot be taken as probability of an event?

   (1) 0 (2) 0.5 (3) 1 (4) -1

9. A particular result of an experiment is called

   (1) Trial (2) Simple event (3) Compound event (4) Outcome

10. A collection of one or more outcomes of an experiment is called

    (1) Event (2) Outcome (3) Sample point (4) None of the above

11. The six faces of the dice are called equally likely if the dice is

(1) Small (2) Fair (3) Six-faced (4) Round

12. A letter is chosen at random from the word "STATISTICS". The probability of getting a vowel is

(1) $\frac{1}{10}$ (2) $\frac{2}{10}$ (3) $\frac{3}{10}$ (4) $\frac{4}{10}$

Points to Remember

- If we are able to predict the outcome of an experiment then it is called deterministic experiment.
- If we cannot predict the outcome of an experiment then it is called random experiment.
- Sample space S for an experiment is the set of all possible outcomes of a random experiment.
- An event is a particular outcome or combination of outcomes of an experiment.
- Empirical probability states that probability of an outcome is close to the percentage of occurrence of the outcome.
- If the likelihood of happening of two events are same then they are known as equally likely events.
- If two events cannot occur simultaneously in single trial then they are said to be mutually exclusive events.
- Two events E and E' are said to be complementary events if $P(E)+P(E')=1$.
- An event which is sure to happen is called certain or sure event. The probability of a sure event is one.
- An event which never happen is called impossible event. The probability of an impossible event is zero.

ANSWERS

1. Algebra

Exercise 1.1

2. Yes   3.(i) $x=-28$ (ii) $x=-\frac{2}{3}$

Exercise 1.3

1.(i) (5,2)   (ii) Infinite number of solutions

(iii) no solution   (iv) $(-3, -3)$ (v) no solution

(vi) Infinite number of solutions   (vii) (1,3)   (viii) $(-3, 3)$

2. 75*km*, 25*km*

Exercise 1.4

1.(i) $(2, -1)$ (ii) (4,2)   (iii) (40,100)   (iv) $\left(\sqrt{8},\sqrt{3}\right)$

(v) (4,9)   (2) 45 (3) 409

Exercise 1.5

1.(i) (2,1)   (ii) (7,2)   (iii) (80,30) (iv) $\left(1,\frac{3}{2}\right)$ (v) $\left(\frac{1}{3},-1\right)$

(vi) (2,1)   (vii) (2,4)   (viii) $(2, -1)$

(2) ₹30000, ₹40000   (3) 75, 15

Exercise 1.6

1.(i) (3,4)   (ii) $(3, -1)$ (iii) $\left(-\frac{1}{2},\frac{1}{3}\right)$

(2) Number of 2 rupee coins 60; Number of 5 rupee coins 20

(3) Larger pipe 20 hours; Smaller pipe 30 hours

Exercise 1.7

1. 8 square units   2. 64   3. $\frac{5}{7}$

4. speed of car A=40km/hr; speed of car B=30km/hr

5. $\angle A=120°$, $\angle B=70°$, $\angle C=60°$, $\angle D=110°$

6. Price of TV = ₹20000; Price of fridge = ₹10000

7. 5:6 8. 40 9. 180 10. 210, 6

11. 10, 3 12. 1 boy – 36 days; 1 man – 18 days

Exercise 1.8

1. (4) infinite solutions 2. (2) -2 3. (3) $3x+5=\frac{2}{3}$ 4. (2) (4,2)

5. (2) $5x-7=6-2x$ 6. (4) 13 7. (3) $a=0,\ b=0,\ c\neq 0$

8.(2) $0x+0y+c=0$ 9. (1) $k=3$ 10. (2) $l \parallel m$ 11.(3) unique

12. (1) No solution

2. Coordinate Geometry

Exercise 2.1

1.(i) $(-4,-1)$ (ii) $(0,-1)$ (iii) $(a+b,a)$ (iv) $(1,-1)$

2. $(-5,-3)$ 3. $P=-15$ 4. $(9,3)(-5,5)$ and $(1,1)$

5. $\left(\frac{9}{2},\frac{3}{2}\right)$ 6. $(1,8)$

Exercise 2.2

1. $(7,3)$ 2. $(2,-3)$ 3. 5:2 4. $(3,4)$

5. $(-2,3),(1,0)$ 6. $\left(\frac{19}{2},\frac{13}{2}\right),\left(\frac{-9}{2},\frac{-15}{2}\right)$ 8. $(3,2)$

Exercise 2.3

1.(i) $(2,-3)$ (ii) $\left(\frac{-8}{3},\frac{-11}{3}\right)$ 2. $(4,-6)$ 3. 5 units

4. 20 5. $3\sqrt{\frac{5}{2}}$ units 6. $(1,0)$ 7. $(5,-2)$

Exercise 2.4

1. (4) $(4,6)$ 2. (1) $(-9,7)$ 3. (3) 1:3 4. (4) $(-9,0)$

5. (2) $-b_1:b_2$ 6. (3) 4:7 7. (2) $(4,0)$, $(2,8)$

8. (2) $(-2a,-b)$ 9. (4) 5:2 10. (2) 5 11. (2) $(2,3)$

3. Trigonometry

Exercise 3.1

1. $\sin B=\frac{9}{41};\cos B=\frac{40}{41};\tan B=\frac{9}{40};\ \text{cosec}B=\frac{41}{9};\sec B=\frac{41}{40};\cot B=\frac{40}{9}$

2. $\sin\theta=\frac{4}{5};\cos\theta=\frac{3}{5};\tan\theta=\frac{4}{3}\ ;\ \text{cosec}\theta=\frac{5}{4};\sec\theta=\frac{5}{3};\cot\theta=\frac{3}{4}$

3.(i) $\sin B=\frac{12}{13}$ (ii) $\sec B=\frac{13}{5}$ (iii) $\cot B=\frac{5}{12}$ (iv) $\cos C=\frac{4}{5}$

(v) $\tan C = \frac{3}{4}$ (vi) $\operatorname{cosec} C = \frac{5}{3}$

4. $\sin\theta = \frac{1}{2}; \cos\theta = \frac{\sqrt{3}}{2}; \tan\theta = \frac{1}{\sqrt{3}}$; $\operatorname{cosec}\theta = \frac{2}{1}; \sec\theta = \frac{2}{\sqrt{3}}; \cot\theta = \sqrt{3}$

5. $\frac{3}{40}$

6. $\sin A = \frac{1-x^2}{1+x^2}; \tan A = \frac{1-x^2}{2x}$

8. $\frac{1}{2}$

9. $\frac{1}{2}$

10. $\sin\alpha = \frac{4}{5}; \cos\beta = \frac{4}{5}; \tan\phi = \frac{4}{3}$

11. 7m

Exercise 3.2

2.(i) 0 (ii) $\frac{7}{4}$ (iii) 3 3. 0 4. 2

Exercise 3.3

1.(i) 0 (ii) 1 (iii) 1 (iv) 2

Exercise 3.4

1.(i) 0.7547 (ii) 0.2648 (iii) 1.3985 (iv) 0.3641

(v) 0.8302 (vi) 2.7907 2.(i) $85°57'$ (or) $85°58'$ (or) $85°59'$

(ii) $47°27'$ (iii) $4°7'$ (iv) $87°39'$ (v) $82°30'$

3.(i) 1.9970 (ii) 2.8659 4. 18.81 cm^2 5. $36°52'$

6. 54.02 m^2

Exercise 3.5

1. (1) $\frac{1}{2}$
2. (2) $53°$
3. (2) 1
4. (3) $\tan 45°$
5. (3) $\tan 60°$
6. (1) $0°$
7. (2) $30°$
8. (3) 5
9. (2) 2
10. (3) 0
11. (4) $\frac{4}{3}$
12. (1) 0
13. (2) 1
14. (2) $90°$
15. (3) 1

4. Mensuration

Exercise 4.1

1.(i) 120 cm^2 (ii) 7.2 m^2 2. 1320 m^2, ₹26400 3. 12000 m^2

4. 1558.8 cm^2 5. 1050 6.(i) 480 cm^2 (ii) 24 cm

7. 240 cm^2

Exercise 4.2

1. 138 cm^2
2. 354cm^2
3. 1536 m^2
4. 672 m^2
5. 86.6 m^2

Exercise 4.3

1.(i) $1160cm^2$, $560cm^2$ (ii) $860\ m^2$, $476\ m^2$ 2. ₹1716 3. ₹3349

4.(i) $384\ m^2$, $256\ m^2$ (ii) $2646\ cm^2$, $1764\ cm^2$ (iii) $337.5\ cm^2$, $225\ cm^2$

5.(i) $1600\ cm^2$ (ii) $486\ cm^2$ 6. $253.50m^2$, ₹6084 7. $224cm^2$, $128cm^2$

Exercise 4.4

1.(i) $576\ cm^3$ (ii) $2250\ m^3$ (iii) $864000\ cm^3$ 2. $630\ cm^3$

3. 25 *cm*, 20 *cm*, 15 *cm* 4. 2624000 *litres* 5. 25000

6. 10 *cm* 7. 12 *m* 8.(i) $125\ cm^3$ (ii) $42.875\ m^3$

(iii) $9261\ cm^3$ 9. $1331cm^3$ 10. 5 *m* 11. 15 *cm*

12. 360000 *litres* 13. $6264\ m^3$

Exercise 4.5

1. (4) $\sqrt{s(s-a)(s-b)(s-c)}$ *sq. units* 2. (3) 30 *cm* 3. (2) $6\ cm^2$

4. (4) $25\sqrt{3}\ cm^2$ 5. (4) $2(lb+bh+lh)$ *sq. units* 6. (3) $576\ cm^2$ 7. (3) $900\ cm^2$

8. (1) $280\ cm^2$ 9. (2) 4:9 10. (3) 20*cm*

11. (4) 75000 *litres* 12. (1) 1000

5. Probability

Exercise 5.1

1. $\frac{1}{7}$ 2. $\frac{3}{13}$ 3. $\frac{1}{2}$ 4.(i) $\frac{5}{24}$

(ii) $\frac{1}{8}$ (iii) $\frac{2}{3}$ 5. $\frac{1}{4}$ 6.(i) 0

(ii) $\frac{1}{12}$ (iii) 1 7. $\frac{1}{280}$ 8. $\frac{1}{5}$

9. $\frac{3}{4}$

Exercise 5.2

1. 0.9975 2. $\frac{209}{400}$ 3. $\frac{15}{8}$ 4. 0.28

5.(i) $\frac{1}{6}$ (ii) $\frac{43}{75}$ (iii) $\frac{1}{75}$

Exercise 5.3

1. (4) Probability 2. (2) 0 and 1 3. (1) Empirical probability

4. (4) less than zero 5. (2) at least two outcomes 6. (1) One

7. (4) $1-P(A)$ 8. (4) -1 9. (4) outcome 10. (1) event

11. (2) fair 12. (3) $\frac{3}{10}$

MATHEMATICAL TERMS

3-dimensional	முப்பரிமாணம்
Adjacent side	அடுத்துள்ள பக்கம்
Angle of elevation	ஏற்றக் கோணம்
Classical probability	தொன்மை நிகழ்தகவு
Coinciding lines	ஒன்றின் மீது ஒன்று பொருந்தும் கோடுகள்
Complementary angles	நிரப்புக் கோணங்கள்
Complementary events	நிரப்பு நிகழ்ச்சிகள்
Consistent	ஒருங்கமைவன
Cross multiplication	குறுக்குப் பெருக்கல் முறை
Cube	கனச் சதுரம்
Cuboid	கனச் செவ்வகம்
Deterministic Experiment	உறுதியான சோதனை
Dice	பகடைகள்
Edge	விளிம்பு
Elimination method	நீக்கல் முறை
Empirical probability	சோதனை நிகழ்தகவு
Equally likely event	சமவாய்ப்பு நிகழ்ச்சி
Equilibrium	சமநிலை
Event	நிகழ்ச்சி
Excentre	வெளிவட்ட மையம்
Externally	வெளிப்புறமாக
Face of a solid	ஒரு திண்மத்தின் முகப்பு
Hypotenuse side	கர்ணம்
Impossible event	இயலா நிகழ்ச்சி
Inconsistent	ஒருங்கமையாத
Internally	உட்புறமாக
Intersecting lines	வெட்டும் கோடுகள்
Lateral surface area	பக்கப் பரப்பு
Linear equations	நேரியச் சமன்பாடுகள்
Mean difference	பொது வித்தியாசம்
Mid point	நடுப்புள்ளி
Mutually exclusive event	ஒன்றையொன்று விலக்கும் நிகழ்ச்சிகள்
Opposite side	எதிர்ப்பக்கம்
Outcome	விளைவு
Parallel lines	இணை கோடுகள்
Probability	நிகழ்தகவு
Random Experiment	சமவாய்ப்பு சோதனை
Sample point	கூறுபுள்ளி
Sample space	கூறுவெளி
Section formula	பிரிவு வாய்ப்பாடு
Substitution method	பிரதியிடும் முறை
Sure event	உறுதியான நிகழ்ச்சி
Total surface area	மொத்தப் பரப்பு (அல்லது) மொத்தப் புறப்பரப்பு
Trial	முயற்சி
Trigonometric ratios	முக்கோணவியல் விகிதங்கள்
Trigonometric table	முக்கோணவியல் அட்டவணை
Trigonometry	முக்கோணவியல்
Uncertainty	உறுதியற்ற (அ) நிச்சயமற்ற
Vertex	முனை
Volume	கன அளவு

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