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TERM III

VOLUME 2

MATHEMATICS

NOT FOR SALE

Untouchability is Inhuman and a Crime

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Preface

The Trimester pattern has been introduced in Tamil Nadu as a milestone in the history of school education. In this method, a voluminous textbook has been divided into three small booklets, one for each term for easy understanding of concepts. The first and second term Mathematics textbooks have already been prepared and distributed to all schools.The units such as Set Theory, Real Number System, Algebra, Geometry, Coordinate Geometry and Practical Geometry have been included for the first term. Among these, Set theory and Coordinate Geometry have been discussed completely. The second term includes topics such as Algebra, Trigonometry, Statistics, and Practical Geometry.

The Mathematics textbook for the third term includes topics such as Real Number System, Scientific notations of Real Numbers and Logarithms, Geometry, Mensuration, Probability and Graphs. As the concepts of Real Number System, Geometry and other topics in the third term are in continuation of the Term I and Term II, the teachers should enable the students to recall the concepts learnt in the first term and second term while teaching higher level concepts in the third term.

At the end of each unit, FA (a) activities have been suggested. The teachers need to select the appropriate activity for explaining and reinforcing the concepts. The same activities can be used for conducting Formative Assessment (a) also. The teachers are free either to use them as such or they can design their own new activities that are appropriate for their students and school setting with the objective of making them learn and enjoy the beauty of mathematics.

-Textbook team

SYMBOLS

$=$	equal to
$\neq$	not equal to
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\approx$	equivalent to
$\cup$	union
$\cap$	intersection
$\mathbb{U}$	universal Set
$\in$	belongs to
$\notin$	does not belong to
$\subset$	proper subset of
$\subseteq$	subset of or is contained in
$\not\subset$	not a proper subset of
$\nsubseteq$	not a subset of or is not contained in
A' (or) A^c	complement of A
$\varnothing$ (or) $\{ \}$	empty set or null set or void set
$n(A)$	number of elements in the set A
$P(A)$	power set of A
$\|\|\|^{ly}$	similarly
$\mathrm{P}(A)$	probability of the event A

Δ	symmetric difference
$\mathbb{N}$	natural numbers
$\mathbb{W}$	whole numbers
$\mathbb{Z}$	integers
$\mathbb{R}$	real numbers
$\triangle$	triangle
$\angle$	angle
$\perp$	perpendicular to
$\parallel$	parallel to
$\Longrightarrow$	implies
$\therefore$	therefore
$\because$	since (or) because
$\vert\ \vert$	absolute value
$\simeq$	approximately equal to
$\vert$ (or) :	such that
$\equiv$ (or) $\cong$	congruent
$\equiv$	identically equal to
π	pi
$\pm$	plus or minus
$\blacksquare$	end of the proof

CONTENTS

1 REAL NUMBER SYSTEM

Life is good for only two things, discovering mathematics and teaching mathematics

- SIMEON POISSON

Main Targets

- To understand the four basic operations in irrational numbers.
- To rationalise the denominator of the given irrational numbers.

1.1 Introduction

There are two major periods in the historical development of the real number system . The first is the period of classical Greek mathematics in which mathematics first emerged as a deductive science. The second is that of the rigourisation of analysis and the formalisation of mathematics which took place mostly in the 19th century. Between these periods mathematics expanded very much in areas which depended on real numbers.

The meaning of the word 'surd' is twofold – in mathematics it refers to a number that is partly rational, partly irrational.

It is known that Al-Khwarizmi identified surds as something special in mathematics, and this mute quality of theirs. In or around 825AD he referred to the rational numbers as 'audible' and irrational as 'inaudible'. It appears that the first European mathematician to adopt the terminology of surds (surdus means 'deaf' or 'mute' in Latin) was Gherardo of Cremona (c. 1150). It also seems that Fibonacci adopted the same term in 1202 to refer to a number that has no root.

Surd is defined as a positive irrational number of the type 'nth root of x', where it is not possible to find 'nth root of x' where x is a positive rational number.

A surd is a radical that is not evaluated, or cannot be precisely evaluated. The radicand is often a constant, such as the square root of two.

AL-KHWARIZMI
(780 C.E. - 850 C.E.)

Al-Khwarizmi was the most illustratrious and most famous of the ancient Arab Mathematicians. He was likely born in Baghdad, now part of Iraq. The algebra treatise Hisab al-jabr w' al-muqobala was the most famous and significant of all of Al-Khwarizmi's works. He also wrote a treatise on Hindu-Arabic numerals. Another important work by him was his work Sindhind zij on astronomy.

1.2 Surds

We know that $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers. These are square roots of rational numbers, which cannot be expressed as squares of any rational number. $\sqrt[3]{2}$, $\sqrt[3]{3}$, $\sqrt[3]{7}$ etc. are the cube roots of rational numbers, which cannot be expressed as cubes of any rational number. This type of irrational numbers are called surds or radicals.

Key Concept **Surds**

If 'a' is a positive rational number and n is a positive integer such that $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called a 'surd' or a 'radical'.

Notation

The general form of a surd is $\sqrt[n]{a}$

$\sqrt{}$ is called the *radical sign*

n is called the *order* of the radical.

a is called the *radicand*.

order — $\sqrt[n]{a}$ — Radical sign

Radicand

1.2.1 Index Form of a Surd

The index form of a surd $\sqrt[n]{a}$ is $a^{\frac{1}{n}}$

For example, $\sqrt[5]{8}$ can be written in index form as $\sqrt[5]{8} = (8)^{\frac{1}{5}}$

Think and Answer !
$a^{\frac{1}{3}}$ and a^{3} differ. why?

In the following table, the index form, order and radicand of some surds are given.

Surd	Index Form	Order	Radicand
$\sqrt{5}$	$5^{\frac{1}{2}}$	2	5
$\sqrt[3]{14}$	$(14)^{\frac{1}{3}}$	3	14
$\sqrt[4]{7}$	$7^{\frac{1}{4}}$	4	7
$\sqrt{50}$	$(50)^{\frac{1}{2}}$	2	50
$\sqrt[5]{11}$	$(11)^{\frac{1}{5}}$	5	11

Remark If $\sqrt[n]{a}$ is a surd, then

(i) a is a positive rational number. (ii) $\sqrt[n]{a}$ is an irrational number.

In the table given below both the columns A and B have irrational numbers.

A	B
$\sqrt{5}$	$\sqrt{2+\sqrt{3}}$
$\sqrt[3]{7}$	$\sqrt[3]{5+\sqrt{7}}$
$\sqrt[3]{100}$	$\sqrt[3]{10-\sqrt[3]{3}}$
$\sqrt{12}$	$\sqrt[4]{15+\sqrt{5}}$

The numbers in Column A are surds and the numbers in Column B are irrationals.

Thus, every surd is an irrational number, but every irrational number need not be a surd.

1.2.2 Reduction of a Surd to its Simplest Form

We can reduce a surd to its simplest form.

For example, consider the surd $\sqrt{50}$

$$\text{Now } \sqrt{50} = \sqrt{25\times 2} = \sqrt{25}\,\sqrt{2} = \sqrt{5^2}\,\sqrt{2} = 5\sqrt{2}$$

Thus $5\sqrt{2}$ is the simplest form of $\sqrt{50}$.

1.2.3 Like and Unlike Surds

Surds in their simplest form are called like surds if their order and radicand are the same. Otherwise the surds are called unlike surds.

For example,

(i) $\sqrt{5}$, $4\sqrt{5}$, $-6\sqrt{5}$ are like surds. (ii) $\sqrt{10}$, $\sqrt[3]{3}$, $\sqrt[4]{5}$, $\sqrt[3]{81}$ are unlike surds.

1.2.4 Pure surds

A Surd is called a pure surd if its rational coefficient is unity

For example, $\sqrt{3}$, $\sqrt[3]{5}$, $\sqrt[4]{12}$, $\sqrt{80}$ are pure surds.

1.2.5 Mixed Surds

A Surd is called a mixed if its rational coefficient is other than unity

For example, $2\sqrt{3}$, $5\sqrt[3]{5}$, $3\sqrt[4]{12}$ are mixed surds.

A mixed surd can be converted into a pure surd and a pure surd may or may not be converted into a mixed surd.

For example,

(i) $\sqrt{80} = \sqrt{16\times 5} = 4\sqrt{5}$ (ii) $3\sqrt{2} = \sqrt{3^2\times 2} = \sqrt{9\times 2} = \sqrt{18}$

(iii) $\sqrt{17}$ is a pure surd, but it cannot be converted into a mixed surd.

Laws of Radicals
For positive integers m, n and positive rational numbers a, b we have
(i) $(\sqrt[n]{a})^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$
(iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

Using (i) we have $(\sqrt{a})^2 = a$, $\sqrt[3]{a^3} = (\sqrt[3]{a})^3 = a$

Example 1.1

Convert the following surds into index form.

(i) $\sqrt{7}$ (ii) $\sqrt[4]{8}$ (iii) $\sqrt[3]{6}$ (iv) $\sqrt[8]{12}$

Solution In index form we write the given surds as follows

(i) $\sqrt{7} = 7^{\frac{1}{2}}$ (ii) $\sqrt[4]{8} = 8^{\frac{1}{4}}$ (iii) $\sqrt[3]{6} = 6^{\frac{1}{3}}$ (iv) $\sqrt[8]{12} = (12)^{\frac{1}{8}}$

Example 1.2

Express the following surds in its simplest form.

(i) $\sqrt[3]{32}$ (ii) $\sqrt{63}$ (iii) $\sqrt{243}$ (iv) $\sqrt[3]{256}$

Solution

(i) $\sqrt[3]{32} = \sqrt[3]{8 \times 4} = \sqrt[3]{8} \times \sqrt[3]{4} = \sqrt[3]{2^3} \times \sqrt[3]{4} = 2\sqrt[3]{4}$

(ii) $\sqrt{63} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3\sqrt{7}$

(iii) $\sqrt{243} = \sqrt{81 \times 3} = \sqrt{81} \times \sqrt{3} = \sqrt{9^2} \times \sqrt{3} = 9\sqrt{3}$

(iv) $\sqrt[3]{256} = \sqrt[3]{64 \times 4} = \sqrt[3]{64} \times \sqrt[3]{4} = \sqrt[3]{4^3} \times \sqrt[3]{4} = 4\sqrt[3]{4}$

Example 1.3

Express the following mixed surds into pure surds.

(i) $16\sqrt{2}$ (ii) $3\sqrt[3]{2}$ (iii) $2\sqrt[4]{5}$ (iv) $6\sqrt{3}$

Solution

(i) $16\sqrt{2} = \sqrt{16^2} \times \sqrt{2}$ $\quad (\because 16 = \sqrt{16^2})$

$= \sqrt{16^2 \times 2} = \sqrt{256 \times 2} = \sqrt{512}$

(ii) $3\sqrt[3]{2} = \sqrt[3]{3^3 \times 2}$ $\quad (\because 3 = \sqrt[3]{3^3})$

$= \sqrt[3]{27 \times 2} = \sqrt[3]{54}$

(iii) $2\sqrt[4]{5} = \sqrt[4]{2^4 \times 5}$ $\quad (\because 2 = \sqrt[4]{2^4})$

$= \sqrt[4]{16 \times 5} = \sqrt[4]{80}$

(iv) $6\sqrt{3} = \sqrt{6^2 \times 3}$ $\quad (\because 6 = \sqrt{6^2})$

$= \sqrt{36 \times 3} = \sqrt{108}$

Example 1.4

Identify whether $\sqrt{32}$ is rational or irrational.

Solution $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$

4 is a rational number and $\sqrt{2}$ is an irrational number.

$\therefore 4\sqrt{2}$ is an irrational number and hence $\sqrt{32}$ is an irrational number.

Example 1.5

Identify whether the following numbers are rational or irrational.

(i) $3 + \sqrt{3}$ (ii) $(4 + \sqrt{2}) - (4 - \sqrt{3})$ (iii) $\dfrac{\sqrt{18}}{2\sqrt{2}}$ (iv) $\sqrt{19} - (2 + \sqrt{19})$
(v) $\dfrac{2}{\sqrt{3}}$ (vi) $\sqrt{12} \times \sqrt{3}$

Solution

(i) $3 + \sqrt{3}$

3 is a rational number and $\sqrt{3}$ is irrational. Hence, $3 + \sqrt{3}$ is irrational.

(ii) $(4 + \sqrt{2}) - (4 - \sqrt{3})$

$= 4 + \sqrt{2} - 4 + \sqrt{3} = \sqrt{2} + \sqrt{3}$, is irrational.

(iii) $\dfrac{\sqrt{18}}{2\sqrt{2}} = \dfrac{\sqrt{9 \times 2}}{2\sqrt{2}} = \dfrac{\sqrt{9} \times \sqrt{2}}{2\sqrt{2}} = \dfrac{3}{2}$, is rational.

(iv) $\sqrt{19} - (2 + \sqrt{19}) = \sqrt{19} - 2 - \sqrt{19} = -2$, is rational.

(v) $\dfrac{2}{\sqrt{3}}$ here 2 is rational and $\sqrt{3}$ is irrational. Hence, $\dfrac{2}{\sqrt{3}}$ is irrational.

(vi) $\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = 6$, is rational.

1.3 Four Basic Operations on Surds

1.3.1 Addition and Subtraction of Surds

Like surds can be added and subtracted.

Example 1.6

Simplify

(i) $10\sqrt{2} - 2\sqrt{2} + 4\sqrt{32}$

(ii) $\sqrt{48} - 3\sqrt{72} - \sqrt{27} + 5\sqrt{18}$

(iii) $\sqrt[3]{16} + 8\sqrt[3]{54} - \sqrt[3]{128}$

Solution

(i) $10\sqrt{2} - 2\sqrt{2} + 4\sqrt{32}$

$$= 10\sqrt{2} - 2\sqrt{2} + 4\sqrt{16 \times 2}$$

$$= 10\sqrt{2} - 2\sqrt{2} + 4 \times 4 \times \sqrt{2}$$

$$= (10 - 2 + 16)\sqrt{2} = 24\sqrt{2}$$

(ii) $\sqrt{48} - 3\sqrt{72} - \sqrt{27} + 5\sqrt{18}$

$$= \sqrt{16 \times 3} - 3\sqrt{36 \times 2} - \sqrt{9 \times 3} + 5\sqrt{9 \times 2}$$

$$= \sqrt{16}\sqrt{3} - 3\sqrt{36}\sqrt{2} - \sqrt{9}\sqrt{3} + 5\sqrt{9}\sqrt{2}$$

$$= 4\sqrt{3} - 18\sqrt{2} - 3\sqrt{3} + 15\sqrt{2}$$

$$= (-18 + 15)\sqrt{2} + (4 - 3)\sqrt{3} = -3\sqrt{2} + \sqrt{3}$$

(iii) $\sqrt[3]{16} + 8\sqrt[3]{54} - \sqrt[3]{128}$

$$= \sqrt[3]{8 \times 2} + 8\sqrt[3]{27 \times 2} - \sqrt[3]{64 \times 2}$$

$$= \sqrt[3]{8}\sqrt[3]{2} + 8\sqrt[3]{27}\sqrt[3]{2} - \sqrt[3]{64}\sqrt[3]{2}$$

$$= 2\sqrt[3]{2} + 8 \times 3 \times \sqrt[3]{2} - 4\sqrt[3]{2}$$

$$= 2\sqrt[3]{2} + 24\sqrt[3]{2} - 4\sqrt[3]{2}$$

$$= (2 + 24 - 4)\sqrt[3]{2} = 22\sqrt[3]{2}$$

1.3.2 Multiplication of Surds

Product of two like surds can simplified using the following law.

$$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$$

Example 1.7

Multiply (i) $\sqrt[3]{13} \times \sqrt[3]{5}$ (ii) $\sqrt[4]{32} \times \sqrt[4]{8}$

Solution

(i) $\sqrt[3]{13} \times \sqrt[3]{5} = \sqrt[3]{13 \times 5} = \sqrt[3]{65}$

(ii) $\sqrt[4]{32} \times \sqrt[4]{8} = \sqrt[4]{32 \times 8}$

$$= \sqrt[4]{2^5 \times 2^3} = \sqrt[4]{2^8} = \sqrt[4]{2^4 \times 2^4} = 2 \times 2 = 4$$

1.3.3 Division of Surds

Like surds can be divided using the law

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

Example 1.8

Simplify (i) $15\sqrt{54} \div 3\sqrt{6}$ (ii) $\sqrt[3]{128} \div \sqrt[3]{64}$

Solution

(i) $15\sqrt{54} \div 3\sqrt{6}$

$$= \frac{15\sqrt{54}}{3\sqrt{6}} = 5\sqrt{\frac{54}{6}} = 5\sqrt{9} = \quad 5 \times 3 = 15$$

(ii) $\sqrt[3]{128} \div \sqrt[3]{64}$

$$= \frac{\sqrt[3]{128}}{\sqrt[3]{64}} = \sqrt[3]{\frac{128}{64}} = \sqrt[3]{2}$$

Note When the order of the surds are different, we convert them to the same order and then multiplication or division is carried out.

Result $\sqrt[n]{a} = \sqrt[m]{a^{\frac{m}{n}}}$

For example, (i) $\sqrt[3]{5} = \sqrt[12]{5^{\frac{12}{3}}} = \sqrt[12]{5^4}$ (ii) $\sqrt[4]{11} = \sqrt[8]{11^{\frac{8}{4}}} = \sqrt[8]{11^2}$

1.3.4 Comparison of Surds

Irrational numbers of the same order can be compared. Among the irrational numbers of same order, the greatest irrational number is the one with the largest radicand.

If the order of the irrational numbers are not the same, we first convert them to the same order. Then, we just compare the radicands.

Example 1.9

Convert the irrational numbers $\sqrt{3}$, $\sqrt[3]{4}$, $\sqrt[4]{5}$ to the same order.

Solution The orders of the given irrational numbers are 2, 3 and 4.

LCM of 2, 3 and 4 is 12

$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Example 1.10

Which is greater ? $\sqrt[4]{5}$ or $\sqrt[3]{4}$

Solution The orders of the given irrational numbers are 3 and 4.

We have to convert each of the irrational number to an irrational number of the same order.

LCM of 3 and 4 is 12. Now we convert each irrational number as of order 12.

$\sqrt[4]{5} = \sqrt[12]{5^3} = \sqrt[12]{125}$

$\sqrt[3]{4} = \sqrt[12]{4^4} = \sqrt[12]{256}$

$\therefore \sqrt[12]{256} > \sqrt[12]{125} \quad \Rightarrow \sqrt[3]{4} > \sqrt[4]{5}$

Example 1.11

Write the irrational numbers $\sqrt[3]{2}$, $\sqrt[4]{4}$, $\sqrt{3}$ in

(i) ascending order (ii) descending order

Solution The orders of the irrational numbers $\sqrt[3]{2}, \sqrt[4]{4}$ and $\sqrt{3}$ are 3, 4 and 2 respectively LCM of 2, 3, and 4 is 12. Now, we convert each irrational number as of order 12.

$$\sqrt[3]{2} = \sqrt[12]{2^4} = \sqrt[12]{16}$$

$$\sqrt[4]{4} = \sqrt[12]{4^3} = \sqrt[12]{64}$$

$$\sqrt{3} = \sqrt[12]{3^6} = \sqrt[12]{729}$$

$\therefore$ Ascending order: $\sqrt[3]{2}$, $\sqrt[4]{4}$, $\sqrt{3}$

Descending order: $\sqrt{3}$, $\sqrt[4]{4}$, $\sqrt[3]{2}$.

Exercise 1.1

1. Identify which of the following are surds and which are not with reasons.

   (i) $\sqrt{8}\times\sqrt{6}$ (ii) $\sqrt{90}$ (iii) $\sqrt{180}\times\sqrt{5}$ (iv) $4\sqrt{5}\div\sqrt{8}$ (v) $\sqrt[3]{4}\times\sqrt[3]{16}$

2. Simplify

   (i) $(10+\sqrt{3})(2+\sqrt{5})$ (ii) $(\sqrt{5}+\sqrt{3})^2$

   (iii) $(\sqrt{13}-\sqrt{2})(\sqrt{13}+\sqrt{2})$ (iv) $(8+\sqrt{3})(8-\sqrt{3})$

3. Simplify the following.

   (i) $5\sqrt{75}+8\sqrt{108}-\frac{1}{2}\sqrt{48}$ (ii) $7\sqrt[3]{2}+6\sqrt[3]{16}-\sqrt[3]{54}$

   (iii) $4\sqrt{72}-\sqrt{50}-7\sqrt{128}$ (iv) $2\sqrt[3]{40}+3\sqrt[3]{625}-4\sqrt[3]{320}$

4. Express the following surds in its simplest form.

   (i) $\sqrt[3]{108}$ (ii) $\sqrt{98}$ (iii) $\sqrt{192}$ (iv) $\sqrt[3]{625}$

5. Express the following as pure surds.

   (i) $6\sqrt{5}$ (ii) $5\sqrt[3]{4}$ (iii) $3\sqrt[4]{5}$ (iv) $\frac{3}{4}\sqrt{8}$

6. Simplify the following.

   (i) $\sqrt{5}\times\sqrt{18}$ (ii) $\sqrt[3]{7}\times\sqrt[3]{8}$ (iii) $\sqrt[4]{8}\times\sqrt[4]{12}$ (iv) $\sqrt[3]{3}\times\sqrt[6]{5}$

   (v) $3\sqrt{35}\div 2\sqrt{7}$ (vi) $\sqrt[4]{48}\div\sqrt[8]{72}$

7. Which is greater ?

   (i) $\sqrt{2}$ or $\sqrt[3]{3}$ (ii) $\sqrt[3]{3}$ or $\sqrt[4]{4}$ (iii) $\sqrt{3}$ or $\sqrt[4]{10}$

8. Arrange in descending and ascending order.

   (i) $\sqrt[4]{5}, \sqrt{3}, \sqrt[3]{4}$ (ii) $\sqrt[3]{2}, \sqrt[3]{4}, \sqrt[4]{4}$ (iii) $\sqrt[3]{2}, \sqrt[9]{4}, \sqrt[6]{3}$

1.4 Rationalization of Surds

Rationalization of Surds

When the denominator of an expression contains a term with a square root or a number under radical sign, the process of converting into an equivalent expression whose denominator is a rational number is called rationalizing the denominator.

If the product of two irrational numbers is rational, then each one is called the **rationalizing factor** of the other.

Let a and b be integers and x, y be positive integers. Then

Remark

(i) $(a+\sqrt{x})$ and $(a-\sqrt{x})$ are rationalizing factors of each other.

(ii) $(a+b\sqrt{x})$ and $(a-b\sqrt{x})$ are rationalizing factors of each other.

(iii) $\sqrt{x}+\sqrt{y}$ and $\sqrt{x}-\sqrt{y}$ are rationalizing factors of each other.

(iv) $a+\sqrt{b}$ is also called the conjugate of $a-\sqrt{b}$ and $a-\sqrt{b}$ is called the conjugate of $a+\sqrt{b}$.

(v) For rationalizing the denominator of a number, we multiply its numerator and denominator by its rationalizing factor.

Example 1.12

Rationalize the denominator of $\frac{2}{\sqrt{3}}$

Solution Multiplying the numerator and denominator of the given number by $\sqrt{3}$, we get

$$\frac{2}{\sqrt{3}} = \frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Example 1.13

Rationalize the denominator of $\frac{1}{5+\sqrt{3}}$

Solution The denominator is $5+\sqrt{3}$. Its conjugate is $5-\sqrt{3}$ or the rationalizing factor is $5-\sqrt{3}$.

$$\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}}\times\frac{5-\sqrt{3}}{5-\sqrt{3}}$$

$$= \frac{5-\sqrt{3}}{5^2-(\sqrt{3})^2} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

Example 1.14

Simplify $\frac{1}{8-2\sqrt{5}}$ by rationalizing the denominator.

Solution Here the denominator is $8-2\sqrt{5}$. The rationalizing factor is $8+2\sqrt{5}$

$$\frac{1}{8-2\sqrt{5}} = \frac{1}{8-2\sqrt{5}} \times \frac{8+2\sqrt{5}}{8+2\sqrt{5}}$$

$$= \frac{8+2\sqrt{5}}{8^2-(2\sqrt{5})^2} \quad = \frac{8+2\sqrt{5}}{64-20}$$

$$= \frac{8+2\sqrt{5}}{44} \quad = \frac{2(4+\sqrt{5})}{44} = \frac{4+\sqrt{5}}{22}$$

Example 1.15

Simplify $\frac{1}{\sqrt{3}+\sqrt{5}}$ by rationalizing the denominator.

Solution Here the denominator is $\sqrt{3}+\sqrt{5}$. So, the rationalizing factor is $\sqrt{3}-\sqrt{5}$

$$\frac{1}{\sqrt{3}+\sqrt{5}} = \frac{1}{\sqrt{3}+\sqrt{5}} \times \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}$$

$$= \frac{\sqrt{3}-\sqrt{5}}{(\sqrt{3})^2-(\sqrt{5})^2} \quad = \frac{\sqrt{3}-\sqrt{5}}{3-5}$$

$$= \frac{\sqrt{3}-\sqrt{5}}{-2} \quad = \frac{\sqrt{5}-\sqrt{3}}{2}$$

Example 1.16

If $\frac{\sqrt{7}-1}{\sqrt{7}+1}+\frac{\sqrt{7}+1}{\sqrt{7}-1} = a+b\sqrt{7}$, find the values of a and b.

Solution

$$\frac{\sqrt{7}-1}{\sqrt{7}+1}+\frac{\sqrt{7}+1}{\sqrt{7}-1} = \frac{\sqrt{7}-1}{\sqrt{7}+1}\times\frac{\sqrt{7}-1}{\sqrt{7}-1}+\frac{\sqrt{7}+1}{\sqrt{7}-1}\times\frac{\sqrt{7}+1}{\sqrt{7}+1}$$

$$= \frac{(\sqrt{7}-1)^2}{(\sqrt{7})^2-1}+\frac{(\sqrt{7}+1)^2}{(\sqrt{7})^2-1}$$

$$= \frac{7+1-2\sqrt{7}}{7-1}+\frac{7+1+2\sqrt{7}}{7-1}$$

$$= \frac{8-2\sqrt{7}}{6}+\frac{8+2\sqrt{7}}{6}$$

$$= \frac{8-2\sqrt{7}+8+2\sqrt{7}}{6}$$

$$= \frac{16}{6} = \frac{8}{3}+0\sqrt{7}$$

$$\therefore \frac{8}{3}+0\sqrt{7} = a+b\sqrt{7} \implies a = \frac{8}{3},\ b = 0.$$

Exmaple 1.17

If $x = 1 + \sqrt{2}$, find $\left(x - \frac{1}{x}\right)^2$

Solution $x = 1 + \sqrt{2}$

$$\Longrightarrow \quad \frac{1}{x} = \frac{1}{1+\sqrt{2}}$$

$$= \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1} = -(1-\sqrt{2})$$

$$\therefore\ x - \frac{1}{x} = (1+\sqrt{2}) - \{-(1-\sqrt{2})\}$$

$$= 1 + \sqrt{2} + 1 - \sqrt{2} = 2$$

Hence, $\left(x - \frac{1}{x}\right)^2 = 2^2 = 4.$

Exercise 1.2

1. Write the rationalizing factor of the following.

   (i) $3\sqrt{2}$ (ii) $\sqrt{7}$ (iii) $\sqrt{75}$ (iv) $2\sqrt[3]{5}$

   (v) $5 - 4\sqrt{3}$ (vi) $\sqrt{2} + \sqrt{3}$ (vii) $\sqrt{5} - \sqrt{2}$ (viii) $2 + \sqrt{3}$

2. Rationalize the denominator of the following

   (i) $\frac{3}{\sqrt{5}}$ (ii) $\frac{2}{3\sqrt{3}}$ (iii) $\frac{1}{\sqrt{12}}$ (iv) $\frac{2\sqrt{7}}{\sqrt{11}}$ (v) $\frac{3\sqrt[3]{5}}{\sqrt[3]{9}}$

3. Simplify by rationalizing the denominator.

   (i) $\frac{1}{11+\sqrt{3}}$ (ii) $\frac{1}{9+3\sqrt{5}}$ (iii) $\frac{1}{\sqrt{11}+\sqrt{13}}$ (iv) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$ (v) $\frac{3-\sqrt{3}}{2+5\sqrt{3}}$

4. Find the values of the following upto 3 decimal places. Given that $\sqrt{2} \approx 1.414$, $\sqrt{3} \approx 1.732$, $\sqrt{5} \approx 2.236$, $\sqrt{10} \approx 3.162$.

   (i) $\frac{1}{\sqrt{2}}$ (ii) $\frac{6}{\sqrt{3}}$ (iii) $\frac{5-\sqrt{3}}{\sqrt{3}}$ (iv) $\frac{\sqrt{10}-\sqrt{5}}{\sqrt{2}}$

   (v) $\frac{3-\sqrt{5}}{3+2\sqrt{5}}$ (vi) $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ (vii) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$ (viii) $\frac{1}{\sqrt{10}+\sqrt{5}}$

5. If $\frac{5+\sqrt{6}}{5-\sqrt{6}} = a + b\sqrt{6}$ find the values of a and b.

6. If $\frac{(\sqrt{3}+1)^2}{4-2\sqrt{3}} = a + b\sqrt{3}$ find the values of a and b.

7. If $\dfrac{\sqrt{5}+1}{\sqrt{5}-1}+\dfrac{\sqrt{5}-1}{\sqrt{5}+1}=a+b\sqrt{5}$, find the values of a and b.

8. If $\dfrac{4+\sqrt{5}}{4-\sqrt{5}}-\dfrac{4-\sqrt{5}}{4+\sqrt{5}}=a+b\sqrt{5}$, find the values of a and b.

9. If $x=2+\sqrt{3}$, find the values of $x^2+\dfrac{1}{x^2}$.

10. If $x=\sqrt{3}+1$, find the values of $\left(x-\dfrac{2}{x}\right)^2$.

1.5 Division Algorithm

A series of well defined steps which gives a procedure for solving a problem is called an algorithm. In this section we state an important property of integers called the division algorithm.

As we know from our earlier classes, when we divide one integer by another non-zero integer, we get an integer quotient and a remainder (generally a rational number). Then we write

$$\text{Fraction} = \text{Quotient} + \frac{\text{remainder}}{\text{divisor}}$$

For example, $$\frac{13}{5} = 2 + \frac{3}{5} \qquad (1)$$

We can rephrase this division, totally in terms of integers, without reference to the division operation.

$13 = 2(5) + 3$

We observe that this expression is obtained by multiplying (1) by the divisor 5. We refer to this way of writing a division of integers as the division algorithm.

> If a and b are any two positive integers, then there exist two non-negative integers q and r such that $a = bq + r$, $0 \le r < b$.

In the above statement q (or) r can be zero.

Example 1.18

Using division algorithm find the quotient and remainder of the following pairs.

(i) 19, 5 (ii) 3, 13 (iii) 30, 6

Solution

(i) 19, 5

We write the given pair in the form $a = bq + r$, $0 \le r < b$ as follows.

$19 = 5(3) + 4$ [5 divides 19 three time and leaves the remainder 4]

$\therefore$ quotient = 3; remainder = 4

(ii) 3, 13

We write the given pair in the form $a = bq + r,\ 0 \le r < b$ as follows.

$3 = 13(0) + 3$

$\therefore$ quotient = 0; remainder = 3

(iii) 30, 6

We write the given pair 30, 6 in the form $a = bq + r,\ 0 \le r < b$ as follows.

$30 = 6(5) + 0$ [6 divides 30 five times and leaves the reaminder 0]

$\therefore$ quotient = 5; remainder = 0

Exercise 1.3

1. Using division algorithm, find the quotient and remainder of the following pairs.

(i) 10, 3 (ii) 5, 12 (iii) 27, 3

Exercise 1.4

Multiple Choice Questions

1. Which one of the following is not a surd?

(A) $\sqrt[3]{8}$ (B) $\sqrt[3]{30}$ (C) $\sqrt[5]{4}$ (D) $\sqrt[8]{3}$

2. The simplest form of $\sqrt{50}$ is

(A) $5\sqrt{10}$ (B) $5\sqrt{2}$ (C) $10\sqrt{5}$ (D) $25\sqrt{2}$

3. $\sqrt[4]{11}$ is equal to

(A) $\sqrt[8]{11^2}$ (B) $\sqrt[8]{11^4}$ (C) $\sqrt[8]{11^8}$ (D) $\sqrt[8]{11^6}$

4. $\frac{2}{\sqrt{2}}$ is equal to

(A) $2\sqrt{2}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) 2

5. The ratioanlising factor of $\frac{5}{\sqrt[3]{3}}$ is

(A) $\sqrt[3]{6}$ (B) $\sqrt[3]{3}$ (C) $\sqrt[3]{9}$ (D) $\sqrt[3]{27}$

6. Which one of the following is not true?

(A) $\sqrt{2}$ is an irrational number

(B) $\sqrt{17}$ is a irrational number

(C) $0.101100111000011110\cdots$ is an irrational number

(D) $\sqrt[4]{16}$ is an irrational number

7. The order and radicand of the surd $\sqrt[8]{12}$ are respectively

(A) 8,12 (B) 12,8 (C) 16,12 (D) 12,16

8. The surd having radicand 9 and order 3 is

(A) $\sqrt[9]{3}$ (B) $\sqrt[3]{27}$ (C) $\sqrt[3]{9}$ (D) $\sqrt[3]{81}$

9. $5\sqrt[3]{3}$ represents the pure surd

(A) $\sqrt[3]{15}$ (B) $\sqrt[3]{375}$ (C) $\sqrt[3]{75}$ (D) $\sqrt[3]{45}$

10. Which one of the following is not true?

(A) $\sqrt{2}$ is an irrational number

(B) If a is a rational number and $\sqrt{b}$ is an irrational number then $a\sqrt{b}$ is irrational number

(C) Every surd is an irrational number.

(D) The square root of every positive integer is always irrational

11. Which one of the following is not true?

(A) When x is not a perfect square, $\sqrt{x}$ is an irrational number

(B) The index form of $\sqrt[m]{x^n}$ is $x^{\frac{n}{m}}$

(C) The radical form of $\left(x^{\frac{1}{n}}\right)^{\frac{1}{m}}$ is $\sqrt[mn]{x}$

(D) Every real number is an irrational number

12. $(\sqrt{5}-2)(\sqrt{5}+2)$ is equal to

(A) 1 (B) 3 (C) 23 (D) 21

Points to Remember

- If 'a' is a positive rational number and n is a positive integer such that $\sqrt[n]{a}$ is an irrational number, then $\sqrt[n]{a}$ is called a 'surd' or a 'radical'.

- For positive integers m, n and positive rational numbers a, b we have

  (i) $(\sqrt[n]{a})^n = a = \sqrt[n]{a^n}$ (ii) $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

  (iii) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ (iv) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

- When the denominator of an expression contains a term with a square root or a number under radical sign, the process of converting to an equivalent expression whose denominator is a rational number is called rationalizing the denominator.

- If the product of two irrational numbers is rational, then each one is called the rationalizing factor of the other.

- If a and b are any two positive integers, there exist two non-negative integers q and r such that $a = bq + r$, $0 \le r < b$. (Division Algorithm)

Activity 1

Complete the following table.

Sl.No.	Order	Radicand	Surd
1.	3	7	
2.		8	$\sqrt[6]{8}$
3.	4		$\sqrt[4]{26}$
4.			$\sqrt[3]{16}$

Activity 2

$\sqrt[3]{19}$ $\quad 3\sqrt[5]{7}$ $\quad \sqrt{21}$

$\sqrt[7]{89}$ $\quad \sqrt[3]{125}$

$8\sqrt{57}$ $\quad \sqrt[4]{256}$ $\quad \sqrt[5]{31}$

Use the diagram to complete the following :

1. The order and radicand of $\sqrt[3]{19}$ are respectively ...., ...
2. ...., ... are not surds.
3. ...., ... are like surds.
4. $3\sqrt[5]{7}$ and $8\sqrt{57}$ are .... surds. Also they are .... surds.
5. The index form of the surd $\sqrt[7]{89}$ is ....

Exercise 1.1

1. (i) Surd (ii) Surd (iii) not a surd (iv) Surd (v) not a surd

2. (i) $20 + 10\sqrt{5} + 2\sqrt{3} + \sqrt{15}$ (ii) $8 + 2\sqrt{15}$ (iii) 11 (iv) 61

3. (i) $71\sqrt{3}$ (ii) $16\sqrt[3]{2}$ (iii) $-37\sqrt{2}$ (iv) $3\sqrt[3]{5}$ **4.** (i) $3\sqrt[3]{4}$ (ii) $7\sqrt{2}$ (iii) $8\sqrt{3}$ (iv) $5\sqrt[3]{5}$

5. (i) $\sqrt{180}$ (ii) $\sqrt[3]{500}$ (iii) $\sqrt[4]{405}$ (iv) $\sqrt{\dfrac{9}{2}}$ **6.** (i) $3\sqrt{10}$ (ii) $2\sqrt[3]{7}$ (iii) $2\sqrt[4]{6}$

(iv) $\sqrt[6]{45}$ (v) $\dfrac{3}{2}\sqrt{5}$ (vi) $\sqrt[8]{32}$ **7.** (i) $\sqrt[3]{3} > \sqrt{2}$ (ii) $\sqrt[3]{3} > \sqrt[4]{4}$ (iii) $\sqrt[4]{10} > \sqrt{3}$

8. (i) descending order : $\sqrt{3},\ \sqrt[3]{4},\ \sqrt[4]{5}$, ascending order : $\sqrt[4]{5},\ \sqrt[3]{4},\ \sqrt{3}$

(ii) descending order : $\sqrt[3]{4},\ \sqrt[4]{4},\ \sqrt[3]{2}$, ascending order : $\sqrt[3]{2},\ \sqrt[4]{4},\ \sqrt[3]{4}$

(iii) descending order : $\sqrt[3]{2},\ \sqrt[6]{3},\ \sqrt[9]{4}$, ascending order : $\sqrt[9]{4},\ \sqrt[6]{3},\ \sqrt[3]{2}$

Exercise 1.2

1. (i) $\sqrt{2}$ (ii) $\sqrt{7}$ (iii) $\sqrt{3}$ (iv) $\sqrt[3]{25}$ (v) $5 + 4\sqrt{3}$ (vi) $\sqrt{2} - \sqrt{3}$ (vii) $\sqrt{5} + \sqrt{2}$

(viii) $2 - \sqrt{3}$ **2.** (i) $\dfrac{3\sqrt{5}}{5}$ (ii) $\dfrac{2\sqrt{3}}{9}$ (iii) $\dfrac{\sqrt{3}}{6}$ (iv) $\dfrac{2\sqrt{77}}{11}$ (v) $\sqrt[3]{15}$

3. (i) $\dfrac{11 - \sqrt{3}}{118}$ (ii) $\dfrac{3 - \sqrt{5}}{12}$ (iii) $\dfrac{\sqrt{13} - \sqrt{11}}{2}$ (iv) $\dfrac{3 + \sqrt{5}}{2}$ (v) $\dfrac{17\sqrt{3} - 21}{71}$

4. (i) 0.707 (ii) 3.464 (iii) 1.887 (iv) 0.655 (v) 0.102 (vi) 4.441

(vii) 3.732 (viii) 0.185 **5.** $a = \dfrac{31}{19},\ b = \dfrac{10}{19}$ **6.** $a = 7,\ b = 4$

7. $a = 3,\ b = 0$ **8.** $a = 0,\ b = \dfrac{16}{11}$ **9.** 14 **10.** 4

Exercise 1.3

1. 3, 1 **2.** 0, 5 **3.** 9, 0

Exercise 1.4

1. A **2.** B **3.** A **4.** B **5.** C **6.** D **7.** A **8.** C **9.** B **10.** D **11.** D **12.** A

2 SCIENTIFIC NOTATIONS OF REAL NUMBERS AND LOGARITHMS

Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers.... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances

- JOHN NAPIER

Main Targets

- To represent the number in Scientific Notation.
- To convert exponential form to logarithmic form and vice-versa.
- To understand the rules of logarithms.
- To apply the rules and to use logarithmic table.
- Base system.

2.1 Scientific Notation

Scientists, engineers and technicians use scientific notations when working with very large or very small numbers. The speed of light is 29,900,000,000 centimeter per second; the distance of sun from earth is about 92,900,000 miles; the mass of an electron is 0.000549 atomic mass units. It is easier to express these numbers in a shorter way called *Scientific Notation*, thus avoiding the writing of many zeros and transposition errors.

For example,

$$29{,}900{,}000{,}000 = 299 \times 10^{8} = 2.99 \times 10^{10}$$

$$92{,}900{,}000 = 929 \times 10^{5} = 9.29 \times 10^{7}$$

$$0.000549 = \frac{549}{1000000} = \frac{5.49}{10000} = 5.49 \times 10^{-4}$$

JOHN NAPIER
(1550 - 1617)

John Napier was born in the Tower of Merciston, which is now at the center of Napier University's Merchiston campus, in 1550. Napier, who is credited with the invention of logarithms, only considered the study of mathematics as a hobby. Napier is placed within a short lineage of mathematical thinkers beginning with Archimedes and more recent geniuses, Sir Issac Newton and Albert Einstein.

That is, the very large or very small numbers are expressed as the product of a decimal number $1 \leq a < 10$ and some integral power of 10.

Key Concept	**Scientific Notation**
A number N is in *scientific notation* when it is expressed as the product of a decimal number between 1 and 10 and some integral power of 10. $N = a \times 10^n$, where $1 \leq a < 10$ and n is an integer.	

To transform numbers from decimal notation to scientific notation, the laws of exponents form the basis for calculations using powers. Let m and n be natural numbers and a is a real number. The laws of exponents are given below:

(i) $a^m \times a^n = a^{m+n}$ (Product law)

(ii) $\dfrac{a^m}{a^n} = a^{m-n}$ (Quotient law)

(iii) $(a^m)^n = a^{mn}$ (Power law)

(iv) $a^m \times b^m = (a \times b)^m$ (Combination law)

For $a \neq 0$, we define $a^{-m} = \dfrac{1}{a^m}$, and $a^0 = 1$.

2.1.1 Writing a Number in Scientific Notation

The steps for converting a number to scientific notation are as follows:

Step 1: Move the decimal point so that there is only one non - zero digit to its left.

Step 2: Count the number of digits between the old and new decimal point. This gives n, the power of 10.

Step 3: If the decimal is shifted to the left, the exponent n is positive. If the decimal is shifted to the right, the exponent n is negative.

Example 2.1

Express 9781 in scientific notation.

Solution In integers, the decimal point at the end is usually omitted.

$$9\underset{3}{\;}7\underset{2}{\;}8\underset{1}{\;}1\,.$$

The decimal point is to be moved 3 places to the left of its original position. So the power of 10 is 3.

$\therefore\ 9781 = 9.781 \times 10^3$

Example 2.2

Express $0 \cdot 000432078$ in scientific notation.

Solution $0\ .\ \underset{1}{0}\ \underset{2}{0}\ \underset{3}{0}\ \underset{4}{4}\ 3\ 2\ 0\ 7\ 8$

The decimal point is to be moved four places to the right of its original position. So the power of 10 is – 4

$\therefore\ 0.000432078 = 4.32078 \times 10^{-4}$

Remark Observe that while converting a given number into the scientific notation, if the decimal point is moved p places to the left, then this movement is compensated by the factor 10^{p} ; and if the decimal point is moved r places to the right, then this movement is compensated by the factor 10^{-r}.

Example 2.3

Write the following numbers in scientific notation.

(i) 9345 (ii) 205852 (iii) 3449098.96

(iv) 0.0063 (v) 0.00008035 (vi) 0.000108

Solution

(i) $9345 = 9\ \underset{3}{3}\ \underset{2}{4}\ \underset{1}{5}\ . = 9.345 \times 10^{3}$, n = 3 because the decimal point is shifted three places to the left.

(ii) $205852 = 2\ \underset{5}{0}\ \underset{4}{5}\ \underset{3}{8}\ \underset{2}{5}\ \underset{1}{2}\ . = 2.05852 \times 10^{5}$, n = 5 because the decimal point is shifted five places to the left .

(iii) $3449098.96 = 3\ \underset{6}{4}\ \underset{5}{4}\ \underset{4}{9}\ \underset{3}{0}\ \underset{2}{9}\ \underset{1}{8}\ .\ 9\ 6 = 3.44909896 \times 10^{6}$, n = 6 because the decimal point is shifted six places to the left.

(iv) $0.0063 = 0\ .\ \underset{1}{0}\ \underset{2}{0}\ \underset{3}{6}\ 3 = 6.3 \times 10^{-3}$, n = –3 because the decimal point is shifted three places to the right.

(v) $0.00008035 = 0\ .\ \underset{1}{0}\ \underset{2}{0}\ \underset{3}{0}\ \underset{4}{0}\ \underset{5}{8}\ 0\ 3\ 5 = 8.035 \times 10^{-5}$, n = –5 because the decimal point is shifted five places to the right.

(vi) $0.000108 = 0\ .\ \underset{1}{0}\ \underset{2}{0}\ \underset{3}{0}\ \underset{4}{1}\ 0\ 8 = 1.08 \times 10^{-4}$, n = –4 because the decimal point is shifted four places to the right.

2.2 Converting Scientific Notation to Decimal Form

Often, numbers in scientific notation need to be written in decimal form. To convert scientific notation to integers we have to follow these steps.

Step 1 : Write the decimal number.

Step 2 : Move the decimal point the number of places specified by the power of ten: to the right if positive, to the left if negative. Add zeros if necessary.

Step 3 : Rewrite the number in decimal form.

Example 2.4

Write the following numbers in decimal form.

(i) 5.236×10^5 (ii) 1.72×10^9 (iii) 6.415×10^{-6} (iv) 9.36×10^{-9}

Solution

(i) 5.236

$5.236 \times 10^5 = 523600$

5 . 2 3 6 0 0
1 2 3 4 5

(ii) 1.72

$1.72 \times 10^9 = 1720000000$

1 . 7 2 0 0 0 0 0 0 0
1 2 3 4 5 6 7 8 9

(iii) 6.415

$6.415 \times 10^{-6} = 0.000006415$

0 0 0 0 0 6 . 4 1 5
6 5 4 3 2 1

(iv) 9.36

$9.36 \times 10^{-9} = 0.00000000936$

0 0 0 0 0 0 0 0 9 . 3 6
9 8 7 6 5 4 3 2 1

2.2.1 Multiplication and Division in Scientific Notation

One can find the product or quotient of very large(googolplex) or very small numbers easily in scientific notion.

Example 2.5

Write the following in scientific notation.

(i) $(4000000)^3$ (ii) $(5000)^4 \times (200)^3$

(iii) $(0.00003)^5$ (iv) $(2000)^2 \div (0.0001)^4$

Solution

(i) First we write the number (within the brackets) in scientific notation.

$$4000000 = 4.0 \times 10^6$$

Now, raising to the power 3 on both sides we get,

$$\therefore (4000000)^3 = (4.0\times 10^6)^3 = (4.0)^3\times(10^6)^3$$
$$= 64\times 10^{18} \quad = 6.4\times 10^1\times 10^{18} = 6.4\times 10^{19}$$

(ii) In scientific notation,

$5000 = 5.0\times 10^3$ and $200 = 2.0\times 10^2$.

$$\therefore (5000)^4\times(200)^3 = (5.0\times 10^3)^4\times(2.0\times 10^2)^3$$
$$= (5.0)^4\times(10^3)^4\times(2.0)^3\times(10^2)^3$$
$$= 625\times 10^{12}\times 8\times 10^6 = 5000\times 10^{18}$$
$$= 5.0\times 10^3\times 10^{18} = 5.0\times 10^{21}$$

(iii) In scientific notation, $0.00003 = 3.0\times 10^{-5}$

$$\therefore (0.00003)^5 = (3.0\times 10^{-5})^5 = (3.0)^5\times(10^{-5})^5$$
$$= 243\times 10^{-25} \quad = 2.43\times 10^2\times 10^{-25} = 2.43\times 10^{-23}$$

(iv) In scientific notation,

$2000 = 2.0\times 10^3$ and $0.0001 = 1.0\times 10^{-4}$

$$\therefore (2000)^2\div(0.0001)^4 = \frac{(2.0\times 10^3)^2}{(1.0\times 10^{-4})^4} = \frac{(2.0)^2\times(10^3)^2}{(1.0)^4\times(10^{-4})^4}$$
$$= \frac{4\times 10^6}{10^{-16}} = 4.0\times 10^{6-(-16)} = 4.0\times 10^{22}$$

Exercise 2.1

1. Represent the following numbers in the scientific notation.

(i) 749300000000 (ii) 13000000 (iii) 105003

(iv) 543600000000000 (v) 0.0096 (vi) 0.0000013307

(vii) 0.0000000022 (viii) 0.0000000000009

2. Write the following numbers in decimal form.

(i) 3.25×10^{-6} (ii) 4.134×10^{-4} (iii) 4.134×10^4

(iv) 1.86×10^7 (v) 9.87×10^9 (vi) 1.432×10^{-9}

3. Represent the following numbers in scientific notation.

(i) $(1000)^2\times(20)^6$ (ii) $(1500)^3\times(0.0001)^2$

(iii) $(16000)^3\div(200)^4$ (iv) $(0.003)^7\times(0.0002)^5\div(0.001)^3$

(v) $(11000)^3\times(0.003)^2\div(30000)$

2.3 Logarithms

Logarithms were originally developed to simplify complex arithmetic calculations. They were designed to transform multiplicative processes into additive ones. Before the advent of calculators, logarithms had great use in multiplying and dividing numbers with many digits since adding exponents was less work than multiplying numbers. Now they are important in nuclear work because many laws governing physical behavior are in exponential form. Examples are radioactive decay, gamma absorption, and reactor power changes on a stable period.

To introduce the notation of logarithm, we shall first introduce the exponential notation for real numbers.

2.3.1 Exponential Notation

Let a be a positive number. We have already introduced the notation a^x, where x is an integer.

We know that $a^{\frac{1}{n}}$ is a positive number whose nth power is equal to a. Now we can see how to define $a^{\frac{p}{q}}$, where p is an integer and q is a positive integer.

Notice that $\frac{p}{q} = p \times \frac{1}{q}$, so if the power rule is to hold then

$$a^{\frac{p}{q}} = \left(a^{\frac{1}{q}}\right)^p = (a^p)^{\frac{1}{q}}$$

So, we define $a^{\frac{p}{q}} = (\sqrt[q]{a})^p$. For example, $8^{\frac{3}{5}} = (\sqrt[5]{8})^3$ and $5^{\frac{-7}{3}} = (\sqrt[3]{5})^{-7}$

Thus, if $a > 0$, we have been able to give suitable meaning to a^x for all rational numbers x. Also for $a > 0$ it is possible to extend the definition of a^x to irrational exponents x so that the laws of exponents remain valid. We will not show how a^x may be defined for irrational x because the definition of a^x requires some advanced topics in mathematics. So, we accept now that, for any $a > 0$, a^x is defined for all real numbers x and satifies the laws of exponents.

2.3.2 Logarithmic Notation

If $a > 0$, $b > 0$ and $a \neq 1$, then the logarithm of b to the base a is the number to which a to be raised to obtain b.

Key Concept **Logarithmic Notation**

Let a be a positive number other than 1 and let x be a real number (positive, negative, or zero). If $a^x = b$, we say that the exponent x is the logarithm of b to the base a and we write $x = \log_a b$.

$x = \log_a b$ is the logarithmic form of the exponential form $b = a^x$. In both the forms, the base is same.

For example,

Exponential Form	Logarithmic Form
$2^4 = 16$	$\log_2 16 = 4$
$8^{\frac{1}{3}} = 2$	$\log_8 2 = \frac{1}{3}$
$4^{-\frac{3}{2}} = \frac{1}{8}$	$\log_4\left(\frac{1}{8}\right) = -\frac{3}{2}$

Example 2.6

Change the following from logarithmic form to exponential form.

(i) $\log_4 64 = 3$ (ii) $\log_{16} 2 = \frac{1}{4}$ (iii) $\log_5\left(\frac{1}{25}\right) = -2$ (iv) $\log_{10} 0.1 = -1$

Solution

(i) $\log_4 64 = 3 \implies 4^3 = 64$

(ii) $\log_{16} 2 = \frac{1}{4} \implies (16)^{\frac{1}{4}} = 2$

(iii) $\log_5\left(\frac{1}{25}\right) = -2 \implies (5)^{-2} = \frac{1}{25}$

(iv) $\log_{10} 0.1 = -1 \implies (10)^{-1} = 0.1$

Example 2.7

Change the following from exponential form to logarithmic form.

(i) $3^4 = 81$ (ii) $6^{-4} = \frac{1}{1296}$ (iii) $\left(\frac{1}{81}\right)^{\frac{3}{4}} = \frac{1}{27}$

(iv) $(216)^{\frac{1}{3}} = 6$ (v) $(13)^{-1} = \frac{1}{13}$

Solution

(i) $3^4 = 81 \implies \log_3 81 = 4$

(ii) $6^{-4} = \frac{1}{1296} \implies \log_6\left(\frac{1}{1296}\right) = -4$

(iii) $\left(\frac{1}{81}\right)^{\frac{3}{4}} = \frac{1}{27} \implies \log_{\frac{1}{81}}\left(\frac{1}{27}\right) = \frac{3}{4}$

(vi) $(216)^{\frac{1}{3}} = 6 \implies \log_{216} 6 = \frac{1}{3}$

(v) $(13)^{-1} = \frac{1}{13} \implies \log_{13}\left(\frac{1}{13}\right) = -1$

Example 2.8

Evaluate (i) $\log_8 512$ (ii) $\log_{27} 9$ (iii) $\log_{16}\left(\frac{1}{32}\right)$

Solution

(i) Let $x = \log_8 512$. Then

Mathematics

$$8^x = 512 \qquad \text{(exponential form)}$$

$$8^x = 8^3 \Longrightarrow x = 3$$

$$\therefore \log_8 512 = 3$$

(ii) Let $x = \log_{27} 9$. Then

$$27^x = 9 \qquad \text{(exponential form)}$$

$$(3^3)^x = (3)^2 \qquad \text{(convert both sides to base three)}$$

$$3^{3x} = 3^2 \Longrightarrow 3x = 2 \Longrightarrow x = \frac{2}{3}$$

$$\therefore \log_{27} 9 = \frac{2}{3}$$

(iii) Let $x = \log_{16}\left(\frac{1}{32}\right)$. Then

$$16^x = \frac{1}{32} \qquad \text{(exponential form)}$$

$$(2^4)^x = \frac{1}{(2)^5} \qquad \text{(convert both sides to base two)}$$

$$2^{4x} = 2^{-5} \Longrightarrow 4x = -5 \Longrightarrow x = -\frac{5}{4}$$

$$\therefore \log_{16}\left(\frac{1}{32}\right) = -\frac{5}{4}$$

Example 2.9

Solve the equations (i) $\log_5 x = -3$ (ii) $x = \log_{\frac{1}{4}} 64$ (iii) $\log_x 8 = 2$

(iv) $\log_x 7^{\frac{1}{6}} = \frac{1}{3}$

Solution

(i) $\log_5 x = -3$

$$5^{-3} = x \qquad \text{(exponential form)}$$

$$x = \frac{1}{5^3} \Longrightarrow x = \frac{1}{125}$$

(ii) $x = \log_{\frac{1}{4}} 64$

$$\left(\frac{1}{4}\right)^x = 64 \qquad \text{(exponential form)}$$

$$\frac{1}{4^x} = 4^3 \Longrightarrow 4^{-x} = 4^3 \Longrightarrow x = -3$$

(iii) $\log_x 8 = 2$

$$x^2 = 8 \qquad \text{(exponential form)}$$

$$x = \sqrt{8} = 2\sqrt{2}$$

(iv) $\log_x 7^{\frac{1}{6}} = \frac{1}{3} \Longrightarrow x^{\frac{1}{3}} = 7^{\frac{1}{6}}$ (exponential form)

We write $7^{\frac{1}{6}} = \left(7^{\frac{1}{2}}\right)^{\frac{1}{3}}$. Then $x^{\frac{1}{3}} = \left(7^{\frac{1}{2}}\right)^{\frac{1}{3}}$

$$\therefore x = 7^{\frac{1}{2}} = \sqrt{7}$$

The Rules of Logarithms

1. **Product Rule:** The logarithm of the product of two positive numbers is equal to sum of their logarithms of the same base. That is,

$$\log_a(M\times N) = \log_a M + \log_a N;\ \text{a M, N are posivive numbers, } a \neq 1.$$

2. **Quotient Rule:** The logarithm of the quotient of two positive numbers is equal to the logarithm of the numerator minus the logarithm of the denominator to the same base. That is,

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N;\ \text{a M, N are posivive numbers, } a \neq 1.$$

3. **Power Rule:** The logarithm of a number in exponential form is equal to the logarithm of the number multiplied by its exponent. That is,

$$\log_a (M)^n = n\log_a M;\ \text{a M are posivive numbers, } a \neq 1.$$

4. **Change of Base Rule:** If M, a and b are positive numbers and $a \neq 1$, $b \neq 1$, then

$$\log_a M = (\log_b M)\times(\log_a b)$$

Remark

(i) If a is a positive number and $a \neq 1$, $\log_a a = 1$

(ii) If a is a positive number and $a \neq 1$, $\log_a 1 = 0$

(iii) If a and b are positive numbers $a \neq 1, b \neq 1$ $(\log_a b)\times(\log_b a) = 1$ and $\log_a b = \dfrac{1}{\log_b a}$

(iv) If a and b are positive numbers and $b \neq 1$, $b^{\log_b a} = a$.

(v) If $a > 0$, $\log_a 0$ is undefined.

(vi) If b ,x and y are positive numbers other than 1 then $\log_b x = \log_b y$ if and only if $x = y$.

(vii) We are avoiding 1 in the base of all logarithms because if we consider one such logarithm, say $\log_1 9$ with 1 in the base, then $x = \log_1 9$ would give $1^x = 9$. We know that there is no real number x, such that $1^x = 9$.

Example 2.10

Simplify (i) $\log_5 25 + \log_5 625$ (ii) $\log_5 4 + \log_5\left(\frac{1}{100}\right)$

Solution

(i) $\log_5 25 + \log_5 625 = \log_5(25\times 625)$ $[\because \log_a(M\times N) = \log_a M + \log_a N]$

$= \log_5(5^2\times 5^4) = \log_5 5^6 = 6\log_5 5$ $[\because \log_a (M)^n = n\log_a M]$

$= 6(1) = 6$ $[\because \log_a a = 1]$

(ii) $\log_5 4 + \log_5\left(\frac{1}{100}\right) = \log_5\left(4 \times \frac{1}{100}\right)$ $[\because \log_a(M \times N) = \log_a M + \log_a N]$

$= \log_5\left(\frac{1}{25}\right) = \log_5\left(\frac{1}{5^2}\right) = \log_5 5^{-2} = -2\log_5 5$ $[\because \log_a (M)^n = n\log_a M]$

$= -2(1) = -2$ $[\because \log_a a = 1]$

Example 2.11

Simplify $\log_8 128 - \log_8 16$

Solution $\log_8 128 - \log_8 16 = \log_8 \frac{128}{16}$ $[\because \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N]$

$= \log_8 8 = 1$ $[\because \log_a a = 1]$

Example 2.12

Prove that $\log_{10} 125 = 3 - 3\log_{10} 2$

Solution $3 - 3\log_{10} 2 = 3\log_{10} 10 - 3\log_{10} 2 = \log_{10} 10^3 - \log_{10} 2^3$

$= \log_{10} 1000 - \log_{10} 8 = \log_{10} \frac{1000}{8}$

$= \log_{10} 125$

$\therefore \log_{10} 125 = 3 - 3\log_{10} 2$

Example 2.13

Prove that $\log_3 2 \times \log_4 3 \times \log_5 4 \times \log_6 5 \times \log_7 6 \times \log_8 7 = \frac{1}{3}$

Solution $\log_3 2 \times \log_4 3 \times \log_5 4 \times \log_6 5 \times \log_7 6 \times \log_8 7$

$= (\log_3 2 \times \log_4 3) \times (\log_5 4 \times \log_6 5) \times (\log_7 6 \times \log_8 7)$

$= \log_4 2 \times \log_6 4 \times \log_8 6 = (\log_4 2 \times \log_6 4) \times \log_8 6$ $[\because \log_a M = \log_b M \times \log_a b]$

$= \log_6 2 \times \log_8 6 = \log_8 2 = \frac{1}{\log_2 8}$ $[\because \log_a b = \frac{1}{\log_b a}]$

$= \frac{1}{\log_2 2^3} = \frac{1}{3\log_2 2}$ $[\because \log_a (M)^n = n\log_a M]$

$= \frac{1}{3}$ $[\because \log_2 2 = 1]$

Example 2.14

Find the value of $25^{-2\log_5 3}$

Solution $25^{-2\log_5 3} = (5^2)^{-2\log_5 3} = 5^{-4\log_5 3}$ $[\because n\log_a M = \log_a M^n]$

$= 5^{\log_5 3^{-4}} = 3^{-4} = \frac{1}{3^4} = \frac{1}{81}$ $[\because b^{\log_b a} = a]$

Example 2.15

Solve $\log_{16}x + \log_4 x + \log_2 x = 7$

Solution $\log_{16}x + \log_4 x + \log_2 x = 7$

$$\Longrightarrow \quad \frac{1}{\log_x 16} + \frac{1}{\log_x 4} + \frac{1}{\log_x 2} = 7 \qquad [\because \log_a b = \frac{1}{\log_b a}]$$

$$\frac{1}{\log_x 2^4} + \frac{1}{\log_x 2^2} + \frac{1}{\log_x 2} = 7$$

$$\frac{1}{4\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{\log_x 2} = 7 \qquad [\because n\log_a M = \log_a M^n]$$

$$\left[\frac{1}{4} + \frac{1}{2} + 1\right]\frac{1}{\log_x 2} = 7 \implies \left[\frac{7}{4}\right]\frac{1}{\log_x 2} = 7$$

$$\frac{1}{\log_x 2} = 7 \times \frac{4}{7}$$

$$\log_2 x = 4 \qquad [\because \log_a b = \frac{1}{\log_b a}]$$

$$2^4 = x \qquad \text{(exponential form)}$$

$$\therefore \; x = 16$$

Example 2.16

Solve $\dfrac{1}{2 + \log_x 10} = \dfrac{1}{3}$

Solution $\dfrac{1}{2 + \log_x 10} = \dfrac{1}{3}$. Cross multiplying, we get

$$2 + \log_x 10 = 3$$

$$\Longrightarrow \log_x 10 = 3 - 2 = 1$$

$$x^1 = 10 \qquad \text{(exponential form)}$$

$$\therefore x = 10$$

Example 2.17

Solve $\log_3(\log_2 x) = 1$

Solution Let $\log_2 x = y$

Then, $\log_3 y = 1$

$$3^1 = y \qquad \text{(exponential form)}$$

$$\therefore \; y = 3$$

Put $y = 3$ in (1). Then $\log_2 x = 3$

$$2^3 = x \qquad \text{(exponential form)}$$

$$\therefore \; x = 8$$

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Example 2.18

Solve $\log_2(3x-1) - \log_2(x-2) = 3$

Solution $\log_2(3x-1) - \log_2(x-2) = 3$

$$\log_2\left(\frac{3x-1}{x-2}\right) = 3 \qquad [\because \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N]$$

$$2^3 = \frac{3x-1}{x-2} \qquad \text{(exponential form)}$$

$$8 = \frac{3x-1}{x-2}$$

Cross multiplying, we get

$$8(x-2) = 3x-1 \Longrightarrow 8x-16 = 3x-1$$

$$8x-3x = -1+16 \Longrightarrow 5x = 15$$

$$\therefore x = 3$$

Example 2.19

Prove that $\log_5 1125 = 2\log_5 6 - \frac{1}{2}\log_5 16 + 6\log_{49} 7$

Solution $2\log_5 6 - \frac{1}{2}\log_5 16 + 6\log_{49} 7$

$$= \log_5 6^2 - \log_5 (16)^{\frac{1}{2}} + 3\times 2\log_{49} 7 = \log_5 36 - \log_5 4 + 3\log_{49} 7^2$$

$$= \log_5\left(\frac{36}{4}\right) + 3\log_{49} 49 = \log_5 9 + 3(1)$$

$$= \log_5 9 + 3\log_5 5 = \log_5 9 + \log_5 (5)^3$$

$$= \log_5 9 + \log_5 125 = \log_5(9\times 125) = \log_5 1125$$

$$\therefore \log_5 1125 = 2\log_5 6 - \frac{1}{2}\log_5 16 + 6\log_{49} 7$$

Example 2.20

Solve $\log_5\sqrt{7x-4} - \frac{1}{2} = \log_5\sqrt{x+2}$

Solution $\log_5\sqrt{7x-4} - \frac{1}{2} = \log_5\sqrt{x+2}$

$$\log_5\sqrt{7x-4} - \log_5\sqrt{x+2} = \frac{1}{2}$$

$$\log_5\left(\frac{\sqrt{7x-4}}{\sqrt{x+2}}\right) = \frac{1}{2} \qquad [\because \log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N]$$

$$\log_5\left(\frac{7x-4}{x+2}\right)^{\frac{1}{2}} = \frac{1}{2}$$

$$\frac{1}{2}\left[\log_5\left(\frac{7x-4}{x+2}\right)\right] = \frac{1}{2} \qquad [\because \log_a M^n = n\log_a M]$$

$$\log_5\left(\frac{7x-4}{x+2}\right) = 1$$

$$5^1 = \frac{7x-4}{x+2} \quad \text{(exponential form)}$$

Cross multiplying, $7x - 4 = 5(x+2)$

$$7x - 4 = 5x + 10 \implies 7x - 5x = 10 + 4$$

$$\implies 2x = 14$$

$$\therefore\ x = 7$$

Exercise 2.2

1. State whether each of the following statements is true or false.

(i) $\log_5 125 = 3$ (ii) $\log_{\frac{1}{2}} 8 = 3$

(iii) $\log_4(6+3) = \log_4 6 + \log_4 3$ (iv) $\log_2\left(\frac{25}{3}\right) = \frac{\log_2 25}{\log_2 3}$

(v) $\log_{\frac{1}{3}} 3 = -1$ (vi) $\log_a(M-N) = \log_a M \div \log_a N$

2. Obtain the equivalent logarithmic form of the following.

(i) $2^4 = 16$ (ii) $3^5 = 243$ (iii) $10^{-1} = 0.1$

(iv) $8^{-\frac{2}{3}} = \frac{1}{4}$ (v) $25^{\frac{1}{2}} = 5$ (vi) $12^{-2} = \frac{1}{144}$

3. Obtain the equivalent exponential form of the following.

(i) $\log_6 216 = 3$ (ii) $\log_9 3 = \frac{1}{2}$ (iii) $\log_5 1 = 0$

(iv) $\log_{\sqrt{3}} 9 = 4$ (v) $\log_{64}\left(\frac{1}{8}\right) = -\frac{1}{2}$ (vi) $\log_{0.5} 8 = -3$

4. Find the value of the following.

(i) $\log_3\left(\frac{1}{81}\right)$ (ii) $\log_7 343$ (iii) $\log_6 6^5$

(iv) $\log_{\frac{1}{2}} 8$ (v) $\log_{10} 0.0001$ (vi) $\log_{\sqrt{3}} 9\sqrt{3}$

5. Solve the following equations.

(i) $\log_2 x = \frac{1}{2}$ (ii) $\log_{\frac{1}{5}} x = 3$ (iii) $\log_3 y = -2$

(iv) $\log_x 125\sqrt{5} = 7$ (v) $\log_x 0.001 = -3$ (vi) $x + 2\log_{27} 9 = 0$

6. Simplify the following.

(i) $\log_{10} 3 + \log_{10} 3$ (ii) $\log_{25} 35 - \log_{25} 10$

(iii) $\log_7 21 + \log_7 77 + \log_7 88 - \log_7 121 - \log_7 24$

(iv) $\log_8 16 + \log_8 52 - \frac{1}{\log_{13} 8}$

(v) $5\log_{10} 2 + 2\log_{10} 3 - 6\log_{64} 4$

(vi) $\log_{10} 8 + \log_{10} 5 - \log_{10} 4$

7. Solve the equation in each of the following.

(i) $\log_4(x+4)+\log_4 8=2$ (ii) $\log_6(x+4)-\log_6(x-1)=1$

(iii) $\log_2 x+\log_4 x+\log_8 x=\frac{11}{6}$ (iv) $\log_4(8\log_2 x)=2$

(v) $\log_{10}5+\log_{10}(5x+1)=\log_{10}(x+5)+1$

(vi) $4\log_2 x-\log_2 5=\log_2 125$ (vii) $\log_3 25+\log_3 x=3\log_3 5$

(viii) $\log_3(\sqrt{5x-2})-\frac{1}{2}=\log_3(\sqrt{x+4})$

8. Given $\log_a 2=x$, $\log_a 3=y$ and $\log_a 5=z$. Find the value in each of the following in terms of x, y and z.

(i) $\log_a 15$ (ii) $\log_a 8$ (iii) $\log_a 30$

(iv) $\log_a\left(\frac{27}{125}\right)$ (v) $\log_a\left(3\frac{1}{3}\right)$ (vi) $\log_a 1.5$

9. Prove the following equations.

(i) $\log_{10}1600=2+4\log_{10}2$ (ii) $\log_{10}12500=2+3\log_{10}5$

(iii) $\log_{10}2500=4-2\log_{10}2$ (iv) $\log_{10}0.16=2\log_{10}4-2$

(v) $\log_5 0.00125=3-5\log_5 10$ (vi) $\log_5 1875=\frac{1}{2}\log_5 36-\frac{1}{3}\log_5 8+20\log_{32}2$

2.4 Common Logarithms

For the purpose of calculations, the most logical number for a base is 10, the base of the decimal number system. Logarithms to the base 10 are called *common logarithms.* Therefore, in the discussion which follows, no base designation is used, i.e., $\log N$ means $\log_{10} N$. Consider the following table.

Number N	0.0001	0.001	0.01	0.1	1	10	100	1000	10000
Exponential Form of N	10^{-4}	10^{-3}	10^{-2}	10^{-1}	10^0	10^1	10^2	10^3	10^4
$\log N$	-4	-3	-2	-1	0	1	2	3	4

So, $\log N$ is an integer if N is an integral power of 10. What about logarithm of 3.16 or 31.6 or 316? For example, $3.16=10^{0.4997}$; $31.6=10^{1.4997}$; $316=10^{2.4997}$

Therefore, $\log 3.16=0.4997$; $\log 31.6=1.4997$; $\log 316=2.4997$.

Notice that logarithm of a number between 1 and 10 is a number between 0 and 1 ; logarithm of a number between 10 and 100 is a number between 1 and 2 and so on.

Every logarithm consists of an integral part called the *characteristic* and a fractional part called the *mantissa*. For example,

$\log 3.16 = 0.4997$; characteristic is 0 and mantissa is 0.4997

$\log 31.6 = 1.4997$; characteristic is 1 and mantissa is 0.4997

$\log 316 = 2.4997$; characteristic is 2 and mantissa is 0.4997

The logarithm of a number less than 1 is negative. It is convenient to keep the mantissa positive even though the logarithm is negative.

Scientific notation provides a convenient method for determining the characteristic. In scientific notation $316 = 3.16 \times 10^2$. Thus, we have

$$\begin{aligned}\log 316 &= \log(3.16 \times 10^2)\\ &= \log 3.16 + \log 10^2\\ &= 0.4997 + 2 = 2.4997.\end{aligned}$$

Thus, the power of 10 determines the characteristic of logarithm.

Example 2.21

Write the characteristic of the following.

(i) log 27.91 (ii) log 0.02871 (iii) log 0.000987 (iv) log 2475.

Solution

(i) In scientific notation, $27.91 = 2.791 \times 10^1$

$\therefore$ The characteristic is 1

(ii) In scientific notation, $0.02871 = 2.871 \times 10^{-2}$

$\therefore$ The characteristic is –2

(iii) In scientific notation, $0.000987 = 9.87 \times 10^{-4}$

$\therefore$ The characteristic is – 4

(iv) In scientific notation, $2475 = 2.475 \times 10^3$

$\therefore$ The characteristic is 3

The characteristic is also determined by inspection of the number itself according to the following rules.

(i) For a number greater than or equal to 1, the characteristic is non-negative and is one less than the number of digits before the decimal point.

(ii) For a number less than 1, the characteristic is negative and is one more than the number of zeros immediately following the decimal point. The negative sign of the characteristic is written above the characteristics as $\bar{1}, \bar{2}$, etc. For example, the characteristic of 0.0316 is $\bar{2}$.

(iii) Mantissa is always positive.

Example 2.22

Given that log 4586 = 3.6615, find (i) log 45.86 (ii) log 45860 (iii) log 0.4586 (iv) log 0.004586 (v) log 0.04586 (vi) log 4.586

Solution The mantissa of log 4586 is 0.6615. Hence,

(i) log 45.86 = 1.6615 (ii) log 45860 = 4.6615

(iii) $\log 0.4586 = -1 + 0.6615 = \bar{1}.6615$ (iv) $\log 0.004586 = -3 + 0.6615 = \bar{3}.6615$

(v) $\log 0.04586 = -2 + 0.6615 = \bar{2}.6615$ (vi) log 4.586 = 0.6615

2.4.1 Method of Finding Logarithm

Tables of logarithm usually contain only mantissas since the characteristic can be readily determined as explained above. Note that the mantissas of logarithms of all the numbers consisting of same digits in same order but differing only in the position of decimal point are the same. The mantissas are given correct to four places of decimals.

A logarithmic table consists of three parts .

(i) First column contains numbers from 1.0, 1.1, 1.2 ,1.3,... upto 9.9

(ii) Next ten columns headed by 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 contain the mantissas.

(iii) After these columns, there are again nine columns under the head mean difference. These columns are marked with serial numbers 1 to 9.

We shall explain how to find the mantissa of a given number in the following example. Suppose, the given number is 40.85. Now $40.85 = 4.085 \times 10^1$. Therefore, the characteristic is 1. The row in front of the number 4.0 in logarithmic table is given below.

										Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
4.0	.6021	.6031	.6042	.6053	.6064	.6075	.6085	.6096	.6107	.6117	1	2	3	4	5	6	8	9	10

We note the number in row beneath the digit 8 in front of N = 4.0. The number is 0.6107. Next the mean difference corresponding to 5 is 0.0005. Thus the required mantissa is 0.6107 + 0.0005 = 0.6112.

Hence, $\log 40.85 = 1.6112$.

2.4.2 Antilogarithms

The number whose logarithm is x, is called the antilogarithm of x and is written as antilog x. Thus, if $\log y = x$, then antilog $x = y$.

2.4.3 Method of Finding Antilogarithm

The antilogarithm of a number is found by using a table named 'ANTILOGARITHMS' given at the end of the book. This table gives the value of the antilogarithm of a number correct to four places of decimal.

For finding antilogarithm, we take into consideration only the mantissa. The characteristic is used only to determine the number of digits in the integral part or the number of zeros immediately after the decimal point.

The method of using the table of antilogarithms is the same as that of the table of logarithms discussed above.

Note Since the logarithmic table given at the end of this text book can be applied only to four digit number, in this section we approximated all logarithmic calculations to four digits

Example 2.23

Find (i) log 86.76 (ii) log 730.391 (iii) log 0.00421526

Solution

(i) $86.76 = 8.676 \times 10^1$ (scientific notation)

$\therefore$ The characteristic is 1. To find mantissa consider the number 8.676.

From the table, log 8.67 is 0.9380

Mean difference of 6 is 0.0003

$\log 8.676 = 0.9380 + 0.0003 = 0.9383$

$\therefore \quad \log 86.76 = 1.9383$

(ii) $730.391 = 7.30391 \times 10^2$ (scientific notation)

$\therefore$ The characteristic is 2. To find mantissa consider the number 7.30391 and approximate it as 7.304 (since 9 in the fourth decimal place is greater than 5)

From the table, $\log 7.30$ is 0.8633

Mean difference of 4 is 0.0002

$\log 7.304 = 0.8633 + 0.0002 = 0.8635$

$\therefore \quad \log 730.391 = 2.8635$

(iii) $0.00421526 = 4.21526 \times 10^{-3}$ (scientific notation)

$\therefore$ The characteristic is -3. To find mantissa consider the number 4.21526 and approximate it as 4.215 (since 2 in the fourth decimal place is less than 5).

From the table, $\log 4.21$ is 0.6243

Mean difference of 5 is 0.0005

$$\log 4.215 = 0.6243 + 0.0005 = 0.6248$$

$\therefore \log 0.00421526 = -3 + 0.6248 = \bar{3}.6248$

Example 2.24

Find (i) antilog 1.8652 (ii) antilog 0.3269 (iii) antilog $\bar{2}.6709$

Solution

(i) Characteristic is 1. So, the number contains two digits in its integral part. Mantissa is 0.8652.

From the table, antilog 0.865 is 7.328

Mean difference of 2 is 0.003

antilog 0.8652 = 7.328 + 0.003 = 7.331

$\therefore$ antilog 1.8652 = 73.31

(ii) Characteristic is 0. So, the number contains one digit in its integral part. Mantissa is 0.3269

From the table, antilog of 0.326 is 2.118

Mean difference of 9 is 0.004

$\therefore$ antilog 0.3269 = 2.118 + 0.004 = 2.122

(iii) Characteristic is -2. So, the number contains one zero immediately following the decimal point. Mantissa is 0.6709

From the table, antilog 0.670 is 4.677

Mean difference of 9 is 0.010

antilog 0.6709 = 4.677 + 0.010 = 4.687

$\therefore$ antilog $\bar{2}.6709$ = 0.04687

Example: 2.25

Find (i) 42.6×2.163 (ii) 23.17×0.009321

Solution

(i) Let $x = 42.6 \times 2.163$. Taking logarithm on both sides, we get

$$\log x = \log(42.6 \times 2.163)$$
$$= \log 42.6 + \log 2.163$$
$$= 1.6294 + 0.3351 = 1.9645$$
$$\therefore\ x = \text{antilog } 1.9645 = 92.15$$

(ii) Let $x = 23.17 \times 0.009321$. Taking logarithm on both sides, we get

$$\log x = \log(23.17 \times 0.009321) = \log 23.17 + \log 0.009321$$
$$= 1.3649 + \bar{3}.9694 = 1 + 0.3649 - 3 + 0.9694$$
$$= -2 + 1.3343 = -2 + 1 + 0.3343$$
$$= -1 + 0.3343 = \bar{1}.3343$$
$$\therefore\quad x = \text{antilog } \bar{1}.3343 = 0.2159$$

Example: 2.26

Find the value of (i) $(36.27)^6$ (ii) $(0.3749)^4$ (iii) $\sqrt[5]{0.2713}$

Solution

(i) Let $x = (36.27)^6$. Taking logarithm on both sides, we get

$$\log x = \log(36.27)^6 = 6 \log 36.27 = 6(1.5595) = 9.3570$$
$$\therefore x = \text{antilog } 9.3570 = 2275000000$$

(ii) Let $x = (0.3749)^4$. Taking logarithm on both sides, we get

$$\log x = \log(0.3749)^4 = 4 \log 0.3749 = 4(\bar{1}.5739) = 4(-1 + 0.5739)$$
$$= -4 + 2.2956 = -4 + 2 + 0.2956 = -2 + 0.2956 = -2.2956$$
$$= \bar{2}.2956$$
$$\therefore\ x = \text{antilog } \bar{2}.2956 = 0.01975$$

(iii) Let $x = \sqrt[5]{0.2713} = (0.2713)^{\frac{1}{5}}$. Taking logarithm on both sides, we get

$$\log x = \log(0.2713)^{\frac{1}{5}} = \frac{1}{5}\log 0.2713$$
$$= \frac{1}{5}(\bar{1}.4335) = \frac{-1 + 0.4335}{5}$$

$$= \frac{(-1-4)+4+0.4335}{5}$$

$$= \frac{-5+4.4335}{5} = \frac{-5}{5} + \frac{4.4335}{5}$$

$$= -1 + 0.8867 = \bar{1}.8867$$

$$\therefore\ x = \text{antilog } \bar{1}.8867 = 0.7703$$

Example : 2.27

Simplify (i) $\dfrac{(46.7)\times\sqrt{65.2}}{(2.81)^3\times(4.23)}$ (ii) $\dfrac{(84.5)^4\times\sqrt[3]{0.0064}}{(72.5)^2\times\sqrt{62.3}}$

Solution

(i) Let $x = \dfrac{(46.7)\times\sqrt{65.2}}{(2.81)^3\times(4.23)} = \dfrac{46.7\times(65.2)^{\frac{1}{2}}}{(2.81)^3\times 4.23}$

Taking logarithm on both sides, we get

$$\log x = \log\left[\frac{46.7\times(65.2)^{\frac{1}{2}}}{(2.81)^3\times 4.23}\right]$$

$$= \log 46.7 + \log (65.2)^{\frac{1}{2}} - \log(2.81)^3 - \log 4.23$$

$$= \log 46.7 + \frac{1}{2}\log 65.2 - 3\log 2.81 - \log 4.23$$

$$= 1.6693 + \frac{1}{2}(1.8142) - 3(0.4487) - 0.6263$$

$$= 1.6693 + 0.9071 - 1.3461 - 0.6263$$

$$= 2.5764 - 1.9724 = 0.6040$$

$$\therefore x = \text{antilog } 0.6040 = 4.018$$

(ii) Let $x = \dfrac{(84.5)^4\times\sqrt[3]{0.0064}}{(72.5)^2\times\sqrt{62.3}} = \dfrac{(84.5)^4\times(0.0064)^{\frac{1}{3}}}{(72.5)^2\times(62.3)^{\frac{1}{2}}}$

Taking logarithm on both sides, we get

$$\log x = \log\left[\frac{(84.5)^4\times(0.0064)^{\frac{1}{3}}}{(72.5)^2\times(62.3)^{\frac{1}{2}}}\right]$$

$$= \log (84.5)^4 + \log(0.0064)^{\frac{1}{3}} - \log (72.5)^2 - \log(62.3)^{\frac{1}{2}}$$

$$= 4\log 84.5 + \frac{1}{3}\log 0.0064 - 2\log 72.5 - \frac{1}{2}\log 62.3$$

$= 4(1.9269) + \frac{1}{3}(\bar{3}.8062) - 2(1.8603) - \frac{1}{2}(1.7945)$

$= 7.7076 + \frac{1}{3}(-3 + 0.8062) - 3.7206 - 0.8973$

$= 3.0897 + (-1+0.2687) = 3+0.0897-1+0.2687$

$= 2+0.3584 = 2.3584$

$\therefore x = \text{antilog}2.3584 = 228.2$

Example 2.28

Find the value of $\log_4 13.26$

Solution $\log_4 13.26 = \log_{10} 13.26 \times \log_4 10$ $\quad [\because \log_a M = \log_b M \times \log_a b]$

$= \log_{10} 13.26 \times \frac{1}{\log_{10} 4}$ $\quad [\because \log_a b = \frac{1}{\log_b a}]$

$= \frac{1.1225}{0.6021} = x$ (say)

Then $x = \frac{1.1225}{0.6021}$. Taking logarithm on both sides, we get

$\log x = \log\left(\frac{1.1225}{0.6021}\right)$

$= \log 1.1225 - \log 0.6021 = 0.0503 - \bar{1}.7797$

$= 0.0503 - (-1 + 0.7797) = 0.0503 + 1 - 0.7797$

$= 1.0503 - 0.7797 = 0.2706$

$\therefore x = \text{antilog}\ 0.2706 = 1.865$

Exercise: 2.3

1. Write each of the following in scientific notation:

(i) 92.43 (ii) 0.9243 (iii) 9243

(iv) 924300 (v) 0.009243 (vi) 0.09243

2. Write the characteristic of each of the following

(i) log 4576 (ii) log 24.56 (iii) log 0.00257

(iv) log 0.0756 (v) log 0.2798 (vi) log 6.453

3. The mantissa of log 23750 is 0.3756. Find the value of the following.

(i) log 23750 (ii) log 23.75 (iii) log 2.375

(iv) log 0.2375 (v) log 23750000 (vi) log 0.00002375

4. Using logarithmic table find the value of the following.

(i) log 23.17 (ii) log 9.321 (iii) log 329.5
(iv) log 0.001364 (v) log 0.9876 (vi) log 6576

5. Using antilogarithmic table find the value of the following.

(i) antilog 3.072 (ii) antilog 1.759 (iii) antilog $\bar{1}.3826$
(iv) antilog $\bar{3}.6037$ (v) antilog 0.2732 (vi) antilog $\bar{2}.1798$

6. Evaluate:

(i) 816.3×37.42 (ii) $816.3 \div 37.42$ (iii) 0.000645×82.3

(iv) $0.3421 \div 0.09782$ (v) $(50.49)^5$ (vi) $\sqrt[3]{561.4}$

(vii) $\dfrac{175.23 \times 22.159}{1828.56}$ (viii) $\dfrac{\sqrt[3]{28} \times \sqrt[5]{729}}{\sqrt{46.35}}$ (ix) $\dfrac{(76.25)^3 \times \sqrt[3]{1.928}}{(42.75)^5 \times 0.04623}$

(x) $\sqrt[3]{\dfrac{0.7214 \times 20.37}{69.8}}$ (xi) $\log_9 63.28$ (xii) $\log_3 7$

2.5 BASE SYSTEM

2.5.1 Base 2 system

The decimal number system that we used everyday contains ten digits, 0 through 9 and the base of this system is 10. There is another very important number system known as Binary Number System which contains only two digits, 0 and 1. So the base of binary number system is 2.

The Binary Number System plays a central role in how information of all kinds is stored on computers. Almost all packaged goods we buy today are marked with Universal Product Code (UPC). An Optical scanner 'reads' the patterns of black and white, thick and thin and converts it to a binary code that is sent to the scanner's computer, which then calls up the appropriate product name, price, etc.,.

On a compact disc (CD), music is digitally encoded on the underside of the disc in a binary system of pits and "lands' (non – pits). If a CD is played, a laser beam traverses along the spiral and is reflected when it hits a land, but it is not reflected by the pits. It is these changes that return binary date. A change is recorded as a 1, and no change is recorded as a 0. The binary sequence is then converted into music.

To avoid confusion while using different numeral systems, the base of each individual number may be specified by writing it as subscript of the number. For example, the decimal number 156 will be written as 156_{10}. The binary number 10011100 will be specified as 10011100_2.

In the binary system, only 2 digits are used, namely 0 and 1.

The positional values in a base 2 system are …., 2^4, 2^3, 2^2, 2^1, 2^0.

Note: There is no 2 in base 2.

For (1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, 1111, …..)

For example, the number 10011 (base 2)

Means: $1\times2^4+0\times2^3+0\times2^2+1\times2^1+1\times2^0 = 16+2+1=19$(base 10)

(A) Now, let us try to convert base 10 numbers to base 2 numbers.

Example 2.29

Convert 35_{10} to base 2.

Solution We go on dividing by 2 and get the remainder.

2	35
2	17-1
2	8-1
2	4-0
2	2-0
	1-0

Therefore $35_{10} = 100011_2$

Example 2.30

Convert 29_{10} to base 2.

Solution

2	29
2	14-1
2	7-0
2	3-1
	1-1

Therefore $29_{10} = 11101_2$

(B) Let us try to convert base 2 numbers to base 10 numbers.

Example 2.31

Convert 1110011_2 to base 10.

Solution

$$1110011_2 = 1\times2^6+1\times2^5+1\times2^4+0\times2^3+0\times2^2+1\times2^1+1\times2^0$$
$$= 64+32+16+0+0+2+1 = 115_{10}$$

Therefore $11100011_2 = 115_{10}$

Example 2.32

Convert 111111_2 to base 10.

Solution

$$111111_2 = 1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$$

$$= 32+16+8+4+2+1 = 63_{10}$$

Therefore $111111_2 = 63_{10}$

2.5.2 Base 5 system

Base 5 systems were used by some primitive tribes in Bolivia, but the tribes are now extinct.

The digits used are 0,1,2,3 and 4. Note that there is no 5 in base 5 system.

Positional values in a base 5 system are $\ldots 5^4, 5^3, 5^2, 5^1, 5^0$

For Example $3421_5 = 3 \times 5^3 + 4 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$

We can very easily understand this system by using our fingers in our hands.

We start to count as 1, 2, 3, 4, one hand, one hand one, one hand two, one hand three, one hand four, two hands,….

[1, 2, 3, 4, 10, 11, 12, 13, 14, 20,…….

3421_5 must be read as Three four two one base 5.

Example 2.33

Convert 624_{10} to base 5.

Solution Go on dividing by 5 and get the remainder.

5	624
5	124 - 4
5	24 - 4
	4 - 4

Therefore $624_{10} = 4444_5$

Example 2.34

Convert 9875_{10} to base 5.

Solution

5	9875
5	1975- 0
5	395- 0
5	79- 0
5	15- 4
	3-0

Therefore $9875_{10} = 304000_5$

Example 2.35

Convert 3421_5 to base 10.

Solution $3421_5 = 3 \times 5^3 + 4 \times 5^2 + 2 \times 5^1 + 1 \times 5^0$

$= 375 + 100 + 10 + 1 = 486_{10}$

Therefore $3421_5 = 486_{10}$

Example 2.36

Convert 40324_5 to base 10

Solution $40324_5 = 4 \times 5^4 + 0 \times 5^3 + 3 \times 5^2 + 2 \times 5^1 + 4 \times 5^0$

$= 4 \times 625 + 0 + 75 \times 10 + 4$

$= 2500 + 0 + 75 + 10 + 4$

$= 2589_{10}$

Therefore $40324_5 = 2589_{10}$

2.5.3 Base 8 system

The digits used in base 8 system are 0, 1, 2, 3, 4. 5, 6 and 7. Note that there is no 8 in base 8 system.

Positional values in a base 8 system are ... $8^7, 8^6, 8^5, 8^4, 8^3, 8^2, 8^1, 8^0$.

For example, $126_8 = 1 \times 8^2 + 2 \times 8^1 + 6 \times 8^0$

$= 1 \times 64 + 2 \times 8 + 6 \times 1$

$= 64 + 16 + 6$

$= 86_{10}$

Example 2.37

Convert 7893_{10} to base 8

Solution

8	7893
8	986 - 5
8	123-2
8	15-3
	1-7

Therefore $7893_{10} = 17325_8$.

Example 2.38

Convert 1347_8 to base 10

Solution $1347_8 = 1 \times 8^3 + 3 \times 8^2 + 4 \times 8^1 + 7 \times 8^0$

$= 1\times 512 + 3\times 64 + 4\times 8 + 7\times 1$

$= 512+192+32+7$

$= 743_{10}$

Therefore $1347_8 = 743_{10}$.

Exercise: 2.4

1. Convert 45_{10} to base 2.
2. Convert 73_{10} to base 2.
3. Convert 1101011_2 to base 10.
4. Convert 111_2 to base 10.
5. Convert 987_{10} to base 5.
6. Convert 1238_{10} to base 5.
7. Convert 10234_5 to base 10.
8. Convert 211423_5 to base 10.
9. Convert 98567_{10} to base 8.
10. Convert 688_{10} to base 8.
11. Convert 47156_8 to base 10.
12. Convert 585_{10} to base 2, 5 and 8.

Exercise: 2.5

Multiple Choice Questions.

1. The scientific notation of 923.4 is

   (A) 9.234×10^{-2} (B) 9.234×10^{2} (C) 9.234×10^{3} (D) 9.234×10^{-3}

2. The scientific notation of 0.00036 is

   (A) 3.6×10^{-3} (B) 3.6×10^{3} (C) 3.6×10^{-4} (D) 3.6×10^{4}

3. The decimal form of 2.57×10^{3} is

   (A) 257 (B) 2570 (C) 25700 (D) 257000

4. The decimal form of 3.506×10^{-2} is

   (A) 0.03506 (B) 0.003506 (C) 35.06 (D) 350.6

5. The logarithmic form of $5^2 = 25$ is

(A) $\log_5 2 = 25$ (B) $\log_2 5 = 25$ (C) $\log_5 25 = 2$ (D) $\log_{25} 5 = 2$

6. The exponential form of $\log_2 16 = 4$ is

(A) $2^4 = 16$ (B) $4^2 = 16$ (C) $2^{16} = 4$ (D) $4^{16} = 2$

7. The value of $\log_{\frac{3}{4}}\left(\frac{4}{3}\right)$ is

(A) – 2 (B) 1 (C) 2 (D) –1

8. The value of $\log_{49} 7$ is

(A) 2 (B) $\frac{1}{2}$ (C) $\frac{1}{7}$ (D) 1

9. The value of $\log_{\frac{1}{2}} 4$ is

(A) – 2 (B) 0 (C) $\frac{1}{2}$ (D) 2

10. $\log_{10} 8 + \log_{10} 5 - \log_{10} 4 =$

(A) $\log_{10} 9$ (B) $\log_{10} 36$ (C) 1 (D) –1

Points to Remember

- A number N is in scientific notation when it is expressed as the product of a decimal number $1 \le a < 10$ and some integral power of 10.

  $N = a \times 10^n$, where $1 \le a < 10$ and n is an integer.

- If $a^x = b \, (a > 0, a \ne 1)$, then x is said to be the logarithm of b to the base a, which is written $x = \log_a b$.
- Product rule : $\log_a (M \times N) = \log_a M + \log_a N$; a, M, N are positive numbers, $a \ne 1$
- Quotient rule : $\log_a \left(\frac{M}{N}\right) = \log_a M - \log_a N$; a, M, N are positive numbers, $a \ne 1$
- Power rule : $\log_a (M)^n = n \log_a M$; a, M are positive numbers, $a \ne 1$
- Change of base rule : $\log_a M = \log_b M \times \log_a b, a \ne 1, b \ne 1$
- In a logarithm the integral part is called the *characteristic* and the fractional part is called the *mantissa*.

Activity 1

With out using logarithm table find approximate value for $\log_{10} 2$ and $\log_{10} 3$.

We know that $2^{10} = 1024$

$$2^{10} \simeq 1000 \quad \text{(Rounded to nearest powers of 10)}$$

$$2^{10} \simeq 10^3$$

$$2 \simeq 10^{\frac{3}{10}}$$

$$2 \simeq 10^{0.3}$$

$$\Rightarrow \quad \log_{10} 2 \simeq 0.3$$

Similarly $3^4 = 81 \simeq 80 = 8 \times 10 = 2^3 \times 10$

$$\therefore \quad 3^4 \simeq 2^3 \times 10$$

Taking logarithm on both sides with respect to base 10

$$\Rightarrow \quad \log_{10} 3^4 \simeq \log_{10}(2^3 \times 10)$$

$$\Rightarrow \quad 4\log_{10} 3 \simeq 3\log_{10} 2 + \log_{10} 10$$

$$\Rightarrow \quad 4\log_{10} 3 \simeq 3 \times 0.3 + 1$$

$$\log_{10} 3 \simeq \frac{1.9}{4}$$

$$\log_{10} 3 \simeq 0.475$$

Like this calculate approximate value of $\log_{10} 4$, $\log_{10} 8$ and $\log_{10} 12$.

Activity 2

Find $\log_{12} 144$ (i) Using logarithm table (ii) Without using logarithm table

Activity 3

Find the number of digits in 2^{10}, 3^{25}, 4^{50}, 5^{80} by using logarithm.

Activity 4

Complete the following table :

S.No.	Numbers	Scientific Notation	Characteristic	Mantisa	Logarithm of the number
1.	25.6	2.56×10^{1}	1	0.4082	1.4082
2.	0.0154	1.54×10^{-2}	–2	0.1875	$\bar{2}.1875$
3.	375.4			0.5745	
4.	4022		3		3.6044
5.	0.234		–1		$\bar{1}.3692$
6.	0.00056			0.7482	

Activity 5

Look at the following TABLE

A	B	C	D	E
1	2	4	8	16
3	3	5	9	17
5	6	6	10	18
7	7	7	11	19
9	10	12	12	20
11	11	13	13	21
13	14	14	14	22
15	15	15	15	23
17	18	20	24	24
19	19	21	25	25
21	22	22	26	26
23	23	23	27	27
25	26	28	28	28
27	27	29	29	29
29	30	30	30	30
31	31	31	31	31

We can find the age of a person 31 years old or younger. He or She needs only tell the columns that contains his or her age.

For Example, suppose John says that his age appears in columns A, B and D only. Add the numbers from the top row of these columns.

Therefore John's age is 1+2+8=11 years.

Suppose Radha's age is 14, in which columns her age will be present?

Convert 14_{10} to base 2.

$$\begin{array}{r|l} 2 & 14 \\ \hline 2 & 7-0 \\ \hline 2 & 3-1 \\ \hline & 1-1 \\ \hline \end{array}$$

A	B	C	D	E
0	1	1	1	-

So, her age can be found in columns B,C,D.

Activity 6

THINK – A-LETTER

Please select one of the twenty six letters of the alphabet. Look for your thought of letter in each of the five columns below. Please tell me the column numbers. I will tell you the letter you thought of

1	2	3	4	5
A	B	D	H	P
C	C	E	I	Q
E	F	F	J	R
G	G	G	K	S
I	J	L	L	T
K	K	M	M	U
M	N	N	N	V
O	O	O	O	W
Q	R	T	X	X
S	S	U	Y	Y
U	V	V	Z	Z
W	W	W	-	-
Y	Z	-	-	-

You can tell me one of the letters of the alphabet. I will tell you the numbers of the columns in which it is found.

If one says that the letter one has chosen is found in columns 1, 2 and 5.

Solution : 1+2+16=19. The 19th letter is 'S'.

Column 1=1, Column 2=2, Column 3=4, Column 4=8, Column 5=16.

If one says that the letter one has chosen is 'U'.

Solution : 'U' is the 21st letter.

2 | 21
2 | 10-1
2 | 5-0
2 | 2-1
  1-0

It is present in column 1,3 and 5.

MATHEMATICS IN LIFE

Why do we study Mathematics?

We must study Mathematics.

- To ADD noble qualities
- To SUBTRACT bad habits
- To MULTIPLY love and friendship
- To DIVIDE equal thoughts among us
- To ROOT-OUT dreadful caste and creed
- To EQUATE rich and poor in the society
- To ELIMINATE social evils
- To DIFFERENTIATE good from bad
- To INTEGRATE People of our country
- To MINIMISE our ignorance
- To MAXIMIZE our IQ and EQ
- To EXPAND our unity among the world
- To SIMPLIFY our difficulties and
- To be RATIONAL and dynamic.

Exercise 2.1

1. (i) 7.493×10^{11} (ii) 1.3×10^{7} (iii) 1.05003×10^{5} (iv) 5.436×10^{14}

(v) 9.6×10^{-3} (vi) 1.3307×10^{-6} (vii) 2.2×10^{-9} (viii) 9.0×10^{-13}

2. (i) 0.00000325 (ii) 0.0004134 (iii) 41340 (iv) 18600000

(v) 9870000000 (vi) 0.000000001432

3. (i) 6.4×10^{13} (ii) 3.375×10^{1} (iii) 2.56×10^{3} (iv) 6.9984×10^{-28} (v) 3.993×10^{2}

Exercise 2.2

1. (i) True (ii) False (iii) False (iv) False (v) True (vi) False

2. (i) $\log_2 16 = 4$ (ii) $\log_3 243 = 5$ (iii) $\log_{10} 0.1 = -1$ (iv) $\log_8\left(\frac{1}{4}\right) = -\frac{2}{3}$

(v) $\log_{25} 5 = \frac{1}{2}$ (vi) $\log_{12}\left(\frac{1}{144}\right) = -2$

3. (i) $6^3 = 216$ (ii) $9^{\frac{1}{2}} = 3$ (iii) $5^0 = 1$ (iv) $(\sqrt{3})^4 = 9$ (v) $(64)^{-\frac{1}{2}} = \frac{1}{8}$ (vi) $(.5)^{-3} = 8$

4. (i) -4 (ii) 3 (iii) 5 (iv) -3 (v) -4 (vi) 5

5. (i) $x = \sqrt{2}$ (ii) $x = \frac{1}{125}$ (iii) $y = \frac{1}{9}$ (iv) $x = \sqrt{5}$ (v) $x = 10$ (vi) $x = -\frac{4}{3}$

6. (i) $\log_{10} 9$ (ii) $\log_{25}\left(\frac{7}{2}\right)$ (iii) 2 (iv) 2 (v) $\log_{10}\left(\frac{72}{25}\right)$ (vi) 1

7. (i) $x = -2$ (ii) $x = 2$ (iii) $x = 2$ (iv) $x = 4$ (v) $x = 3$ (vi) $x = 5$ (vii) $x = 5$

(viii) $x = 7$ **8.** (i) $y + z$ (ii) $3x$ (iii) $x + y + z$ (iv) $3(y - z)$ (v) $x - y + z$ (vi) $y - x$

Exercise 2.3

1. (i) 9.243×10^{1} (ii) 9.243×10^{-1} (iii) 9.243×10^{3} (iv) 9.243×10^{5} (v) 9.243×10^{-3}

(vi) 9.243×10^{-2} **2.** (i) 3 (ii) 1 (iii) -3 (iv) -2 (v) -1 (vi) 0

3. (i) 4.3756 (ii) 1.3756 (iii) 0.3756 (iv) $\bar{1}.3756$ (v) 7.3756 (vi) $\bar{5}.3756$

4. (i) 1.3649 (ii) 0.9694 (iii) 2.5179 (iv) $\bar{3}.1348$ (v) $\bar{1}.9946$ (vi) 3.8180

5. (i) 1180 (ii) 57.41 (iii) 0.2413 (iv) 0.004015 (v) 1.876 (vi) 0.01513

6. (i) 30550 (ii) 21.82 (iii) 0.05309 (iv) 3.497 (v) 328100000 (vi) 8.249
(vii) 2.122 (viii) 1.666 (ix) 0.08366 (x) 0.5948 (xi) 1.888 (xii) 1.772

Exercise 2.4

1. 101101_2 2. 1001001_2 3. 107_{10} 4. 7_{10} 5. 12422_5

6. 14423_5 7. 694_{10} 8. 7113_{10} 9. 300407_8 10. 1260_8

11. 20038_{10} 12. 1001001001_2, 4320_5, 1111_8.

Exercise 2.5

1. B 2. C 3. B 4. A 5. C 6. A 7. D 8. B 9. A 10. C

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
1.0	0.0000	0.0043	0.0086	0.0128	0.0170	0.0212	0.0253	0.0294	0.0334	0.0374	4	8	12	17	21	25	29	33	37
1.1	0.0414	0.0453	0.0492	0.0531	0.0569	0.0607	0.0645	0.0682	0.0719	0.0755	4	8	11	15	19	23	26	30	34
1.2	0.0792	0.0828	0.0864	0.0899	0.0934	0.0969	0.1004	0.1038	0.1072	0.1106	3	7	10	14	17	21	24	28	31
1.3	0.1139	0.1173	0.1206	0.1239	0.1271	0.1303	0.1335	0.1367	0.1399	0.1430	3	6	10	13	16	19	23	26	29
1.4	0.1461	0.1492	0.1523	0.1553	0.1584	0.1614	0.1644	0.1673	0.1703	0.1732	3	6	9	12	15	18	21	24	27
1.5	0.1761	0.1790	0.1818	0.1847	0.1875	0.1903	0.1931	0.1959	0.1987	0.2014	3	6	8	11	14	17	20	22	25
1.6	0.2041	0.2068	0.2095	0.2122	0.2148	0.2175	0.2201	0.2227	0.2253	0.2279	3	5	8	11	13	16	18	21	24
1.7	0.2304	0.2330	0.2355	0.2380	0.2405	0.2430	0.2455	0.2480	0.2504	0.2529	2	5	7	10	12	15	17	20	22
1.8	0.2553	0.2577	0.2601	0.2625	0.2648	0.2672	0.2695	0.2718	0.2742	0.2765	2	5	7	9	12	14	16	19	21
1.9	0.2788	0.2810	0.2833	0.2856	0.2878	0.2900	0.2923	0.2945	0.2967	0.2989	2	4	7	9	11	13	16	18	20
2.0	0.3010	0.3032	0.3054	0.3075	0.3096	0.3118	0.3139	0.3160	0.3181	0.3201	2	4	6	8	11	13	15	17	19
2.1	0.3222	0.3243	0.3263	0.3284	0.3304	0.3324	0.3345	0.3365	0.3385	0.3404	2	4	6	8	10	12	14	16	18
2.2	0.3424	0.3444	0.3464	0.3483	0.3502	0.3522	0.3541	0.3560	0.3579	0.3598	2	4	6	8	10	12	14	15	17
2.3	0.3617	0.3636	0.3655	0.3674	0.3692	0.3711	0.3729	0.3747	0.3766	0.3784	2	4	6	7	9	11	13	15	17
2.4	0.3802	0.3820	0.3838	0.3856	0.3874	0.3892	0.3909	0.3927	0.3945	0.3962	2	4	5	7	9	11	12	14	16
2.5	0.3979	0.3997	0.4014	0.4031	0.4048	0.4065	0.4082	0.4099	0.4116	0.4133	2	3	5	7	9	10	12	14	15
2.6	0.4150	0.4166	0.4183	0.4200	0.4216	0.4232	0.4249	0.4265	0.4281	0.4298	2	3	5	7	8	10	11	13	15
2.7	0.4314	0.4330	0.4346	0.4362	0.4378	0.4393	0.4409	0.4425	0.4440	0.4456	2	3	5	6	8	9	11	13	14
2.8	0.4472	0.4487	0.4502	0.4518	0.4533	0.4548	0.4564	0.4579	0.4594	0.4609	2	3	5	6	8	9	11	12	14
2.9	0.4624	0.4639	0.4654	0.4669	0.4683	0.4698	0.4713	0.4728	0.4742	0.4757	1	3	4	6	7	9	10	12	13
3.0	0.4771	0.4786	0.4800	0.4814	0.4829	0.4843	0.4857	0.4871	0.4886	0.4900	1	3	4	6	7	9	10	11	13
3.1	0.4914	0.4928	0.4942	0.4955	0.4969	0.4983	0.4997	0.5011	0.5024	0.5038	1	3	4	6	7	8	10	11	12
3.2	0.5051	0.5065	0.5079	0.5092	0.5105	0.5119	0.5132	0.5145	0.5159	0.5172	1	3	4	5	7	8	9	11	12
3.3	0.5185	0.5198	0.5211	0.5224	0.5237	0.5250	0.5263	0.5276	0.5289	0.5302	1	3	4	5	6	8	9	10	12
3.4	0.5315	0.5328	0.5340	0.5353	0.5366	0.5378	0.5391	0.5403	0.5416	0.5428	1	3	4	5	6	8	9	10	11
3.5	0.5441	0.5453	0.5465	0.5478	0.5490	0.5502	0.5514	0.5527	0.5539	0.5551	1	2	4	5	6	7	9	10	11
3.6	0.5563	0.5575	0.5587	0.5599	0.5611	0.5623	0.5635	0.5647	0.5658	0.5670	1	2	4	5	6	7	8	10	11
3.7	0.5682	0.5694	0.5705	0.5717	0.5729	0.5740	0.5752	0.5763	0.5775	0.5786	1	2	3	5	6	7	8	9	10
3.8	0.5798	0.5809	0.5821	0.5832	0.5843	0.5855	0.5866	0.5877	0.5888	0.5899	1	2	3	5	6	7	8	9	10
3.9	0.5911	0.5922	0.5933	0.5944	0.5955	0.5966	0.5977	0.5988	0.5999	0.6010	1	2	3	4	5	7	8	9	10
4.0	0.6021	0.6031	0.6042	0.6053	0.6064	0.6075	0.6085	0.6096	0.6107	0.6117	1	2	3	4	5	6	8	9	10
4.1	0.6128	0.6138	0.6149	0.6160	0.6170	0.6180	0.6191	0.6201	0.6212	0.6222	1	2	3	4	5	6	7	8	9
4.2	0.6232	0.6243	0.6253	0.6263	0.6274	0.6284	0.6294	0.6304	0.6314	0.6325	1	2	3	4	5	6	7	8	9
4.3	0.6335	0.6345	0.6355	0.6365	0.6375	0.6385	0.6395	0.6405	0.6415	0.6425	1	2	3	4	5	6	7	8	9
4.4	0.6435	0.6444	0.6454	0.6464	0.6474	0.6484	0.6493	0.6503	0.6513	0.6522	1	2	3	4	5	6	7	8	9
4.5	0.6532	0.6542	0.6551	0.6561	0.6571	0.6580	0.6590	0.6599	0.6609	0.6618	1	2	3	4	5	6	7	8	9
4.6	0.6628	0.6637	0.6646	0.6656	0.6665	0.6675	0.6684	0.6693	0.6702	0.6712	1	2	3	4	5	6	7	7	8
4.7	0.6721	0.6730	0.6739	0.6749	0.6758	0.6767	0.6776	0.6785	0.6794	0.6803	1	2	3	4	5	5	6	7	8
4.8	0.6812	0.6821	0.6830	0.6839	0.6848	0.6857	0.6866	0.6875	0.6884	0.6893	1	2	3	4	4	5	6	7	8
4.9	0.6902	0.6911	0.6920	0.6928	0.6937	0.6946	0.6955	0.6964	0.6972	0.6981	1	2	3	4	4	5	6	7	8
5.0	0.6990	0.6998	0.7007	0.7016	0.7024	0.7033	0.7042	0.7050	0.7059	0.7067	1	2	3	3	4	5	6	7	8
5.1	0.7076	0.7084	0.7093	0.7101	0.7110	0.7118	0.7126	0.7135	0.7143	0.7152	1	2	3	3	4	5	6	7	8
5.2	0.7160	0.7168	0.7177	0.7185	0.7193	0.7202	0.7210	0.7218	0.7226	0.7235	1	2	2	3	4	5	6	7	7
5.3	0.7243	0.7251	0.7259	0.7267	0.7275	0.7284	0.7292	0.7300	0.7308	0.7316	1	2	2	3	4	5	6	6	7
5.4	0.7324	0.7332	0.7340	0.7348	0.7356	0.7364	0.7372	0.7380	0.7388	0.7396	1	2	2	3	4	5	6	6	7

Mathematics

LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
5.5	0.7404	0.7412	0.7419	0.7427	0.7435	0.7443	0.7451	0.7459	0.7466	0.7474	1	2	2	3	4	5	5	6	7
5.6	0.7482	0.7490	0.7497	0.7505	0.7513	0.7520	0.7528	0.7536	0.7543	0.7551	1	2	2	3	4	5	5	6	7
5.7	0.7559	0.7566	0.7574	0.7582	0.7589	0.7597	0.7604	0.7612	0.7619	0.7627	1	2	2	3	4	5	5	6	7
5.8	0.7634	0.7642	0.7649	0.7657	0.7664	0.7672	0.7679	0.7686	0.7694	0.7701	1	1	2	3	4	4	5	6	7
5.9	0.7709	0.7716	0.7723	0.7731	0.7738	0.7745	0.7752	0.7760	0.7767	0.7774	1	1	2	3	4	4	5	6	7
6.0	0.7782	0.7789	0.7796	0.7803	0.7810	0.7818	0.7825	0.7832	0.7839	0.7846	1	1	2	3	4	4	5	6	6
6.1	0.7853	0.7860	0.7868	0.7875	0.7882	0.7889	0.7896	0.7903	0.7910	0.7917	1	1	2	3	4	4	5	6	6
6.2	0.7924	0.7931	0.7938	0.7945	0.7952	0.7959	0.7966	0.7973	0.7980	0.7987	1	1	2	3	3	4	5	6	6
6.3	0.7993	0.8000	0.8007	0.8014	0.8021	0.8028	0.8035	0.8041	0.8048	0.8055	1	1	2	3	3	4	5	5	6
6.4	0.8062	0.8069	0.8075	0.8082	0.8089	0.8096	0.8102	0.8109	0.8116	0.8122	1	1	2	3	3	4	5	5	6
6.5	0.8129	0.8136	0.8142	0.8149	0.8156	0.8162	0.8169	0.8176	0.8182	0.8189	1	1	2	3	3	4	5	5	6
6.6	0.8195	0.8202	0.8209	0.8215	0.8222	0.8228	0.8235	0.8241	0.8248	0.8254	1	1	2	3	3	4	5	5	6
6.7	0.8261	0.8267	0.8274	0.8280	0.8287	0.8293	0.8299	0.8306	0.8312	0.8319	1	1	2	3	3	4	5	5	6
6.8	0.8325	0.8331	0.8338	0.8344	0.8351	0.8357	0.8363	0.8370	0.8376	0.8382	1	1	2	3	3	4	4	5	6
6.9	0.8388	0.8395	0.8401	0.8407	0.8414	0.8420	0.8426	0.8432	0.8439	0.8445	1	1	2	2	3	4	4	5	6
7.0	0.8451	0.8457	0.8463	0.8470	0.8476	0.8482	0.8488	0.8494	0.8500	0.8506	1	1	2	2	3	4	4	5	6
7.1	0.8513	0.8519	0.8525	0.8531	0.8537	0.8543	0.8549	0.8555	0.8561	0.8567	1	1	2	2	3	4	4	5	5
7.2	0.8573	0.8579	0.8585	0.8591	0.8597	0.8603	0.8609	0.8615	0.8621	0.8627	1	1	2	2	3	4	4	5	5
7.3	0.8633	0.8639	0.8645	0.8651	0.8657	0.8663	0.8669	0.8675	0.8681	0.8686	1	1	2	2	3	4	4	5	5
7.4	0.8692	0.8698	0.8704	0.8710	0.8716	0.8722	0.8727	0.8733	0.8739	0.8745	1	1	2	2	3	4	4	5	5
7.5	0.8751	0.8756	0.8762	0.8768	0.8774	0.8779	0.8785	0.8791	0.8797	0.8802	1	1	2	2	3	3	4	5	5
7.6	0.8808	0.8814	0.8820	0.8825	0.8831	0.8837	0.8842	0.8848	0.8854	0.8859	1	1	2	2	3	3	4	5	5
7.7	0.8865	0.8871	0.8876	0.8882	0.8887	0.8893	0.8899	0.8904	0.8910	0.8915	1	1	2	2	3	3	4	4	5
7.8	0.8921	0.8927	0.8932	0.8938	0.8943	0.8949	0.8954	0.8960	0.8965	0.8971	1	1	2	2	3	3	4	4	5
7.9	0.8976	0.8982	0.8987	0.8993	0.8998	0.9004	0.9009	0.9015	0.9020	0.9025	1	1	2	2	3	3	4	4	5
8.0	0.9031	0.9036	0.9042	0.9047	0.9053	0.9058	0.9063	0.9069	0.9074	0.9079	1	1	2	2	3	3	4	4	5
8.1	0.9085	0.9090	0.9096	0.9101	0.9106	0.9112	0.9117	0.9122	0.9128	0.9133	1	1	2	2	3	3	4	4	5
8.2	0.9138	0.9143	0.9149	0.9154	0.9159	0.9165	0.9170	0.9175	0.9180	0.9186	1	1	2	2	3	3	4	4	5
8.3	0.9191	0.9196	0.9201	0.9206	0.9212	0.9217	0.9222	0.9227	0.9232	0.9238	1	1	2	2	3	3	4	4	5
8.4	0.9243	0.9248	0.9253	0.9258	0.9263	0.9269	0.9274	0.9279	0.9284	0.9289	1	1	2	2	3	3	4	4	5
8.5	0.9294	0.9299	0.9304	0.9309	0.9315	0.9320	0.9325	0.9330	0.9335	0.9340	1	1	2	2	3	3	4	4	5
8.6	0.9345	0.9350	0.9355	0.9360	0.9365	0.9370	0.9375	0.9380	0.9385	0.9390	1	1	2	2	3	3	4	4	5
8.7	0.9395	0.9400	0.9405	0.9410	0.9415	0.9420	0.9425	0.9430	0.9435	0.9440	0	1	1	2	2	3	3	4	4
8.8	0.9445	0.9450	0.9455	0.9460	0.9465	0.9469	0.9474	0.9479	0.9484	0.9489	0	1	1	2	2	3	3	4	4
8.9	0.9494	0.9499	0.9504	0.9509	0.9513	0.9518	0.9523	0.9528	0.9533	0.9538	0	1	1	2	2	3	3	4	4
9.0	0.9542	0.9547	0.9552	0.9557	0.9562	0.9566	0.9571	0.9576	0.9581	0.9586	0	1	1	2	2	3	3	4	4
9.1	0.9590	0.9595	0.9600	0.9605	0.9609	0.9614	0.9619	0.9624	0.9628	0.9633	0	1	1	2	2	3	3	4	4
9.2	0.9638	0.9643	0.9647	0.9652	0.9657	0.9661	0.9666	0.9671	0.9675	0.9680	0	1	1	2	2	3	3	4	4
9.3	0.9685	0.9689	0.9694	0.9699	0.9703	0.9708	0.9713	0.9717	0.9722	0.9727	0	1	1	2	2	3	3	4	4
9.4	0.9731	0.9736	0.9741	0.9745	0.9750	0.9754	0.9759	0.9763	0.9768	0.9773	0	1	1	2	2	3	3	4	4
9.5	0.9777	0.9782	0.9786	0.9791	0.9795	0.9800	0.9805	0.9809	0.9814	0.9818	0	1	1	2	2	3	3	4	4
9.6	0.9823	0.9827	0.9832	0.9836	0.9841	0.9845	0.9850	0.9854	0.9859	0.9863	0	1	1	2	2	3	3	4	4
9.7	0.9868	0.9872	0.9877	0.9881	0.9886	0.9890	0.9894	0.9899	0.9903	0.9908	0	1	1	2	2	3	3	4	4
9.8	0.9912	0.9917	0.9921	0.9926	0.9930	0.9934	0.9939	0.9943	0.9948	0.9952	0	1	1	2	2	3	3	4	4
9.9	0.9956	0.9961	0.9965	0.9969	0.9974	0.9978	0.9983	0.9987	0.9991	0.9996	0	1	1	2	2	3	3	3	4

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.00	1.000	1.002	1.005	1.007	1.009	1.012	1.014	1.016	1.019	1.021	0	0	1	1	1	1	2	2	2
0.01	1.023	1.026	1.028	1.030	1.033	1.035	1.038	1.040	1.042	1.045	0	0	1	1	1	1	2	2	2
0.02	1.047	1.050	1.052	1.054	1.057	1.059	1.062	1.064	1.067	1.069	0	0	1	1	1	1	2	2	2
0.03	1.072	1.074	1.076	1.079	1.081	1.084	1.086	1.089	1.091	1.094	0	0	1	1	1	1	2	2	2
0.04	1.096	1.099	1.102	1.104	1.107	1.109	1.112	1.114	1.117	1.119	0	1	1	1	1	2	2	2	2
0.05	1.122	1.125	1.127	1.130	1.132	1.135	1.138	1.140	1.143	1.146	0	1	1	1	1	2	2	2	2
0.06	1.148	1.151	1.153	1.156	1.159	1.161	1.164	1.167	1.169	1.172	0	1	1	1	1	2	2	2	2
0.07	1.175	1.178	1.180	1.183	1.186	1.189	1.191	1.194	1.197	1.199	0	1	1	1	1	2	2	2	2
0.08	1.202	1.205	1.208	1.211	1.213	1.216	1.219	1.222	1.225	1.227	0	1	1	1	1	2	2	2	3
0.09	1.230	1.233	1.236	1.239	1.242	1.245	1.247	1.250	1.253	1.256	0	1	1	1	1	2	2	2	3
0.10	1.259	1.262	1.265	1.268	1.271	1.274	1.276	1.279	1.282	1.285	0	1	1	1	1	2	2	2	3
0.11	1.288	1.291	1.294	1.297	1.300	1.303	1.306	1.309	1.312	1.315	0	1	1	1	2	2	2	2	3
0.12	1.318	1.321	1.324	1.327	1.330	1.334	1.337	1.340	1.343	1.346	0	1	1	1	2	2	2	2	3
0.13	1.349	1.352	1.355	1.358	1.361	1.365	1.368	1.371	1.374	1.377	0	1	1	1	2	2	2	3	3
0.14	1.380	1.384	1.387	1.390	1.393	1.396	1.400	1.403	1.406	1.409	0	1	1	1	2	2	2	3	3
0.15	1.413	1.416	1.419	1.422	1.426	1.429	1.432	1.435	1.439	1.442	0	1	1	1	2	2	2	3	3
0.16	1.445	1.449	1.452	1.455	1.459	1.462	1.466	1.469	1.472	1.476	0	1	1	1	2	2	2	3	3
0.17	1.479	1.483	1.486	1.489	1.493	1.496	1.500	1.503	1.507	1.510	0	1	1	1	2	2	2	3	3
0.18	1.514	1.517	1.521	1.524	1.528	1.531	1.535	1.538	1.542	1.545	0	1	1	1	2	2	2	3	3
0.19	1.549	1.552	1.556	1.560	1.563	1.567	1.570	1.574	1.578	1.581	0	1	1	1	2	2	3	3	3
0.20	1.585	1.589	1.592	1.596	1.600	1.603	1.607	1.611	1.614	1.618	0	1	1	1	2	2	3	3	3
0.21	1.622	1.626	1.629	1.633	1.637	1.641	1.644	1.648	1.652	1.656	0	1	1	2	2	2	3	3	3
0.22	1.660	1.663	1.667	1.671	1.675	1.679	1.683	1.687	1.690	1.694	0	1	1	2	2	2	3	3	3
0.23	1.698	1.702	1.706	1.710	1.714	1.718	1.722	1.726	1.730	1.734	0	1	1	2	2	2	3	3	4
0.24	1.738	1.742	1.746	1.750	1.754	1.758	1.762	1.766	1.770	1.774	0	1	1	2	2	2	3	3	4
0.25	1.778	1.782	1.786	1.791	1.795	1.799	1.803	1.807	1.811	1.816	0	1	1	2	2	2	3	3	4
0.26	1.820	1.824	1.828	1.832	1.837	1.841	1.845	1.849	1.854	1.858	0	1	1	2	2	3	3	3	4
0.27	1.862	1.866	1.871	1.875	1.879	1.884	1.888	1.892	1.897	1.901	0	1	1	2	2	3	3	3	4
0.28	1.905	1.910	1.914	1.919	1.923	1.928	1.932	1.936	1.941	1.945	0	1	1	2	2	3	3	4	4
0.29	1.950	1.954	1.959	1.963	1.968	1.972	1.977	1.982	1.986	1.991	0	1	1	2	2	3	3	4	4
0.30	1.995	2.000	2.004	2.009	2.014	2.018	2.023	2.028	2.032	2.037	0	1	1	2	2	3	3	4	4
0.31	2.042	2.046	2.051	2.056	2.061	2.065	2.070	2.075	2.080	2.084	0	1	1	2	2	3	3	4	4
0.32	2.089	2.094	2.099	2.104	2.109	2.113	2.118	2.123	2.128	2.133	0	1	1	2	2	3	3	4	4
0.33	2.138	2.143	2.148	2.153	2.158	2.163	2.168	2.173	2.178	2.183	0	1	1	2	2	3	3	4	4
0.34	2.188	2.193	2.198	2.203	2.208	2.213	2.218	2.223	2.228	2.234	1	1	2	2	3	3	4	4	5
0.35	2.239	2.244	2.249	2.254	2.259	2.265	2.270	2.275	2.280	2.286	1	1	2	2	3	3	4	4	5
0.36	2.291	2.296	2.301	2.307	2.312	2.317	2.323	2.328	2.333	2.339	1	1	2	2	3	3	4	4	5
0.37	2.344	2.350	2.355	2.360	2.366	2.371	2.377	2.382	2.388	2.393	1	1	2	2	3	3	4	4	5
0.38	2.399	2.404	2.410	2.415	2.421	2.427	2.432	2.438	2.443	2.449	1	1	2	2	3	3	4	4	5
0.39	2.455	2.460	2.466	2.472	2.477	2.483	2.489	2.495	2.500	2.506	1	1	2	2	3	3	4	5	5
0.40	2.512	2.518	2.523	2.529	2.535	2.541	2.547	2.553	2.559	2.564	1	1	2	2	3	4	4	5	5
0.41	2.570	2.576	2.582	2.588	2.594	2.600	2.606	2.612	2.618	2.624	1	1	2	2	3	4	4	5	5
0.42	2.630	2.636	2.642	2.649	2.655	2.661	2.667	2.673	2.679	2.685	1	1	2	2	3	4	4	5	6
0.43	2.692	2.698	2.704	2.710	2.716	2.723	2.729	2.735	2.742	2.748	1	1	2	3	3	4	4	5	6
0.44	2.754	2.761	2.767	2.773	2.780	2.786	2.793	2.799	2.805	2.812	1	1	2	3	3	4	4	5	6
0.45	2.818	2.825	2.831	2.838	2.844	2.851	2.858	2.864	2.871	2.877	1	1	2	3	3	4	5	5	6
0.46	2.884	2.891	2.897	2.904	2.911	2.917	2.924	2.931	2.938	2.944	1	1	2	3	3	4	5	5	6
0.47	2.951	2.958	2.965	2.972	2.979	2.985	2.992	2.999	3.006	3.013	1	1	2	3	3	4	5	5	6
0.48	3.020	3.027	3.034	3.041	3.048	3.055	3.062	3.069	3.076	3.083	1	1	2	3	4	4	5	6	6
0.49	3.090	3.097	3.105	3.112	3.119	3.126	3.133	3.141	3.148	3.155	1	1	2	3	4	4	5	6	6

ANTI LOGARITHM TABLE

											Mean Difference								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.50	3.162	3.170	3.177	3.184	3.192	3.199	3.206	3.214	3.221	3.228	1	1	2	3	4	4	5	6	7
0.51	3.236	3.243	3.251	3.258	3.266	3.273	3.281	3.289	3.296	3.304	1	2	2	3	4	5	5	6	7
0.52	3.311	3.319	3.327	3.334	3.342	3.350	3.357	3.365	3.373	3.381	1	2	2	3	4	5	5	6	7
0.53	3.388	3.396	3.404	3.412	3.420	3.428	3.436	3.443	3.451	3.459	1	2	2	3	4	5	6	6	7
0.54	3.467	3.475	3.483	3.491	3.499	3.508	3.516	3.524	3.532	3.540	1	2	2	3	4	5	6	6	7
0.55	3.548	3.556	3.565	3.573	3.581	3.589	3.597	3.606	3.614	3.622	1	2	2	3	4	5	6	7	7
0.56	3.631	3.639	3.648	3.656	3.664	3.673	3.681	3.690	3.698	3.707	1	2	3	3	4	5	6	7	8
0.57	3.715	3.724	3.733	3.741	3.750	3.758	3.767	3.776	3.784	3.793	1	2	3	3	4	5	6	7	8
0.58	3.802	3.811	3.819	3.828	3.837	3.846	3.855	3.864	3.873	3.882	1	2	3	4	4	5	6	7	8
0.59	3.890	3.899	3.908	3.917	3.926	3.936	3.945	3.954	3.963	3.972	1	2	3	4	5	5	6	7	8
0.60	3.981	3.990	3.999	4.009	4.018	4.027	4.036	4.046	4.055	4.064	1	2	3	4	5	6	6	7	8
0.61	4.074	4.083	4.093	4.102	4.111	4.121	4.130	4.140	4.150	4.159	1	2	3	4	5	6	7	8	9
0.62	4.169	4.178	4.188	4.198	4.207	4.217	4.227	4.236	4.246	4.256	1	2	3	4	5	6	7	8	9
0.63	4.266	4.276	4.285	4.295	4.305	4.315	4.325	4.335	4.345	4.355	1	2	3	4	5	6	7	8	9
0.64	4.365	4.375	4.385	4.395	4.406	4.416	4.426	4.436	4.446	4.457	1	2	3	4	5	6	7	8	9
0.65	4.467	4.477	4.487	4.498	4.508	4.519	4.529	4.539	4.550	4.560	1	2	3	4	5	6	7	8	9
0.66	4.571	4.581	4.592	4.603	4.613	4.624	4.634	4.645	4.656	4.667	1	2	3	4	5	6	7	9	10
0.67	4.677	4.688	4.699	4.710	4.721	4.732	4.742	4.753	4.764	4.775	1	2	3	4	5	7	8	9	10
0.68	4.786	4.797	4.808	4.819	4.831	4.842	4.853	4.864	4.875	4.887	1	2	3	4	6	7	8	9	10
0.69	4.898	4.909	4.920	4.932	4.943	4.955	4.966	4.977	4.989	5.000	1	2	3	5	6	7	8	9	10
0.70	5.012	5.023	5.035	5.047	5.058	5.070	5.082	5.093	5.105	5.117	1	2	4	5	6	7	8	9	11
0.71	5.129	5.140	5.152	5.164	5.176	5.188	5.200	5.212	5.224	5.236	1	2	4	5	6	7	8	10	11
0.72	5.248	5.260	5.272	5.284	5.297	5.309	5.321	5.333	5.346	5.358	1	2	4	5	6	7	9	10	11
0.73	5.370	5.383	5.395	5.408	5.420	5.433	5.445	5.458	5.470	5.483	1	3	4	5	6	8	9	10	11
0.74	5.495	5.508	5.521	5.534	5.546	5.559	5.572	5.585	5.598	5.610	1	3	4	5	6	8	9	10	12
0.75	5.623	5.636	5.649	5.662	5.675	5.689	5.702	5.715	5.728	5.741	1	3	4	5	7	8	9	10	12
0.76	5.754	5.768	5.781	5.794	5.808	5.821	5.834	5.848	5.861	5.875	1	3	4	5	7	8	9	11	12
0.77	5.888	5.902	5.916	5.929	5.943	5.957	5.970	5.984	5.998	6.012	1	3	4	5	7	8	10	11	12
0.78	6.026	6.039	6.053	6.067	6.081	6.095	6.109	6.124	6.138	6.152	1	3	4	6	7	8	10	11	13
0.79	6.166	6.180	6.194	6.209	6.223	6.237	6.252	6.266	6.281	6.295	1	3	4	6	7	9	10	11	13
0.80	6.310	6.324	6.339	6.353	6.368	6.383	6.397	6.412	6.427	6.442	1	3	4	6	7	9	10	12	13
0.81	6.457	6.471	6.486	6.501	6.516	6.531	6.546	6.561	6.577	6.592	2	3	5	6	8	9	11	12	14
0.82	6.607	6.622	6.637	6.653	6.668	6.683	6.699	6.714	6.730	6.745	2	3	5	6	8	9	11	12	14
0.83	6.761	6.776	6.792	6.808	6.823	6.839	6.855	6.871	6.887	6.902	2	3	5	6	8	9	11	13	14
0.84	6.918	6.934	6.950	6.966	6.982	6.998	7.015	7.031	7.047	7.063	2	3	5	6	8	10	11	13	15
0.85	7.079	7.096	7.112	7.129	7.145	7.161	7.178	7.194	7.211	7.228	2	3	5	7	8	10	12	13	15
0.86	7.244	7.261	7.278	7.295	7.311	7.328	7.345	7.362	7.379	7.396	2	3	5	7	8	10	12	13	15
0.87	7.413	7.430	7.447	7.464	7.482	7.499	7.516	7.534	7.551	7.568	2	3	5	7	9	10	12	14	16
0.88	7.586	7.603	7.621	7.638	7.656	7.674	7.691	7.709	7.727	7.745	2	4	5	7	9	11	12	14	16
0.89	7.762	7.780	7.798	7.816	7.834	7.852	7.870	7.889	7.907	7.925	2	4	5	7	9	11	13	14	16
0.90	7.943	7.962	7.980	7.998	8.017	8.035	8.054	8.072	8.091	8.110	2	4	6	7	9	11	13	15	17
0.91	8.128	8.147	8.166	8.185	8.204	8.222	8.241	8.260	8.279	8.299	2	4	6	8	9	11	13	15	17
0.92	8.318	8.337	8.356	8.375	8.395	8.414	8.433	8.453	8.472	8.492	2	4	6	8	10	12	14	15	17
0.93	8.511	8.531	8.551	8.570	8.590	8.610	8.630	8.650	8.670	8.690	2	4	6	8	10	12	14	16	18
0.94	8.710	8.730	8.750	8.770	8.790	8.810	8.831	8.851	8.872	8.892	2	4	6	8	10	12	14	16	18
0.95	8.913	8.933	8.954	8.974	8.995	9.016	9.036	9.057	9.078	9.099	2	4	6	8	10	12	15	17	19
0.96	9.120	9.141	9.162	9.183	9.204	9.226	9.247	9.268	9.290	9.311	2	4	6	8	11	13	15	17	19
0.97	9.333	9.354	9.376	9.397	9.419	9.441	9.462	9.484	9.506	9.528	2	4	7	9	11	13	15	17	20
0.98	9.550	9.572	9.594	9.616	9.638	9.661	9.683	9.705	9.727	9.750	2	4	7	9	11	13	16	18	20
0.99	9.772	9.795	9.817	9.840	9.863	9.886	9.908	9.931	9.954	9.977	2	5	7	9	11	14	16	18	20

3 GEOMETRY

Truth can never be told so as to be understood, and not to be believed

- William Blake

Main Targets

- To understand theorems on circles.

EUCLID
(325 - 265 BC)

Euclid (325-265), also known as Euclid of Alexandria, was a Greek mathematician, often referred to as the "Father of Geometry". He was active in Alexandria during the reign of Ptolemy 1 (323-293 BC). His elements is one of the most influential works in the history of mathematics, seving as the main textbook for teaching mathematics (especially geometry) from the time of its publication until the late 19th or early 20th century. In the Elements, Euclid deduced the principles of what is now called Euclidean geometry and Number theory.

3.1 Introduction

The Geometry universally used today is referred to as the Euclidean Geometry. Geometry was already in existence in 3000 B.C. in ancient civilizations of Babylon and Egypt but was a nameless mathematical system. It can be concluded that the inventor of geometry came from any of these civilizations. Ancient Indian civilizations also contained records of the earlier versions of geometry. During the Vedic period, several sutras were made that contained geometric instructions on how to construct fire altars. Euclid of Alexandria wrote the book called "Elements", which now becomes the foundation for our modern day geometry.

3.2 Circles

Locus

Locus is a path traced out by a moving point which satisfies certain geometrical conditions.

For example, the locus of a point equidistant from two fixed points is the perpendicular bisector of the line segment joining the two points.

Circles

The locus of a point which moves in such that the distance from a fixed point is always a constant is a circle.

The fixed point is called its centre and the constant distance is called its radius.

The boundary of a circle is called its circumference.

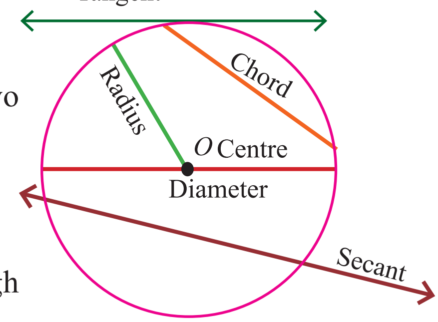


Fig. 3.1

Chord

A chord of a circle is a line segment joining any two points on its circumference.

Diameter

A diameter is a chord of the circle passing through the centre of the circle.

Diameter is the longest chord of the circle.

Secant

A line which intersects a circle in two distinct points is called a secant of the circle.

Tangent

A line that touches the circle at only one point is called a tangent to the circle.

The point at which the tangent meets the circle is its point of contact.

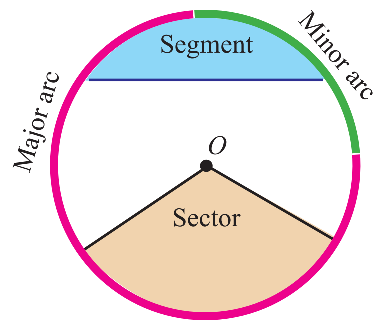


Fig. 3.2

Arc of a Circle

A continuous piece of a circle is called an arc of the circle.

The whole circle has been divided into two pieces, namely, major arc, minor arc.

Concentric Circles

Circles which have the same centre but different radii are called concentric circles.

In the given figure, the two circles are concentric circles having the same centre O but different radii r and R respectively.

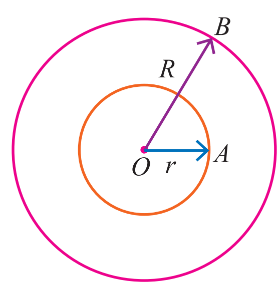


Fig. 3.3

Congruent Arcs

Two arcs $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ of a circle are said to be congruent if they substend same angle at the centre and we write $\overset{\frown}{AB} \equiv \overset{\frown}{CD}$. So,

$$\overset{\frown}{AB} \equiv \overset{\frown}{CD} \Longleftrightarrow m\overset{\frown}{AB} = m\overset{\frown}{CD} \Longleftrightarrow \angle AOB = \angle COD$$

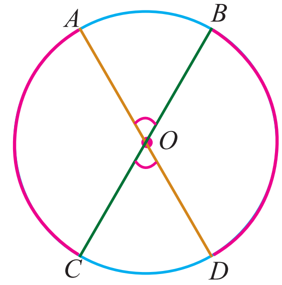


Fig. 3.4

3.2.1 Properties of Chords of a Circle

Result

Equal chords of a circle subtend equal angles at the centre.

In the Fig. 3.5., chord AB = chord CD $\Longrightarrow \angle AOB = \angle COD$

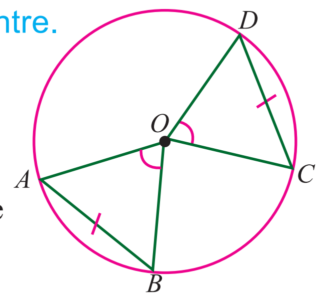


Fig. 3.5

Converse of the result

If the angles subtended by two chords at the centre of a circle are equal, then the chords are equal.

$\angle AOB = \angle COD \Longrightarrow$ chord AB = chord CD

Theorem 1

Perpendicular from the centre of a circle to a chord bisects the chord.

Given : A circle with centre O and AB is a chord of the circle other than the diameter and $OC \perp AB$

To prove: $AC = BC$

Construction: Join OA and OB

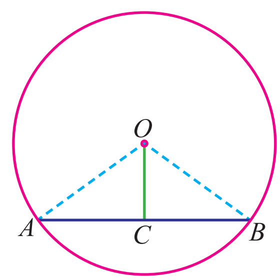


Fig. 3.6

Proof:

In Δs OAC and OBC

(i)	$OA = OB$	(Radii of the same circle.)
(ii)	OC is common	
(iii)	$\angle OCA = \angle OCB$	(Each 90°, since $OC \perp AB$.)
(iv)	$\Delta OAC \equiv \Delta OBC$	(RHS congruency.)

$\therefore AC = BC$ ■

Converse of Theorem 1 : The line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.

Theorem 2

Equal chords of a circle are equidistant from the centre.

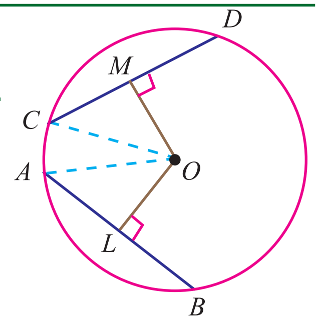


Fig. 3.7

Given: A cirle with centre O and radius r such that chord AB = chord CD.

To prove: $OL = OM$

Construction: Draw $OL \perp AB$ and $OM \perp CD$. Join OA and OC

Proof:

(i) $AL = \frac{1}{2}AB$ and $CM = \frac{1}{2}CD$ (Perpendicular from the centre of a circle to the chord bisects the chord.)

$AB = CD \Longrightarrow \frac{1}{2}AB = \frac{1}{2}CD \Longrightarrow AL = CM$

(ii) $OA = OC$ (radii)

(iii) $\angle OMC = \angle OLA$ (Each 90°)

(iii) $\Delta OLA \equiv \Delta OMC$ (RHS congruence.)

$\therefore\ OL = OM$

Hence AB and CD are equidistant from O .

Converse of Theorem 2 : The chords of a circle which are equidistant from the centre are equal.

Example 3.1

A chord of length 16 cm is drawn in a circle of radius 10 cm. Find the distance of the chord from the centre of the circle.

Solution AB is a chord of length 16 cm

C is the midpoint of AB.

OA is the radius of length 10 cm

$AB = 16$ cm

$AC = \frac{1}{2} \times 16 = 8$ cm

$OA = 10$ cm

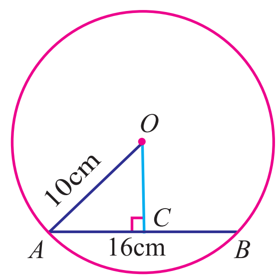


Fig. 3.8

In a right triangle OAC.

$OC^2 = OA^2 - AC^2$

$= 10^2 - 8^2 = 100 - 64 = 36$ cm

$\therefore OC = 6$ cm

Hence, the distance of the chord from the centre is 6 cm.

Example 3.2

In two concentric circles, chord AB of the outer circle cuts the inner circle at C and D. Prove that $AC = BD$.

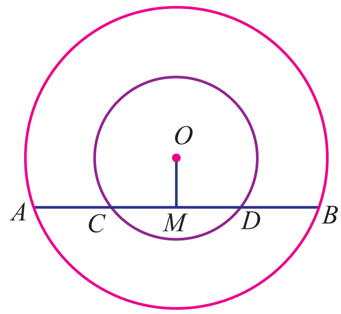


Fig. 3.9

Solution Given: Chord AB of the outer circle cuts the inner circle at C and D.

To prove: $AC = BD$

Construction: Draw $OM \perp AB$

Proof :

Since $OM \perp AB$ (by construction)

OM also $\perp CD$ ($ACDB$ is a line)

In the outer circle

$AM = BM$ (1) ($\because$ OM bisects the chord AB)

In the inner circle

$CM = DM$ (2) ($\because$ OM bisects the chord CD)

From (1) and (2), we get

$AM - CM = BM - DM$

$AC = BD$

3.2.2 Angles Subtended by Arcs

Theorem 3

The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

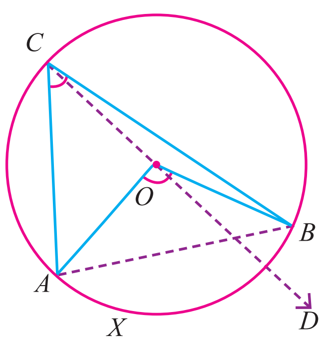


Fig. 3.10

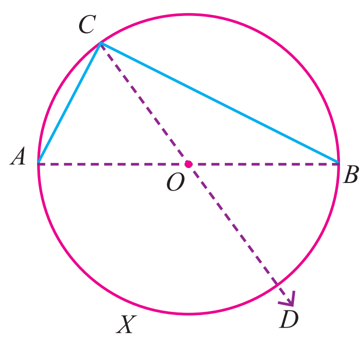


Fig. 3.11

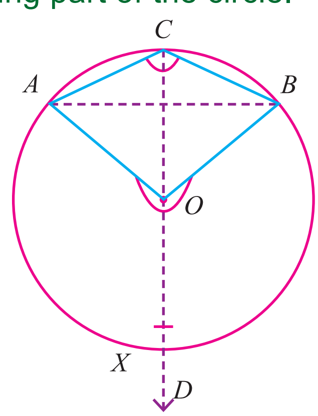


Fig. 3.12

Given : O is the centre of the circle. AXB is the arc. $\angle AOB$ is the angle subtended by the arc $\overset{\frown}{AXB}$ at the centre. $\angle ACB$ is the angle subtended by the arc $\overset{\frown}{AXB}$ at a point on the remaining part of the circle.

To prove : $\angle AOB = 2\angle ACB$

Construction : Join CO and produce it to D

Proof :

(i) $OA = OC$ (radii)

(ii) $\angle OCA = \angle OAC$ (angles opposite to equal sides are equal.)

(iii) In ΔAOC
$\angle AOD = \angle OCA + \angle OAC$ (exterior angles of a triangle = sum of interior opposite angles.)

(iv) $\angle AOD = \angle OCA + \angle OCA$ (substituting $\angle OAC$ by $\angle OCA$)

(v) $\angle AOD = 2\angle OCA$ (by addition)

(vi) similarly in ΔBOC
$\angle BOD = 2\angle OCB$

(vii) $\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$ ($\because \angle AOD + \angle BOD = \angle AOB$
$= 2(\angle OCA + \angle OCB)$ $\angle OCA + \angle OCB = \angle ACB$)

(viii) $\angle AOB = 2\angle ACB$ ■

Note
(i) An angle inscribed in a semicircle is a right angle.
(ii) Angles in the same segment of a circle are equal.

3.2.3 Cyclic Quadrilaterals

Theorem 4

Opposite angles of a cyclic quadrilateral are supplementary (or)
The sum of opposite angles of a cyclic quadrilateral is 180°

Given : O is the centre of circle. $ABCD$ is the cyclic quadrilateral.

To prove : $\angle BAD + \angle BCD = 180^\circ$, $\angle ABC + \angle ADC = 180^\circ$

Construction : Join OB and OD

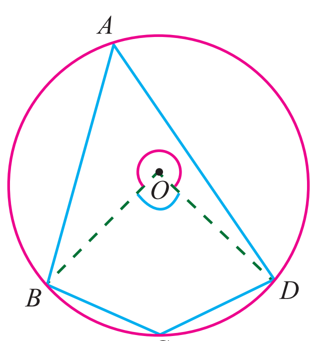


Fig. 3.13

Proof:

(i) $\angle BAD = \frac{1}{2}\angle BOD$ (The angle substended by an arc at the centre is double the angle on the circle.)

(ii) $\angle BCD = \frac{1}{2}$ reflex $\angle BOD$

(iii) $\therefore \angle BAD + \angle BCD = \frac{1}{2}\angle BOD + \frac{1}{2}$ reflex $\angle BOD$ (add (i) and (ii)) i.e., $\angle BAD + \angle BCD = \frac{1}{2}(\angle BOD + \text{reflex } \angle BOD)$

i.e., $\angle BAD + \angle BCD = \frac{1}{2}(360^\circ)$ (Complete angle at the centre is 360°)

i.e., $\angle BAD + \angle BCD = 180^\circ$

(iv) Similarly $\angle ABC + \angle ADC = 180^\circ$ ■

Converse of Theorem 4 : If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.

Theorem 5 (Exterior - angle property of a cyclic quadrilateral)

If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

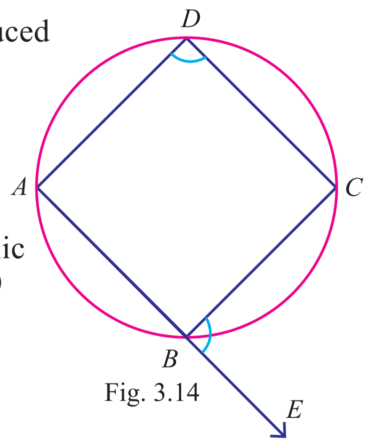


Fig. 3.14

Given : A cyclic quadrilateral $ABCD$, whose side AB is produced to E.

To prove : $\angle CBE = \angle ADC$

Proof :

(i) $\angle ABC + \angle CBE = 180^\circ$ (linear pair)

(ii) $\angle ABC + \angle ADC = 180^\circ$ (Opposite angles of a cyclic quadrilateral)

from (i)and (ii)

(iii) $\angle ABC + \angle CBE = \angle ABC + \angle ADC$

(iv) $\therefore \angle CBE = \angle ADC$ ■

Example: 3.3

Find the value of x in the following figure.

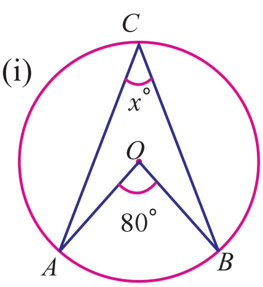
(i)


Fig. 3.15

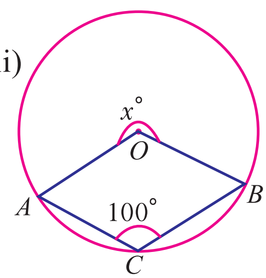
(ii)


Fig. 3.16

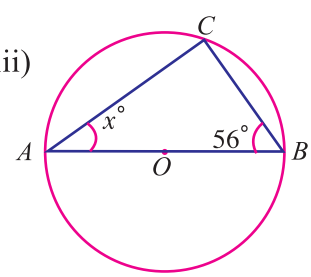
(iii)


Fig. 3.17

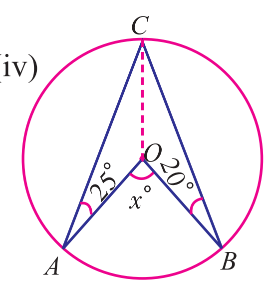
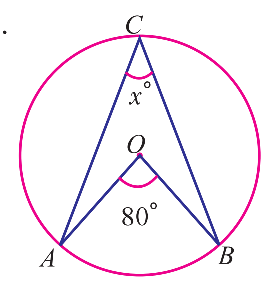
(iv)


Fig. 3.18

Solution Using the theorem the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



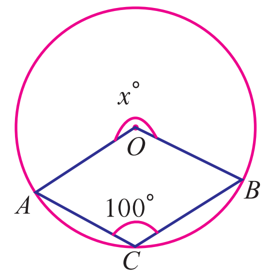
(i) $\angle AOB = 2\angle ACB$

$\angle ACB = \frac{1}{2}\angle AOB$

$= \frac{1}{2} \times 80^\circ = 40^\circ$

(ii) $reflex \angle AOB = 2\angle ACB$

$$x = 2 \times 100^\circ = 200^\circ$$



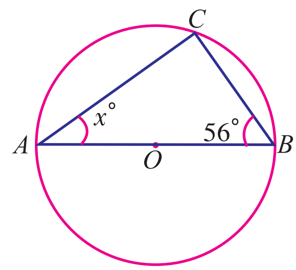
(iii) $\angle ABC + \angle BCA + \angle CAB = 180^\circ$

$$56^\circ + 90^\circ + \angle CAB = 180^\circ$$

$$(\because \angle BCA = \text{angle on a semicircle} = 90^\circ)$$

$$\angle CAB = 180^\circ - 146^\circ$$

$$x = 34^\circ$$



(iv) $OA = OB = OC$ (radius)

$$\angle OCA = \angle OAC = 25^\circ$$

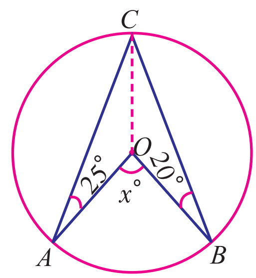
$$\angle OBC = \angle OCB = 20^\circ$$

$$\angle ACB = \angle OCA + \angle OCB$$

$$= 25^\circ + 20^\circ = 45^\circ$$

$$AOB = 2\angle ACB$$

$$x = 2 \times 45^\circ = 90^\circ$$



Example 3.4

In the Fig.7.27, O is the centre of a circle and $\angle ADC = 120^\circ$. Find the value of x.

Solution $ABCD$ is a cyclic quadrilateral.

we have

$$\angle ABC + \angle ADC = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ = 60^\circ$$

Also $\angle ACB = 90^\circ$ (angle on a semi circle)

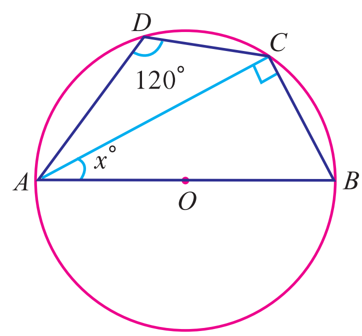


Fig. 3.19

In $\triangle ABC$ we have

$$\angle BAC + \angle ACB + \angle ABC = 180^\circ$$

$$\angle BAC + 90^\circ + 60^\circ = 180^\circ$$

$$\angle BAC = 180^\circ - 150^\circ = 30^\circ$$

Exercise 3.1

1. The radius of a circle is 15 cm and the length of one of its chord is 18 cm. Find the distance of the chord from the centre.

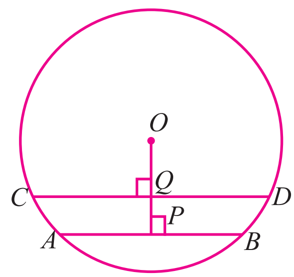
2. The radius of a circles 17 cm and the length of one of its chord is 16 cm. Find the distance of the chord from the centre.

3. A chord of length 20 cm is drawn at a distance of 24 cm from the centre of a circle. Find the radius of the circle.

4. A chord is 8 cm away from the centre of a circle of radius 17 cm. Find the length of the chord.

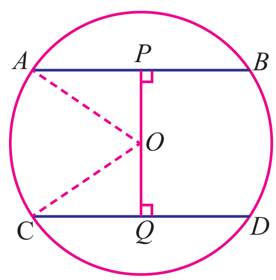
5. Find the length of a chord which is at a distance of 15 cm from the centre of a circle of radius 25 cm.

6. In the figure at right, AB and CD are two parallel chords of a circle with centre O and radius 5 cm such that $AB = 6$ cm and $CD = 8$ cm. If $OP \perp AB$ and $CD \perp OQ$ determine the length of PQ.



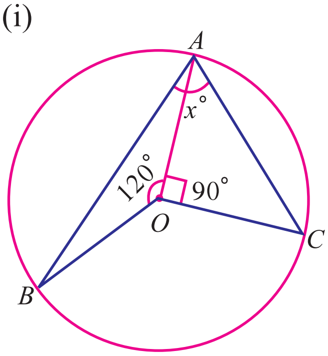
7. AB and CD are two parallel chords of a circle which are on either sides of the centre. Such that $AB = 10$ cm and $CD = 24$ cm. Find the radius if the distance between AB and CD is 17 cm.

8. In the figure at right, AB and CD are two parallel chords of a circle with centre O and radius 5 cm. Such that $AB = 8$ cm and $CD = 6$ cm. If $OP \perp AB$ and $OQ \perp CD$ determine the length PQ.

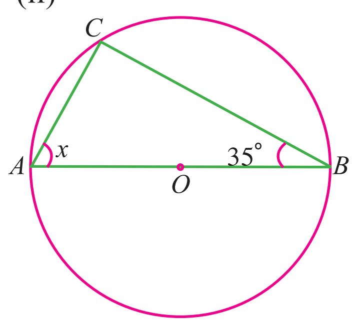


9. Find the value of x in the following figures.

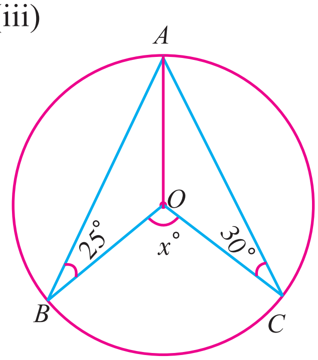
(i)



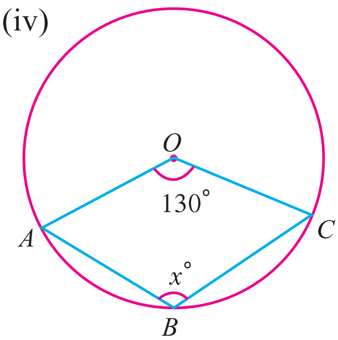
(ii)



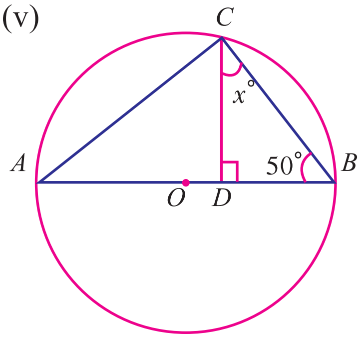
(iii)



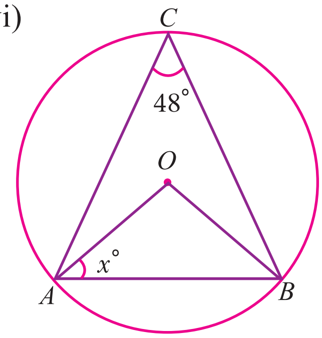
(iv)



(v)

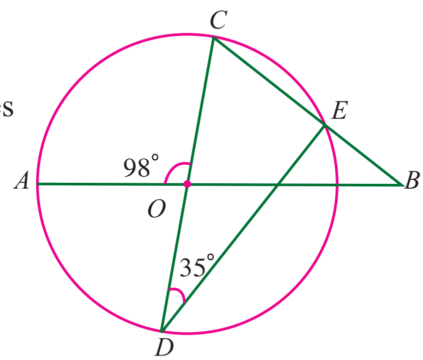


(vi)

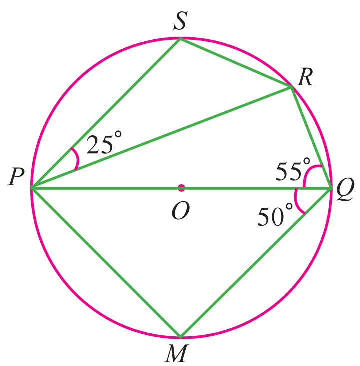


10. In the figure at right, AB and CD are straight lines through the centre O of a circle. If $\angle AOC = 98^\circ$ and $\angle CDE = 35^\circ$

find (i) $\angle DCE$ (ii) $\angle ABC$



11.

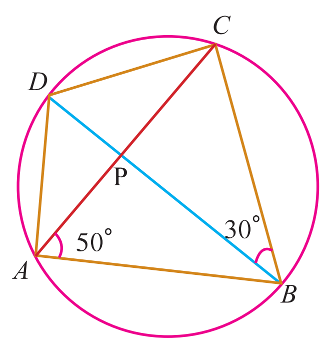


In the figure at left, PQ is a diameter of a circle with centre O. If $\angle PQR = 55^\circ$, $\angle SPR = 25^\circ$ and $\angle PQM = 50^\circ$.

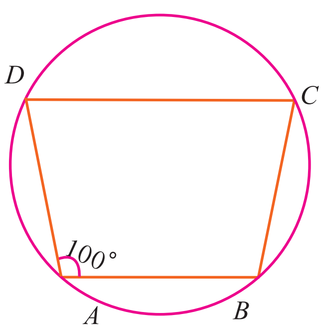
Find (i) $\angle QPR$, (ii) $\angle QPM$ and (iii) $\angle PRS$.

12. In the figure at right, $ABCD$ is a cyclic quadrilateral whose diagonals intersect at P such that $\angle DBC = 30^\circ$ and $\angle BAC = 50^\circ$.

Find (i) $\angle BCD$ (ii) $\angle CAD$



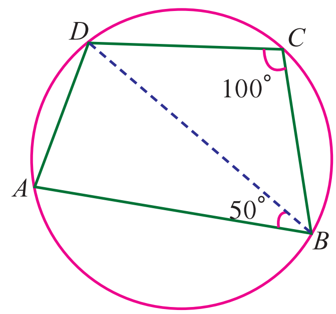
13.

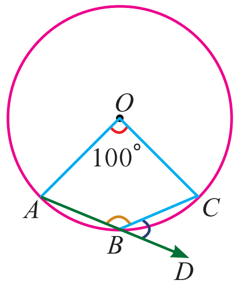


In the figure at left , $ABCD$ is a cyclic quadrilateral in which $AB \parallel DC$. If $\angle BAD = 100^\circ$

find (i) $\angle BCD$ (ii) $\angle ADC$ (iii) $\angle ABC$.

14. In the figure at right, $ABCD$ is a cyclic quadrilateral in which $\angle BCD = 100^\circ$ and $\angle ABD = 50^\circ$ find $\angle ADB$



15. 

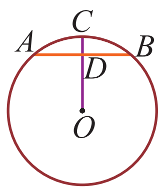
In the figure at left, O is the centre of the circle, $\angle AOC = 100^\circ$ and side AB is produced to D.

Find (i) $\angle CBD$ (ii) $\angle ABC$

Exercise 3.2

Multiple Choice Questions.

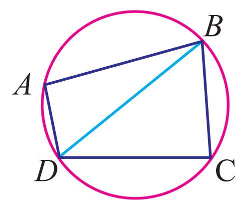
1. O is the centre of the circle. AB is the chord and D is mid-point of AB. If the length of CD is 2cm and the length of chord is12 cm, what is the radius of the circle



(A) 10cm (B) 12cm

(C) 15cm (D) 18cm

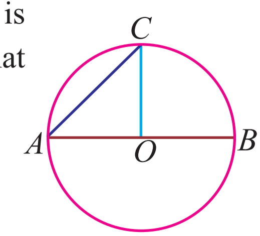
2. $ABCD$ is a cyclic quadrilateral. Given that $\angle ADB + \angle DAB = 120^\circ$ and $\angle ABC + \angle BDA = 145^\circ$. Find the value of $\angle CDB$



(A) 75° (B) 115°

(C) 35° (D) 45°

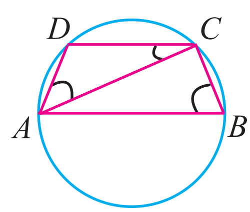
3. In the given figure, AB is one of the diameters of the circle and OC is perpendicular to it through the center O. If AC is $7\sqrt{2}\,cm$, then what is the area of the circle in cm^2



(A) 24.5 (B) 49

(C) 98 (D) 154

4. In the given figure, AB is a diameter of the circle and points C and D are on the circumference such that $\angle CAD = 30^\circ$ and $\angle CBA = 70^\circ$ what is the measure of ACD?



(A) 40° (B) 50°

(C) 30° (D) 90°

5. Angle in a semi circle is

(A) obtuse angle (B) right angle (C) an acute angle (D) supplementary

6. Angle in a minor segment is

(A) an acute angle (B) an obtuse angle (C) a right angle (D)a reflexive angle

7. In a cyclic quadrilateral $ABCD$, $\angle A = 5x, \angle C = 4x$ the value of x is

(A) 12° (B) 20° (C) 48° (D) 36°

8. Angle in a major segment is

(A) an acute angle (B) an obtuse angle

(C) a right triangle (D) a reflexive angle

9. If one angle of a cyclic quadrilateral is 70°, then the angle opposite to it is

(A) 20° (B) 110° (C) 140° (D) 160°

Points to Remember

- Equal chords of a circle subtend equal angles at the centre.
- If two arcs of a circle are congruent then the corresponding chords are equal.
- Perpendicular from the centre of a circle to a chord bisects the chord.
- Equal chords of a circle are equidistant from the centre.
- The angle substended by an are of a circle at the centre is double the angel subtended by it at any point on the remaining part of the circle.
- The angle in a semi circle is a right angle.
- Angle in the same segment of a circle are equal.
- The sum of either pair of opposite angle of opposite angles of a cyclic quadrilateral is 180°
- If one side of a cyclic quadrilateral is produced then the exterior angle is equal to the interior opposite angle.

Through the technique of paper folding you will discover several properties of circles.

Chords of a Circle

- ✪ Make a cut out of a circle in a chart paper.
- ✪ Find the centre of the circle using two diameters
- ✪ Draw several lines (chords) parallel to one of these diameter folds
- ✪ Which chord is the longest chord?______________
- ✪ Complete the following statement:

  "As the length of the chord increases, the distance from the centre ____________"
- ✪ Flip the circle over and draw 3 chords of the same length on the circle
- ✪ Fold each of these chords with a perpendicular bisector to find the centre of the circle
- ✪ Measure the distance from each chord to the centre of the circle

Complete the following statements:

1. "If chords are congruent (equal), then they must be ______________ from the centre of the circle"

OR

2. "If two chords are equidistant from the centre of a circle then they must be ____________"

3. "The perpendicular bisector of a chord goes through the _____________ of acircle"

4. "The perpendicular to a chord through the _______ of a circle __________ the chord"

Activity 2

Central Angles and Inscribed Angles

- ✪ Make a cut out of a circle in a chart paper.
- ✪ Fold the circle with two chords to create an inscribed angle
- ✪ Create two more chords that open on the same arc as the chords above
- ✪ Measure the inscribed angles created

Complete the following statement:

" The measure of inscribed angles opening onto the same arc or chord are ____________"

- ✪ Fold to make a chord on the circle and then fold to make another chord that shares one endpoint of the first chord
- ✪ You should have an inscribed angle from these two cords; trace the angle with a pencil.
- ✪ Find the centre of the circle, by folding along two diameter lines that share one endpoint of one of the chords.
- ✪ Measure the i) inscribed angle ii) central angle

Exercise 3.1

1. 12cm **2.** 15cm **3.** 26cm **4.** 30cm **5.** 40cm **6.** 1cm **7.** 13cm **8.** 7cm

9. (i) 75° (ii) 55° (iii) 110° (iv) 115° (v) 40° (vi) 42° **10.** (i) 55°, (ii) 43°

11. (i) 35° (ii) 40° (iii) 30° **12.** (i) 100° (ii) 30° **13.** (i) 80° (ii) 80° (iii) 100

14. 50° **15.** (i) 50° (ii) 130°

Exercise 3.2

1. A **2.** C **3.** D **4.** A **5.** B **6.** B **7.** B **8.** A **9.** B

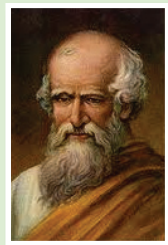
4 MENSURATION

The most beautiful plane figure is – the circle and the most beautiful solid figure – the sphere

- PYTHAGORAS

Main Targets

- To find the length of arc, area and perimeter of sectors of circles.
- To find the surface area and volume of cubes.
- To find the surface area and volume of cuboids.



Archimedes
287 - 212 B.C.

One of the very great mathematicians of all time was Archimedes, a native of the Greek city of Syracus on the island of Sicily. He was born about 287 B.C. It was Archimedes who inaugurated the classical method of computing π by the use of regular polygons inscribed in and circumscribed about a circle. He is responsible for the correct formulas for the area and volume of a sphere. He calculated a number of interesting curvilinear areas, such as that of a parabolic segment and of a sector of the now so called Archimedean spiral. In a number of his works he laid foundations of mathematical physics.

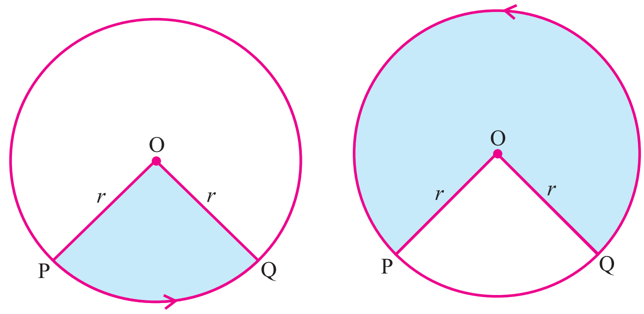
4.1 Introduction

Every day, we see various shapes like triangles, rectangles, squares, circles, spheres and so on all around us, and we are already familiar with some of their properties: like area and perimeter. The part of Mathematics that deals with measurements of geometrical shapes is known as *Mensuration*. It is considered very important because there are various fields of life where geometry is considered as an important field of study.

Perimeter, Area and Volume plays a vital role in architecture and carpentry. Perimeter, Area and volume can be used to analyze real-world situations. It is necessary for everyone to learn formulas used to find the perimeter ,areas of two-dimensional figures and the surface areas and volumes of three dimensional figures for day- to-day life. In this chapter we deal with arc length and area of sectors of circles and area and volume of cubes and cuboids.

4.2 Sectors

Two points P and Q on a circle with centre O determine an arc PQ denoted by $\overset{\frown}{PQ}$, an angle $\angle POQ$ and a sector POQ. The arc starts at P and goes counterclockwise to Q along the circle. The sector POQ is the region bounded by the arc $\overset{\frown}{PQ}$ and the radii OP and OQ. As Fig.4.1 shows, the arcs $\overset{\frown}{PQ}$ and $\overset{\frown}{QP}$ are different.



Arc $\overset{\frown}{PQ}$
Sector POQ

Arc $\overset{\frown}{QP}$
Sector QOP

Fig. 4.1

Key Concept	**Sector**
A sector is the part of a circle enclosed by any two radii of the circle and their intercepted arc.	

4.2.1 Central Angle or Sector Angle of a Sector

Key Concept	**Central Angle**
Central Angle is the angle subtended by the arc of the sector at the centre of the circle in which the sector forms a part.	

In fig.4.2, the angle subtended by the arc $\overset{\frown}{PQ}$ at the centre is θ. So the central angle of the sector POQ is θ.

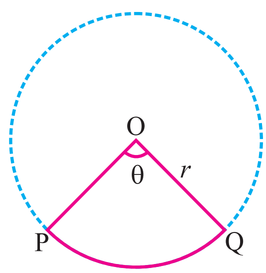


Fig. 4.2

For example,

1. A semi- circle is a sector whose central angle is 180°.
2. A quadrant of a circle is a sector whose central angle is 90°.

4.2.2 Length of Arc (Arc Length) of a Sector

In fig.4.3, arc length of a sector POQ is the length of the portion on the circumference of the circle intercepted between the bounding radii (OP and OQ) and is denoted by l.

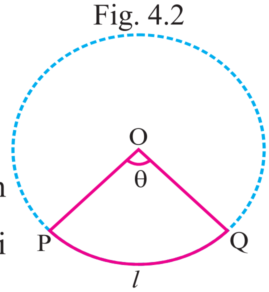


Fig. 4.3

For example,

1. Length of arc of a circle is its circumference. i.e., $l=2\pi r$ units, where r is the radius.
2. Length of arc of a semicircle is $l=2\pi r\times\frac{180^\circ}{360^\circ}=\pi r$ units, where r is the radius and the central angle is 180°.
3. Length of arc of a quadrant of a circle is $l=2\pi r\times\frac{90^\circ}{360^\circ}=\frac{\pi r}{2}$ units, where r is the radius and the central angle is 90°.

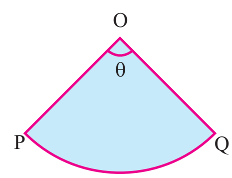
Key Concept — **Length of Arc**

If θ is the central angle and r is the radius of a sector, then its arc length is given by $l=\frac{\theta}{360^\circ}\times 2\pi r$ units

4.2.3 Area of a Sector

Area of a sector is the region bounded by the bounding radii and the arc of the sector.

For Example,



1. Area of a circle is πr^2 square units.
2. Area of a semicircle is $\frac{\pi r^2}{2}$ square units.
3. Area of a quadrant of a circle is $\frac{\pi r^2}{4}$ square units.

Key Concept — **Area of a Sector**

If θ is the central angle and r is the radius of a sector, then the area of the sector is $\frac{\theta}{360^\circ}\times\pi r^2$ square units.

Let us find the relationship between area of a sector, its arc length l and radius r.

$$\text{Area}=\frac{\theta}{360^\circ}\times\pi r^2$$

$$=\frac{\theta}{360^\circ}\times\frac{2\pi r}{2}\times r$$

$$=\frac{1}{2}\times\left(\frac{\theta}{360^\circ}\times 2\pi r\right)\times r$$

$$=\frac{1}{2}\times lr$$

$$\boxed{\text{Area of sector}=\frac{lr}{2}\text{ square units.}}$$

4.2.4 Perimeter of a Sector

The perimeter of a sector is the sum of the lengths of all its boundaries. Thus, perimeter of a sector is $l + 2r$ units.

Key Concept	Perimeter of a Sector
If l is the arc length and r is the radius of a sector, then its perimeter P is given by the formula $P = l + 2r$ units.	

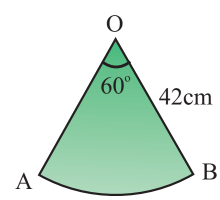
For example,

1. Perimeter of a semicircle is $(\pi + 2)r$ units.
2. Perimeter of a quadrant of a circle is $\left(\frac{\pi}{2} + 2\right)r$ units.

Note
1. Length of an arc and area of a sector are proportional to the central angle.
2. As π is an irrational number, we use its approximate val1ue $\frac{22}{7}$ or 3.14 in our calculations.

Example 4.1

The radius of a sector is 42 cm and its sector angle is 60°. Find its arc length, area and perimeter.



Solution Given that radius $r = 42$ cm and $\theta = 60^\circ$. Therefore,

$$\text{length of arc } l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{60^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 42 = 44 \text{ cm.}$$

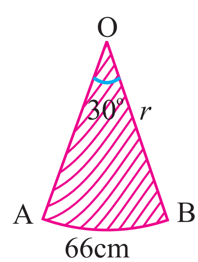
$$\text{Area of the sector} = \frac{lr}{2} = \frac{44 \times 42}{2} = 924 \text{ cm}^2.$$

$$\text{Perimeter} = l + 2r$$

$$= 44 + 2(42) = 128 \text{ cm.}$$

Example 4.2

The arc length of a sector is 66 cm and the central angle is 30°. Find its radius.



Solution Given that $\theta = 30^\circ$ and $l = 66$ cm. So,

$$\frac{\theta}{360^\circ} \times 2\pi r = l$$

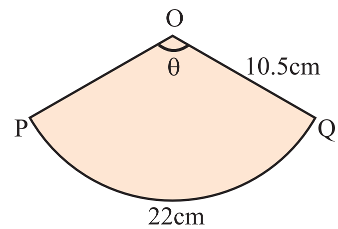
$$\text{i. e., } \frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r = 66$$

$$\therefore r = 66 \times \frac{360^\circ}{30^\circ} \times \frac{1}{2} \times \frac{7}{22} = 126 \text{ cm}$$

Example 4.3

The length of arc of a sector is 22 cm and its radius is 10.5 cm. Find its central angle.

Solution Given that $r = 10.5$ cm and $l = 22$ cm.



$$\frac{\theta}{360^\circ} \times 2\pi r = l$$

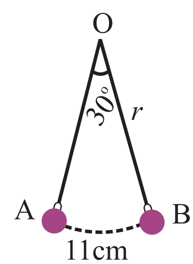
$$\text{i. e., } \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 10.5 = 22$$

$$\therefore \theta = 22 \times 360^\circ \times \frac{1}{2} \times \frac{7}{22} \times \frac{1}{10.5} = 120^\circ$$

Example 4.4

A pendulum swings through an angle of 30° and describes an arc length of 11 cm. Find the length of the pendulum.

Solution If the pendulum swings once, then it forms a sector and the radius of the sector is the length of the pendulum. So,



$\theta = 30^\circ$, $l = 11$ cm

Using the formula $\frac{\theta}{360^\circ} \times 2\pi r = l$, we have

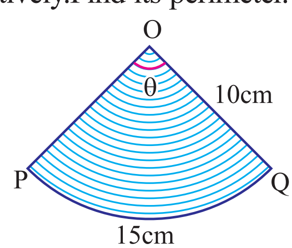
$$\frac{30^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r = 11$$

$$\therefore r = 11 \times \frac{360^\circ}{30^\circ} \times \frac{1}{2} \times \frac{7}{22} = 21 \text{ cm}$$

Example 4.5

The radius and length of arc of a sector are 10 cm and 15 cm respectively.Find its perimeter.

Solution Given that $r = 10$ cm, $l = 15$ cm



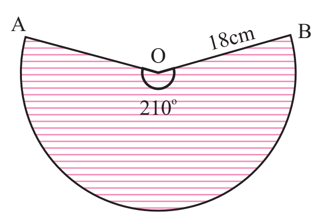
Perimeter of the sector $= l + 2r = 15 + 2(10)$

$= 15 + 20 = 35$ cm

Example 4.6

Find the perimeter of a sector whose radius and central angle are 18 cm and 210° respectively.

Solution Given that $r = 18$ cm, $\theta = 210^\circ$. Hence,



$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$= \frac{210^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 18 = 66 \text{ cm}$$

$\therefore$ Perimeter of the sector $= l + 2r = 66 + 2(18) = 66 + 36 = 102$ cm

Mathematics

Example 4.7

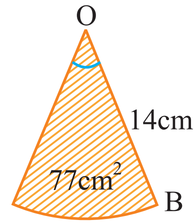
The area of a sector of a circle of radius 14 cm is 77 cm^2. Find its central angle.

Solution Given that $r = 14$ cm, area $= 77$ cm^2

$$\frac{\theta}{360^\circ} \times \pi r^2 = \text{Area of the sector}$$

$$\frac{\theta}{360^\circ} \times \frac{22}{7} \times 14 \times 14 = 77$$

$$\therefore\ \theta = \frac{77 \times 360^\circ \times 7}{22 \times 14 \times 14} = 45^\circ$$



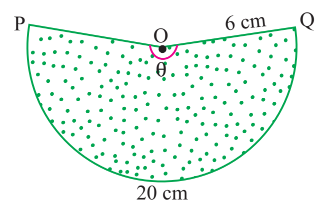
Example 4.8

Calculate the area of a sector whose radius and arc length are 6 cm and 20 cm respectively.

Solution Given that $r = 6$ cm, $l = 20$ cm

$$\text{Area} = \frac{lr}{2} \text{ square units}$$

$$= \frac{20 \times 6}{2} = 60 \text{ cm}^2$$



Example 4.9

If the perimeter and radius of a sector are 38 cm and 9 cm respectively, find its area.

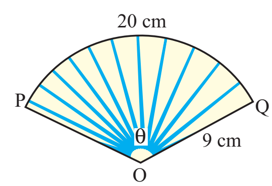
Solution Given, $r = 9$ cm, perimeter $= 38$ cm

$$\text{Perimeter} = l + 2r = 38$$

$$\text{i.e.,}\quad l + 18 = 38$$

$$l = 38 - 18 = 20 \text{ cm}$$

$$\therefore\ \text{Area} = \frac{lr}{2} = \frac{20 \times 9}{2} = 90 \text{ cm}^2$$



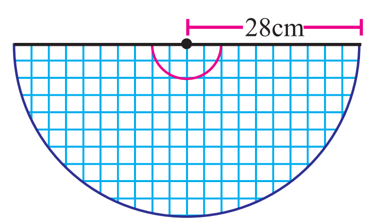
Example 4.10

Find the perimeter and area of a semicircle of radius 28 cm.

Solution Given, $r = 28$ cm

$$\text{Perimeter} = (\pi + 2)\, r = (\frac{22}{7} + 2)\, 28 = 144 \text{ cm}$$

$$\text{Area} = \frac{\pi r^2}{2} = \frac{22}{7} \times \frac{28 \times 28}{2} = 1232 \text{ cm}^2$$



Example 4.11

Find the radius, central angle and perimeter of a sector whose arc length and area are 27.5 cm and 618.75 cm^2 respectively.

Solution Given that $l = 27.5$ cm and Area $= 618.75$ cm^2. So,

$$\text{Area} = \frac{lr}{2} = 618.75 \text{ cm}^2$$

$$\text{i.e.} \frac{27.5 \times r}{2} = 618.75$$

$$\therefore r = 45 \text{ cm}$$

Hence, perimeter is $l + 2r = 27.5 + 2(45) = 117.5$cm

Now, arc length is given by $\frac{\theta}{360^\circ} \times 2\pi r = l$

$$\text{i.e. } \frac{\theta}{360^\circ} \times 2 \times \frac{22}{7} \times 45 = 27.5$$

$$\therefore \theta = 35^\circ$$

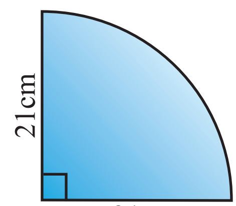
Example 4.12

Calculate the area and perimeter of a quadrant of a circle of radius 21 cm.

Solution Given that $r = 21$ cm, $\theta = 90^\circ$

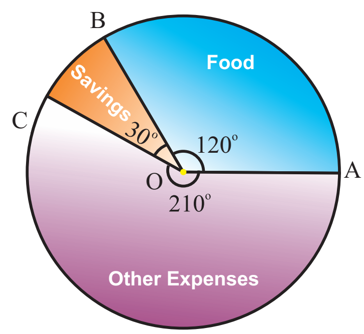
$$\text{Perimeter} = \left(\frac{\pi}{2} + 2\right)r = \left(\frac{22}{7 \times 2} + 2\right) \times 21 = 75 \text{ cm}$$

$$\text{Area} = \frac{\pi r^2}{4} = \frac{22}{7 \times 4} \times 21 \times 21 = 346.5 \text{ cm}^2$$



Example 4.13

Monthly expenditure of a person whose monthly salary is ₹ 9,000 is shown in the adjoining figure. Find the amount he has (i) spent for food (ii) in his savings



Solution Let ₹ 9,000 be represented by the area of the circle, i. e., $\pi r^2 = 9000$

(i) Area of sector $AOB = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{120^\circ}{360^\circ} \times 9000 = 3,000$$

Amount spent for food is ₹ 3,000.

(ii) Area of sector $BOC = \frac{\theta}{360^\circ} \times \pi r^2$

$$= \frac{30^\circ}{360^\circ} \times 9,000 = 750$$

Amount saved in savings is ₹ 750.

Example 4.14

Three equal circles of radius 3 cm touch one another. Find the area enclosed by them.

Solution Since the radius of the circles are equal and the circles touch one another, in the figure, ABC is an equilateral triangle and the area of the sectors DAF, DBE and ECF are equal. Hence,

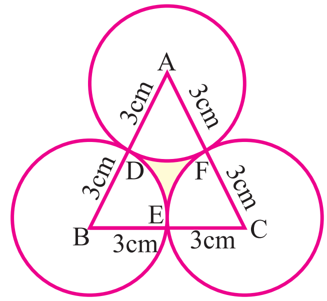
Area enclosed = area of the equilateral triangle ABC − 3 times area of the sector

$$= \frac{\sqrt{3}}{4}a^2 - 3\times\frac{\theta}{360^\circ}\times\pi r^2$$

$$= \frac{\sqrt{3}}{4}\times 6\times 6 - 3\times\frac{60^\circ}{360^\circ}\times\frac{22}{7}\times 3\times 3$$

$$= 9\sqrt{3} - \frac{99}{7} = 15.59 - 14.14 = 1.45\text{ cm}^2$$

$\therefore$ Area = 1.45 cm^2



Example 4.15

Find the area of the shaded portion in the following figure $[\pi = 3.14]$

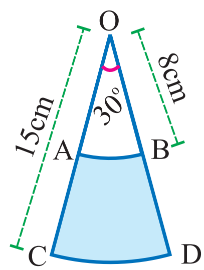
Solution Let R and r denote the radius of sector COD and sector AOB respectively.

Area of the shaded portion = Area of sector COD − Area of sector AOB

$$= \frac{\theta}{360^\circ}\times\pi R^2 - \frac{\theta}{360^\circ}\times\pi r^2$$

$$= \frac{30^\circ}{360^\circ}\times 3.14\times 15\times 15 - \frac{30^\circ}{360^\circ}\times 3.14\times 8\times 8$$

$$= 58.875 - 16.747 = 42.128\text{ cm}^2$$



Exercise: 4.1

1. Find the arc length, area and perimeter of the sector with
   (i) radius 21 cm and central angle 60°
   (ii) radius 4.9 cm and central angle 30°
   (iii) radius 14 cm and sector angle 45°
   (iv) radius 15 cm and sector angle 63°
   (v) radius 21 dm and sector angle 240°
2. (i) Find the angle subtended by an arc 88 cm long at the centre of a circle of radius 42 cm.
   (ii) The arc length of a sector of a circle of radius 14 cm is 22 cm. Find its central angle.
   (iii) Find the radius of a sector of a circle having a central angle 70° and an arc length of 44 cm.
3. Find the area and perimeter of the sector with
   (i) radius 10 cm and arc length 33 cm.
   (ii) radius 55 cm and arc length 80 cm.
   (iii) radius 12 cm and arc length 15.25 cm.
   (iv) radius 20 cm and arc length 25 cm.

4. (i) Find the arc length of the sector of radius 14 cm and area 70 cm^2

   (ii) Find the radius of the sector of area 225 cm^2 and having an arc length of 15 cm

   (iii) Find the radius of the sector whose central angle is 140° and area 44 cm^2.

5. (i) The perimeter of a sector of a circle is 58 cm. Find the area if its diameter is 9 cm.

   (ii) Find the area of a sector whose radius and perimeter are 20 cm and 110 cm respectively.

6. Find the central angle of a sector of a circle having

   (i) area 352 cm^2 and radius 12 cm

   (ii) area 462 cm^2 and radius 21 cm

7. (i) Calculate the perimeter and area of the semicircle whose radius is 14 cm.

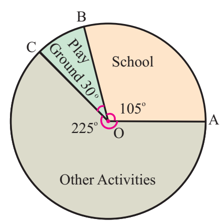
   (ii) Calculate the perimeter and area of a quadrant circle of radius 7 cm.

8. (i) Calculate the arc length of a sector whose perimeter and radius are 35 cm and 8 cm respectively.

   (ii) Find the radius of a sector whose perimeter and arc length are 24 cm and 7 cm respectively.

9. Time spent by a student in a day is shown in the figure. Find how much time is spent in

   (i) school (ii) play ground (iii) other activities



10. Three coins each 2 cm in diameter are placed touching one another. Find the area enclosed by them.

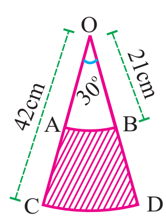
11. Four horses are tethered with ropes measuring 7 m each to the four corners of a rectangular grass land 21 m × 24 m in dimension. Find

    (i) the maximum area that can be grazed by the horses and

    (ii) the area that remains ungrazed.

12. Find the area of card board wasted if a sector of maximum possible size is cut out from a square card board of size 24 cm. [$\pi = 3.14$].

13. Find the area of the shaded portion in the adjoining figure



14. Find the radius, central angle and perimeter of a sector whose length of arc and area are 4.4 m and 9.24 m^2 respectively.

Mathematics

4.3 Cubes

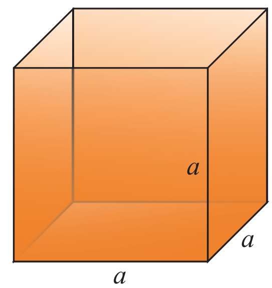
You have learnt that a cube is a solid having six square faces. Example: Die.

In this section you will learn about surface area and volume of a cube.

4.3.1 Surface Area of a Cube

The sum of the areas of all the six equal faces is called the *Total Surface Area* (T.S.A) of the cube.

In the adjoining figure, let the side of the cube measure a units. Then the area of each face of the cube is a^2 square units. Hence, the total surface area is $6a^2$ square units.



In a cube, if we don't consider the top and bottom faces, the remaining area is called the *Lateral Surface Area* (L.S.A). Hence, the lateral surface area of the cube is $4a^2$ square units.

Key Concept — Surface Area of Cube

Let the side of a cube be a units. Then:

(i) The Total Surface Area (T.S.A) = $6a^2$ square units.

(ii) The Lateral Surface Area (L.S.A) = $4a^2$ square units.

4.3.2 Volume of a Cube

Key Concept — Volume of Cube

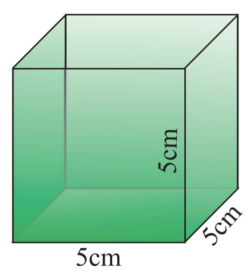
If the side of a cube is a units, then its volume V is given by the formula

$V = a^3$ cubic units

Note Volume can also be defined as the number of unit cubes required to fill the entire cube.

Example 4.16

Find the L.S.A, T.S.A and volume of a cube of side 5 cm.



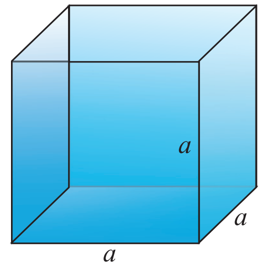
Solution L.S.A $= 4a^2 = 4(5^2) = 100$ sq. cm

T.S.A $= 6a^2 = 6\,(5^2) = 150$ sq. cm

Volume $= a^3 = 5^3 = 125$ cm^3

Example: 4.17

Find the length of the side of a cube whose total surface area is 216 square cm.



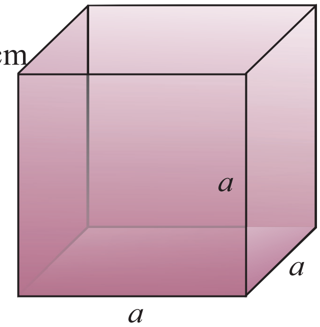
Solution Let a be the side of the cube. Given that T.S.A = 216 sq. cm

i. e., $6a^2 = 216 \implies a^2 = \frac{216}{6} = 36$

$\therefore \quad a = \sqrt{36} = 6$ cm

Example 4.18

A cube has a total surface area of 384 sq. cm. Find its volume.



Solution Let a be the side of the cube. Given that T.S.A = 384 sq. cm

$6a^2 = 384 \implies a^2 = \frac{384}{6} = 64$

$\therefore \quad a = \sqrt{64} = 8$ cm

Hence, Volume $= a^3 = 8^3 = 512$ cm^3

Example 4.19

A cubical tank can hold 27,000 litres of water. Find the dimension of its side.

Solution Let a be the side of the cubical tank. Volume of the tank is 27,000 litres. So,

$V = a^3 = \frac{27,000}{1,000} m^3 = 27\ m^3 \qquad \therefore\ a = \sqrt[3]{27} = 3\ m$

Exercise 4.2

1. Find the Lateral Surface Area (LSA), Total Surface Area (TSA) and volume of the cubes having their sides as

   (i) 5.6 cm (ii) 6 dm (iii) 2.5 m (iv) 24 cm (v) 31 cm

2. (i) If the Lateral Surface Area of a cube is 900 cm^2, find the length of its side.

   (ii) If the Total Surface Area of a cube is 1014 cm^2, find the length of its side.

   (iii) The volume of the cube is 125 dm^3. Find its side.

3. A container is in the shape of a cube of side 20 cm. How much sugar can it hold?
4. A cubical tank can hold 64,000 litres of water. Find the length of the side of the tank.
5. Three metallic cubes of side 3 cm, 4 cm and 5 cm respectively are melted and are recast into a single cube. Find the total surface area of the new cube.
6. How many cubes of side 3 cm are required to build a cube of side 15 cm?
7. Find the area of card board required to make an open cubical box of side 40 cm. Also find the volume of the box.
8. What is the total cost of oil in a cubical container of side 2 m if it is measured and sold using a cubical vessel of height 10 cm and the cost is ₹ 50 per measure.
9. A container of side 3.5m is to be painted both inside and outside. Find the area to be painted and the total cost of painting it at the rate of ₹ 75 per square meter.

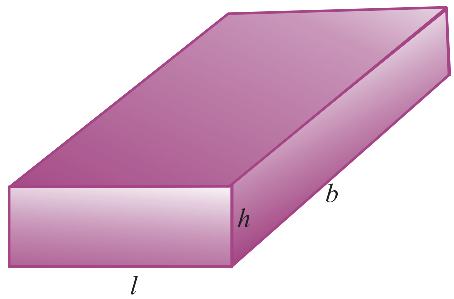
4.4 Cuboids

A cuboid is a three dimensional solid having six rectangular faces.

Example: Bricks, Books etc.,

4.4.1 Surface Area of a Cuboid

Let l, b and h be the length, breadth and height of a cuboid respectively. To find the total surface area, we split the faces into three pairs.



(i) The total area of the front and back faces is $lh + lh = 2lh$ square units.

(ii) The total area of the side faces is

$bh + bh = 2bh$ square units.

(iii) The total area of the top and bottom faces is $lb + lb = 2lb$ square units.

The Lateral Surface Area (L.S.A) = $2(l + b)h$ square units.

The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ square units.

Key Concept **Surface Area of a Cuboid**

Let l, b and h be the length, breadth and height of a cuboid respectively. Then

(i) The Lateral Surface Area (L.S.A) = $2(l + b)h$ square units

(ii) The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ sq. units

Note L.S.A. is also equal to the product of the perimeter of the base and the height.

4.4.2 Volume of a Cuboid

Key Concept **Volume of a Cuboid**

If the length, breadth and height of a cuboid are l, b and h respectively, then the volume V of the cuboid is given by the formula

$$V = l \times b \times h \text{ cu. units}$$

Example: 4.20

Find the total surface area of a cuboid whose length, breadth and height are 20 cm, 12 cm and 9 cm respectively.

Solution

Given that $l = 20$ cm, $b = 12$ cm, $h = 9$ cm

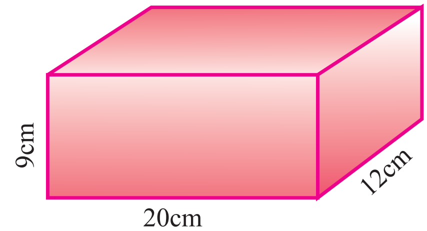
$\therefore$ T.S.A $= 2(lb + bh + lh)$

$= 2[(20 \times 12) + (12 \times 9) + (20 \times 9)]$

$= 2(240 + 108 + 180)$

$= 2 \times 528$

$= 1056 \text{ cm}^2$



Example: 4.21

Find the L.S.A of a cuboid whose dimensions are given by $3\text{m} \times 5\text{m} \times 4\text{m}$.

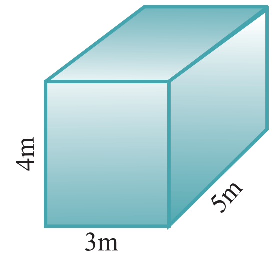
Solution Given that $l = 3$ m, b = 5 m, h = 4 m

L.S.A $= 2(l + b)h$

$= 2 \times (3+5) \times 4$

$= 2 \times 8 \times 4$

$= 64$ sq. m



Example: 4.22

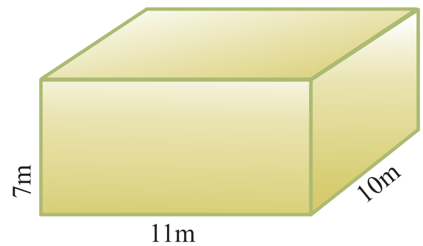
Find the volume of a cuboid whose dimensions are given by 11 m, 10 m and 7 m.

Solution Given that $l = 11$ m, b = 10 m, h = 7 m

volume $= lbh$

$= 11 \times 10 \times 7$

$= 770$ cu.m.



Example: 4.23

Two cubes each of volume 216 cm^3 are joined to form a cuboid as shown in the figure. Find the T.S.A of the resulting cuboid.

Solution Let the side of each cube be a. Then $a^3 = 216$

$\therefore a = \sqrt[3]{216} = 6$ cm

Now the two cubes of side 6 cm are joined to form a cuboid. So,

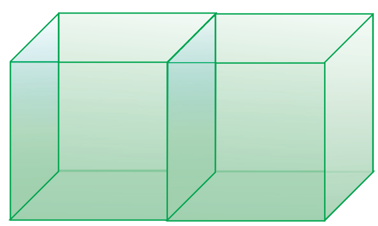
$\therefore l = 6 + 6 = 12$ cm, $b = 6$ cm, $h = 6$ cm

$\therefore$ T.S.A $= 2(lb + bh + lh)$

$= 2[(12 \times 6) + (6 \times 6) + (12 \times 6)]$

$= 2[72 + 36 + 72]$

$= 2 \times 180 = 360 \text{ cm}^2$

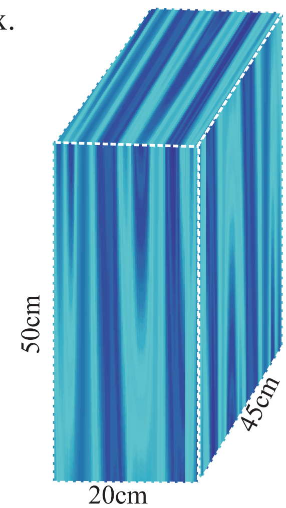


Example 4.24

Johny wants to stitch a cover for his C.P.U whose length, breadth and height are 20 cm, 45 cm and 50 cm respectively. Find the amount he has to pay if it costs ₹ 50 per sq. m

Solution The cover is in the shape of a one face open cuboidal box.

$l = 20$ cm $= 0.2$ m, b $= 45$ cm $=0.45$ m, h $= 50$ cm $= 0.5$ m



$\therefore$ Area of cloth required $=$ L.S.A + area of the top

$$= 2\,(l + b)\,h + lb$$

$$= 2\,(0.2 + 0.45)\,0.5 + (0.2 \times 0.45)$$

$$= 2 \times 0.65 \times 0.5 + 0.09$$

$$= 0.65 + 0.09$$

$$= 0.74 \text{ sq.m}$$

Given that cost of 1 sq. m of cloth is ₹ 50

$\therefore$ cost of 0.74 sq.m of cloth is $50 \times 0.74 =$ ₹ 37.

Example: 4.25

Find the cost for filling a pit of dimensions $5\text{m} \times 2\text{m} \times 1\text{m}$ with soil if the rate of filling is ₹ 270 per cu. m

Solution The pit is in the shape of a cuboid having $l = 5$m, b $= 2$m and h $= 1$m.

$\therefore$ volume of the pit $=$ volume of the cuboid

$$= lbh$$

$$= 5 \times 2 \times 1$$

$$= 10 \text{ cu.m}$$

Given that cost for filling 1 cu. m is ₹ 270

$\therefore$ cost for filling 10 cu. m is

$270 \times 10 =$ ₹ 2700

Exercise 4.3

1. Find the L.S.A, T.S.A and volume of the cuboids having the length, breadth and height respectively as

   (i) 5 cm, 2 cm , 11cm (ii) 15 dm, 10 dm, 8 dm

   (iii) 2 m, 3 m, 7 m (iv) 20 m, 12 m, 8 m

2. Find the height of the cuboid whose length, breadth and volume are 35 cm, 15 cm and 14175 cm^3 respectively.

3. Two cubes each of volume 64 cm^3 are joined to form a cuboid. Find the L.S.A and T.S.A of the resulting solid.

4. Raju planned to stitch a cover for his two speaker boxes whose length, breadth and height are 35 cm, 30 cm and 55 cm respectively. Find the cost of the cloth he has to buy if it costs ₹ 75 per sq.m.

5. Mohan wanted to paint the walls and ceiling of a hall. The dimensions of the hall is $20\text{m} \times 15\text{m} \times 6\text{m}$. Find the area of surface to be painted and the cost of painting it at ₹ 78 per sq. m.
6. How many hollow blocks of size $30\text{cm} \times 15\text{cm} \times 20\text{cm}$ are needed to construct a wall 60m in length, 0.3m in breadth and 2m in height.
7. Find the cost of renovating the walls and the floor of a hall that measures $10\text{m}\times45\text{m}\times6\text{m}$ if the cost is ₹ 48 per square meter.

Exercise 4.4

Multiple Choice Questions.

1. The length of the arc of a sector having central angle 90° and radius 7 cm is

   (A) 22 cm (B) 44 cm (C) 11 cm (D) 33 cm

2. If the radius and arc length of a sector are 17 cm and 27 cm respectively, then the perimeter is

   (A) 16 cm (B) 61 cm (C) 32 cm (D) 80 cm

3. If the angle subtended by the arc of a sector at the center is 90°, then the area of the sector in square units is

   (A) $2\pi r^2$ (B) $4\pi r^2$ (C) $\frac{\pi r^2}{4}$ (D) $\frac{\pi r^2}{2}$

4. Area of a sector having radius 12 cm and arc length 21 cm is

   (A) 126 cm^2 (B) 252 cm^2 (C) 33 cm^2 (D) 45 cm^2

5. The area of a sector with radius 4 cm and central angle 60° is

   (A) $\frac{2\pi}{3}$cm^2 (B) $\frac{4\pi}{3}$cm^2 (C) $\frac{8\pi}{3}$cm^2 (D) $\frac{16\pi}{3}$cm^2

6. If the area and arc length of the sector of a circle are 60 cm^2 and 20 cm respectively, then the diameter of the circle is

   (A) 6 cm (B) 12 cm (C) 24 cm (D) 36 cm

7. The perimeter of a sector of a circle is 37cm. If its radius is 7cm, then its arc length is

   (A) 23 cm (B) 5.29 cm (C) 32 cm (D) 259 cm

8. A solid having six equal square faces is called a

(A) cube (B) cuboid (C) square (D) rectangle

9. The quantity of space occupied by a body is its

(A) area (B) length (C) volume (D) T.S.A

10. The LSA of a cube of side 1dm is

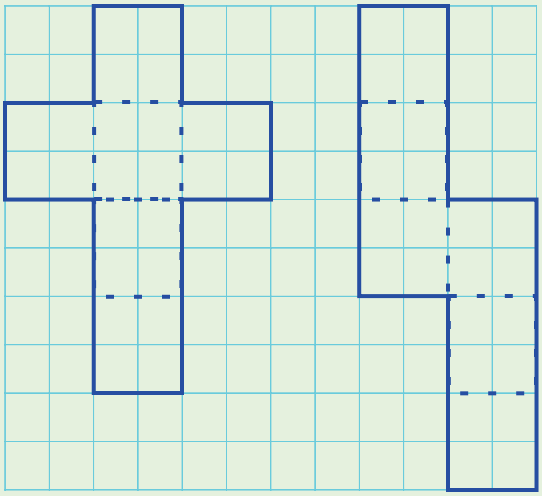
(A) 16 dm^2 (B) 4 dm^2 (C) 2 dm^2 (D) 1 dm^2

Points to Remember

- A sector is the part of a circle enclosed by any two radii of the circle and their intercepted arc.
- Central Angle is the angle subtended by the arc of the sector at the centre of the circle in which the sector forms a part.
- If θ is the central angle and r is the radius of a sector, then its arc length is given by $l = \frac{\theta}{360} \times 2\pi r$ units
- If θ is the central angle and r is the radius of a sector, then the area of the sector is $\frac{\theta}{360} \times \pi r^2$ square units.
- If l is the arc length and r is the radius of a sector, then its perimeter P is given by the formula $P = l + 2r$ units.
- Let the side of a cube be a units. Then:
  (i) The Total Surface Area (T.S.A) = $6a^2$ square units.
  (ii) The Lateral Surface Area (L.S.A) = $4a^2$ square units.
- If the side of a cube is a units, then its volume V is given by the formula, $V = a^3$ cubic units
- Let l, b and h be the length, breadth and height of a cuboid respectively. Then:
  (i) The Lateral Surface Area (L.S.A) = $2(l + b)h$ squre units
  (ii) The Total Surface Area (T.S.A) = $2(lb + bh + lh)$ sq. units
- If the length, breadth and height of a cuboid are l, b and h respectively, then the volume V of the cuboid is given by the formula $V = l \times b \times h$ cu. units

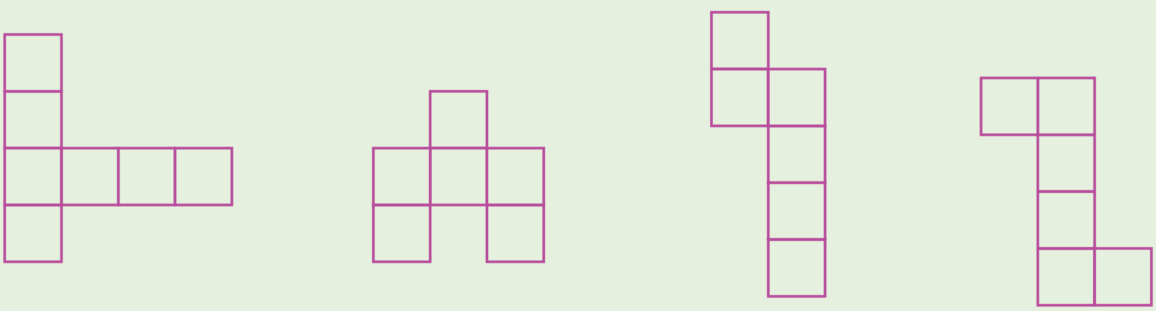
Activity 1

Copy and cut out larger versions of the following nets. Fold and glue them to obtain cubes. Do not forget to add tabs to the nets.



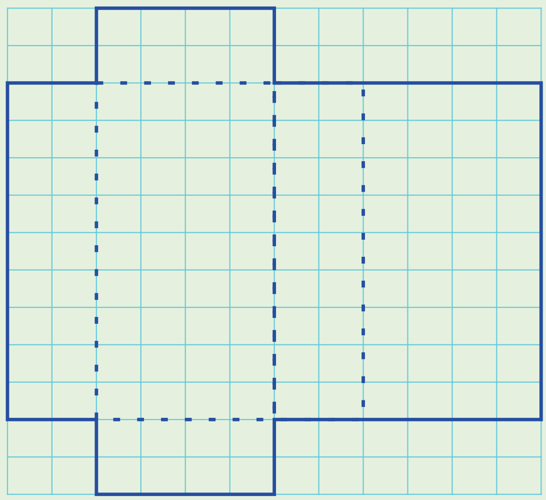
Activity 2

Which one of these nets can be folded to make a cube?



Activity 3

Copy each net shown below make it into a solid. State the name of the solid that you make, if it has one.



Activity 4

Construct a cuboid with 64 unit cubes, as shown opposite.

Does the surface area change if the arrangement (configuration) of cubes changes?

Find the surface area of this cuboid.

Using 64 unit cubes to make a cuboid, what is the configuration that gives the

minimum surface area,

maximum surface area?

Exercise 4.1

1. (i) 22 cm, 231 cm^2, 64 cm (ii) 2.57 cm, 6.3 cm^2, 12.37 cm (iii) 11 cm, 77 cm^2, 39 cm
(iv) 16.5 cm, 123.75 cm^2, 46.5 cm (v) 88 dm, 924 dm^2,130 dm

2. (i) 120° (ii) 90° (iii) 36 cm **3.** (i) 165 cm^2, 53 cm (ii) 2200 cm^2, 190 cm
(iii) 91.5 cm^2, 39.25cm (iv) 250 cm^2,65 cm **4.** (i) 10 cm (ii) 30 cm (iii) 6 cm

5. (i) 110.25 cm^2 (ii) 700 cm^2 **6.** (i) 280° (ii) 120° **7.** (i) 72 cm, 308cm^2 (ii) 25cm, 38.5 cm^2

8. (i) 19 cm, (ii) 8.5 cm **9.** (i) 7 hrs, (ii) 2 hrs, (iii) 15 hrs **10.** 0.16 cm^2

11. (i) 154 m^2, (ii) 350 m^2, **12.** 123.84 cm^2 **13.** 346.5 cm^2 **14.** 4.2 m, 60°,12.8 m

Exercise 4.2

1. (i) 125.44 cm^2, 188.16 cm^2, 175.62 cm^3, (ii) 144 dm^2, 216 dm^2, 216 dm^3
(iii) 25 m^2, 37.5 m^2, 15.625 m^3, (iv) 2304 cm^2,3456 cm^2, 13824 cm^3
(v) 3844 cm^2,5766 cm^2, 29791 cm^3 **2.** (i) 15 cm (ii) 13 cm (iii) 5 dm

3. 8000 cm^3, **4.** 4 m **5.** 216 cm^2 **6.** 125 cubes **7.** 8000 cm^2, 64000 cm^3

8. ₹ 4,00,000 **9.** 147 m^2, ₹ 11,025

Exercise 4.3

1. (i) 154 cm^2, 174 cm^2, 110 cm^3 (ii) 400 dm^2, 700 dm^2, 1200 dm^3
(iii) 70 m^2, 82 m^2, 42 m^3 (iv) 512 m^2,992 m^2, 1920 m^3

2. 27 cm **3.** 96 cm^2, 160 cm^2 **4.** ₹ 123 **5.** 720 m^2 , ₹ 56,160,

6. 4000 hollow blocks, **7.** ₹ 53,280

Exercise 4.4

1. C **2.** B **3.** C **4.** A **5.** C **6.** B **7.** A **8.** C **9.** C **10.** B

5 PROBABILITY

All business proceeds on beliefs, or judgments of probabilities, and not on certainty - CHARLES ELIOT

Main Targets

- To understand repeated experiments and observed frequency approach of Probability
- To understand Empirical Probability

Richard Von Mises
(1883-1953)

The statistical, or empirical, attitude toward probability has been developed mainly by R.F. Fisher and R. Von Mises. The notion of sample space comes from R.Von Mises. This notion made it possible to build up a strictly mathematical theory of probability based on measure theory. Such an approach emerged gradually in the last century under the influence of many authors. An axiomatic treatment representing the modern development was given by A. Kolmogorov.

5.1 Introduction

From dawn to dusk any individual makes decisions regarding the possible events that are governed at least in part by chance. Few examples are: "Should I carry an umbrella to work today?", "Will my cellphone battery last until tonight?", and "Should I buy a new brand of laptop?". Probability provides a way to make decisions when the person is uncertain about the things, quantities or actions involved in the decision. Though probability started with gambling, it has been used extensively, in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Insurance, Investments, Weather Forecasting and in various other emerging areas.

Consider the statements:

- ***Probably*** Kuzhalisai will stand first in the forth coming annual examination.
- ***Possibly*** Thamizhisai will catch the train today.
- The prices of essential commodities are ***likely*** to be stable.
- There is a ***chance*** that Leela will win today's Tennis match.

The words "***Probably***", "***Possibly***" , "***Likely***", "***Chance***" , etc., will mean "the lack of certainty" about the events mentioned above. To measure "the lack of certainty or uncertainty", there is no perfect yardstick, i.e., uncertainty is not perfectly quantifiable one. But based on some assumptions uncertainty can be measured mathematically. This numerical measure is referred to as probability. It is a purposeful technique used in decision making depending on, and changing with, experience. Probability would be effective and useful even if it is not a single numerical value.

5.2 Basic Concepts and Definitions

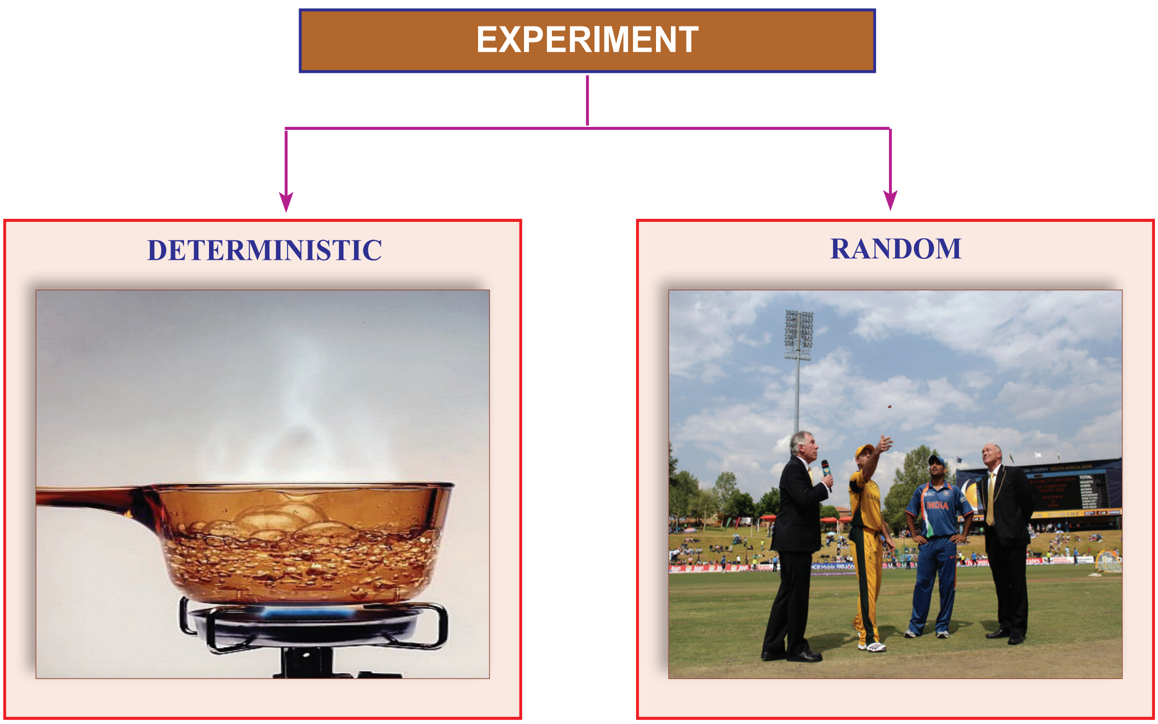
Before we start the theory on Probability, let us define some of the basic terms required for it.

- Experiment
- Random Experiment
- Trial
- Sample Space
- Sample Point
- Events

Key Concept **Experiment**

An ***experiment*** is defined as a process whose result is well defined

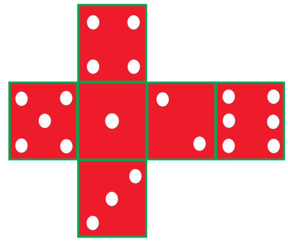
Experiments are classified broadly into two ways:



1. ***Deterministic Experiment*** : It is an experiment whose outcomes can be predicted with certainty, under identical conditions.

For example, in the cases-when we heat water it evaporates, when we keep a tray of water into the refrigerator it freezes into ice and while flipping an unusual coin with heads on both sides getting head - the outcomes of the experiments can be predicted well in advance. Hence these experiments are deterministic.

2. ***Random Experiment*** : It is an experiment whose all possible outcomes are known, but it is not possible to predict the exact outcome in advance.



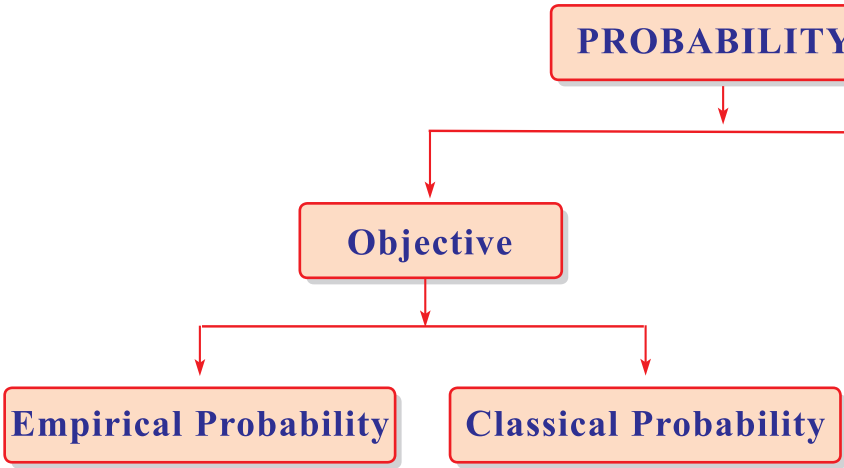
For example, consider the following experiments:

(i) A coin is flipped (tossed)

(ii) A die is rolled.

These are random experiments, since we cannot predict the outcome of these experiments.

Key Concept		
Trial	A Trial is an action which results in one or several outcomes.	For example, " Flipping" a coin and "Rolling" a die are trials
Sample Space	A sample space S is the set of all possible outcomes of a random experiment.	For example, While flipping a coin the sample space, S = { Head, Tail} While rolling a die, sample space S = { 1, 2, 3, 4, 5, 6}
Sample Point	Each outcome of an experiment is called a sample point.	While flipping a coin each outcome {Head}, {Tail} are the sample points. While rolling a die each outcome, {1} {2} {3} {4} {5} and {6} are are corresponding sample points
Event	Any subset of a sample space is called an event.	For example, When a die is rolled some of the possible events are {1, 2, 3}, {1, 3}, {2, 3, 5, 6}



5.3 Classification of Probability

According to various concepts of probability, it can be classified mainly in to three types as given below:

(1) Subjective Probability

(2) Classical Probability

(3) Empirical Probability

5.3.1 Subjective Probability

Subjective probabilities express the strength of one's belief with regard to the uncertainties. It can be applied especially when there is a little or no direct evidence about the event desired, there is no choice but to consider indirect evidence, educated guesses and perhaps intuition and other subjective factors to calculate probability .

5.3.2 Classical Probability

Classical probability concept is originated in connection with games of chance. It applies when all possible outcomes are equally likely. If there are n equally likely possibilities of which one must occur and s of them are regarded as favorable or as a *success* then the probability of a *success* is given by $\left({}^{s}\!/_{n}\right)$.

5.3.3 Empirical Probability

It relies on actual experience to determine the likelihood of outcomes.

5.4 Probability - An Empirical Approach

In this chapter, we shall discuss only about empirical probability. The remaining two approaches would be discussed in higher classes. *Empirical* or *experimental* or *Relative frequency Probability* relies on actual experience to determine the likelihood of outcomes.

Empirical approach can be used whenever the experiment can be repeated many times and the results observed. Empirical probability is the most accurate scientific 'guess' based on the results of experiments about an event.

For example, the decision about people buying a certain brand of a soap, cannot be calculated using classical probability since the outcomes are not equally likely. To find the probability for such an event, we can perform an experiment such as you already have or conduct a survey. This is called collecting experimental data. The more data we collect the better the estimate is.

Key Concept **Empirical Probability**

Let m be the number of trials in which the event E happened (number of observations favourable to the event E) and n be the total number of trials (total number of observations) of an experiment. The empirical probability of happening of an event E, denoted by P(E), is given by

$$P(E) = \frac{\textit{Number of trials in which the event happened}}{\textit{Total number of trials}}$$

(or)

$$P(E) = \frac{\textit{Number of favourable observations}}{\textit{Total number of observations}}$$

$$\therefore\ P(E) = \frac{m}{n}$$

Clearly $0 \le m \le n \implies 0 \le \frac{m}{n} \le 1$, hence $0 \le P(E) \le 1$.

$$\boxed{0 \le P(E) \le 1}$$

i.e.the probability of happening of an event always lies from 0 to 1.

Probability is its most general use is a measure of our degree of confidence that a thing will happen. If the probability is 1.0, we know the thing happen certainly, and if probability is high say 0.9, we feel that the event is likely to happen. A probability of 0.5 denotes that the event is a equally likely to happen or not and one of 0 means that it certainly will not. This interpretation applied to statistical probabilities calculated from frequencies is the only way of expecting what we know of the individual from our knowledge of the populations.

Remark

If $P(E) = 1$ then E is called **Certain event** or **Sure event.**

If $P(E) = 0$ then E is known is an **Impossible event**.

If $\mathrm{P}(E)$ is the probability of an event, then the probability of not happening of E is denoted by $\mathrm{P}(E')$ or $\mathrm{P}(\overline{E})$

We know, $\mathrm{P}(E) + \mathrm{P}(E') = 1; \Rightarrow \mathrm{P}(E') = 1 - P(E)$

$$\boxed{\mathrm{P}(E') = 1 - P(E)}$$

We shall calculate a few typical probabilities, but it should be kept in mind that numerical probabilities are not the principal object of the theory. Our aim is to learn axioms, laws, concepts and to understand the theory of probability easily in higher classes.

Illustration

A coin is flipped several times. The number of times head and tail appeared and their ratios to the number of flips are noted below.

Number of Tosses (n)	Number of Heads (m_1)	P(H) = $\frac{m_1}{n}$	Number of Tails (m_2)	P(T) = $\frac{m_2}{n}$
50	29	$\frac{29}{50}$	21	$\frac{21}{50}$
60	34	$\frac{34}{60}$	26	$\frac{26}{60}$
70	41	$\frac{41}{70}$	29	$\frac{29}{70}$
80	44	$\frac{44}{80}$	36	$\frac{36}{80}$
90	48	$\frac{48}{90}$	42	$\frac{42}{90}$
100	52	$\frac{52}{100}$	48	$\frac{48}{100}$

From the above table we observe that as we increase the number of flips more and more, the probability of getting of heads and the probability of getting of tails come closer and closer to each other.

Activity (1) Flipping a coin:

Each student is asked to flip a coin for 10 times and tabulate the number of heads and tails obtained in the following table.

Outcome	Tally Marks	Number of heads or tails for 10 flips.
Head		
Tail		

Repeat the experiment for 20, 30, 40, 50 times and tabulate the results in the same manner as shown in the above example. Write down the values of the following fractions.

$$\frac{Number\ of\ times\ head\ turn\ up}{Total\ number\ of\ times\ the\ coin\ is\ flipped} = \frac{\square}{\square}$$

$$\frac{Number\ of\ times\ tail\ turn\ up}{Total\ number\ of\ times\ the\ coin\ is\ flipped} = \frac{\square}{\square}$$

Activity (2) Rolling a die:

Roll a die 20 times and calculate the probability of obtaining each of six outcomes.

Outcome	Tally Marks	Number of outcome for 20 rolls.	$\frac{No.\ of\ times\ corresponding\ outcomes\ come\ up}{Total\ no.\ of\ times\ the\ die\ is\ rolled}$
1			
2			
3			
4			
5			
6			

Repeat the experiment for 50, 100 times and tabulate the results in the same manner.

Activity (3) Flipping two coins:

Flip two coins simultaneously 10 times and record your observations in the table.

Outcome	Tally	Number of outcomes for 10 times	$\frac{No.\ of\ times\ corresponding\ outcomes\ comes\ up}{Total\ no.\ of\ times\ the\ two\ coins\ are\ flipped}$
Two Heads			
One head and one tail			
No head			

In Activity (1) each flip of a coin is called a trial. Similarly in Activity (2) each roll of a die is called a trial and each simultaneous flip of two coins in Activity (3) is also a trial.

In Activity (1) the getting a head in a particular flip is an event with outcome “head”. Similarly, getting a tail is an event with outcome tail.

In Activity (2) the getting of a particular number say " 5" is an event with outcome 5.

The value $\dfrac{\text{Number of heads comes up}}{\text{Total number of times the coins fliped}}$ is called an experimental or empirical probability.

Example 5.1

A manufacturer tested 1000 cell phones at random and found that 25 of them were defective. If a cell phone is selected at random, what is the probability that the selected cellphone is a defective one.

Solution Total number of cell phones tested = 1000 i.e., $n = 1000$

Let E be the event of selecting a defective cell phone.

$n(E) = 25$ i.e., $m = 25$

$$P(E) = \frac{\textit{Number of defective cellphones}}{\textit{Total number of cellphones tested}}$$

$$= \frac{m}{n} = \frac{25}{1000} = \frac{1}{40}$$

Example 5.2

In T-20 cricket match, Raju hit a "Six" 10 times out of 50 balls he played. If a ball was selected at random find the probability that he would not have hit a "Six".

Solution Total Number of balls Raju faced = 50i.e., $n = 50$

Let E be the event of hit a "Six" by Raju

$n(E) = 10$ i.e., $m=10$

$$P(E) = \frac{\textit{Number of times Raju hits a "Six"}}{\textit{Total number of balls faced}}$$

$$= \frac{m}{n} = \frac{10}{50} = \frac{1}{5}$$

P (Raju does not hit a Six) = $P(E') = 1 - P(E)$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

Example 5.3

The selection committee of a cricket team has to select a team of players. If the selection is made by using the past records scoring more than 40 runs in a match, then find the probability of selecting these two players whose performance are given below?

The performance of their last 30 matches are

Name of the player	More than 40 runs
Kumar	20 times
Kiruba	12 times

Solution Total number of matches observed = 30 i.e., $n = 30$

Let E_1 be the event of Kumar scoring more than 40 runs.

$$\text{n}(E_1) = 20 \qquad \text{i.e., } m_1 = 20$$

Let E_2 be the event of Kiruba scoring more than 40 runs.

$$\text{n}(E_2) = 12 \qquad \text{i.e., } m_2 = 12$$

$$\text{P}(E_1) = \frac{m_1}{n} = \frac{20}{30}$$

$$\text{P}(E_2) = \frac{m_2}{n} = \frac{12}{30}$$

The probability of Kumar being selected is $= \frac{20}{30} = \frac{2}{3}$

The probability of Kiruba being selected is $= \frac{12}{30} = \frac{2}{5}$

Example 5.4

On a particular day a policeman observed vehicles for speed check. The frequency table shows the speed of 160 vehicles that pass a radar speed check on dual carriage way.

Speed (Km/h)	20-29	30-39	40-49	50-59	60-69	70 & above
No. of Vehicles	14	23	28	35	52	8

Find the probability that the speed of a vehicle selected at random is

(i) faster than 69 *km/h*. (ii) between 20 - 39 *km/h*.

(iii) less than 60 *km/h*. (iv) between 40 - 69 *km/h*.

Solution

(i) Let E_1 be the event of a vehicle travelling faster than 69 *km/h*.

$$\text{n}(E_1) = 8 \qquad \text{i.e. } m_1 = 8$$

Total number of vehicles = 160. i.e. $n = 160$

$$\text{P}(E_1) = \frac{m_1}{n} = \frac{8}{160} = \frac{1}{20}$$

(ii) Let E_2 be the event of a vehicle travelling the speed between 20 - 39 *km/h*.

$$\text{n}(E_2) = 14+23 = 37 \qquad \text{i.e. } m_2 = 37$$

$$\text{P}(E_2) = \frac{m_2}{n} = \frac{37}{160}$$

(iii) Let E_3 be the event of a vehicle travelling the speed less than 60 *km/h*.

$$n(E_3) = 14+23+28+35 = 100 \qquad \text{i.e. } m_3 = 100$$

$$P(E_3) = \frac{m_3}{n} = \frac{100}{160} = \frac{5}{8}$$

(iv) Let E_4 be the event of a vehicle travelling the speed between 40-69 *km/h*.

$$n(E_4) = 28+35+52 = 115 \qquad \text{i.e. } m_4 = 115$$

$$P(E_4) = \frac{m_4}{n} = \frac{115}{160} = \frac{23}{32}$$

Example 5.5

A researcher would like to determine whether there is a relationship between a student's interest in statistics and his or her ability in mathematics. A random sample of 200 students is selected and they are asked whether their ability in mathematics and interest in statistics is low, average or high. The results were as follows:

		Ability in mathematics		
		Low	Average	High
Interest in statistics	Low	60	15	15
	Average	15	45	10
	High	5	10	25

If a student is selected at random, what is the probability that he / she

(i) has a high ability in mathematics (ii) has an average interest in statistics

(iii) has a high interest in statistics (iv) has high ability in mathematics and high interest in statistics and (v) has average ability in mathematics and low interest in statistics.

Solution

Total number of students $= 80+70+50=200$. i.e. $n = 200$

(i) Let E_1 be the event that he/she has a high ability in mathematics .

$$n(E_1) = 15+10+25= 50 \qquad \text{i.e. } m_1 = 50$$

$$P(E_1) = \frac{m_1}{n} = \frac{50}{200} = \frac{1}{4}$$

(ii) Let E_2 be the event that he/she has an average interest in statistics.

$$n(E_2) = 15+45+10 = 70 \qquad \text{i.e. } m_2 = 70$$

$$P(E_2) = \frac{m_2}{n} = \frac{70}{200} = \frac{7}{20}$$

Mathematics

(iii) Let E_3 be the event that he/she has a high interest in statistics.

$$n(E_3) = 5+10+25 = 40 \qquad \text{i.e. } m_3 = 40$$

$$P(E_3) = \frac{m_3}{n} = \frac{40}{200} = \frac{1}{5}$$

(iv) Let E_4 be the event has high ability in mathematics and high interest in statistics.

$$n(E_4) = 25 \qquad \text{i.e. } m_4 = 25$$

$$P(E_4) = \frac{m_4}{n} = \frac{25}{200} = \frac{1}{8}$$

(v) Let E_5 be the event has has average ability in mathematics and low interest in statistics.

$$n(E_5) = 15 \qquad \text{i.e. } m_5 = 15$$

$$P(E_5) = \frac{m_5}{n} = \frac{15}{200} = \frac{3}{40}$$

Example 5.6

A Hospital records indicated that maternity patients stayed in the hospital for the number of days as shown in the following.

No. of days stayed	3	4	5	6	more than 6
No. of patients	15	32	56	19	5

If a patient was selected at random find the probability that the patient stayed

(i) exactly 5 days (ii) less than 6 days

(iii) at most 4 days (iv) at least 5 days

Solution

Total number of patients of observed = 127 i.e., $n = 127$

(i) Let E_1 be the event of patients stayed exactly 5 days.

$$n(E_1) = 56 \qquad \text{i.e., } m_1 = 56$$

$$P(E_1) = \frac{m_1}{n} = \frac{56}{127}$$

(ii) Let E_2 be the event of patients stayed less than 6 days.

$$n(E_2) = 15 + 32 + 56 = 103 \quad \text{i.e., } m_2 = 103$$

$$P(E_2) = \frac{m_2}{n} = \frac{103}{127}$$

(iii) Let E_3 be the event of patients stayed atmost 4 days (3 and 4 days only).

$$n(E_3) = 15 + 32 = 47 \qquad \text{i.e., } m_3 = 47$$

$$P(E_3) = \frac{m_3}{n} = \frac{47}{127}$$

(iv) Let E_4 be the event of patients stayed atleast 5 days (5, 6 and 7 days only).

$$n(E_4) = 56 + 19 + 5 = 80 \qquad \text{i.e., } m_4 = 80$$

$$P(E_4) = \frac{m_4}{n} = \frac{80}{127}$$

Exercise 5.1

1. A probability experiment was conducted. Which of these cannot be considered as a probability of an outcome?

   i) 1/3 ii) – 1/5 iii) 0.80 iv) – 0.78 v) 0

   vi) 1.45 vii) 1 viii) 33% ix) 112%

2. Define: i) experiment ii) deterministic experiment iii) random experiment iv) sample space v) event vi) trial

3. Define empirical probability.

4. During the last 20 basket ball games, Sangeeth has made 65 and missed 35 free throws. What is the empirical probability if a ball was selected at random that Sangeeth make a foul shot?

5. The record of a weather station shows that out of the past 300 consecutive days, its weather was forecasted correctly 195 times. What is the probability that on a given day selected at random, (i) it was correct (ii) it was not correct.

6. Gowri asked 25 people if they liked the taste of a new health drink. The responses are,

Responses	Like	Dislike	Undecided
No. of people	15	8	2

   Find the probability that a person selected at random

   (i) likes the taste (ii) dislikes the taste (iii) undecided about the taste

7. In the sample of 50 people, 21 has type "O" blood, 22 has type "A" blood, 5 has type "B" blood and 2 has type "AB" blood. If a person is selected at random find the probability that

(i) the person has type "O" blood (ii) the person does not have type "B" blood
(iii) the person has type "A" blood (iv) the person does not have type "AB" blood.

8. A die is rolled 500 times. The following table shows that the outcomes of the die.

Outcomes	1	2	3	4	5	6
Frequencies	80	75	90	75	85	95

Find the probability of getting an outcome (i) less than 4 (ii) less than 2
(iii) greater than 2 (iv) getting 6 (v) not getting 6.

9. 2000 families with 2 children were selected randomly, and the following data were recorded.

Number of girls in a family	2	1	0
Number of families	624	900	476

Find the probability of a family, chosen at random, having (i) 2 girls (ii) 1 girl (iii) no girl

10. The follwing table gives the lifetime of 500 CFL lamps.

Life time (months)	9	10	11	12	13	14	more than 14
Number of Lamps	26	71	82	102	89	77	53

A bulb is selected at random. Find the probability that the life time of the selected bulb is
(i) less than 12 months (ii) more than 14 months
(iii) at most 12 months (iv) at least 13 months

11. On a busy road in a city the number of persons sitting in the cars passing by were observed during a particular interval of time. Data of 60 such cars is given in the following table.

No. of persons in the car	1	2	3	4	5
No. of Cars	22	16	12	6	4

Suppose another car passes by after this time interval. Find the probability that it has
(i) only 2 persons sitting in it (ii) less than 3 persons in it
(iii) more than 2 persons in it (iv) at least 4 persons in it

12. Marks obtained by Insuvai in Mathematics in ten unit tests are listed below.

Unit Test	I	II	III	IV	V	VI	VII	VIII	IX	X
Marks obtained (%)	89	93	98	99	98	97	96	90	98	99

Based on this data find the probability that in a unit test Insuvai get

(i) more than 95% (ii) less than 95% (iii) more than 98%

13. The table below shows the status of twenty residents in an apartment

Gender \ Status	College Students	Employees
Male	5	3
Female	4	8

If one of the residents is chosen at random, find the probability that the chosen resident will be (i) a female (ii) a college student (iii) a female student (iv) a male employee

14. The following table shows the results of a survey of thousand customers who bought a new or used cars of a certain model

Type \ Satisfaction level	Satisfied	Not Satisfied
New	300	100
Used	450	150

If a customer is selected at random, what is the probability that the customer

(i) bought a new car (ii) was satisfied (iii) bought an used car but not satisfied

15. A randomly selected sample of 1,000 individuals were asked whether they were planning to buy a new cellphone in the next 12 months. A year later the same persons were interviewed again to find out whether they actually bought a new cellphone. The response of both interviews is given below

	Buyers	Non-buyers
Plan to buy	200	50
No plan to buy	100	650

If a person was selected at random, what is the probability that he/she (i) had a plan to buy (ii) had a plan to buy but a non-buyer (iii) had no plan to buy but a buyer.

16. The survey has been undertaken to determine whether there is a relationship between the place of residence and ownership of an automobile. A random sample of car owners, 200 from large cities, 150 from suburbs and 150 from rural areas were selected and tabulated as follow

Car ownership \ Type of Area	Large city	Suburb	Rural
Own a foreign car	90	60	25
Do not own a foreign car	110	90	125

If a car owner was selected at random, what is the probability that he/she

(i) owns a foreign car.

(ii) owns a foreign car and lives in a suburb.

(iii) lives in a large city and does not own a foreign car.

(iv) lives in large city and owns a foreign car.

(v) neither lives in a rural area nor owns a foreign car.

17. The educational qualifications of 100 teachers of a Government higher secondary school are tabulated below

Age \ Education	M.Phil	Master Degree Only	Bachelor Degree Only
below 30	5	10	10
30 - 40	15	20	15
above 40	5	5	15

If a teacher is selected at random what is the probability that the chosen teacher has (i) master degree only (ii) M.Phil and age below 30 (iii) only a bachelor degree and age above 40 (iv) only a master degree and in age 30-40 (v) M.Phil and age above 40

18. A random sample of 1,000 men was selected and each individual was asked to indicate his age and his favorite sport. The results were as follows.

Age \ Sports	Volleyball	Basket ball	Hockey	Football
Below 20	26	47	41	36
20 - 29	38	84	80	48
30 - 39	72	68	38	22
40 - 49	96	48	30	26
50 and above	134	44	18	4

If a respondent is selected at random, what is the probability that

(i) he prefers Volleyball (ii) he is between 20 - 29 years old

(iii) he is between 20 and 29 years old and prefers Basketball

(iv) he doesn't prefer Hockey (v) he is at most 49 of age and prefers Football.

19. On one Sunday Muhil observed the vehicles at a Tollgate in the NH-45 for his science project about air pollution from 7 am. to 7 pm. The number of vehicles crossed are tabulated below.

Vehicles \ Time interval	7 a.m. to 11 a.m.	11 a.m. to 3 p.m.	3 p.m. to 7 p.m.
Bus	300	120	400
Car	200	130	250
Two Wheeler	500	250	350

A vehicle is selected at random. Find the probability that the vehicle chosen is a
(i) a bus at the time interval 7 a.m. to 11 a.m. (ii) a car at the time interval 11 a.m. to 7 p.m.
(iii) a bus at the time interval 7 a.m. to 3 p.m. (iv) a car at the time interval 7 a.m. to 7 p. m.
(v) not a two wheeler at the time interval 7 a.m. to 7 p.m.

Exercise 5.2

Multiple Choice Questions.

1. Probability of sure event is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) 2

2. Which one can represent a probability of an event

(A) $\frac{7}{4}$ (B) -1 (C) $-\frac{2}{3}$ (D) $\frac{2}{3}$

3. Probability of impossible event is

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) -1

4. Probability of any event x lies

(A) $0 < x < 1$ (B) $0 \le x < 1$ (C) $0 \le x \le 1$ (D) $1 < x < 2$

5. $P(E')$ is

(A) $1 - P(E)$ (B) $P(E) - 1$ (C) 1 (D) 0

Points to Remember

- Uncertainty or probability can be measured numerically.
- Experiment is defined as a process whose result is well defined.
- Deterministic Experiment : It is an experiment whose outcomes can be predicted with certainty, under identical conditions.
- Random Experiment is an experiment whose all possible outcomes are known, but it is not possible to predict the exact outcome in advance.
- A trial is an action which results in one or several outcomes.
- A sample space S is a set of all possible outcomes of a random experiment.
- Each outcome of an experiment is called a sample point.
- Any subset of a sample space is called an event.
- Classification of probability

  (1) Subjective probability (2) Classical probability (3) Empirical probability
- The empirical probability of happening of an event E, denoted by P(E), is given by

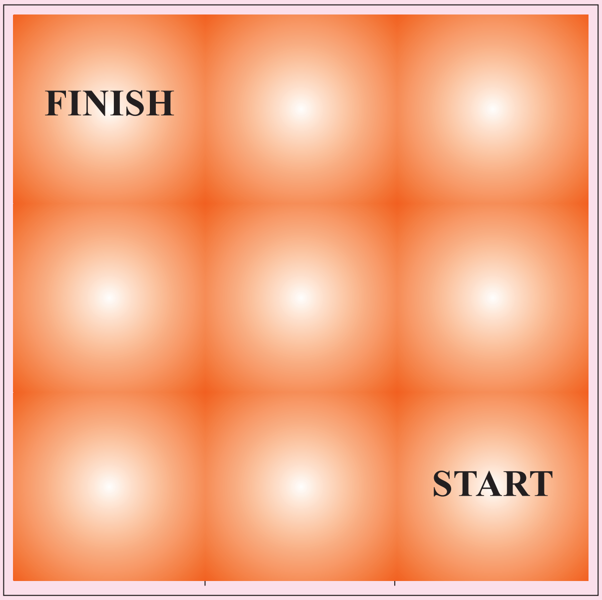
$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$$

$$\text{(or)}\quad P(E) = \frac{\text{Number of favourable observations}}{\text{Total number of observations}} \qquad \text{(or)}\quad P(E) = \frac{m}{n}$$

- $0 \leq P(E) \leq 1$
- $P(E') = 1 - P(E)$, where E' is the complementary event of E.

Activity 1

This is a simple game, where you throw a dice which controls the position of your counter on a 3×3 board.



Place your counter at the START square. Throw a dice.

If you get an EVEN number, you move your counter one square upwards.

If you get an ODD number, you move your counter one square left.

If your counter moves off any side of the board, you lose!

If your counter reaches the FINISH square, you have won.

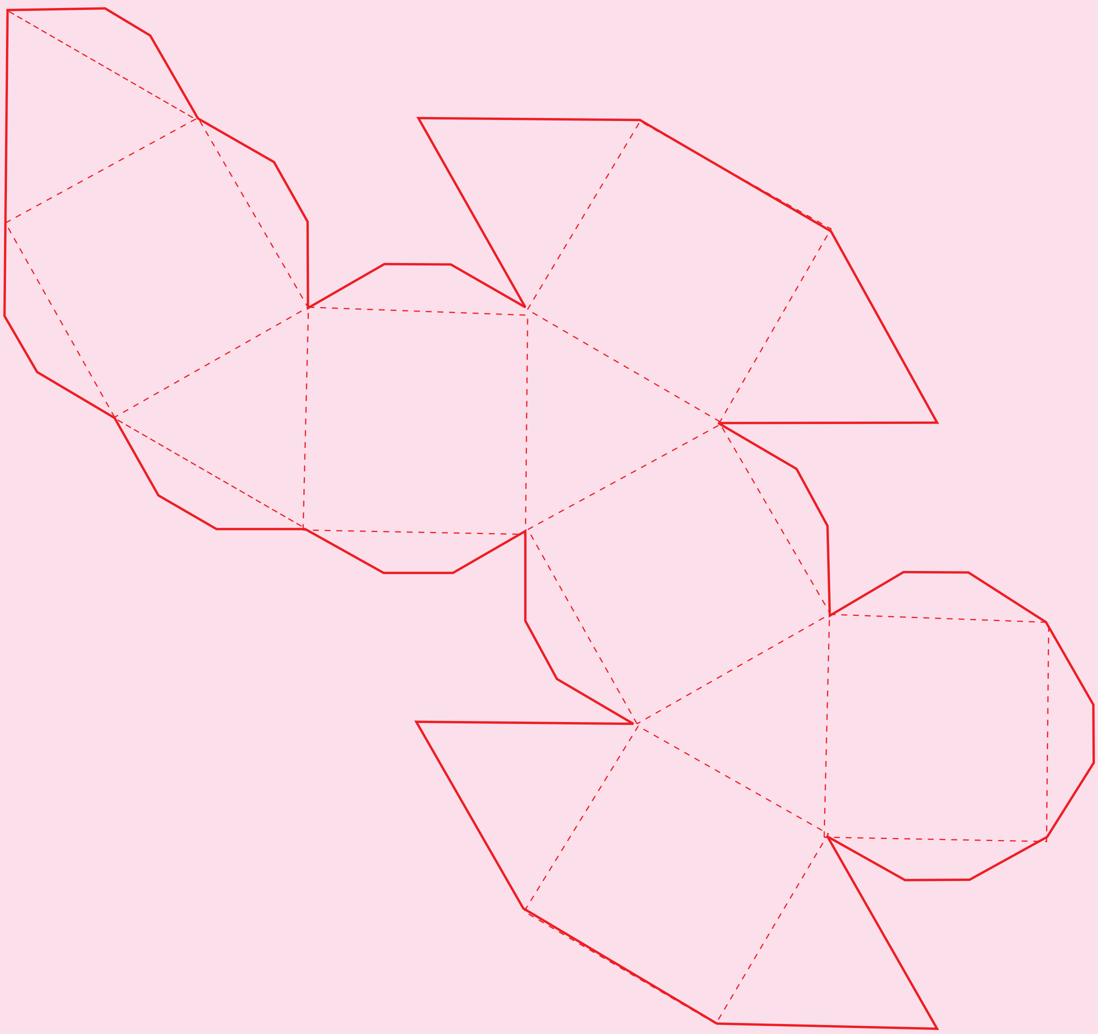
Play the game a few times and see if you win.

How many 'odds' and how many 'evens' do you need to get to win?

What is the probability of winning?

Activity 2

The net of a cuboctahedron is given below. It consists of 6 squares and 8 triangles. Make this 3-dimentsional object using card.



If this object is thrown, what do you think will be to probability of it landing on

(i) one of its square faces

(ii) one of its triangular faces?

Throw the object (at least 100 times) and estimate these probabilities.

How close are they to your original estimates?

Exercise 5.1

1. (ii) $\frac{-1}{5}$ (iv) -0.78 (vi) 1.45 (ix) 112% **4.** $\frac{13}{20}$ **5.** (i) $\frac{13}{20}$ (ii) $\frac{7}{20}$

6. (i) $\frac{3}{5}$ (ii) $\frac{8}{25}$ (iii) $\frac{2}{25}$ **7.** (i) $\frac{21}{50}$ (ii) $\frac{9}{10}$ (iii) $\frac{11}{25}$ (iv) $\frac{24}{25}$

8. (i) $\frac{49}{100}$ (ii) $\frac{4}{25}$ (iii) $\frac{69}{100}$ (iv) $\frac{19}{100}$ (v) $\frac{81}{100}$ **9.** (i) $\frac{39}{125}$ (ii) $\frac{9}{20}$ (iii) $\frac{119}{500}$

10. (i) $\frac{179}{500}$ (ii) $\frac{53}{500}$ (iii) $\frac{281}{500}$ (iv) $\frac{219}{500}$

11. (i) $\frac{4}{15}$ (ii) $\frac{19}{30}$ (iii) $\frac{11}{30}$ (iv) $\frac{1}{6}$

12. (i) $\frac{7}{10}$ (ii) $\frac{3}{10}$ (iii) $\frac{1}{5}$ **13.** (i) $\frac{3}{5}$ (ii) $\frac{9}{20}$ (iii) $\frac{1}{5}$ (iv) $\frac{3}{20}$

14. (i) $\frac{2}{5}$ (ii) $\frac{3}{4}$ (iii) $\frac{3}{20}$ **15.** (i) $\frac{1}{4}$ (ii) $\frac{1}{20}$ (iii) $\frac{1}{10}$

16. (i) $\frac{7}{20}$ (ii) $\frac{3}{25}$ (iii) $\frac{11}{50}$ (iv) $\frac{9}{50}$ (v) $\frac{3}{4}$

17. (i) $\frac{7}{20}$ (ii) $\frac{1}{20}$ (iii) $\frac{3}{20}$ (iv) $\frac{1}{5}$ (v) $\frac{1}{20}$

18. (i) $\frac{183}{500}$ (ii) $\frac{1}{4}$ (iii) $\frac{21}{250}$ (iv) $\frac{793}{1000}$ (v) $\frac{33}{250}$

19. (i) $\frac{3}{25}$ (ii) $\frac{19}{125}$ (iii) $\frac{21}{125}$ (iv) $\frac{29}{125}$ (v) $\frac{14}{25}$

Exercise 5.2

1. A **2.** D **3.** B **4.** C **5.** A

6 GRAPHS

A mathematical theory can be regarded as perfect only if you are prepared to present its contents to the first man in the street – D.HILBERT

Main Targets

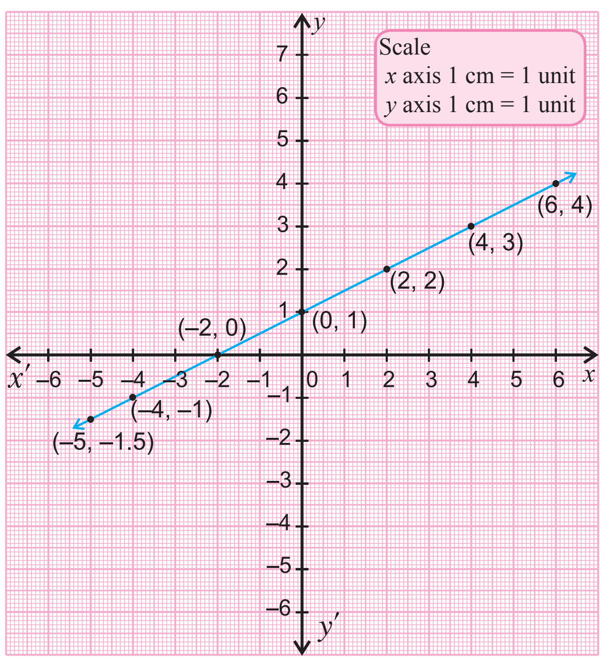
- To understand the concept of graph.
- To graph linear equations.
- To solve linear equations in two variables.

6.1 Introduction

This chapter covers the basic ideas of graphs. Almost everyday you see diagrams and graphs in newspapers, magazines, books etc. The purpose of the graph is to show numerical facts in visual form so that they can be understood quickly, easily and clearly. In this chapter you will learn how to use graphs to give a visual representation of the relationship between two variables and find solutions of equations in two variables.

6.2 Linear Graph

An equation such as $x - 2y = -2$ is an example of a linear equation in the two variables x and y. A solution of this equation is an ordered pair of numbers (x_0, y_0) so that x_0 and y_0 satisfy the equation $x - 2y = -2$, in the sense that $x_0 - 2y_0 = -2$. We observe that in this situation, it is easy to find all the solutions with a prescribed first number x_0 or a prescribed second number y_0. Relative to a pair of coordinate axes in the plane, the collection of all the points (x_0, y_0) in the coordinate plane so that each pair (x_0, y_0) is a solution of the equation $x - 2y = -2$ is called the *graph* of $x - 2y = -2$ in the plane. Using the above method of getting all the solutions of the equation $x - 2y = -2$, we can plot as many points of the graph as we please to get a good idea of the graph. For example, the adjacent picture contains the following points (given by the dots) on the graph, going from left to right: (−5,−1.5), (−4,−1), (−2, 0), (0, 1), (2, 2), (4, 3), (6, 4).



These points strongly suggest that the graph of $x - 2y = -2$ is a straight line.

Thus, a first degree equation in two variables always represents a straight line. Hence we can take general equation of a straight line as $ax + by + c = 0$, with at least one of a or b not equal to zero. For the sake of simplicity to draw lines in graphs we consider $y = mx + c$ as another simple form of the equation of straight line. For each value of x, the equation $y = mx + c$ gives a value of y and we can obtain an ordered pair (x, y).

Note The general equation of a straight line is $ax + by + c = 0$

(i) If $c = 0$, then the equation becomes $ax + by = 0$ and the line passes through the origin

(ii) If $a = 0$, then the equation becomes $by + c = 0$ and the line is parallel to x-axis

(iii) If $b = 0$, then the equation becomes $ax + c = 0$ and the line is parallel to y-axis

6.2.1 Procedure to Draw a Linear Graph

When graphing an equation, we usually begin by creating a table of x and y values. We do this by choosing three x values and computing the corresponding y values. Although two points are sufficient to sketch the graph of a line, we usually choose three points so that we can check our work.

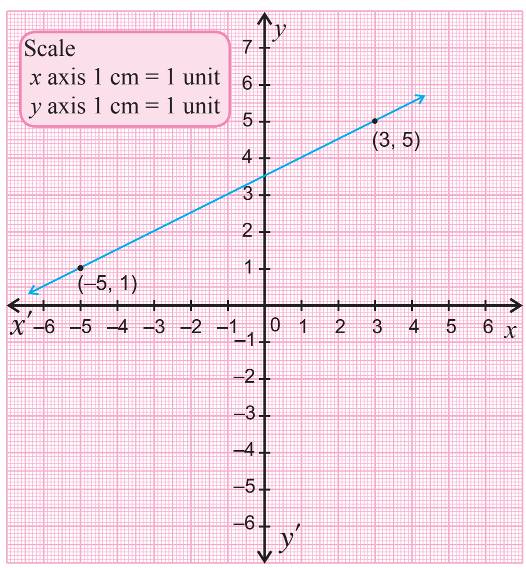
Step 1: Using the given equation construct a table of with x and y values.
Step 2: Draw x-axis and y-axis on the graph paper.
Step 3: Select a suitable scale on the coordinate axes.
Step 4: Plot the points
Step 5: Join the points and extend it to get the line.

6.2.2 Draw Straight Lines

Example 6.1

Draw the graph of the line joining the points (3, 5) and (– 5, 1)

Solution



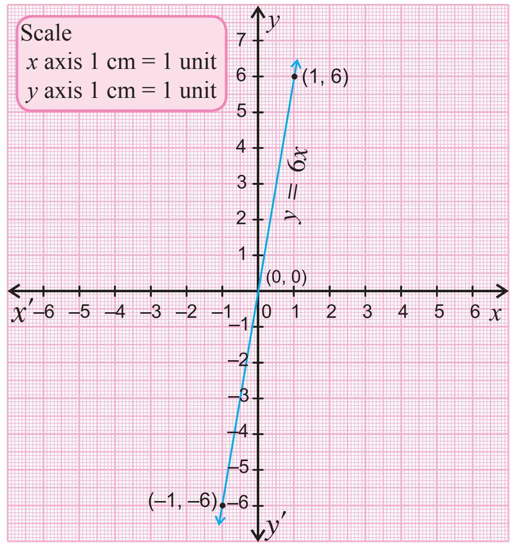
1. Draw the x-axis and y-axis on a graph sheet with 1 cm = 1 unit on both axes.
2. We plot the two given points (3, 5), (– 5, 1) on the graph sheet.
3. We join the points by a line segment and extend it on either direction.
4. We get the required linear graph.

Example 6.2

Draw the graph of $y = 6x$

Solution Substituting the values $x = -1, 0, 1$ in the equation of the line, we find the values of y as follows

$y = 6x$			
x	-1	0	1
y	-6	0	6



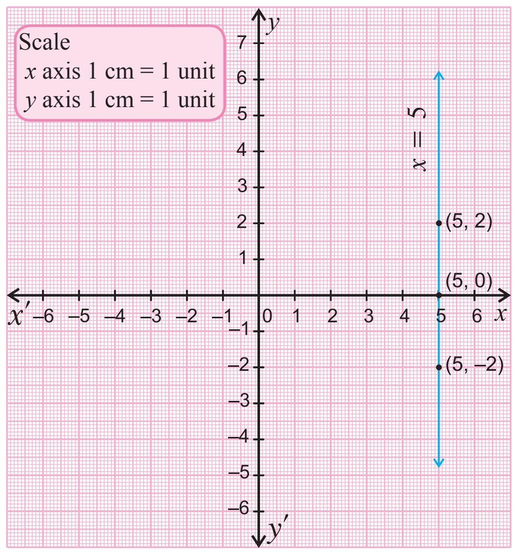
In a graph, plot the points $(-1, -6)$, $(0, 0)$ and $(1, 6)$ and draw a line passing through the plotted points. This is the required linear graph.

Example 6.3

Draw the graph of $x = 5$

Solution The line $x = 5$ is parallel to y-axis. On this line $x = 5$, a constant. So, any point on this line is of the form $(5, y)$. Taking the values $y = -2, 0, 2$ we get the points $(5, -2)$, $(5, 0)$ and $(5, 2)$.

$x = 5$			
x	5	5	5
y	-2	0	2



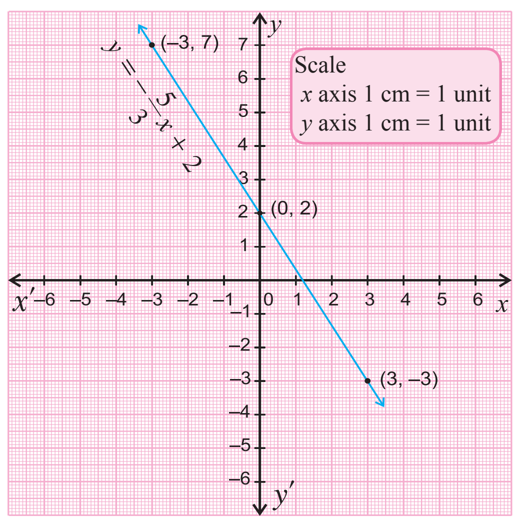
In a graph sheet, plot these points and draw a line passing through the points. Thus we get the required linear graph.

Example 6.4

Draw the graph of the line $y = -\frac{5}{3}x + 2$.

Solution Substituting $x = -3, 0, 3$ in the equation of the line, we find the values of y as follows

$y = -\frac{5}{3}x + 2$			
x	-3	0	3
$-\frac{5}{3}x$	5	0	-5
$y = -\frac{5}{3}x + 2$	7	2	-3



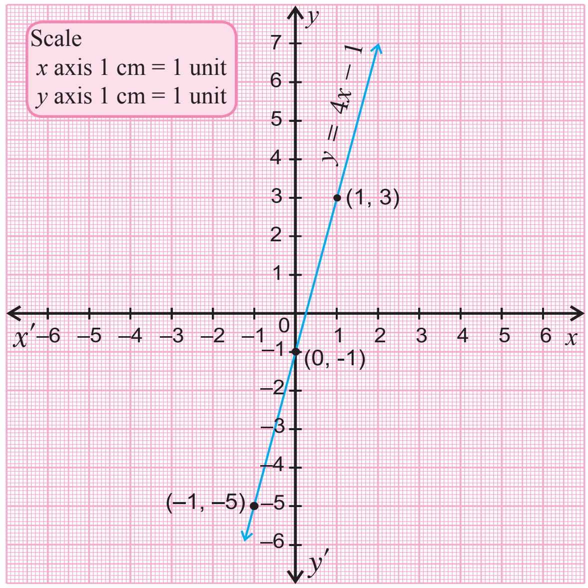
Plot the points $(-3, 7)$, $(0, 2)$ and $(3, -3)$ and draw a line passing through the plotted points. This is the required graph of the equation $y = \frac{-5}{3}x + 2$.

Example 6.5

Draw the graph of $y = 4x - 1$.

Solution Substituting the values $x = -1, 0, 1$ in the given equation of line, we find the values of y as follows

$y = 4x - 1$			
x	-1	0	1
$4x$	-4	0	4
$y = 4x - 1$	-5	-1	3



Plot the points $(-1, -5)$, $(0, -1)$ and $(1, 3)$ in a graph sheet and draw a line passing through the plotted points. We now get the required linear graph.

Example 6.6

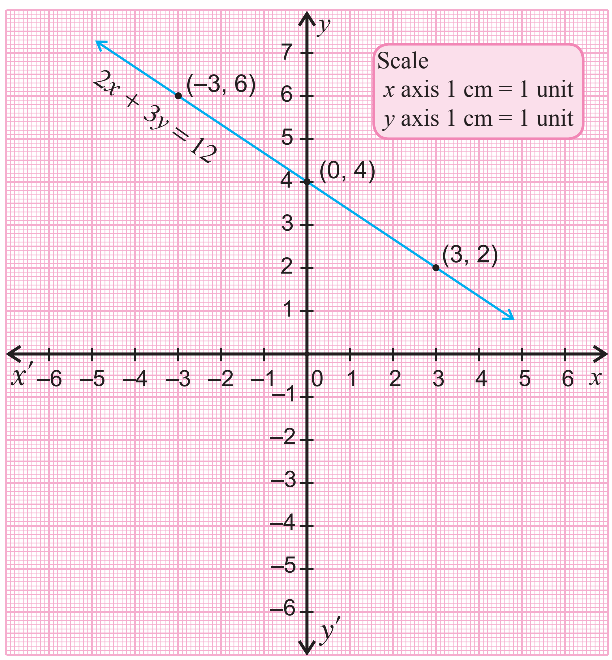
Draw the graph of $2x + 3y = 12$

Solution First, we rewrite the equation $2x + 3y = 12$ in the form of $y = mx + c$.

$2x + 3y = 12$ implies $y = -\frac{2}{3}x + 4$

Substituting $x = -3, 0, 3$ in the above equation, we find the values of y as follows

$y = -\frac{2}{3}x + 4$			
x	-3	0	3
$-\frac{2}{3}x$	2	0	-2
$y = -\frac{2}{3}x + 4$	6	4	2



Plot the points $(-3, 6)$, $(0, 4)$ and $(3, 2)$ and draw a line passing through these points. Now we get the required graph.

Exercise 6.1

1. Draw the linear graph joining the points

   (i) $(2, 3)$ and $(-6, -5)$ (ii) $(-2, -4)$ and $(-1, 6)$

   (iii) $(5, -7)$ and $(-1, 5)$ (iv) $(-3, 9)$ and $(5, -6)$ (v) $(4, -5)$ and $(6, 10)$

2. Draw the graph of the following

   (i) $y = 5$ (ii) $y = -6$ (iii) $x = 3$

   (iv) $x = -5$ (v) $2x + 7 = 0$ (vi) $6 + 3y = 0$

3. Draw the graph of the following

   (i) $y = 4x$ (ii) $3x + y = 0$ (iii) $x = -2y$

   (iv) $y - 3x = 0$ (v) $9y - 3x = 0$

4. Draw the linear graph of the following equations

   (i) $y = 3x + 1$ (ii) $4y = 8x + 2$ (iii) $y - 4x + 3 = 0$

   (iv) $x = 3y + 3$ (v) $x + 2y - 6 = 0$ (vi) $x - 2y + 1 = 0$

   (vii) $3x + 2y = 12$

5. Draw the graph of the equation $y = mx + c$, where

   (i) $m = 2$ and $c = 3$ (ii) $m = -2$ and $c = -2$ (iii) $m = -4$ and $c = 1$

   (iv) $m = 3$ and $c = -4$ (v) $m = \frac{1}{2}$ and $c = 3$ (vi) $m = \frac{-2}{3}$ and $c = 2$

6.3 Application of Graphs

By a system of linear equations in two variables we mean a collection of more than one linear equations in two variables. The solutions of system of linear equations is the set of ordered pairs that satisfy all the equations in that system. In this section you will learn to solve graphically a pair of two linear equations in two variables.

Here three cases arise:

(i) The two graphs coincide, that is, the graphs are one and the same. In this case there are infinitely many solutions.

(ii) The two graphs do not coincide but they are parallel. That is, do not meet at all. So, there is no common point and hence there is no solution.

(iii) The two graphs intersect exactly at one point. In this case the equations have a unique solution.

Example 6.7

Solve graphically the pair of equations $x+2y=4;\ 2x+4y=8$.

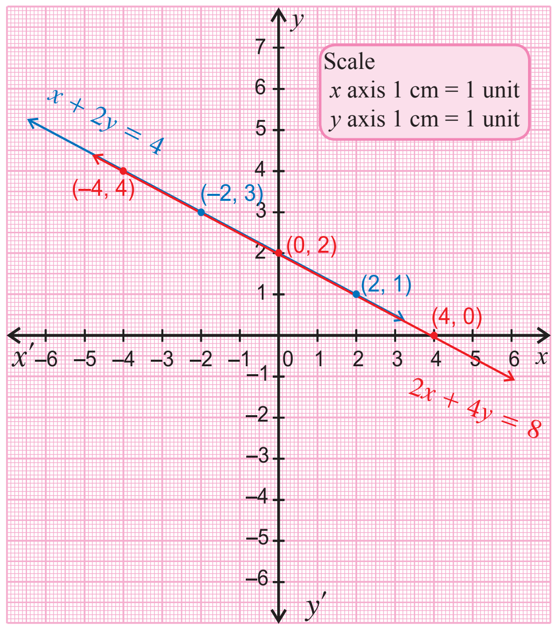
Solution We find three points for each equation, by choosing three valuesof x and computing the corresponding y values. We show our results in tables.

Line 1: $x+2y=4$

$$2y=-x+4 \Longrightarrow y=-\frac{x}{2}+2$$

Substituting $x=-2,0,2$ in the above equation, we get the corresponding y values as

$y=-\frac{x}{2}+2$			
x	-2	0	2
$-\frac{x}{2}$	1	0	-1
$y=-\frac{x}{2}+2$	3	2	1



Line 2: $2x+4y=8$

$$4y=-2x+8 \Longrightarrow y=-\frac{x}{2}+2$$

Substituting $x=-4,0,4$ in the above equation, we get y values as

$y=-\frac{x}{2}+2$			
x	-4	0	4
$-\frac{x}{2}$	2	0	-2
$y=-\frac{x}{2}+2$	4	2	0

We plot these points in a graph paper and draw the lines. Then we find that both the lines coincide. Any point on one line is also a point on the other. That is all points on the line are common points. Therefore each point on the line is a solution.Hence there are infinitely many solution

Example 6.8

Solve graphically $x-3y=6;\ x-3y+9=0$

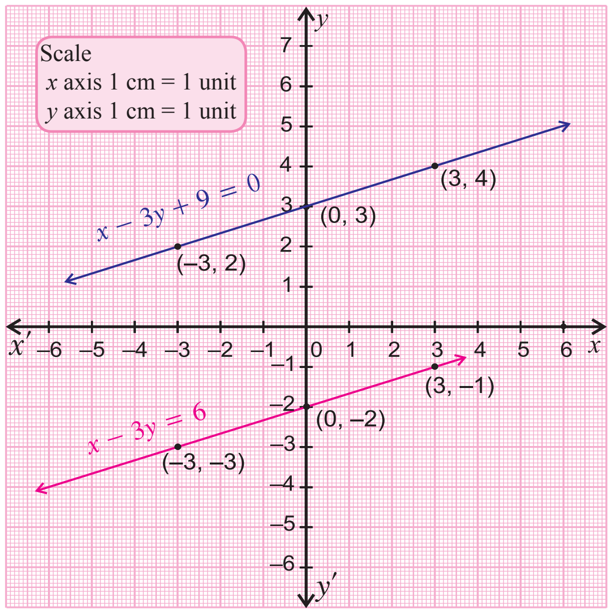
Solution let us find three points for each equation, by choosing three x values and computing the corresponding y values. We present our results in the tables.

Line 1: $x - 3y = 6$

$3y = x - 6 \Longrightarrow y = \frac{x}{3} - 2$

Substituting $x = -3, 0, 3$ in the above equation, we get the values of y as follows

$y = \frac{x}{3} - 2$			
x	-3	0	3
$\frac{x}{3}$	-1	0	1
$y = \frac{x}{3} - 2$	-3	-2	-1



Line 2: $x - 3y + 9 = 0$

$$3y = x + 9$$

$$y = \frac{x}{3} + 3$$

Substituting $x = -3, 0, 3$ in above equation, we get

$y = \frac{x}{3} + 3$			
x	-3	0	3
$\frac{x}{3}$	-1	0	1
$y = \frac{x}{3} + 3$	2	3	4

We plot the points $(-3, -3)$ $(0, -2)$ and $(3, -1)$ in the graph sheet and draw the line through them. Next, we plot the points $(-3, 2)$ $(0, 3)$ and $(3, 4)$ in the same graph sheet and draw the line through them. We find that the two graphs are parallel. That is no point is common to both lines. So, the system of equations has no solution.

Example 6.9

Solve graphically the equations $2x - y = 1$; $x + 2y = 8$

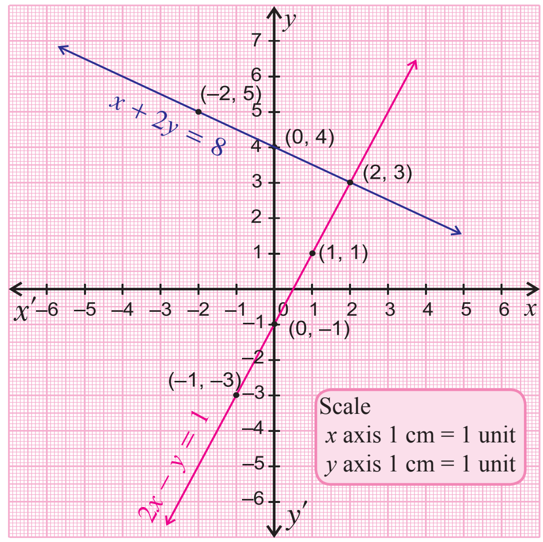
Solution We find three points for each equation, by choosing three values of x and computing the corresponding y values. We'll put our results in tables.

Line 1: $2x - y = 1$

$$y = 2x - 1$$

Substituting $x = -1, 0, 1$ in the above equation, we find

$y = 2x - 1$			
x	-1	0	1
$2x$	-2	0	2
$y = 2x - 1$	-3	-1	1



Line 2: $x + 2y = 8$

$$2y = -x + 8 \Longrightarrow y = -\frac{x}{2} + 4$$

Substituting $x = -2, 0, 2$ in above equation, we get

$y = -\frac{x}{2} + 4$			
x	-2	0	2
$-\frac{x}{2}$	1	0	-1
$y = -\frac{x}{2} + 4$	5	4	3

We plot the points $(-1, -3)$, $(0, -1)$ and $(1, 1)$ in a graph sheet and draw the line through them. Next, we plot the points $(-2, 5)$, $(0, 4)$ and $(2, 3)$ in the same graph sheet and draw the line through them. We find that the two graphs are intersecting at the point (2, 3). Hence, the system of equations has only one solution (unique solution) and the solution is $x=2$, $y=3$

Therefore the solution is (2, 3)

Exercise 6.2

Solve Graphically the following pairs of equations.

1. $3x - y = 0;\ x - 2 = 0$
2. $2x + y = 4;\ 4x + 2y = 8$
3. $2x = y + 1;\ x + 2y - 8 = 0$
4. $x + y = 5;\ x - y = 1$
5. $x - 2y = 6;\ x - 2y = -6$
6. $4x - y - 5 = 0;\ x + y - 5 = 0$
7. $3x + 2y = 4;\ 9x + 6y - 12 = 0$
8. $y = 2x + 1;\ y + 3x - 6 = 0$
9. $y - 2x + 2 = 0;\ y = 4x - 4$
10. $x - y = 0;\ y + 3 = 0$
11. $2x - 4 = 0;\ 4x + y + 4 = 0$
12. $\frac{x}{2} + \frac{y}{4} = 1;\ \frac{x}{2} + \frac{y}{4} = 2$

Exercise 6.2

1. (2, 6) **2.** many solution **3.** (2, 3) **4.** (3, 2) **5.** no solution

6. (2, 3) **7.** many solution **8.** (1, 3) **9.** (1, 0) **10.** (–3, –3)

11. (2, –12) **12.** no solution

'I can, I did'

Student's Activity Record

Subject :

Sl. No.	Date	Lesson No.	Topic of the Lesson	Activities	Remarks