GOVERNMENT OF TAMIL NADU

STANDARD EIGHT

TERM - III

VOLUME - 2

MATHEMATICS

A publication under Free Textbook Programme of Government of Tamil Nadu

Department Of School Education

Untouchability is Inhuman and a Crime

Government of Tamil Nadu

First Edition - 2019

(Published under New Syllabus in

Trimester Pattern)

NOT FOR SALE

Content Creation

State Council of Educational
Research and Training

Printing & Publishing

Tamil Nadu Textbook and Educational
Services Corporation

www.textbooksonline.tn.nic.in

Mathematics is a unique symbolic language in which the whole world works and acts accordingly. This text book is an attempt to make learning of Mathematics easy for the students community.

Mathematics is not about numbers, equations, computations or algorithms; it is about understanding

— William Paul Thurston

The main goal of Mathematics in School Education is to mathematise the child's thought process. It will be useful to know how to mathematise than to know a lot of Mathematics.

CONTENTS

CHAPTER	TITLE	PAGE No.	MONTH
1	NUMBERS	1-30	January
2	LIFE MATHEMATICS	31-47	February
3	GEOMETRY	48-73	February & March
4	STATISTICS	74-99	March
5	INFORMATION PROCESSING	100-119	April
	ANSWERS	120-122	
	MATHEMATICAL TERMS	123	

D6Q6W6
E-Book

R1S8B4
Assessment

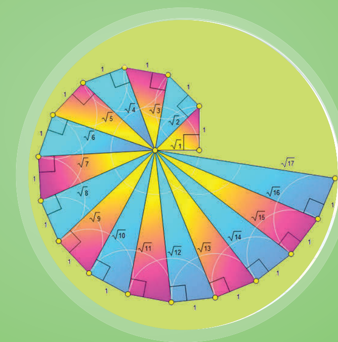
B358_8_MATHS_EM
DIGI Links

QR GUIDE

Let's use the QR code in the text books!

- Download DIKSHA app from the Google Play Store.
- Tap the QR code icon to scan QR codes in the textbook.
- Point the device and focus on the QR code.
- On successful scan, content linked to the QR code gets listed.
- Note: For ICT corner, Digi Links QR codes use any other QR scanner.

1 NUMBERS



Learning Objectives

- To compute squares of numbers.
- To find the square root of a number by (i) factor method and (ii) division method.
- To find cubes of numbers.
- To find the cube root of a number by factor method.
- To make a rough estimate of square roots and cube roots.
- To express numbers in exponential form with integral powers.
- To understand the laws of exponents with integral powers.
- To identify and express in scientific notation.

Q3C1L2

1.1 Introduction to square numbers

1	1 2 3 4	1 2 3 4 5 6 7 8 9
This is a square of side 1 unit. It is 1 squared. We write this as 1^2 $\mathbf{1^2 = 1}$	This is a square with side 2 units. It is 2 squared. We write this as 2^2 $\mathbf{2^2 = 2 \times 2 = 4}$	This is a square with side 3 units. It is 3 squared. We write this as 3^2 $\mathbf{3^2 = 3 \times 3 = 9}$

More often we write like this:

$$4^2 = 16$$

This says "4 squared is 16"

The 2 at the top stands for squared and it indicates the number of times the number 4 appears in the product ($4 \times 4 = 4^2 = 16$).

The numbers 1, 4, 9, 16, ... are all square numbers (also called perfect square numbers). Each of them is made up of the product of same two factors.

A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.

Is 49 a square number? Yes, because it can be written as 7^2. Is 50 a square number? The following table gives the squares of numbers up to 20.

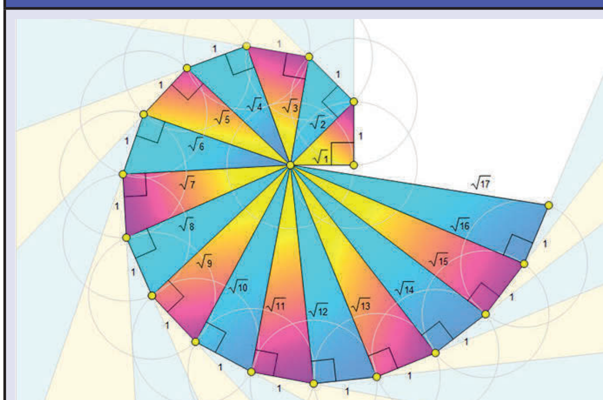
Number	Its square	Number	Its square	Number	Its square	Number	Its square
1	1	6	36	11	121	16	256
2	4	7	49	12	144	17	289
3	9	8	64	13	169	18	324
4	16	9	81	14	196	19	361
5	25	10	100	15	225	20	400

Try to extend the table up to the square number of 50.

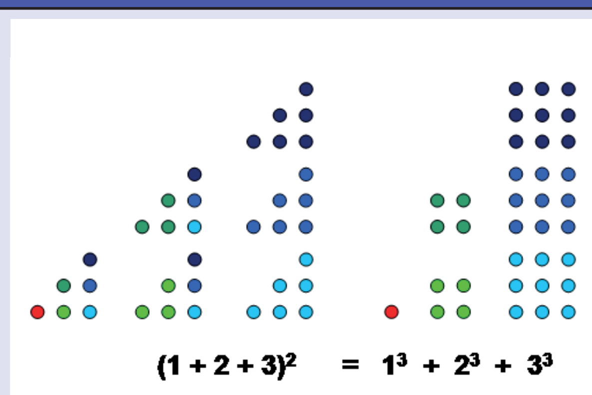
We can now easily verify the following properties of square numbers from the table given above:

- The square numbers end in 0, 1, 4, 5, 6 or 9 only.
- If a number ends with 1 or 9, its square ends with 1.
- If a number ends with 2 or 8, its square ends with 4.
- If a number ends with 3 or 7, its square ends with 9.
- If a number ends with 4 or 6, its square ends with 6.
- If a number ends with 5 or 0 its square also ends with 5 or 0 respectively.
- Square of an odd number is always odd and the square of an even number is always even.
- Numbers that end with 2,3,7 and 8 are not perfect squares.

MATHEMATICS ALIVE - NUMBERS IN REAL LIFE



Finding the square root of numbers in the form spiral using the Pythagoras Theorem from Geometry



Patterns connecting the squares and the cubes of numbers and illustrating $(1 + 2 + 3 + ... + n)^2 = 1^3 + 2^3 + 3^3 + ... + n^3$

1.1.1 Summing up odd numbers from one results in a square:

Study the following pattern of odd numbers:

First odd number	= 1 which is the same as		1^2
Sum of first two odd numbers	= 1 + 3	= 4	= 2^2
Sum of first three odd numbers	= 1 + 3 + 5	= 9	= 3^2
Sum of first four odd numbers	= 1 + 3 + 5 + 7	= 16	= 4^2
Sum of first five numbers	= 1 + 3 + 5 + 7 + 9	= 25	= 5^2

Continuing this,what will be the sum of n consecutive odd numbers starting with 1?

> The sum of the first n consecutive odd natural numbers is n^2.

1.1.2 Summing up two consecutive natural numbers results in a square:

Consider any odd natural number, say, 7. Its square is $7^2 = 49$.

We can write

$$7^2 = 24 + 25 \text{ (which is the same as } \frac{7^2 - 1}{2} + \frac{7^2 + 1}{2}\text{)}$$

Take a few more odd natural numbers and we can see that each of their squares can be written as the sum of two consecutive natural numbers. Thus, the square of an odd number can be written as the sum of two consecutive natural numbers.

Think

1. Is the square of a prime number, prime?
2. Will the sum of two perfect squares, always a perfect square? What about their difference and their product?

Try these

Consider the following square numbers:

(i) 441 (ii) 225 (iii) 289 (iv) 1089

Express each of them as the sum of two consecutive positive integers.

Example 1.1

Find the sum of the odd natural numbers 1 + 3 + 5 + 7 + … … … + 99.

Solution:

Here, there are 50 odd numbers from 1 to 99.

The sum of the first n consecutive odd natural numbers $= n^2$

The sum of the first 50 consecutive odd natural numbers $= 50^2$

$= 50 \times 50 = 2500$

DO YOU KNOW?

A perfect number (recall) cannot be a square number. Perfect numbers such as 6, 28, 496, 8128 etc., are not square numbers.

Think

1. Take an even natural number, say, 46 (or any other even number of your choice). Try to express it as a sum of consecutive odd numbers starting with 1. Do you succeed?
2. The square of an odd number can always be written as the sum of two consecutive natural numbers. Can the reverse statement be true? Is the sum of any two consecutive natural numbers a perfect square of a number?

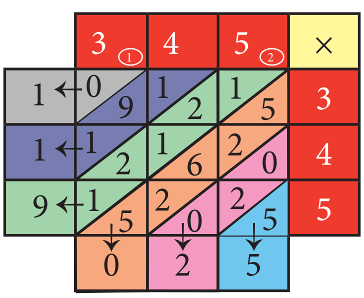
1.1.3 Finding the square of a number using diagonal method:

Using the diagonal method, we can find the square of a number easily as given in the following example.

Example 1.2

Find the square of 345 using diagonal method.

Solution:



Create a 3×3 square below and to the left of 345 (red). Make a diagonal in each of the 9 squares created. Place the product value, say, $5 \times 3 = 15$, 1 in the upper part and 5 in the lower part of the square. After filling all the squares, add the numbers through coloured diagonals. The arrow indicated numbers from the left 119025 square of 345.

$\therefore \; 345^2 = 119025.$

Try these

(i) Which among 256, 576, 960, 1025, 4096 are perfect square numbers? (Hint: Try to extend the table of squares already seen).

(ii) One can judge just by look, that the each of the following numbers: 82, 113, 2057, 24353, 8888, 1972 is not a perfect square. Explain why?

(iii) Find the squares by diagonal method and also the ones digit in the squares of the following numbers: 11, 27, 42, 79, 146, 324, 520.

Note

If a perfect square number ends in zero, it must end with even number of zeros always –We can verify this for a few numbers in the table given below.

Number	10	20	30	40	...	90	100	110	...	200	...
Its Square	100	400	900	1600	...	8100	10000	12100	...	40000	...

Think

Consider the claim: "Between the squares of the consecutive numbers n and $(n+1)$, there are $2n$ non-square numbers". Can it be true? How many non-square numbers are there between 2500 and 2601 and verify the claim.

Activity

Verify the following statements:

(i) The square of a natural number, other than 1, is either a multiple of 3 or exceeds a multiple of 3 by 1.

(ii) The square of a natural number, other than 1, is either a multiple of 4 or exceeds a multiple of 4 by 1.

(iii) The remainder of a perfect square when divided by 3, is either 0 or 1 but never 2.

(iv) The remainder of a perfect square, when divided by 4, is either 0 or 1 but never 2 and 3.

(v) When a perfect square number is divided by 8, the remainder is either 0 or 1 or 4, but never be equal to 2, 3, 5, 6 or 7.

1.1.4 Pythagorean Triplets:

The concept of *squares* is used to describe what is known as Pythagorean Triplet.

Triplet means a set of three numbers. Any three numbers that can constitute the numerical measure of the sides of a right angled triangle are said to make a Pythagorean triplet.

Mathematically, three numbers a, b, c make a Pythagorean triplet, if the sum of the squares of any two equals the square of the third. This means,

$$a^2 + b^2 = c^2 \;\; (or) \;\; b^2 + c^2 = a^2 \;(or)\;\; c^2 + a^2 = b^2$$

For example, the following are Pythagorean Triplets

Triplet	Reason
(3, 4, 5)	$3^2 + 4^2 = 9 + 16 \quad = 25$ which is 5^2.
(5, 12, 13)	$5^2 + 12^2 = 25 + 144 \quad = 169$ which is 13^2
(20, 21, 29)	$20^2+21^2 = 400 + 441 = 841$ which is 29^2

Example 1.3

Examine if (8, 15, 17) is a Pythagorean triplet.

Solution:

We have, $8^2 = 8 \times 8 = 64$, $15^2 = 15 \times 15 = 225$ and $17^2 = 17 \times 17 = 289$

We find, $8^2 + 15^2 = 64 + 225 = 289 = 17^2$

Thus, $8^2 + 15^2 = 17^2$

$\therefore$ (8, 15, 17) is a Pythagorean triplet.

Activity

Consider any natural number $m > 1$.We find that $(2m, m^2 - 1, m^2 + 1)$ will form a Pythagorean triplet. (A little algebra can help you to verify this!).

With this formula, generate a few Pythagorean triplets. One is done for you!

Now, we take $2m = 14$. Here, $2m$ has to be an even integer (why?).

$$2m = 14 \Rightarrow (2m)^2 = 14^2 = 196$$

This shows that
$$m^2 - 1 = 7^2 - 1 = 48 \Rightarrow (m^2 - 1)^2 = 48^2 = 2304$$
$$m^2 + 1 = 7^2 + 1 = 50 \Rightarrow (m^2 + 1)^2 = 50^2 = 2500$$
$$196 + 2304 = 2500$$
$$(or)\ (2m)^2 + (m^2 - 1)^2 = (m^2 + 1)$$

Thus (14, 48, 50) is a Pythagorean triplet.

1.2 Square Root

Squaring a number is a mathematical operation like addition, subtraction, multiplication etc., Most mathematical operations have 'inverse' (meaning 'opposite') operations. For example, subtraction is the inverse of addition, division is the inverse of multiplication etc., Squaring also has an inverse operation namely finding the Square root.

The square root of a number n, written $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself.

For example, $\sqrt{81}$ is 9, because $9 \times 9 = 81$.

In the adjacent table, we have square roots of all the perfect squares starting from 1 up to 100.

Square root	Reason
$\sqrt{1} = 1$	$1^2 = 1$
$\sqrt{4} = 2$	$2^2 = 4$
$\sqrt{9} = 3$	$3^2 = 9$
$\sqrt{16} = 4$	$4^2 = 16$
$\sqrt{25} = 5$	$5^2 = 25$
$\sqrt{36} = 6$	$6^2 = 36$
$\sqrt{49} = 7$	$7^2 = 49$
$\sqrt{64} = 8$	$8^2 = 64$
$\sqrt{81} = 9$	$9^2 = 81$
$\sqrt{100} = 10$	$10^2 = 100$

If we know that $324 = 18^2$, we can immediately tell that $\sqrt{324}$ is 18.

If $11^2 = 121$, what is $\sqrt{121}$?

If $529 = 23^2$, what is the square root of 529?

We have, $1^2 = 1$ and so 1 is a square root of 1. $(-1)^2$ is also =1. So (-1) is also a square root of 1.

$2^2 = 4$ and so 2 is a square root of 4. $(-2)^2$ is also = 4. So (-2) is also a square root of 4.

Similarly,

$3^2 = 9$ and so 3 is a square root of 9. $(-3)^2$ is also = 9. So (-3) is also a square root of 9.

The above examples suggest that there are two integral square roots for a perfect square number. However, in working out the problems, we will take up only positive square root of a natural number.

The positive square root of a number is always denoted by the symbol $\sqrt{}$.

Thus, $\sqrt{4}$ is 2 (and not –2); $\sqrt{9}$ is 3 (and not –3). We have to remember that this is a universally accepted notation.

1.2.1 Square root by repeated subtraction of successive odd numbers :

We know that a square number can be written as the sum of successive odd numbers starting from 1. Therefore, we can hit upon the square root of a number by repeatedly subtracting successive odd numbers (starting from 1) from the given square number, till we reach zero.

Example 1.4

Find the square root of 64 by repeated subtraction method.

Solution:

We proceed as follows:

Step 1: $64 - 1 = 63$
Step 2: $63 - 3 = 60$
Step 3: $60 - 5 = 55$
Step 4: $55 - 7 = 48$
Step 5: $48 - 9 = 39$
Step 6: $39 - 11 = 28$
Step 7: $28 - 13 = 15$
Step 8: $15 - 15 = 0$

Try these

Use the method of successive subtraction of odd numbers (starting from 1) to examine the following numbers and find if they are perfect squares or not. For perfect square numbers that you find, identify the square root.

(i) 144 (ii) 256 (iii) 360

We have subtracted successive odd numbers (starting from 1) repeatedly from the given number 64. We get zero in the 8^{th} step. Therefore $\sqrt{64} = 8$. If we don't get zero, then the given number is not a perfect square.

Given that 685584 is a perfect square number. Is it possible for us to find out its square root by the method of successive subtraction of odd numbers?

Yes, we can, but it will be time consuming and tedious. So we need to consider other methods of finding the root of square numbers.

1.2.2 Square root through Prime Factorisation:

Study the following table giving the prime factors of numbers and those of their squares.

Numbers	Prime factorisation of the numbers	The square of given numbers	Prime factorisation of their squares
6	$6 = 2 \times 3$	$6^2 = 36$	$36 = 2 \times 2 \times 3 \times 3 = (2 \times 3)^2$
8	$8 = 2 \times 2 \times 2$	$8^2 = 64$	$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = (2 \times 2 \times 2)^2$
12	$12 = 2 \times 2 \times 3$	$12^2 = 144$	$44 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = (2 \times 2 \times 3)^2$
15	$15 = 3 \times 5$	$15^2 = 225$	$225 = 3 \times 3 \times 5 \times 5 = (3 \times 5)^2$

Look at 6 and its prime factors. How many times do 2 occur in the list?

Now, look at its square 36 and its prime factors. How many times do 2 occur here?

Repeat the above task in the cases of other numbers 8, 12, and 15. (You may also choose your own number and its square). What do you find?

$$\left.\begin{array}{l}\text{The number of times a prime}\\ \text{factor occurs in the square}\end{array}\right\} = \left\{\begin{array}{l}\text{twice the number of times it occurs in}\\ \text{the prime factorisation of the number.}\end{array}\right.$$

We use this idea to find the square root of a square number. First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

Example 1.5

Find the square root of 324 by prime factorisation.

Solution:

First, resolve the given number into prime factors. Group the identical factors in pairs and then take one from them to find the square root.

$$\begin{aligned} \text{Now, } 324 &= 2\times2\times3\times3\times3\times3 \\ &= 2^2\times3^2\times3^2 \\ &= (2\times3\times3)^2 \\ \therefore \sqrt{324} &= \sqrt{(2\times3\times3)^2} \\ &= 2\times3\times3 \\ &= 18 \end{aligned}$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 1.6

Is 108 a perfect square number?

Solution:

$$\begin{aligned} \text{Here, } 108 &= 2\times2\times3\times3\times3 \\ &= 2^2\times3^2\times3 \end{aligned}$$

Here, the second prime factor 3 does not have a pair. Hence, 108 is not a perfect square number.

2	108
2	54
3	27
3	9
3	3
	1

Think

In this case, if we want to find the smallest factor with which we can multiply or divide 108 to get a square number, what should we do?

Example 1.7

Find the least number by which 250 is to be multiplied (or) divided so that the resulting number is a perfect square. Also, find the square root in that case.

Solution:

We find $250 = 5 \times 5 \times 5 \times 2$

$= 5^2 \times 5 \times 2$

5	250
5	50
5	10
	2

Here, the prime factors 5 and 2 do not have pairs.

Therefore, we can either divide 250 by 10 (5 × 2) or multiply 250 by 10.

(i) If we multiply 250 by 10, we get $2500 = 5^2 \times 5 \times 2 \times 5 \times 2$ and therefore the square root for 2500 would be $5 \times 5 \times 2 = 50$.

(ii) If we divide 250 by 10 we get 25 and in that case we get $\sqrt{25} = \sqrt{5^2} = 5$.

Exercise 1.1

1. Fill in the blanks:

(i) The ones digit in the square of 77 is____________.

(ii) The number of non-square numbers between 24^2 and 25^2 is _______.

(iii) If a number ends with 5, its square ends with _______.

(iv) A square number will not end with numbers _______.

(v) The number of perfect square numbers between 300 and 500 is _______.

2. Say True or False:

(i) When a square number ends in 6, its square root will have 6 in the unit's place.

(ii) A square number will not have odd number of zeros at the end.

(iii) The number of zeros in the square of 961000 is 9.

(iv) (7, 24, 25) is a Pythagorean triplet.

(v) The square root of 221 is 21.

3. What will be the ones' digit in the squares of the following numbers?

(i) 36 (ii) 252 (iii) 543

4. Study the given numbers and justify why each of them obviously cannot be a perfect square.

(i) 1000 (ii) 34567 (iii) 408

5. Find the sum without actually adding the following odd numbers:

(i) $1 + 3 + 5 + 7 + \ldots\ldots\ldots + 35$

(ii) The first 99 odd natural numbers.

6. Express (i) 15^2 and (ii) 19^2 as the sum of two consecutive positive integers.

7. Write (i) 10^2 and (ii) 11^2 as the sum of consecutive odd natural numbers.

8. Find a Pythagorean triplet whose

(i) largest member is 65 (ii) smallest member is 10

9. Find the square root of the following by repeated subtraction method.

(i) 144 (ii) 256 (iii) 784

10. Find the square root by prime factorisation method.

(i) 1156 (ii) 4761 (iii) 9025

11. Examine if each of the following is a perfect square:

(i) 725 (ii) 190 (iii) 841 (iv) 1089

12. Find the least number by which 1800 should be multiplied so that it becomes a perfect square. Also, find the square root of the perfect square thus obtained.
13. Find the smallest number by which 10985 should be divided so that the quotient is a perfect square.
14. Is 2352 a perfect square? If not, find the smallest number by which 2352 must be multiplied so that the product is a perfect square. Find the square root of the new number.
15. Find the least square number which is divisible by each of the numbers 8,12 and 15.

1.2.3 Finding the square root of a number by long division method:

When we come across numbers with large number of digits, finding their square roots by factorisation becomes lengthy and difficult. Use of long division helps us in such cases. Let us look into the method with a few Illustration.

Illustration 1

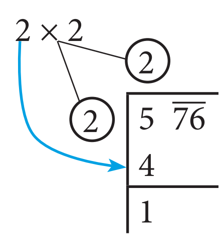
Find the square root of 576 by long division method.

Step 1:

Group the digits in pairs, starting with the digit in the unit's place. Each pair and the remaining digit (if any) is called a period. Put a bar over every pair of digits starting from the right of the given number. If there are odd number of digits, the extreme left digit will be without a bar sign above it.

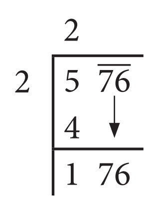
So, here we have $5\ \overline{76}$

Step 2:



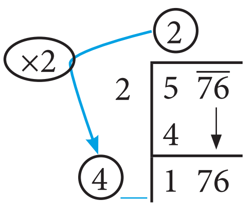
Think of the largest number whose square is equal to or just less than the first period. Take this number as the divisor and also as the quotient. The left extreme number here is 5. The largest number whose square is less than or equal to 5 is 2. This is our divisor and the quotient.

Step 3:



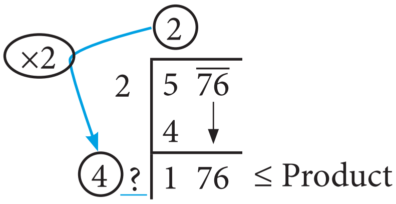
Bring 76 down and write it down to the right of the remainder 1.
Now, the new dividend is 176.

Step 4:



To find the new divisor, multiply the earlier quotient (2) by 2 (always) and write it leaving a blank space next to it.

Step 5:



The new divisor is 4 followed by a digit. We should choose this digit next to 4 such that the new quotient multiplied by the new divisor will be less than or equal to 176.

Step 6:

Clearly, the required digit here has to be 4 or 6. (why?)

When we calculate,

$\underline{46} \times \underline{6} = 276$ whereas $4\underline{4} \times \underline{4} = 176$.

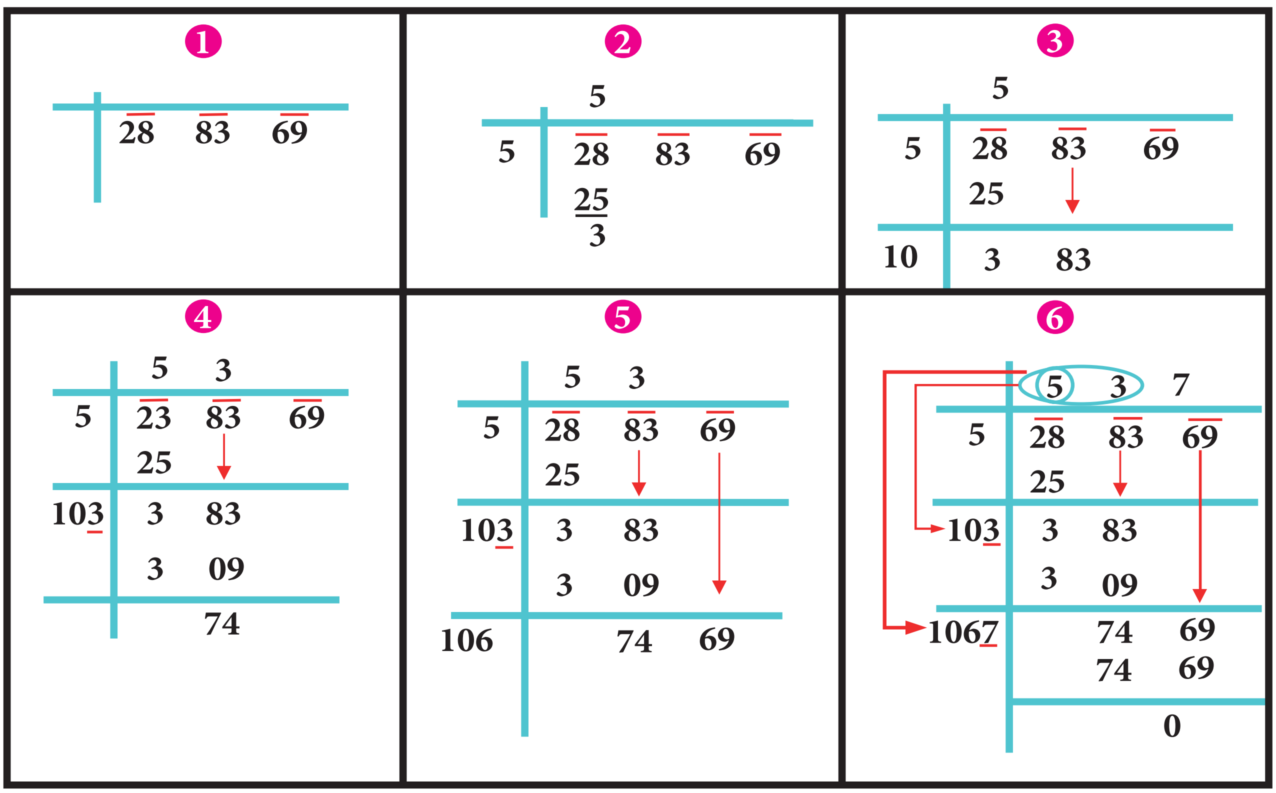
Therefore, we put 4 in the blank space and write 176 below 176 and subtract to get the remainder 0.

$\therefore \sqrt{576} = 24$.

$$
\begin{array}{r|l}
 & \;\;\;2\;\;4 \\
\hline
2 & \;\;\;5\;\;\overline{76} \\
\hline
44 & \;\;1\;\;76 \\
 & \;\;1\;\;76 \\
\hline
 & \;\;\;\;\;\;\;0
\end{array}
$$

Illustration 2

In the following example, the gradual computation, stage by stage, of computing the square root of 288369. Follow the figures one after another and try to understand what each figure explains.



You find that $\sqrt{288369} = 537$.

Try these

Find the square root by long division method.

(i) 400 (ii) 1764 (iii) 4356

1.2.4 Number of digits in the square root of a perfect square number:

We made use of bars to find the square root in the division method. This marking bars help us to find the number of digits in the square root of a perfect square number. Observe the following examples (with bars shown as if we compute square root by division procedure).

Square root of numbers	No. of bars	No. of digits in the square root	Square root of numbers	No. of bars	No. of digits in the square root
$\sqrt{\overline{1}\,\overline{69}} = 13$	2	2	$\sqrt{\overline{43}\,\overline{56}} = 66$	2	2
$\sqrt{\overline{4}\,\overline{41}} = 21$	2	2	$\sqrt{\overline{60}\,\overline{84}} = 78$	2	2
$\sqrt{\overline{1}\,\overline{25}\,\overline{44}} = 112$	3	3	$\sqrt{\overline{2}\,\overline{72}\,\overline{25}} = 165$	3	3

Hence, we conclude that the number of bars indicates the number of digits in the square root.

Note

If n is the number of digits of a number then, the number of digits in the square root of that number is $= \begin{cases} \dfrac{n+1}{2} & \text{if } n \text{ is odd} \\ \dfrac{n}{2} & \text{if } n \text{ is even.} \end{cases}$

Try these

Without calculating the square root, guess the number of digits in the square root of the following numbers:
(i) 14400 (ii) 100000000 (iii) 390625

1.2.5 Square root of decimal numbers:

To compute the square root of numbers in the decimal form, we simply follow the following steps

Step 1:

Put up the number of decimal places even by affixing a zero on the extreme right of the digit in the decimal part (only if required).

To find $\sqrt{42.25}$

Step 2:

The number has an integral part and a decimal part. In the integral part, mark the bars as done in the case of division method to find the square root of a perfect square number.

We put the bars as $\sqrt{\overline{42}.\overline{25}}$

Step 3:

In the decimal part, mark the bars on every pair of digits beginning with the first decimal place.

Try these

Find the square root of
(i) 19.36 (ii) 1.2321 (iii) 116.64

Step 4:

Now, calculate the square root by long division method.

$$\begin{array}{r|l} & 6\ \ 5 \\ \hline 6 & \overline{42}\ \overline{25} \\ & 36 \\ \hline 125 & \ \ 6\ 25 \\ & \ \ 6\ 25 \\ \hline & \ \ \ \ 0 \end{array}$$

Step 5:

Put the decimal point in the square root as soon as the integral part is exhausted.

$\therefore \sqrt{42.25} = 6.5$

1.2.6 Square root of product and quotient of numbers:

For any two positive numbers p and q. we have

(i) $\sqrt{pq} = \sqrt{p} \times \sqrt{q}$ and (ii) $\sqrt{\dfrac{p}{q}} = \dfrac{\sqrt{p}}{\sqrt{q}}$

Example 1.8

Find the value of $\sqrt{256}$

Solution:

$\sqrt{256}=\sqrt{16\times16}=\sqrt{16}\times\sqrt{16}=4\times4=16.$ *or* $\sqrt{256}=\sqrt{64\times4}=\sqrt{64}\times\sqrt{4}=8\times2=16.$

Try to fill up the following table of similar problems using $\sqrt{a\times b}=\sqrt{a}\times\sqrt{b}$.

Think

Fill in the table:

$\sqrt{36}=6$	$\sqrt{9}\times\sqrt{4}=3\times2=6$	Is $\sqrt{36}=\sqrt{9}\times\sqrt{4}$?	$\sqrt{81}=?$	$\sqrt{9}\times\sqrt{9}=$ __ $\times$ __ $=$ __	Is $\sqrt{81}=\sqrt{9}\times\sqrt{9}$?
$\sqrt{144}=?$	$\sqrt{9}\times\sqrt{16}=$ __ $\times$ __ $=$ __	Is $\sqrt{144}=\sqrt{9}\times\sqrt{16}$?	$\sqrt{144}=?$	$\sqrt{36}\times\sqrt{4}=$ __ $\times$ __ $=$ __	Is $\sqrt{144}=\sqrt{36}\times\sqrt{4}$?
$\sqrt{100}=?$	$\sqrt{25}\times\sqrt{4}=$ __ $\times$ __ $=$ __	Is $\sqrt{100}=\sqrt{25}\times\sqrt{4}$?	$\sqrt{1225}=?$	$\sqrt{25}\times\sqrt{49}=$ __ $\times$ __ $=$ __	Is $\sqrt{1225}=\sqrt{25}\times\sqrt{49}$?

Activity

Attempt to prepare a table of square root problems as in the above case to show that if a and b are two perfect square numbers, then $\sqrt{\dfrac{a}{b}}=\dfrac{\sqrt{a}}{\sqrt{b}}\ (b\neq0)$

We can use this idea to compute easily certain square-root problems.

Example 1.9

Find the value of $\sqrt{42.25}$.

Solution:

We can write this as $\sqrt{42.25}=\sqrt{\dfrac{4225}{100}}=\dfrac{\sqrt{4225}}{\sqrt{100}}$

Now, it is easy to compute the square root of the whole number 4225, without any botheration of decimal symbol.

$\sqrt{4225}=65$ and so, we now get $\sqrt{42.25}=\sqrt{\dfrac{4225}{100}}=\dfrac{\sqrt{4225}}{\sqrt{100}}=\dfrac{65}{10}=$

This is another way of tackling problems of square root of decimal numbers instead of long division method.

Try these

Using the above method, find the square root of 1.2321 and 11.9025.

Example 1.10

Simplify: $\sqrt{12}\times\sqrt{3}$

Solution:

Remembering the rule, $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$,

$\sqrt{12}\times\sqrt{3}=\sqrt{12\times3}=\sqrt{36}=\sqrt{6^2}=6$

Example 1.11

Simplify: $\sqrt{\dfrac{98}{162}}$

Solution:

Remembering the rule, $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}\ (b \neq 0)$

$$\sqrt{\frac{98}{162}} = \sqrt{\frac{2\times 49}{2\times 81}} = \sqrt{\frac{49}{81}} = \sqrt{\frac{7^2}{9^2}} = \frac{7}{9}$$

Example 1.12

Simplify: (i) $\sqrt{2\dfrac{7}{9}}$ (ii) $\sqrt{1\dfrac{9}{16}}$

Solution:

(i) $\sqrt{2\dfrac{7}{9}} = \sqrt{\dfrac{25}{9}} = \sqrt{\dfrac{5^2}{3^2}} = \dfrac{5}{3} = 1\dfrac{2}{3}$ (ii) $\sqrt{1\dfrac{9}{16}} = \sqrt{\dfrac{25}{16}} = \sqrt{\dfrac{5^2}{4^2}} = \dfrac{5}{4} = 1\dfrac{1}{4}$

Remark: In the case of the second problem one may be tempted to give immediately the answer as $1\dfrac{3}{4}$, but this is not correct since, you have to convert the mixed fraction into an improper fraction and then use the rule $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}\ (b \neq 0)$

1.2.7 Approximating square roots:

Can you write the given numbers $\sqrt{40}$, 6 and 7 in ascending order? Here $\sqrt{40}$ is not a square number and so we cannot determine its root easily. However, we can estimate an approximation to $\sqrt{40}$ and use it here.

We know that the two closest squares surrounding 40 are 36 and 49.

Thus, 36 < 40 < 49 which can be written as $6^2 < 40 < 7^2$.

Considering the square root, we have $6 < \sqrt{40} < 7$.

Try these

Put the numbers (i) 4, $\sqrt{14}$, 5 and (ii) 7, $\sqrt{65}$, 8 in ascending order.

Exercise 1.2

1. **Fill in the blanks:**

   (i) If a number has 5 or 6 digits in it then, its square root will have ____________ digits.

   (ii) The value of $\sqrt{180}$ lies between integers _______ and _______.

   (iii) $\sqrt{10} \times \sqrt{6} \times \sqrt{15} =$ _______.

   (iv) $\dfrac{\sqrt{300}}{\sqrt{192}} =$ _______.

   (v) $\sqrt{65.61} =$ _______.

2. Estimate the value of the following square roots to the nearest whole number:

   (i) $\sqrt{440}$ (ii) $\sqrt{800}$ (iii) $\sqrt{1020}$

3. Find the least number that must be added to 1300 so as to get a perfect square. Also, find the square root of the perfect square.
4. Find the least number that must be subtracted to 6412 so as to get a perfect square. Also find the square root of the perfect square.
5. Find the square root by long division method.
   (i)17956 (ii) 11025 (iii) 6889 (iv) 1764 (v) 418609
6. Find the square root of the following decimal numbers:
   i) 2.89 *ii*) 1.96 *iii*) 67.24 *iv*) 31.36 *v*) 2.0164 *vi*) 13.987
7. Find the square root of each of the following fractions:
   i) $\frac{144}{221}$ *ii*) $7\frac{18}{49}$ *iii*) $6\frac{1}{4}$ *iv*) $4\frac{25}{36}$
8. **Say True or False:**
   (*i*) $\frac{\sqrt{32}}{\sqrt{8}} = 2$ (*ii*) $\sqrt{\frac{625}{1024}} = \frac{25}{32}$ (*iii*) $\sqrt{28} \times \sqrt{7} = 2\sqrt{7}$
   (*iv*) $\sqrt{225} + \sqrt{64} = \sqrt{289}$ (*v*) $\sqrt{1\frac{400}{441}} = 1\frac{20}{21}$

Objective Type Questions

9. $\sqrt{48}$ is approximately equal to
   (a) 5 (b) 6 (c) 7 (d) 8
10. $\sqrt{128} - \sqrt{98} + \sqrt{18} =$
   (a) $\sqrt{2}$ (b) $\sqrt{8}$ (c) $\sqrt{48}$ (d) $\sqrt{32}$
11. $\sqrt{22 + \sqrt{7 + \sqrt{4}}} =$
   (a) $\sqrt{25}$ (b) $\sqrt{33}$ (c) $\sqrt{31}$ (d) $\sqrt{29}$
12. The number of digits in the square root of 123454321 is
   (a) 4 (b) 5 (c) 6 (d) 7

1.3 Cubes and Cube Numbers

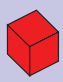
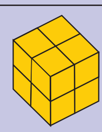
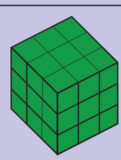
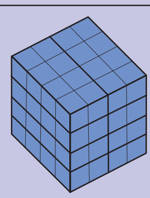
If you multiply a number by itself, and then by itself again, the result is a cube number. This means that a cube number is a number that is the product of three identical numbers. If n is a number, its cube is represented by n^3.

Cube numbers can be represented visually as 3D cubes comprising of single unit cubes. Cube numbers are also called as perfect **cubes**. The perfect cubes of natural numbers are 1, 8, 27, 64, 125, 216, ... and so on.

DO YOU KNOW?

Ramanujan Number - *$1729 = 12^3+1^3 = 10^3+9^3$*

Once Professor Hardy went to see Ramanujan when he was ill at Putney, riding in taxi cab number 1729 and said that the number seemed a dull one, and hoped it was not an unfavorable omen. "No," replied Ramanujan, "It is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways." 4104, 13832, 20683 are few more examples of Ramanujan-Hardy numbers.

Geometrical Representation	Product Representation	Notation	Perfect cube
	$1 \times 1 \times 1$	1^3	1
	$2 \times 2 \times 2$	2^3	8
	$3 \times 3 \times 3$	3^3	27
	$4 \times 4 \times 4$	4^3	64

1.3.1 Properties of cubes of numbers:

	Properties	Examples
1	The cube of a positive number is positive.	$6^3 = 6 \times 6 \times 6 = 216$.
2	The cube of a negative number is negative.	$(-7)^3 = (-7) \times (-7) \times (-7) = -$
3	The cube of every even number is even.	$2^3, 4^3, 6^3, 8^3 \ldots$ are all even
4	The cube of every odd number is odd.	$1^3, 3^3, 5^3, 7^3, 9^3 \ldots$ are all odd
5	If a natural number ends with 0, 1, 4, 5, 6 or 9, its cube also ends with the same 0,1, 4, 5, 6 or 9 respectively.	$10^3 = 1000$, $11^3 = 1331$, $14^3 = 2744$ $15^3 = 3375$, $16^3 = 4096$, $19^3 = 6859$
6	If a natural number ends with 2 or 8, its cube ends with 8 or 2 respectively.	$12^3 = 1728$, $18^3 = 5832$
7	If a natural number ends with 3 or 7, its cube ends with 7 or 3 respectively.	$13^3 = 2197$, $17^3 = 4913$
8	The sum of the cubes of first n natural numbers is equal to the square of their sum.	$1^3 + 2^3 + 3^3 + \ldots.. + n^3 = (1+2+3+\ldots..+n)^2$ Check that, $1^3 + 2^3 + 3^3 + 4^3 = (1+2+3+4)^2$

Note

- A perfect cube does not end with two zeroes.
- The cube a two digit number may have 4 or 5 or 6 digits in it.

Try these

Find the ones digit in the cubes of each of the following numbers.

(i) 17 (ii) 12 (iii) 38

(iv) 53 (v) 71 (vi) 84

1.3.2 Cube root:

The cube root of a number is the value that, when cubed, gives the original number.

For example, the cube root of 27 is 3, because when 3 is cubed we get 27.

Notation:

The cube root of a number x is denoted as

$$\sqrt[3]{x}\ (or)\ x^{\frac{1}{3}}.$$

Here are some more cubes and cube roots:

$\sqrt[3]{1} = 1$ since $1^3 = 1$, $\sqrt[3]{8} = 2$ since $2^3 = 8$,

$\sqrt[3]{27} = 3$ since $3^3 = 27$, $\sqrt[3]{64} = 4$ since $4^3 = 64$,

$\sqrt[3]{125} = 5$ since $5^3 = 125$ and so on

Cubes	Cube Roots	Cubes	Cube Roots
1	1	729	9
8	2	1000	10
27	3	1331	11
64	4	1728	12
125	5	2197	13
216	6	2744	14
343	7	3375	15
512	8	4096	16

Example 1.13

Is 400 a perfect cube?

Solution:

By prime factorisation, we have $400 = \underbrace{2 \times 2 \times 2} \times 2 \times 5 \times 5$.

There is only one triplet. To make further triplets, we will need two more 2's and one more 5. Therefore, 400 is not a perfect cube.

Example 1.14

Find the smallest number by which 675 must be multiplied to obtain a perfect cube.

Solution:

3	675
3	225
3	75
5	25
5	1
	1

We find that, $675 = 3 \times 3 \times 3 \times 5 \times 5$ ....(i)

Grouping the prime factors of 675 as triplets, we are left over with 5×5.

We need one more 5 to make it a perfect cube.

To make 675 a perfect cube, multiply both sides of (i) by 5.

$675 \times 5 = (3 \times 3 \times 3 \times 5 \times 5) \times 5$

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Now, 3375 is a perfect cube. Thus, the smallest required number to multiply 675 such that the new number perfect cube is 5.

Think

If in the above question, the word 'multiplied' is replaced by the word 'divided', how will the solution vary?

1.3.3 Cube root of a given number by Prime Factorisation:

Step 1: Resolve the given number as the product of prime factors.

Step 2: Make triplet groups of same primes.

Step 3: Choosing one from each triplet, find the product of primes.

Example 1.15

Find the cube root of 27000.

Solution:

By prime factorisation, we have $27000 = 2\times2\times2\times3\times3\times3\times5\times5\times5$

$$\therefore \sqrt[3]{27000} = 2\times3\times5 = 30.$$

Example 1.16

Evaluate: i) $\sqrt[3]{\dfrac{9261}{8000}}$ ii) $\sqrt[3]{\dfrac{1728}{729}}$

Solution:

i) $\sqrt[3]{\dfrac{9261}{8000}} = \dfrac{\sqrt[3]{9261}}{\sqrt[3]{8000}} = \dfrac{(21\times21\times21)^{1/3}}{(20\times20\times20)^{1/3}} = \dfrac{(21^3)^{1/3}}{(20^3)^{1/3}} = \dfrac{21}{20} =$

ii) $\sqrt[3]{\dfrac{1728}{729}} = \dfrac{\sqrt[3]{1728}}{\sqrt[3]{729}} = \dfrac{(12\times12\times12)^{1/3}}{(9\times9\times9)^{1/3}} = \dfrac{(12^3)^{1/3}}{(9^3)^{1/3}} = \dfrac{12}{9} = \dfrac{4}{3} = 1\dfrac{1}{3}$

Exercise 1.3

1. **Fill in the blanks:**
   - (i) The ones digits in the cube of 73 is____________.
   - (ii) The maximum number of digits in the cube of a two digit number is ________.
   - (iii) The cube root of 540×50 is ____________.
   - (iv) The cube root of 0.000004913 is ____________.
   - (v) The smallest number to be added to 3333 to make it a perfect cube is ____________.
2. **Say True or False:**
   - (i) The cube of 0.0012 is 0.000001728.
   - (ii) The cube root of 250047 is 63.
   - (iii) 79570 is not a perfect cube.

Q5F3Z8

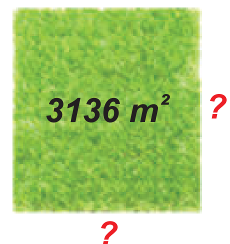
3. Show that 1944 is not a perfect cube.
4. Find the cube root of 24 × 36 × 80 × 25.
5. Find the smallest number by which 10985 should be divided so that the quotient is a perfect cube.
6. Find two smallest perfect square numbers which when multiplied together gives a perfect cube number.
7. If the cube of a squared number is 729, find the square root of that number.

8. What is the square root of cube root of 46656?
9. Find the cube root of 729 and 6859 by prime factorisation.
10. Find the smallest number by which 200 should be multiplied to make it a perfect cube.

Miscellaneous Examples:

Example 1.17

The area of a square field is 3136 m^2. Find its perimeter.



Solution:

Given that the area of the square field $= 3136\ m^2$.

$\therefore$ The side of square field $= \sqrt{3136}\ m = 56\ m$ (find it!)

$\therefore$ The perimeter of the square field $= 4 \times$ side

$= 4 \times 56$

$= 224\ m$

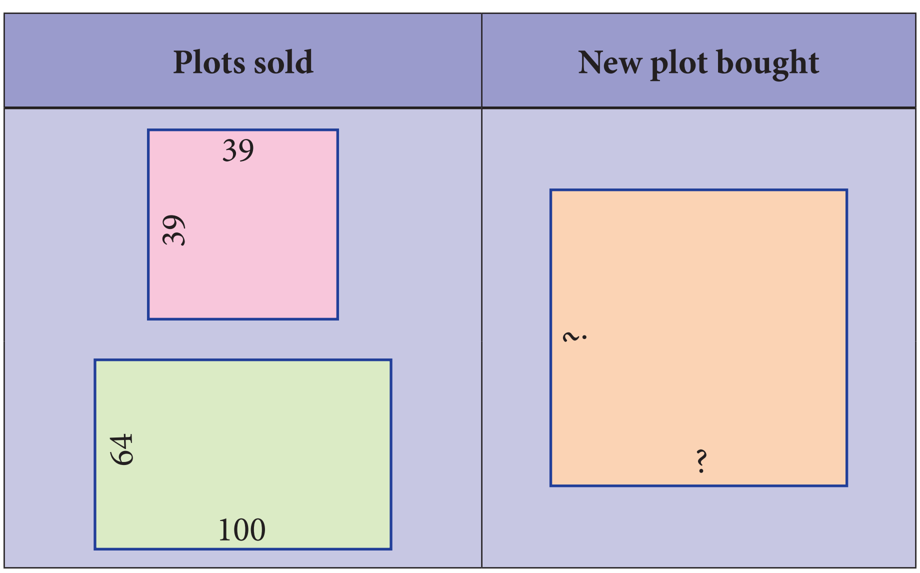
	5 6
5	31 36
	25
106	6 36
	6 36
	0

Example 1.18

A real estate owner had two plots, a square plot of side 39 m and a rectangular plot of dimensions 100 m length and 64 m breadth. He sells both these and acquires a new square plot of the same area. What is the length of side of his new plot?

Solution:

The transactions can be summarized as follows:



Area of the square plot bought $=$ Area of the square plot sold $+$ Area of the rectangular plot sold

$= 39 \times 39 + 100 \times 64$

$= 1521 + 6400$

$= 7921\ m^2$

Length of a side of the new square plot $= \sqrt{7921}$

$= 89\ m$

	8 9
8	79 21
	64
169	15 21
	15 21
	0

1.4 Exponents and Powers

We know how to express some numbers as squares and cubes. For example, we write 5^2 for 25 and 5^3 for 125.

In general terms, an expression that represents repeated multiplication of the same factor is called a **power**.

exponent

5^2

base

We say this as 5 raised to the power 2

The number 5 is called the **base** and the number 2 is called the **exponent** or power.

The exponent corresponds to the number of times the base is used as a factor.

1.4.1 Powers with positive exponents:

Value of powers given by whole number exponents often quite rapidly increase. Observe the following example.

$$2^1 = 2$$
$$2^2 = 2\times2=4$$
$$2^3 = 2\times2\times2=8$$
$$2^4 = 2\times2\times2\times2 =16$$
$$2^5 = 2\times2\times2\times2\times2=32$$
$$2^6 = 2\times2\times2\times2\times2\times2=64$$
$$2^7 = 2\times2\times2\times2\times2\times2\times2=128$$
$$2^8 = 2\times2\times2\times2\times2\times2\times2\times2=256$$
$$2^9 = 2\times2\times2\times2\times2\times2\times2\times2\times2=512$$
$$2^{10} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2=1024$$

At this rate, what do you think 2^{100} will be?

In fact, **2^{100}=1,267, 650, 600, 228, 229, 401, 496, 703, 205, 376**

Thus, we understand that the positive exponential notation with positive power could be useful when we deal with large numbers.

1.4.2 Powers with zero and negative exponents:

Observe this pattern: Starting from the beginning, what happens in the successive steps? We find that the result is half of that of the previous step. So, what can we say about 2^0 ?

$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = ?$

If we prepare a table like this for 3^5, 3^4, 3^3, and so on what will it tell us about 3^0 ?

We can use the same process as in this pattern, to show that any non-zero number raised to the zero exponent must result in 1.

$$\boxed{a^0 = 1, \text{ where } a \neq 0}$$

Let us see what happens if we extend the above pattern further downward.

As before, starting from the beginning, in the successive steps, we find that the result is half of that of the previous step.

Since $2^0 = 1$, the next step is $\mathbf{2^{-1}}$, whose value is the previous step's value 1, divided by 2; that is. $\frac{1}{2}$ Next is $\mathbf{2^{-2}}$, which is same as $\frac{1}{2}$ divided by 2, that is $\frac{1}{4}$ etc.,

$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$
$2^{-1} = \frac{1}{2}$
$2^{-2} = \frac{1}{4}$
$2^{-3} = \frac{1}{8}$

In general, $a^{-m} = \dfrac{1}{a^m}$, where m is an integer

1.4.3 Expanded form of numbers using exponents:

In the lower classes, we have learnt how to write a whole number in expanded form. For example,

$$\begin{aligned} 5832 &= (5\times1000) + (8\times100) + (3\times10) + (2\times1) \\ &= 5\times10^3 + 8 \times 10^2 + 3 \times 10^1 + 2, \text{ (when we use exponential notation).} \end{aligned}$$

What will we do if we get decimal places? Our powers of 10 with negative exponents come to our rescue!

5 8 3 2

$2 \times 1 = 2$
$3 \times 10 = 30$
$8 \times 100 = 800$
$5 \times 1000 = 5000$

$$\begin{aligned} \text{Thus, } 58.32 &= 50 + 8 + \frac{3}{10} + \frac{2}{100} \\ &= (5\times10) + (8\times1) + \left(3\times\frac{1}{10}\right) + \left(2\times\frac{1}{100}\right) \\ &= (5\times10^1) + (8\times10^0) + (3\times10^{-1}) + (2\times10^{-2}) \end{aligned}$$

Try these

Expand the following numbers using exponents:

(i) 8120 (ii) 20,305 (iii) 3652.01 (iv) 9426.521

1.4.4 Laws of exponents:

Laws of exponents arise out of certain basic ideas. The exponent indicate **how many times** we use the number in a multiplication. A **negative exponent** suggests us **to divide**, since the opposite of multiplying is dividing.

Product law

According to this law, when multiplying two powers that have the same base, we can add the exponents. That is,

$$a^m \times a^n = a^{m+n}$$

where a ($a \neq 0$), m, n are integers. Note that the base should be the same in both the quantities.

Examples:

a)	$2^3 \times 2^2 = 2^5$ (meaning $8 \times 4 = 32$; note that it is not $2^{3\times 2}$)
b)	$(-2)^{-4} \times (-2)^{-3} = \frac{1}{(-2)^4} \times \frac{1}{(-2)^3} = \frac{1}{(-2)^4 \times (-2)^3} = \frac{1}{(-2)^{4+3}} = \frac{1}{(-2)^7} = (-2)^{-7}$ *or* $(-2)^{-4} \times (-2)^{-3} = (-2)^{(-4)+(-3)} = (-2)^{-7}$
c)	$(-5)^3 \times (-5)^{-3} = (-5)^{3-3} = (-5)^o = 1$

Quotient law

This rule tells that we can divide two powers with the same base by subtracting the exponents.

$$\frac{a^m}{a^n} = a^{m-n}$$

where a, $(a \neq 0)$, m, n are integers. Note that the base should be the same in both the quantities.

How does it work? Study the examples that follows:

Examples:

a)	$\frac{(-3)^5}{(-3)^2} = \frac{(-3)\times(-3)\times(-3)\times(-3)\times(-3)}{(-3)\times(-3)} = (-3)\times(-3)\times(-3) = (-3)^3$ (or) $\frac{(-3)^5}{(-3)^2} = (-3)^5 \times (-3)^{-2} = (-3)^{5-2} = (-3)^3$ (or) $\frac{(-3)^5}{(-3)^2} = (-3)^{5-2}$ *by rule* and $= (-3)^3$ [The simplified answer is -27].
b)	$\frac{(-7)^{100}}{(-7)^{98}} = \frac{(-7)\times(-7)\times(-7)\times\ldots 100 \text{ times}}{(-7)\times(-7)\times(-7)\times\ldots\ 98 \text{ times}} = (-7)\times(-7) = 49$ (or) $\frac{(-7)^{100}}{(-7)^{98}} = (-7)^{100-98}$ *by rule* and $= (-7)^2 = 49$.

Power law

The power rule declares that to raise a power to a power, just multiply the exponents.

$$(a^m)^n = a^{mn}$$

where a, $(a \neq 0)$, m, n are integers.

Example:

$$[(-2)^3]^2 = [(-2)\times(-2)\times(-2)]^2 = [-8]^2 = 64$$

(or)

$$[(-2)^3]^2 = [-8]^2 = (-8)\times(-8) = 64$$

(or)

$$[(-2)^3]^2 = (-2)^{3\times 2} = (-2)^6 = 64 \text{ (using power rule)}$$

Try these

Verify the following rules (as we did above). Here, a,b are non-zero integers and m, n are any integers.

1. Zero exponent rule: $a^0 = 1$.
2. Product of same powers to power of product rule: $a^m \times b^m = (ab)^m$
3. Quotient of same powers to power of quotient rule: $\dfrac{a^m}{b^m} = \left(\dfrac{a}{b}\right)^m$

Example 1.19

Find the value of (i) 4^{-3} (ii) $\dfrac{1}{2^{-3}}$

Solution:

(i) $4^{-3} = \dfrac{1}{4^3} = \dfrac{1}{4\times4\times4} = \dfrac{1}{64}$ (ii) $\dfrac{1}{2^{-3}} = 2^3 = 2\times2\times2 = 8$

Example 1.20

Simplify: (i) $(-2)^5 \times (-2)^{-3}$ (ii) $\dfrac{3^2}{3^{-2}}$

Solution:

(i) $(-2)^5\times(-2)^{-3} = (-2)^{5-3} = (-2)^2 = -2\times-2 = 4$.

(ii) $\dfrac{3^2}{3^{-2}} = 3^2\times3^2 = 9\times9 = 81$

Example 1.21

Express 4^{-5} as a power with the base 2.

Solution:

Since $4 = 2^2$, we write $4^{-5} = (2^2)^{-5} = 2^{2\times(-5)} = 2^{-10}$ [Here, we use $(a^m)^n = a^{mn}$]

Example 1.22

Simplify and write the answer in exponential form:

(i) $(3^5 \div 3^8)^5 \times 3^{-5}$ (ii) $(-3)^4 \times \left(\dfrac{5}{3}\right)^4$

Solution:

(i) $\left(\frac{3^5}{3^8}\right)^5 \times 3^{-5} = (3^{5-8})^5 \times 3^{-5} = (3^{-3})^5 \times 3^{-5} = 3^{-3\times5} \times 3^{-5} = 3^{-15} \times 3^{-5} = 3^{-15-5} = 3^{-20} = \frac{1}{3^{20}}$

(ii) $(-3)^4 \times \left(\frac{5}{3}\right)^4 = 3^4 \times \frac{5^4}{3^4} = 5^4 = 625$

Example 1.23

Find x so that $(-7)^{x+2} \times (-7)^5 = (-7)^{10}$

Solution:

$$(-7)^{x+2} \times (-7)^5 = (-7)^{10}$$

$$(-7)^{x+2+5} = (-7)^{10}$$

Since the bases are equal, we equate the exponents and get

$$x + 7 = 10$$

$$x = 10 - 7 = 3$$

1.4.5 Standard notation and Scientific notation:

Standard notation for a number is just the number as we normally write it. We use *expanded notation* to show the value of each digit that is, it is exhibited as a sum of each digit duly multiplied by its matching place value (like ones, tens, hundreds, etc.)

Example:

195 is in standard notation.

$$\underset{\text{Standard Notation}}{195} = \underbrace{1 \times 100 + 9 \times 10 + 5}_{\text{Expanded notation}}$$

Astronomers, biologists, engineers, physicists and many others come across quantities whose measures require very small or very large numbers. If they write the numbers in standard form, it may not help others to understand or make computations. Scientific notation is a way to make these numbers easier to work with.

A number in scientific notation is given as the product of a number (integer or decimal) and a power of 10. We move the decimal place forward or backward until we have a number between 1 and 10. Then we add a power of ten that tells how many places you moved the decimal forward or backward.

Note

1. The positive exponent in 1.3×10^{12} indicates that it is a large number.
2. The negative exponent in 7.89×10^{-21} indicates that it is a small number.

Examples:

Standard Form	Scientific Notation	Standard Form	Scientific Notation
0.00123	1.23×10^{-3}	123	1.23×10^{2}
0.0123	1.23×10^{-2}	1,230	1.23×10^{3}
0.123	1.23×10^{-1}	12,300	1.23×10^{4}
1.23	1.23×10^{0}	1,23,000	1.23×10^{5}
12.3	1.23×10^{1}	12,30,000	1.23×10^{6}

To write in scientific notation, follow the form $\mathbf{N} \times \mathbf{10}^{a}$

where **N** is a number between 1 and 10, but not 10 itself, and *a* is an integer (positive or negative number).

Some more examples:

(a) The diameter of the earth is 12756000 miles. The scientific form to express this number is 1.2756×10^{7} miles.

(b) The volume of Jupiter is about 143,300,000,000,000 km^3. This can be easily written in scientific form as $1.433 \times 10^{14}\ km^3$.

(c) The size of a bacterium is 0.00000085 mm. This size can be easily written in scientific form as $8.5 \times 10^{-7}\ mm$.

Example 1.24

Write in scienticfic form: (i) 1642.398 (ii) $(7 \times 10^{2})(5.2 \times 10^{7})$ (iii) $(3.7 \times 10^{-5})(2 \times 10^{-3})$

Solution:

(i) $1642.398 = 1.642398 \times 10^{3}$

(ii) $(7 \times 10^{2})(5.2 \times 10^{7}) = 36.4 \times 10^{9} = 3.64 \times 10^{10}$

(iii) $(3.7 \times 10^{-5})(2 \times 10^{-3}) = 7.4 \times 10^{-8}$

Example 1.25

Write in standard form: (i) 2.27×10^{-4} (ii) Light travels at 1.86×10^{5} miles per second.

Solution:

(i) $2.27 \times 10^{-4} = 0.000227$.

(ii) Light travels at 1.86×10^{5} miles per second= 186000 miles per second

Try these

1. Write in standard form: Mass of planet Uranus is 8.68×10^{25} *kg*.
2. Write in scientific form: (i) 0.000012005 (ii) 4312.345 (iii) 0.10524
   (iv) The distance between the Sun and the planet Saturn 1.4335×10^{12} miles.

Exercise 1.4

1. **Fill in the blanks:**
   (i) $(-1)^{\text{even integer}}$ is ____________.
   (ii) For $a \neq 0$, a^0 is ____________.
   (iii) $4^{-3} \times 5^{-3} =$ ____________.
   (iv) $(-2)^{-7} =$ ____________.
   (v) $\left(-\dfrac{1}{3}\right)^{-5} =$ ____________.

2. **Say True or False:**
   (i) If $8^x = \dfrac{1}{64}$, the value of x is -2.
   (ii) The simplified form of $(256)^{\frac{-1}{4}}$ is $\dfrac{1}{4}$.
   (iii) The standard form of 2×10^{-4} is 0.0002.
   (iv) The scientific form of 123.456 is 1.23456×10^{-2}.
   (v) The multiplicative inverse of 3^{-7} is 3^7.

3. Evaluate: (i) $\left(\dfrac{1}{2}\right)^3$ (ii) $\left(\dfrac{1}{2}\right)^{-5}$ (iii) (-3) (iv) $(-3)^4$ (v) $\left(\dfrac{-5}{6}\right)^{-3}$
   (vi) $\left(2^{-5} \div 2^7\right) \times 2^{-2}$ (vii) $\left(2^{-1} \times 3^{-1}\right) \div 6^{-2}$ (viii) $\left(-\dfrac{1}{3}\right)^{-2}$

4. Evaluate: (i) $\left(\dfrac{2}{5}\right)^4 \times \left(\dfrac{2}{5}\right)^2$ (ii) $\left(\dfrac{4}{5}\right)^{-2} \times \left(\dfrac{4}{5}\right)^{-3}$ (iii) $\left(\dfrac{1}{2}\right)^{-3} \times \left(\dfrac{1}{2}\right)^7$

5. Evaluate: (i) $\left(5^0 + 6^{-1}\right) \times 3^3$ (ii) $\left(2^{-1} \times 3^{-1}\right) \div 6^{-1}$ (iii) $\left(3^{-1} + 4^{-2} + 5^{-3}\right)^0$

6. Simplify: (i) $(3^2)^3 \times (2 \times 3^5)^{-2} \times (18)^2$ (ii) $\dfrac{9^2 \times 7^3 \times 2^5}{84^3}$ (iii) $\dfrac{2^{-8} \times 4^2 \times 3^5}{3^7 \times 2^5}$

7. Solve for x : (i) $\dfrac{10^x}{10^{-3}}=10^9$ (ii) $\dfrac{2^{2x-1}}{2^{x+2}}=4$ (iii) $\dfrac{5^5\times5^{-4}\times5^x}{5^{12}}=5^{-5}$

8. Expand using exponents: (i) 6054.321 (ii) 897.14

9. Find the number is standard form:

(i) $8\times10^4+7\times10^3+6\times10^2+5\times10^1+2\times1+4\times10^{-2}+7\times10^{-4}$

(ii) $5\times10^3+5\times10^1+5\times10^{-1}+5\times10^{-3}$

10. The radius of a hydrogen atom is 2.5×10^{-11} m. Express this number in standard notation.

11. Write the following numbers in scientific notation:

(i) 467800000000 (ii) 0.000001972

12. Write in scientific notation:

(i) Earth's volume is about 1,083,000,000,000 cubic kilometres.

(ii) If you fill a bucket with dirt, the portion of the whole Earth that is in the bucket will be 0.0000000000000000000000016 kg.

Objective Type Questions

13. By what number should $(-4)^{-1}$ be multiplied so that the product becomes 10^{-1}?

(a) $\dfrac{2}{3}$ (b) $\dfrac{-2}{5}$ (c) $\dfrac{5}{2}$ (d) $\dfrac{-5}{2}$

14. 0.0000000002020 in scientific form is

(a) 2.02×10^9 (b) 2.02×10^{-9} (c) 2.02×10^{-8} (d) 2.02×10^{-10}

15. $(-2)^{-3}\times(-2)^{-2}$ is

(a) $\dfrac{-1}{32}$ (b) $\dfrac{1}{32}$ (c) 32 (d) –32

16. Which is not correct?

(a) $\left(\dfrac{-1}{4}\right)^2=4^{-2}$ (b) $\left(\dfrac{-1}{4}\right)^2=\left(\dfrac{1}{2}\right)^4$ (c) $\left(\dfrac{-1}{4}\right)^2=16^{-1}$ (d) $-\left(\dfrac{1}{4}\right)^2=16^{-1}$

17. If $\left(\dfrac{p}{q}\right)^{1-3x}=\left(\dfrac{q}{p}\right)^{\frac{1}{2}}$, then x is

(a) 4^{-1} (b) 3^{-1} (c) 2^{-1} (d) 1^{-1}

Exercise 1.5

Miscellaneous and Practice Problems

1. A square carpet covers an area of 1024 m^2 of a big hall. It is placed in the middle of the hall. What is the length of a side of the carpet?

2. There is a large square portrait of a leader that covers an area of 4489 cm^2. If each side has a 2 cm liner, what would be its area?

3. 2401 plants are planted in a garden such that each contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

4. If $\sqrt[3]{1906624} \times \sqrt{x} = 3100$, find x.

5. If $(625)^x = 15625$, find x^2 and x^3.

6. If $2^{m-1} + 2^{m+1} = 640$, then find m.

7. Simplify: $\dfrac{16 \times 10^2 \times 64}{4^2 \times 2^4}$

8. Give the answer in scientific notation:
A human heart beats at an average of 80 beats per minute. How many times does it beat in i) an hour? ii)a day? iii)a year? iv) 100 years?

Challenging Problems

9. A greeting card has an area 90 cm^2. Between what two whole numbers is the length of its side?

10. 225 square shaped mosaic tiles, each of area 1 square decimetre exactly cover a square shaped verandah. How long is each side of the square shaped verandah?

11. A group of 1536 cadets wanted to have a parade forming a square design. Is it possible? If it is not possible how many more cadets would be required?

12. Find the decimal fraction which when multiplied by itself gives 176.252176.

13. Evaluate: $\sqrt{286225}$ and use it to compute $\sqrt{2862.25} + \sqrt{28.6225}$

14. The speed of light in glass is about $2 \times 10^8\, m/\text{sec}$. Use the formula, $time = \dfrac{distance}{speed}$ to find the time (in hours) for a pulse of light to travel 7200 km in glass.

15. Simplify: $(3.769 \times 10^5) + (4.21 \times 10^5)$

16. Order the following from the least to the greatest: $16^{25}, 8^{100}, 3^{500}, 4^{400}, 2^{600}$

Activity

Observe that

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Find the value of $15^3 - 14^3$ in the above pattern.

Think

$$1^3 = 1 = 1$$
$$2^3 = 8 = 3 + 5$$
$$3^3 = 27 = 7 + 9 + 11$$

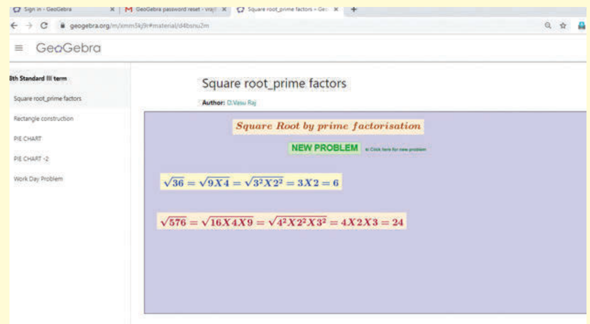
Observe and continue this pattern to find the value of 7^3 as the sum of consecutive odd numbers.

Summary

- A natural number n is called a square number, if we can find another natural number m such that $n = m^2$.
- The sum of the first n consecutive odd natural numbers is n^2.
- The square of an odd number can be written as the sum of two consecutive natural numbers.
- Mathematically, three numbers a, b, c make a Pythagorean triplet, if the sum of the squares of any two equals the square of the third.
- The square root of a number n, written as $\sqrt{n}$ (or) $n^{\frac{1}{2}}$, is the number that gives n when multiplied by itself.
- The number of times a prime factor occurs in the square is equal to twice the number of times it occurs in the prime factorization of the number.
- If n is the number of digits of a number then the number, of digits of in the square root of that number is $= \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$
- For any two positive numbers p and q. we have (i) $\sqrt{pq} = \sqrt{p} \times \sqrt{q}$ and (ii) $\sqrt{\frac{p}{q}} = \frac{\sqrt{p}}{\sqrt{q}}$
- If you multiply a number by itself, and then by itself again, the result is a cube number.
- The cube root of a number is the value, that when cubed, gives the original number.
- An expression that represents repeated multiplication of the same factor is called a power.
- The exponent corresponds to the number of times the base is used as a factor.
- Laws of exponents: (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$ (iii) $(a^m)^n = a^{mn}$
- Other results: (i) $a^0 = 1$ (ii) $a^{-m} = \frac{1}{a^m}$ (iii) $a^m \times b^m = (ab)^m$ (iv) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$
- To write in scientific notation, we follow the form $N \times 10^a$ where N is a number between 1 and 10, but not 10 itself, and a is an integer (positive or negative number).

ICT CORNER

Expected Outcome

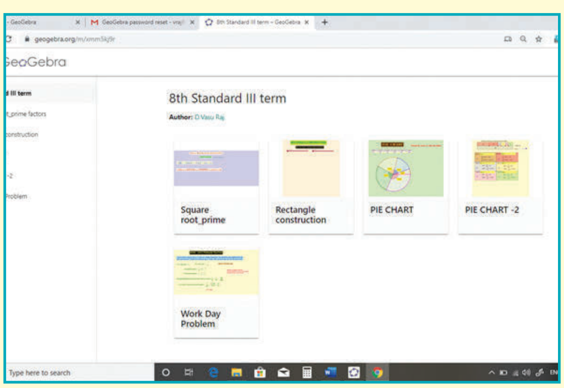


Step – 1

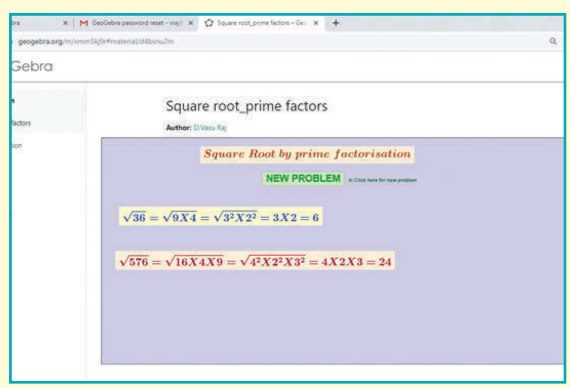
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Square root_prime factors"

Step - 2

Click on " NEW PROBLEM" and check the calculation.



Step – 1



Step – 2

Browse in the link

NUMBERS:

https://www.geogebra.org/m/xmm5kj9r or
Scan the QR Code.

B358_8_MATHS_EM

2 LIFE MATHEMATICS

Learning Objectives

- To recall direct and inverse proportions.
- To know compound variation and do problems on it.
- To solve time and work problems.

2.1 Introduction

The following conversation takes place in the Math class of Std VIII.

Teacher: Students, before we could learn about what Compound Variation is, let me ask you a few questions on direct and inverse (indirect) proportions which you have already learnt in Std VII. Can anyone of you tell me what direct proportion is?

Bharathi: Yes, teacher. If one quantity increases or decreases depending on the increase or decrease of another quantity simultaneously, then it is direct proportion.

Teacher: Good Bharathi, give me an example too.

Bharathi: Well teacher, if I plan to give 2 pens to each of my friends in the birthday party, the number of pens to be bought will be in direct proportion with the number of friends who will attend the party. The following table will help us understand clearly, teacher.

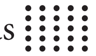
Number of friends	1	2	5	12	15
Number of pens	2	4	10	24	30

Teacher: Very good example Bharathi. Students give her a big hand. (The class applauds). Mukesh, can you tell about inverse proportion?

Mukesh: Yes teacher, if our class of 30 students goes on streets in our village for health awareness campaign in an orderly manner then, we can see an inverse proportion in the number of rows and columns, Teacher, this is easily understood from the following table.

Number of students in columns	1	2	3	5	6
Number of students in rows	30	15	10	6	5

Teacher: Fine Mukesh, you have explained it nicely with a good example.

Mukesh: We can map a few of these arrangements as  and  also see the opposite variations in rows and columns, teacher.

Teacher: Well done Mukesh. Students, I hope you have now understood clearly by these two examples about direct and inverse proportions which you have already learnt in Std VII. Let me now explain what a Compound Variation is? Some problems may involve a chain of two or more variations in them what we call as **Compound Variation.**

Ragini: Teacher, Can you explain the Compound Variation with an example?

Teacher: Yes, Ragini, I will. Before I could explain that, let me ask you all another question. If Kani can finish a given work in 2 hours and Viji in 3 hours, then in what time can they finish it working together?

Bharathi: I think, they will finish it in $2\frac{1}{2}$ hours. Am I correct teacher?

Teacher: Not really Bharathi. I will tell you the correct answer. These types of questions which come under the heading **Time and Work** need some explanation and we will learn all these topics in this term.

Now, let us recall the concepts about the direct and inverse proportions.

2.2 Direct Proportion

If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly. In other words, x and y are said to vary directly if $\frac{x}{y} = k$ always, where k is a positive constant.

Examples of Direct Proportion:

1. Distance –Time (under constant speed): If distance increases, the time taken to reach that distance will also increase and vice- versa.
2. Purchase – Spending: If the purchase on utilities for a family during the festival time increases,the spending limit also increases and vice versa.
3. Work Time – Earnings: If the number of hours worked is less, the pay earned will be less and vice-versa.

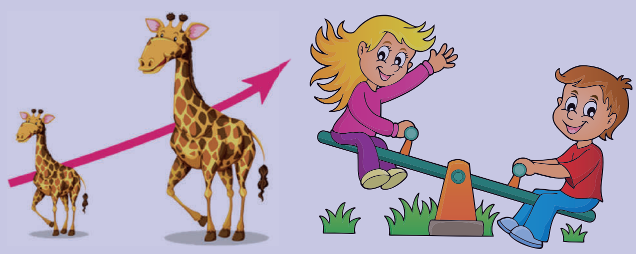
2.3 Inverse Proportion

If two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely. In other words, x and y are said to vary inversely, if $xy = k$ always, where k is a positive constant.

Examples of Inverse Proportion:

1. Price – Consumption: If the price of an article increases, then its consumption will naturally decrease and vice-versa.

2. Workers – Time: If more workers are employed to complete a work, then the time taken to complete will be less and vice-versa.
3. Speed – Time (Fixed Distance): If we travel with less speed,the time taken to cover a given distance will be more and vice-versa.

MATHEMATICS ALIVE – LIFE MATHEMATICS IN REAL LIFE	
	
The growth of a giraffe over time is an example for direct proportion and see-saw is an example for inverse proportion	If 3 persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x}:\frac{1}{y}:\frac{1}{z}$.

Try these

Classify the given examples as direct or inverse proportion:

(i) Weight of pulses to their cost.
(ii) Distance travelled by bus to the price of ticket.
(iii) Speed of the athelete to cover a certain distance.
(iv) Number of workers employed to complete a construction in a specified time.
(v) Volume of water flown through a pipe to its pressure.
(vi) Area of a circle to its radius.

Use the concept of direct and inverse proportions and try to answer the following questions:

1. A student can type 21 pages in 15 minutes. At the same rate, how long will it take the student to type 84 pages?
2. The weight of an iron pipe varies directly with its length. If 8 feet of an iron pipe weighs 3.2 kg, find the proportionality constant k and determine the weight of a 36 feet iron pipe.
3. A car covers a distance of 765 km in 51 litres of petrol. How much distance would it cover in 30 litres of petrol?
4. If x and y vary inversely and $x = 24$ when $y = 8$, find x when $y = 12$.
5. If 35 women can do a piece of work in 16 days, in how many days will 28 women do the same work?

6. A farmer has food for 14 cows which can last for 39 days. How long would the food last, if 7 more cows join his cattle?

7. Identify the type of proportion and fill in the blank boxes:

x	1	2		4	6	8		12	15		24
y	20		60		120		180		300	360	

8. Identify the type of proportion and fill in the blank boxes:

x	1	2		4	6	8		12		18	24
y	144		48		24		16		9	8	

2.4 Compound Variation

There will be problems which may involve a chain of two or more variations in them. This is called as compound variation. The different possibilities of two variations are: **Direct-Direct, Direct-Inverse, Inverse- Direct, Inverse- Inverse.**

Note

There are situations where neither direct proportion nor indirect proportion can be applied. For example, if one can see a parrot at a distance through one eye, it does not mean that he can see two parrots at the same distance through both the eyes. Also, if it takes 5 minutes to fry a vadai, it does not mean that 20 vadais will take 100 minutes to fry!

Let us now solve a few problems on compound variation. Here, we compare the known quantity with the unknown (x). There are a few methods in practice by which problems on compound variation are solved. They are:

2.4.1 Proportion Method:

In this method, we shall compare the given data and find whether they are in direct or indirect proportion. By finding the proportion, we can use the fact that

the product of the extremes = the product of the means

and get the value of the unknown (x).

2.4.2 Multiplicative Factor Method:

Illustration:

Men	Hours	Days
a D I x	c D d	e I f

Here, the unknown (x) in men is compared to the known, namely hours and days. If men and hours are in direct proportion (D) then, take the multiplying factor as is $\frac{d}{c}$ (take the reciprocal). Also, if men and days are in inverse proportion (I), then take the multiplying factor as $\frac{e}{f}$ (no change). Thus, we can find the unknown (x) in men as $x = a \times \frac{d}{c} \times \frac{e}{f}$.

Note

- When the number of days is constant, work and persons are directly proportional to each other and vice-versa.

  i.e., increase (↑) in work means increase (↑) in persons with same number of days.
- When the number of persons is constant, work and days are directly proportional to each other and vice-versa.

  i.e., increase (↑) in work means increase (↑) in days with same number of persons.
- When the work is constant, the number of persons and days are inversely proportional to each other and vice-versa.

  i.e., increase (↑) persons means decrease (↓) in days with constant work.

2.4.3 Formula Method:

Identify the data from the given statement as Persons (P), Days (D), Hours (H) and Work (W) and use the formula,

$$\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$$

where the suffix 1 contains the complete data from the first statement of the given problem and the suffix 2 contains the unknown data in the second statement to be found out in the problem. That is, this formula says, P_1 men doing W_1 units of work in D_1 days working H_1 hours per day is the same as P_2 men doing W_2 units of work in D_2 days working H_2 hours per day. Identifying the work W_1 and W_2 correctly is more important in these problems. This method will be easy for finding the unknown (x) quickly.

Example 2.1 (Direct – Direct Variation)

If a company pays ₹6 lakh for 15 workers for 20 days, what would it pay for 5 workers for 12 days?

Solution:

Proportion Method:

Workers	Payment (Work)	Days
D 15 5	D 6 D x	20 D 12

Here, the unknown is the payment (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less payment. So, it is a direct proportion.

$\therefore$ The proportion is $20:12::6:x$ $\longrightarrow$ (1)

Step 2:

Also, less workers means less payment. So, it is a direct proportion again.

$\therefore$ The proportion is $15:5::6:x$ $\longrightarrow$ (2)

Step 3:

Combining (1) and (2)

$$\left.\begin{array}{l}20:12\\15:5\end{array}\right\}::6:x$$

We know that the product of the extremes = the product of the means

Extremes		Means		Extremes
20	:	12 : 6	:	x
15	:	5		

So, $20\times15\times x=12\times6\times5 \Rightarrow x=\dfrac{12\times6\times5}{20\times15}$ = ₹ 1.2 lakh.

Multiplicative Factor Method:

Workers	Payment (Work)	Days
15	6	20
D 5	D x D	12 D

Here, the unknown is the payment (x). It is to be compared with the workers and the days.

Step 1:

Here, less days means less payment. So, it is a direct proportion.

$\therefore$ The multiplying factor is $\dfrac{12}{20}$ (take the reciprocal).

Step 2:

Also, less workers means less payment. So, it is a direct proportion again.

$\therefore$ The multiplying factor is $\dfrac{5}{15}$ (take the reciprocal).

Step 3:

$$\therefore x=6\times\frac{12}{20}\times\frac{5}{15}$$

x = ₹ 1.2 lakh

Formula Method

Here, $P_1=15, D_1=20$ and $W_1=6$

$P_2=5, D_2=12$ and $W_2=x$

Using the formula, $\dfrac{P_1\times D_1}{W_1}=\dfrac{P_2\times D_2}{W_2}$

We have, $\frac{15\times 20}{6}=\frac{5\times 12}{x}$

$$\Rightarrow x=\frac{5\times 12\times 6}{15\times 20}=₹\ 1.2 \text{ lakh}.$$

Example 2.2 (Direct – Inverse Variation)

A mat of length 180 m is made by 15 women in 12 days. How long will it take for 32 women to make a mat of length 512 m?

Solution:

Proportion Method:

Length (Work)	Women	Days
180	15	12
512	32	x
D	*I*	*D* *I*

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is a direct proportion.

$\therefore$ The proportion is 180 : 512 :: 12 : x $\longrightarrow$ (1)

Step 2:

Also, more women means less days. So, it is an inverse proportion.

$\therefore$ The proportion is 32 : 15 :: 12 : x $\longrightarrow$ (2)

Step 3:

Combining (1) and (2)

$$\left.\begin{matrix}180:512\\32:15\end{matrix}\right\}::12:x$$

We know that the product of the extremes = the product of the means

Extremes		Means		Extremes
180	:	512 : 12	:	x
32	:	15		

So, $180\times 32\times x=512\times 12\times 15 \Rightarrow x=\frac{512\times 12\times 15}{180\times 32}=16$ days.

Multiplicative Factor Method:

Length (Work)	Women	Days
180	15	12
512	32	x
D	*I*	*D* *I*

Here, the unknown is the days (x). It is to be compared with the length and the women.

Step 1:

Here, more length means more days. So, it is a direct proportion.

$\therefore$ The multiplying factor is $\frac{512}{180}$ (take the reciprocal).

Step 2:

Also, more women means less days. So, it is an inverse proportion.

$\therefore$ The multiplying factor is $\frac{15}{32}$ (no change).

Step 3:

$\therefore x = 12 \times \frac{512}{180} \times \frac{15}{32} = 16$ days.

Formula Method:

Here, $P_1 = 15,\ D_1 = 12$ and $W_1 = 180$

$P_2 = 32,\ D_2 = x$ and $W_2 = 512$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

We have, $\frac{15 \times 12}{180} = \frac{32 \times x}{512}$

$\Rightarrow\ 1 = \frac{32 \times x}{512}\quad \Rightarrow\ x = \frac{512}{32} = 16$ days.

Remark: Students may answer in any of the three given methods dealt here.

Try these

1. When $x = 5$ and $y = 5$ find k, if x and y vary directly.
2. When x and y vary inversely, find the constant of variation when $x = 64$ and $y = 0.75$
3. You draw a circle of a given radius. Then, draw its radii in such a way that the angles between any pair of radii are equal. You start with drawing 3 radii and end with drawing 12 radii in the circle. Prepare a table for the number of radii to the angle between a pair of consecutive radii and check whether they are in inverse proportion. What is the proportionality constant?

Think

(i) When x and y are in direct proportion and if y is doubled, then what happens to x?

(ii) If $\frac{x}{y-x} = \frac{6}{7}$ what is $\frac{x}{y}$?

Example 2.3 (Inverse – Direct Variation)

If 81 students can do a painting on a wall of length 448 m in 56 days. How many students can do the painting on a similar type of wall of length 160 m in 27 days?

Solution:

Multiplicative Factor Method:

Students	Days	Length of the wall (Work)
81	56	448
I *D*	*I*	*D*
x	27	160

Step 1:

Here, less days means more students. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{56}{27}$.

Step 2:

Also, less length means less students. So, it is a direct variation.

$\therefore$ The multiplying factor is $\frac{160}{448}$.

Step 3:

$$\therefore x = 81 \times \frac{56}{27} \times \frac{160}{448}$$

$x = 60$ students.

Formula Method:

Here, $P_1 = 81$, $D_1 = 56$ and $W_1 = 448$

$P_2 = x$, $D_2 = 27$ and $W_2 = 160$

Using the formula, $\frac{P_1 \times D_1}{W_1} = \frac{P_2 \times D_2}{W_2}$

We have, $\frac{81 \times 56}{448} = \frac{x \times 27}{160}$

$$\Rightarrow x = \frac{81 \times 56}{448} \times \frac{160}{27}$$

$x = 60$ students.

Example 2.4 (Inverse – Inverse Variation)

If 48 men working 7 hours a day can do a work in 24 days, then in how many days will 28 men working 8 hours a day can complete the same work?

Solution:

Multiplicative Factor Method:

Men	Hours	Days
48	7	24
I	*I*	*I* *I*
28	8	x

Step 1:

Here, less men means more days. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{48}{28}$.

Step 2:

Also, more hours means less days. So, it is an inverse variation.

$\therefore$ The multiplying factor is $\frac{7}{8}$.

Step 3:

$\therefore x = 24 \times \frac{48}{28} \times \frac{7}{8} = 36$ days.

Formula Method:

Here, $P_1 = 48, D_1 = 24, H_1 = 7$ and $W_1 = 1$ (Why?)

$P_2 = 28, D_2 = x, H_2 = 8$ and $W_2 = 1$ (Why?)

Using the formula, $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$

We have, $\frac{48 \times 24 \times 7}{1} = \frac{28 \times x \times 8}{1}$

$$\Rightarrow x = \frac{48 \times 24 \times 7}{28 \times 8} = 36 \text{ days.}$$

Try these

Identify the different variations present in the following questions:

1. 24 men can make 48 articles in 12 days. Then, 6 men can make ______ articles in 6 days.
2. 15 workers can lay a road of length 4 km in 4 hours. Then, ______ workers can lay a road of length 8 km in 8 hours.
3. 25 women working 12 hours a day can complete a work in 36 days. Then, 20 women must work ______ hours a day to complete the same work in 30 days.
4. In a camp, there are 420 kg of rice sufficient for 98 persons for 45 days. The number of days that 60 kg of rice will last for 42 persons is ______.

Example 2.5

If 15 men take 40 days to complete a work, how long will it take if 15 more men join them to complete the same work?

Solution:

If 15 men can complete the work in 40 days, then the work measured in terms of person days = $15 \times 40 = 600$ person days.

If the same work is to be done by 30 (15 + 15) men, then the number of days they will take is $\frac{600}{30} = 20$ days.

Note

- The concept of *person days* is important here. The number of *persons* multiplied by the number of *days* required to complete the work gives the *person days*. Here, work is measured in terms of *person days*.
- If x women or y men can complete a piece of work in p days, then a women and b men can complete the same work in $\frac{xyp}{xb+ya}$ (or) $\frac{p}{\frac{a}{x}+\frac{b}{y}}$ days.

Example 2.6

6 women or 8 men can construct a room in 86 days. How long will it take for 7 women and 5 men to do the same type of room?

Solution:

Person days Method:

Here, let M and W denote a men and a women respectively.

Given that, 6W = 8M $\Rightarrow 1\text{W} = \frac{8}{6}\text{M} = \frac{4}{3}\text{M}$.

Now, $7\text{W}+5\text{M} = 7\times\frac{4}{3}\text{M}+5\text{M} = \frac{43\text{M}}{3}$

If 8M can construct the room in 86 days, then $\frac{43\text{M}}{3}$ can construct the same type of room in $8\text{M}\times 86 \div \frac{43\text{M}}{3} = 8\text{M}\times 86\times\frac{3}{43\text{M}} = 48$ days.

Formula Method:

$$\text{Required time to construct the room} = \frac{xyp}{xb+ya}$$

$$= \frac{6\times 8\times 86}{6\times 5+8\times 7} = \frac{6\times 8\times 86}{30+56} = \frac{6\times 8\times 86}{86} = 48 \text{ days.}$$

(or)

$$\text{Required time to construct the room} = \frac{p}{\frac{a}{x}+\frac{b}{y}}$$

$$= \frac{86}{\frac{7}{6}+\frac{5}{8}} = \frac{86\times 48}{86} = 48 \text{ days.}$$

2.5 Time and Work

Work to be done is usually considered as one unit. Work can be in any form like building a wall, making a road, filling or emptying a tank, or even eating a certain amount of food.

Time is measured in hours, days etc., Certain assumptions are made that the work so done is uniform and each person shares the same work time in case of group work in completing the work.

Unitary Method:

If two persons X and Y can do some work individually in a and b days, then their one day's work is $\frac{1}{a}$ and $\frac{1}{b}$ respectively.

Also, their one day's work together $= \frac{1}{a} + \frac{1}{b} = \frac{a+b}{ab}$

Thus, X and Y together can complete the work in $\frac{ab}{a+b}$ days.

Example 2.7

A and B together can do a piece of work in 16 days and A alone can do it in 48 days. How long will B done take to complete the work?

Solution:

$(A+B)$'s 1 day's work $= \frac{1}{16}$

A's 1 day's work $= \frac{1}{48}$

$\therefore$ B's 1 day's work $= \frac{1}{16} - \frac{1}{48}$

$= \frac{3-1}{48} = \frac{2}{48} = \frac{1}{24}$

$\therefore$ B alone can complete the work in 24 days.

Note

The time taken to complete a work or task depends on various factors such as number of persons, their capacity to do the work, the amount of work and the time spent per day for the completion of work.

DO YOU KNOW?

If A is $\frac{a}{b}$ times as good a worker as B, then A will take $\frac{b}{a}$ of the time taken by B to complete the work.

Example 2.8

A works 3 times as fast as B and is able to complete a task in 24 days less than the days taken by B. Find the time in which they can complete the work together.

Solution:

If B does the work in 3 days, then A will do it in 1 day. That is, the difference is 2 days. Here, given that the difference between A and B in completing the work is 24 days. Therefore, A will take $\frac{24}{2} = 12$ days and B will take $3 \times 12 = 36$ days to complete the work separately.

Hence, the time taken by A and B together to complete the work $= \frac{ab}{a+b}$ days.

$$= \frac{12 \times 36}{12+36} = \frac{12 \times 36}{48} = 9 \text{ days.}$$

Example 2.9

P and Q can do a piece of work in 20 days and 30 days respectively. They started the work together and Q left after some days of work and P finished the remaining work in 5 days. After how many days from the start did Q leave?

Solution:

$$\text{P's 1 day's work} = \frac{1}{20} \text{ and Q's 1 day's work} = \frac{1}{30}$$

$$\text{P's work for 5 days} = \frac{1}{20} \times 5 = \frac{5}{20} = \frac{1}{4}$$

Therefore, the remaining work $= 1 - \frac{1}{4} = \frac{3}{4}$ (Total work is always 1)

This remaining work was done by both P and Q.

Work done by P and Q in a day $= \frac{1}{20} + \frac{1}{30} = \frac{5}{60} = \frac{1}{12}$

Therefore, the number of days they worked together $= \dfrac{3/4}{1/12} = \frac{3}{4} \times \frac{12}{1} = 9$ days

So, Q left after 9 day from the days the work started.

Example 2.10

A and B can do a piece of work in 12 days and 9 days respectively. They work on alternate days starting with A on the first day. In how many days will the work be completed?

Solution:

Since they work on alternate days, let us consider a period of two days.

In the period of 2 days, work done by A and B = $\frac{1}{12} + \frac{1}{9} = \frac{7}{36}$

If we consider 5 such time periods for the fraction $\frac{7}{36}$ (we consider 5 periods because 7 goes 5 times completely in 36),

work done by A and B in $5 \times 2\ (=10)$ days = $5 \times \frac{7}{36} = \frac{35}{36}$

Therefore, the remaining work $= 1 - \frac{35}{36} = \frac{1}{36}$.

This is done by A (why?) in $\frac{1}{36} \times 12 = \frac{1}{3}$ days

So, the total time taken = 10 days + $\frac{1}{3}$ days $= 10\frac{1}{3}$ days.

2.6 Sharing of the money for work

When a group of people do some work together, based on their individual work, they get a share of money themselves. In general, **money earned** is shared by people, who worked together, in the ratio of the **total work** done by each of them.

DO YOU KNOW?

- If the ratio of the time taken by A and B in doing a work is $x : y$, then the ratio of work done by A and B is $\frac{1}{x} : \frac{1}{y} = y : x$. This is the ratio for their separate wages too.
- If three persons A, B and C can do a work in x, y and z days respectively, then the ratio in which their wages will be distributed to them is $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$.

Example 2.11

X, Y and Z can do a piece of job in 4, 6 and 10 days respectively. If X,Y and Z work together to complete, then find their separate shares if they will be paid ₹ 3100 for completing the job.

Solution:

Since they all work for the same number of days, the ratio in which they share the money is equal to the ratio of their work done per day.

That is, $\frac{1}{4}:\frac{1}{6}:\frac{1}{10}=\frac{15}{60}:\frac{10}{60}:\frac{6}{60}=15:10:6$

Here, the total parts = 15 + 10 + 6 = 31

Hence, A's share = $\frac{15}{31}\times 3100=₹1500$, B's share = $\frac{10}{31}\times 3100=₹1000$ and

C's share is ₹ 3100 – (₹ 1500 + ₹ 1000)= ₹ 600.

Try these

1. Vikram can do one-third of work in p days. He can do $\frac{3}{4}$ th of work in ________ days.
2. If m persons can complete a work in n days ,then $4m$ persons can complete the same work in ________days and $\frac{m}{4}$ persons can complete the same work in ________ days.

Exercise 2.1

1. Fill in the blanks:

(i) A can finish a job in 3 days where as B finishes it in 6 days. The time taken to complete the job together is ___________days.

(ii) If 5 persons can do 5 jobs in 5 days, then 50 persons can do 50 jobs in _________ days.

(iii) A can do a work in 24 days. A and B together can finish the work in 6 days. Then B alone can finish the work in _________ days.

(iv) A alone can do a piece of work in 35 days. If B is 40% more efficient than A, then B will finish the work in ____________days.

(v) A alone can do a work in 10 days and B alone in 15 days. They undertook the work for ₹200000. The amount that A will get is _________.

Unit -2 Life Mathematics.indd 44
07-11-2019 16:22:34

2. 210 men working 12 hours a day can finish a job in 18 days. How many men are required to finish the job in 20 days working 14 hours a day?
3. A cement factory makes 7000 cement bags in 12 days with the help of 36 machines. How many bags can be made in 18 days using 24 machines?
4. A soap factory produces 9600 soaps in 6 days working 15 hours a day. In how many days will it produce 14400 soaps working 3 hours more a day?
5. If 6 container lorries can transport 135 tonnes of goods in 5 days, how many more lorries are required to transport 180 tonnes of goods in 4 days?
6. A can do a piece of work in 12 hours, B and C can do it 3 hours whereas A and C can do it in 6 hours. How long will B alone take to do the same work?
7. A and B can do a piece of work in 12 days, while B and C can do it in 15 days whereas A and C can do it in 20 days. How long would each take to do the same work?
8. Carpenter A takes 15 minutes to fit the parts of a chair while Carpenter B takes 3 minutes more than A to do the same work. Working together, how long will it take for them to fit the parts for 22 chairs?
9. A man takes 10 days to finish a job where as a woman takes 6 days to finish the same job. Together they worked for 3 days and then the woman left. In how many days will the man complete the remaining job?
10. A is thrice as fast as B. If B can do a piece of work in 24 days, then find the number of days they will take to complete the work together.

W4I6H4

Exercise 2.2

Miscellaneous and Practice Problems

1. 5 boys or 3 girls can do a science project in 40 days. How long will it take for 15 boys and 6 girls to do the same project?
2. If 32 men working 12 hours a day can do a work in 15 days, how many men working 10 hours a day can do double that work in 24 days?
3. Amutha can weave a saree in 18 days. Anjali is twice as good a weaver as Amutha. If both of them weave together, in how many days can they complete weaving the saree?
4. A, B and C can complete a work in 5 days. If A and C can complete the same work in $7\frac{1}{2}$ days and A alone in 15 days, then in how many days can B and C finish the work?
5. P and Q can do a piece of work in 12 days and 15 days respectively. P started the work alone and then, after 3 days Q joined him till the work was completed. How long did the work last?

Challenging Problems

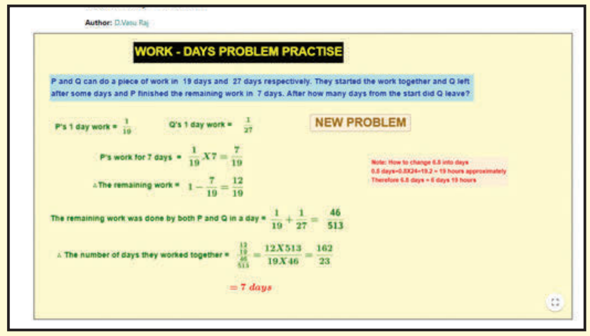
6. A camp had provisions for 490 soldiers for 65 days. After 15 days, more soldiers arrived and the remaining provisions lasted for 35 days. How many soldiers joined the camp?
7. A small–scale company undertakes an agreement to make 540 motor pumps in 150 days and employs 40 men for the work. After 75 days, the company could make only 180 motor pumps. How many more men should the company employ so that the work is completed on time as per the agreement?
8. A can do a work in 45 days. He works at it for 15 days and then, B alone finishes the remaining work in 24 days. Find the time taken to complete 80% of the work, if they work together.
9. P alone can do $\frac{1}{2}$ of a work in 6 days and Q alone can do $\frac{2}{3}$ of the same work in 4 days. In how many days working together, will they finish $\frac{3}{4}$ of the work?
10. X alone can do a piece of work in 6 days and Y alone in 8 days. X and Y undertook the work for ₹4800. With the help of Z, they completed the work in 3 days. How much is Z's share?

Summary

- If two quantities are such that an increase or decrease in one quantity makes a corresponding increase or decrease (same effect) in the other quantity, then they are said to be in direct proportion or said to vary directly.
- x and y are said to vary directly if $\frac{x}{y} = k$ always, where k is a positive constant.
- If two quantities are such that an increase or decrease in one quantity makes a corresponding decrease or increase (opposite effect) in the other quantity, then they are said to be in inverse (indirect) proportion or said to vary inversely.
- x and y are said to vary inversely, if $xy = k$ always, where k is a positive constant.
- There will be problems which involve a chain of two or more variations in them. This is called as compound variation.
- By finding the proportion, we can use the fact that the product of the extremes is equal to the product of the means to find the unknown (x) in the problem.
- By using the formula $\frac{P_1 \times D_1 \times H_1}{W_1} = \frac{P_2 \times D_2 \times H_2}{W_2}$, we can find the unknown (x).
- We can find the unknown (x) by Multiplicative Factor Method also.
- If two persons X and Y can do some work individually in a and b days, their one day's work is $\frac{1}{a}$ and $\frac{1}{b}$ respectively.
- X and Y together can complete the work in $\frac{ab}{a+b}$ days.

ICT CORNER

Expected Outcome

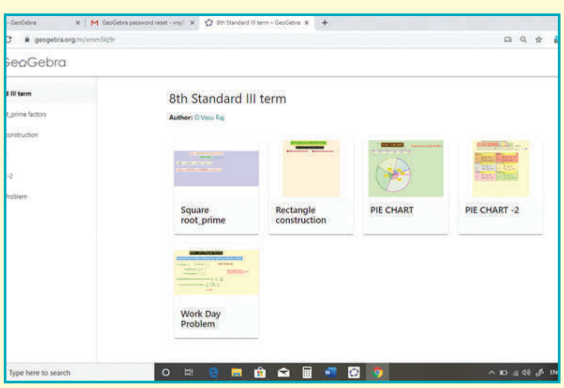


Step – 1

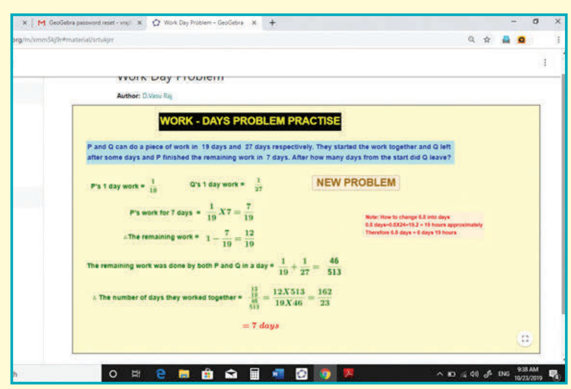
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "Work Day Problem"

Step - 2

Click on " NEW PROBLEM". Check the calculation and work out yourself.



Step – 1



Step – 2

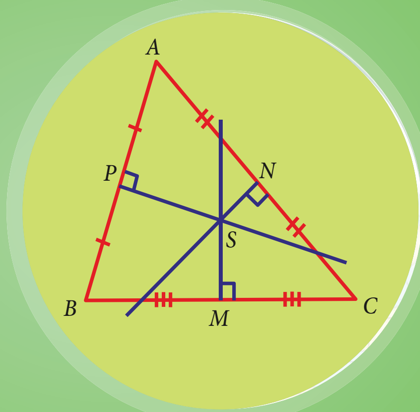
Browse in the link

LIFE MATHEMATICS:

https://www.geogebra.org/m/xmm5kj9r or
Scan the QR Code.

B358_8_MATHS_EM

3 GEOMETRY

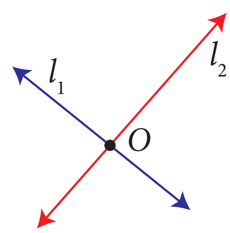


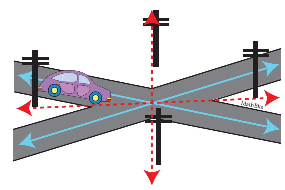
Learning Objectives

- To know and understand the concurrency of medians, altitudes, angle bisectors and perpendicular bisectors in a triangle.
- To construct rhombus, rectangle and square.

Y7B9C7

3.1 Introduction



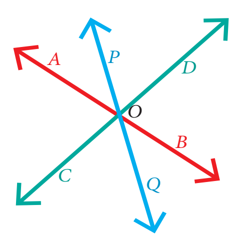


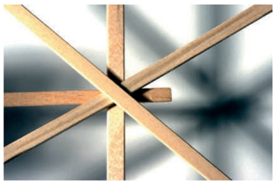
Intersecting Roads

When two lines in a plane cross each other, they are called *intersecting lines.* Here, lines l_1 and l_2 intersect at point O and hence it is called the *point of intersection* of l_1 and l_2.

Three or more lines in a plane are said to be concurrent, if all of them pass through the same point.

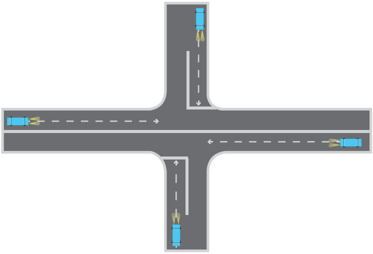
In this figure, $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ and $\overleftrightarrow{PQ}$ are concurrent lines and O is the *point of concurrency*.

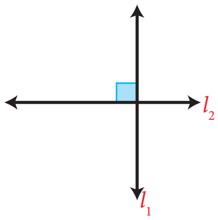




Concurring wooden pieces.

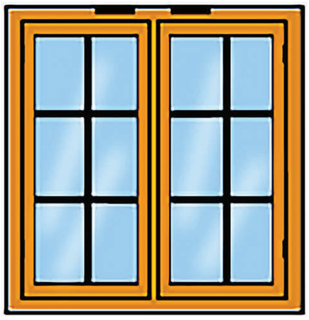
Here you find two roads intersecting in a special way; they cross each other at 90°. They resemble lines at right angle. Such lines are called *perpendicular lines*.

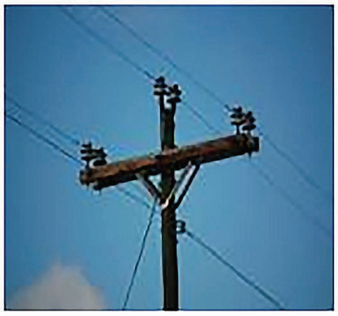


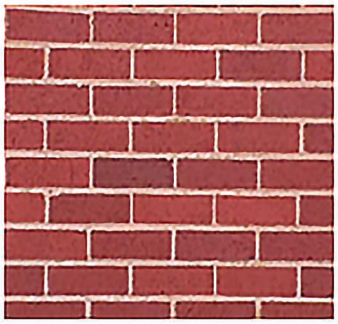


In the figure, two perpendicular lines l_1 and l_2 are seen. We write this as $l_1 \perp l_2$.

You can see perpendicular lines in plenty around you.

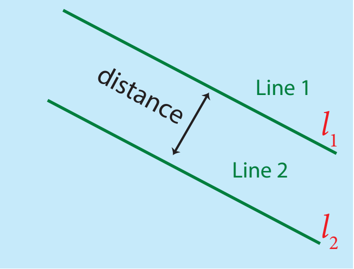






Are all intersecting lines perpendicular lines? What if two lines in a plane do not intersect? Is it possible? Yes, we have plenty of examples around us.

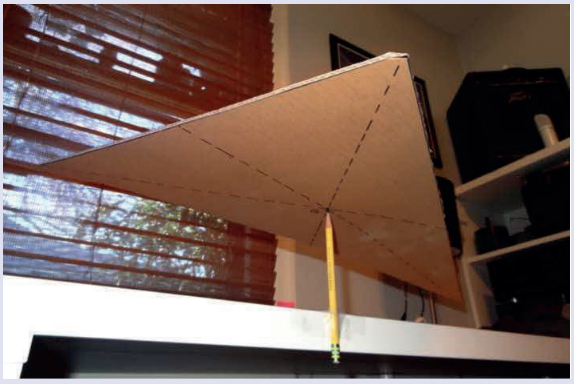
Lines in a plane that never meet are called parallel lines. They are at the same distance apart. In the figure lines l_1 and l_2 are parallel. We write this as $l_1 \parallel l_2$.



Activity

Take a piece of paper of any shape and fold
(i) a pair of perpendicular lines. (ii) a pair of parallel lines.

MATHEMATICS ALIVE - GEOMETRY IN REAL LIFE

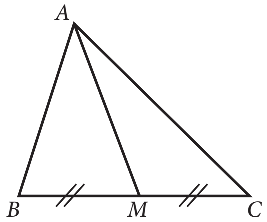


Triangle is balanced at centroid

Rhombus shape seen in a lift ladder

3.2 Median of a triangled

A median of a triangle is a line segment from a vertex to the midpoint of the side opposite that vertex.



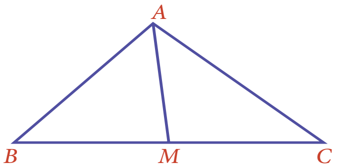
In the figure $\overline{AM}$ is a median of $\Delta\ ABC$.

Are there any more medians for $\Delta\ ABC$?

Since there are three vertices in a triangle, one can identify three medians in a triangle.

Example 3.1

In the figure, ABC is a triangle and AM is one of its medians. If BM = 3.5 cm, find the length of the side BC.

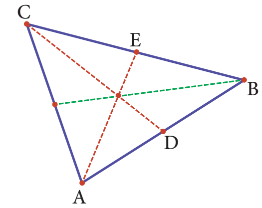


Solution:

AM is median $\Rightarrow$ M is the midpoint of BC.

Given that, $BM = 3.5$ cm, hence BC = twice the length $BM = 2 \times 3.5$ cm $= 7$ cm.

Example 3.2



In the figure, ABC is a triangle and CD is one of its medians. If AD = 9x – 13 and BD = 4x + 2, find the length of the side AB.

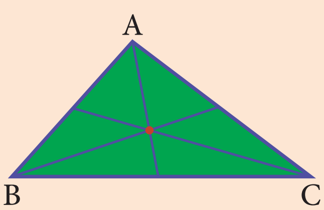
Solution:

AD = DB, since D is the midpoint of AB. (Why?)

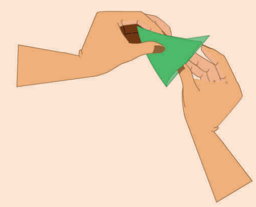
Therefore, $9x - 13 = 4x + 2$. Solving this (try!) simple equation, we get $x = 3$.

Hence, AB = $2(9x - 13) = 2(9(3) - 13) = 28$ units.

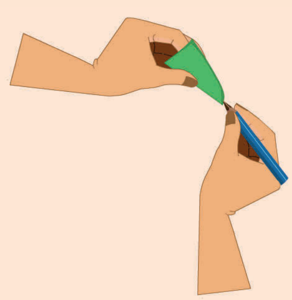
Activity



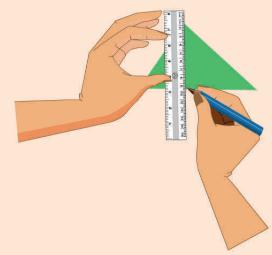
1. Consider a paper cut-out of a triangle. (Let us have an acute-angled triangle, to start with). Name it, say ABC.



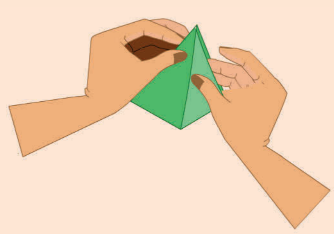
2. Fold the paper along the line that passes through the point A and meets the line BC such that point B falls on C. Make a crease and unfold the sheet.



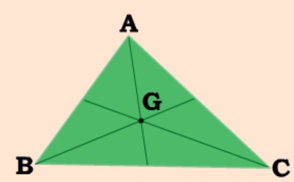
3. Mark the mid point M of BC.



4. You can now draw the median AM, if you want to see it clearly. (Or you can leave it as a fold).



5. In the same way, fold and draw the other two medians.

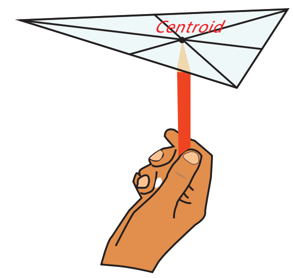


6. Do the medians pass through the same point?

Now you can repeat this activity for an obtuse-angled triangle and a right triangle. What is the conclusion?

The three medians of *any* triangle are concurrent.

3.2.1 Centroid:



The point of concurrence of the three medians in a triangle is called its **Centroid**, denoted by the letter **G**. Interestingly, it happens to be the centre of mass of the triangle. One can easily verify this fact. Take a stiff cut out of triangle of paper. It can be balanced horizontally at this point on a finger tip or a pencil tip.

Should you fold all the three medians to find the centroid? Now you can explore among yourself the following questions:

(i) How can you find the centroid of a triangle?

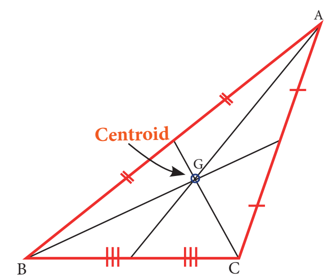
(ii) Is the centroid equidistant from the vertices?

(iii) Is the centroid of a triangle always in its interior?

(iv) Is there anything special about the medians of an
  (a) Isosceles triangle? (b) Equilateral triangle?

3.2.2 Properties of the Centroid of a triangle:

The location of the Centroid of a triangle exhibits some nice properties.



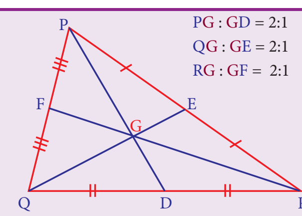
- It is always *located inside the triangle*.
- We have already seen that it *serves as the Centre of gravity* for any triangular lamina.
- Observe the figure given. The lines drawn from each vertex to G form the three triangles $\triangle ABG$, $\triangle BCG$, and $\triangle CAG$. Surprisingly, the areas of these triangles are equal.

The medians of a triangle divide it into three smaller triangles of equal area!

DO YOU KNOW?

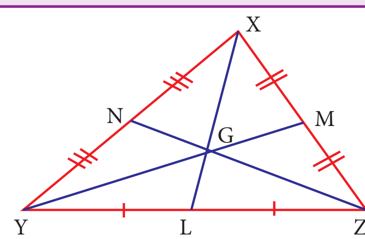
The centroid of a triangle splits each of the medians in two segments, the one closer to the vertex being twice as long as the other one.

This means the centroid divides each median in a ratio of 2:1. (For example, GD is ⅓ of PD). *(Try to verify this by paper folding)*.



Example 3.3

In the figure G is the centroid of the triangle XYZ.



(i) If GL = 2.5 cm, find the length XL.

(ii) If YM = 9.3 cm, find the length GM.

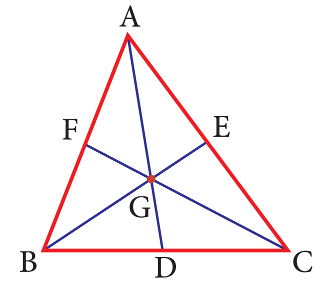
Solution:

(i) Since G is the centroid, XG : GL = 2 : 1 which gives XG : 2.5 = 2 : 1.
Therefore, we get 1 × (XG) = 2 × (2.5) ⇒ XG = 5 cm.
Hence, length XL = XG + GL = 5 + 2.5 = 7.5 cm.

(ii) If YG is of 2 parts then GM will be 1 part. (Why?)
This means YM has 3 parts.
3 parts is 9.3 cm long. So GM (made of 1 part) must be 9.3 ÷ 3 = 3.1 cm.

Example 3.4

ABC is a triangle and G is its centroid. If AD=12 cm, BC=8 cm and BE=9 cm, find the perimeter of ΔBDG.



Solution:

ABC is a triangle and G is its centroid.

If, $AD = 12\text{ cm} \Rightarrow GD = \frac{1}{3}$ of $AD = \frac{1}{3}(12) = 4\text{ cm}$ and

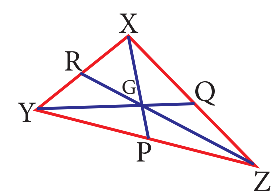
$$BE = 9\text{ cm} \Rightarrow BG = \frac{2}{3} \text{ of } BE = \frac{2}{3}(9) = 6\text{ cm}.$$

Also D is a midpoint of BC $\Rightarrow BD = \frac{1}{2}$ of $BC = \frac{1}{2}(8) = 4$c.

$\therefore$ The perimeter of ΔBDG = BD+GD+BG = 4+4+6 = 14 cm

Example 3.5

XYZ is a triangle and G is its centroid. If GP=2.5 cm, GY=6 cm and ZR=12 cm, find XP, QY and GR.



Solution:

Centroid G divides each median in the ratio 2:1

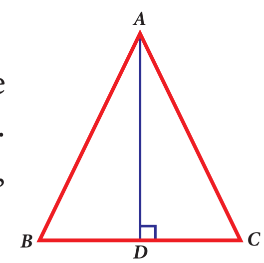
$\therefore$ XG = 2GP $\Rightarrow$ XG = 2(2.5) = 5 cm and XP = XG + GP = 5 + 2.5 = 7.5 cm.

Similarly, GY = 2GQ $\Rightarrow$ 6 = 2GQ $\Rightarrow$ GQ = 3 cm and QY = 3 + 6 = 9 cm.

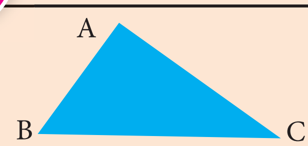
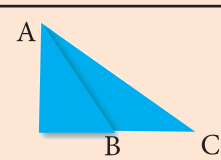
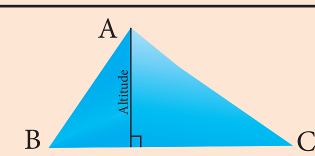
Now, ZR = 12 cm $\Rightarrow$ GR + GZ = 12 cm $\Rightarrow$ GR+2GR = 12 cm $\Rightarrow$ GR = 4 cm.

3.3 Altitude of a triangle

Altitude of a triangle also known as the height of the triangle, is the perpendicular drawn from the vertex of the triangle to the opposite side. The altitude makes a right angle with the base of a triangle. Here, in $\triangle ABC$, $AD \perp BC$ is one of the altitudes.



Activity

		
1. Consider a paper cut-out of an acute angled triangle. Name it, say ABC.	2. Fold the triangle so that a side overlaps itself and the fold contains the vertex opposite to that side.	3. You can now draw the altitude AM, if you want to see it clearly.

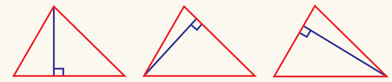
In the same way, you find altitudes of other two sides. Also, with the help of your teacher, you find altitudes of right angled triangle and obtuse angled triangle. Do the altitudes of triangle pass through the same point? What is your conclusion?

The three altitudes of *any* triangle are concurrent.

The point of concurrence is known as its Orthocentre, denoted by the letter H.

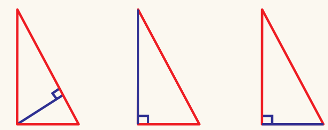
Think

1. **In any acute angled triangle,** all three altitudes are inside the triangle. Where will be the orthocentre? In the interior of the triangle or in its exterior?



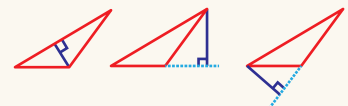
Altitude of an acute triangle

2. **In any right angled triangle,** the altitude perpendicular to the hypotenuse is inside the triangle; the other two altitudes are the legs of the triangle. Can you identify the orthocentre in this case?



Altitude of a right triangle

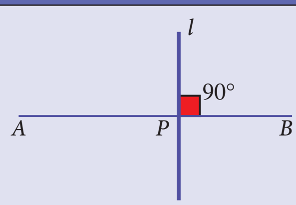
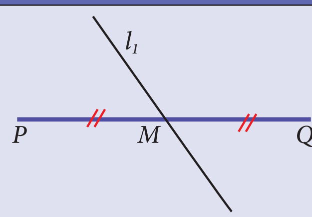
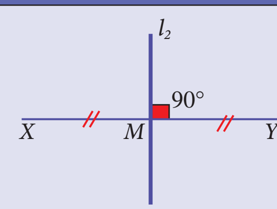
3. **In any obtuse angled triangle,** the altitude connected to the obtuse vertex is inside the triangle, and the two altitudes connected to the acute vertices are outside the triangle. Can you identify the orthocentre in this case?



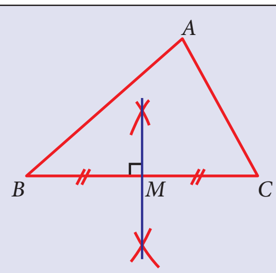
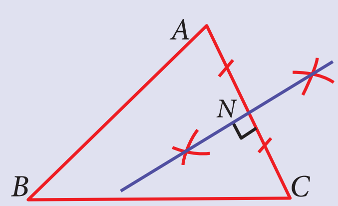
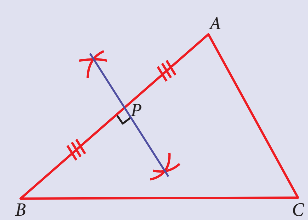
Altitude of an obtuse triangle

3.4 Perpendicular bisector

Let us first recall the following ideas.

Perpendicular	Bisector	Perpendicular bisector
		
$\overline{AB}$ is a line segment. l is perpendicular to $\overline{AB}$. P is the foot of the $\perp^r$. Note that $\overline{AP} \neq \overline{PB}$ here.	$\overline{PQ}$ is a line segment. l_1 is a bisector to $\overline{PQ}$. M is the midpoint of $\overline{PQ}$. l_1 need not be $\perp^r$ to $\overline{PQ}$	$\overline{XY}$ is a line segment. l_2 is a bisector to $\overline{XY}$. l_2 is also $\perp^r$ to $\overline{XY}$. M is the midpoint of $\overline{XY}$.

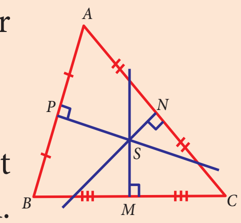
Consider a triangle ABC. It has three sides. For each side you can have a perpendicular bisector as follows:

		
Perpendicular bisector of side BC	Perpendicular bisector of side AC	Perpendicular bisector of side AB
M is the midpoint of $\overline{BC}$ We have a perpendicular at M.	N is the midpoint of $\overline{AC}$ We have a perpendicular at N.	P is the midpoint of $\overline{AB}$ We have a perpendicular at P.

Surprisingly, all the three perpendicular bisectors of the sides of a triangle are concurrent at a point!

Activity

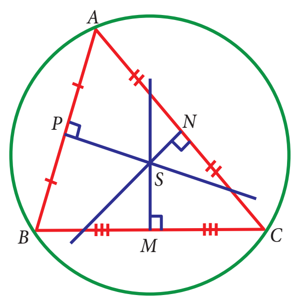
One can visualize the point of concurrence of the perpendicular bisectors, through simple paper folding. Try it!



The perpendicular bisectors of the sides of *any* triangle are concurrent.

As done in the earlier activity on Centroid, you can repeat the experiment for various types of triangle, acute, obtuse, right, isosceles and equilateral. Do you find anything special with the equilateral triangle in this case?

3.4.1 Circumcentre:



The point of concurrence of the three perpendicular bisectors of a triangle is called as its **Circumcentre,** denoted by the letter **S**.

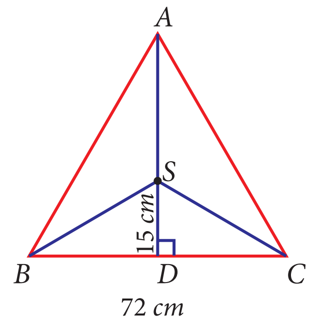
Why should it be called so? Because one can draw a circle exactly passing through the three vertices of the triangle, with centre at the point of concurrence of the perpendicular bisectors of sides. Thus, the circumcentre is equidistant from the vertices of the triangle.

Activity

Check if the following are true by paper-folding:

1. The circumcentre of an acute angled triangle lies in the interior of the triangle.
2. The circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
3. The circumcentre of a right triangle lies at the midpoint of its hypotenuse.

Example 3.6



In ΔABC, S is the circumcentre, BC = 72 cm and DS = 15 cm. Find the radius of its circumcircle.

Solution:

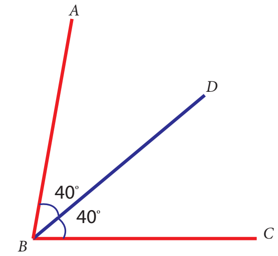
As S is the circumcentre of ΔABC, it is equidistant from A,B and C. So AS=BS=CS=radius of its circumcircle. As AD is the perpendicular bisector of BC, $BD = \frac{1}{2} \times BC = \frac{1}{2} \times 72 = 36\text{cm}$

In right angled triangle BDS, by Pythagoras theorem,

$$BS^2 = BD^2 + SD^2 = 36^2 + 15^2 = 1521 = 39^2 \Rightarrow BS = 39\,\text{cm}.$$

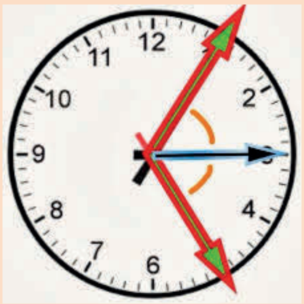
$\therefore$ The radius of the circumcircle of ΔABC is 39 cm.

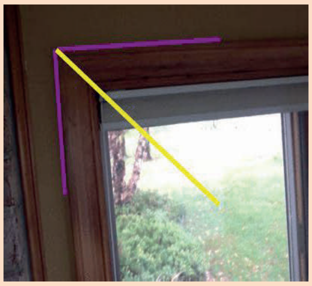
3.5 Angle bisector

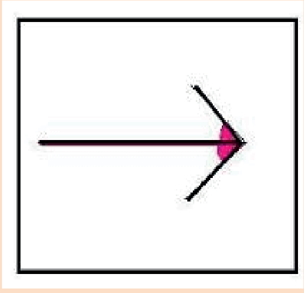


We have learnt about angle bisectors in the previous class. An **angle bisector** is a line or ray that divides an angle into two congruent angles. In the figure, $\angle$ABC is bisected by the line BD such that $\angle$ABD = $\angle$CBD.

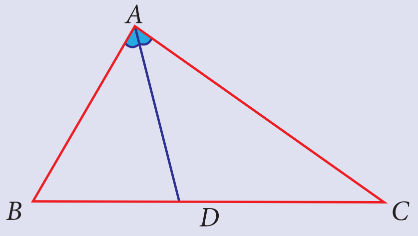
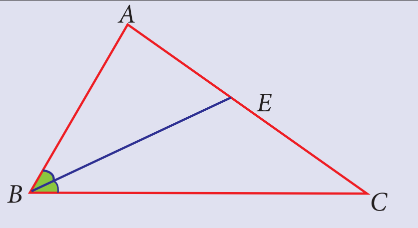
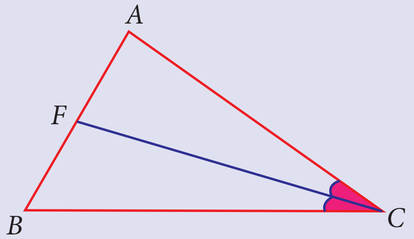
Some examples are shown where one can see angle bisectors in daily life.



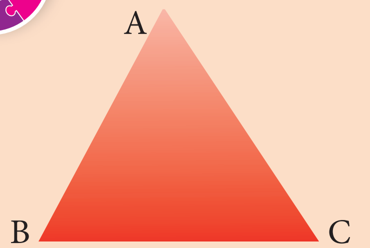
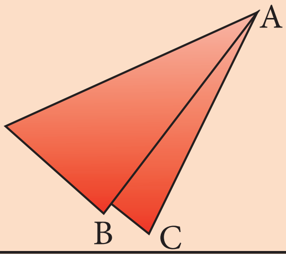
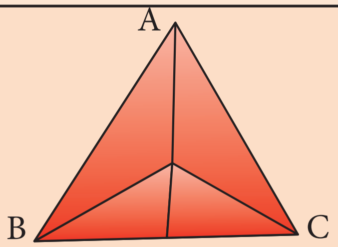




Consider a triangle ABC. How many angles does a triangle have ? 3 angles. For each angle you can have an angle bisector as follows:

		
AD bisects ∠A into two congruent angles. Hence it is an angle bisector of ∠A.	BE bisects ∠B into two congruent angles. Hence it is an angle bisector of ∠B.	CF bisects ∠C into two congruent angles. Hence it is an angle bisector of ∠C.

Activity

		
1. Consider a paper cut-out of a triangle. Name it, say ABC.	2. Fold the triangle so that the opposite sides meet and contain the vertex. Repeat the same to find angle bisectors of other two angles also.	3. Trace all of the folds. Do the angle bisectors pass through the same point?

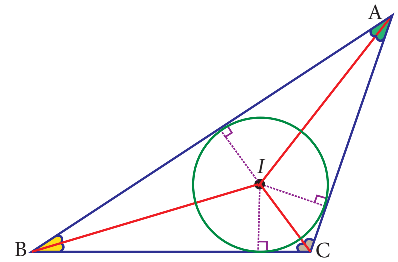
Now you can repeat this activity for an obtuse-angled triangle and a right angled triangle. What is the conclusion?

Do the angle bisectors pass through the same point in all the cases?

The three angle bisectors of any triangle are concurrent.

3.5.1 Incentre:

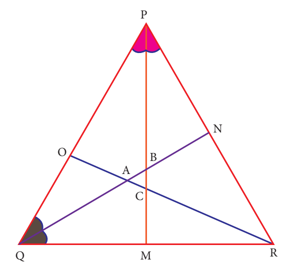
The point of concurrence of the three angle bisectors of a triangle is called as its **incentre,** denoted by the letter **I**.



Why should it be called so? Because one can draw a circle inside of the triangle so that it touches all three sides internally, with centre at the point of concurrence of the angle bisectors. The lengths of a perpendicular line drawn from incentre to each side is found to be same. Thus, the incentre is equidistant from the sides of the triangle.

Example 3.7

Identify the incentre of the triangle PQR.



Solution:

Incentre is the point of intersection of angle bisectors.

Here, PM and QN are angle bisectors of $\angle P$ and $\angle Q$ respectively, intersecting at B.

So, the incentre of the triangle PQR is B.

(Can A and C be the incentre of ΔABC? Why?)

DO YOU KNOW?

The position of the **CENTROID, ORTHOCENTRE, CIRCUMCENTRE** and **INCENTRE** differs depending on the type of triangles given. The following points will help us in locating and remembering these.

(i) For all types of triangles, **CENTROID (G)** and **INCENTRE (I)** will be **inside** the triangle.

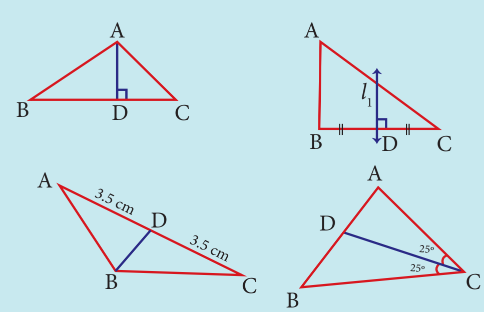
(ii) The **ORTHOCENTRE (H)** will be **inside** in an acute angled triangle, **outside** in an obtuse angled triangle and **on the vertex containing 90°** in a right angled triangle.

(iii) The **CIRCUMCENTRE (S)** will be **inside** in an acute angled triangle, **outside** in an obtuse angled triangle and **on the hypotenuse** in a right-angled triangle.

Try these

Identify the type of segment required in each triangle:

(median, altitude, perpendicular bisector, angle bisector)



(i) AD =__________ (ii) l_1 =__________

(iii) BD =__________ (iv) CD =__________

Think

1. By paper folding, find the centroid, orthocentre, circumcentre and incentre of an equilateral triangle. Do they coinside?

2. By paper folding, find the centroid (G), orthocentre(H), circumcentre (S) and incentre(I) of a triangle. Join G,H,S and I. Are they collinear?

Exercise 3.1

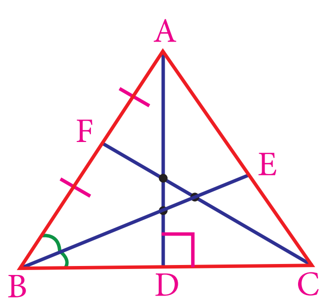
1. **Fill in the blanks:**
   (i) The altitudes of a triangle intersect at ____________.
   (ii) The medians of a triangle cross each other at ____________.
   (iii) The meeting point of the angle bisectors of a triangle is ____________.
   (iv) The perpendicular bisectors of the sides a triangle meet at ________.
   (v) The centroid of a triangle divides each medians in the ratio ____________

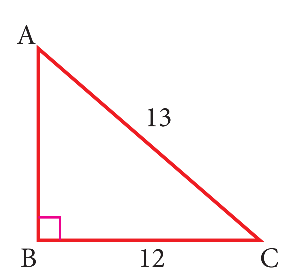
2. **Say True or False:**
   (i) In any triangle the Centroid and the Incentre are located inside the triangle.
   (ii) The centroid, orthocentre, and incentre of a triangle are collinear.
   (iii) The incentre is equidistant from all the vertices of a triangle.

3. a) **Where does the circumcentre lie in the case of**
   (i) An acute-angled triangle.
   (ii) An obtuse-angled triangle.
   (iii) A right angled triangle.

   b) **Where does the orthocentre lie in the case of**
   (i) An acute-angled triangle.
   (ii) An obtuse-angled triangle.
   (iii) A right angled triangle.

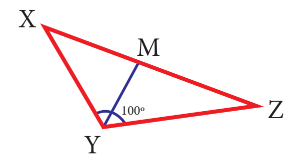
4. Fill in the blanks:
   In the triangle ABC,
   (i) The angle bisector is ________
   (ii) The altitude is ________
   (iii) The median is _______



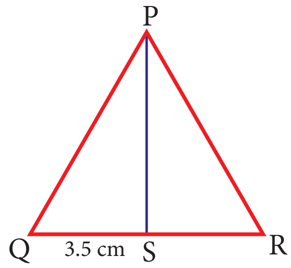


5. In right triangle ABC, what is the length of altitude drawn from the vertex A to BC?

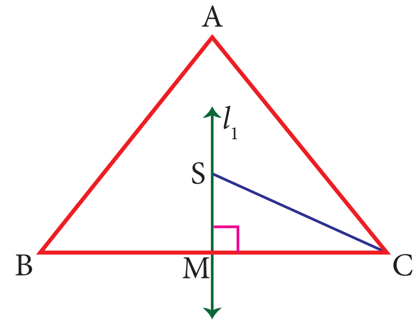
6. In triangle XYZ, YM is the angle bisector of $\angle Y$ and $\angle Y$ is $100°$. Find $\angle XYM$ and $\angle ZYM$.



7. In triangle PQR, PS is a median and QS=3.5 cm, then find QR?



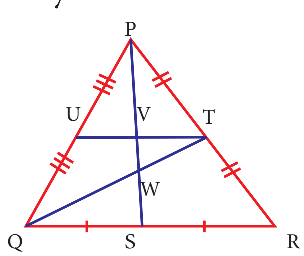
8. In triangle ABC, line l_1 is a perpendicular bisector of BC. If BC=12 cm, SM=8 cm, find CS.



Exercise 3.2

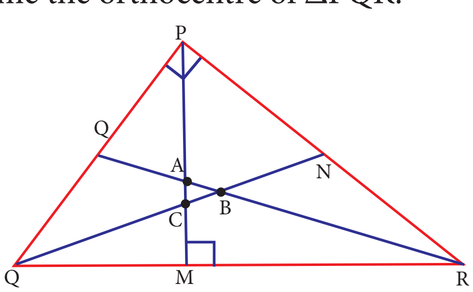
Miscellaneous and Practice Problems

1. Identify the centroid of ΔPQR.

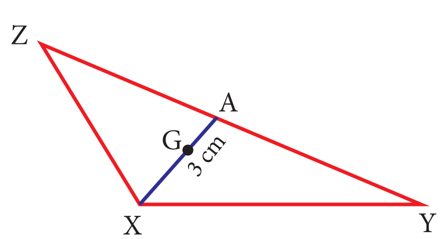


X8V2M5

2. Name the orthocentre of ΔPQR.

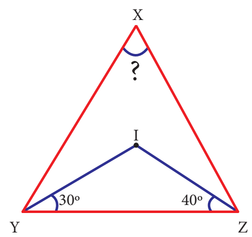


3. In the given figure, A is the midpoint of YZ and G is the centroid of the triangle XYZ. If the length of GA is 3 cm, find XA.

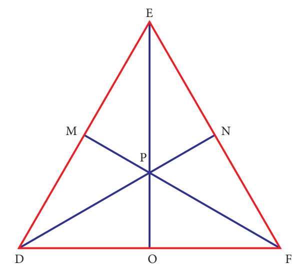


Challengeing Problems

4. Find the length of an altitude on the hypotenuse of a right angled triangle of legs of length 15 feet and 20 feet.
5. If I is the incentre of ΔXYZ, $\angle IYZ = 30^\circ$ and $\angle IZY = 40^\circ$, find $\angle YXZ$.



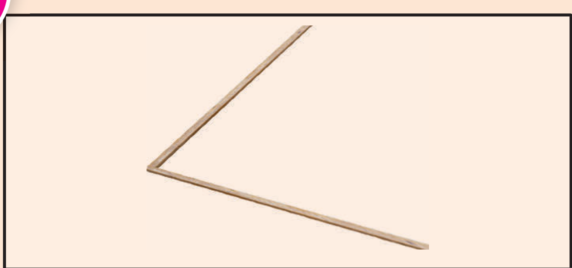
6. In ΔDEF, DN, EO, FM are medians and point P is the centroid. Find the following.
   (i) IF DE = 44, then DM = ?
   (ii) IF PD = 12, then PN = ?
   (iii) If DO = 8, then FD = ?
   (iv) IF OE = 36 then EP = ?



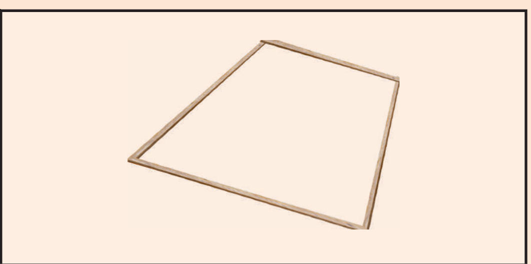
3.6 Construction of certain quadrilaterals

Before we begin to learn constructing certain quadrilaterals, it is essential to recall their basic properties that would help us during the process. We will try to do this by performing some activities and then sum them up.

Activity



1. Place a pair of *unequal* sticks (say pieces of broomstick) such that they have their end points joined at one end.



2. Now place another such pair meeting the free ends of the first pair.

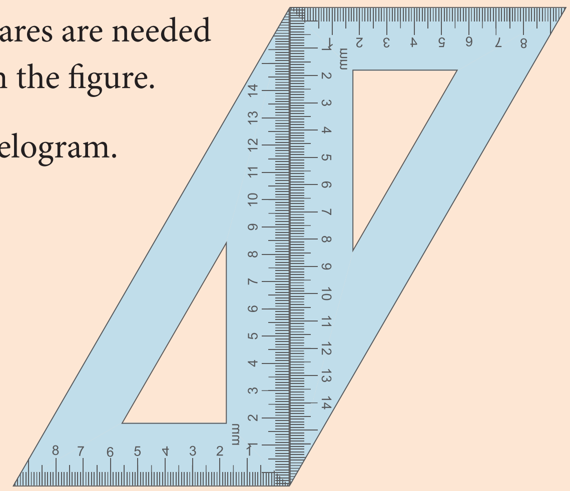
What is the figure enclosed? It is a quadrilateral. Name it as ABCD. How many sides are there? What are its diagonals? Are the diagonals equal? Are the angles equal?

In the above activity can you get a quadrilateral in which

(i) All the four angles are acute.
(ii) One of the angles is obtuse.
(iii) Two of the angles are obtuse.
(iv) One of the angles is a right angle.
(v) Two of the angles are right angles.
(vi) The diagonals are mutually $\perp^r$.

Activity

1. A pair of identical 30º–60º–90º set-squares are needed for this activity. Place them as shown in the figure.



(i) What is the shape we get? It is a parallelogram.

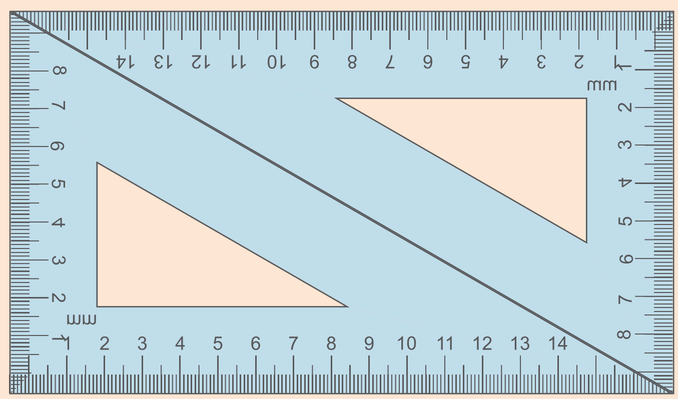
(ii) Are the opposite sides parallel?

(iii) Are the opposite sides equal?

(iv) Are the diagonals equal?

(v) Can you get this shape by using any other pair of identical set-squares?

2. We need a pair of 30º–60º–90º set- squares for this activity. Place them as shown in the figure.



(i) What is the shape we get?

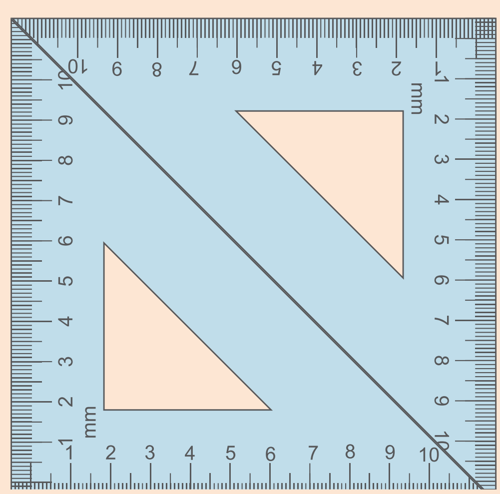
(ii) Is it a parallelogram?

It is a quadrilateral; infact it is a rectangle. (How?)

(iii) What can we say about its lengths of sides, angles and diagonals?

Discuss and list them out.

3. Repeat the above activity, this time with a pair of 45º–45º–90º set-squares.



(i) How does the figure change now? Is it a parallelogram? It becomes a square! (How did it happen?)

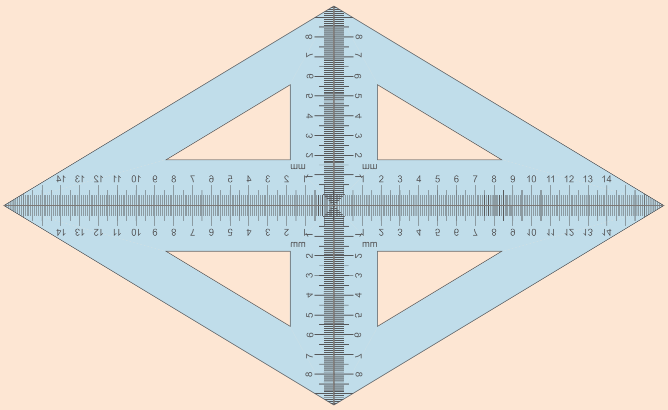
(ii) What can we say about its lengths of sides, angles and diagonals?

Discuss and list them out.

(iii) How does it differ from the list we prepared for the rectangle?

Activity

4. We again use four identical 30°–60°–90° set- squares for this activity. Note carefully how they are placed touching one another.



   (i) Do we get a parallelogram now?

   (ii) What can we say about its lengths of sides, angles and diagonals?

   (iii) What is special about their diagonals?

Based on the outcome of the above activities, we can list out the various properties of the above quadrilaterals, all of which happen to be parallelograms!

Special Quadrilaterals	All sides	All angles	Opposite Sides		All angles	Opposite angles	Diagonals	
	Equal	Equal	Equal	Parallel	90°	Supple-mentary	Bisect each other	Cut at rt.angles
(i) Parallelogram	Some times	Some times	Always	Always	Some times	Some times	Always	Some times
(ii) Rhombus	Always	Some times	Always	Always	Always	Some-times	Always	Always
(iii) Rectangle	Some times	Always	Always	Always	Always	Always	Always	Some times
(iv) Square	Always	Always	Always	Always	Always	Always	Always	Always

Try these

1. Say True or False:

(a) A square is a special rectangle.

(b) A square is a parallelogram.

(c) A square is a special rhombus.

(d) A rectangle is a parallelogram

2. Name the quadrilaterals

(a) which have diagonals bisecting each other.

(b) In which the diagonals are perpendicular bisectors of each other.

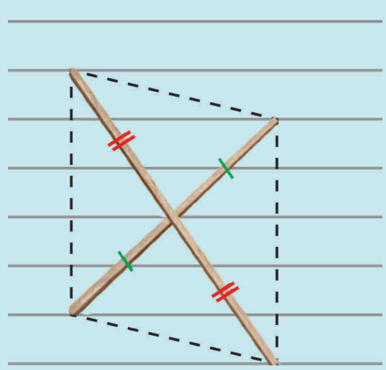
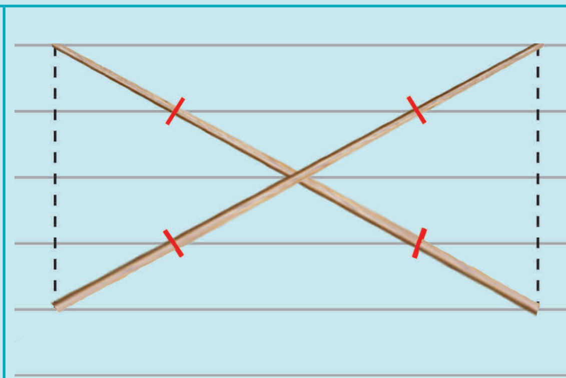
(c) Which have diagonals of different lengths.

(d) Which have equal diagonals.

(e) Which have parallel opposite sides.

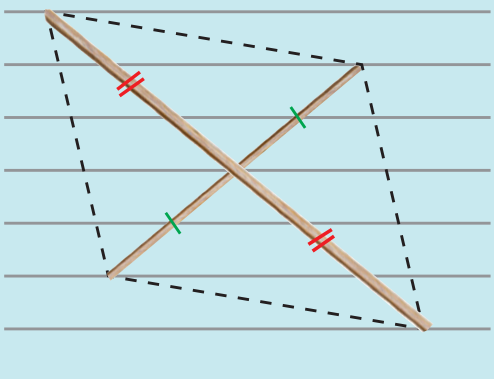
(f) In which opposite angles are equal.

3. Two sticks are placed on a ruled sheet as shown. What figure is formed if the four corners of the sticks are joined?

(a)	 Two unequal sticks. Placed such that their midpoints coincide.	(b)	 Two equal sticks. Placed such that their midpoints coincide.

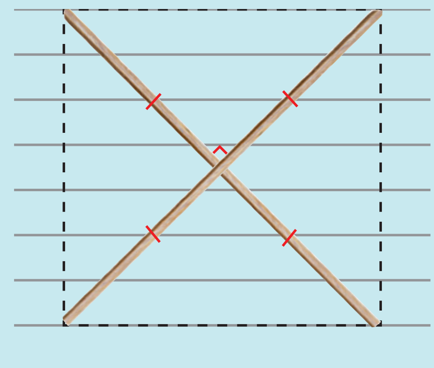
Try these

(c)



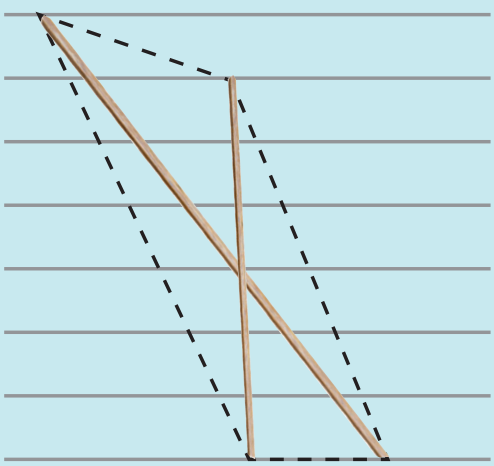
Two unequal sticks. Placed intersecting at mid points perpendicularly.

(d)



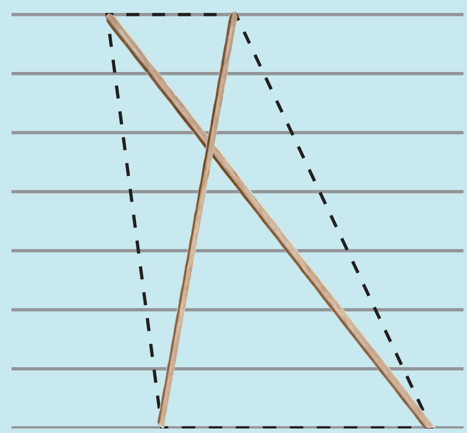
Two equal sticks. Placed intersecting at mid points perpendicularly.

(e)



Two unequal sticks. Tops are not on the same ruling. Bottoms on the same ruling. Not cutting at the mid point of either.

(f)



Two unequal sticks. Tops on the same ruling. Bottoms on the same ruling. Not *necessarily* cutting at the mid point of either.

3.7 Construction of a rhombus

Let us now construct a rhombus with the given measurements

(i) One side and one diagonal

(ii) One side and one angle

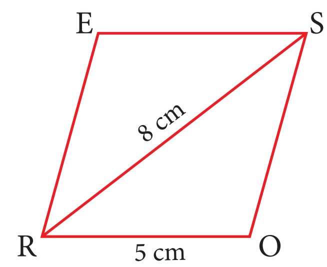
(iii) Two diagonals

(iv) One diagonal and one angle

3.7.1 Construction of a rhombus when one side and one diagonal are given:

Example 3.8

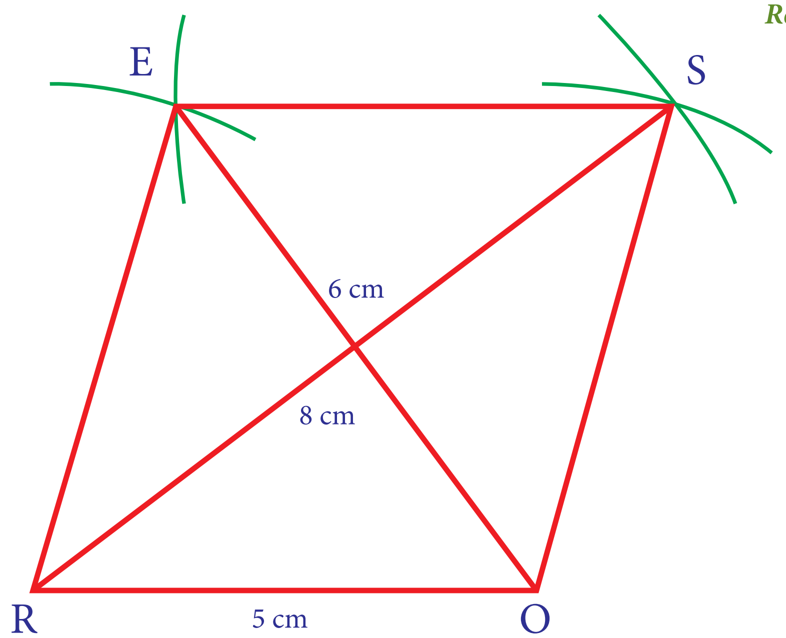
Construct a rhombus ROSE with RO = 5 cm and RS = 8 cm. Also find its area.



Rough diagram

Solution:

Given: RO = 5 cm and RS = 8 cm



Steps:

(i) Draw a line segment RO = 5 cm.

(ii) With R and O as centres, draw arcs of radii 8 cm and 5 cm respectively and let them cut at S.

(iii) Join RS and OS.

(iv) With R and S as centres, draw arcs of radius 5 cm each and let them cut at E.

(v) Join RE and SE.

(vi) ROSE is the required rhombus.

Calculation of area:

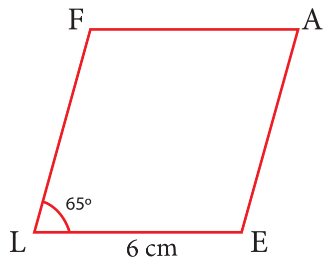
Area of rhombus ROSE $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 8 \times 6 = 24 sq.cm$

3.7.2 Construction of a rhombus when one side and one angle are given:

Example 3.9

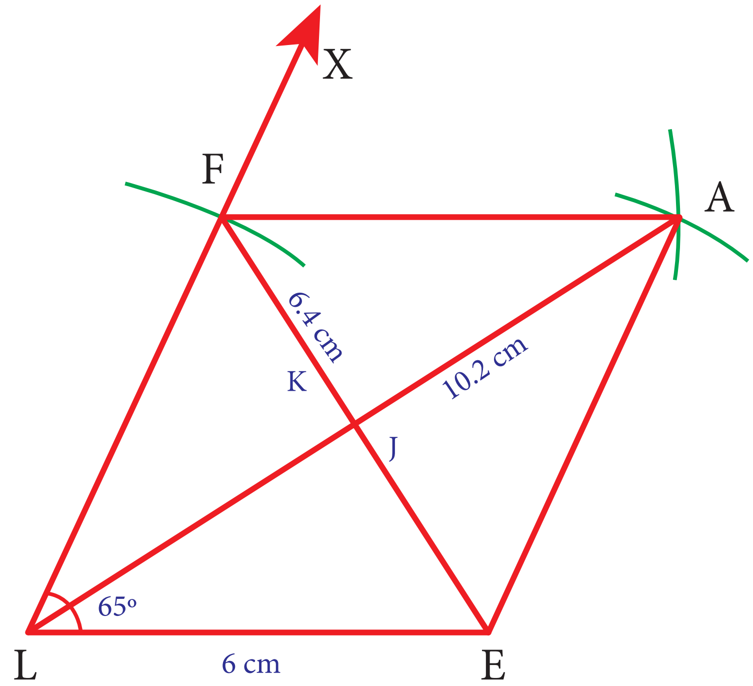
Construct a rhombus LEAF with LE = 6 cm and $\angle L = 65°$. Also find its area.



Rough diagram

Solution:

Given: LE = 6 cm and $\angle L = 65°$



Steps:

(i) Draw a line segment LE = 6 cm.

(ii) At L on LE, make $\angle ELX = 65°$.

(iii) With L as centre draw an arc of radius 6 cm. Let it cut LX at F.

(iv) With E and F as centres, draw arcs of radius 6 cm each and let them cut at A.

(v) Join EA and AF.

(vi) LEAF is the required rhombus.

Calculation of area:

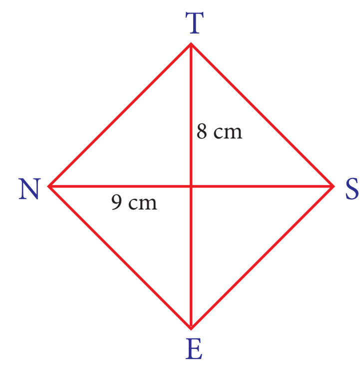
Area of rhombus LEAF $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 6.4 \times 10.2 = 32.64 \; sq.cm$

3.7.3 Construction of a rhombus when two diagonals are given:

Example 3.10

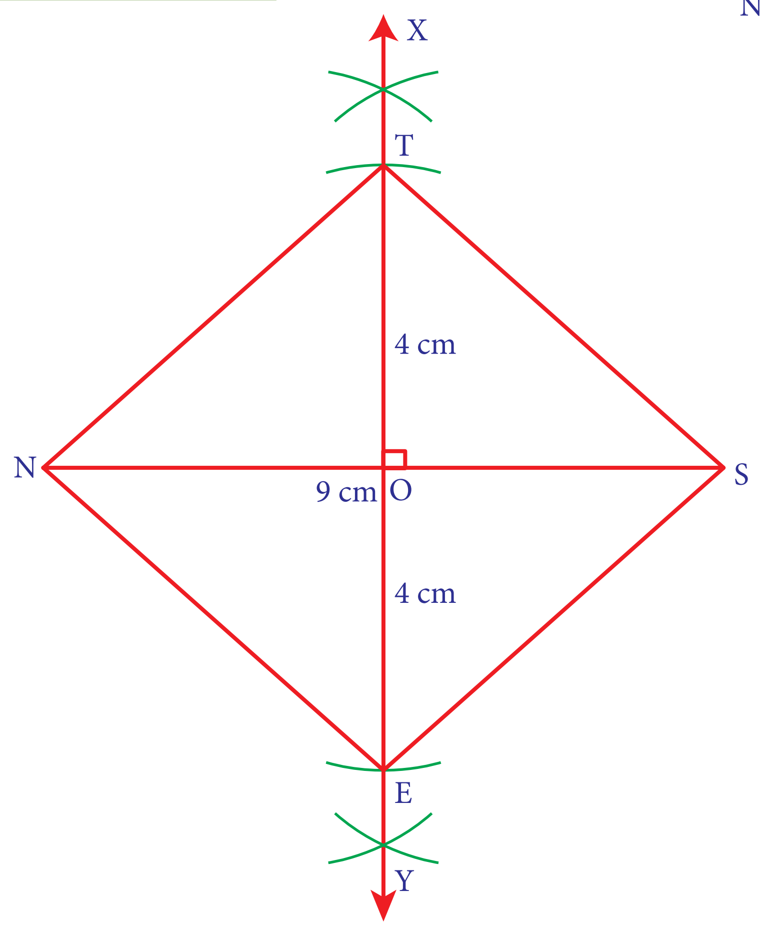
Construct a rhombus NEST with NS = 9 cm and ET = 8 cm. Also find its area.



Rough diagram

Solution:

Given: NS = 9 cm and ET = 8 cm



Steps:

(i) Draw a line segment NS = 9 cm.

(ii) Draw the perpendicular bisector XY to NS. Let it cut NS at O.

(iii) With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at T and OY at E.

(iv) Join NE, ES, ST and TN.

(v) NEST is the required rhombus.

Calculation of area:

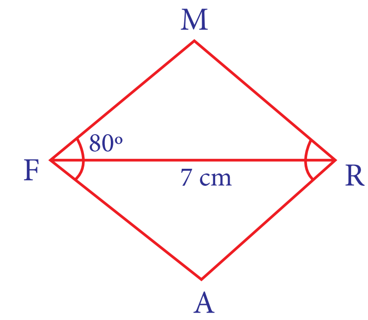
Area of rhombus NEST $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 9 \times 8 = 36 \; sq.cm$

3.7.4 Construction of a rhombus when one diagonal and one angle are given:

Example 3.11

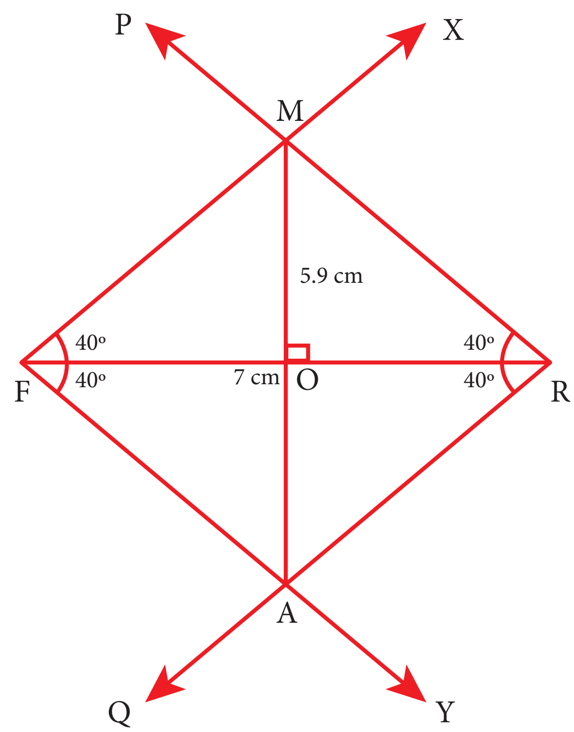
Construct a rhombus FARM with FR = 7 cm and $\angle F = 80^\circ$. Also find its area.



Rough diagram

Solution:

Given: FR = 7 cm and $\angle F = 80^\circ$



Steps:

(i) Draw a line segment FR = 7 cm.

(ii) At F, make $\angle RFX = \angle RFY = 40^\circ$ on either side of FR.

(iii) At R, make $\angle FRP = \angle FRQ = 40^\circ$ on either side of FR.

(iv) Let FX and RP cut at M and FY and RQ cut at A.

(v) FARM is the required rhombus.

Calculation of area:

Area of rhombus FARM $= \frac{1}{2} \times d_1 \times d_2 \; sq.units$

$= \frac{1}{2} \times 7 \times 5.9 = 20.65 \; sq.cm$

3.8 Construction of a rectangle

Let us now construct a rectangle with the given measurements

(i) length and breadth

(ii) a side and a diagonal

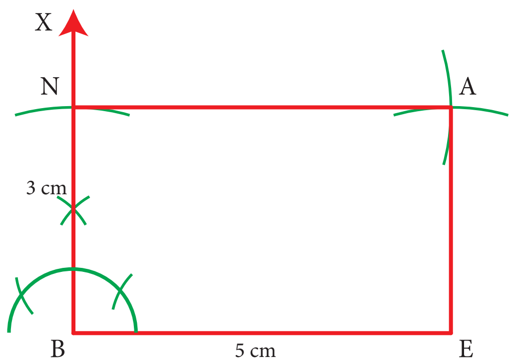
3.8.1 Construction of a rectangle when its length and breadth are given:

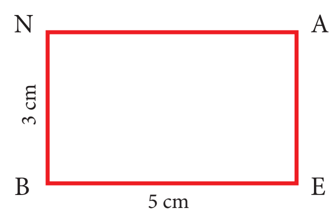
Example 3.12

Construct a rectangle BEAN with BE = 5 cm and BN = 3 cm. Also find its area.

Solution:

Given: BE = 5 cm and BN = 3 cm





Rough diagram

Steps:

(i) Draw a line segment BE = 5 cm.

(ii) At B, construct $BX \perp BE$.

(iii) With B as centre, draw an arc of radius 3 cm and let it cut BX at N.

(iv) With E and N as centres, draw arcs of radii 3 cm and 5 cm respectively and let them cut at A.

(v) Join EA and NA.

(vi) BEAN is the required rectangle.

Calculation of area:

Area of rectangle BEAN $= l \times b\ sq.units$

$= 5 \times 3 = 15\ sq.cm$

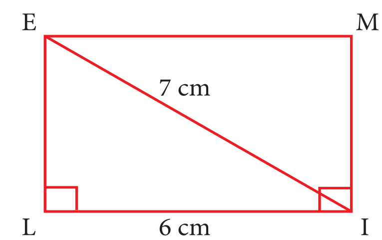
3.8.2 Construction of a rectangle when length and breadth are given:

Example 3.13

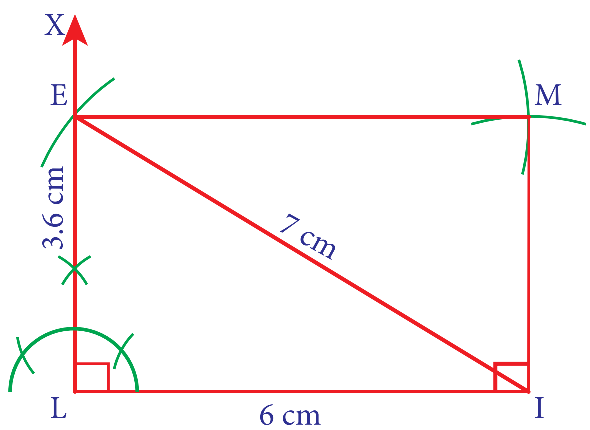
Construct a rectangle LIME with LI = 6 cm and IE = 7 cm. Also find its area.

Solution:

Given: LI = 6 cm and IE = 7 cm



Rough diagram



Steps:

(i) Draw a line segment LI = 6 cm.

(ii) At L, construct $LX \perp LI$.

(iii) With I as centre, draw an arc of radius 7 cm and let it cut LX at E.

(iv) With I as centre and LE as radius draw an arc. Also, with E as centre and LI as radius draw an another arc. Let them cut at M.

(v) Join IM and EM.

(vi) LIME is the required rectangle.

Calculation of area:

Area of rectangle LIME $= l \times b \; sq.units$

$= 6 \times 3.6 = 21.6 \; sq.cm$

3.9 Construction of a square

Let us now construct a square when (i) its side is given and (ii) its diagonal is given

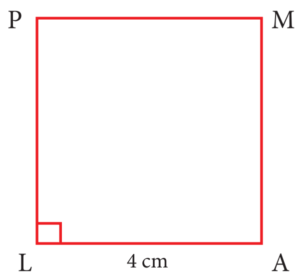
3.9.1 Construction of a square when its side is given:

Example 3.14

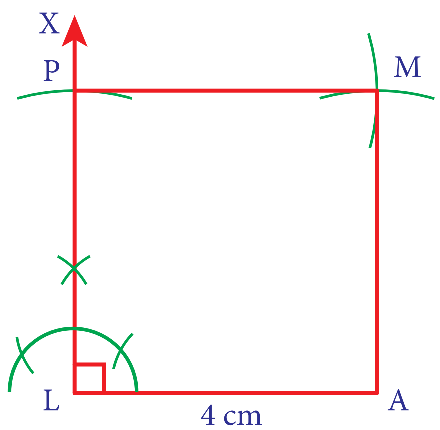
Construct a square LAMP of side 4 cm. Also find its area.

Solution:

Given: side = 4 cm



Rough diagram



Steps:

(i) Draw a line segment LA = 4 cm.

(ii) At L, construct $LX \perp LA$.

(iii) With L as centre, draw an arc of radius 4 cm and let it cut LX at P.

(iv) With A and P as centres, draw arcs of radius 4 cm each and let them cut at M.

(v) Join AM and PM. LAMP is the required square.

Calculation of area:

Area of square LAMP $= a^2$ *sq.units*

$= 4 \times 4 = 16$ *sq.cm*

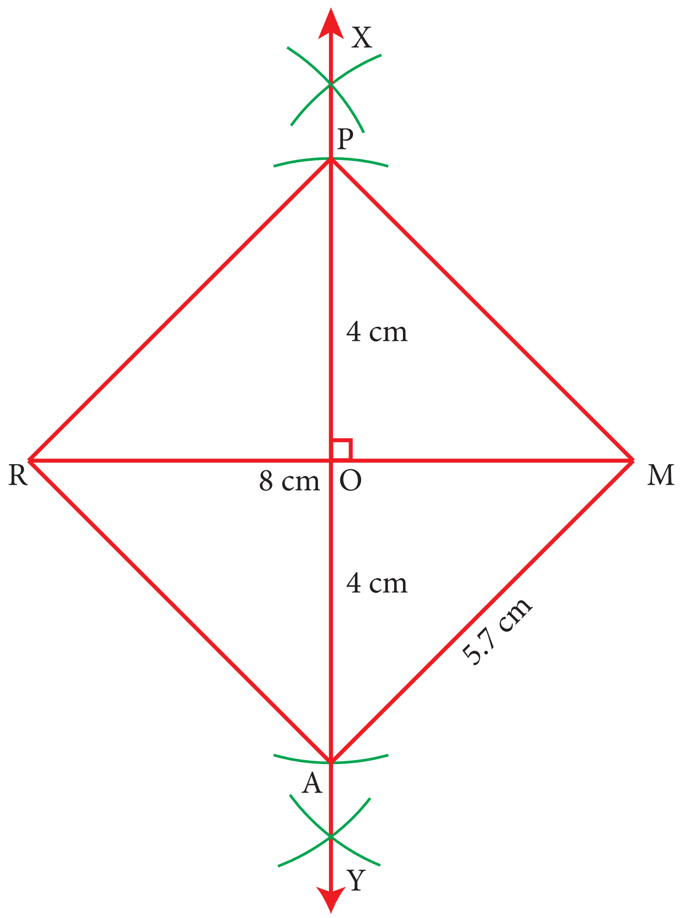
3.9.2 Construction of a square when its diagonal is given:

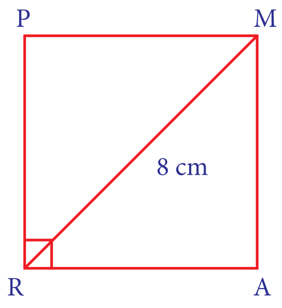
Example 3.15

Construct a square RAMP of a diagonal 8 cm. Also find its area.

Solution:

Given: diagonal = 8 cm





Rough diagram

E5Y1R6

Steps:

(i) Draw a line segment RM = 8 cm.

(ii) Draw the perpendicular bisector XY to RM. Let it bisect RM at O.

(iii) With O as centre, draw arcs of radius 4 cm on either side of O which cut OX at P and OY at A.

(iv) Join RA, AM, MP and PR.

(v) RAMP is the required square.

Calculation of area:

Area of square RAMP $= a^2$ *sq.units*

$= 5.7 \times 5.7 = 32.49\ sq.cm$

Exercise 3.3

I. Construct the following rhombuses with the given measurements and also find their area.

(i) FACE, FA= 6 cm and FC = 8 cm

(ii) RACE, RA= 5.5 cm and AE = 7 cm

(iii) CAKE, CA=5 cm and $\angle A = 65°$

(iv) MAKE, MA= 6.4 cm and $\angle M = 80°$

(v) LUCK, LC = 7.8 cm and UK = 6 cm

(vi) DUCK, DC = 8 cm and UK = 6 cm

(vii) PARK, PR = 9 cm and $\angle P = 70°$

(viii) MARK, AK =7.5 cm and $\angle A = 80°$

II. Construct the following rectangles with the given measurements and also find their area.

(i) HAND, HA = 7 cm and AN = 4 cm

(ii) SAND, SA =5.6 cm and SN = 4.4 cm

(iii) LAND, LA = 8 cm and AD = 10 cm

(iv) BAND, BA = 7.2 cm and BN = 9.7 cm

III. Construct the following squares with the given measurements and also find their area.

(i) EAST, EA = 6.5 cm

(ii) WEST, ST = 6 cm

(iii) BEST, BS = 7.5 cm

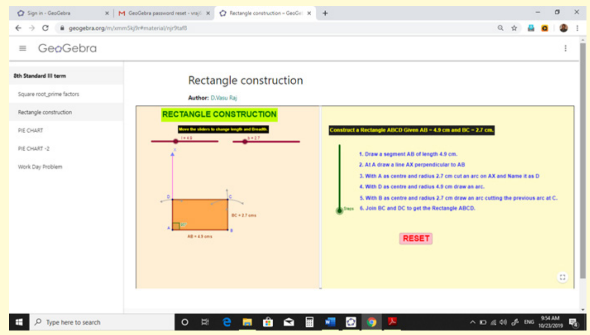
(iv) REST, ET = 8 cm

Summary

- Three or more lines in a plane are said to be *concurrent*, if all of them pass through the same point.
- The three medians of any triangle are concurrent. The point of concurrence of the three medians in a triangle is called its Centroid, denoted by the letter G.
- The three altitudes of any triangle are concurrent. The point of concurrence of the three altitudes of a triangle is called as its Orthocentre, denoted by the letter H.
- The three perpendicular bisectors of the sides of any triangle are concurrent. The point of concurrence of the three perpendicular bisectors of a triangle is called as its Circumcentre, denoted by the letter S.
- The three angle bisectors of any triangle are concurrent. The point of concurrence of the three angle bisectors of a triangle is called as its Incentre, denoted by the letter I.
- Rhombus is a parallelogram in which all its sides are congruent.
- Rectangle is a parallelogram whose all its angles are right angles.
- Square is a parallelogram in which all its sides and angles are equal.

ICT CORNER

Expected Outcome

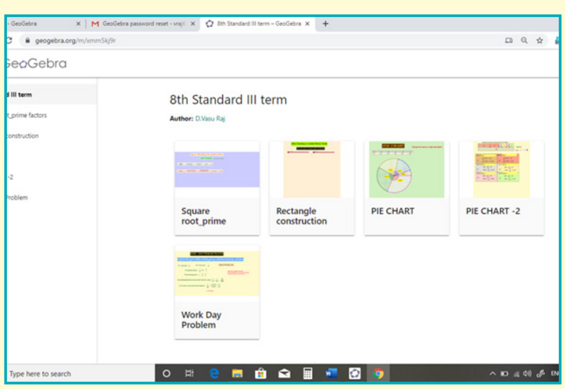


Step – 1

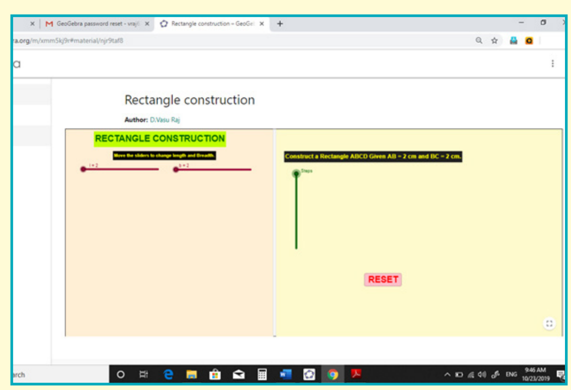
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named “8th Standard III term” will open. Select the work sheet named “Rectangle Construction”

Step - 2

Move the sliders on left side to change the length and breadth of the rectangle. Drag the slider step by step on right side to see the steps for construction.



Step – 1



Step – 2

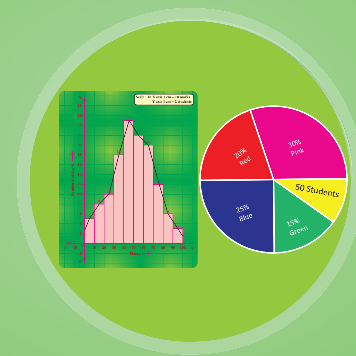
Browse in the link

GEOMETRY:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

B358_8_MATHS_EM

4 STATISTICS



Learning Objectives

B1K5L9

- To recall the formation of frequency tables.
- To construct simple Pie-charts for the given data.
- To know how to draw Histogram and Frequency Polygon for grouped data.

4.1 Introduction

Before we learn on Pie charts, Histograms and Frequency Polygons, let us recall what we have studied in the previous classes like data (primary and secondary) and frequency tables for ungrouped data.

Kamaraj! Go and collect II-term Math marks of all the students from our class.
Geetha! You go and note down the heights of all the students from the cumulative record.
Students, here the marks collected by Kamaraj and heights noted by Geetha are called 'Data'.

4.1.1 Data:

Data is the basic unit in Statistics. Data is a collection of facts such as numbers, words, measurements and observations. It must be organised, for it to be useful and to get information. Data can be collected in many ways. Among all the ways, direct observation is one of the simplest way to collect the data.

For example, if you want to find the number of types of houses in a village, what do you do? You can count the types of houses in the village, in person similarly,

(i) Collection of brand wise motorcycles in your place.
Brand A – 25, Brand B – 40, Brand, C –14 and Brand D – 37

(ii) Collection of term marks in Mathematics of your class mates.
39, 20, 19, 47, 50, 26, 35, 40, 17, 25, 41.

(iii) Number of students playing different sports from your class.
Volley ball –12 Kabaddi – 10 Hockey – 9 Cricket – 7 Badminton – 7

(iv) Staff's age in a company
27, 51, 19, 21, 46, 35, 52, 25, 57, 29.

The above facts are some more examples for data,

Students, let us now see the kinds of data. There are two kinds of data namely primary data and secondary data.

Primary data:

These are the data that are collected in person for the first time for a specific purpose. Here, Kamaraj has collected the data of math marks from the students in person. It is called primary data.

Also, (i) Census in a village

(ii) Collection of colours which the students like in a class are some examples of primary data.

Secondary data:

These are the data that are sourced from some places that has originally collected it. This kind of data has already been collected by some other persons. The statistical operation may have been performed on them already. Here, Geetha also collected the data but she took it from a record which had already collected them. This is called secondary data.

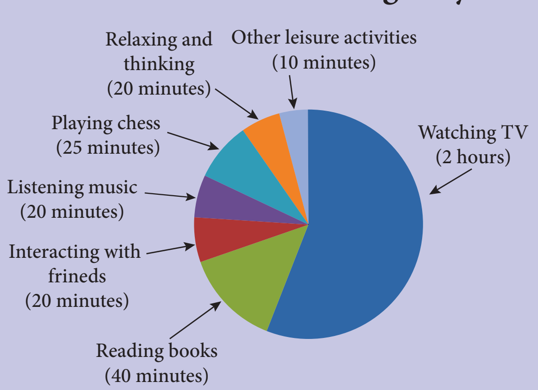
Also, (i) The details of 'PATTA' for a land can be had from the registration office.

(ii) Birth–Death details data can be got from concern office are some examples of secondary data.

From these primary and secondary data , we can't get any specific or required information directly like, how many students have got more than 50 marks? how many students got marks between 30 and 40? how many of them are with height 125 cm? If we need answer for these questions, we have to tabulate the data.

MATHEMATICS ALIVE – STATISTICS IN OUR REAL LIFE

Leisure time on an average day



Pie charts are used in our daily life activities

Stock Market Return Distribution

Histograms are used in stock market to represent the returns of stock buyer.

4.2 Data in Tabular form

To make the given data easily understandable, we tabulate the data in the form of tables or charts. A table has three columns that contains

(i) Variable /Class (ii) Tally Marks (iii) Frequency

Variable / Class:

Arrange the given data from the lowest to the highest in the first column under the heading variable or class.

Tally Marks:

A vertical line(|) which is marked against each item falling in the variable /class is called tally marks.

Frequency:

The number of times an observation occurs in the given data is called the frequency of the observation. This is easily counted from the tally marks column.

For example:

Variable (Marks)	Tally marks	Frequency (f) (no.of students)
10	|||	3
14	卌	5
17	卌 |||	8
20	||||	4
	Total	20

Note

Range: The difference between the largest and the smallest values of the data given. If 5, 15, 10, 7, 20, 18 are the data then, Range = 20–5=15

From the table, we understand that three students got 10 marks, five students got 14 marks and so on.

Ungrouped data or Discrete Data:

An ungrouped data can assume only whole numbers and exact measurement. These are the data that cannot have a range of values. A usual way to represent this is by using Bar graphs

Examples: 1. The number of teachers in a school.

2. The number of players in a game.

Grouped data or Continuous Data:

A grouped data is any value within a certain interval. The data can take values between certain range with the highest and the lowest value. Continuous data can be tabulated in what is called as frequency distribution. They can be graphically represented using Histograms.

Example: 1. The age of persons in a village.

2. The height and the weight of the students of your class.

4.3 Frequency distribution table

Frequency distribution:

A frequency distribution is the arrangement of the given data in the form of the table showing frequency with which each variable occurs.

If we have more number of students in the class , it would be very difficult to understand and to get information unless it is organised. For this reason, we organise larger data into a table called the frequency distribution table. Therefore, the tabular arrangement which shows the observations and their frequency of occurrences is called the frequency distribution table. There are two types of distribution table namely

(i) frequency distribution table for ungrouped data and

(ii) frequency distribution table for grouped data.

Try these

1. Arrange the given data in ascending and descending order:
   9,34,4,13,42,10,25,7,31,4,40
2. Find the range of the given data : 53, 42, 61, 9, 39, 63, 14, 20, 06, 26, 31, 4, 57

4.3.1 Construction of frequency distribution table for ungrouped data.

Example 4.1

Form an ungrouped frequency distribution table for the weight of 25 students in STD IV given below and answer the following questions.

25, 24, 20, 25, 16, 15, 18, 20, 25, 16, 20, 16, 15, 18, 25, 16, 24, 18, 25, 15, 27, 20, 20, 27, 25.

(i) Find the range of the weights.

(ii) How many of the students has the highest weight in the class?

(iii) What is the weight to which more number of students belong to?

(iv) How many of them belong to the least weight?

Solution:

To form a distribution table, arrange the given data in ascending order under Weight column then, put a vertical mark against each variable under Tally marks column and count the number of tally marks against the variable and enter it in Frequency column as given below. Hence, the distribution table is

<table>
<tr><th>Weight</th><th>Tally Marks</th><th>Frequency</th></tr>
<tr><td>15</td><td>|||</td><td>3</td></tr>
<tr><td>16</td><td>||||</td><td>4</td></tr>
<tr><td>18</td><td>|||</td><td>3</td></tr>
<tr><td>20</td><td>~~||||~~</td><td>5</td></tr>
<tr><td>24</td><td>||</td><td>2</td></tr>
<tr><td>25</td><td>~~||||~~ |</td><td>6</td></tr>
<tr><td>27</td><td>||</td><td>2</td></tr>
<tr><td></td><td>Total</td><td>25</td></tr>
</table>

Thus, we can tabulate the above table as follows.

Weight (kg)	15	16	18	20	24	25	27
Frequency	3	4	3	5	2	6	2

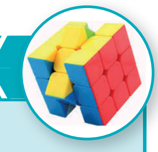
(i) The range of the given data is the difference between the largest and the smallest value. Here, the range = 27−15 =12**.**

(ii) From this table, two of the students have the highest weight of 27 kg.

(iii) 6 students belong to 25 kg weight.

(iv) 3 students belong to the least weight of 15 kg.

So, when we tabulate the given data, it is easy to get the information at a glance, Isn't it?



Try these

Collect the blood group of your classmates. Complete the table and analyse.

Blood group	Tally marks	No. of students
A+		
B+		
AB+		
O+		
A-		
B-		
AB-		
O-		

4.3.2 Construction of frequency distribution table for grouped data:

Now, we will consider a situation, if we collect data of marks for 50 students, it becomes very difficult to put tally for each and every marks of all the 50 students. Because if we arrange the marks in a table, it will be very large in length and not understandable at once. In this case, we use class intervals. In this table, consider the groups of data in the form of class intervals to tally the frequency for the given data.

Class Interval:

The range of the variable is grouped into number of classes, and each group is known as class interval (C.I). The difference between the upper limit (U) and the lower limit (L) of the class is known as class size.

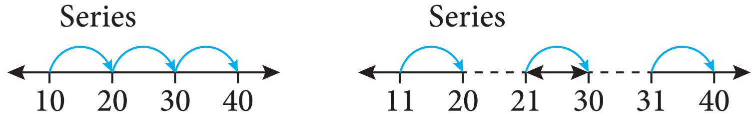
i.e. C.I = Upper limit – Lower limit

For example,

Marks for the C.I 10 to 20 can be written as 10-20, whose class size is 20–10=10

Class interval

(a) Continuous Series | (b) Discontinuous Series



(e.g.)	(e.g.)
10 - 20	11 - 20 gap
20 - 30	21 - 30
30 - 40	31 - 40
40 - 50	41 - 50

(a) While distributing the frequency, we follow the counting as given below. Suppose the classes are 10-20, 20-30, 30-40, 40-50 ..... This represent a continuous series. Here, 20 is included in the class 20-30 and 30 is included in 30-40, likewise for the other classes also.

(b) In case the given series has a gap between the limits of any two adjacent classes, this gap may be filled up by extending the two limits of each class by taking half of the value of the gap. Half of the gap is called the adjustment factor.

Conversion of a discontinuous series into continuous series:

In case the given series is a discontinuous, we can make it as continuous as follows,

Illustration: 1

11 - 20 gap

21 - 30

31 - 40

41 - 50

difference in the gap = 21 – 20

= 1

Lower boundary = lower limit –half of the gap

$$= 11 - \frac{1}{2}(1)$$

= 11 – 0.5 = 10.5

Upper boundary = upper limit + half of the gap

$$= 20 + \frac{1}{2}(1)$$

= 20 + 0.5

= 20.5 and so on for other classes too.

Think

How will you change the given series as continuous series

15-25

28-38

41-51

54-64

Therefore, the class interval can be changed into a continuous one as given in the following table,

Discontinuous series	Continuous series
-0.5 +0.5 11-20	10.5-20.5
21-30	20.5-30.5
31-40	30.5-40.5
41-50	40.5-50.5

Illustration: 2

How can you make it continuous, if the classes are

0-10

12-22 gap

24-34

36-46

48-58

difference in the gap = 24–22

= 2

Take half of the gap.

So, 1 is the adjustment factor.

Hence, subtract 1 from the lower limit and add 1 to the upper limit to make it as a continuous series as given below

Discontinuous series	Continuous series
-1 +1 0-10	-1-11
12-22	11-23
24-34	23-35
36-46	35-47
48-58	47-59

(i) Construction of grouped frequency distribution table – Continuous series .

Example 4.2

The EB bill(in ₹) of each of the 26 houses in a village are given below. Construct the frequency table.

215	200	120	350	800	600	350	400	180	210	170	305	204
220	425	540	315	640	700	790	340	586	660	785	290	300

Solution:

Maximum bill amount = ₹ 800

Minimum bill amount = ₹ 120

Range = maximum value – minimum value

Range = 800 – 120 = ₹ 680

Suppose if we want to take class size as 100, then

$$\text{the number of possible class intervals} = \frac{Range}{Class\ size}$$

$$= \frac{680}{100} = 6.8 \simeq 7$$

Class Intervals	Tally Marks	Frequency
100-200	$\vert\vert\vert$	3
200-300	$\cancel{\vert\vert\vert\vert}\ \vert$	6
300-400	$\cancel{\vert\vert\vert\vert}\ \vert$	6
400-500	$\vert\vert$	2
500-600	$\vert\vert$	2
600-700	$\vert\vert\vert$	3
700-800	$\vert\vert\vert\vert$	4
	Total	26

Note

Generally we may take class size in multiples of 10 or multiples of 5.

Think

If we want to represent the given data by 5 classes, then how shall we find the interval?

Activity-1

Observe the last alphabet in the names of your classmates, tabulate them and answer the following questions.

Alphabet	Tally marks	No. of students (f)

1. In which letter do the names end the most?
2. In which letter do the names end the least?
3. What are the letters in which the names do not end with?
4. Girl names mostly end with -----------letter(s).
5. Boy names mostly end with ---------letter(s).

Note

Inclusive series:

In the class-intervals, if the upper limit and lower limit are included in that class interval then it is called inclusive series. For example, 11-20, 21-30 , 31-40, 41-50 etc is an inclusive series.

Here, the data 11 and 20 are included in the class (11-20) and so on. Clearly, it is a discontinuous series.

Exclusive series:

In the class intervals, if the upper limit of one class interval is the lower limit of the next class interval then it is called exclusive series. For example, 10-15, 15-20, 20-25, 25-30 etc., is an exclusive series.

Here, 15 is included in the class 15-20 and 20 is included in 20-30. Clearly, it is a continuous series.

(ii) Construction of grouped frequency distribution table - Discontinuous series.

Example 4.3

Construct a continuous series frequency distribution table.

Class	0-5	6-11	12-17	18-23	24-29
Frequency(f)	7	10	9	5	12

Solution:

As told above, first we should fill the gap by extending the two limits of each class by half of the value of the gap. Here the gap is 1, so subtracting and adding half of the gap i.e 0.5 to the lower and the upper limit of each class makes it as a continuous series.

Class	-0.5-5.5	5.5-11.5	11.5-17.5	17.5-23.5	23.5-29.5
Frequency(f)	7	10	9	5	12

Try these

1. Prepare a frequency table for the data :
   3, 4, 2, 4, 5, 6, 1, 3, 2, 1, 5, 3, 6, 2, 1, 3, 2, 4
2. Prepare a grouped frequency table for the data :
   10, 9, 3, 29, 17, 34, 23, 20, 39, 42, 5, 12, 19, 47, 18, 19, 27, 7, 13, 40, 38, 24, 34, 15, 40

4.4 Graphical representation of the frequency distribution for ungrouped data

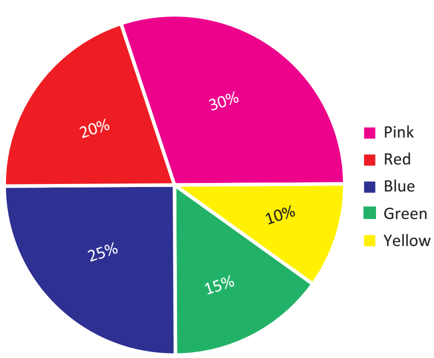
A graphical representation is the geometrical image of a set of data. It is a mathematical picture. It enables us to think about a statistical problem in visual terms. A picture is said to be more effective than words for describing a particular thing. The graphical representation of data is more effective for understanding. In the previous classes, we have studied some graphical representations of ungrouped data such as Line graph, Bar graph, and Pictograph. Now, we are going to represent the given ungrouped data in the circular form namely the pie diagram or the pie chart.

4.5 Pie chart (or) Pie diagram

A pie chart is a circular graph which shows the total value with its components. The area of a circle represents the total value and the different sectors of the circle represent the different components. The circle is divided into sectors and the area of the sectors is proportional to the information given. In the 'pie chart' the data are mostly expressed in percentage. Each component is expressed as percentage of the total value.

The Pie diagram is so called because the entire graph looks like an American food 'pie' and the components resemble slices cut from 'pie'.

Example



American food 'pie'

4.5.1 Method of constructing a pie chart:

In a pie chart, we know that the various components are represented by the sectors of a circle and the whole circle represents the sum of the value of all the components. Therefore, the total angle of 360° at the centre of the circle is divided into different sectors according to the value of the components.

$$\textbf{The central angle of a component} = \frac{\text{value of the component}}{\text{total value}} \times 360^\circ$$

Sometimes, the value of the components are expressed in percentage. In such cases,

$$\textbf{The central angle of a component} = \frac{\textit{percentage value of the component}}{100} \times 360^\circ$$

Steps for construction of the pie chart:

1) Calculate the central angle for each component using the above formula and tabulate it.
2) Draw a circle of convenient radius and mark one horizontal radius in it.
3) Draw radius making central angle of first component with horizontal radius. This sector represents the first component. From this radius, draw next radius with central angle of the second component and so on, until the completion of all components.
4) For identification of each sector, shade with different colours.
5) Label each sector.

Here are given some examples, let us draw the pie chart for the given data.

Example 4.4

The number of hours spent by a school student on various activities on a working day is given below. Construct a pie chart.

Activity	Sleep	School	Play	Home work	Other
No of hours	8	6	2	3	5

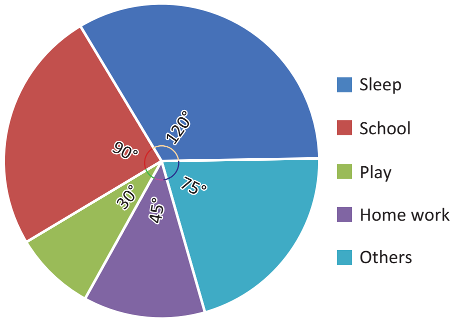
1. Find the percentage of sleeping hours.
2. By what angle is home work more than play?
3. By what angle are other activities less than sleep?

Solution:

Number of hours spent in different activities in a day of 24 hours are converted into components parts of 360°.

Activity	Duration in hours	Central angle
Sleep	8	$\frac{8}{24} \times 360^\circ = 120^\circ$
School	6	$\frac{6}{24} \times 360^\circ = 90^\circ$
Play	2	$\frac{2}{24} \times 360^\circ = 30^\circ$
Home work	3	$\frac{3}{24} \times 360^\circ = 45^\circ$
Others	5	$\frac{5}{24} \times 360^\circ = 75^\circ$
Total	24	360°

The time spent by a school student during a day (24 hours)



1. The percentage of sleeping hours = $\frac{8}{24} \times 100 = 33.33\%$
2. Home work is $45°–30°=15°$ more than play
3. Other activities are $120°–75°=45°$ less than sleep.

Example 4.5

Draw a pie diagram to represent the following data, which shows the expenditure of paddy cultivation in 2 acres of land.

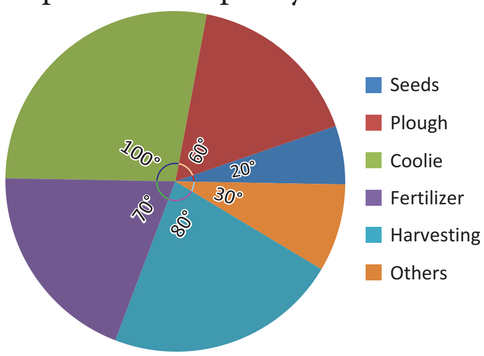
Particulars	Seeds	Ploughing	Wages	Fertilizer	Harvest	Others
Expenses (₹)	2000	6000	10000	7000	8000	3000

Also, 1. Find the percentage of the head in which he had spent more?
2. What percentage of money was spent for seeds?

Solution:

Particulars	**Expenses**	**Central angle**
Seeds	2000	$\frac{2000}{36000} \times 360° = 20°$
Plough	6000	$\frac{6000}{36000} \times 360° = 60°$
Coolie	10000	$\frac{10000}{36000} \times 360° = 100°$
Fertilizer	7000	$\frac{7000}{36000} \times 360° = 70°$
Harvesting	8000	$\frac{8000}{36000} \times 360° = 80°$
Others	3000	$\frac{3000}{36000} \times 360° = 30°$
Total	36000	360°

Expenditure of paddy cultivation in 2 acres.



1. He spent more for wages `10,000. Converting into percentage, We have

$$\text{Wages } = \frac{10000}{36000} \times 100\% = 27.7\%$$

2. He spent ₹2000 for seeds. Converting into percentage, We have,

$$\text{Seeds } = \frac{2000}{36000} \times 100\% = 5.55\%$$

Example 4.6

Draw a suitable pie chart for the following data relating to the cost of construction of a house.

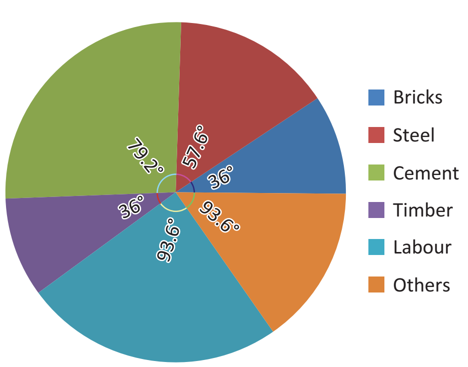
Particulars	Bricks	Steel	Cement	Timber	Labour	Others
Expenses:	10%	16%	22%	10%	26%	16%

Also, find how much was spent on labour if ₹55000 was spent for cement.

Solution:

Particulars	**Expenses**	**Central angle**
Bricks	10 %	$\frac{10}{100} \times 360° = 36°$
Steel	16 %	$\frac{16}{100} \times 360° = 57.6°$
Cement	22 %	$\frac{22}{100} \times 360° = 79.2°$
Timber	10 %	$\frac{10}{100} \times 360° = 36°$
Labour	26 %	$\frac{26}{100} \times 360° = 93.6°$
Others	16 %	$\frac{16}{100} \times 360° = 57.6°$
	Total	360°

Cost of construction of a house.



If the expenses on cement is ₹ 55000 then, it represents 22 % and he spent 26 % on labour

$$\text{Therefore, the expense on Labour} = \frac{26}{22} \times 55000$$

%	Expenses
22	55000
26	?
Direct proportion	

$= ₹\ 65{,}000$

Note

Uses of pie chart:

1. Pie charts are widely used by the business and the media people.
2. With the help of Pie charts, one can show how the expenditure of the Government or Industry is distributed over different heads.
3. Research people use these type of charts to show their results.

Merits and Demerits:

Merits:

1. Simple to create.
2. Pie charts are visually simple than other types of graphs.
3. Pie charts are easy to understand information quickly.

Demerits:

1. Pie charts are inconvenient for comparing more than one sample.
2. Separate Pie charts have to be used for different samples.
3. It becomes less effective if there are more components in the data.

Exercise 4.1

1. Fill in the blanks

(i) Data has already been collected by some other person is _____________ data.

(ii) The upper limit of the class interval (25-35) is _____________.

(iii) The range of the data 200, 15, 20, 103, 3, 197, is _____________.

(iv) If a class size is 10 and range is 80 then the number of classes are _________.

(v) Pie chart is a __________ graph.

2. Say True or False

(i) Inclusive series is a continuous series.

(ii) Pie charts are easy to understand.

(iii) Same pie chart can be used for different samples.

(iv) Media and business people use pie charts.

(v) A pie diagram is a circle broken down into component sectors.

J5T9E2

3. The continuous series of

10 -20	
24-34	
38-48	
52-62	

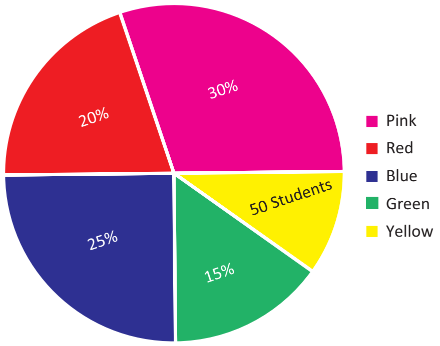
4. Represent the following data in ungrouped frequency table which gives the number of children in 25 families.

1, 3, 0, 2, 5, 2, 3, 4, 1, 0, 5, 4, 3, 1, 3, 2, 5, 2, 1, 1, 2, 6, 2, 1, 4

5. Form a continuous frequency distribution table for the marks obtained by 30 students in a X std public examination.

328, 470, 405, 375, 298, 326, 276, 362, 410, 255, 391, 370, 455, 229, 300, 183, 283, 366, 400, 495, 215, 157, 374, 306, 280, 409, 321, 269, 398, 200.

6. A paint company asked a group of students about their favourite colours and made a pie chart of their findings. Use the information to answer the following questions.



(i) What percentage of the students like red colour?

(ii) How many students liked green colour?

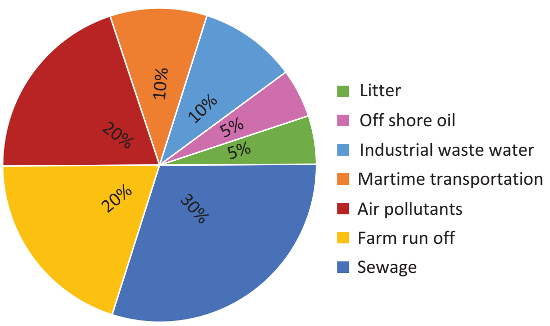
(iii) What fraction of the students liked blue?

(iv) How many students did not like red colour?

(v) How many students liked pink or blue?

(vi) How many students were asked about their favourite colours?

7. Write any five points from the given pie chart information regarding pollutants entering in the oceans.



8. A survey gives the following information of food items preferred by people. Draw a Pie chart.

Items	Vegetables	Meat	Salad	Fruits	Sprouts	Bread
No.of people	160	90	80	50	30	40

9. Draw a pie chart for the following information.

Ocean	Pacific	Atlantic	Indian	Arctic	Antarctic
Water	46 %	24 %	20 %	4 %	6 %

10. Income from various sources for Government of India from a rupee is given below. Draw a pie chart.

Source	Corporation tax	Income tax	Customs	Excise duties	Service Tax	Others
Income (in paise)	19	16	9	14	10	32

11. Monthly expenditure of Kumaran's family is given below. Draw a suitable Pie chart.

Particulars	Food	Education	Rent	Transport	Miscellaneous
Expenses (in %)	50 %	20 %	15 %	5 %	10 %

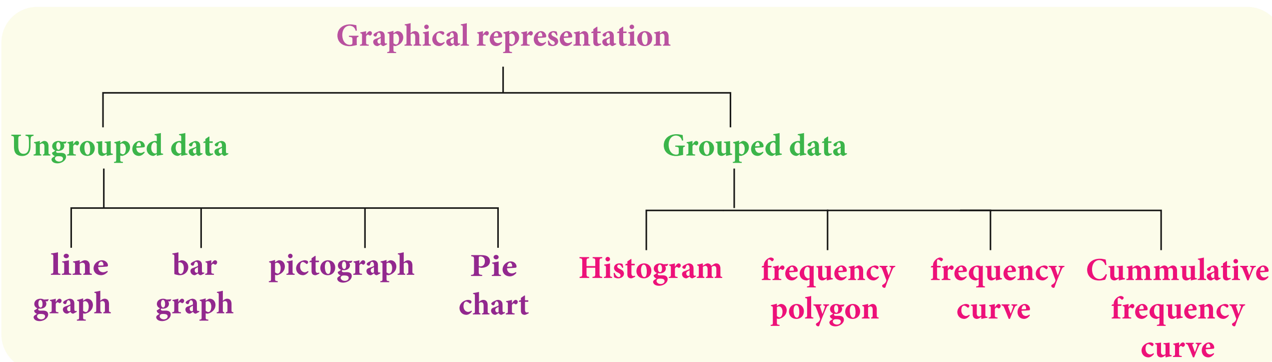
Also

1. Find the amount spent for education if Kumaran spends ₹6000 for Rent.
2. What is the total salary of Kumaran?
3. How much did he spend more for food than education?

4.6 Graphical representation of the frequency distribution for grouped data

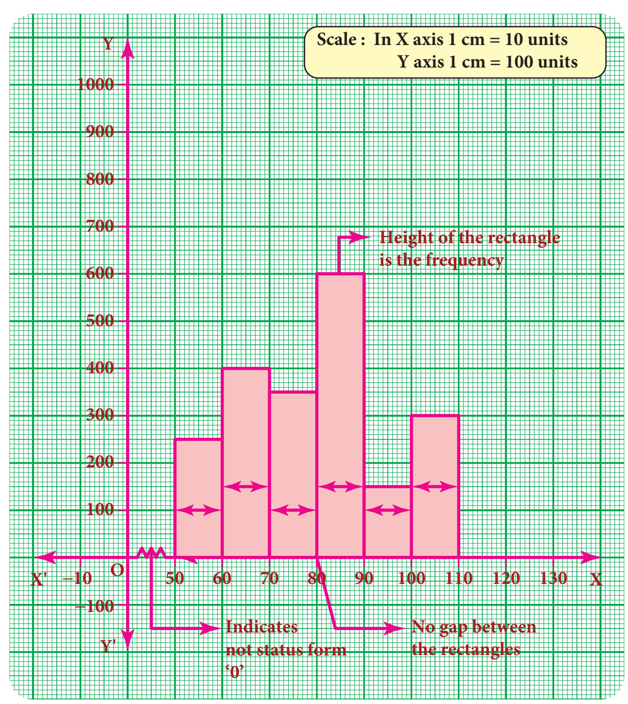
The Line graph, Bar graph, Pictograph and the Pie chart are the graphical representations of the frequency distribution for ungrouped data. Histogram, Frequency polygon, Frequency curve, Cumulative frequency curves (Ogives) are some of the graphical representations of the frequency distribution for grouped data.

In this class, we are going to represent the grouped data frequency by Histogram and Frequency polygon only. You will learn the other type of representations in the higher classes.



4.7 Histogram

A histogram is a graph of a continuous frequency distribution. Histogram contains a set of rectangles, the base of which is the length of the class interval and the frequency in each class interval is its height. i.e the class intervals are represented on the horizontal axis (x- axis) and the frequencies are represented on the vertical axis (y-axis).



The area of each rectangle is proportional to the frequency in the respective class interval and the total area of the histogram is proportional to the total frequency. Because of the continuous frequency distribution, the rectangles are placed continuously side by side with no gap between adjacent rectangles.

Steps to construct a Histogram:

1. Represent the data in the continuous form (exclusive form) if it is in discontinuous form (inclusive form) by converting it using the adjustment factor.
2. Select the appropriate units along the x-axis and y-axis.
3. Plot the lower limits of all class interval on the x –axis.
4. Plot the frequencies of the distribution on the y – axis.
5. Construct the rectangles with class intervals as bases and corresponding frequencies as heights. Each class has lower and upper values. This gives us two equal vertical lines representing the frequencies. The upper ends of the lines are joined together and this process will give us rectangles.

Note

1. If class intervals do not start from '0' then, it is indicated by drawing a kink (Zig-Zag) mark ( ) on the x-axis near the origin. If necessary, the kink mark ( ) may be made on y-axis or on both the axes. i.e it indicates that we do not have data starting from the origin (O)

Differences between a Bar graph and a Histogram

	Bar graph	Histogram
1	Used for Ungrouped data	Used for Grouped data
2	Gap between the bars	No gap between rectangles
3	Height of each bar is important and not its width	Height and width of each rectangle are equally important

4.7.1 Construction of a histogram for continuous frequency distribution:

Example 4.7

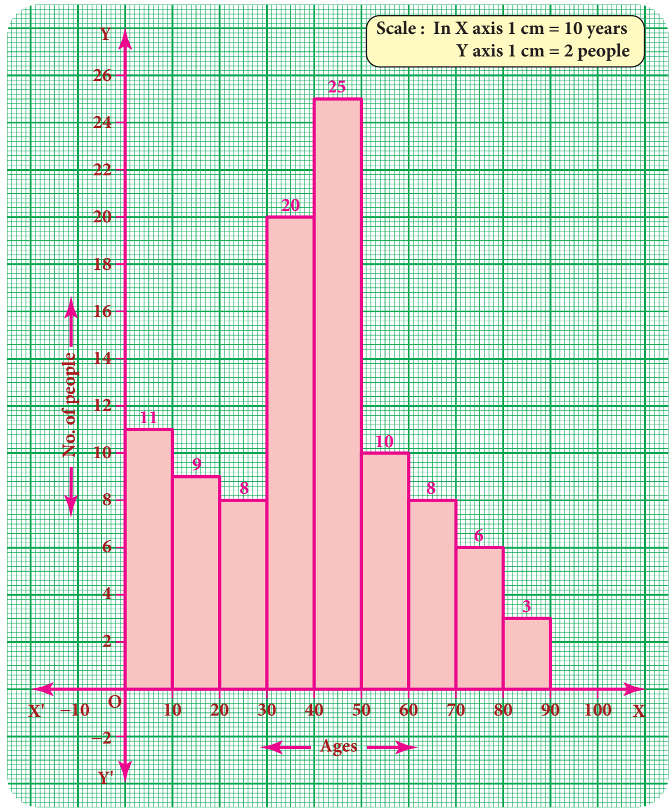
Draw a histogram for the following table which represents the age groups from 100 people in a village.

Ages	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of people	11	9	8	20	25	10	8	6	3

Solution:

The given data is a continuous frequency distribution. The class intervals are drawn on x-axis and their respective frequencies on y-axis. Classes (ages) and its frequencies (number of people) are taken together to form a rectangle.

The histogram is constructed as given below.



4.7.2 Construction of histogram for discontinuous frequency distribution:

Example 4.8

The following table gives the number of literate females in the age group 10 to 45 years in a town.

Age group	10-15	16-21	22-27	28-33	34-39	40-45
No. of females	350	920	850	480	230	200

Draw a histogram to represent the above data

Solution:

The given distribution is discontinuous. If we represent the given data as it is by a graph we shall get a bar graph, as there will be gaps in between the classes. So, convert this into a continuous distribution using the adjustment factor .

That is, lower boundary $=$ lower limit $-\frac{1}{2}$ (difference in gap)

$$= 10 - \frac{1}{2}(1)$$

$$= 10 - 0.5 = 9.5$$

Upper boundary $=$ upper limit $+\frac{1}{2}$ (difference in gap)

$$= 15 + \frac{1}{2}(1)$$

$$= 15 + 0.5$$

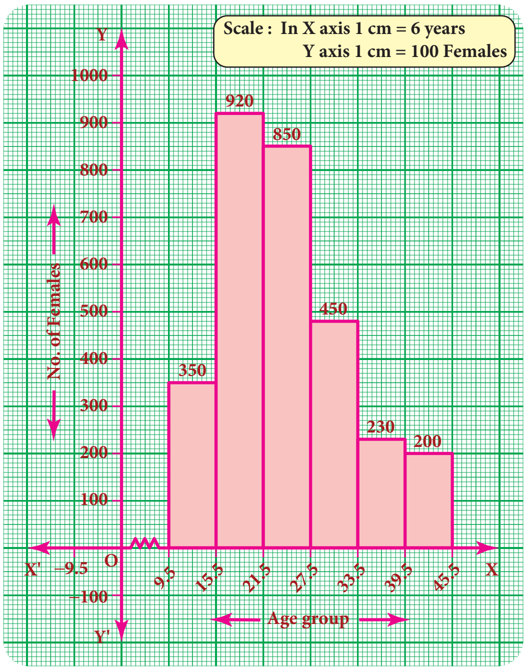
$$= 15.5$$

The first class interval can be written as 9.5-15.5 and the remaining class intervals are changed in the same way. There are no changes in frequencies.

The new continuous frequency table is

Age group	9.5-15.5	15.5-21.5	21.5-27.5	27.5-33.5	33.5-39.5	39.5-45.5
No of females	350	920	830	480	230	200

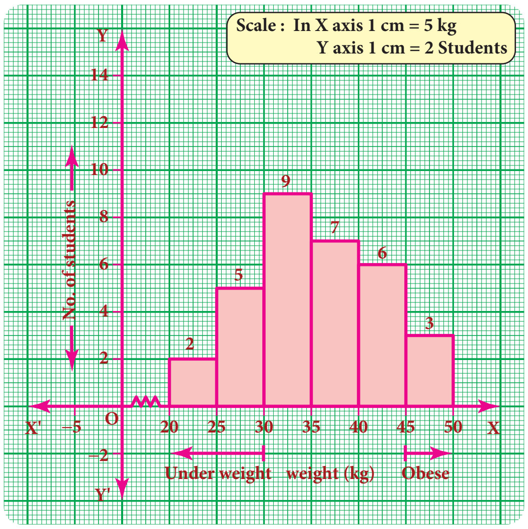
The histogram is constructed as below



Example 4.9

Observe the given histogram and answer the following questions

1. What information does the histogram represent?
   The histogram represents the collection of weights from std VIII.
2. Which group has maximum number of students?
   There are maximum 9 students in 30-35 kg weight.
3. How many of them are under weight?
   There are $7(=2+5)$ students who are under weight.
4. How many students are obese?
   There are 3 students who are obese.
5. How many students are in the weight group of 30-40 kg?
   There are $16(=9+7)$ students in the 30-40 kg weight group.



6. What is your suggestion?
   Underweight students should eat healthy food and take more protein. Obese students should do regular exercise to reduce their weight and the rest of the students are asked to continue their healthy food habits and regular exercise activities.

4.8 Frequency Polygon

A frequency polygon is a line graph for the graphical representation of the frequency distribution. If we mark the midpoints on the top of the rectangles in a histogram and join them by straight lines, the figure so formed is called a frequency polygon. It is called a polygon as it consists of a number of lines as the sides of a polygon.

A frequency polygon is useful in comparing two or more frequency distributions. A frequency polygon for a grouped frequency distribution can be constructed in two ways.

i) Using a histogram
ii) Without using a histogram

4.8.1 To construct a frequency polygon using a histogram:

1. Draw a histogram from the given data.
2. Join the consecutive midpoints of the upper sides of the adjacent rectangles of the histogram by the line segments.
3. It is assumed that the class interval preceding the first rectangle and the class interval succeeding the last rectangle exists in the histogram and the frequency of each extreme class interval is zero. These class intervals are known as imagined class intervals.
4. To get frequency polygon, join the midpoints of these imagined classes with the corresponding midpoints of the upper sides of the first and last rectangles of the histogram.

Example 4.10

The following is the distribution of pocket money of 200 students in a school.

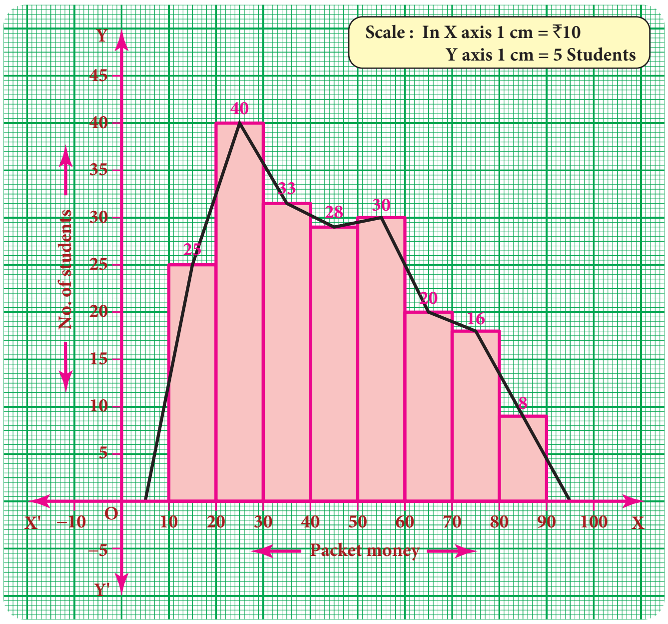
Pocket money	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Number of Students	25	40	33	28	30	20	16	8

Draw a frequency polygon using histogram.

Solution:

Represent the pocket money along x- axis and number of students along the y–axis.

Draw a histogram for the given data. Now, mark the midpoints of the upper sides of the consecutive rectangles. Also mark the midpoints of two imagined class intervals 0-10 and 90-100 whose frequency is 0 on x- axis. Now, join all the midpoints with the help of ruler. We get a frequency polygon imposed on the histogram.



Note

Sometimes imagined class intervals do not exist. For example, in case of marks obtained by the students in a test, we cannot go below zero and beyond maximum marks on the two sides. In such cases, the extreme line segments meet at the mid points of the vertical left and right sides of first and last rectangles respectively.

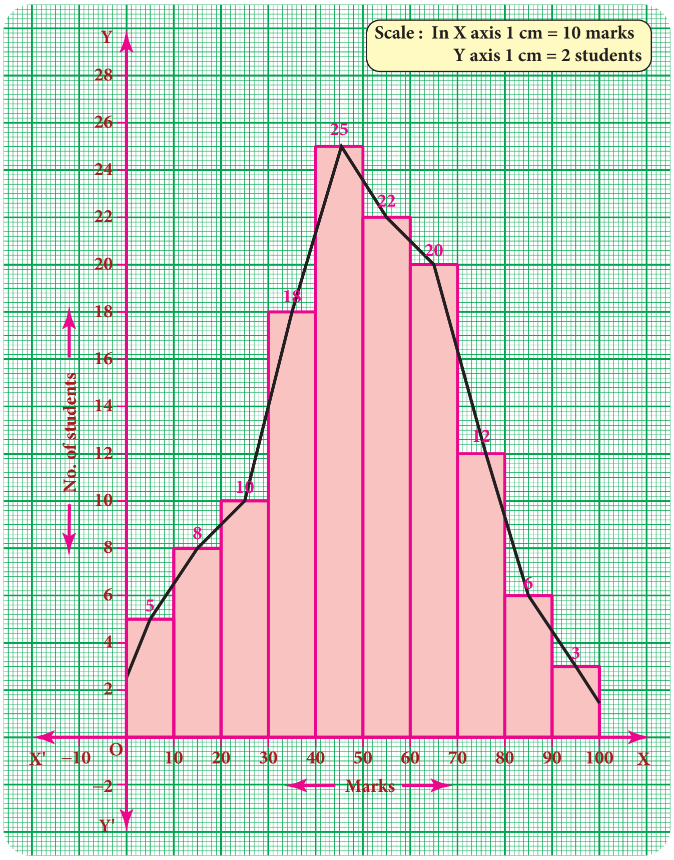
Example 4.11

Draw a frequency polygon for the following data using histogram.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Number of students	5	8	10	18	25	22	20	13	6	3

Solution:

Mark the class intervals along the x-axis and the number of students along the y-axis . Draw a histogram for the given data and mark the midpoints of the rectangles and join them by lines. We get frequency polygon. Note that the first and last edges of the frequency polygon meet at the mid points of the left and right vertical edges of first and last rectangles. Because imagined class intervals do not exist in the marks (refer the above note).



4.8.2 To draw a frequency polygon without using a histogram:

(1) Find the midpoints of the class intervals and tabulate it.

(2) Mark the midpoints of the class intervals on x-axis and frequencies on y-axis.

(3) Plot the points corresponding to the frequencies at each midpoints.

(4) Join the points using a ruler, to get the frequency polygon.

Example 4.12

Draw a frequency polygon for the following data without using histogram.

Class interval (Marks)	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90
Frequency	4	6	8	12	10	14	5	7

Solution:

Find the midpoint of the class intervals and tabulate it.

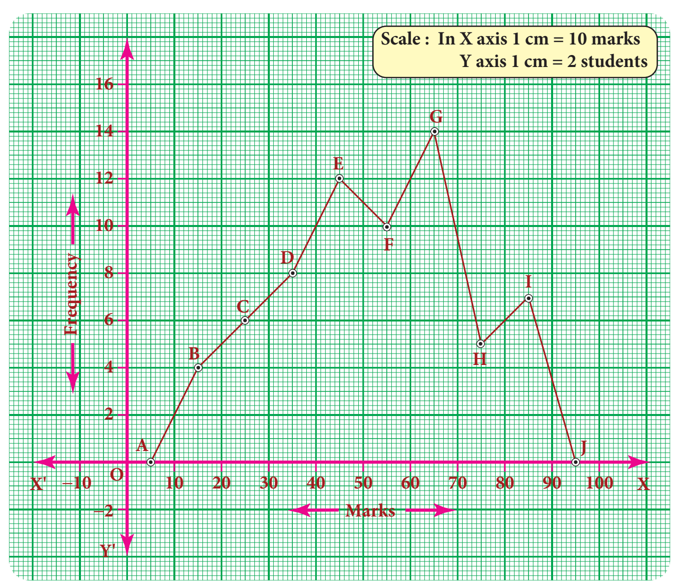
Class interval (C.I)	Mid point (x)	Frequency (f)
10 – 20	15	4
20 – 30	25	6
30 – 40	35	8
40 – 50	45	12
50 – 60	55	10
60 – 70	65	14
70 – 80	75	5
80 – 90	85	7

The points are (15,4) (25,6) (35,8) (45,12) (55,10) (65,14) (75,5) (85,7).

In the graph sheet, mark the midpoints along the x- axis and the frequency along the y- axis.

We take the imagined class as 0 – 10 at the beginning and 90 – 100 at the end , each with frequency 'zero'.

From the table, plot the points. We draw the line segments AB, BC, CD, DE, EF, FG, GH, HI, IJ to obtain the required frequency polygon ABCDEFGHIJ.



DO YOU KNOW?

Advantage of frequency polygon

In comparing two or more distributions by plotting two or more graphs on the same axis, the frequency polygon is more useful than the histogram.

Think

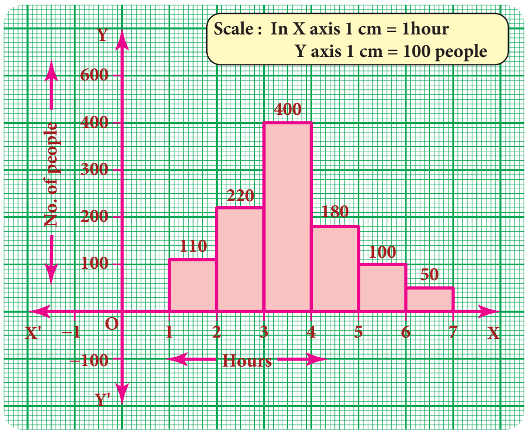
When joining two adjacent midpoints without using a ruler, can you get a polygon?

Exercise 4.2

1. Which of the following data can be represented in a histogram?
   (i) The number of mountain climbers in the age group 20 to 60.
   (ii) Production of cycles in different years.
   (iii) The number of students in each class of a school.
   (iv) The number votes polled from 7 am to 6 pm in an election.
   (v) The wickets fallen from 1 over to 50th over in a one day cricket match.

2. Fill in the blanks
   (i) The area of the retangles are proportional to the _________ given.
   (ii) The total area of the histogram is _________ to the total frequency of the given data.
   (iii) _____________ is a graphical representation of continuous frequency distribution with rectangles.
   (iv) Histogram is a graphical representation of ___________ data.

3. In a village, there are 570 people who have cell phones. An NGO survey their cell phone usage. Based on this survey a histogram is drawn. Answer the following questions.



   (i) How many people use the cell phone for less than 3 hours?
   (ii) How many of them use the cell phone for more than 5 hours?
   (iii) Are people using cell phone for less than 1 hour?
   (iv) Give your suggestions on the data.

4. Draw a histogram for the following data.

Class Interval	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	15	23	20	10	7

5. Construct a histogram from the following distribution of total marks of 40 students in a class.

Marks	90-110	110-130	130-150	150-170	170-190	190-210
No. of Students	9	5	10	7	4	6

6. The distribution of heights (in cm) of 100 people is given below. Construct a histogram and the frequency polygon imposed on it.

Height (in cm)	125-135	136-146	147-157	158-168	169-179	180-190	191-201
Frequency	12	22	18	24	15	7	2

7. In a study of dental problem, the following data were obtained.

Ages	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of patients	5	13	25	14	30	35	43	50

Represent the above data by a frequency polygon.

8. The marks obtained by 50 students in Mathematics are given below (i) Make a frequency distribution table taking a class size of 10 marks (ii) Draw a histogram and a frequency polygon.

52	33	56	52	44	59	47	61	49	61
47	52	67	39	89	57	64	58	63	65
32	64	50	54	42	48	22	37	59	63
36	35	48	48	55	62	74	43	41	51
08	71	30	18	43	28	20	40	58	49

Objective Type Questions

9. Data is a collection of _________
   (a) numbers (b) words (c) measurements (d) all the three

10. The number of times an observation occurs in the given data is called _________
   (a) tally marks (b) data (c) frequency (d) none of these

11. The difference between the largest value and the smallest value of the given data is _________
   (a) range (b) frequency (c) variable (d) none of these

12. The data that can take values between a certain range is called_________
   (a) ungrouped (b) grouped (c) frequency (d) none of these

13. Inclusive series is a _________series.
   (a) continuous (b) discontinuous (c) both (d) none of these

14. In a class interval the upper limit of one class is the lower limit of the other class. This is _________series.
   (a) Inclusive (b) exclusive (c) ungrouped (d) none of these

15. The graphical representation of ungrouped data is _________
   (a) histogram (b) frequency polygon (c)pie chart (d) all the three

16. Histogram is a graph of a _________ frequency distribution.
   (a) continuous (b) discontinuous (c) discrete (d) none of these

17. A _________ is a line graph for the graphical representation of the continuous frequency distribution.
   (a) frequency polygon (b) histogram (c) pie chart (d) bar graph

18. The graphical representation of grouped data is -----
   (a) bar graph (b) pictograph (c) pie chart (d) histogram

Exercise 4.3

Miscellaneous and Practice Problems

1. Draw a pie chart for the given table.

Continent	Asia	Africa	North America	South America	Europe	Australia	Antarctica
Area	30 %	20 %	16 %	12 %	7 %	6 %	9 %

2. The data on modes of transport used by the students to come to school are given below. Draw a pie chart for the data.

Mode of transport	Bus	Cycle	Walking	Scooter	Car
Percentage of students	40 %	30 %	15 %	10 %	5 %

3. Draw a histogram for the given frequency distribution.

Age	41-45	46-50	51-55	56-60	61-65	66-70	71-75
Frequency	4	9	17	25	15	8	2

4. Draw a histogram and the frequency polygon in the same diagram to represent the following data.

Weight (in kg)	50-55	56-61	62-67	68-73	74-79	80-85	86-91
No.of persons	15	8	12	17	9	10	6

5. The daily income of men and women is given below, draw a separate histogram for men and women.

Income(in Rs.)		200-300	300-400	400-500	500-600	600-700	700-800	800-900
No.of persons	Men	20	45	50	40	35	25	15
	Women	16	30	55	35	40	20	10

Challenging problems

6. Form a continuous frequency distribution table and draw histogram from the following data.

Age (in years)	No. of persons
Under 5	1
Under 10	12
Under 15	19
Under 20	26
Under 25	27
Under 30	35
Under 35	38
Under 40	45
Under 45	48
Under 50	53

7. A rupee spent in a cloth manufacturing company is distributed as follows. Represent this in a pie chart.

Particulars	Paise
Farmer	20
Spinner	34
Dyer	12
Weaver	14
Printer	09
Salary	11

8. Draw a histogram for the following data.

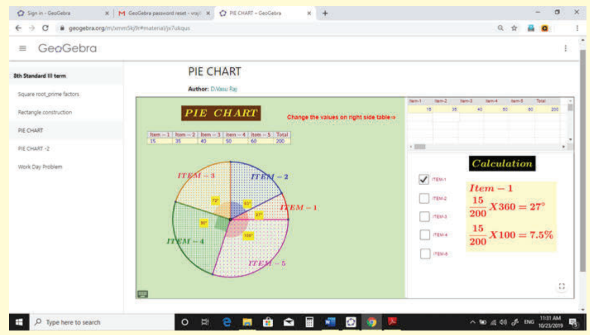
Mid Value (x)	15	25	35	45	55	65	75
Frequency (f)	12	24	30	18	26	10	8

Summary

- Data is the basic unit in Statistics. Data is a collection of facts such as numbers, words, measurements and observations.
- These are the data that are collected in person for the first time for a specific purpose is called primary data.
- The data that are sourced from some place that has originally collected it. This kind of data has already been collected by some other persons is called secondary data.
- The number of times an observation occurs in the given data is called the frequency of the observation.
- A frequency distribution is the arrangement of the given data in the form of the table showing frequency with which each variable occurs.
- The range of the variable is grouped into number of classes, and each group is known as class interval (C.I). The difference between the upper limit (U) and the lower limit (L) of the class is known as class size.
- In the class-intervals, if the upper limit and lower limit are included in that class interval then it is called inclusive series.
- In the class intervals, if the upper limit of one class interval is the lower limit of the next class interval then it is called exclusive series.
- A pie chart is a circular graph which shows the total value with its components.
- A histogram is a graph of a continuous frequency distribution.
- A frequency polygon is a line graph for the graphical representation of the frequency distribution.
- A frequency polygon is useful in comparing two or more frequency distributions.

ICT CORNER

Expected Outcome

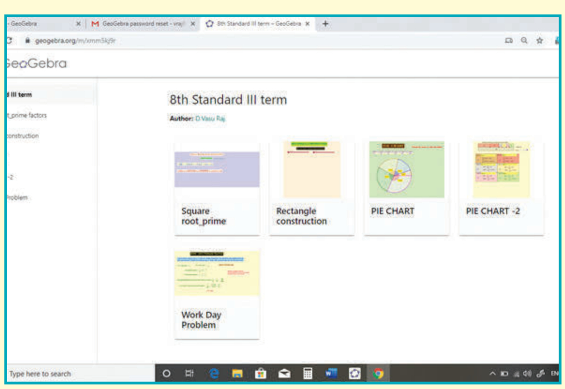


Step – 1

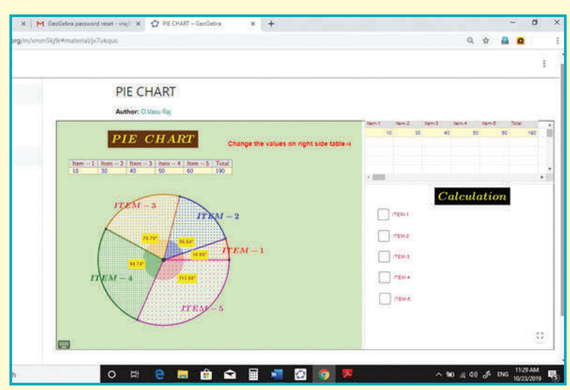
Open the Browser type the URL Link given below (or) Scan the QR Code. GeoGebra work sheet named "8th Standard III term" will open. Select the work sheet named "PIE CHART"

Step - 2

Type your values in the check box on right side. You can observe the change in the pie chart. Click on the check boxes to see respective calculations.



Step – 1



Step – 2

Browse in the link

STATISTICS:

https://www.geogebra.org/m/xmm5kj9r or Scan the QR Code.

B358_8_MATHS_EM

5 INFORMATION PROCESSING

G8R4L9

Learning Objectives

- Shopping – To consider alternatives before making a purchase, calculate the unit price for each items and make purchase in limited budget.
- Packing – To understand how to fit things efficiently in a given space and find the optimal solution.

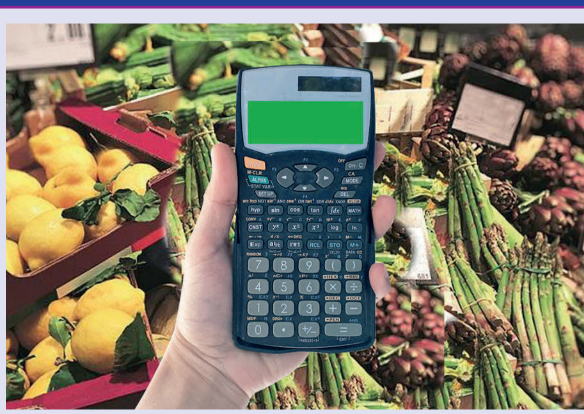
5.1 Introduction

Students, I hope you all having an experience on shopping. Could you share your experience with? I would like to rise some questions on your shopping experience. Will you shop the things you need, by (i) attractive colour or (ii) best price or (iii) big in size or (iv) on seeing. What ever it may be, among all this things, one more important point should be noted. What is that? Yes, that is expiry date. Have you ever noticed the expiry date on all the packed goods? It is very important to see that and one more the best way of shopping is comparing goods means its price, quality, quantity, offers, discount and other considerable things.

Before spending your money to shop any items from a market or a departmental store, consider the best prices, the best quality and other reliable things. This is wise shopping.

Here we learn, how to be a wise consumer before shopping a product from the following situation.

MATHEMATICS ALIVE—INFORMATION PROCESSING IN OUR REAL LIFE



Calculating the unit price while shopping

Mason packing bricks for constructing a wall between two pillars

Shopping comparison:

Situation 1:

Priya wants to buy a helmet to wear while riding her scooty. In a shop, 3 brands of helmet are displayed and features of those helmets also tabulated as below. Priya wants to buy at a affordable cost with good quality. Which will be her wise selection of helmet?

Helmet Brands	Price		Desired features					
			Chin Strap and buckle		Proper ventilation		Reliable protection	
	Reasonable	High	Okay	Excellent	Okay	Excellent	Okay	Excellent
A	✓	-	-	✓	✓	-	-	✓
B	-	✓	-	✓	-	✓	✓	-
C	✓	-	✓	-	-	✓	✓	-

Yes **Brand A**, because according to Priya's requirements, Brand A is the best one,since it provides all the features she needs, at a reasonable price. Brand B is a good helmet, but too expensive. Brand C is also a good helmet, but the chin strap and buckle is only okay to the level and not so good as Brand A. So, buying Brand A helmet is the wise decision.

Situation 2:

Imagine that the teacher appoints you and your friend to be an incharge of the fruit section of your school canteen for a week. she also instructs the following steps, and she can help you when needed.

Shopping list

1. 20 kg apples
2. 20 kg of guavas
3. 30 boxes of strawberries
4. 20 dozens of bananas

* Now you have to buy fruits for 2 days as per your shopping list.
* One of you should go to the market and the other should go to the departmental store to know the cost of the fruits before shopping.
* Estimate yourselves which place will give you the best deal.

After that,

* Check your shopping list to see how much fruits you required.
* Compare the weight and price for each item from both places.
* Select the best deal for all items in only one place .

* Discuss and compare the price list so that you decide where to buy the required list of fruits.

For example, the collected model price list from both shops is given in the table below:

S. No	Fruit name	Departmental store		Market price	
		Quantity	Price (₹)	Quantity	Price (₹)
1	Apple	1 kg	120	1 kg	110
2	Guava	1 kg	50	1 kg	40
3	Strawberry	1 box	80	1 box	85
4	Banana	1 dozen	60	1 kg	50

Now, we will calculate the total price of the required and quantity of fruits from both the departmental store and market.

Calculating the Departmental store Price:

Fruit Name	Cost of the required fruits	Total Price (₹)
Apple 	Cost of 1 kg of apples = ₹120 Cost of 20 kg of apples = 20×120 = ₹2400	2400
Guava 	Cost of 1 kg guavas = ₹50 Cost of 20 kg guavas = 20×50 = ₹1000	1000
Strawberry 	Cost of 1 box of strawberries = ₹80 Cost of 30 boxes of strawberries = 30×80= ₹2400	2400
Banana 	Cost of 1dozen of bananas = ₹60 Cost of 20 kg of bananas = 20×60 = ₹1200	1200

Calculating the Market Price:

Fruit Name	Cost of the required fruits	Total Price (₹)
Apple 	Cost of 1 kg apples = ₹110 Cost of 20 kg of apples = 20 × 110 = ₹2200	2200
Guava 	Cost of 1 kg guavas = ₹40 Cost of 20 kg guavas = 20 × 40 = ₹800	800
Strawberry 	Cost of 1 box of strawberries = ₹85 Cost of 30 boxes of strawberries = 30 × 85 = ₹2550	2550
Banana 	Cost of 1 kg of bananas = ₹50 Cost of 20 kg of bananas = 20 × 50 = ₹1000	1000

Now, let us compare the shopping price of the Departmental store to that of the Market shop price.

Fruits	Cost of items as per your requirement (₹)	
	Departmental Store	Market
20 kg Apples	2400	2200
20 kg Guavas	1000	800
30 boxes of Strawberries	2400	2550
20 dozens Bananas	1200	1000
Total cost of shopping price	7000	6550

From the above comparison, we find that shopping made at the Market shop is the best deal quantity wise as well as in price and hence it is a wise decision to shop in the Market.

Activity-1

Consider that you are going to a store with your total budget of ₹220 to buy things without changing the quantity of the items given in the list below with the following conditions.

Conditions:

(i) First you have to complete the price list given.

(ii) You have to buy three items as per the given price list but within your budget **₹220**.

(iii) You won't carry exceeding **5kg** because you have to walk home carrying them, so they cannot be bulky.

Now, answer the following questions:

1. In how many ways can you buy your items? Complete the price lists given below. One is done for you.
2. Which one is the best purchase price list and why?

Price List

S.No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
1	Rice	37.50	2.50	
2	Toor Dal	62.00	1.00	
3	Sugar	32.50	1.50	
4	Wheat	26.50	1.00	
	Total Bill Amount			

Price List

S.No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
1	Rice	37.50	2.50	93.75
2	Toor Dal	62.00	1.00	62.00
3	Wheat	26.50	1.00	26.50
	Total Bill Amount			182.25

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
	Total Bill Amount			

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
	Total Bill Amount			

Price List

S. No.	Description	Price / 1 kg (₹)	Quantity kg	Amount (₹)
	Total Bill Amount			

Comparing containers of different size:

Many times, items are packed in different size of containers.

- Sometimes shoppers save money by selecting a larger container of the same item. For example, 5 units of 200ml pack of milk often costs more than 1 litre of milk.
- Sometimes a store has two prices for the same item. One price is for buying a single item, while the other price is for buying more than one of that item. For example, groundnut oil may cost ₹135 for 1 litre bottle and ₹240 for 2 litre bottles. In this case, if you buy two 1 litre bottles, you will pay more. Sometimes, buying in quantity saves money.

Some times the consumer may not be able to use up the larger size of an item before it becomes stale or outdated. To find out which size container is the best to buy, you will need to know the price of single pack of the contents.

Activity-2

Consider that you want to buy 12 litres of same quality of edible oil at your budget price of ₹250 per litre. In a supermarket, there are a lot of offers on various oil brands. Some of the offers are given below. Complete the table and find which one is the best offer for you and how much you will save for your total purchase.

Which one is the best deal?

Product (Edible oil)	Size (in litres)	Regular Price (₹)	Offer	Special Price (₹)	Saving Price (₹)	Cost of 1 litre (₹)	Cost of 12 litres(₹)
	1	293	₹ 50 off	243		243	
	2	850	1 *l* +1 *l* combo	499	351 (850-499)	249.50	
	5+1 = 6	2000	Buy 5 *l* get 1 *l* free	1500			3000
	2+2 = 4	1486	Buy 1 get 1 free	743		185.75	
	1+1= 2	850	Spl. offer 1 *l* pack of 2 ₹ 390	390		195	
	12 (1) = 12	5100	1 *l* pack of 12	1650	3450		

Best offer price for you ________________

Amount that you saved for your total purchase ______________

Try these

The teacher divides the class into four groups and setup a mock market in the class room and ask the students to involve in role play as two groups of businessmen and two groups of consumers. Consumers have to buy products at different shops and prepare a price list.

The two supermarkets in which the two groups buy are Star Food Mart and Super Provisions. This week they each have got a special deal on some products. At Star Food Mart, you can buy items at discount prices. At Super Provisions, there are some "**BUY ONE GET ONE**" deals. Have a look at their deal:

Star Food Mart

Chocolate Biscuits worth ₹114 per packet at ₹30 off

Peanut candies worth ₹90 now at ₹20 off

Protein milk worth ₹60 now at ₹20 off

Badam nuts worth ₹450 now at ₹150 off

Super Provisions

Chocolate Biscuits worth ₹180 per packet Buy one get one free

Peanut candies worth ₹150 Buy one get one free

Protein milk worth ₹80 Buy one get one free

Badam nuts worth ₹580 Buy one get one free

Now, answer the following questions.

I. Here is your shopping list:

4 bottles of Protein Milk (200 ml size), 2 packets of Peanut candies(200 gm), 1 packet of Chocolate biscuits and 1 packet of Badam nuts (500 gm)

(i) If you buy all the items in one shop, where will you get the best price?

(ii) If you buy the items from different shops, how will you do it to spend the least amount of money?

II. You have ₹1000/- to spend to buy the following shopping list:

6 bottles of Protein Milk (200 ml size), 3 packets of Peanut candies(200 gm), 3 packets of Chocolate biscuits and 1 packet of Badam nuts (250 gm).

(i) How can you do this so that you don't go over your budget amount ₹1000?

(ii) Which shop offers you the best value for money on each item?

(iii) Is the "**BUY ONE GET ONE**" deal at Super Provisions the same as "**50% off**" deal?

Exercise 5.1

1. **Choose the correct answer:**

(i) Online or television advertisements influence on spending decisions by

(a) using special music
(b) making me think I need the item
(c) using attractive pictures
(d) all the above

(ii) When I go shopping, I will buy

(a) something that looks attractive
(b) something my friend has
(c) something that I need to purchase
(d) the first thing I see in the store

(iii) The best shopping choice is

(a) always shop at brand name stores
(b) compare the choices before buying
(c) the same thing my friends bought
(d) always buy at a regular shop

2. **Say true or false:**

(i) Wise consumers take time to compare two or three shops before spending money.

(ii) Taking time to analyse advertisements cannot save money when shopping.

(iii) One cannot shop on double and triple coupon discounts in available days.

(iv) Every time one must make a shopping list and stick on within his/her budget.

3. **Find the best buy of the following purchases:**

(i) A pack of 5 chocolate bars for ₹175 or 3 chocolate bars for ₹114?

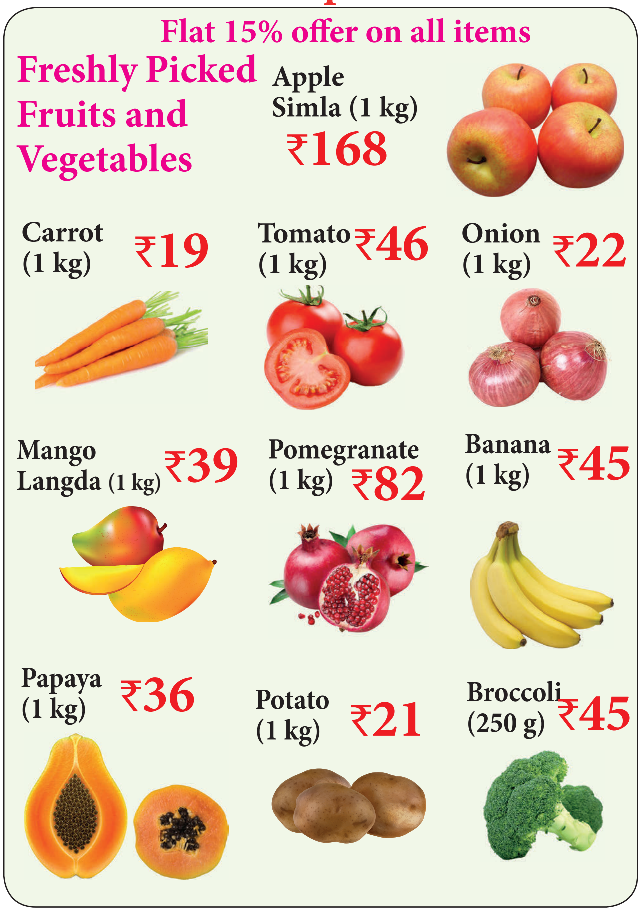
(ii) Basker buy 1 1/2 dozen of eggs for ₹81 and Aruna buy 15 eggs for ₹64.5?

4. Using the given picture find the total special offer price of fresh sweets and bakery products to buy 1/2 kg laddu, 1 kg cake, 6 pockets of bread.

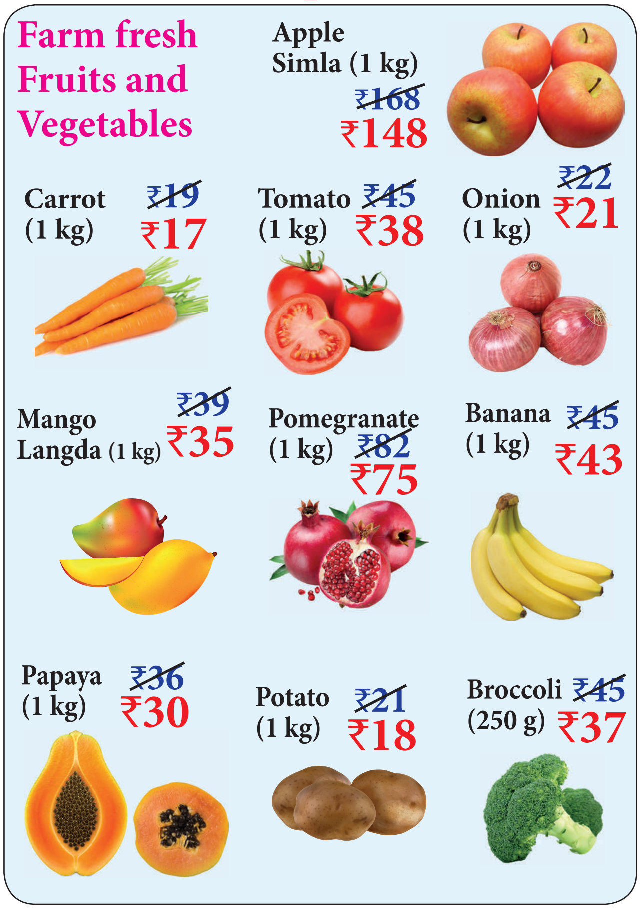
5. Using given picture prepare a price list.

Suppose you plan to buy 1½ kg of apple, 2 kg of pomegranate, 2 kg of banana, 3 kg of mango, ½ kg of papaya, 3 kg of onion, 1½ kg of tomato, and 1 kg of carrot in shop 1, how much will you save compared to shop 2.

Shop 1



Shop 2



6. When you plan to buy a shirt, one shop offers a discount of ₹200 on MRP ₹1000 and another shop offers 15% discount on the same MRP. Where would you buy?

7. Amazing park is offers a package deal of 5 entrance passes for ₹130. If one entrance pass normally costs ₹30, how much will you save by taking advantage of this special deal?

8. Consider that you are going to buy the toys that are given below in the price list. Prepare a comparison price chart table and find where will you get the best offer price and also find how much you save?

Golden Toys - Price List			
Product name	Product	Original price	Special Offer
Teddy Bear		1699	45%
Tetromino Blocks		1150	30%
Space Shuttle Rocket Launch Station Centre Educational Block Construction Toy		3650	45%
Throw Ball		710	18%

Toys and Trades Mart - Price List			
Product name	Product	Original price	Special Offer
Traddy Bear		1798	53%
Tetromino Blocks		1003	25%
Space Shuttle Rocket Launch Station Centre Educational Block Construction Toy		3499	47%
Throw Ball		720	20%

[The amount to the nearest rupee as ₹789.84 is rounded off to ₹790]

Comparison Chart Table											
		Golden Toys					Toys & Trades Mart				
S.No.	Product Name	Original Price	Offer %	Amount that you saved	Round the amount you saved to the nearest rupees	Sale Price	Original Price	Offer %	Amount that you saved	Round the amount you saved to the nearest rupees	Sale Price
1											
2											
3											
4											
	Total Amount										

5.2 Packing

When we are packing something in a box or suitcase or cupboard, first we have to decide how we are going to pack. How many items can be fitted into that fixed space? A good example of this is, before you go to school, you try to pack everything you need (like your books, notebooks, geometry box, sports equipment, food and water bottle) into your school bag. At that time, you are very clear that your books should not be damaged and you are able to carry everything yourself. Think! The same rules apply to posting a package to a friend or family member or others.

Apart from these, the packaging method is used in many cases. Such as cutting of sheets, glass, paper, wood, cloth or other materials and room allotment, seating arrangement in the particular space provided, parking vehicles with proper lanes and saving data in hard disk, CD, pen drive and so on.

Using some packing methods, from the following situations and examples, let us try to understand how best to fit the items into the space in the containers or in rooms or in boxes etc.,

5.3 Packaging Methods

Fractional Method:

Here, when we fill items in bags or in containers, we determine the weight, value and number of each item with the condition that the total weight of the container is less than or equal to a given limit and the total value is as large as possible. Fractional method uses the technique of buying things fractionally and admits buying of more items within a given budget. Let us learn more about this method from the following situation.

U1Q3M1

Situation 1

Suppose that you want to buy some vegetables and fruits that are given in the picture with their weights and price and you have a bag that capacity of carrying 15 kg. The objective is to buy the more items within your budget ₹550 and also weight should not exceeding 15 kg.

You cannot buy all the items, because if you calculate the total weight of all the items, then it would be greater than 15 kg (extreme capacity of your bag is 15 kg). So, let us try some approaches, to find how you can select more items so that you will buy them with maximum price within your budget of ₹550. For that let us tabulate the items with their weights and price you want to buy.

Items							
Weight (kg)	1	3	5	4	1	3	2
Price (₹)	60	105	150	70	80	90	40

I Approach - Selecting items with maximum price

In this approach, we select the items according to the maximum price. Here the maximum price in the table is ₹150/-. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

Items	Price (₹)	Weight (kg)	Remaining weight to buy
	150.00	5	15−5=10
	105.00	3	10−3=7
	90.00	3	7−3=4
	80.00	1	4−1=3
	$70 \times \frac{3}{4} = 52.50$	3	3−3=0
Total price	**472.50**	**15 kg**	

Here, 3kg of papaya is enough as the total weight reaches 15kg. So, 3kg of papaya costs ₹52.50. Hence, in this approach, we will spend maximum ₹472.50 to buy 15kg of vegetables and fruits.

II Approach - Selecting items with minimum weight

In this approach, we select the items according to the minimum weight. Here, we can select more and more items. Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15kg.

Items	Price (₹)	Weight(kg)	Remaining weight to buy
	60.00	1	15−1=14
	80.00	1	14−1=13
	40.00	2	13−2=11
	105.00	3	11−3=8
	90.00	3	8−3=5
	70.00	4	5−4=1
	$\mathbf{150 \times \frac{1}{5} = 30.00}$	**1**	1−1=0
Total price	**475.00**	**15 kg**	

Here, 1 kg of Sapotta is enough to complete 15 kg with minimum price of ₹30 per kg. Hence in this approach, we will spend maximum ₹475 to buy 15 kg of vegetables and fruits.

III. Approach - Finding the maximum price to weight ratio.

In this approach, we select the items according to the maximum price to weight ratio (find the rate of 1kg). Now, let us tabulate to find the total price and how much can we buy vegetables and fruits within our budget and not exceeding 15 kg.

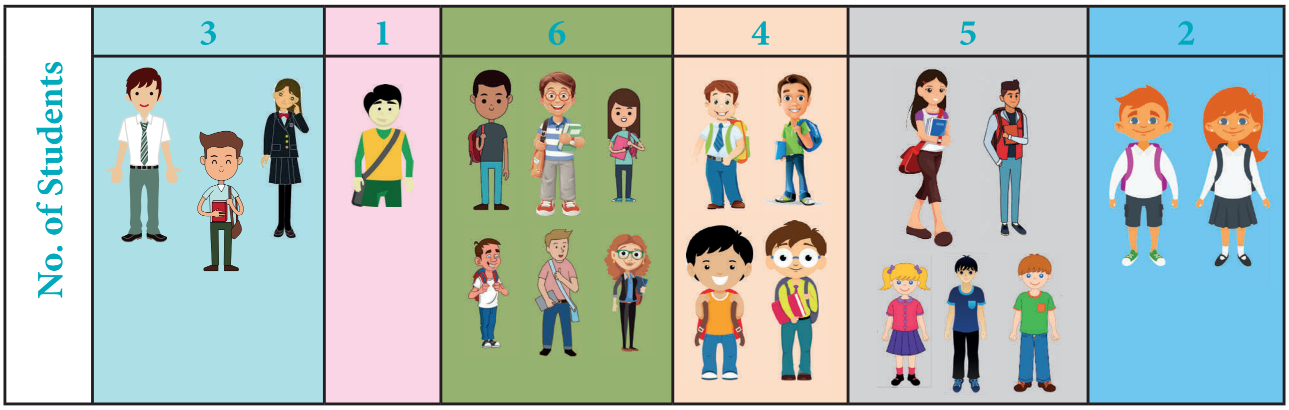
Items	Price of 1kg	Price (₹)	Weight (kg)	Remaining weight to buy
	80.00	80.00	1	15−1=14
	60.00	60.00	1	14−1=13
	35.00	105.00	3	13−3=10
	30.00	150.00	5	10−5=5
	30.00	90.00	3	5−3=2
	20.00	40.00	2	3−2=0
Total price		**520.00**	**15 kg**	

In this approach, we can buy all vegetables and fruits except papaya as we need with maximum price within our budget and not exceeding 15 kg. Comparatively, in the II approach we can buy more items but spend minimum amount only. So, we can say third approach is best one. Isn't it?

Sorting Method:

Situation 2:

Consider that you are going on a field trip in your school and you have six groups of students of group sizes as given below.



You need to fit the group of students in a cab that has a capacity of seven members. How many cabs would you need to arrange so that each group stays together?

To solve this problem, we have to remember two things, one is minimum number of cabs to be used and another thing is each group of students stays together. For these purposes, the packing methods will help us.

There are two packing methods in common use. They are:

(i) First-fit method (ii) First-fit decreasing method

Before we try to solve this problem using one of these packing methods, one thing we need to know is finding the minimum required.

What is the minimum required in this problem is the number of cabs. So, to calculate the minimum number required, we have to add up the total number students and divide by the seeking capacity of a cab.

Here, the total number of students = 3+1+6+4+5+2 = 21

Capacity of seater of a cab = 7

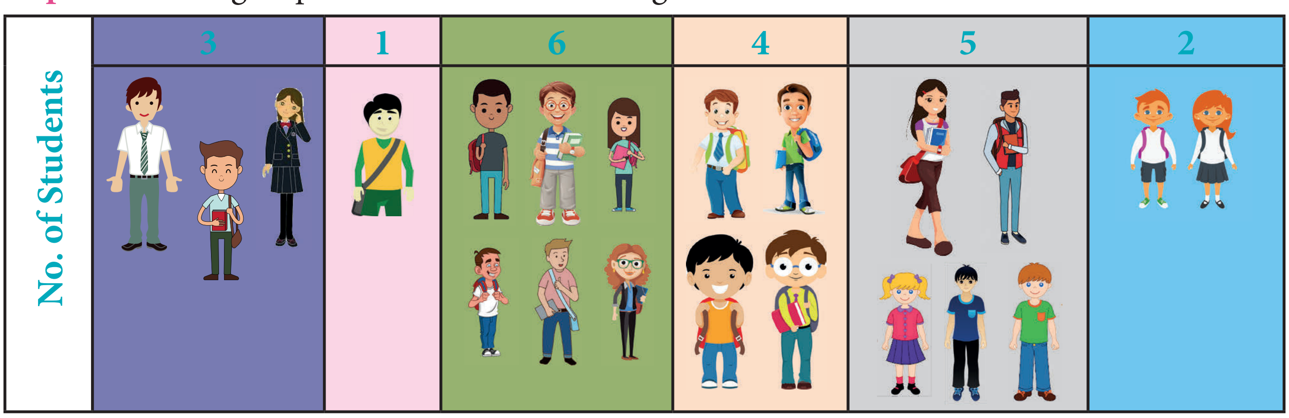
Therefore, the cab minimum required = 21÷ 7 = 3 cabs.

Now, we don't know whether 3 cabs can be an answer to this problem. 3 cabs may or may not be enough to accommodate when the group of students stays together.

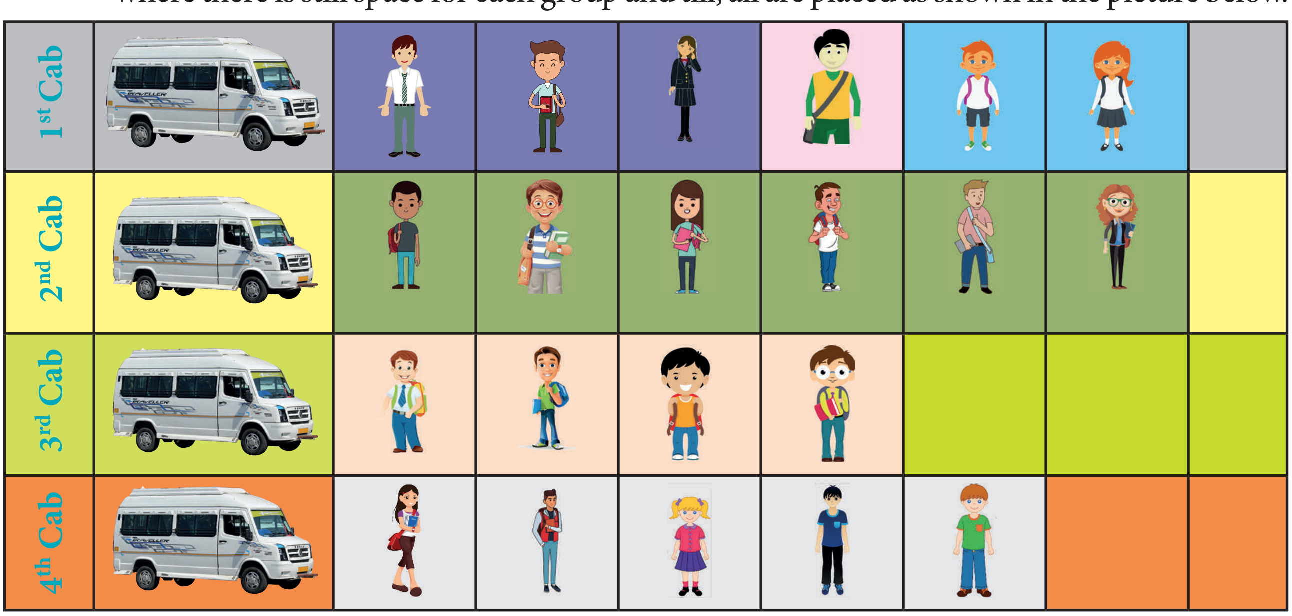
Let us go on to apply the methods now one by one.

First-fit method:

Step 1. Take the group of students in the order given.



Step 2. Place each group of students in the first cab and continue trying to a fit them in the cabs where there is still space for each group and till, all are placed as shown in the picture below.



From the above picture, observe the following:

Group 1 – 3 students - accommodate them into 1st cab, so that the remaining seats are 4.

Group 2 – 1 student - also accommodate him into 1st cab,so that the remaining seats are 3.

Group 3 – 6 students - As there are no enough seats in the 1st cabs and so accommodate them into 2nd cab, so that remaining seat is 1.

Group 4 – 4 students - Since there are no enough seats in the first 2 cabs and so accommodate them into 3rd cab, so that remaining seats are 3.

Group 5 – 2 students – As there are enough seats in the 1st cab, accommodate them into 1st cab, so that remaining seat is 1.

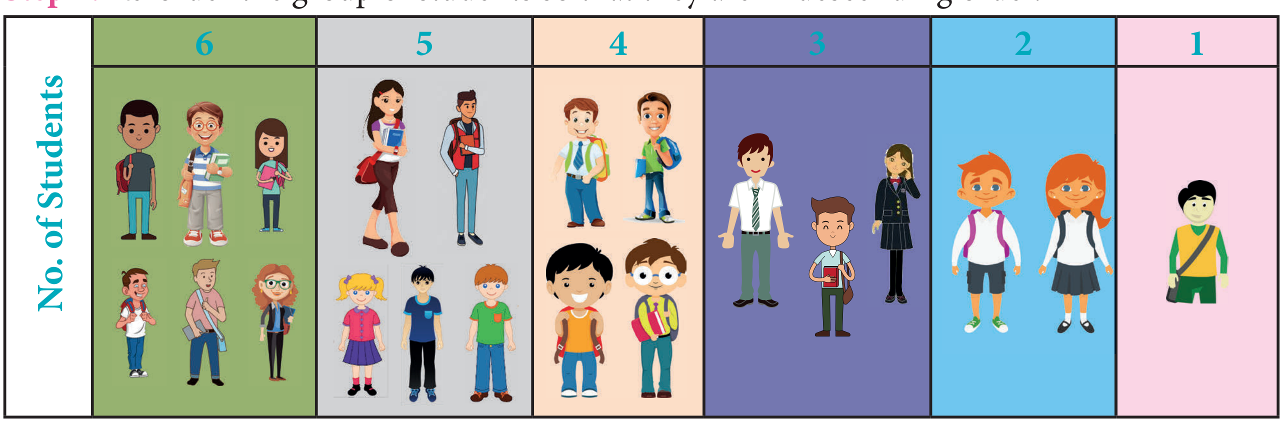
Group 6 – 5 students -Since there are no enough seats in all the 3 cabs and so accommodate them into 4th cab, so that remaining seat is 1.

Using this First- fit method, we need **4 cabs** and there are 1+1+3+2=7 seats still remaining to be filled and we can say that the seats are not utilized to the optimum level.

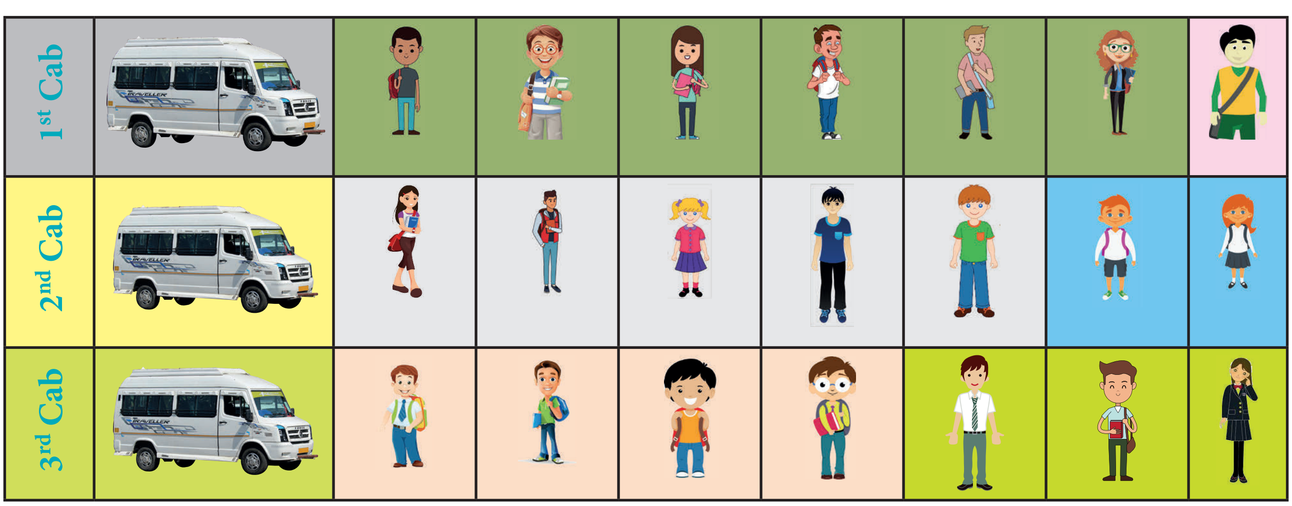
Let us now see the other method.

First-fit decreasing method:

Step 1. Re-order the group of students so that they are in descending order.



Step 2. Do the same process of the first fit method to the re-ordered group. Place each group of students in the first cab and continue trying to fit them in the cabs where there is still space and all are placed as shown in the picture below.



From the above picture, we observe that:

Group 3 – 6 students - accommodate them into 1st cab so that the remaining seat is 1.
Group 6 – 5 students - As there are no enough seats in the 1st cab and so accommodate them into 2nd cab and the remaining seats are 2.
Group 4 – 4 students - As there are no enough seats in the first 2 cabs and so accommodate them into 3rd cab and the remaining seats are 3.
Group 1 – 3 students - Since there enough seats in the 3rd cab, accommodate them into 3rd cab and the remaining seat is 0.
Group 5 – 2 students - Since there are enough seats in the 2nd cab, accommodate them into 2nd cab and the remaining seat is 0.
Group 2 – 1 student - Since there is a seat left in the 1st cab, accommodate him into 1st cab and the remaining seat is 0.
Using this First- fit decreasing method, we need **3 cabs** and there are **no remaining seats** and hence seats are used to the optimum level .

	Advantage	Disadvantages
First-fit method	Quick and easy to do	Less likely to give a good solution
First-fit decreasing method	Easy to do	Usually better solution than first fit method

Example 1:

Kumaran is a trainee carpenter. He has to cut the following length of wood in the table given below. The available length of wood in the market is 8 ft. Help him to cut without wasting any of the woods.

Length of wood (in feet)	2	3	4	6
Length of required wooden pieces	2 feet 2 feet 2 feet 2 feet	3 feet 3 feet	4 feet 4 feet 4 feet	6 feet

Solution:

For that, first we have to calculate minimum required.

Here, the total length of wooden pieces required

$$= (2\text{ft} \times 4) + (3\text{ft} \times 2) + (4\text{ft} \times 3) + (6\text{ft} \times 1) = 8 + 6 + 12 + 6 = 32 \text{ feet}$$

Available length of wood = 8 feet

Therefore, minimum required wood = $32 \div 8 = 4$

Avaiable Length of wood (in feet)	8 feet			
Length of required wooden pieces	2 feet 2 feet 2 feet 2 feet	3 feet 3 feet	4 feet 4 feet 4 feet	6 feet
Length required (in feet)	2	3	4	6
Number of wooden pieces	4	2	3	1
Total wooden pieces (in feet)	8	6	12	6
Total feet of wooden pieces required = 8 + 6 + 12 + 6 = 32 feet				

So, kumaran needs 4 pieces of woods to fullfil his requirement.

Let us check, how can Kumaran cut exactly four-piece of woods with no wastage using the first-fit decreasing method of packing.

The following picture shows how Kumaran could cut the length of wood without any wastage.

Avaiable Length of wood (in feet)	8 feet
Length of wooden pieces cut by him from the 4 log	6 feet, 2 feet 4 feet, 4 feet 3 feet, 3 feet, 2 feet 4 feet, 2 feet, 2 feet

Try this

If Kumaran cut the woods using first fit method then find the wastage pieces.

Activity-3

Seva Sangam has decided to deliver some aids to flood victims via lorries with a maximum capacity of 5000kg. All of these items that are given below are to be packed and sent in the lorry.

No. of Items	1	2	3	4	5	6	7	8	9	10
Weight of the Items (kg)	1969	1211	1996	1999	1508	2007	1520	1485	1005	300

Use the first-fit method and first-fit decreasing method to deliver the aids to flood victims. Find the number of lorries used and the amount of wasted space.

First-fit method

Number of lorries used = kg and the amount of wasted space = kg

First-fit decreasing method

Number of lorries used = kg and the amount of wasted space = kg

Therefore, the total number of the lorries needed =

Exercise 5.2

I. Answer the following questions:

P8J7K2

1. The sizes in MB, for nine computer files are given below.

53 82 61 38 23 41 16 34 42

The files are to be grouped into folders. Each folder may contain a maximum limit of 100 MB.

(a) Determine the minimum number of folders required.

(b) Use the first-fit method to group the files into folders.

(c) Use the first-fit decreasing method to group the files into folders.

2. A parcel delivery company has 4 motorcycles. Each motorcycle can carry a maximum load of 30kg. The weights of the parcels, (in kg), in the order they are waiting to be delivered are given below.

Items	1	2	3	4	5	6	7	8	9
Weight of the Items (kg)	16	20	8	14	7	6	2	5	12

(a) Determine the minimum number of motorcycles required .

(b) Use the first-fit method to show how the parcels could be allocated to the motorcycles.

(c) Use the first-fit decreasing method to show how the parcels could be allocated to the motorcycles.

3 A plumber wishes to cut the following sections from standard size of pipe each of length 6m.

No. of pipes	1	2	3	4	5	6	7	8	9	10	11	12
Size of length	1	1	1.5	1.5	1.5	1.5	2	2	2	3	3.5	3.5

Find the

(a) To cut according to the first-fit method calculate the wastage of the pipe length.

(b) To cut according to the first-fit decreasing method calculate the wastage of the pipe length.

ANSWERS

NUMBERS

Exercise 1.1

1. (i) 9 (ii) 48 (iii) 5 iv) 2, 3, 7, 8 v) 5
2. (i) True (ii) True (iii) False (iv) True (v) False
3. (i) 6 (ii) 4 (iii) 9
4. (i) odd number of zeroes (ii) cannot end with 7 (iii) cannot end with 8
5. (i) 324 (ii) 9801 6. (i) $15^2 = 112 + 113$ (ii) 180 +181
7. (i) 1+ 3 +....+ 19 (ii) 1 + 3+... + 41 8. (i) 16, 63, 65 (ii) 10,24,26
9. (i) 12 (ii) 16 (iii) 28 10. (i) 34 (ii) 69 (iii) 95 11. (i) No (ii) No (iii) Yes (iv) Yes
12. 2, 60 13. 65 14. 3, 84 15. 3600

Exercise 1.2

1. (i) 3 (ii) 13, 14 (iii) 30 (iv) $\frac{5}{4}$ (v) 8.1
2. (i) 21 (ii) 28 (iii) 32 3. 69, 37 4. 171, 79
5. (i) 134 (ii) 105 (iii) 83 (iv) 42 (v) 647
6. (i) 1.7 (ii) 1.4 (iii) 8.2 (iv) 5.6 (v) 1.42 (vi) 3.74
7. (i) $\frac{12}{15}$ (ii) $2\frac{5}{7}$ (iii) $\frac{5}{2}$ (iv) $2\frac{1}{6}$
8. (i) True (ii) True (iii) False (iv) False (v) False

Objective Type Questions

9. (c) 7 10. (d) $\sqrt{32}$ 11. (a) $\sqrt{25}$ 12. (b) 5

Exercise 1.3

1. (i) 7 (ii) 6 (iii) 90 (iv) 0.017 (v) 42
2. (i) False (ii) True (iii) True 4. 120 5. 5 6. 4, 16
7. $\sqrt{3} = 1.732$ 8. $\sqrt{36} = 6$ 9. 9,19 10. 5

Exercise 1.4

1. (i) 1 (ii) 1 (iii) 20^{-3} (iv) $\frac{-1}{128}$ (v) -243
2. (i) True (ii) True (iii) True (iv) False (v) True

3. (a) $\frac{1}{8}$ (ii) 32 (iii) $\frac{4}{9}$ (iv) 81 (v) $\frac{-216}{125}$ (vi) 21^{-14} (vii) 6 (viii) -9

4. (i) $\left(\frac{2}{5}\right)^6$ (ii) $\left(\frac{4}{5}\right)^{-5}$ (iii) $\left(\frac{1}{2}\right)^4$ 5. (i) $\frac{63}{2}$ (ii) 1 (iii) 1

6. (i) 1 (ii) $\frac{3}{2}$ (iii) $\frac{2^{-9}}{3^2}$ 7. (i) $x = 6$ (ii) $x = 5$ (iii) $x = 6$

8. (i) $6 \times 10^3 + 5 \times 10^1 + 4 \times 10^0 + 3 \times 10^{-1} + 2 \times 10^{-2} + 1 \times 10^{-3}$
(ii) $8 \times 10^2 + 9 \times 10^1 + 7 \times 10^0 + 1 \times 10^{-1} + 4 \times 10^{-2}$

9. (i) 87652.0407
(ii) 5050.505

10. 0.000000000025 11. 4.678×10^{11} (ii) 1.975×10^{-6}

12. (i) 1.083×10^{12} cubic. km (ii) 1.6×10^{-24}

Exercise 1.5

1. 32 m 2. 552 cm^2 3. 49 4. 625 5. $\frac{9}{4}, \frac{27}{8}$ 6. 8

7. 400 8. (i) 4.8×10^3 (ii) 1.152×10^5 (iii) 4.2048×10^7 (iii) 4.2048×10^9

Challenging Problems

9. 9 cm and 10 cm 10. 15 decimetre 11. No, 64 12. 13.276 (or) $\frac{3319}{50}$

13. 58.85 14. 10^{-5} hours 15. 7.978×10^5 16. $8^{100}, 2^{600}, 3^{500}, 4^{400}, 16^{25}$

LIFE MATHEMATICS

Exercise 2.1

1. (i) 25 (ii) 2 (iii) 8 (iv) 5 (v) ₹1,20,000
2. 162 3. 7000 4. 15 5. 4 more lorries 6. 4 hours
7. A- 30 days B -20 days C-60 days 8. 180 min / 3 hours 9. 2 days 10. 6 days

Exercise 2.2

1. 8 days 2. 48 men 3. 6 days 4. $7\frac{1}{2}$ days 5. 8 days

Challenging Problems

6. 210 soldiers 7. 20 more men 8. 16 days 9. 3 days 10. ₹ 600

GEOMETRY

Exercise 3.1

1. (i) Orthocentre (ii) Centroid (iii) Incentre (iv) Circumcentre (v) 2:1
2. (i) True (ii) True (iii) false
3. (a) (i) Interior (ii) Exterior (iii) On the hypotenuse
(b) (i) Interior (ii) Exterior (iii) On the vertices containing 90°
4. (i) BE (ii) AD (iii) CF

5. AB=5cm 6. $\angle XYM = \angle ZYM = 50°$ 7. 7 cm 8. 10 cm

Exercise 3.2

1. W   2. P   3. 9cm   4. 12 feet   5. 40°

6. (i) 22   (ii) 6   (iii) 16   (iv) 24

STATISTICS

Exercise 4.1

1.	(i) Secondary	(ii)35	(iii)197	(iv) 8	(v) circular	
2.	(i) False	(ii) True	(iii) False	(iv) True	(v) True	
3.	8-22	22-36	36-50	50-64		
6.	(i) 20%	(ii) 75	(iii) 1/4	(iv) 400	(v) 275	(vi) 500
11.	(i) ₹8000	(ii) ₹40000	(iii) ₹12000			

Exercise 4.2

1.	(i) yes	(ii) No	(iii) No	(iv) yes	(v) yes
2.	(i) Frequency	(ii) Proportional	(iii) Histogram	(iv) grouped	
3.	(i) 330	(ii) 150	(iii) No		

9. (d) all the three   10. (c) Frequency   11. (a) range   12. (b) grouped

13. (b) discontinuous   14. (b) Exclusive   15. (c) pie chart   16. (a) continuous

17. (a) frequency polygon   18. (d) histogram

INFORMATION PROCESSING

Exercise 5.1

1. (i) (d) all the above   (ii) (c) Something that I need to purchase   (iii) (b) Compare the choices before buying

2. (i) True   (ii) False   (iii) False   (iv) True

3. (i) 5 chocolate bars for ₹175   (ii) 15 eggs for ₹64.5

4. ₹634   5. ₹39.25   6. buying in first shop is better   7. ₹20   8. ₹301

Exercise 5.2

1. (a) 4 folders

(b)

Folder 1 (MB)	Folder 2 (MB)	Folder 3 (MB)	Folder 4 (MB)	Folder 5 (MB)
53	82	61	41	42
38	16	23	34	
91	98	84	75	42

(c)

Folder 1 (MB)	Folder 2 (MB)	Folder 3 (MB)	Folder 4 (MB)
82	61	53	41
16	38	42	34
			23
98	99	95	98

2. (a) 3 motor cycles

(b)

Motor 1 (kg)	Motor 2 (kg)	Motor 3 (kg)	Motor 4 (kg)
16	20	14	12
8	7	5	-
6	2	-	-
30	29	19	12

(c)

Motor 1(kg)	Motor 2(kg)	Motor 3(kg)
20	16	12
8	14	7
2	-	6
-	-	5
30	30	30

3. (a) 12 m
(b) 6 m

Mathematical Terms

advertisement	விளம்பரம்
alternate	அடுத்தடுத்த
altitudes	செங்குத்து
angle biscetor	கோண இருசமவெட்டி
approximate	தோராயமான
centroid	நடுக்கோட்டு மையம்
circumcentre	சுற்று வட்டமையம்
class interval	பிரிவு இடைவெளி
class size	பிரிவு அளவு
coastant	மாறிலி
coinside	ஒருங்கமைவு
column	நிரல்
caculation	கணக்கிடுதல்
compound variation	கூட்டு மாறல்
congruent	சர்வசமம்
consecutive	அடுத்தடுத்த
consumption	நுகர்வு
cube	கனம்
cube root	கன மூலம்
cuncurrent lines	ஒரு புள்ளி வழிச் செல்லும் கோடுகள்
data	தரவு
decimal	தசம
deposit	முதலீடு
digits	இலக்கங்கள்
direct proportion	நேர் விகிதம்
estimate	மதிப்பீடு
expanded form	விரிவான வடிவம்
exponent	அடுக்கு விதிகள்
extremes	முனை உறுப்புகள்
factor	காரணி
first-fit method	வரிசைப்படி முன்னுரிமை முறை
first-fit decreasing method	சரிவிகித முன்னுரிமை முறை
frequency	நிகழ்வெண்
frequency distribution	நிகழ்வெண் பரவல்
frequency polygon	நிகழ்வு பலகோணம்
grouped data	தொகுக்கப்பட்ட தரவு
histogram	நிகழ்வு செவ்வகம்
incentre	உள்வட்டமையம்
inclusive series	உள்ளடங்கிய தொடர்
increase	ஏறு/ உயர்வு/அதிகம்
individually	தனித்தனியே
interest	வட்டி
intersecting lines	வெட்டும் கோடுகள்
inverse	தலை கீழி
inverse proportion	எதிர் விகிதம்
laws of exponent	அடுக்கு விதிகள்
long division	நீள் வகுத்தல்
lower limit	கீழ் வரம்பு
means	சராசரிகள்
median	நடுக்கோடு
multiple	மடங்கு
multiplication factor	பெருக்கல் காரணி
natural number	இயல் எண்
online	இணைய வழி
operation	செயல்பாடு
orthocentre	செங்கோட்டு மையம்
packing	அடைத்தல்
parallel lines	இணைக்கோடுகள்
pattern	அமைப்பு
perfect square	முழு வர்க்கம்
perpendicular bisector	மையக்குத்துக்கோடு
perpendicular lines	செங்கோட்டு கோடுகள்
pie chart	வட்ட விளக்கப்படம்
point of concurrency	ஒருங்கமைப் புள்ளி
power	படி
primary data	முதல்நிலைத் தரவு
prime factonisation	பகா காரணிப்படுத்துதல்
product	பெருக்கற்பலன்
purchase	கொள்முதல்
pythagoream triplet	பிதாகரஸின் மூன்றன் தொகுதி
range	வீச்சு
remaining work	மீதமுள்ள வேலை
repeated	மீண்டும் மீண்டும்
row	நிரை
scientific notation	அறிவியல் குறியீடு
secondary data	இரண்டாம் நிலைத் தரவு
Shopping	பொருள்களை வாங்குதல்
spending	வாங்குதல்
square	வர்க்கம்
square root	வர்க்கம் மூலம்
successive	தொடர்ச்சியாக
suffix	பின்னொட்டு
tally marks	குறிகள்
upper limit	மேல் வரம்பு

Upper Primary Mathematics - Class 8 Term III
Text Book Development Team

Reviewers
Dr. R. Ramanujam
Professor,
Institute of Mathematical Science,
Chennai.

R. Athmaraman
Educational Consultant,
Association of Mathematics Teachers of India,
Chennai.

Content Writers
G. Kamalanathan
B.T. Assistant,
GHSS, Arpakkam,
Kancheepuram.

K. Gunasekar
B.T. Assistant,
PUMS, Vallavanur West,
Koliyanur Block,
Villupuram.

G.Palani
B.T. Assistant,
GHS, Jagadab,
Krishnagiri.

P. Malarvizhi
B.T. Assistant,
Chennai High School,
Strahans Road, Pattalam, Chennai.

ICT Coordinator
D. Vasuraj
P.G. Assistant, KRM Public School,
Sembiyam, Chennai.

Typist
K. Punitha
Triplicane, Chennai.

Academic Advisor
Dr. Pon. Kumar
Joint Director (Syllabus),
SCERT, Chennai.

Academic Co-ordinator
V. Ilayarani Mohan
Assistant Professor,
SCERT, Chennai.

Art and Design Team
Layout and Illustration
R. Mathan Raj, C. Prasanth,
P. Arun Kamaraj, B. Yogesh,
V. Vinoth Kumar, V. Sridhar
In-House QC
C. Jerald Wilson
Rajesh Thangappan,
Kamatchi Balan Arumugam

Wrapper Design
Kathir Aarumugam

Layout Co-ordinator
Ramesh Munisamy

QR Code Management Team
R. Jaganathan
SGT, PUMS - Ganesapuram, Thiruvannamalai.
S. Albert ValavanBabu,
B.T. Asst., G.H.S, Paramakudi, Ramanathapuram.
M. Murugesan,
B.T. Asst., PUMS, Pethavelankottagam, Thiruvarur

This book has been printed on 80 GSM
Elegant Maplitho paper
Printed by offset at :