Government of Tamilnadu

STANDARD NINE

TERM I

VOLUME 2

MATHEMATICS

NOT FOR SALE

A Publication Under
Free Textbook Programme of
Government of Tamilnadu

Department of School Education

First Edition - 2013
Reprint - 2015
(Published under Uniform System of School Education Scheme in Trimester Pattern)

Textbook Preparation

State Council of Educational Research and Training

College Road, Chennai - 600 006.

Textbook Printing

Tamil Nadu Textbook and Educational Services Corporation

College Road, Chennai - 600 006.

This book has been printed on 80 G.S.M. Maplitho Paper

Price : Rs.

Printed by Web Offset at :

Textbook available at

www.textbooksonline.tn.nic.in

Preface

சீரிளமைத் திறம் வியந்து செயல் மறந்து வாழ்த்துதுமே!

-'மனோன்மணீயம்' பெ. சுந்தரனார்

The Government of Tamil Nadu has decided to evolve a uniform system of school education in the state to ensure social justice and provide quality education to all the schools of the state. With due consideration to this view and to prepare the students to face new challenges in the field of Mathematics, this textbook is well designed within the frame work of NCF2005 by the textbook committee of subject experts and practicing teachers in schools and colleges.

Mathematics is a language which uses easy words for hard ideas. With the aid of Mathematics and imagination the nano or the googolplex all things may be brought within man's domain. This laptop of handbook is an important collection of twelve topics. A brief and breezy explanation of each chapter proceeds with an introduction to the topics, significant contributions made by the great Mathematicians, concise definitions, key concepts, relevant theorems, practice problems and a brief summary at the end of the lesson written with wit and clarity to motivate the students. This book helps the student to complete the transition from usual manipulation to little rigorous Mathematics.

Real life examples quoted in the text help in the easy grasp of meaning and in understanding the necessity of mathematics. These examples will shape the abstract key concepts, definitions and theorems in simple form to understand clearly. But beyond finding these examples, one should examine the reason why the basic definitions are given. This leads to a split into streams of thought to solve the complicated problems easily in different ways.

By means of colourful visual representation, we hope the charming presents in our collection will invite the students to enjoy the beauty of Mathematics to share their views with others and to become involved in the process of creating new ideas. A mathematical theory is not to be considered complete until it has been made so clear that the student can explain it to the first man whom he or she meets on the street. It is a fact that mathematics is not a mere manipulation of numbers but an enjoyable domain of knowledge.

To grasp the meaning and necessity of Mathematics, to appreciate its beauty and its value, it is time now to learn the depth of fundamentals of Mathematics given in this text. Anyone who penetrates into it will find that it proves both charming and exciting. Learning and creating Mathematics is indeed a worthwhile way to spend one's life.

Mathematics is not a magic it is a music ; play it, enjoy! bloom!! and flourish!!!

-Textbook team

SYMBOLS

$=$	equal to
$\neq$	not equal to
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater than or equal to
$\approx$	equivalent to
$\cup$	union
$\cap$	intersection
$\mathbb{U}$	universal Set
$\in$	belongs to
$\notin$	does not belong to
$\subset$	proper subset of
$\subseteq$	subset of or is contained in
$\not\subset$	not a proper subset of
$\nsubseteq$	not a subset of or is not contained in
A' (or) A^c	complement of A
$\emptyset$ (or) $\{\ \}$	empty set or null set or void set
$n(A)$	number of elements in the set A
$P(A)$	power set of A
$\|\|\|^{ly}$	similarly
$\mathrm{P}(A)$	probability of the event A

Δ	symmetric difference
$\mathbb{N}$	natural numbers
$\mathbb{W}$	whole numbers
$\mathbb{Z}$	integers
$\mathbb{R}$	real numbers
$\triangle$	triangle
$\angle$	angle
$\perp$	perpendicular to
$\parallel$	parallel to
$\Longrightarrow$	implies
$\therefore$	therefore
$\because$	since (or) because
$\lvert\ \rvert$	absolute value
$\simeq$	approximately equal to
$\mid$ (or) :	such that
$\equiv$ (or) $\cong$	congruent
$\equiv$	identically equal to
π	pi
$\pm$	plus or minus
$\blacksquare$	end of the proof

CONTENT

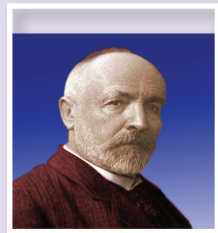
1 THEORY OF SETS

No one shall expel us from the paradise that Cantor has created for us

- DAVID HILBERT

Main Targets

- To describe a set
- To represent sets in descriptive form, set builder form and roster form
- To identify different kinds of sets
- To understand and perform set operations
- To use Venn diagrams to represent sets and set operations
- To use the formula involving $n(A \cup B)$ simple word problems



GEORG CANTOR
(1845-1918)

*The basic ideas of set theory were developed by the German mathematician **Georg Cantor** (1845-1918). He worked on certain kinds of infinite series particularly on Fourier series. Most mathematicians accept set theory as a basis of modern mathematical analysis. Cantor's work was fundamental to the later investigation of Mathematical logic.*

1.1 Introduction

The concept of set is vital to mathematical thought and is being used in almost every branch of mathematics. In mathematics, sets are convenient because all mathematical structures can be regarded as sets. Understanding set theory helps us to see things in terms of systems, to organize things into sets and begin to understand logic. In chapter 2, we will learn how the natural numbers, the rational numbers and the real numbers can be defined as sets. In this chapter we will learn about the concept of set and some basic operations of set theory.

1.2 Description of Sets

We often deal with a group or a collection of objects, such as a collection of books, a group of students, a list of states in a country, a collection of coins, etc. Set may be considered as a mathematical way of representing a collection or a group of objects.

Key Concept	**Set**
A set is a collection of well defined objects. The objects of a set are called elements or members of the set.	

The main property of a set in mathematics is that it is well defined. This means that given any object, it must be clear whether that object is a member (element) of the set or not.

The objects of a set are all distinct, i.e., no two objects are the same.

Which of the following collections are well defined?

(1) The collection of male students in your class.

(2) The collection of numbers 2, 4, 6, 10 and 12.

(3) The collection of districts in Tamil Nadu.

(4) The collection of all good movies.

(1), (2) and (3) are well defined and therefore they are sets. (4) is not well defined because the word good is not defined. Therefore, (4) is not a set.

Generally, sets are named with the capital letters A, B, C, etc. The elements of a set are denoted by the small letters a, b, c, etc.

Reading Notation

$\in$	'is an element of' or 'belongs to'

If x is an element of the set A, we write $x \in A$.

$\notin$	'is not an element of' or 'does not belong to'

If x is not an element of the set A, we write $x \notin A$.

For example,

Consider the set $A = \{1, 3, 5, 9\}$.

1 is an element of A, written as $1 \in A$

3 is an element of A, written as $3 \in A$

8 is not an element of A, written as $8 \notin A$

Example 1.1

Let $A = \{1, 2, 3, 4, 5, 6\}$. Fill in the blank spaces with the appropriate symbol $\in$ or $\notin$.

(i) 3 .... A (ii) 7 .... A

(iii) 0 ...... A (iv) 2 ...... A

Solution (i) $3 \in A$ ($\because$ 3 is an element of A)

(ii) $7 \notin A$ ($\because$ 7 is not an element of A)

(iii) $0 \notin A$ ($\because$ 0 is not an element of A)

(iv) $2 \in A$ ($\because$ 2 is an element of A)

1.3 Representation of a Set

A set can be represented in any one of the following three ways or forms.

(i) Descriptive Form

(ii) Set-Builder Form or Rule Form

(iii) Roster Form or Tabular Form

1.3.1 Descriptive Form

Key Concept **Descriptive Form**

One way to specify a set is to give a verbal description of its elements. This is known as the Descriptive form of specification.

The description must allow a concise determination of which elements belong to the set and which elements do not.

For example,

(i) The set of all natural numbers.

(ii) The set of all prime numbers less than 100.

(iii) The set of all letters in English alphabets.

1.3.2 Set-Builder Form or Rule Form

Key Concept — **Set-Builder Form**

Set-builder notation is a notation for describing a set by indicating the properties that its members must satisfy.

Reading Notation

'|' or ':'	such that

A = $\{x : x$ is a letter in the word *dictionary*$\}$

We read it as

"A is the set of all x such that x is a letter in the word *dictionary*"

For example,

(i) $\mathbb{N} = \{x : x$ is a natural number$\}$

(ii) $P = \{x : x$ is a prime number less than $100\}$

(iii) $A = \{x : x$ is a letter in the English alphabet$\}$

1.3.3 Roster Form or Tabular Form

Key Concept — **Roster Form**

Listing the elements of a set inside a pair of braces { } is called the roster form.

For example,

(i) Let A be the set of even natural numbers less than 11.
In roster form we write $A = \{2, 4, 6, 8, 10\}$

(ii) $A = \{x : x$ is an integer and $-1 \leq x < 5\}$
In roster form we write $A = \{-1, 0, 1, 2, 3, 4\}$

Remark

(i) In roster form each element of the set must be listed exactly once. By convention, the elements in a set should not be repeated.

(ii) Let A be the set of letters in the word "coffee", i.e, A={ c, o, f, e }. So, in roster form of the set A the following are invalid.

$\{c, o, e\}$ (not all elements are listed)

$\{c, o, f, f, e\}$ (element 'f' is listed twice)

(iii) In a roster form the elements in a set can be written in any order.

The following are valid roster form of the set containing the elements 2, 3 and 4.

$\{2, 3, 4\}$ $\{2, 4, 3\}$ $\{4, 3, 2\}$

Each of them represents the same set

(iv) If there are either infinitely many elements or a large finite number of elements, then three consecutive dots called *ellipsis* are used to indicate that the pattern of the listed elements continues, as in $\{5, 6, 7, \cdots\}$ or $\{3, 6, 9, 12, 15, \cdots, 60\}$.

(v) Ellipsis can be used only if enough information has been given so that one can figure out the entire pattern.

Representation of sets in Different Forms

Descriptive Form	Set - Builder Form	Roster Form
The set of all vowels in English alphabet	$\{x : x$ is a vowel in the English alphabet$\}$	$\{a, e, i, o, u\}$
The set of all odd positive integers less than or equal to 15	$\{x : x$ is an odd number and $0 < x \leq 15\}$	$\{1, 3, 5, 7, 9, 11, 13, 15\}$
The set of all positive cube numbers less than 100	$\{x : x$ is a cube number and $0 < x < 100\}$	$\{1, 8, 27, 64\}$

Example 1.2

List the elements of the following sets in Roster form:

(i) The set of all positive integers which are multiples of 7.

(ii) The set of all prime numbers less than 20.

Solution (i) The set of all positive integers which are multiples of 7 in roster form is $\{7, 14, 21, 28, \cdots\}$

(ii) The set of all prime numbers less than 20 in roster form is $\{2, 3, 5, 7, 11, 13, 17, 19\}$

Example 1.3

Write the set $A = \{x : x$ is a natural number $\leq 8\}$ in roster form.

Solution $A = \{x : x$ is a natural number $\leq 8\}$.

So, the set contains the elements 1, 2, 3, 4, 5, 6, 7, 8.

Hence in roster form $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Example 1.4

Represent the following sets in set-builder form

(i) $X = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

(ii) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots\right\}$

Solution (i) $X = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}\}$

The set X contains all the days of a week.

Hence in set builder form, we write

$X = \{x : x \text{ is a day in a week}\}$

(ii) $A = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \cdots\right\}$. The denominators of the elements are 1, 2, 3, 4, $\cdots$

$\therefore$ The set-builder form is $A = \left\{x : x = \frac{1}{n}, n \in \mathbb{N}\right\}$

1.3.4 Cardinal Number

Key Concept	**Cardinal Number**

The number of elements in a set is called the cardinal number of the set.

Reading Notation

$n(A)$	number of elements in the set A

The cardinal number of the set A is denoted by $n(A)$.

For example,

Consider the set $A = \{-1, 0, 1, 2, 3, 4, 5\}$. The set A has 7 elements.

$\therefore$ The cardinal number of A is 7 i.e., $n(A) = 7$.

Example 1.5

Find the cardinal number of the following sets.

(i) $A = \{x : x \text{ is a prime factor of } 12\}$

(ii) $B = \{x : x \in \mathbb{W}, x \leq 5\}$

Solution (i) Factors of 12 are 1, 2, 3, 4, 6, 12. So, the prime factors of 12 are 2, 3.

We write the set A in roster form as $A = \{2, 3\}$ and hence $n(A) = 2$.

(ii) $B = \{x : x \in \mathbb{W}, x \leq 5\}$. In Tabular form, $B = \{0, 1, 2, 3, 4, 5\}$.

The set B has six elements and hence $n(B) = 6$

1.4 Different Kinds of Sets

1.4.1 The Empty Set

Key Concept — **Empty Set**

A set containing no elements is called the empty set or null set or void set.

Reading Notation

$\varnothing$ or $\{\ \}$	Empty set or Null set or Void set

The empty set is denoted by the symbol $\varnothing$ or $\{\ \}$

For example,

Consider the set $A = \{x : x < 1, x \in \mathbb{N}\}$.

There is no natural number which is less than 1.

$\therefore\ A = \{\ \}$

Think and Answer !
What is $n(\varnothing)$?

Note The concept of empty set plays a key role in the study of sets just like the role of the number *zero* in the study of number system.

1.4.2 Finite Set

Key Concept — **Finite Set**

If the number of elements in a set is zero or finite, then the set is called a finite set.

For example,

(i) Consider the set A of natural numbers between 8 and 9.
There is no natural number between 8 and 9. So, $A = \{\ \}$ and $n(A) = 0$.

$\therefore\ A$ is a finite set

(ii) Consider the set $X = \{x : x \text{ is an integer and } -1 \le x \le 2\}$.

$X = \{-1, 0, 1, 2\}$ and $n(X) = 4$

$\therefore\ X$ is a finite set

Note The cardinal number of a finite set is finite

1.4.3 Infinite Set

Key Concept — **Infinite Set**

A set is said to be an infinite set if the number of elements in the set is not finite.

For example,

Let $\mathbb{W}$ = The set of all whole numbers. i. e., $\mathbb{W} = \{0, 1, 2, 3, \cdots\}$

The set of all whole numbers contain infinite number of elements

$\therefore$ $\mathbb{W}$ is an infinite set.

Note The cardinal number of an infinite set is not a finite number.

Example 1.6

State whether the following sets are finite or infinite

(i) $A = \{x : x \text{ is a multiple of } 5,\ x \in \mathbb{N}\}$

(ii) $B = \{x : x \text{ is an even prime number}\}$

(iii) The set of all positive integers greater than 50.

Solution (i) $A = \{x : x \text{ is a multiple of } 5,\ x \in \mathbb{N}\} = \{5, 10, 15, 20, ...\}$

$\therefore$ A is an infinite set.

(ii) $B = \{x : x \text{ is even prime numbers}\}$. The only even prime number is 2.

$\therefore$ $B = \{\ 2\ \}$ and hence B is a finite set.

(iii) Let X be the set of all positive integers greater than 50.

Then $X = \{51, 52, 53, ...\}$

$\therefore$ X is an infinite set.

1.4.4 Singleton Set

Key Concept — **Singleton Set**

A set containing only one element is called a singleton set

For example,

Consider the set $A = \{x : x \text{ is an integer and } 1 < x < 3\}$.

$A = \{\ 2\ \}$ i.e., A has only one element

$\therefore$ A is a singleton set.

Remark

It is important to recognise that the following sets are not equal.

(i) The null set $\varnothing$

(ii) The set having the null set as its only element $\{\varnothing\}$

(iii) The set having zero as its only element $\{ 0 \}$

1.4.5 Equivalent Set

Key Concept — **Equivalent Set**

Two sets A and B are said to be equivalent if they have the same number of elements

In other words, A and B are equivalent if $n(A) = n(B)$.

Reading Notation

$\approx$	Equivalent

'A and B are equivalent' is written as $A \approx B$

For example,

Consider the sets $A = \{ 7, 8, 9, 10 \}$ and $B = \{ 3, 5, 6, 11 \}$.

Here $n(A) = 4$ and $n(B) = 4$ $\quad\therefore\ A \approx B$

1.4.6 Equal Sets

Key Concept — **Equal Sets**

Two sets A and B are said to be equal if they contain exactly the same elements, regardless of order. Otherwise the sets are said to be unequal.

In other words, two sets A and B are said to be equal if

(i) every element of A is also an element of B and

(ii) every element of B is also an element of A.

Reading Notation

$=$	Equal	When two sets A and B are equal we write $A = B$.
$\neq$	Not equal	When they are unequal, we write $A \neq B$.

For example,

Consider the sets

$A = \{ a, b, c, d \}$ and $B = \{ d, b, a, c \}$

Set A and set B contain exactly the same elements $\therefore A = B$

If two sets A and B are equal, then $n(A) = n(B)$.

But, if $n(A) = n(B)$, then A and B need not be equal

Thus equal sets are equivalent but equivalent sets need not be equal

Example 1.7

Let $A = \{2, 4, 6, 8, 10, 12, 14\}$ and

$B = \{x : x \text{ is a multiple of } 2,\ x \in \mathbb{N} \text{ and } x \leq 14\}$

State whether $A = B$ or not.

Solution $A = \{2, 4, 6, 8, 10, 12, 14\}$ and

$B = \{x : x \text{ is a multiple of } 2,\ x \in \mathbb{N} \text{ and } x \leq 14\}$

In roster form, $B = \{2, 4, 6, 8, 10, 12, 14\}$

Since A and B have exactly the same elements, $A = B$

1.4.7 Subset

Key Concept **Subset**

A set X is a subset of set Y if every element of X is also an element of Y. In symbol we write $X \subseteq Y$

Reading Notation

$\subseteq$	is a subset of (or) is contained in

Read $X \subseteq Y$ as 'X is a subset of Y' or 'X is contained in Y'

$\nsubseteq$	is not a subset of (or) is not contained in

Read $X \nsubseteq Y$ as 'X is not a subset of Y' or 'X is not contained in Y'

For example,

Consider the sets

$X = \{7, 8, 9\}$ and $Y = \{ 7, 8, 9, 10 \}$

We see that every element of X is also an element of Y.

$\therefore$ X is a subset of Y.

i.e. $X \subseteq Y$.

Note

(i) Every set is a subset of itself i.e. $X \subseteq X$ for any set X

(ii) The empty set is a subset of any set i.e., $\varnothing \subseteq X$, for any set X

(iii) If $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$.

The converse is also true i.e. if $X = Y$ then $X \subseteq Y$ and $Y \subseteq X$

(iv) Every set (except $\varnothing$) has atleast two subsets, $\varnothing$ and the set itself.

1.4.8 Proper Subset

Key Concept — **Proper Subset**

A set X is said to be a proper subset of set Y if $X \subseteq Y$ and $X \neq Y$. In symbol we write $X \subset Y$. Y is called super set of X.

Reading Notation

$\subset$	is a proper subset of

Read $X \subset Y$ as, X is a proper subset of Y

For example,

Consider the sets $X = \{5, 7, 8\}$ and $Y = \{5, 6, 7, 8\}$

Every element of X is also an element of Y and $X \neq Y$

$\therefore$ X is a proper subset of Y

Remark

(i) Proper subsets have atleast one element less than its superset.

(ii) No set is a proper subset of itself.

(iii) The empty set $\varnothing$ is a proper subset of every set except itself ($\varnothing$ has no proper subset). i.e., $\varnothing \subset X$ if X is a set other than $\varnothing$.

(iv) It is important to distinguish between $\in$ and $\subseteq$. The notation $x \in X$ denotes x is an element of X. The notation $X \subseteq Y$ means X is a subset of Y.

Thus $\varnothing \subseteq \{a, b, c\}$ is true, but $\varnothing \in \{a, b, c\}$ is not true.

It is true that $x \in \{x\}$, but the relations $x = \{x\}$ and $x \subseteq \{x\}$ are not correct.

Example 1.8

Write $\subseteq$ or $\nsubseteq$ in each blank to make a true statement.

(a) $\{4, 5, 6, 7\}$ ----- $\{4, 5, 6, 7, 8\}$ (b) $\{a, b, c\}$ ----- $\{b, e, f, g\}$

Solution (a) $\{4, 5, 6, 7\}$ ----- $\{4, 5, 6, 7, 8\}$

Since every element of $\{4, 5, 6, 7\}$ is also an element of $\{4, 5, 6, 7, 8\}$, place $\subseteq$ in the blank.

$\therefore \{4, 5, 6, 7\} \subseteq \{4, 5, 6, 7, 8\}$

(b) The element a belongs to $\{a, b, c\}$ but not to $\{b, e, f, g\}$

So, place $\nsubseteq$ in the blank

$\therefore \{a, b, c\} \nsubseteq \{b, e, f, g\}$

Example 1.9

Decide which of these symbols $\subset, \subseteq$ both can be placed in each of the following blank.

(i) $\{8, 11, 13\}$ ----- $\{8, 11, 13, 14\}$

(ii) $\{a, b, c\}$ ------ $\{a, c, b\}$

Solution (i) Every element of the set $\{8, 11, 13\}$ is also an element in the set $\{8, 11, 13, 14\}$

So, place $\subseteq$ in the blank

$\therefore \{8, 11, 13\} \subseteq \{8, 11, 13, 14\}$

Also, the element 14 belongs to $\{8, 11, 13, 14\}$ but does not belong to $\{8, 11, 13\}$

$\therefore$ $\{8, 11, 13\}$ is proper subset of $\{8, 11, 13, 14\}$.

So, we can also place $\subset$ in the blank. $\therefore \{8, 11, 13\} \subset \{8, 11, 13, 14\}$

(ii) Every element of $\{a, b, c\}$ is also an element of $\{a, c, b\}$

and hence they are equal. So, $\{a, b, c\}$ is not a proper subset of $\{a, c, b\}$

Hence we can only place $\subseteq$ in the blank.

1.4.9 Power Set

Key Concept	**Power Set**
The set of all subsets of A is said to be the power set of the set A.	
Reading Notation	
$P(A)$	Power set of A
The power set of a set A is denoted by $P(A)$	

For example,

Let $A = \{-3, 4\}$

The subsets of A are $\varnothing, \{-3\}, \{4\}, \{-3, 4\}$.

Then the power set of A is $P(A) = \{\varnothing, \{-3\}, \{4\}, \{-3, 4\}\}$

Example 1.10

Write down the power set of $A = \{3, \{4, 5\}\}$

Solution $A = \{3, \{4, 5\}\}$

The subsets of A are

$\varnothing, \{3\}, \{\{4, 5\}\}, \{3, \{4, 5\}\}$

$\therefore P(A) = \{\varnothing, \{3\}, \{\{4, 5\}\}, \{3, \{4, 5\}\}\}$

Number of Subsets of a Finite Set

For a set containing a very large number of elements, it is difficult to find the number of subsets of the set. Let us find a rule to tell how many subsets are there for a given finite set.

(i) The set $A = \varnothing$ has only itself as a subset

(ii) The set $A = \{5\}$ has subsets $\varnothing$ and $\{5\}$

(iii) The set $A = \{5, 6\}$ has subsets $\varnothing, \{5\}, \{6\}, \{5, 6\}$

(iv) The set $A = \{5, 6, 7\}$ has subsets $\varnothing, \{5\}, \{6\}, \{7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}$ and $\{5, 6, 7\}$

This information is shown in the following table

Number of Elements	0	1	2	3
Number of subsets	$1 = 2^0$	$2 = 2^1$	$4 = 2^2$	$8 = 2^3$

This table suggests that as the number of elements of the set increases by one, the number of subsets doubles. i.e. the number of subsets in each case is a power of 2.

Thus we have the following generalization

The number of subsets of a set with *m* elements is 2^m

$$\boxed{n(A) = m \Rightarrow n[P(A)] = 2^{n(A)} = 2^m}$$

The 2^m subsets includes the given set itself.

$\therefore$ **The number of proper subsets of a set with *m* elements is** $2^m - 1$

Example 1.11

Find the number of subsets and proper subsets of each set

(i) $A = \{3, 4, 5, 6, 7\}$ (ii) $A = \{1, 2, 3, 4, 5, 9, 12, 14\}$

Solution (i) $A = \{3, 4, 5, 6, 7\}$. So, $n(A) = 5$. Hence,

The number of subsets $= n[P(A)] = 2^5 = 32$.

The number of proper subsets $= 2^5 - 1 = 32 - 1 = 31$

(ii) $A = \{1, 2, 3, 4, 5, 9, 12, 14\}$. Now, $n(A) = 8$.

$\therefore$ The number of subsets $= 2^8 = 2^5 \times 2^3 = 32 \times 2 \times 2 \times 2 = 256$

The number of proper subsets $= 2^8 - 1 = 256 - 1 = 255$

Exercise 1.1

1. Which of the following are sets? Justify your answer.

(i) The collection of good books

(ii) The collection of prime numbers less than 30

(iii) The collection of ten most talented mathematics teachers.

(iv) The collection of all students in your school

(v) The collection of all even numbers

2. Let $A = \{0, 1, 2, 3, 4, 5\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces

(i) 0 ----- A (ii) 6 ----- A (iii) 3 ----- A

(iv) 4 ----- A (v) 7 ----- A

3. Write the following sets in Set-Builder form

(i) The set of all positive even numbers

(ii) The set of all whole numbers less than 20

(iii) The set of all positive integers which are multiples of 3

(iv) The set of all odd natural numbers less than 15

(v) The set of all letters in the word 'computer'

4. Write the following sets in Roster form

(i) $A = \{x : x \in \mathbb{N}, \ 2 < x \le 10\}$

(ii) $B = \left\{x : x \in \mathbb{Z}, \ -\frac{1}{2} < x < \frac{11}{2}\right\}$

(iii) C = $\{x : x$ is a prime number and a divisor of $6\}$

(iv) $X = \{x : x = 2^n, n \in \mathbb{N}$ and $n \leq 5\}$

(v) $M = \{x : x = 2y - 1, y \leq 5, y \in \mathbb{W}\}$

(vi) $P = \{x : x$ is an integer, $x^2 \leq 16\}$

5. Write the following sets in Descriptive form

(i) $A = \{a, e, i, o, u\}$

(ii) $B = \{1, 3, 5, 7, 9, 11\}$

(iii) $C = \{1, 4, 9, 16, 25\}$

(iv) $P = \{x : x$ is a letter in the word 'set theory'$\}$

(v) $Q = \{x : x$ is a prime number between 10 and 20$\}$

6. Find the cardinal number of the following sets

(i) $A = \{x : x = 5^n, n \in \mathbb{N}$ and $n < 5\}$

(ii) $B = \{x : x$ is a consonant in English Alphabet$\}$

(iii) $X = \{x : x$ is an even prime number$\}$

(iv) $P = \{x : x < 0, x \in \mathbb{W}\}$

(v) $Q = \{x : -3 \leq x \leq 5, x \in \mathbb{Z}\}$

7. Identify the following sets as finite or infinite

(i) $A = \{4, 5, 6, ...\}$

(ii) $B = \{0, 1, 2, 3, 4, ... 75\}$

(iii) $X = \{x : x$ is an even natural number$\}$

(iv) $Y = \{x : x$ is a multiple of 6 and $x > 0\}$

(v) $P =$ The set of letters in the word 'freedom'

8. Which of the following sets are equivalent?

(i) $A = \{2, 4, 6, 8, 10\}$, B $= \{1, 3, 5, 7, 9\}$

(ii) $X = \{x : x \in \mathbb{N}, 1 < x < 6\}$, $Y = \{x : x$ is a vowel in the English Alphabet$\}$

(iii) $P = \{x : x$ is a prime number and $5 < x < 23\}$

$Q = \{x : x \in \mathbb{W}, 0 \leq x < 5\}$

9. Which of the following sets are equal?

(i) $A = \{1, 2, 3, 4\}$, $B = \{4, 3, 2, 1\}$

(ii) $A = \{4, 8, 12, 16\}$, $B = \{8, 4, 16, 18\}$

(iii) $X = \{2, 4, 6, 8\}$

$Y = \{x : x$ is a positive even integer $0 < x < 10\}$

(iv) $P = \{x : x$ is a multiple of 10, $x \in \mathbb{N}\}$

$Q = \{10, 15, 20, 25\ 30, ... \}$

10. From the sets given below, select equal sets.

$A = \{12, 14, 18, 22\}$, $B = \{11, 12, 13, 14\}$, $C = \{14, 18, 22, 24\}$

$D = \{13, 11, 12, 14\}$, $E = \{-11, 11\}$, $F = \{10, 19\}$, $G = \{11, -11\}$, $H = \{10, 11\}$

11. Is $\varnothing = \{\varnothing\}$? Why ?

12. Which of the sets are equal sets? State the reason.

$\varnothing, \{0\}, \{\varnothing\}$

13. Fill in the blanks with $\subseteq$ or $\nsubseteq$ to make each statement true.

(i) $\{3\}$ ----- $\{0, 2, 4, 6\}$ (ii) $\{a\}$ ----- $\{a, b, c\}$

(iii) $\{8, 18\}$ ----- $\{18, 8\}$ (iv) $\{d\}$ ----- $\{a, b, c\}$

14. Let $X = \{-3, -2, -1, 0, 1, 2\}$ and $Y = \{x : x$ is an integer and $-3 \le x < 2\}$

(i) Is X a subset of Y? (ii) Is Y a subset of X?

15. Examine whether $A = \{x : x$ is a positive integer divisible by $3\}$ is a subset of

B $= \{x : x$ is a multiple of 5, $x \in \mathbb{N}\}$

16. Write down the power sets of the following sets.

(i) A $= \{x, y\}$ (ii) X $= \{a, b, c\}$ (iii) A $= \{5, 6, 7, 8\}$ (iv) $A = \varnothing$

17. Find the number of subsets and the number of proper subsets of the following sets.

(i) $A = \{13, 14, 15, 16, 17, 18\}$

(ii) $B = \{a, b, c, d, e, f, g\}$

(iii) $X = \{x : x \in \mathbb{W}, x \notin \mathbb{N}\}$

18. (i) If $A = \varnothing$, find $n[P(A)]$ (ii) If $n(A) = 3$, find $n[P(A)]$

(iii) If $n[P(A)] = 512$ find n(A)

(iv) If $n[P(A)] = 1024$ find n(A)

19. If $n[P(A)] = 1$, what can you say about the set A?

20. Let $A = \{x : x \text{ is a natural number} < 11\}$

$B = \{x : x \text{ is an even number and } 1 < x < 21\}$

$C = \{x : x \text{ is an integer and } 15 \leq x \leq 25\}$

(i) List the elements of A, B, C

(ii) Find $n(A)$, $n(B)$, $n(C)$.

(iii) State whether the following are True (T) or False (F)

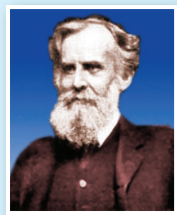
(a) $7 \in B$ ☐ (b) $16 \notin A$ ☐

(c) $\{15, 20, 25\} \subset C$ ☐ (d) $\{10, 12\} \subset B$ ☐

1.5 SET OPERATIONS

1.5.1 Venn Diagrams

We use diagrams or pictures in geometry to explain a concept or a situation and sometimes we also use them to solve problems. In mathematics, we use diagrammatic representations called Venn Diagrams to visualise the relationships between sets and set operations.



John Venn
(1834-1883)

John Venn (1834-1883) a British mathematician used diagrammatic representation as an aid to visualize various relationships between sets and set operations.

1.5.2 The Universal Set

Sometimes it is useful to consider a set which contains all elements pertinent to a given discussion.

Key Concept **Universal Set**

The set that contains all the elements under consideration in a given discussion is called the universal set. The universal set is denoted by U or ξ.

For example,

If the elements currently under discussion are integers, then the universal set U is the set of all integers. i.e., $U = \{n : n \in \mathbb{Z}\}$

Remark The universal set may change from problem to problem.

In Venn diagrams, the universal set is generally represented by a rectangle and its proper subsets by circles or ovals inside the rectangle. We write the names of its elements inside the figure.

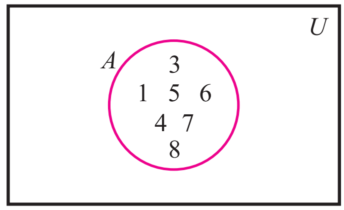


Fig. 1.1

1.5.3 Complement of a Set

Key Concept — **Complement Set**

The set of all elements of U (universal set) that are not elements of $A \subseteq U$ is called the complement of A. The complement of A is denoted by A' or A^c.

Reading Notation

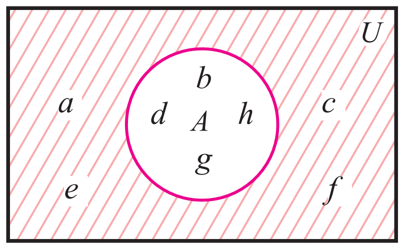
In symbol, $A' = \{x : x \in U \text{ and } x \notin A\}$

For example,

Let $U = \{a, b, c, d, e, f, g, h\}$ and $A = \{b, d, g, h\}$.

Then $A' = \{a, c, e, f\}$

In Venn diagram A', the complement of set A is represented as shown in Fig. 1.2



A' (shaded portion)
Fig. 1.2

Note (i) $(A')' = A$ (ii) $\varnothing' = U$ (iii) $U' = \varnothing$

1.5.4 Union of Two Sets

Key Concept — **Union of Sets**

The union of two sets A and B is the set of elements which are in A or in B or in both A and B. We write the union of sets A and B as $A \cup B$.

Reading Notation

$\cup$	Union

Read $A \cup B$ as 'A union B'

In symbol, $A \cup B = \{x : x \in A \text{ or } x \in B\}$

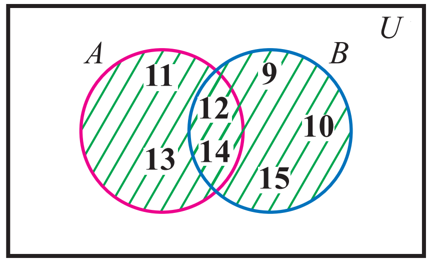
For example,

Let $A = \{11, 12, 13, 14\}$ and

$B = \{9, 10, 12, 14, 15\}$.

Then $A \cup B = \{9, 10, 11, 12, 13, 14, 15\}$

The union of two sets can be represented by a Venn diagram as shown in Fig. 1.3



$A \cup B$ (shaded portion)
Fig. 1.3

Note

(i) $A \cup A = A$ (ii) $A \cup \varnothing = A$ (iii) $A \cup A' = U$

(iv) If A is any subset of U then $A \cup U = U$

(v) $A \subseteq B$ if and only if $A \cup B = B$ (vi) $A \cup B = B \cup A$

Example 1.12

Find the union of the following sets.

(i) $A = \{1,2,3,5,6\}$ and $B = \{4,5,6,7,8\}$

(ii) $X = \{3,4,5\}$ and $Y = \varnothing$

Think and Answer !
Can we say
$A \subset (A \cup B)$ and
$B \subset (A \cup B)$?

Solution (i) $A = \{1, 2, 3, 5, 6\}$ and $B = \{4, 5, 6, 7, 8\}$

1, 2, 3, 5, 6 ; 4, 5, 6, 7, 8 (repeated)

$\therefore A \cup B = \{1,2,3,4,5,6,7,8\}$

(ii) $X = \{3,4,5\}$, $Y = \varnothing$. There are no elements in Y

$\therefore X \cup Y = \{3,4,5\}$

1.5.5 Intersection of Two Sets

Key Concept **Intersection of Sets**

The intersection of two sets A and B is the set of all elements common to both A and B. We denote it as $A \cap B$.

Reading Notation

$\cap$	Intersection

Read $A \cap B$ as 'A intersection B'

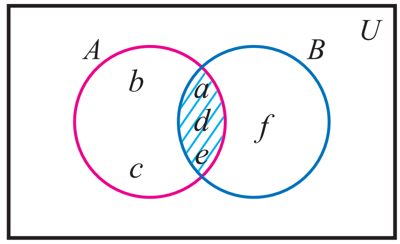
Symbolically, we write $A \cap B = \{x : x \in A \text{ and } x \in B\}$

For example,

Let $A = \{a, b, c, d, e\}$ and $B = \{a, d, e, f\}$.

$\therefore A \cap B = \{a, d, e\}$

The intersection of two sets can be represented by a Venn diagram as shown in Fig. 1.4



$A \cap B$ (shaded portion)
Fig. 1.4

Note

(i) $A \cap A = A$ (ii) $A \cap \varnothing = \varnothing$

(iii) $A \cap A' = \varnothing$ (iv) $A \cap B = B \cap A$

(v) If A is any subset of U, then $A \cap U = A$

(vi) If $A \subseteq B$ if and only if $A \cap B = A$

Think and Answer !
Can we say $(A \cap B) \subset A$ and $(A \cap B) \subset B$?

Example 1.13

Find $A \cap B$ if (i) $A = \{10, 11, 12, 13\}$, $B = \{12, 13, 14, 15\}$

(ii) $A = \{5, 9, 11\}$, $B = \varnothing$

Solution (i) $A = \{10, 11, 12, 13\}$ and $B = \{12, 13, 14, 15\}$.

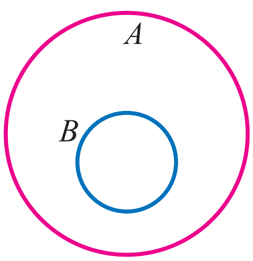
12 and 13 are common in both A and B.

$\therefore A \cap B = \{12, 13\}$

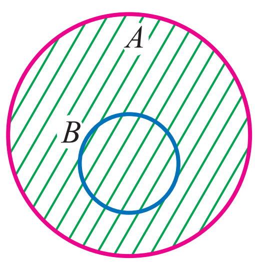
(ii) $A = \{5, 9, 11\}$ and $B = \varnothing$.

There is no element in common and hence $A \cap B = \varnothing$

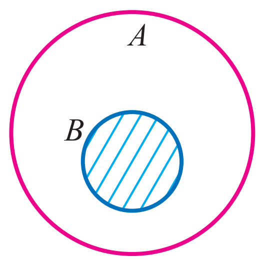
Remark When $B \subseteq A$, the union and intersection of two sets A and B are represented in Venn diagram as shown in Fig.1.6 and in Fig.1.7 respectively



$B \subseteq A$
Fig.1.5



$A \cup B$ (shaded portion)
Fig.1.6



$A \cap B$ (shaded portion)
Fig.1.7

1.5.6 Disjoint Sets

Key Concept	**Disjoint Sets**

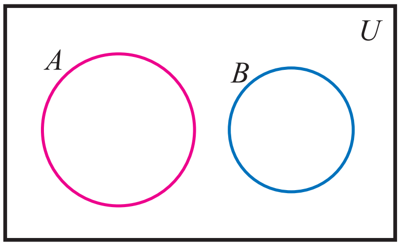
Two sets A and B are said to be disjoint if there is no element common to both A and B.

In other words, if A and B are disjoint sets, then $A \cap B = \varnothing$

For example,

Consider the sets $A = \{5, 6, 7, 8\}$ and $B = \{11, 12, 13\}$.

We have $A \cap B = \varnothing$. So A and B are disjoint sets.



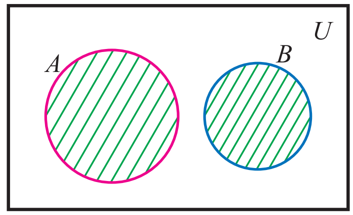
Disjoint sets
Fig.1.8

Two disjoint sets A and B are represented in Venn diagram as shown in Fig.1.8

Note

(i) The union of two disjoint sets A and B are represented in Venn diagram as shown in Fig.1.9

(ii) If $A \cap B \neq \varnothing$, then the two sets A and B are said to be overlapping sets



$A \cup B$ (shaded portion)
Fig.1.9

Example 1.14

Given the sets $A = \{4, 5, 6, 7\}$ and $B = \{1, 3, 8, 9\}$. Find $A \cap B$.

Solution $A = \{4, 5, 6, 7\}$ and $B = \{1, 3, 8, 9\}$. So $A \cap B = \varnothing$.

Hence A and B are disjoint sets.

1.5.7 Difference of Two Sets

Key Concept — **Difference of two Sets**

The difference of the two sets A and B is the set of all elements belonging to A but not to B. The difference of the two sets is denoted by $A - B$ or $A \backslash B$.

Reading Notation

$A - B$ or $A \backslash B$	A difference B

In symbol, we write : $A - B = \{x : x \in A \text{ and } x \notin B\}$

Similarly, we write : $B - A = \{x : x \in B \text{ and } x \notin A\}$

For example,

Consider the sets $A = \{2, 3, 5, 7, 11\}$ and $B = \{5, 7, 9, 11, 13\}$

To find $A - B$, we remove the elements of B from A.

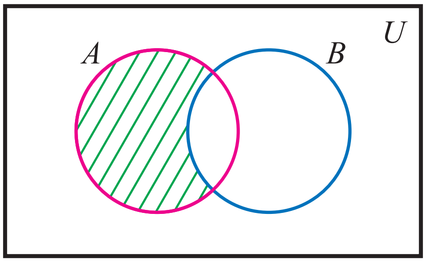
$\therefore \; A - B = \{2, 3\}$

Note

(i) Generally, $A - B \neq B - A$. (ii) $A - B = B - A \Leftrightarrow A = B$

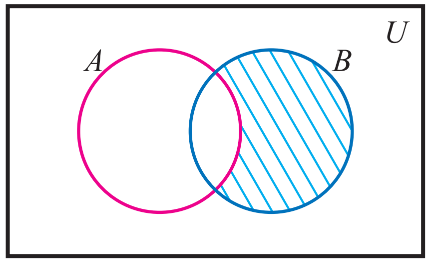
(iii) $U - A = A'$ (iv) $U - A' = A$ (v) $A - \varnothing = A$

The difference of two sets A and B can be represented by Venn diagram as shown in Fig.1.10 and in Fig.1.11. The shaded portion represents the difference of the two sets



$A - B$

Fig.1.10



$B - A$

Fig.1.11

Example 1.15

If $A = \{-2, -1, 0, 3, 4\}$, $B = \{-1, 3, 5\}$, find (i) $A - B$ (ii) $B - A$

Solution $A = \{-2, -1, 0, 3, 4\}$ and $B = \{-1, 3, 5\}$.

(i) $A - B = \{-2, 0, 4\}$ (ii) $B - A = \{5\}$

1.5.8 Symmetric Difference of Sets

Key Concept **Symmetric Difference of Sets**

The symmetric difference of two sets A and B is the union of their differences and is denoted by $A \,\Delta\, B$.

Reading Notation

$A \,\Delta\, B$	A symmetric B

Thus, $A \,\Delta\, B = (A - B) \cup (B - A)$

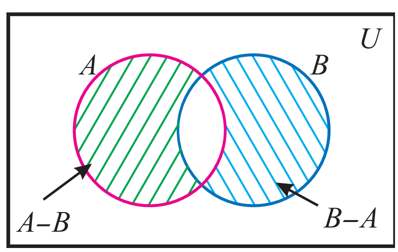
For example,

Consider the sets $A = \{a, b, c, d\}$ and $B = \{b, d, e, f\}$. We have

$$A - B = \{a, c\} \text{ and } B - A = \{e, f\}$$

$$\therefore\ A \,\Delta\, B = (A - B) \cup (B - A) = \{a, c, e, f\}$$

The symmetric difference of two sets A and B can be represented by Venn diagram as shown in Fig.1.12. The shaded portion represents the symmetric difference of the two sets A and B.



$A \,\Delta\, B = (A - B) \cup (B - A)$

Fig.1.12

Note (i) $A \triangle A = \varnothing$ (ii) $A \triangle B = B \triangle A$

(iii) From the Venn diagram 1.12, we can write

$A \triangle B = \{x : x \in A, x \in B \text{ and } x \notin A \cap B\}$

So, we can find the elements of $A \triangle B$, by listing the elements which are not common to both A and B.

Example 1.16

If $A = \{2, 3, 5, 7, 11\}$ and $B = \{5, 7, 9, 11, 13\}$, find $A \triangle B$.

Solution Given $A = \{2, 3, 5, 7, 11\}$ and $B = \{5, 7, 9, 11, 13\}$. So

$A - B = \{2, 3\}$ and $B - A = \{9, 13\}$. Hence

$A \triangle B = (A - B) \cup (B - A) = \{2, 3, 9, 13\}$

Exercise 1.2

1. Find $A \cup B$ and $A \cap B$ for the following sets.

   (i) $A = \{0, 1, 2, 4, 6\}$ and $B = \{-3, -1, 0, 2, 4, 5\}$

   (ii) $A = \{2, 4, 6, 8\}$ and $B = \varnothing$

   (iii) $A = \{x : x \in \mathbb{N}, x \leq 5\}$ and $B = \{x : x \text{ is a prime number less than } 11\}$

   (iv) $A = \{x : x \in \mathbb{N}, 2 < x \leq 7\}$ and $B = \{x : x \in \mathbb{W}, 0 \leq x \leq 6\}$

2. If $A = \{x : x \text{ is a multiple of } 5,\ x \leq 30 \text{ and } x \in \mathbb{N}\}$

   $B = \{1, 3, 7, 10, 12, 15, 18, 25\}$,

   Find (i) $A \cup B$ (ii) $A \cap B$

3. If $X = \{x : x = 2n, x \leq 20 \text{ and } n \in \mathbb{N}\}$ and $Y = \{x : x = 4n, x \leq 20 \textit{ and } n \in \mathbb{W}\}$

   Find (i) $X \cup Y$ (ii) $X \cap Y$

4. $U = \{1, 2, 3, 6, 7, 12, 17, 21, 35, 52, 56\}$,

   $P = \{\text{numbers divisible by } 7\}$, $Q = \{\text{prime numbers}\}$

   List the elements of the set $\{x : x \in P \cap Q\}$

5. State which of the following sets are disjoint

   (i) $A = \{2, 4, 6, 8\}$; $B = \{x : x \text{ is an even number} < 10,\ x \in \mathbb{N}\}$

   (ii) $X = \{1, 3, 5, 7, 9\}$, $Y = \{0, 2, 4, 6, 8, 10\}$

   (iii) $P = \{x : x \text{ is a prime} < 15\}$

   $Q = \{x : x \text{ is a multiple of 2 and } x < 16\}$

   (iv) $R = \{a, b, c, d, e\}$, $S = \{d, e, a, b, c\}$

6. (i) If $U = \{x : 0 \le x \le 10, x \in \mathbb{W}\}$ and $A = \{x : x$ is a multiple of $3\}$, find A'

   (ii) If U is the set of natural numbers and A' is the set of all composite numbers, then what is A ?

7. If $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d\}$ and $B = \{b, d, f, g\}$,

   find (i) $A \cup B$ (ii) $(A \cup B)'$ (iii) $A \cap B$ (iv) $(A \cap B)'$

8. If $U = \{x : 1 \le x \le 10, x \in \mathbb{N}\}$, $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 9, 10\}$,

   find (i) A' (ii) B' (iii) $A' \cup B'$ (iv) $A' \cap B'$

9. Given that $U = \{3, 7, 9, 11, 15, 17, 18\}$, $M = \{3, 7, 9, 11\}$ and $N = \{7, 11, 15, 17\}$,

   find (i) $M - N$ (ii) $N - M$ (iii) $N' - M$ (iv) $M' - N$

   (v) $M \cap (M - N)$ (vi) $N \cup (N - M)$ (vii) $n(M - N)$

10. If $A = \{3, 6, 9, 12, 15, 18\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12\}$ and $D = \{5, 10, 15, 20, 25\}$, find

    (i) $A - B$ (ii) $B - C$ (iii) $C - D$ (iv) $D - A$ (v) $n(A - C)$

11. Let $U = \{x : x$ is a positive integer less than $50\}$, $A = \{x : x$ is divisible by $4\}$ and

    $B = \{x : x$ leaves a remainder 2 when divided by $14\}$.

    (i) List the elements of U, A and B

    (ii) Find $A \cup B$, $A \cap B$, $n(A \cup B)$, $n(A \cap B)$

12. Find the symmetric difference between the following sets.

    (i) $X = \{a, d, f, g, h\}$, $Y = \{b, e, g, h, k\}$

    (ii) $P = \{x : 3 < x < 9, x \in \mathbb{N}\}$, $Q = \{x : x < 5, x \in \mathbb{W}\}$

    (iii) $A = \{-3, -2, 0, 2, 3, 5\}$, $B = \{-4, -3, -1, 0, 2, 3\}$

13. Use the Venn diagram to answer the following questions

    (i) List the elements of $U, E, F, E \cup F$ and $E \cap F$

    (ii) Find $n(U)$, $n(E \cup F)$ and $n(E \cap F)$

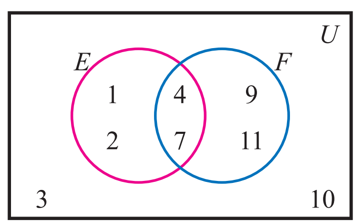


Fig. 1.13

14. Use the Venn diagram to answer the following questions

    (i) List U, G and H

    (ii) Find G', H', $G' \cap H'$, $n(G \cup H)'$ and $n(G \cap H)'$

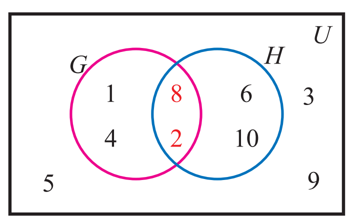


Fig. 1.14

1.6 Representation of Set Operations Using Venn Diagram

We shall now give a few more representations of set operations in Venn diagrams.

(a) $A \cup B$

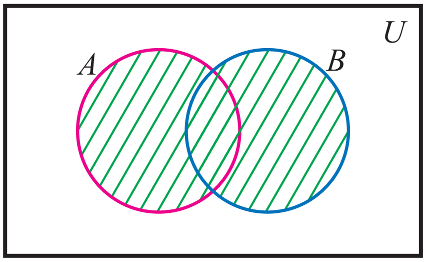


Fig. 1.15

(b) $(A \cup B)'$

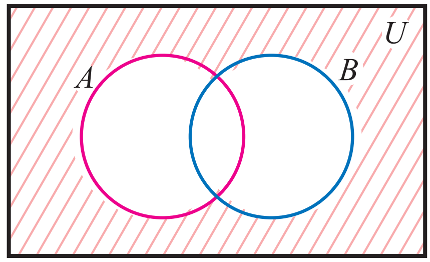
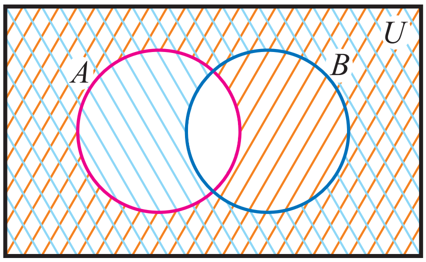


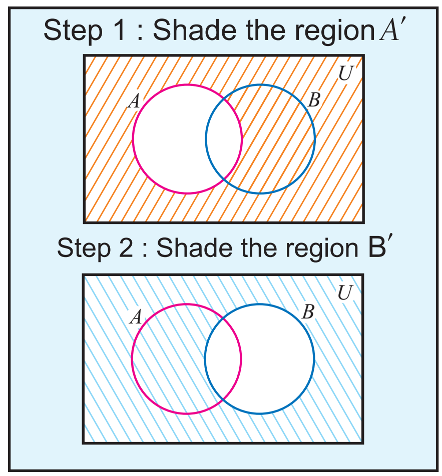
Fig. 1.16

(c) $A' \cup B'$

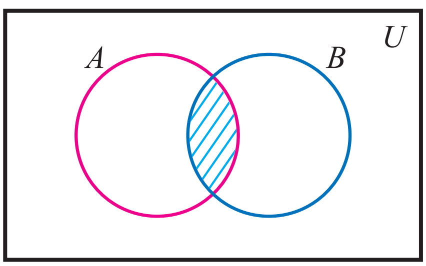


$A' \cup B'$ (shaded portion)

Fig. 1.17

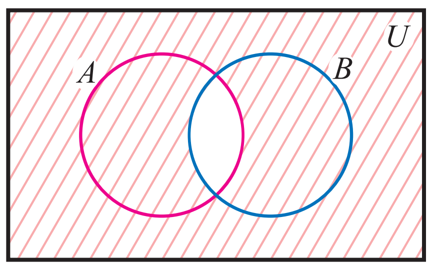


Similarly the shaded regions represent each of the following set operations.



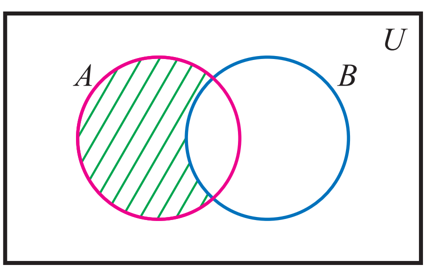
$A \cap B$ (shaded portion)

Fig. 1.18



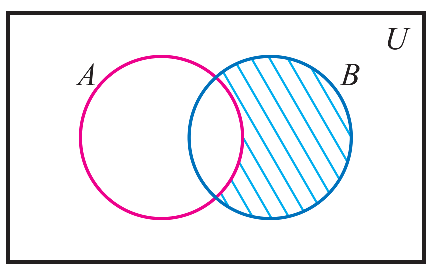
$(A \cap B)'$ (shaded portion)

Fig. 1.19



$A \cap B'$ (shaded portion)

Fig 1.20.



$A' \cap B$ (shaded portion)

Fig. 1.21

Remark We can also make use of the following idea to represent sets and set operations in Venn diagram.

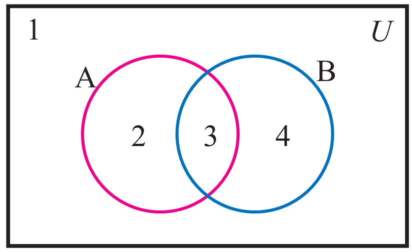


Fig. 1.22

In Fig. 1.22 the sets A and B divide the universal set into four regions. These four regions are numbered for reference. This numbering is arbitrary.

Region 1	Contains the elements outside of both the sets A and B
Region 2	Contains the elements of the set A but not in B
Region 3	Contains the elements common to both the sets A and B
Region 4	Contains the elements of the set B but not in A

Example 1.17

Draw a Venn diagram similar to one at the side and shade the regions representing the following sets

(i) A' (ii) B' (iii) $A' \cup B'$ (iv) $(A \cup B)'$ (v) $A' \cap B'$

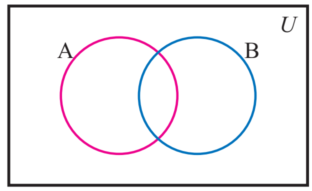
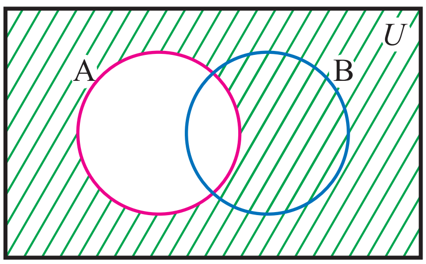


Fig. 1.23

Solution

(i) A'



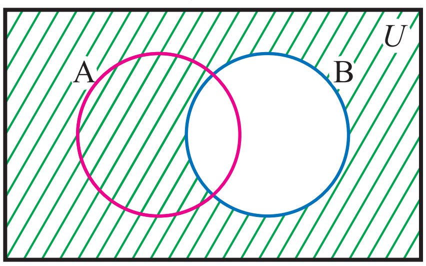
A' (shaded portion)

Fig. 1.24

Tip to shade

Set	Shaded Region
A'	1 and 4

(ii) B'



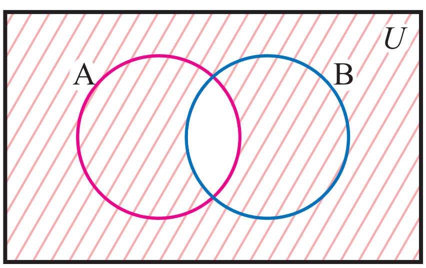
B' (shaded portion)

Fig. 1.25

Tip to shade

Set	Shaded Region
B'	1 and 2

(iii) $A' \cup B'$



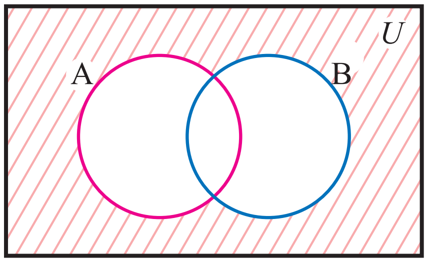
$A' \cup B'$ (shaded portion)

Fig. 1.26

Tip to shade

Set	Shaded Region
A'	1 and 4
B'	1 and 2
$A' \cup B'$	1, 2 and 4

(iv) $(A \cup B)'$

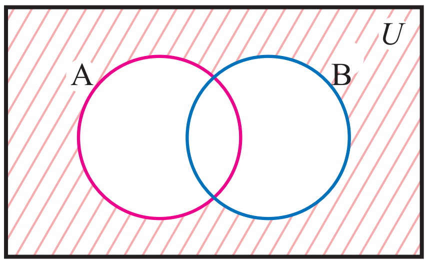


$(A \cup B)'$ (shaded portion)
Fig. 1.27

Tip to shade

Set	Shaded Region
$A \cup B$	2, 3 and 4
$(A \cup B)'$	1

(v) $A' \cap B'$



$A' \cap B'$ (shaded portion)
Fig. 1.28

Tip to shade

Set	Shaded Region
A'	1 and 4
B'	1 and 2
$A' \cap B'$	1

Important Results

For any two finite sets A and B, we have the following useful results

(i) $n(A) = n(A - B) + n(A \cap B)$

(ii) $n(B) = n(B - A) + n(A \cap B)$

(iii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

(iv) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(v) $n(A \cup B) = n(A) + n(B)$, when $A \cap B = \varnothing$

(vi) $n(A) + n(A') = n(U)$

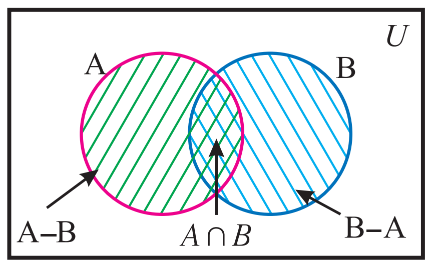


Fig. 1.29

Example 1.18

From the given Venn diagram, find the following

(i) A (ii) B (iii) $A \cup B$ (iv) $A \cap B$

Also verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

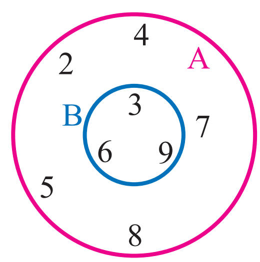


Fig. 1.30

Solution From the Venn diagram

(i) $A = \{2, 3, 4, 5, 6, 7, 8, 9\}$, (ii) $B = \{3, 6, 9,\}$,

(iii) $A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$ and (iv) $A \cap B = \{3, 6, 9\}$

We have $n(A) = 8,\ n(B) = 3,\ n(A \cup B) = 8,\ n(A \cap B) = 3$. Now

$$n(A) + n(B) - n(A \cap B) = 8 + 3 - 3 = 8$$

Hence, $\quad n(A) + n(B) - n(A \cap B) = n(A \cup B)$

Example 1.19

From the given Venn diagram find

(i) A (ii) B (iii) $A \cup B$ (iv) $A \cap B$

Also verify that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

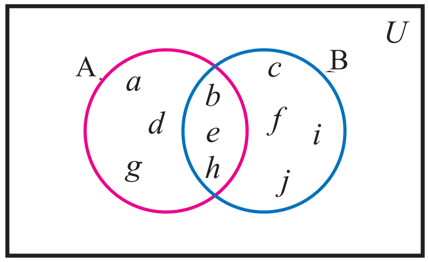


Fig. 1.31

Solution From the Venn diagram

(i) $A = \{a, b, d, e, g, h\}$, (ii) $B = \{b, c, e, f, h, i, j\}$,

(iii) $A \cup B = \{a, b, c, d, e, f, g, h, i, j\}$ and (iv) $A \cap B = \{b, e, h\}$

So, $n(A) = 6,\ n(B) = 7,\ n(A \cup B) = 10,\ n(A \cap B) = 3$. Now

$n(A) + n(B) - n(A \cap B) = 6 + 7 - 3 = 10$

Hence, $n(A) + n(B) - n(A \cap B) = n(A \cup B)$

Example 1.20

If $n(A) = 12, n(B) = 17$ and $n(A \cup B) = 21$, find $n(A \cap B)$

Solution Given that $n(A) = 12, n(B) = 17$ and $n(A \cup B) = 21$

By using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$$n(A \cap B) = 12 + 17 - 21 = 8$$

Example 1.21

In a city 65% of the people view Tamil movies and 40% view English movies, 20% of the people view both Tamil and English movies. Find the percentage of people do not view any of these two movies.

Solution Let the population of the city be 100. Let T denote the set of people who view Tamil movies and E denote the set of people who view English movies. Then $n(T) = 65, n(E) = 40, n(T \cap E) = 20$. So, the number of people who view either of these movies is

$$\begin{aligned} n(T \cup E) &= n(T) + n(E) - n(T \cap E) \\ &= 65 + 40 - 20 = 85 \end{aligned}$$

Hence the number of people who do not view any of these movies is $100 - 85 = 15$

Hence the percentage of people who do not view any of these movies is 15

Aliter

From the Venn diagram the percentage of people who view at least one of these two movies is

$$45 + 20 + 20 = 85$$

Hence, the percentage of people who do not view any of these movies $= 100 - 85 = 15$

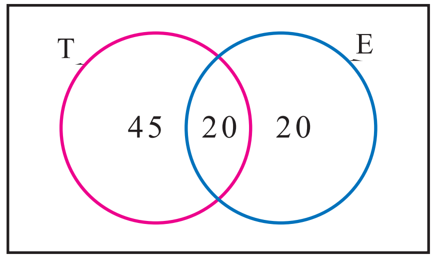


Fig. 1.32

Example 1.22

In a survey of 1000 families, it is found that 484 families use electric stoves, 552 families use gas stoves. If all the families use atleast one of these two types of stoves, find how many families use both the stoves?

Solution Let E denote the set of families using electric stove and G denote the set of families using gas stove. Then $n(E) = 484, n(G) = 552,\ n(E \cup G) = 1000$. Let x be the number of families using both the stoves . Then $n(E \cap G) = x$.

Using the results

$$n(E \cup G) = n(E) + n(G) - n(E \cap G)$$

$$1000 = 484 + 552 - x$$

$$\Rightarrow \quad x = 1036 - 1000 = 36$$

Hence, 36 families use both the stoves.

Aliter

From the Venn diagram,

$$484 - x + x + 552 - x = 1000$$

$$\Rightarrow \quad 1036 - x = 1000$$

$$\Rightarrow \quad -x = -36$$

$$x = 36$$

Hence, 36 families use both the stoves.

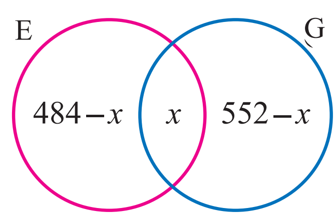


Fig. 1.33

Example 1.23

In a class of 50 students, each of the students passed either in mathematics or in science or in both. 10 students passed in both and 28 passed in science. Find how many students passed in mathematics?

Solution Let M = The set of students passed in Mathematics

S = The set of students passed in Science

Then, $n(S) = 28,\ \ n(M \cap S) = 10,\ \ n(M \cup S) = 50$

We have $n(M \cup S) \quad = n(M) + n(S) - n(M \cap S)$

$50 \quad = n(M) + 28 - 10$

$\Rightarrow \quad n(M) \ = 32$

Aliter

From the Venn diagram

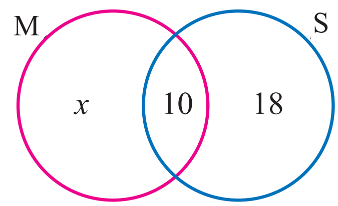


Fig. 1.34

$x + 10 + 18 = 50$

$x \ = 50 - 28 = 22$

Number of students passed in Mathematics $= x + 10 = 22 + 10 = 32$

Exercise 1.3

1. Place the elements of the following sets in the proper location on the given Venn diagram.

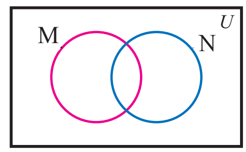
   

   Fig. 1.35

   $U = \{5, 6, 7, 8, 9, 10, 11, 12, 13\}$

   $M = \{5, 8, 10, 11\}$, $\ N = \{5, 6, 7, 9, 10\}$

2. If A and B are two sets such that A has 50 elements, B has 65 elements and $A \cup B$ has 100 elements, how many elements does $A \cap B$ have?

3. If A and B are two sets containing 13 and 16 elements respectively, then find the minimum and maximum number of elements in $A \cup B$?

4. If $n(A \cap B) = 5,\ n(A \cup B) = 35, n(A) = 13$, find $n(B)$.

5. If $n(A) = 26,\ n(B) = 10,\ n(A \cup B) = 30, n(A') = 17$, find $n(A \cap B)$ and $n(\cup)$.

6. If $n(\cup) = 38, n(A) = 16, n(A \cap B) = 12, n(B') = 20$, find $n(A \cup B)$.

7. Let A and B be two finite sets such that $n(A - B) = 30, n(A \cup B) = 180$. Find $n(B)$.

8. The population of a town is 10000. Out of these 5400 persons read newspaper A and 4700 read newspaper B. 1500 persons read both the newspapers. Find the number of persons who do not read either of the two papers.

9. In a school, all the students play either Foot ball or Volley ball or both. 300 students play Foot ball, 270 students play Volley ball and 120 students play both games. Find

   (i) the number of students who play Foot ball only

   (ii) the number of students who play Volley ball only

   (iii) the total number of students in the school

10. In an examination 150 students secured first class in English or Mathematics. 115 students secured first class in Mathematics. How many students secured first class in English only?

11. In a group of 30 persons, 18 take tea. Find how many take coffee but not tea, if each person takes atleast one of the drinks.

12. In a village there are 60 families. Out of these 28 families speak only Tamil and 20 families speak only Urdu. How many families speak both Tamil and Urdu.

13. In a School 150 students passed X Standard Examination. 95 students applied for Group I and 82 students applied for Group II in the Higher Secondary course. If 20 students applied neither of the two, how many students applied for both groups?

14. Pradeep is a Section Chief for an electric utility company. The employees in his section cut down tall trees or climb poles. Pradeep recently reported the following information to the management of the utility.

    Out of 100 employees in my section, 55 can cut tall trees, 50 can climb poles, 11 can do both, 6 can't do any of the two. Is this information correct?

15. A and B are two sets such that $n(A-B) = 32 + x, n(B-A) = 5x$ and $n(A \cap B) = x$ Illustrate the information by means of a Venn diagram. Given that $n(A) = n(B)$. Calculate (i) the value of x (ii) $n(A \cup B)$.

16. The following table shows the percentage of the students of a school who participated in Elocution and Drawing competitions.

Competition	Elocution	Drawing	Both
Percentage of Students	55	45	20

Draw a Venn diagram to represent this information and use it to find the percentage of the students who

(i) participated in Elocution only

(ii) participated in Drawing only

(iii) did not participate in any one of the competitions.

17. A village has total population of 2500 people. Out of which 1300 people use brand A soap and 1050 people use brand B soap and 250 people use both brands. Find the percentage of population who use neither of these soaps.

Exercise 1.4

Multiple Choice Questions.

1. If $A = \{5, \{5, 6\}, 7\}$, which of the following is correct?

   (A) $\{5, 6\} \in A$ (B) $\{5\} \in A$ (C) $\{7\} \in A$ (D) $\{6\} \in A$

2. If $X = \{a, \{b, c\}, d\}$, which of the following is a subset of X?

   (A) $\{a, b\}$ (B) $\{b, c\}$ (C) $\{c, d\}$ (D) $\{a, d\}$

3. Which of the following statements are true?

   (i) For any set A, A is a proper subset of A

   (ii) For any set A, $\varnothing$ is a subset of A

   (iii) For any set A, A is a subset of A

   (A) (i) and (ii) (B) (ii) and (iii) (C) (i) and (iii) (D) (i) (ii) and (iii)

4. If a finite set A has m elements, then the number of non-empty proper subsets of A is

   (A) 2^m (B) $2^m - 1$ (C) 2^{m-1} (D) $2(2^{m-1} - 1)$

5. The number of subsets of the set $\{10, 11, 12\}$ is

   (A) 3 (B) 8 (C) 6 (D) 7

6. Which one of the following is correct?

   (A) $\{x : x^2 = -1, x \in \mathbb{Z}\} = \varnothing$ (B) $\varnothing = 0$

   (C) $\varnothing = \{0\}$ (D) $\varnothing = \{\varnothing\}$

7. Which one of the following is incorrect?

   (A) Every subset of a finite set is finite

   (B) $P = \{x : x - 8 = -8\}$ is a singleton set

   (C) Every set has a proper subset

   (D) Every non - empty set has at least two subsets, $\varnothing$ and the set itself

8. Which of the following is a correct statement?

   (A) $\varnothing \subseteq \{a, b\}$ (B) $\varnothing \in \{a, b\}$ (C) $\{a\} \in \{a, b\}$ (D) $a \subseteq \{a, b\}$

9. Which one of the following is a finite set?

   (A) $\{x : x \in \mathbb{Z}, x < 5\}$ (B) $\{x : x \in \mathbb{W}, x \geq 5\}$

   (C) $\{x : x \in \mathbb{N}, x > 10\}$ (D) $\{x : x \text{ is an even prime number}\}$

10. Given $A = \{5,6,7,8\}$. Which one of the following is incorrect?

(A) $\varnothing \subseteq A$ (B) $A \subseteq A$ (C) $\{7,8,9\} \subseteq A$ (D) $\{5\} \subset A$

11. If $A = \{3,4,5,6\}$ and $B = \{1,2,5,6\}$, then $A \cup B =$

(A) $\{1,2,3,4,5,6\}$ (B) $\{1,2,3,4,6\}$ (C) $\{1,2,5,6\}$ (D) $\{3,4,5,6\}$

12. The number of elements of the set $\{x : x \in \mathbb{Z}, x^2 = 1\}$ is

(A) 3 (B) 2 (C) 1 (D) 0

13. If $n(X) = m,\ n(Y) = n$ and $n(X \cap Y) = p$ then $n(X \cup Y) =$

(A) $m + n + p$ (B) $m + n - p$ (C) $m - p$ (D) $m - n + p$

14. If $U = \{1,2,3,4,5,6,7,8,9,10\}$ and $A = \{2,5,6,9,10\}$ then A' is

(A) $\{2,5,6,9,10\}$ (B) $\varnothing$ (C) $\{1,3,5,10\}$ (D) $\{1,3,4,7,8\}$

15. If $A \subseteq B$, then $A - B$ is

(A) B (B) A (C) $\varnothing$ (D) $B - A$

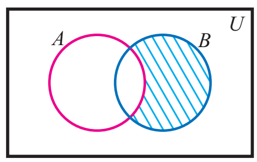
16. If A is a proper subset of B, then $A \cap B =$

(A) A (B) B (C) $\varnothing$ (D) $A \cup B$

17. If A is a proper subset of B, then $A \cup B$

(A) A (B) $\varnothing$ (C) B (D) $A \cap B$

18. The shaded region in the adjoint diagram represents

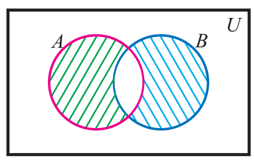


(A) $A - B$ (B) A' (C) B' (D) $B - A$

19. If $A = \{a,b,c\}$, $B = \{e,f,g\}$, then $A \cap B =$

(A) $\varnothing$ (B) A (C) B (D) $A \cup B$

20. The shaded region in the adjoining diagram represents



(A) $A - B$ (B) $B - A$ (C) $A \triangle B$ (D) A'

Points to Remember

- A set is a well defined collection of distinct objects
- Set is represented in three forms (i) Descriptive Form (ii) Set-builder Form (iii) Roster Form
- The number of elements in a set is said to be the cardinal number of the set.
- A set containing no element is called the empty set
- If the number of elements in a set is zero or finite, the set is called a finite set. Otherwise, the set is an infinite set.
- Two sets A and B are said to be equal if they contain exactly the same elements.
- A set X is a subset of a set Y if every element of X is also an element of Y.
- A set X is a proper subset of set Y if $X \subseteq Y$ and $X \neq Y$
- The power set of the set A is the set of all subsets of A. It is denoted by $P(A)$.
- The number of subsets of a set with m elements is 2^m.
- The number of proper subsets of a set with m elements is $2^m - 1$
- The set of all elements of the universal set that are not elements of a set A is called the complement of A. It is denoted by A'.
- The union of two sets A and B is the set of elements which are in A or in B or in both A and B.
- The intersection of two sets A and B is the set of all elements common to both A and B.
- If A and B are disjoint sets, then $A \cap B = \varnothing$
- The difference of two sets A and B is the set of all elements belonging to A but not to B.
- Symmetric difference of two sets A and B is defined as $A \triangle B = (A - B) \cup (B - A)$
- For any two finite sets A and B, we have

  (i) $n(A) = n(A - B) + n(A \cap B)$

  (ii) $n(B) = n(B - A) + n(A \cap B)$

  (iii) $n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$

  (iv) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

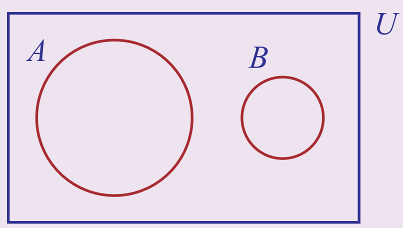
  (v) $n(A \cup B) = n(A) + n(B)$, when $A \cap B = \varnothing$

Activity 1

Represent $A' \cap B$, $A \cap B$ and $A \cap B'$ in Venn diagram. What will be the union of these sets?

Activity 2

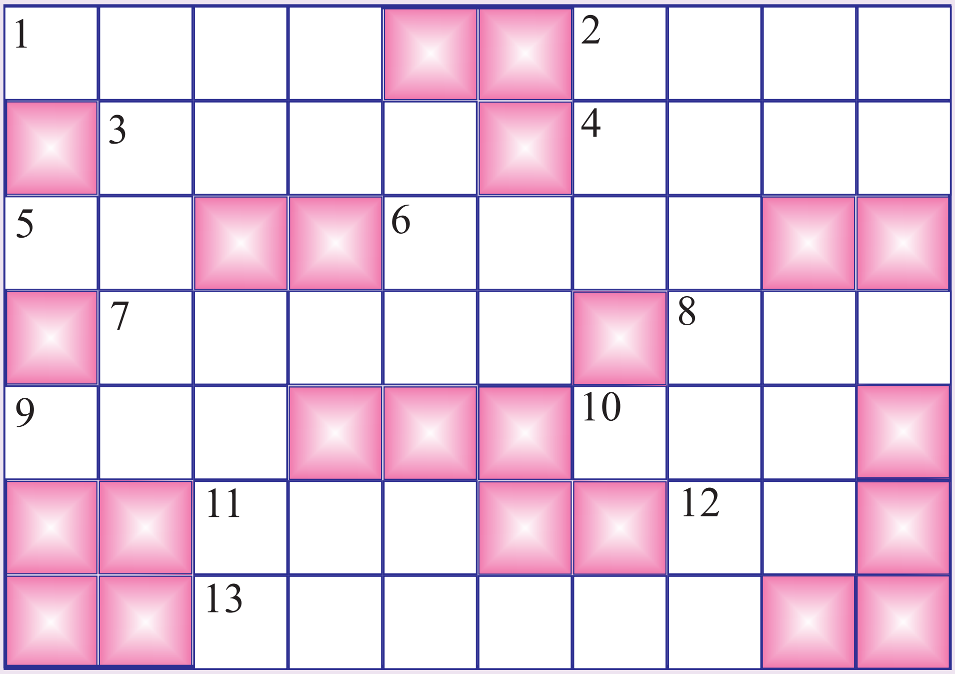
State which of the following statements are True or False using the given Venn diagram.



S.No.	Set Operation	True or False
1.	$A \subset B$	
2.	$B \subset A'$	
3.	$A \cap B = \{\ \}$	
4.	$A' \cap B = A'$	
5.	$B' \subset A$	
6.	$\varnothing \subset A$	
7.	$A' \cap B' = A$	
8.	$A \cup B' = B'$	

Activity 3

Complete the following jumble word puzzle

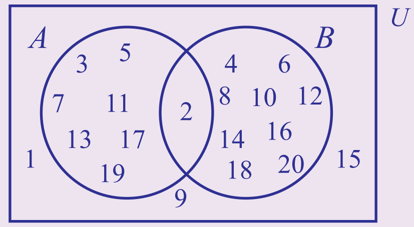


All clues apply to words formed horizontally

1. $A = \{o,r,y\},\ B = \{r,u,y\},\ A \cup B =$
2. $C = \{k, m, o, r, w, z\},\ D = \{k, o, r, t, w\}.\ C \cap D =$
3. $U = \{h, i, k, p, t. w\}, W = \{k, p\},\ W' =$
4. $M = \{t,e,s\}, N = \{s\}, M \cup N$ *and* $M \cap N =$
5. $P = \{h,i,k,s\}, Q = \{g,i,m,s\}, P \cap Q =$
6. $U = \{a,b,e,m,r,u,v,y\}, L = \{a,b,e,m,u\}, K = \{a,b,m,r,u,y\}, (L \cap K)' =$
7. $X = \{d,e,n,o,p,s,u\}, Y = \{d,h,n,o,s,u\}, X \cap Y =$
8. $A = \{a,b,c,d,n,s\}, B = \{a,d,h,n,t\},\ A \cap B =$
9. $U = \{o,p,r,s,u,v,y\}, D = \{p,r,s,v\}, D' =$
10. $E = \{a,e\}, F = \{r,e,a\}, E \cup F =$
11. $K = \{n,o,v,w,x\}, L = \{m,n,o,p,w,r\}, K \cap L =$
12. $P = \{a,d,e,n,p\}, Q = \{a,b,c,m,n,o\},\ P \cap Q =$
13. $X = \{e,h,r,p,s,t,x,v\}, Y = \{e,m,p,u,x\}, Z = \{e,n,r,t,w\}, (X \cap Y)$ and $(X \cap Z) =$

Activity 4

Using the following Venn diagram, represent the given sets in Descriptive Form, Set-Builder Form and Tabular Form.



(i) A (ii) B (iii) U (iv) B' (v) $A \cap B$

Activity 5

Assume that B is the set of letters in the word 'statistics'. State whether each of the following statements is True (T) or False (F).

(i) $\{t\} \in B$ ☐

(ii) $\{a, c\} \subset B$ ☐

(iii) $\{\ \} \subset B$ ☐

(iv) $n(B) = 10$ ☐

Project 1

Collect FA(a1) and FA(b1) first term Grade Sheets of Students studying 8th standard in your school from your mathematics teacher. Form two sets P and Q by listing the roll number of the students who secured Grade A2 in FA (a1) and FA(b1) respectively. Verify $n(P \cup Q) = n(P) + n(Q) - n(P \cap Q)$

Project 2

Conduct a survey in your street to find out the number of families using washing machine, number of families using Computer and the number of families using both. Represent the collected data in a Venn Diagram.

Exercise 1.1

1. (i) Not a set (ii) Set (iii) Not a set (iv) Set (v) Set

2. (i) $0 \in A$ (ii) $6 \notin A$ (iii) $3 \in A$ (iv) $4 \in A$ (v) $7 \notin A$

3. (i) $\{x : x \text{ is a positive even number}\}$ (ii) $\{x : x \text{ is a whole number and } x < 20\}$

(iii) $\{x : x \text{ is a positive integer and multiple of } 3\}$

(iv) $\{x : x \text{ is an odd natural number and } x < 15\}$

(v) $\{x : x \text{ is a letter in the word 'computer'}\}$

4. (i) $A = \{3, 4, 5, 6, 7, 8, 9, 10\}$ (ii) $B = \{0, 1, 2, 3, 4, 5\}$ (iii) $C = \{2, 3\}$

(iv) $X = \{2, 4, 8, 16, 32\}$ (v) $M = \{-1, 1, 3, 5, 7, 9\}$

(vi) $P = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$

5. (i) A is the set of all vowels in the English alphabet

(ii) B is the set of all odd natural numbers less than or equal to 11

(iii) C is the set of all square numbers less than 26.

(iv) P is the set of all letters in the word 'set theory'

(v) Q is the set of all prime numbers between 10 and 20

6. (i) 4 (ii) 21 (iii) 1 (iv) 0 (v) 9 **7.** (i) infinite (ii) finite (iii) infinite (iv) infinite (v) finite

8. (i) equivalent (ii) not equivalent (iii) equivalent

9. (i) equal (ii) not equal (iii) equal (iv) not equal **10.** $B = D$ and $E = G$

11. No, $\varnothing$ contains no element but $\{\varnothing\}$ contains one element,

12. Each one is different from others.

$\varnothing$ contains no element $\{0\}$ contains one element i.e., 0.

$\{\varnothing\}$ contains one element, i.e., the null set

13. (i) $\nsubseteq$ (ii) $\subseteq$ (iii) $\subseteq$ (iv) $\nsubseteq$ **14.** (i) X is not a subset of Y (ii) Y is a subset of X

15. A is not a subset of B

16. (i) $P(A) = \{\varnothing, \{x\}, \{y\}, \{x, y\}\}$ (ii) $P(X) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

(iii) $P(A) = \{\varnothing, \{5\}, \{6\}, \{7\}, \{8\}, \{5,6\}, \{5,7\}, \{5,8\}, \{6,7\}, \{6,8\}, \{7,8\}, \{5,6,7\},$
$\{5,6,8\}, \{5,7,8\}, \{6,7,8\}, \{5,6,7,8\}\}$ (iv) $P(A) = \{\phi\}$

17. (i) 64, 63 (ii) 128, 127 (iii) 2, 1

18. (i) 1 (ii) 8 (iii) 9 (iv) 10 **19.** A is the empty set

20. (i) $A = \{1,2,3,4,5,6,7,8,9,10\}$, $B = \{2,4,6,8,10,12,14,16,18,20\}$

$C = \{15,16,17,18,19,20,21,22,23,24,25\}$

(ii) $n(A) = 10$, $n(B) = 10$, $n(C) = 11$ (iii) a) F b) T c) T d) T

Exercise 1.2

1. (i) $A \cup B = \{-3,-1,0,1,2,4,5,6\}$, $A \cap B = \{0,2,4\}$

(ii) $A \cup B = \{2,4,6,8\}$, $A \cap B = \phi$

(iii) $A \cup B = \{1,2,3,4,5,7\}$, $A \cap B = \{2,3,5\}$

(iv) $A \cup B = \{0,1,2,3,4,5,6,7\}$, $A \cap B = \{3,4,5,6\}$

2. (i) $A \cup B = \{1,3,5,7,10,12,15,18,20,25,30\}$ (ii) $A \cap B = \{10,15,25\}$

3. (i) $X \cup Y = \{0,2,4,6,8,10,12,14,16,18,20\}$, $X \cap Y = \{4,8,12,16,20\}$

4. (i) $\{7\}$ **5.** (ii) X and Y are disjoint sets

6. (i) $A' = \{0,1,2,4,5,7,8,10\}$ (ii) A is the set of all prime numbers and 1

7. (i) $A \cup B = \{a,b,c,d,f,g\}$ (ii) $(A \cup B)' = \{e,h\}$ (iii) $A \cap B = \{b,d\}$

(iv) $(A \cap B)' = \{a,c,e,f,g,h\}$ **8.** (i) $A' = \{2,4,6,8,10\}$ (ii) $B' = \{1,4,6,7,8\}$

(iii) $A' \cup B' = \{1,2,4,6,7,8,10\}$ (iv) $A' \cap B' = \{4,6,8\}$

9. (i) $M - N = \{3,9\}$ (ii) $N - M = \{15,17\}$ (iii) $N' - M = \{18\}$ (iv) $M' - N = \{18\}$

(v) $M \cap (M - N) = \{3,9\}$ (vi) $N \cup (N - M) = \{7,11,15,17\}$ (vii) $n(M - N) = 2$

10. (i) $A - B = \{3,6,9,15,18\}$ (ii) $B - C = \{16,20\}$ (iii) $C - D = \{2,4,6,8,12\}$

(iv) $D - A = \{5,10,20,25\}$ (v) $n(A - C) = 4$

11. (i) $\text{U} = \{1,2,3\cdots 49\}$, $A = \{4,8,12,16,20,24,28,32,36,40,44,48\}$,

$B = \{16,30,44\}$ (ii) $A \cup B = \{4,8,12,16,20,24,28,30,32,36,40,44,48\}$,

$A \cap B = \{16,44\}$, $n(A \cup B) = 13$, $n(A \cap B) = 2$

12. (i) $X \triangle Y = \{a,b,d,e,f,k\}$ (ii) $P \triangle Q = \{0,1,2,3,5,6,7,8\}$

(iii) $A \triangle B = \{-4,-2,-1,5\}$

13. (i) $\text{U} = \{1,2,3,4,7,9,10,11\}$, $E = \{1,2,4,7\}$, $F = \{4,7,9,11\}$

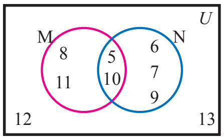
$E \cup F = \{1,2,4,7,9,11\}$, $E \cap F = \{4,7\}$

(ii) $n(\cup) = 8$, $n(E \cup F) = 6$, $n(E \cap F) = 2$

14. (i) $U = \{1, 2, 3, 4, 5, 6, 8, 9, 10\}$, $G = \{1, 2, 4, 8\}$, $H = \{2, 6, 8, 10\}$

(ii) $G' = \{3, 5, 6, 9, 10\}$, $H' = \{1, 3, 4, 5, 9\}$, $G' \cap H' = \{3, 5, 9\}$, $n(G \cup H)' = 3$, $n(G \cap H)' = 7$

Exercise 1.3

1. 

2. $n(A \cap B) = 15$ **3.** 16, 29 **4.** $n(B) = 27$

5. $n(A \cap B) = 6$, $n(\cup) = 43$ **6.** $n(A \cup B) = 22$ **7.** 150 **8.** 1400

9. (i) 180 (ii) 150 (iii) 450 **10.** 35 **11.** 12 **12.** 12 **13.** 47 **14.** Yes, correct

15. (i) $x = 8$ (ii) $n(A \cup B) = 88$ **16.** (i) 35 (ii) 25 (iii) 20 **17.** 16%

Exercise 1.4

1. A **2.** D **3.** B **4.** D **5.** B **6.** A **7.** C **8.** A **9.** D **10.** C **11.** A **12.** B

13. B **14.** D **15.** C **16.** A **17.** C **18.** D **19.** A **20.** C

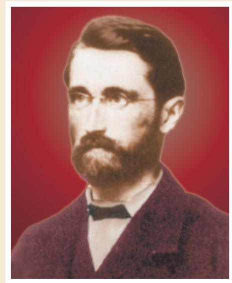
2 REAL NUMBER SYSTEM

Life is good for only two things, discovering mathematics and teaching mathematics

- SIMEON POISSON

Main Targets

- To recall Natural numbers, Whole numbers, Integers.
- To classify rational numbers as recurring / terminating decimals.
- To understand the existence of non-terminating and non recurring decimals.
- To represent terminating / non-terminating decimals on the number line.



RICHARD DEDEKIND
(1831-1916)

Richard Dedekind (1831-1916) belonged to an elite group of mathematicians who had been students of the legendary mathematician Carl Friedrich Gauss.

He did important work in abstract algebra, algebraic number theory and laid the foundations for the concept of the real numbers. He was one of the few mathematicians who understood the importance of set theory developed by Cantor. While teaching calculus for the first time at Polytechnic, Dedekind came up with the notion now called a Dedekind cut, a standard definition of the real numbers.

2.1 Introduction

All the numbers that we use in normal day-to-day activities to represent quantities such as distance, time, speed, area, profit, loss, temperature, etc., are called Real Numbers. The system of real numbers has evolved as a result of a process of successive extensions of the system of natural numbers. The extensions became inevitable as the science of Mathematics developed in the process of solving problems from other fields. Natural numbers came into existence when man first learnt counting. The Egyptians had used fractions around 1700 BC; around 500 BC, the Greek mathematicians led by Pythagoras realized the need for irrational numbers. Negative numbers began to be accepted around 1600 A.D. The development of calculus around 1700 A.D. used the entire set of real numbers without having defined them clearly. Georg Cantor can be considered the first to suggest a rigorous definition of real numbers in 1871 A.D.

Mathematics

In this chapter we discuss some properties of real numbers. First, let us recall various types of numbers that you have learnt in earlier classes.

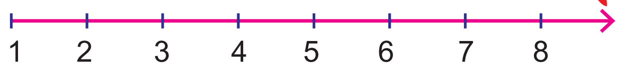
2.1.1 Natural Numbers

The counting numbers 1, 2, 3, $\cdots$ are called natural numbers.

The set of all natural numbers is denoted by $\mathbb{N}$.

i.e., $\mathbb{N} = \{1, 2, 3, \cdots\}$

The line extends endlessly only to the right side of 1.



Remark The smallest natural number is 1, but there is no largest number as it goes up continuously.

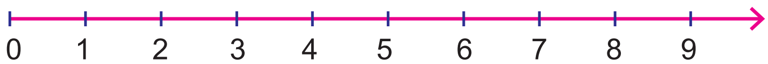
2.1.2 Whole Numbers

The set of natural numbers together with zero forms the set of whole numbers.

The set of whole numbers is denoted by $\mathbb{W}$.

$\mathbb{W} = \{ 0, 1, 2, 3, \cdots \}$

The line extends endlessly only to the right side of 0.



The smallest whole number is 0

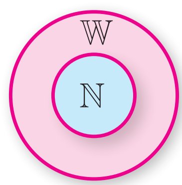
Remark

1) Every natural number is a whole number.

2) Every whole number need not be a natural number.

For, $0 \in \mathbb{W}$, but $0 \notin \mathbb{N}$

3) $\mathbb{N} \subset \mathbb{W}$



2.1.3 Integers

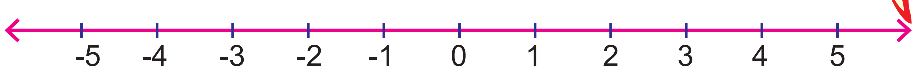
The natural numbers, their negative numbers together with zero are called integers.

$\mathbb{Z}$ is derived from the German word 'Zahlen', means 'to count'

The set of all integers is denoted by $\mathbb{Z}$

$\mathbb{Z} = \{ \cdots -3, -2, -1, 0, 1, 2, 3, \cdots \}$

The line extends endlessly on both sides of 0.



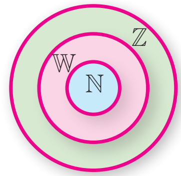
1, 2, 3 $\cdots$ are called positive integers.

$-1, -2, -3$ $\cdots$ are called negative integers.

Think and Answer !
Is zero a positive integer or a negative integer?

Remark

1) Every natural number is an integer.
2) Every whole number is an integer.
3) $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z}$



2.1.4 Rational Numbers

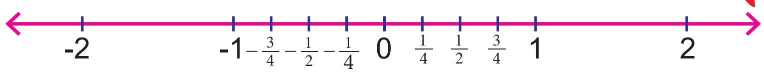
A number of the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$ is called a rational number.

For example, $3 = \frac{3}{1}, -\frac{5}{6}, \frac{7}{8}$ are rational numbers.

The set of all rational numbers is denoted by $\mathbb{Q}$.

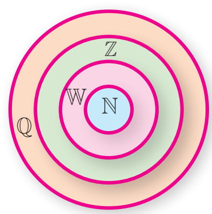
$$\mathbb{Q} = \left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}, \text{and } q \neq 0\right\}$$

We find rational numbers in between two integer



Remark

1) A rational number may be positive, negative or zero.
2) Every integer n is also a rational number, since we can write n as $\frac{n}{1}$.
3) $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q}$



Important Results

1) If a and b are two distinct rational numbers, then $\frac{a+b}{2}$ is a rational number between a and b such that $a < \frac{a+b}{2} < b$.

2) There are infinitely many rational numbers between any two given rational numbers.

Think and Answer !
Can you correlate the word ratio with rational numbers ?

Example 2.1

Find any two rational numbers between $\frac{1}{4}$ and $\frac{3}{4}$.

Solution A rational number between $\frac{1}{4}$ and $\frac{3}{4} = \frac{1}{2}\left(\frac{1}{4}+\frac{3}{4}\right) = \frac{1}{2}\,(1) = \frac{1}{2}$

Another rational number between $\frac{1}{2}$ and $\frac{3}{4} = \frac{1}{2}\left(\frac{1}{2}+\frac{3}{4}\right) = \frac{1}{2}\times\frac{5}{4} = \frac{5}{8}$

The rational numbers $\frac{1}{2}$ and $\frac{5}{8}$ lie between $\frac{1}{4}$ and $\frac{3}{4}$

Note There are infinite number of rationals between $\frac{1}{4}$ and $\frac{3}{4}$. The rationals $\frac{1}{2}$ and $\frac{5}{8}$ that we have obtained in Example 2.1 are two among them

Exercise 2.1

1. State whether the following statements are true or false.

   (i) Every natural number is a whole number.

   (ii) Every whole number is a natural number.

   (iii) Every integer is a rational number.

   (iv) Every rational number is a whole number.

   (v) Every rational number is an integer.

   (vi) Every integer is a whole number.

2. Is zero a rational number ? Give reasons for your answer.

3. Find any two rational numbers between $-\frac{5}{7}$ and $-\frac{2}{7}$.

2.2 Decimal Representation of Rational Numbers

If we have a rational number written as a fraction $\frac{p}{q}$, we get the decimal representation by long division.

When we divide p by q using long division method either the remainder becomes zero or the remainder never becomes zero and we get a repeating string of remainders.

Case (i) The remainder becomes zero

Let us express $\frac{7}{16}$ in decimal form. Then $\frac{7}{16} = 0.4375$

In this example, we observe that the remainder becomes zero after a few steps.

Also the decimal expansion of $\frac{7}{16}$ terminates.

$$\begin{array}{r} 0.4375 \\ 16\overline{)7.0000} \\ \underline{6\,4} \\ 60 \\ \underline{48} \\ 120 \\ \underline{112} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

Similarly, using long division method we can express the following rational numbers in decimal form as

$$\frac{1}{2} = 0.5,\ \frac{7}{5} = 1.4,\ -\frac{8}{25} = -0.32,\ \frac{9}{64} = 0.140625,\ \frac{527}{500} = 1.054$$

In these examples, the decimal expansion terminates or ends after a finite number of steps.

Key Concept — **Terminating Decimal**

When the decimal expansion of $\frac{p}{q}$, $q \neq 0$ terminates (i.e., comes to an end), the decimal expansion is called terminating.

Case (ii) The remainder never becomes zero

Does every rational number has a terminating decimal expansion? Before answering the question, let us express $\frac{5}{11}$, $\frac{7}{6}$ and $\frac{22}{7}$ in decimal form.

$$\begin{array}{r} 0.4545\cdots \\ 11\overline{)5.0000} \\ \underline{4\,4} \\ 60 \\ \underline{55} \\ 50 \\ \underline{44} \\ 60 \\ \underline{55} \\ 50 \\ \ddots \end{array} \qquad \begin{array}{r} 1.1666\cdots \\ 6\overline{)7.0000} \\ \underline{6\,0} \\ 10 \\ \underline{6} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 40 \\ \ddots \end{array} \qquad \begin{array}{r} 3.142857\,142857\cdots \\ 7\overline{)22.00000000} \\ \underline{21} \\ 10 \\ \underline{7} \\ 30 \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 50 \\ \underline{49} \\ 10 \\ \ddots \end{array}$$

$$\therefore \frac{5}{11} = 0.4545\cdots, \qquad \frac{7}{6} = 1.1666\cdots, \qquad \frac{22}{7} = 3.142857\,1\cdots$$

Thus, the decimal expansion of a rational number need not terminate.

In the above examples, we observe that the remainders never become zero. Also we note that the remainders repeat after some steps. So, we have a repeating (recurring) block of digits in the quotient.

Key Concept **Non-terminating and Recurring**

In the decimal expansion of $\frac{p}{q}, q \neq 0$ when the remainder never becomes zero, we have a repeating (recurring) block of digits in the quotient. In this case, the decimal expansion is called non-terminating and recurring.

To simplify the notation, we place a bar over the first block of the repeating (recurring) part and omit the remaining blocks.

So, we can write the expansion of $\frac{5}{11}$, $\frac{7}{6}$ and $\frac{22}{7}$ as follows.

$\frac{5}{11} = 0.4545\cdots = 0.\overline{45}$, $\quad \frac{7}{6} = 1.16666\cdots = 1.1\overline{6}$

$\frac{22}{7} = 3.142857\ 142857\ \cdots = 3.\overline{142857}$

The following table shows decimal representation of the reciprocals of the first ten natural numbers. We know that the reciprocal of a number n is $\frac{1}{n}$. Obviously, the reciprocals of natural numbers are rational numbers.

Number	Reciprocal	Type of Decimal
1	1.0	Terminating
2	0.5	Terminating
3	$0.\overline{3}$	Non-terminating and recurring
4	0.25	Terminating
5	0.2	Terminating
6	$0.1\overline{6}$	Non-terminating and recurring
7	$0.\overline{142857}$	Non-terminating and recurring
8	0.125	Terminating
9	$0.\overline{1}$	Non-terminating and recurring
10	0.1	Terminating

Thus we see that,

A rational number can be expressed by either a terminating or a non-terminating and recurring (repeating) decimal expansion.

The converse of this statement is also true.

That is, if the decimal expansion of a number is terminating or non-terminating and recurring, then the number is a rational number.

We shall illustrate this with examples.

2.2.1 Representing a Terminating Decimal Expansion in the form $\frac{p}{q}$

Terminating decimal expansion can easily be expressed in the form $\frac{p}{q}$ ($p, q \in \mathbb{Z}$ and $q \neq 0$). This method is explained in the following example.

Example 2.2

Express the following decimal expansion in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) 0.75 (ii) 0.625 (iii) 0.5625 (iv) 0.28

Solution (i) $0.75 = \frac{75}{100} = \frac{3}{4}$

(ii) $0.625 = \frac{625}{1000} = \frac{5}{8}$

(iii) $0.5625 = \frac{5625}{10000} = \frac{45}{80} = \frac{9}{16}$

(iv) $0.28 = \frac{28}{100} = \frac{7}{25}$

2.2.2 Representing a Non-terminating and Recurring Decimal Expansion in the form $\frac{p}{q}$

The expression of non-terminating and recurring decimal expansions in the form $\frac{p}{q}$ ($p, q \in \mathbb{Z}$ and $q \neq 0$) is not quite easy and the process is explained in the next example.

Example 2.3

Express the following in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

(i) $0.\overline{47}$ (ii) $0.\overline{001}$ (iii) $0.5\overline{7}$ (iv) $0.2\overline{45}$ (v) $0.\overline{6}$ (vi) $1.\overline{5}$

Solution (i) Let $x = 0.\overline{47}$. Then $x = 0.474747\cdots$

Since two digits are repeating, multiplying both sides by 100, we get

$$100x = 47.474747\cdots = 47 + 0.474747\cdots = 47 + x$$

$$99x = 47$$

$$x = \frac{47}{99} \qquad \therefore\ 0.\overline{47} = \frac{47}{99}$$

(ii) Let $x = 0.\overline{001}$. Then $x = 0.001001001\cdots$

Since three digits are repeating, multiplying both sides by 1000, we get

$$1000x = 1.001001001\cdots = 1 + 0.001001001\cdots = 1 + x$$

$$1000x - x = 1$$

$$999x = 1$$

$$x = \frac{1}{999} \qquad \therefore 0.\overline{001} = \frac{1}{999}$$

(iii) Let $x = 0.5\overline{7}$. Then $x = 0.57777\cdots$

Multiplying both sides by 10, we get

$$10x = 5.7777\cdots = 5.2 + 0.57777\cdots = 5.2 + x$$

$$9x = 5.2$$

$$x = \frac{5.2}{9}$$

$$x = \frac{52}{90} \qquad \therefore 0.5\overline{7} = \frac{52}{90} = \frac{26}{45}$$

(iv) Let $x = 0.2\overline{45}$. Then $x = 0.2454545\cdots$

Multiplying both sides by 100, we get

$$100x = 24.545454\cdots = 24.3 + 0.2454545\cdots = 24.3 + x$$

$$99x = 24.3$$

$$x = \frac{24.3}{99}$$

$$0.2\overline{45} = \frac{243}{990} = \frac{27}{110}$$

(v) Let $x = 0.\overline{6}$. Then $x = 0.66666\cdots$

Multiplying both sides by 10, we get

$$10x = 6.66666\cdots = 6 + 0.6666\cdots = 6 + x$$

$$9x = 6$$

$$x = \frac{6}{9} = \frac{2}{3} \qquad \therefore 0.\overline{6} = \frac{2}{3}$$

(vi) Let $x = 1.\overline{5}$. Then $x = 1.55555\cdots$

Multiplying both sides by 10, we get

$$10x = 15.5555\cdots = 14 + 1.5555\cdots = 14 + x$$

$$9x = 14$$

$$x = \frac{14}{9} \qquad \therefore 1.\overline{5} = 1\frac{5}{9}$$

So, every number with a non-terminating and recurring decimal expansion can be expressed in the form $\frac{p}{q}$, where p and q are integers and q not equal to zero

To determine whether the decimal form of a rational number will terminate or non-terminate, we can make use of the following rule.

> If a rational number $\frac{p}{q}$, $q \neq 0$ can be expressed in the form $\frac{p}{2^m \times 5^n}$, where $p \in \mathbb{Z}$ and $m, n \in \mathbb{W}$, then the rational number will have a terminating decimal expansion. Otherwise, the rational number will have a non-terminating and recurring decimal expansion.

This result is based on the fact that the decimal system uses ten as its base and the prime factors of 10 are 2 and 5.

Example 2.4

Without actual division, classify the decimal expansion of the following numbers as terminating or non-terminating and recurring.

(i) $\frac{7}{16}$ (ii) $\frac{13}{150}$ (iii) $\frac{-11}{75}$ (iv) $\frac{17}{200}$

Solution

(i) $16 = 2^4$

$\frac{7}{16} = \frac{7}{2^4} = \frac{7}{2^4 \times 5^0}$. So, $\frac{7}{16}$ has a terminating decimal expansion.

(ii) $150 = 2 \times 3 \times 5^2$

$\frac{13}{150} = \frac{13}{2 \times 3 \times 5^2}$

Since it is not in the form $\frac{p}{2^m \times 5^n}$, $\frac{13}{150}$ has a non-terminating and recurring decimal expansion.

(iii) $\frac{-11}{75} = \frac{-11}{3 \times 5^2}$

Since it is not in the form $\frac{p}{2^m \times 5^n}$, $\frac{-11}{75}$ has a non-terminating and recurring decimal expansion.

(iv) $\frac{17}{200} = \frac{17}{8 \times 25} = \frac{17}{2^3 \times 5^2}$. So $\frac{17}{200}$ has a terminating decimal expansion.

Example 2.5

Convert $0.\overline{9}$ into a rational number.

Solution Let $x = 0.\overline{9}$. Then $x = 0.99999\cdots$

Multiplying by 10 on both sides, we get

$$10x = 9.99999\cdots = 9 + 0.9999\cdots = 9 + x$$

$$\Longrightarrow \quad 9x = 9$$

$$\Longrightarrow \quad x = 1. \text{ That is, } 0.\overline{9} = 1 \qquad (\because 1 \text{ is rational number})$$

For your Thought

We have proved $0.\overline{9} = 1$. Isn't it surprising?

Most of us think that $0.9999\cdots$ is less than 1. But this is not the case. It is clear from the above argument that $0.\overline{9} = 1$. Also this result is consistent with the fact that $3 \times 0.333\cdots = 0.999\cdots$, while $3 \times \frac{1}{3} = 1$.

Similarly, it can be shown that any terminating decimal can be represented as a non-terminating and recurring decimal expansion with an endless blocks of 9s.

For example $6 = 5.9999\cdots$, $2.5 = 2.4999\cdots$.

Exercise 2.2

1. Convert the following rational numbers into decimals and state the kind of decimal expansion.

   (i) $\frac{42}{100}$ (ii) $8\frac{2}{7}$ (iii) $\frac{13}{55}$ (iv) $\frac{459}{500}$

   (v) $\frac{1}{11}$ (vi) $-\frac{3}{13}$ (vii) $\frac{19}{3}$ (viii) $-\frac{7}{32}$

2. Without actual division, find which of the following rational numbers have terminating decimal expansion.

   (i) $\frac{5}{64}$ (ii) $\frac{11}{12}$ (iii) $\frac{27}{40}$ (iv) $\frac{8}{35}$

3. Express the following decimal expansions into rational numbers.

   (i) $0.\overline{18}$ (ii) $0.\overline{427}$ (iii) $0.\overline{0001}$

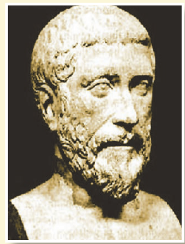
   (iv) $1.\overline{45}$ (v) $7.\overline{3}$ (vi) $0.4\overline{16}$

4. Express $\frac{1}{13}$ in decimal form. Find the number of digits in the repeating block.

5. Find the decimal expansions of $\frac{1}{7}$ and $\frac{2}{7}$ by division method. Without using the long division method, deduce the decimal expressions of $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ from the decimal expansion of $\frac{1}{7}$.

2.3 Irrational Numbers

Let us have a look at the number line again. We have represented rational numbers on the number line. We have also seen that there are infinitely many rational numbers between any two given rational numbers. In fact there are infinitely many more numbers left on the number line, which are not rationals.



Pythagoras
569BC - 500 BC

Around 500 BC, the pythagorians, followers of the famous Greek mathematician Pythagoras, were the first to discover the numbers which cannot be written in the form of a fraction. These numbers are called irrational numbers.

In other words there are numbers whose decimal expansions are non-terminating and non-recurring. Thus, there is a need to extend the system of rational numbers. Consider the following decimal expansion

$0.808008000800008\cdots$ (1) .

This is non-terminating. Is it recurring?

It is true that there is a pattern in this decimal expansion, but no block of digits repeats endlessly and so it is not recurring.

Thus, this decimal expansion is non-terminating and non-repeating (non-recurring). So it cannot represent a rational number. Numbers of this type are called irrational numbers.

Key concept	**Irrational Number**

A number having a non-terminating and non-recurring decimal expansion is called an irrational number. So, it cannot be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

For example,

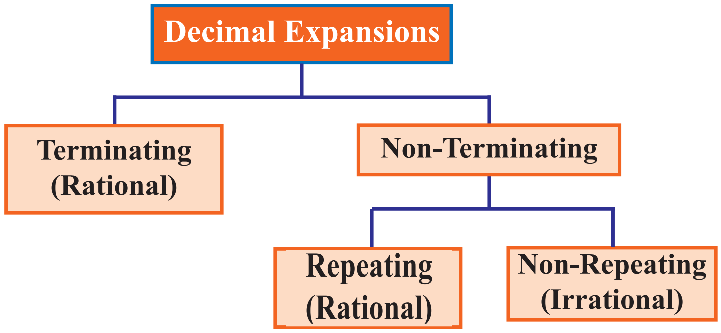
$\sqrt{2}, \sqrt{3}, \sqrt{5}, e, \pi, \sqrt{17}, 0.2020020002\cdots$ are a few examples of irrational numbers.

Note In fact, we can generate infinitely many non-terminating and non-recurring decimal expansions by replacing the digit 8 in (1) by any natural number as we like.

Know about π: In the late 18th century Lambert and Legendre proved that π is irrational.

We usually take $\frac{22}{7}$ (a rational number) as an approximate value for π (an irrational number).

Classification of Decimal Expansions



2.4 Real Numbers

> ***Key Concept*** **Real Numbers**
>
> The union of the set of all rational numbers and the set of all irrational numbers forms the set of all real numbers.
>
> Thus, every real number is either a rational number or an irrational number.
>
> In other words, if a real number is not a rational number, then it must be an irrational number.

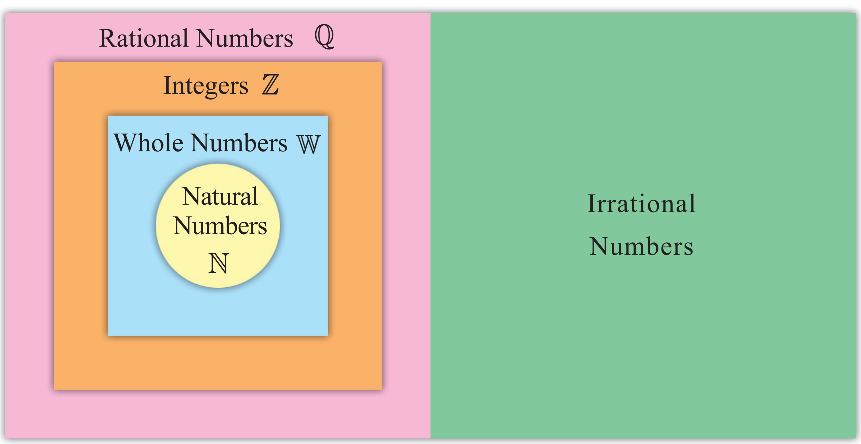
The set of all real numbers is denoted by $\mathbb{R}$.

German mathematicians, Georg Cantor and R. Dedekind proved independently that corresponding to every real number, there is a unique point on the real number line and corresponding to every point on the number line there exists a unique real number.

Thus, on the number line, each point corresponds to a unique real number. And every real number can be represented by a unique point on the number line.

The following diagram illustrates the relationships among the sets that make up the real numbers

REAL NUMBERS $\mathbb{R}$



Let us find the square root of 2 by long division method.

	1.4142135⋯
1	2.00 00 00 00 00
	1
24	1 00
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
282841	383600
	282841
2828423	10075900
	8485269
28284265	159063100
	141421325
	17641775
	⋮

$\therefore \sqrt{2} = 1.4142135\cdots$

If we continue this process, we observe that the decimal expansion has non-terminating and non-recurring digits and hence $\sqrt{2}$ is an irrational number.

Note

(i) The decimal expansions of $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}, \cdots$ are non-terminating and non-recurring and hence they are irrational numbers.

(ii) The square root of every positive integer is not always irrational.
For example, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{25} = 5, \cdots$. Thus $\sqrt{4}, \sqrt{9}, \sqrt{25}, \cdots$ are rational numbers.

(iii) The square root of every positive but a not a perfect square number is an irrational number

2.4.1 Representation of Irrational Numbers on the Number Line

Let us now locate the irrational numbers $\sqrt{2}$ and $\sqrt{3}$ on the number line.

(i) Locating $\sqrt{2}$ on the number line.

Draw a number line. Mark points O and A such that O represents the number zero and A represents the number 1. i.e., $OA = 1$ unit Draw $AB \perp OA$ such that $AB = 1$unit. Join OB.

In right triangle OAB, by Pythagorean theorem,

$$OB^2 = OA^2 + AB^2$$
$$= 1^2 + 1^2$$
$$OB^2 = 2$$
$$OB = \sqrt{2}$$

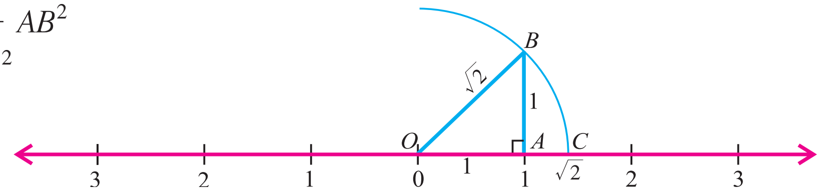


Fig. 2.6

With O as centre and radius OB, draw an arc to intersect the number line at C on the right side of O. Clearly $OC = OB = \sqrt{2}$. Thus, C corresponds to $\sqrt{2}$ on the number line.

(ii) Locating $\sqrt{3}$ on the number line.

Draw a number line. Mark points O and C on the number line such that O represents the number zero and C represents the number $\sqrt{2}$ as we have seen just above.

$\therefore$ $OC = \sqrt{2}$ unit. Draw $CD \perp OC$ such that $CD = 1$ unit. Join OD

In right triangle OCD, by Pythagorean theorem,

$$OD^2 = OC^2 + CD^2$$
$$= (\sqrt{2})^2 + 1^2 = 3$$
$$\therefore\ OD = \sqrt{3}$$

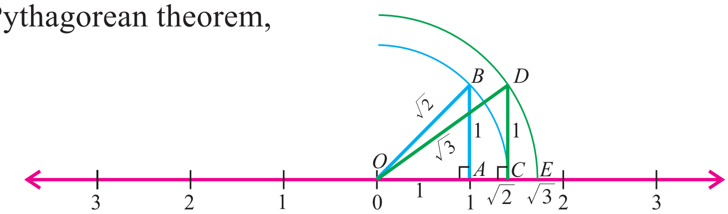


Fig. 2.7

With O as centre and radius OD, draw an arc to intersect the number line at E on the right side of O. Clearly $OE = OD = \sqrt{3}$. Thus, E represents $\sqrt{3}$ on the number line.

Example 2.6

Classify the following numbers as rational or irrational.

(i) $\sqrt{11}$ (ii) $\sqrt{81}$ (iii) 0.0625 (iv) $0.8\overline{3}$ (v) $1.505500555\cdots$

Solution

(i) $\sqrt{11}$ is an irrational number. (11is not a perfect square number)

(ii) $\sqrt{81} = 9 = \frac{9}{1}$, a rational number.

(iii) 0.0625 is a terminating decimal.

$\therefore$ 0.0625 is a rational number.

(iv) $0.8\overline{3} = 0.8333\cdots$

The decimal expansion is non-terminating and recurring.

$\therefore$ $0.8\overline{3}$ is a rational number.

(v) The decimal number is non-terminating and non-recurring.

$\therefore$ $1.505500555\cdots$ is an irrational number.

Example 2.7

Find any three irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$.

Solution

$$\begin{array}{r|l} & 0.714285\cdots \\ 7 & 5.000000 \\ & 4\,9 \\ \hline & \;\;10 \\ & \;\;\;\;7 \\ \hline & \;\;\;\;30 \\ & \;\;\;\;28 \\ \hline & \;\;\;\;\;\;20 \\ & \;\;\;\;\;\;14 \\ \hline & \;\;\;\;\;\;\;\;60 \\ & \;\;\;\;\;\;\;\;56 \\ \hline & \;\;\;\;\;\;\;\;\;\;40 \\ & \;\;\;\;\;\;\;\;\;\;35 \\ \hline & \;\;\;\;\;\;\;\;\;\;\;\;50 \\ & \;\;\;\;\;\;\;\;\;\;\;\;\;\;\ddots \end{array} \qquad \begin{array}{r|l} & 0.8181\cdots \\ 11 & 9.0000 \\ & 8\,8 \\ \hline & \;\;20 \\ & \;\;11 \\ \hline & \;\;\;\;90 \\ & \;\;\;\;88 \\ \hline & \;\;\;\;\;\;20 \\ & \;\;\;\;\;\;\;\;\ddots \end{array}$$

$$\frac{5}{7} = 0.\overline{714285} \qquad \frac{9}{11} = 0.8181\cdots = 0.\overline{81}$$

To find three irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$ (i.e., between 0.714285... and 0.8181...) We find three numbers whose decimal expansions are non-terminating and non-recurring. Infact, there are infinitely many such numbers. Three such numbers are

$0.72022002220002\cdots$

$0.73033003330003\cdots$

$0.75055005550005\cdots$

Example 2.8

In the following equations determine whether x, y, z represent rational or irrational numbers.

(i) $x^3 = 8$ (ii) $x^2 = 81$ (iii) $y^2 = 3$ (iv) $z^2 = 0.09$

Solution

(i) $x^3 = 8 = 2^3$ (8 is a perfect cube)

$\Rightarrow x = 2$, a rational number.

(ii) $x^2 = 81 = 9^2$ (81 is a perfect square)

$\Rightarrow x = 9$, a rational number.

(iii) $y^2 = 3 \quad \Rightarrow y = \sqrt{3}$, an irrational number.

(iv) $z^2 = 0.09 = \frac{9}{100} = \left(\frac{3}{10}\right)^2$

$\Rightarrow z = \frac{3}{10}$, a rational number.

Exercise 2.3

1. Locate $\sqrt{5}$ on the number line.
2. Find any three irrational numbers between $\sqrt{3}$ and $\sqrt{5}$.
3. Find any two irrational numbers between 3 and 3.5.
4. Find any two irrational numbers between 0.15 and 0.16.
5. Insert any two irrational numbers between $\frac{4}{7}$ and $\frac{5}{7}$.
6. Find any two irrational numbers between $\sqrt{3}$ and 2.
7. Find a rational number and also an irrational number between $1.1011001110001\cdots$ and $2.1011001110001\cdots$
8. Find any two rational numbers between $0.12122122212222\cdots$ and $0.2122122212222\cdots$

2.4.2 Representation of Real Numbers on the Number Line

We have seen that any real number can be represented as a decimal expansion. This will help us to represent a real number on the number line.

Let us locate 3.776 on the number line. We know that 3.776 lies between 3 and 4.

Let us look closely at the portion of the number line between 3 and 4.

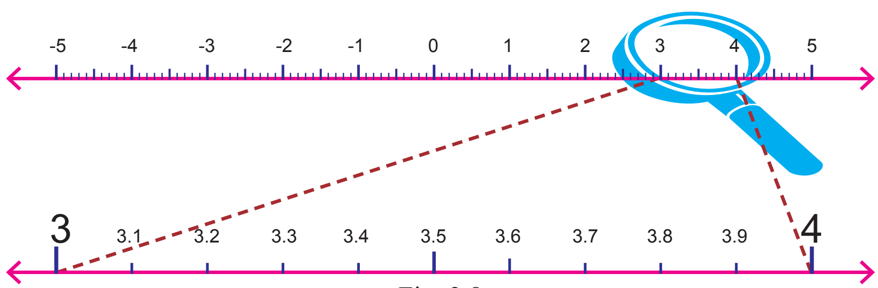


Fig. 2.8

Divide the portion between 3 and 4 into 10 equal parts and mark each point of division as in Fig 2.8. Then the first mark to the right of 3 will represent 3.1, the second 3.2, and so on. To view this clearly take a magnifying glass and look at the portion between 3 and 4. It will look like as shown in Fig. 2.8. Now 3.776 lies between 3.7 and 3.8. So, let us focus on the portion between 3.7 and 3.8 (Fig. 2.9)

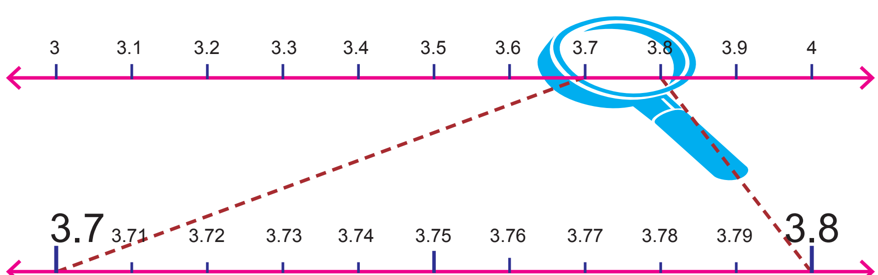


Fig. 2.9

Again divide the portion between 3.7 and 3.8 into 10 equal parts. The first mark will represent 3.71, the next 3.72, and so on. To view this portion clearly, we magnify the portion between 3.7 and 3.8 as shown in Fig 2.9

Again, 3.776 lies between 3.77 and 3.78. So, let us divide this portion into 10 equal parts. We magnify this portion, to see clearly as in Fig. 2.10.

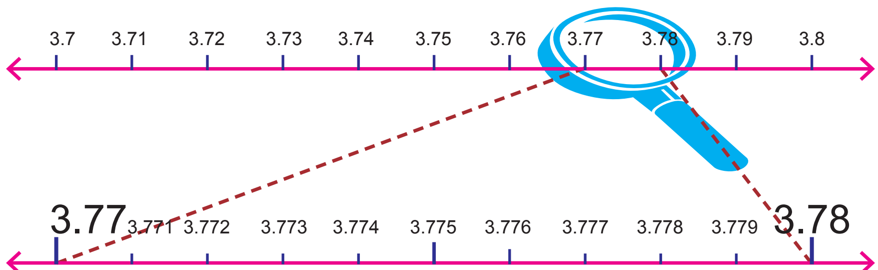


Fig. 2.10

The first mark represents 3.771, the next mark 3.772, and so on. So 3.776 is the 6th mark in this sub division.

This process of visualisation of representation of numbers on the number line, through a magnifying glass is known as the process of successive magnification.

So, we can visualise the position of a real number with a terminating decimal expansion on the number line, by sufficient successive magnifications.

Now, let us consider a real number with a non-terminating recurring decimal expansion and try to visualise the position of it on the number line.

Example 2.9

Visualise $4.\overline{26}$ on the number line, upto 4 decimal places, that is upto 4.2626

Solution We locate $4.\overline{26}$ on the number line, by the process of successive magnification. This has been illustrated in Fig. 2.11

Step 1: First we note that $4.\overline{26}$ lies between 4 and 5

Step 2: Divide the portion between 4 and 5 into 10 equal parts and use a magnifying glass to visualise that $4.\overline{26}$ lies between 4.2 and 4.3

Step 3: Divide the portion between 4.2 and 4.3 into 10 equal parts and use a magnifying glass to visualise that $4.\overline{26}$ lies between 4.26 and 4.27

Step 4: Divide the portion between 4.26 and 4.27 into 10 equal parts and use a magnifying glass to visualise that $4.\overline{26}$ lies between 4.262 and 4.263

Step 5: Divide the portion between 4.262 and 4.263 into 10 equal parts and use a magnifying glass to visualise that $4.\overline{26}$ lies between 4.2625 and 4.2627

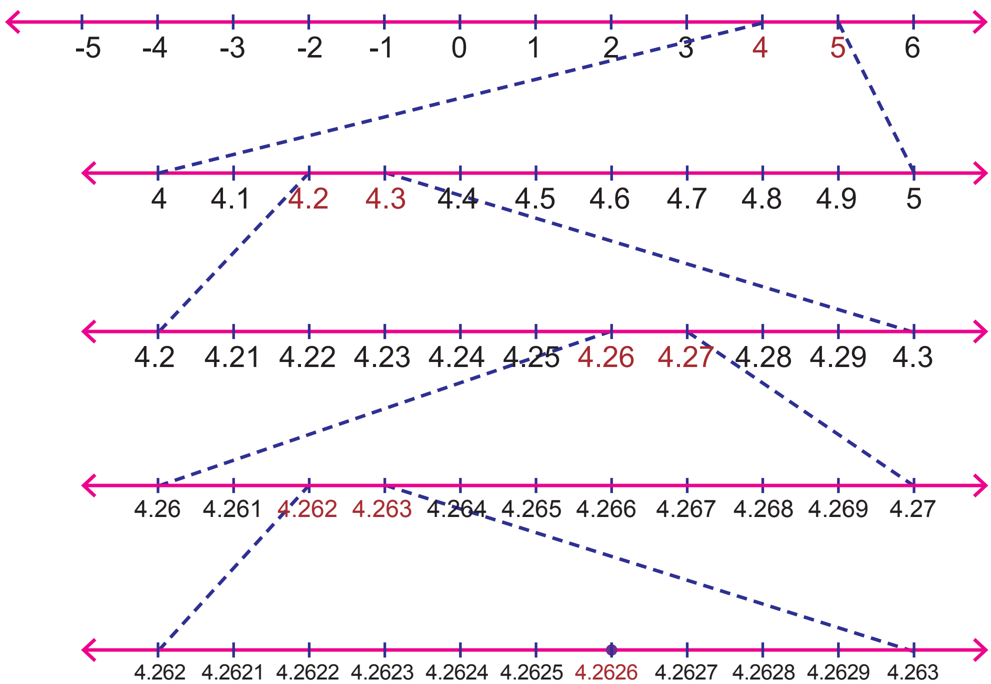


Fig. 2.11

We note that $4.\overline{26}$ is visualised closer to 4.263 than to 4.262.

The same procedure can be used to visualise a real number with a non-terminating and non-recurring decimal expansion on the number line to a required accuracy.

From the above discussions and visualisations we conclude again that every real number is represented by a unique point on the number line. Further every point on the number line represents one and only one real number.

.

2.4.3 Properties of Real Numbers

- For any two real numbers a and b, $a = b$ or $a > b$ or $a < b$
- The sum, difference, product of two real numbers is also a real number.
- The division of a real number by a non-zero real number is also a real number.
- The real numbers obey closure, associative, commutative and distributive laws under addition and under multiplication that the rational numbers obey.
- Every real number has its negative real number. The number zero is its own negative and zero is considered to be neither negative nor positive.

Further the sum, difference, product and quotient (except division by zero) of two rational numbers, will be rational number. However, the sum, difference, product and quotient of two irrational numbers may sometimes turnout to be a rational number.

Let us state the following facts about rational numbers and irrational numbers.

Key Concept
1. The sum or difference of a rational number and an irrational number is always an irrational number
2. The product or quotient of non-zero rational number and an irrational number is also an irrational number.
3. Sum, difference, product or quotient of two irrational numbers need not be irrational. The result may be rational or irrational.

Remark If a is a rational number and $\sqrt{b}$ is an irrational number then

(i) $a+\sqrt{b}$ is irrational (ii) $a-\sqrt{b}$ is irrational

(iii) $a\sqrt{b}$ is irrational (iv) $\dfrac{a}{\sqrt{b}}$ is irrational (v) $\dfrac{\sqrt{b}}{a}$ is irrational

For example, (i) $2+\sqrt{3}$ is irrational (ii) $2-\sqrt{3}$ is irrational

(iii) $2\sqrt{3}$ is irrational (iv) $\dfrac{2}{\sqrt{3}}$ is irrational

2.4.4 Square Root of Real Numbers

Let $a > 0$ be a real number. Then $\sqrt{a} = b$ means $b^2 = a$ and $b > 0$.

2 is a square root of 4 because $2 \times 2 = 4$, but -2 is also a square root of 4 because $(-2) \times (-2) = 4$. To avoid confusion between these two we define the symbol $\sqrt{\ }$, to mean the principal or positive square root.

Let us now mention some useful identities relating to square roots.

Let a and b be positive real numbers. Then	
1	$\sqrt{ab} = \sqrt{a}\sqrt{b}$
2	$\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$
3	$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$
4	$(a+\sqrt{b})(a-\sqrt{b}) = a^2-b$
5	$(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d}) = \sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$
6	$(\sqrt{a}+\sqrt{b})^2 = a+b+2\sqrt{ab}$

Example 2.10

Give two irrational numbers so that their

(i) sum is an irrational number.

(ii) sum is not an irrational number.

(iii) difference is an irrational number.

(iv) difference is not an irrational number.

(v) product is an irrational number.

(vi) product is not an irrational number.

(vii) quotient is an irrational number.

(viii) quotient is not an irrational number.

Solution

(i) Consider the two irrational numbers $2+\sqrt{3}$ and $\sqrt{3}-2$.

Their sum $= 2+\sqrt{3}+\sqrt{3}-2 = 2\sqrt{3}$ is an irrational number.

(ii) Consider the two irrational numbers $\sqrt{2}$ and $-\sqrt{2}$.

Their sum $= \sqrt{2}+(-\sqrt{2}) = 0$ is a rational number.

(iii) Consider the two irrational numbers $\sqrt{3}$ and $\sqrt{2}$.

Their difference $= \sqrt{3}-\sqrt{2}$ is an irrational number.

(iv) Consider the two irrational numbers $5+\sqrt{3}$ and $\sqrt{3}-5$.

Their difference $= (5+\sqrt{3})-(\sqrt{3}-5) = 10$ is a rational number.

(v) Consider the two irrational numbers $\sqrt{3}$ and $\sqrt{5}$.

Their product $= \sqrt{3}\times\sqrt{5} = \sqrt{15}$ is an irrational number.

(vi) Consider the two irrational numbers $\sqrt{18}$ and $\sqrt{2}$.

Their product $= \sqrt{18}\times\sqrt{2} = \sqrt{36} = 6$ is a rational number.

(vii) Consider the two irrational numbers $\sqrt{15}$ and $\sqrt{3}$.

Their quotient $= \dfrac{\sqrt{15}}{\sqrt{3}} = \sqrt{\dfrac{15}{3}} = \sqrt{5}$ is an irrational number.

(viii) Consider the two irrational numbers $\sqrt{75}$ and $\sqrt{3}$.

Their quotient $= \dfrac{\sqrt{75}}{\sqrt{3}} = \sqrt{\dfrac{75}{3}} = 5$ is a rational number.

Exercise 2.4

1. Using the process of successive magnification
   (i) Visualise 3.456 on the number line.
   (ii) Visualise $6.7\overline{3}$ on the number line, upto 4 decimal places

Exercise 2.5

Multiple Choice Questions.

1. A number having non-terminating and recurring decimal expansion is

   (A) an integer (B) a rational number
   (C) an irrational number (D) a whole number

2. If a number has a non-terminating and non-recurring decimal expansion, then it is

   (A) a rational number (B) a natural number
   (C) an irrational number (D) an integer.

3. Decimal form of $-\frac{3}{4}$ is

   (A) – 0.75 (B) – 0.50 (C) – 0.25 (D) – 0.125

4. The $\frac{p}{q}$ form of $0.\overline{3}$ is

   (A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

5. Which one of the following is not true?

   (A) Every natural number is a rational number
   (B) Every real number is a rational number
   (C) Every whole number is a rational number
   (D) Every integer is a rational number.

6. Which one of the following has a terminating decimal expansion?

   (A) $\frac{5}{32}$ (B) $\frac{7}{9}$ (C) $\frac{8}{15}$ (D) $\frac{1}{12}$

7. Which one of the following is an irrational number?

   (A) π (B) $\sqrt{9}$ (C) $\frac{1}{4}$ (D) $\frac{1}{5}$

8. Which of the following are irrational numbers?

(i) $\sqrt{2+\sqrt{3}}$ (ii) $\sqrt{4+\sqrt{25}}$ (iii) $\sqrt[3]{5+\sqrt{7}}$ (iv) $\sqrt{8-\sqrt[3]{8}}$

(A) (ii),(iii) and (iv) (B) (i),(ii) and (iv)

(C) (i),(ii) and (iii) (D) (i),(iii) and (iv)

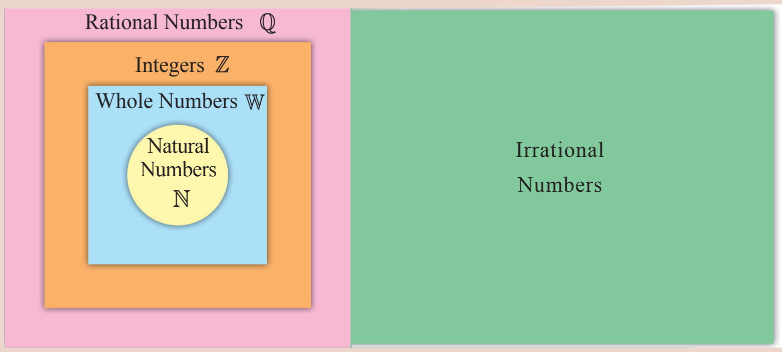
Points to Remember

- When the decimal expansion of $\frac{p}{q}$, $q \neq 0$ terminates i.e., comes to an end, the decimal is called a terminating decimal.
- In the decimal expansion of $\frac{p}{q}$, $q \neq 0$ when the remainder is not zero, we have a repeating (recurring) block of digits in the quotient. In this case, the decimal expansion is called non-terminating and recurring.
- If a rational number $\frac{p}{q}$, $q \neq 0$ can be expressed in the form $\frac{p}{2^m \times 5^n}$, where $p \in \mathbb{Z}$ and, $m, n \in \mathbb{W}$ then the rational number will have a terminating decimal. Otherwise, the rational number will have a non-terminating repeating (recurring) decimal.
- A rational number can be expressed by either a terminating or a non-terminating repeating decimal.
- An irrational number is a non-terminating and non-recurring decimal, i.e., it cannot be written in the form $\frac{p}{q}$, where p and q are both integers and $q \neq 0$.
- The union of all rational numbers and all irrational numbers is called the set of real numbers.
- Every real number is either a rational number or an irrational number.
- If a real number is not a rational number, then it must be an irrational number.
- The sum or difference of a rational number and an irrational number is always an irrational number
- The product or quotient of non-zero rational number and an irrational number is also an irrational number.
- Sum, difference, product or quotient of two irrational numbers need not be irrational. The result may be rational or irrational.

Activity 1

Using the following figure state with reason and example whether each of the following statements is True or False.

Real Numbers $\mathbb{R}$



(i) Every natural number is an integer

(ii) 0 is a natural number

(iii) Every integer is a natural number

(iv) $\sqrt{3}$ is a rational number

(v) Every real number is an irrational number

Activity 2

Complete each of the following statements with an appropriate symbol $=, \subset, \cup$ or $\cap$.
Here $\mathbb{N}$ - the set of natural numbers, $\mathbb{W}$ - the set of whole numbers, $\mathbb{Z}$ - the set of integers, $\mathbb{Q}$ - the set of rational numbers, $\mathbb{T}$ - the set of irrational numbers and $\mathbb{R}$ - the set of real numbers.

(i) $\mathbb{N}$ __________ $\mathbb{Z}$ (ii) $\mathbb{N}$ __________ $\mathbb{R} = \mathbb{R}$

(iii) $\mathbb{N}$ __________ $\mathbb{W} = \mathbb{N}$ (iv) $\mathbb{Q}$ __________ $\mathbb{T} = \varnothing$

(v) $\mathbb{T}'$ __________ $\mathbb{Q}$ (vi) $\mathbb{Z}$ __________ $\mathbb{R} = \mathbb{Z}$

(vii) $\mathbb{T}$ __________ $\mathbb{Q} = \mathbb{R}$

Activity 3

Complete the following chart.

Number	Natural	Whole	Integer	Rational	Irrational	Real
8	Yes					
-11				Yes		
0					No	
$\frac{1}{4}$						Yes
π					Yes	
$\sqrt{7}$	No					
6.32	No					
$1.555\cdots$	No	No	No			
$2.\overline{91}$			No			
$\sqrt{16}$					No	

Project 1

Make a square root spiral using ruler and compass on a sheet of paper till you get $\sqrt{10}$ as the hypotenuse of the triangles generated.

(*or*)

Make a square root spiral by paper folding till you get $\sqrt{10}$ as the hypotenuse of the triangles generated.

Project 2

Examine the history of the number π. List the sources used to collect the data

(or)

Know the significance of Ramanujan number 1729

Exercise 2.1

1. (i) True (ii) False (iii) True (iv) False (v) False (vi) False

2. Yes, For $0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3} = \frac{0}{-1} = \cdots$ **3.** $-\frac{4}{7}, -\frac{3}{7}$

Exercise 2.2

1. (i) 0.42, terminating (ii) $8.\overline{285714}$, nonterminating and recurring

(iii) $0.2\overline{36}$,non-terminating and recurring (iv) 0.918, terminatting

(v) $0.\overline{09}$,non-terminating and recurring

(vi) $-0.\overline{230769}$,non-terminating and recurring

(vii) $6.\overline{3}$, non-terminating and recurring (viii) -0.21875, terminating

2. (i) terminating (ii) non-terminating (iii) terminating (iv) non-terminating

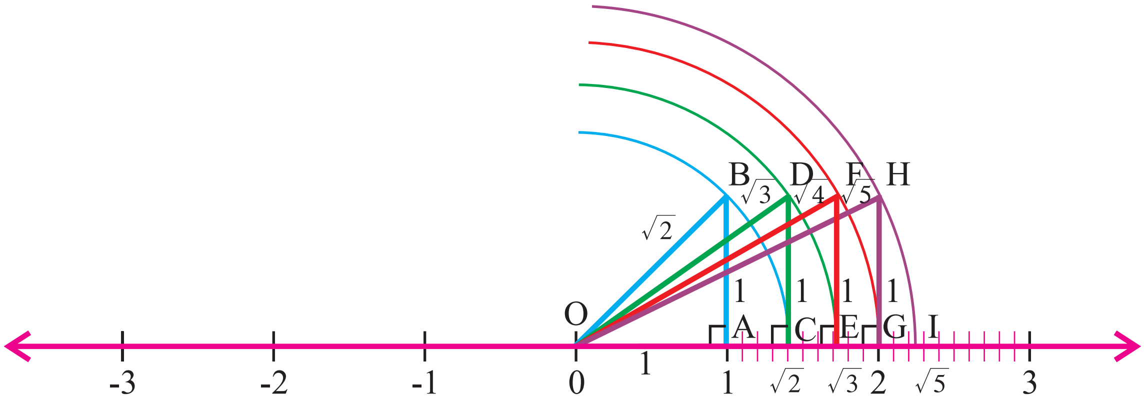
3. (i) $\frac{2}{11}$ (ii) $\frac{427}{999}$ (iii) $\frac{1}{9999}$ (iv) $\frac{16}{11}$ (v) $\frac{22}{3}$ (vi) $\frac{206}{495}$ **4.** $0.\overline{076923}$, 6

5. $\frac{1}{7} = 0.\overline{142857}$, $\frac{2}{7} = 0.\overline{285714}$, $\frac{3}{7} = 0.\overline{428571}$, $\frac{4}{7} = 0.\overline{571428}$,

$\frac{5}{7} = 0.\overline{714285}$, $\frac{6}{7} = 0.\overline{857142}$

Exercise 2.3

1.



2. 1.83205⋯, 1.93205⋯, 2.03205⋯

3. 3.10110011100011110⋯, 3.2022002220002222⋯

4. 0.1510100110001110⋯, 0.1530300330003330⋯

5. 0.58088008880⋯, 0.59099009990⋯

6. 1.83205⋯, 1.93205⋯

7. One rational number : 1.102, An irrational number : 1.9199119991119⋯

8. 0.13, 0.20 [Note: Questions from 2 to 8 will have infinitely many solutions]

Exercise 2.5

1. B **2.** C **3.** A **4.** C **5.** B **6.** A **7.** A **8.** D

3 ALGEBRA

Mathematics is as much an aspect of
culture as it is a collection of algorithms

- CARL BOYER

Main Targets

- To classify polynomials.
- To use Remainder Theorem.
- To use Factor Theorem.

DIOPHANTUS
(200 to 284 A.D. or
214 to 298 A.D.)

Diophantus was a Hellenistic mathematician who lived circa 250 AD, but the uncertainty of this date is so great that it may be off by more than a century. He is known for having written Arithmetica, a treatise that was originally thirteen books but of which only the first six have survived. Arithmetica has very little in common with traditional Greek mathematics since it is divorced from geometric methods, and it is different from Babylonian mathematics in that Diophantus is concerned primarily with exact solutions, both determinate and indeterminate, instead of simple approximations

3.1 Introduction

The language of algebra is a wonderful instrument for expressing shortly, perspicuously, suggestively and the exceedingly complicated relations in which abstract things stand to one another. The history of algebra began in ancient Egypt and Babylon, where people learned to solve linear ($ax = b$) and quadratic ($ax^2 + bx = c$) equations, as well as indeterminate equations such as $x^2 + y^2 = z^2$, whereby several unknowns are involved. Algebra has been developed over a period of 4000 years. But, only by the middle of the 17th Century the representation of elementary algebraic problems and relations looked much as it is today. By the early decades of the twentieth century, algebra had evolved into the study of axiomatic systems. This axiomatic approach soon came to be called modern or abstract algebra. Important new results have been discovered and the subject has found applications in all branches of mathematics and in many of the sciences as well.

3.2 Algebraic Expressions

An algebraic expression is an expression formed from any combination of numbers and variables by using the operations of addition, subtraction, multiplication, division, exponentiation (raising powers), or extraction of roots.

For instance, 7, x, $2x-3y+1$, $\frac{5x^3-1}{4xy+1}$, πr^2 and $\pi r\sqrt{r^2+h^2}$ are algebraic expressions. By an algebraic expression in certain variables, we mean an expression that contains only those variables. A constant, we mean an algebraic expression that contains no variables at all. If numbers are substituted for the variables in an algebraic expression, the resulting number is called the value of the expression for these values of variables.

If an algebraic expression consists of part connected by plus or minus signs, it is called an algebraic sum. Each part, together with the sign preceding it is called a term. For instance, in the algebraic sum $3x^2y-\frac{4xz^2}{y}+\pi x^{-1}y$, the terms are $3x^2y$, $-\frac{4xz^2}{y}$ and $\pi x^{-1}y$.

Any part of a term that is multiplied by the remaining part of the term is called the coefficient of the remaining part. For instance, in the term $-\frac{4xz^2}{y}$, the coefficient of $\frac{z^2}{y}$ is $-4x$, whereas the coefficient of $\frac{xz^2}{y}$ is -4. A coefficient such as -4, which involves no variables, is called a numerical coefficient. Terms such as $5x^2y$ and $-12x^2y$, which differ only in their numerical coefficients, are called like terms or similar terms.

An algebraic expression such as $4\pi r^2$ can be considered as an algebraic expression consisting of just one term. Such a one-termed expression is called a monomial. An algebraic expression with two terms is called a binomial and an algebraic expression with three terms is called a trinomial. For instance, the expression $3x^2+2xy$ is a binomial, whereas $-2xy^{-1}+3\sqrt{x}-4$ is a trinomial. An algebraic expression with two or more terms is called a multinomial.

3.3 Polynomials

A polynomial is an algebraic expression, in which no variables appear in denominators or under radical signs and all variables that do appear are powers of positive integers. For instance, the trinomial $-2xy^{-1}+3\sqrt{x}-4$ is not a polynomial; however, the trinomial $3x^2y^4+\sqrt{2}\,xy-\frac{1}{2}$ is a polynomial in the variables x and y. A term such as $-\frac{1}{2}$ which contains no variables is called a constant term of the polynomial. The numerical coefficients of the terms in a polynomial are called the coefficients of the polynomial. The coefficients of the polynomial above are 3, $\sqrt{2}$ and $-\frac{1}{2}$.

The degree of a term in a polynomial is the sum of the exponents of all the variables in that term. In adding exponents, one should regard a variable with no exponent as being power one. For instance, in the polynomial $9xy^7-12x^3yz^2+3x-2$, the term $9xy^7$ has degree $1+7=8$, the term $-12x^3yz^2$ has degree $3+1+2=6$, and the term $3x$ has degree one. The constant term is always regarded as having degree zero.

The degree of the highest degree term that appears with non-zero coefficients in a polynomial is called the degree of the polynomial.

For instance, the polynomial considered above has degree 8. Although the constant monomial 0 is regarded as a polynomial, this particular polynomial is not assigned a degree.

3.3.1 Polynomials in One Variable

In this section we consider only polynomials in one variable.

Key Concept **Polynomial in One Variable**

A polynomial in one variable x is an algebraic expression of the form

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \quad a_n \neq 0$$

where $a_0, a_1, a_2, \cdots, a_{n-1}, a_n$ are constants and n is a non negative integer.

Here n is the degree of the polynomial and $a_1, a_2, \cdots, a_{n-1}, a_n$ are the coefficients of $x, x^2, \cdots x^{n-1}, x^n$ respectively. a_0 is the constant term. $a_n x^n, a_{n-1} x^{n-1}, \cdots, a_2 x^2, a_1 x, a_0$ are the terms of the polynomial $p(x)$.

For example, in the polynomial $5x^2 + 3x - 1$, the coefficient of x^2 is 5, the coefficient of x is 3 and –1 is the constant term. The three terms of the polynomial are $5x^2, 3x$ and -1.

3.3.2 Types of Polynomials

Key Concept **Types of Polynomials Based on Number of Terms**

Monomial

Polynomials which have only one term are known as monomials.

Binomial

Polynomials which have only two terms are called binomials.

Trinomial

Polynomials which have only three terms are named as trinomials.

Note

1. A binomial is the sum of two monomials of different degrees.
2. A trinomial is the sum of three monomials of different degrees.
3. A polynomial is a monomial or the sum of two or more monomials.

Key Concept — **Types of Polynomials Based on the Degree**

Constant Polynomial

A polynomial of degree zero is called a constant polynomial.

General form : $p(x) = c$, where c is a real number.

Linear Polynomial

A polynomial of degree one is called a linear polynomial.

General form : $p(x) = ax+b$, where a and b are real numbers and $a \neq 0$.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.

General form: $p(x)=ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.

General form : $p(x) = ax^3 + bx^2 + cx + d$, where a,b,c and d are real numbers and $a \neq 0$.

Example 3.1

Classify the following polynomials based on number of terms.

(i) $x^3 - x^2$ (ii) $5x$ (iii) $4x^4 + 2x^3 + 1$ (iv) $4x^3$

(v) $x + 2$ (vi) $3x^2$ (vii) $y^4 + 1$ (viii) $y^{20} + y^{18} + y^2$

(ix) 6 (x) $2u^3 + u^2 + 3$ (xi) $u^{23} - u^4$ (xii) y

Solution

$5x$, $3x^2$, $4x^3$, y and 6 are monomials because they have only one term.

$x^3 - x^2$, $x + 2$, $y^4 + 1$ and $u^{23} - u^4$ are binomials as they contain only two terms.

$4x^4 + 2x^3 + 1$, $y^{20} + y^{18} + y^2$ and $2u^3 + u^2 + 3$ are trinomials as they contain only three terms.

Example 3.2

Classify the following polynomials based on their degree.

(i) $p(x) = 3$ (ii) $p(y) = \frac{5}{2}y^2 + 1$ (iii) $p(x) = 2x^3 - x^2 + 4x + 1$

(iv) $p(x) = 3x^2$ (v) $p(x) = x + 3$ (vi) $p(x) = -7$

(vii) $p(x) = x^3 + 1$ (viii) $p(x) = 5x^2 - 3x + 2$ (ix) $p(x) = 4x$

(x) $p(x) = \frac{3}{2}$ (xi) $p(x) = \sqrt{3}x + 1$ (xii) $p(y) = y^3 + 3y$

Solution

$p(x) = 3,\ p(x) = -7,\ p(x) = \dfrac{3}{2}$ are constant polynomials.

$p(x) = x + 3,\ p(x) = 4x,\ p(x) = \sqrt{3}\,x + 1$ are linear polynomials, since the highest degree of the variable x is one.

$p(x) = 5x^2 - 3x + 2,\ p(y) = \dfrac{5}{2}y^2 + 1,\ p(x) = 3x^2$ are quadratic polynomials, since the highest degree of the variable is two.

$p(x) = 2x^3 - x^2 + 4x + 1,\ p(x) = x^3 + 1,\ p(y) = y^3 + 3y$ are cubic polynomials, since the highest degree of the variable is three.

Exercise: 3.1

1. State whether the following expressions are polynomials in one variable or not. Give reasons for your answer.

   (i) $2x^5 - x^3 + x - 6$ (ii) $3x^2 - 2x + 1$ (iii) $y^3 + 2\sqrt{3}$

   (iv) $x - \dfrac{1}{x}$ (v) $\sqrt[3]{t} + 2t$ (vi) $x^3 + y^3 + z^6$

2. Write the coefficient of x^2 and x in each of the following.

   (i) $2 + 3x - 4x^2 + x^3$ (ii) $\sqrt{3}\,x + 1$ (iii) $x^3 + \sqrt{2}\,x^2 + 4x - 1$

   (iv) $\dfrac{1}{3}x^2 + x + 6$

3. Write the degree of each of the following polynomials.

   (i) $4 - 3x^2$ (ii) $5y + \sqrt{2}$ (iii) $12 - x + 4x^3$ (iv) 5

4. Classify the following polynomials based on their degree.

   (i) $3x^2 + 2x + 1$ (ii) $4x^3 - 1$ (iii) $y + 3$

   (iv) $y^2 - 4$ (v) $4x^3$ (vi) $2x$

5. Give one example of a binomial of degree 27 and monomial of degree 49 and trinomial of degree 36.

3.3.3 Zeros of a Polynomial

Consider the polynomial $p(x) = x^2 - x - 2$. Let us find the values of $p(x)$ at $x = -1$, $x = 1$ and $x = 2$.

$p(-1) = (-1)^2 - (-1) - 2 = 1 + 1 - 2 = 0$

$p(1) = (1)^2 - 1 - 2 = 1 - 1 - 2 = -2$

$p(2) = (2)^2 - 2 - 2 = 4 - 2 - 2 = 0$

That is, 0, –2 and 0 are the values of the polynomial $p(x)$ at $x = -1$, 1 and 2 respectively.

If the value of a polynomial is zero for some value of the variable then that value is known as zero of the polynomial.

Since $p(-1) = 0$, $x = -1$ is a zero of the polynomial $p(x) = x^2 - x - 2$.

Similarly, $p(2) = 0$ at $x = 2$, $\Longrightarrow$ 2 is also a zero of $p(x)$.

Key Concept	**Zeros of Polynomial**
Let $p(x)$ be a polynomial in x. If $p(a) = 0$, then we say that a is a zero of the polynomial $p(x)$.	

Note Number of zeros of a polynomial $\leq$ the degree of the polynomial. Carl Friedrich Gauss (1777-1855) had proven in his doctoral thesis of 1798 that the polynomial equations of any degree n must have exactly n solutions in a certain very specific sense. This result was so important that it became known as the fundamental theorem of algebra. The exact sense in which that theorem is true is the subject of the other part of the story of algebraic numbers.

Example 3.3

If $p(x) = 5x^3 - 3x^2 + 7x - 9$, find (i) $p(-1)$ (ii) $p(2)$.

Solution Given that $p(x) = 5x^3 - 3x^2 + 7x - 9$

(i) $p(-1) = 5(-1)^3 - 3(-1)^2 + 7(-1) - 9 = -5 - 3 - 7 - 9$

$\therefore\ p(-1) = -24$

(ii) $p(2) = 5(2)^3 - 3(2)^2 + 7(2) - 9 = 40 - 12 + 14 - 9$

$\therefore\ p(2) = 33$

Example 3.4

Find the zeros of the following polynomials.

(i) $p(x) = 2x - 3$ (ii) $p(x) = x - 2$

Solution

(i) Given that $p(x) = 2x - 3 = 2\left(x - \frac{3}{2}\right)$. We have

$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2} - \frac{3}{2}\right) = 2(0) = 0$

Hence $\frac{3}{2}$ is the zero of $p(x)$.

(ii) Given that $p(x) = x - 2$. Now,

$p(2) = 2 - 2 = 0$

Hence 2 is the zero of $p(x)$.

3.3.4 Roots of a Polynomial Equations

Let $p(x)$ be a polynomial expression in x. Then $p(x) = 0$ is called a polynomial equation in x.

Consider the polynomial $p(x) = x - 1$. Clearly 1 is the zero of the polynomial $p(x) = x - 1$.Now, consider the polynomial equation $p(x) = 0$, that is, $x - 1 = 0$. Now, $x - 1 = 0$ implies $x = 1$. The value $x = 1$ is called the root of the polynomial equation $p(x) = 0$.

Hence zeros of a polynomial are the roots of the corresponding polynomial equation.

Key Concept	**Root of a Polynomial Equation**
If $x = a$ satisfies the polynomial equation $p(x) = 0$, then $x = a$ is called a root of the polynomial equation $p(x) = 0$.	

Example 3.5

Find the roots of the following polynomial equations

(i) $x - 6 = 0$ (ii) $2x + 1 = 0$

Solution

(i) Given that $x - 6 = 0 \Longrightarrow x = 6$

$\therefore$ $x = 6$ is a root of $x - 6 = 0$

(ii) Given that $2x + 1 = 0 \Longrightarrow 2x = -1 \Longrightarrow x = -\frac{1}{2}$

$\therefore$ $x = -\frac{1}{2}$ is a root of $2x + 1 = 0$

Example 3.6

Verify whether the following are roots of the polynomial equations indicated against them.

(i) $2x^2 - 3x - 2 = 0$; $x = 2, 3$

(ii) $x^3 + 8x^2 + 5x - 14 = 0$; $x = 1, 2$

Solution

(i) Let $p(x) = 2x^2 - 3x - 2$.

$p(2) = 2(2)^2 - 3(2) - 2 = 8 - 6 - 2 = 0$

Hence, $x = 2$ is a root of $2x^2 - 3x - 2 = 0$

But $p(3) = 2(3)^2 - 3(3) - 2 = 18 - 9 - 2 = 7 \neq 0$

Hence, $x = 3$ is not a root of $2x^2 - 3x - 2 = 0$

(ii) Let $p(x) = x^3 + 8x^2 + 5x - 14$

$p(1) = (1)^3 + 8(1)^2 + 5(1) - 14 = 1 + 8 + 5 - 14 = 0$

$x = 1$ is a root of $x^3 + 8x^2 + 5x - 14 = 0$

But $p(2) = (2)^3 + 8(2)^2 + 5(2) - 14 = 8 + 32 + 10 - 14 = 36 \neq 0$

$x = 2$ is not a root of $x^3 + 8x^2 + 5x - 14 = 0$

Exercise 3.2

1. Find the zeros of the following polynomials.

   (i) $p(x) = 4x - 1$ (ii) $p(x) = 3x + 5$ (iii) $p(x) = 2x$ (iv) $p(x) = x + 9$

2. Find the roots of the following polynomial equations.

   (i) $x - 3 = 0$ (ii) $5x - 6 = 0$ (iii) $11x + 1 = 0$ (iv) $-9x = 0$

3. Verify Whether the following are roots of the polynomial equations indicated against them.

   (i) $x^2 - 5x + 6 = 0; \quad x = 2, 3$ (ii) $x^2 + 4x + 3 = 0; \quad x = -1, 2$

   (iii) $x^3 - 2x^2 - 5x + 6 = 0; \ x = 1, -2, 3$ (iv) $x^3 - 2x^2 - x + 2 = 0; \ x = -1, 2, 3$

3.3.5 Division of Polynomials

The division of polynomials $p(x)$ and $g(x)$ is expressed by the following "division algorithm" of algebra.

Key Concept — **Division Algorithm for Polynomials**

Let $p(x)$ and $g(x)$ be two polynomials such that degree of $p(x) \geq$ degree of $g(x)$ and $g(x) \neq 0$. Then there exists unique polynomials $q(x)$ and $r(x)$ such that

$$p(x) = g(x)\,q(x) + r(x) \qquad \ldots (1)$$

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.

The polynomial $p(x)$ is the dividend, $g(x)$ is the divisor, $q(x)$ is the quotient and $r(x)$ is the remainder.

$(1) \Longrightarrow$ Dividend = (Divisor $\times$ Quotient) + Remainder

Example 3.7

Find the quotient and the remainder when $10 - 4x + 3x^2$ is divided by $x - 2$.

Solution Let us first write the terms of each polynomial in descending order (or ascending order). Thus, the given problem becomes $(3x^2 - 4x + 10) \div (x - 2)$

$$\begin{array}{r|l} & \quad 3x + 2 \\ \hline x - 2 & 3x^2 - 4x + 10 \\ & 3x^2 - 6x \\ & \underline{- \quad +} \\ & \qquad 2x + 10 \\ & \qquad 2x - 4 \\ & \qquad \underline{- \quad +} \\ & \qquad\qquad 14 \end{array}$$

(i) $\dfrac{3x^2}{x} = 3x$

(ii) $3x(x - 2) = 3x^2 - 6x$

(iii) $\dfrac{2x}{x} = 2$

(iv) $2(x - 2) = 2x - 4$

$\therefore$ Quotient $= 3x + 2$ and Remainder$= 14$

(i.e. $3x^2 - 4x + 10 = (x - 2)(3x + 2) + 14$ and is in the form

Dividend = (Divisor $\times$ quotient)+Remainder)

Example 3.8

Find the quotient and the remainder $(4x^3 + 6x^2 - 23x - 15) \div (3 + x)$.

Solution Write the given polynomials in ascending or decending order.
i.e. $(4x^3 + 6x^2 - 23x - 15) \div (x + 3)$

$$
\begin{array}{r|l}
 & 4x^2 - 6x - 5 \\ \hline
x + 3 & 4x^3 + 6x^2 - 23x - 15 \\
 & 4x^3 + 12x^2 \\
 & (-) \quad (-) \\ \hline
 & \quad -6x^2 - 23x \\
 & \quad -6x^2 - 18x \\
 & \quad (+) \quad (+) \\ \hline
 & \qquad -5x - 15 \\
 & \qquad -5x - 15 \\
 & \qquad (+) \quad (+) \\ \hline
 & \qquad\qquad 0
\end{array}
$$

(i) $\dfrac{4x^3}{x} = 4x^2$

(ii) $4x^2(x + 3) = 4x^3 + 12x^2$

(iii) $\dfrac{-6x^2}{x} = 6x$

(iv) $-6x(x + 3) = -6x^2 - 18x$

(v) $\dfrac{-5x}{x} = -5$

(vi) $-5(x + 3) = -5x - 15$

$\therefore$ Quotient $= 4x^2 - 6x - 5$

Remainder $= 0$

Example 3.9

If $8x^3 - 14x^2 - 19x - 8$ is divided by $4x + 3$ then find the quotient and the remainder.

Solution

$$
\begin{array}{r|l}
 & 2x^2 - 5x - 1 \\ \hline
4x + 3 & 8x^3 - 14x^2 - 19x - 8 \\
 & 8x^3 + 6x^2 \\
 & (-) \quad (-) \\ \hline
 & \quad -20x^2 - 19x \\
 & \quad -20x^2 - 15x \\
 & \quad (+) \quad (+) \\ \hline
 & \qquad -4x - 8 \\
 & \qquad -4x - 3 \\
 & \qquad (+) \quad (+) \\ \hline
 & \qquad\qquad -5
\end{array}
$$

(i) $\dfrac{8x^3}{4x} = 2x^2$

(ii) $2x^2(4x + 3) = 8x^3 + 6x^2$

(iii) $\dfrac{-20x^2}{4x} = -5x$

(iv) $-5x(4x + 3) = -20x^2 - 15x$

(v) $\dfrac{-4x}{4x} = -1$

(vi) $-1(4x + 3) = -4x - 3$

$\therefore$ Quotient $= 2x^2 - 5x - 1$, Remainder $= -5$

Exercise 3.3

1. Find the quotient the and remainder of the following division.

(1) $(5x^3 - 8x^2 + 5x - 7) \div (x - 1)$

(2) $(2x^2 - 3x - 14) \div (x + 2)$

(3) $(9 + 4x + 5x^2 + 3x^3) \div (x + 1)$

(4) $(4x^3 - 2x^2 + 6x + 7) \div (3 + 2x)$

(5) $(-18 - 9x + 7x^2) \div (x - 2)$

3.4 Remainder Theorem

Remainder Theorem

Let $p(x)$ be any polynomial and a be any real number. If $p(x)$ is divided by the linear polynomial $x-a$, then the remainder is $p(a)$.

Note

1. If $p(x)$ is divided by $(x+a)$, then the remainder is $p(-a)$.
2. If $p(x)$ is divided by $(ax-b)$, then the remainder is $p\left(\frac{b}{a}\right)$.
3. If $p(x)$ is divided by $(ax+b)$, then the remainder is $p\left(-\frac{b}{a}\right)$.
4 Here $-a$, $\frac{b}{a}$ and $-\frac{b}{a}$ are the zeros of the divisors $x+a$, $ax-b$ and $ax+b$ respectively.

Example 3.10

Find the remainder when $4x^3-5x^2+6x-2$ is divided by $x-1$.

Solution Let $p(x)=4x^3-5x^2+6x-2$. The zero of $x-1$ is 1.

When $p(x)$ is divided by $(x-1)$ the remainder is $p(1)$. Now,

$$p(1)=4(1)^3-5(1)^2+6(1)-2$$
$$=4-5+6-2=3$$

$\therefore$The remainder is 3.

Example 3.11

Find the remainder when x^3-7x^2-x+6 is divided by $(x+2)$.

Solution Let $p(x)=x^3-7x^2-x+6$. The zero of $x+2$ is -2.

When $p(x)$ is divided by $x+2$, the remainder is $p(-2)$. Now,

$$p(-2)=(-2)^3-7(-2)^2-(-2)+6$$
$$=-8-7(4)+2+6$$
$$=-8-28+2+6=-28$$

$\therefore$The remainder is -28.

Example 3.12

Find the value of a if $2x^3-6x^2+5ax-9$ leaves the remainder 13 when it is divided by $x-2$.

Solution Let $p(x) = 2x^3 - 6x^2 + 5ax - 9$.

When $p(x)$ is divided by $(x-2)$ the remainder is $p(2)$.

$$
\begin{aligned}
\text{Given that } p(2) &= 13 \\
\Longrightarrow \quad 2(2)^3 - 6(2)^2 + 5a(2) - 9 &= 13 \\
2(8) - 6(4) + 10a - 9 &= 13 \\
16 - 24 + 10a - 9 &= 13 \\
10a - 17 &= 13 \\
10a &= 30 \\
\therefore \quad a &= 3
\end{aligned}
$$

Example 3.13

Find the remainder when $x^3 + ax^2 - 3x + a$ is divided by $x + a$.

Solution

Let $p(x) = x^3 + ax^2 - 3x + a$.

When $p(x)$ is divided by $(x+a)$ the remainder is $p(-a)$.

$p(-a) = (-a)^3 + a(-a)^2 - 3(-a) + a = -a^3 + a^3 + 4a = 4a$

$\therefore$ The remainder is $4a$.

Example 3.14

Find the remainder when $f(x) = 12x^3 - 13x^2 - 5x + 7$ is divided by $(3x+2)$.

Solution $f(x) = 12x^3 - 13x^2 - 5x + 7$.

When $f(x)$ is divided by $(3x+2)$ the remainder is $f\left(-\frac{2}{3}\right)$. Now,

$$
\begin{aligned}
f\left(-\frac{2}{3}\right) &= 12\left(-\frac{2}{3}\right)^3 - 13\left(-\frac{2}{3}\right)^2 - 5\left(-\frac{2}{3}\right) + 7 \\
&= 12\left(-\frac{8}{27}\right) - 13\left(\frac{4}{9}\right) + \frac{10}{3} + 7 \\
&= -\frac{32}{9} - \frac{52}{9} + \frac{10}{3} + 7 = \frac{9}{9} = 1
\end{aligned}
$$

$\therefore$ The remainder is 1.

Example 3.15

If the polynomials $2x^3 + ax^2 + 4x - 12$ and $x^3 + x^2 - 2x + a$ leave the same remainder when divided by $(x-3)$, find the value of a. Also find the remainder.

Solution Let $p(x) = 2x^3 + ax^2 + 4x - 12$,

$q(x) = x^3 + x^2 - 2x + a$

When $p(x)$ is divided by $(x-3)$ the remainder is $p(3)$. Now,

$$p(3) = 2(3)^3 + a(3)^2 + 4(3) - 12$$
$$= 2(27) + a(9) + 12 - 12$$
$$= 54 + 9a \qquad (1)$$

When $q(x)$ is divided by $(x-3)$ the remainder is $q(3)$. Now,

$$q(3) = (3)^3 + (3)^2 - 2(3) + a$$
$$= 27 + 9 - 6 + a$$
$$= 30 + a \qquad (2)$$

Given that $p(3) = q(3)$. That is,

$$54 + 9a = 30 + a \quad (\text{ By (1) and (2) })$$
$$9a - a = 30 - 54$$
$$8a = -24$$
$$\therefore a = -\frac{24}{8} = -3$$

Substituting $a = -3$ in $p(3)$, we get

$$p(3) = 54 + 9(-3) = 54 - 27 = 27$$

$\therefore$ The remainder is 27.

Exercise 3.4

1. Find the remainder using remainder theorem, when
   (i) $3x^3 + 4x^2 - 5x + 8$ is divided by $x - 1$
   (ii) $5x^3 + 2x^2 - 6x + 12$ is divided by $x + 2$
   (iii) $2x^3 - 4x^2 + 7x + 6$ is divided by $x - 2$
   (iv) $4x^3 - 3x^2 + 2x - 4$ is divided by $x + 3$
   (v) $4x^3 - 12x^2 + 11x - 5$ is divided by $2x - 1$
   (vi) $8x^4 + 12x^3 - 2x^2 - 18x + 14$ is divided by $x + 1$
   (vii) $x^3 - ax^2 - 5x + 2a$ is divided by $x - a$

2. When the polynomial $2x^3 - ax^2 + 9x - 8$ is divided by $x - 3$ the remainder is 28. Find the value of a.

3. Find the value of m if $x^3 - 6x^2 + mx + 60$ leaves the remainder 2 when divided by $(x + 2)$.

4. If $(x-1)$ divides $mx^3 - 2x^2 + 25x - 26$ without remainder find the value of m.
5. If the polynomials $x^3 + 3x^2 - m$ and $2x^3 - mx + 9$ leave the same remainder when they are divided by $(x-2)$, find the value of m. Also find the remainder.

3.5 Factor Theorem

Factor Theorem

Let $p(x)$ be a polynomial and a be any real number. If $p(a) = 0$, then $(x-a)$ is a factor of $p(x)$.

Note If $(x-a)$ is a factor of $p(x)$, then $p(a) = 0$

Example 3.16

Determine whether $(x-5)$ is a factor of the polynomial $p(x) = 2x^3 - 5x^2 - 28x + 15$.

Solution By factor theorem, if $p(5) = 0$, then $(x-5)$ is a factor of $p(x)$. Now,

$$\begin{aligned} p(5) &= 2(5)^3 - 5(5)^2 - 28(5) + 15 \\ &= 2(125) - 5(25) - 140 + 15 \\ &= 250 - 125 - 140 + 15 = 0 \end{aligned}$$

$\therefore$ $(x-5)$ is a factor of $p(x) = 2x^3 - 5x^2 - 28x + 15$.

Example 3.17

Determine whether $(x-2)$ is a factor of the polynomial $2x^3 - 6x^2 + 5x + 4$.

Solution Let $p(x) = 2x^3 - 6x^2 + 5x + 4$. By factor theorem,$(x-2)$ is a factor of $p(x)$ if $p(2) = 0$. Now,

$$\begin{aligned} p(2) &= 2(2)^3 - 6(2)^2 + 5(2) + 4 = 2(8) - 6(4) + 10 + 4 \\ &= 16 - 24 + 10 + 4 = 6 \neq 0 \end{aligned}$$

$\therefore$ $(x-2)$ is not a factor of $2x^3 - 6x^2 + 5x + 4$.

Example 3.18

Determine whether $(2x-3)$ is a factor of $2x^3 - 9x^2 + x + 12$.

Solution Let $p(x) = 2x^3 - 9x^2 + x + 12$. By factor theorem, $(2x-3)$ is a factor of $p(x)$ if $p\left(\frac{3}{2}\right) = 0$. Now,

$$p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \frac{3}{2} + 12 = 2\left(\frac{27}{8}\right) - 9\left(\frac{9}{4}\right) + \frac{3}{2} + 12$$

$$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 \qquad = \frac{27 - 81 + 6 + 48}{4} = 0$$

$\therefore$ $(2x - 3)$ is a factor of $2x^3 - 9x^2 + x + 12$.

Example 3.19

Determine the value of m if $(x - 1)$ is a factor of $x^3 + 5x^2 + mx + 4$.

Solution Let $p(x) = x^3 + 5x^2 + mx + 4$. Since $(x - 1)$ is a factor of $p(x)$, the remainder $p(1) = 0$. Now,

$$p(1) = 0 \Longrightarrow (1)^3 + 5(1)^2 + m(1) + 4 = 0$$

$$\Longrightarrow 1 + 5 + m + 4 = 0$$

$$m + 10 = 0$$

$$\therefore m = -10$$

Exercise 3.5

1. Determine whether $(x + 1)$ is a factor of the following polynomials.

   (i) $6x^4 + 7x^3 - 5x - 4$ (ii) $2x^4 + 9x^3 + 2x^2 + 10x + 15$

   (iii) $3x^3 + 8x^2 - 6x - 5$ (iv) $x^3 - 14x^2 + 3x + 12$

2. Determine whether $(x + 4)$ is a factor of $x^3 + 3x^2 - 5x + 36$.
3. Using factor theorem show that $(x - 1)$ is a factor of $4x^3 - 6x^2 + 9x - 7$.
4. Determine whether $(2x + 1)$ is a factor of $4x^3 + 4x^2 - x - 1$.
5. Determine the value of p if $(x + 3)$ is a factor of $x^3 - 3x^2 - px + 24$.

Exercise 3.6

Multiple Choice Questions.

1. The coefficients of x^2 and x in $2x^3 - 3x^2 - 2x + 3$ are respectively

   (A) 2,3 (B) –3,–2 (C) –2,–3 (D) 2,–3

2. The degree of the polynomial $4x^2 - 7x^3 + 6x + 1$ is

   (A) 2 (B) 1 (C) 3 (D) 0

3. The polynomial $3x - 2$ is a

   (A) linear polynomial (B) quadratic polynomial

   (C) cubic polynomial (D) constant polynomial

4. The polynomial $4x^2 + 2x - 2$ is a

   (A) linear polynomial (B) quadratic polynomial

   (C) cubic polynomial (D) constant polynomial

5. The zero of the polynomial $2x - 5$ is

(A) $\frac{5}{2}$ (B) $-\frac{5}{2}$ (C) $\frac{2}{5}$ (D) $-\frac{2}{5}$

6. The root of the polynomial equation $3x - 1 = 0$ is

(A) $x = -\frac{1}{3}$ (B) $x = \frac{1}{3}$ (C) $x = 1$ (D) $x = 3$

7. The roots of the polynomial equation $x^2 + 2x = 0$ are

(A) $x = 0, 2$ (B) $x = 1, 2$ (C) $x = 1, -2$ (D) $x = 0, -2$

8. If a polynomial $p(x)$ is divided by $(ax + b)$, then the remainder is

(A) $p\left(\frac{b}{a}\right)$ (B) $p\left(-\frac{b}{a}\right)$ (C) $p\left(\frac{a}{b}\right)$ (D) $p\left(-\frac{a}{b}\right)$

9. If the polynomial $x^3 - ax^2 + 2x - a$ is divided $(x - a)$, then remainder is

(A) a^3 (B) a^2 (C) a (D) $-a$

10. If $(ax - b)$ is a factor of $p(x)$, then

(A) $p(b) = 0$ (B) $p\left(-\frac{b}{a}\right) = 0$ (C) $p(a) = 0$ (D) $p\left(\frac{b}{a}\right) = 0$

11. One of the factors of $x^2 - 3x - 10$ is

(A) $x - 2$ (B) $x + 5$ (C) $x - 5$ (D) $x - 3$

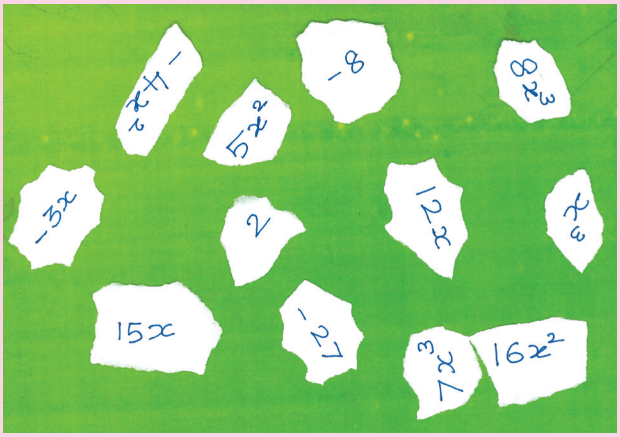
12. One of the factors of $x^3 - 2x^2 + 2x - 1$ is

(A) $x - 1$ (B) $x + 1$ (C) $x - 2$ (D) $x + 2$

Points to Remember

- A polynomial in one variable x is an algebraic expression of the form $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$, $a_n \neq 0$ where $a_0, a_1, a_2, \cdots, a_{n-1}, a_n$ are constants and n is a non negative integer .
- Let $p(x)$ be a polynomial. If $p(a) = 0$ then we say that a is a zero of the polynomial $p(x)$
- If $x = a$ satisfies the polynomial equation $p(x) = 0$ then $x = a$ is called a root of the polynomial equation $p(x) = 0$.
- Remainder Theorem : Let $p(x)$ be any polynomial and a be any real number. If $p(x)$ is divided by the linear polynomial $x - a$, then the remainder is $p(a)$.
- Factor Theorem : Let $p(x)$ be a polynomial and a be any real number. If $p(a) = 0$, then $(x - a)$ is a factor of $p(x)$.

Activity 1



From the terms given in the left side:

(i) Construct atleast three different polynomials in each type based on number of terms.

(ii) Construct at least three different polynomials in each type based on their degrees.

Activity 2

Using long division method find the remainder when $4x^3 - 5x^2 + 7x + 6$ is divided by $(x - 1)$. Verify your answer by using remainder theorem.

Activity 3

Using long division method find the remainder when $2x^3 - 6x^2 + 5x - 2$ is divided by $(x - 2)$. What do you infer?

Think it over !

A polynomial is divided by any other polynomial

Exercise 3.1

1. (i) Polynomial in one variable (ii) Polynomial in one variable
   (iii) Polynomial in one variable
   (iv) Since the exponent of x is not a whole number is not a polynomial.
   (v) Since the exponent of t is not a whole number is not a polynomial.
   (vi) Polynomial in three variables.

2. (i) $-4, 3$ (ii) $0, \sqrt{3}$ (iii) $\sqrt{2}, 4$ (iv) $\frac{1}{3}, 1$

3. (i) 2 (ii) 1 (iii) 3 (iv) 0

4. (i) quadratic polynomial (ii) cubic polynomial (iii) linear polynomial
   (iv) quadratic polynomial (v) cubic polynomial (vi) linear polynomial

5. $ax^{27} + b, \ \ cx^{49}, \ \ lx^{36} + mx^{35} + nx^{2}$

Exercise 3.2

1. (i) $x = \frac{1}{4}$ (ii) $x = -\frac{5}{3}$ (iii) $x = 0$ (iv) $x = -9$
2. (i) $x = 3$ (ii) $x = \frac{6}{5}$ (iii) $x = -\frac{1}{11}$ (iv) $x = 0$
3. (i) $x = 2$ is a root, $x = 3$ is a root (ii) $x = -1$ is a root, $x = 2$ is not a root
   (iii) $x = 1$ is a root, $x = -2$ is a root, $x = 3$ is a root
   (iv) $x = -1$ is a root, $x = 2$ is a root, $x = 3$ is not a root

Exercise 3.3

1. quotient $5x^2 - 3x + 2$, remainder -5
2. quotient $2x - 7$, remainder 0
3. quotient $3x^2 + 2x + 2$, remainder 7
4. quotient $2x^2 - 4x + 9$, remainder -20
5. quotient $7x + 5$, remainder -8

Exercise 3.4

1. (i) 10 (ii) -8 (iii) 20 (iv) -145 (v) -2 (vi) 26 (vii) $-3a$

2. $a = 5$ **3.** $m = 13$ **4.** $m = 3$ **4.** $m = 5$, remainder is 15.

Exercise 3.5

1. (i) Factor (ii) Factor (iii) Not a factor (iv) Not a factor

2. Not a factor **4.** Factor **5.** $p = 10$

Exercise 3.6

1. B **2.** C **3.** A **4.** B **5.** A **6.** B **7.** D **8.** B **9.** C **10.** D **11.** C **12.** A

4 GEOMETRY

Truth can never be told so as to be understood,
and not to be believed

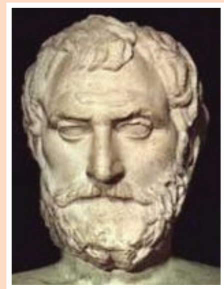
- William Blake

Main Targets

- To recall the basic concepts of geometry.
- To understand theorems on parallelograms.

4.1 Introduction

The very name ***Geometry*** is derived from two greek words meaning *measurement of earth*. Over time geometry has evolved into a beautifully arranged and logically organized body of knowledge. It is concerned with the properties and relationships between points, lines, planes and figures. The earliest records of geometry can be traced to ancient Egypt and the Indus Valley from around 3000 B.C. Geometry begins with undefined terms, definitions, and assumptions; these lead to theorems and constructions. It is an abstract subject, but easy to visualize and it has many concrete practical applications. Geometry has long been important for its role in the surveying of land and more recently, our knowledge of geometry has been applied to help build structurally sound bridges, experimental space stations, and large athletic and entertainment arenas, just to mention a few examples. The geometrical theorem of which a particular case involved in the method just described in the first book of Euclid's *Elements***.**

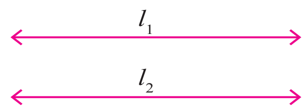
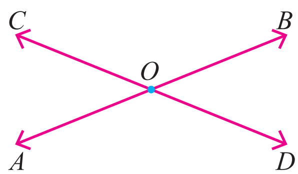
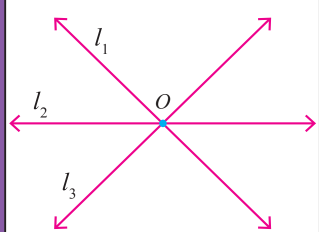
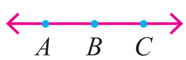


THALES
(640 - 546 BC)

Thales (pronounced THAY-lees) was born in the Greek city of Miletus. He was known for his theoretical and practical understanding of geometry, especially triangles. He established what has become known as Thales' Theorem, whereby if a triangle is drawn within a circle with the long side as a diameter of the circle then the opposite angle will always be a right angle. Thales used geometry to solve problems such as calculating the height of pyramids and the distance of ships from the shore. He is credited with the first use of deductive reasoning applied to geometry, by deriving four corollaries to Thales' Theorem. As a result, he has been hailed as the first true mathematician and is the first known individual to whom a mathematical discovery has been attributed. He was one of the so-called Seven Sages or Seven Wise Men of Greece, and many regard him as the first philosopher in the Western tradition.

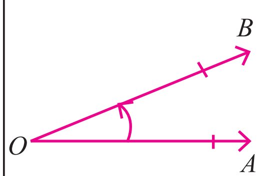
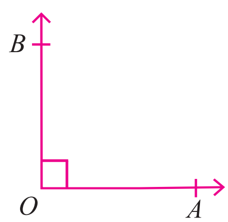
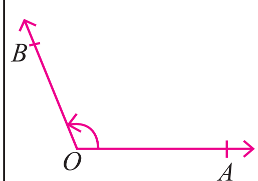
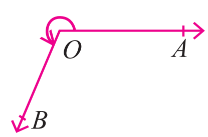
4.2 Geometry Basics

The purpose of this section is to recall some of the ideas that you have learnt in the earlier classes.

Term	Diagram	Description
Parallel lines		Lines in the same plane that do not intersect are called parallel lines. The distance between two parallel lines always remains the same.
Intersecting lines		Two lines having a common point are called intersecting lines. The point common to the two given lines is called their point of intersection. In the figure, the lines AB and CD intersect at a point O.
Concurrent lines		Three or more lines passing through the same point are said to be concurrent. In the figure, lines l_1, l_2, l_3 pass through the same point O and therefore they are concurrent.
Collinear points		If three or more points lie on the same straight line, then the points are called collinear points. Otherwise they are called non-collinear points.

4.2.1 Kinds of Angle

Angles are classified and named with reference to their degree of measure.

Name	Acute Angle	Right Angle	Obtuse Angle	Reflex Angle
Diagram				
Measure	$\angle AOB < 90^\circ$	$\angle AOB = 90^\circ$	$90^\circ < \angle AOB < 180^\circ$	$180^\circ < \angle AOB < 360^\circ$

Complementary Angles

Two angles are said to be complementary to each other if sum of their measures is 90°

For example, if $\angle A = 52^\circ$ and $\angle B = 38^\circ$, then angles $\angle A$ and $\angle B$ are complementary to each other.

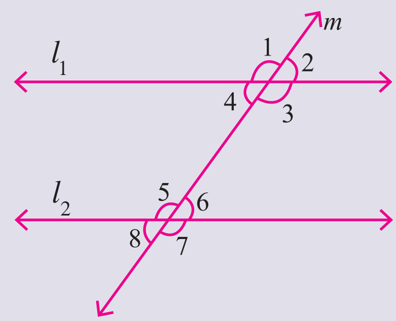
Supplementary Angles

Two angles are said to be supplementary to each other if sum of their measures is 180°. For example, the angles whose measures are 112° and 68° are supplementary to each other.

4.2.2 Transversal

A straight line that intersects two or more straight lines at distinct points is called a transversal.

Suppose a transversal intersects two parallel lines. Then:

Name	Angle	Diagram
Vertically opposite angles are equal.	$\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$, $\angle 6 = \angle 8$	
Corresponding angles are equal.	$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$	
Alternate interior angles are equal.	$\angle 3 = \angle 5$, $\angle 4 = \angle 6$	
Alternate exterior angles are equal.	$\angle 1 = \angle 7$, $\angle 2 = \angle 8$	
Consecutive interior angles are supplementary.	$\angle 3 + \angle 6 = 180^\circ$; $\angle 4 + \angle 5 = 180^\circ$	

4.2.3 Triangles

The sum of the angles of a triangle is 180°.

In the Fig. 4.1., $\angle A + \angle B + \angle C = 180^\circ$

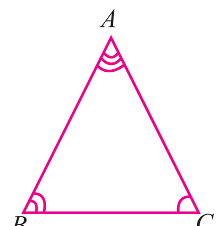


Fig. 4.1

Remarks

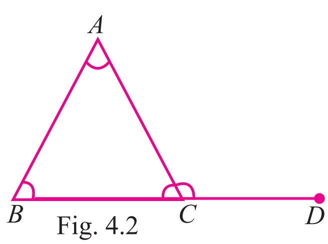


Fig. 4.2

(i) If a side of a triangle is produced , then the exterior angle so formed is equal to the sum of its interior opposite angles.

$\angle ACD = \angle BAC + \angle ABC$

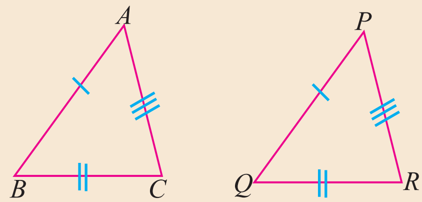
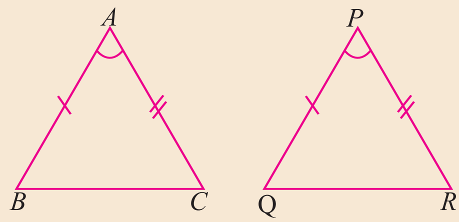
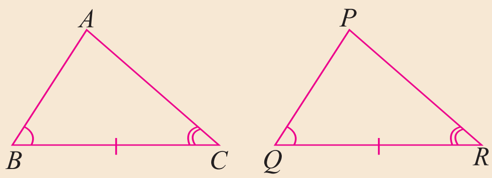
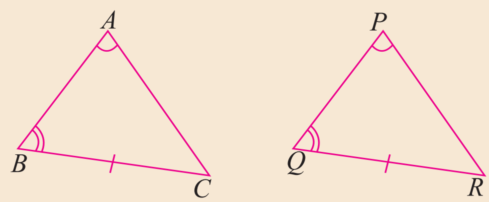
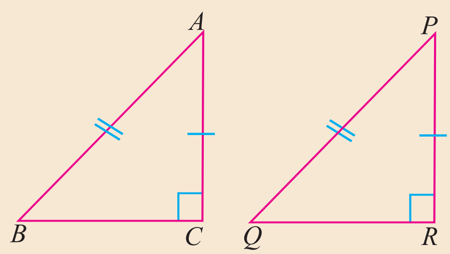
(ii) An exterior angle of a triangle is greater than either of the interior opposite angles.

(iii) In any triangle, the angle opposite to the largest side has the greatest angle.

Congruent Triangles

Two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.

For congruence, we use the symbol '$\equiv$'

	Description	Diagram
SSS	If the three sides of a triangle are equal to three sides of another triangle, then the two triangles are congruent.	 $\Delta ABC \equiv \Delta PQR$
SAS	If two sides and the included angle of a triangle are equal to two sides and the included angle of another triangle, then the two triangles are congruent.	 $\Delta ABC \equiv \Delta PQR$
ASA	If two angles and the included side of a triangle are equal to two angles and the included side of another triangle, then the two triangles are congruent.	 $\Delta ABC \equiv \Delta PQR$
AAS	If two angles and any side of a triangle are equal to two angles and a side of another triangle, then the two triangles are congruent.	 $\Delta ABC \equiv \Delta PQR$
RHS	If one side and the hypotenuse of a right triangle are equal to a side and the hypotenuse of another right triangle, then the two triangles are congruent	 $\Delta ABC \equiv \Delta PQR$

Exercise 4.1

1. Find the complement of each of the following angles.

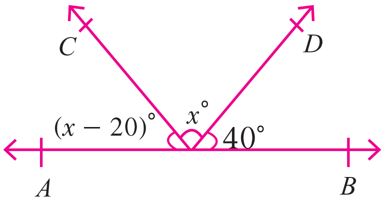
   (i) 63° (ii) 24° (iii) 48° (iv) 35° (v) 20°

2. Find the supplement of each of the following angles.

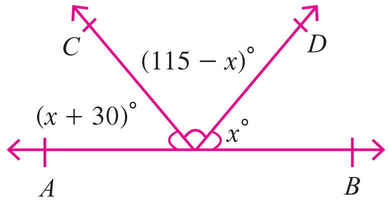
   (i) 58° (ii) 148° (iii) 120° (iv) 40° (v) 100°

3. Find the value of x in the following figures.

(i)



(ii)



4. Find the angles in each of the following.

i) The angle which is two times its complement.

ii) The angle which is four times its supplement.

iii) The angles whose supplement is four times its complement.

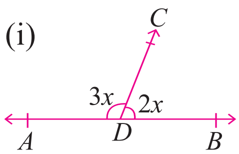
iv) The angle whose complement is one sixth of its supplement.

v) Supplementary angles are in the ratio 4:5.

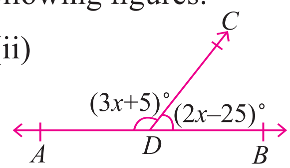
vi) Two complementary angles are in the ratio 3:2.

5. Find the values of x, y in the following figures.

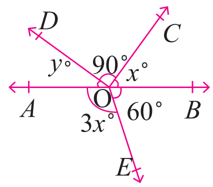
(i)



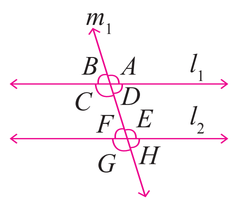
(ii)



(iii)

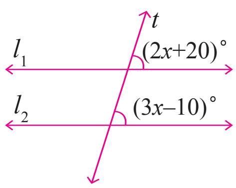


6. Let $l_1 \parallel l_2$ and m_1 is a transversal. If $\angle F = 65^\circ$, find the measure of each of the remaining angles.

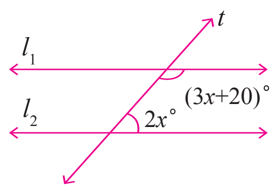


7. For what value of x will l_1 and l_2 be parallel lines.

(i)



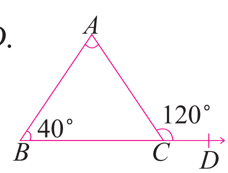
(ii)



8. The angles of a triangle are in the ratio of 1:2:3. Find the measure of each angle of the triangle.

9. In $\triangle ABC$, $\angle A + \angle B = 70^\circ$ and $\angle B + \angle C = 135^\circ$. Find the measure of each angle of the triangle.

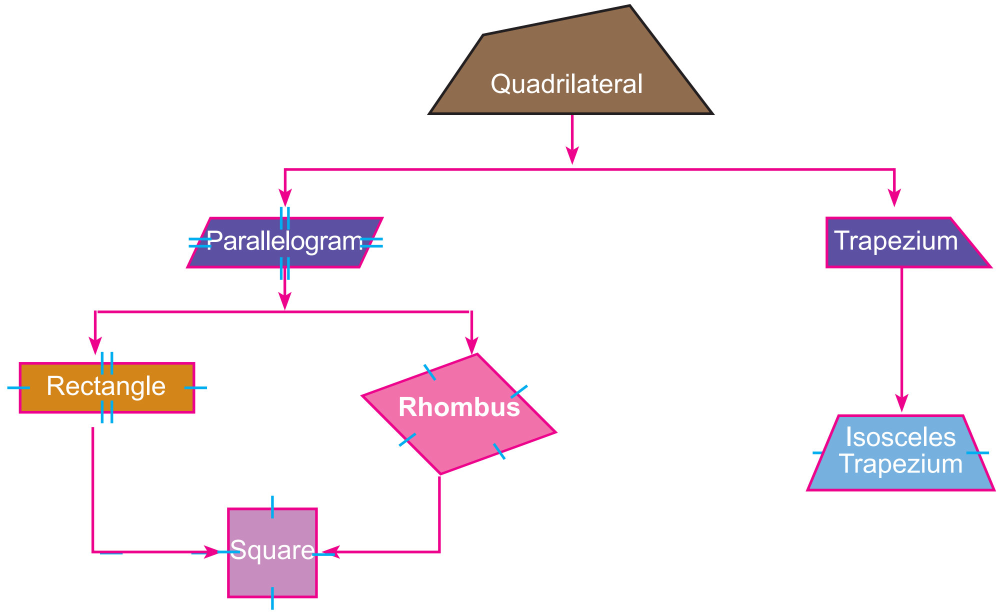
10. In the given figure at right, side BC of $\triangle ABC$ is produced to D. Find $\angle A$ and $\angle C$.



4.3 Quadrilateral

A closed geometric figure with four sides and four vertices is called a quadrilateral.

The sum of all the four angles of a quadrilateral is 360˚.



4.3.1 Properties of Trapezium, Parallelogram and Rhombus

Trapezium	Sides	One pair of opposite sides is parallel.
	Angles	The angles at the ends of each non-parallel side are supplementary.
	Diagonals	Diagonals need not be equal.
Isosceles Trapezium	Sides	One pair of opposite sides is parallel, the other pair of sides is equal in length.
	Angles	The angles at the ends of each parallel side are equal.
	Diagonals	Diagonals are equal in length.
Parallelogram	Sides	Opposite sides are parallel and equal.
	Angles	Opposite angles are equal and sum of any two adjacent angles is 180˚.
	Diagonals	Diagonals bisect each other.
Rhombus	Sides	All sides are equal and opposite sides are parallel.
	Angles	Opposite angles are equal and sum of any two adjacent angles is 180˚.
	Diagonals	Diagonals bisect each other at right angles.

Note

(i) A rectangle is an equiangular parallelogram.

(ii) A rhombus is an equilateral parallelogram.

(iii) A square is an equilateral and equiangular parallelogram.

(iv) Thus a square is a rectangle, a rhombus and a parallelogram.

4.4 Parallelogram

A quadrilateral in which the opposite sides are parallel is called a parallelogram.

4.4.1 Properties of Parallelogram

Property 1 : In a parallelogram, the opposite sides are equal.

Given : $ABCD$ is a parallelogram. So, $AB \parallel DC$ and $AD \parallel BC$

To prove : $AB = CD$ and $AD = BC$

Construction : Join BD

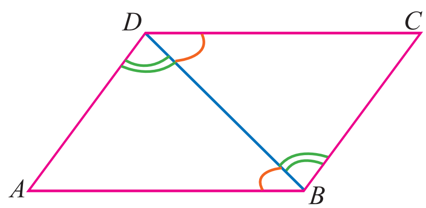


Fig. 4.3

Proof :

Consider the ΔABD and the ΔBCD.

(i) $\angle ABD = \angle BDC$ ($AB \parallel DC$ and BD is a transversal. So, alternate interior angles are equal.)

(ii) $\angle BDA = \angle DBC$ ($AD \parallel BC$ and BD is a transversal. So, alternate interior angles are equal.)

(iii) BD is common side

$\therefore \Delta ABD \equiv \Delta BCD$ (By ASA property)

Thus, $AB = DC$ and $AD = BC$ (Corresponding sides are equal) ■

Converse of Property 1: If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.

Property 2 : In a parallelogram, the opposite angles are equal.

Given : $ABCD$ is a parallelogram, where $AB \parallel DC$, $AD \parallel BC$

To prove : $\angle ABC = \angle ADC$ and $\angle DAB = \angle BCD$

Construction : Join BD

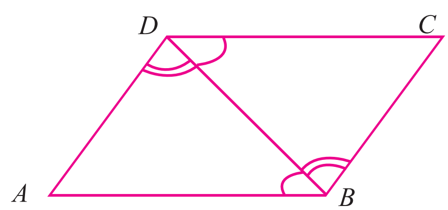


Fig. 4.4

Proof :

(i) $\angle ABD = \angle BDC$ ($AB \parallel DC$ and BD is a transversal. So, alternate interior angles are equal.)

(ii) $\angle DBC = \angle BDA$ ($AD \parallel BC$ and BD is a transversal. So, alternate interior angles are equal.)

(iii) $\angle ABD + \angle DBC = \angle BDC + \angle BDA$

$\therefore \angle ABC = \angle ADC$

Similarly, $\angle BAD = \angle BCD$ ■

Converse of Property 2: If the opposite angles in a quadrilateral are equal, then the quadrilateral is a parallelogram.

Property 3 : The diagonals of a parallelogram bisect each other.

Given : $ABCD$ is a parallelogram, in which $AB \parallel DC$ and $AD \parallel BC$

To prove : M is the midpoint of diagonals AC and BD .

Proof :

Consider the ΔAMB and ΔCMD

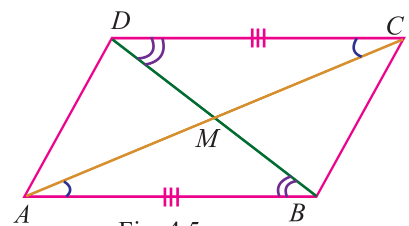


Fig. 4.5

(i) $AB = DC$ Opposite sides of the parallelogram are equal

(ii) $\angle MAB = \angle MCD$ Alternate interior angles ($\because AB \parallel DC$)

$\angle ABM = \angle CDM$ Alternate interior angles ($\because AB \parallel DC$)

(iii) $\Delta AMB \equiv \Delta CMD$ (By ASA property)

$\therefore AM = CM$ and $BM = DM$

i.e., M is the mid point of AC and BD

$\therefore$ The diagonals bisect each other ■

Converse of Property 3: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Note

(i) A diagonal of a parallelogram divides it into two triangles of equal area.

(ii) A parallelogram is a rhombus if its diagonals are perpendicular.

(iii) Parallelograms on the same base and between the same parallels are equal in area.

Example 4.1

If the measures of three angles of a quadrilateral are 100°, 84° and 76° then, find the measure of fourth angle.

Solution Let the measure of the fourth angle be x°.

The sum of the angles of a quadrilateral is 360°. So,

$$100^\circ + 84^\circ + 76^\circ + x^\circ = 360^\circ$$
$$260^\circ + x^\circ = 360^\circ$$
$$x^\circ = 100^\circ$$

Hence, the measure of the fourth angle is 100°.

Example 4.2

In the parallelogram $ABCD$ if $\angle A = 65^\circ$, find $\angle B$, $\angle C$ and $\angle D$.

Solution Let $ABCD$ be a parallelogram in which $\angle A = 65^\circ$.

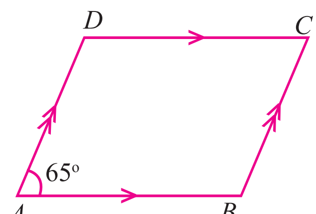


Fig. 4.6

Since $AD \parallel BC$ we can treat AB as a transversal. So,

$$\angle A + \angle B = 180^\circ$$
$$65^\circ + \angle B = 180^\circ$$
$$\angle B = 180^\circ - 65^\circ$$
$$\angle B = 115^\circ$$

Since the opposite angles of a parallelogram are equal, we have

$$\angle C = \angle A = 65^\circ \text{ and } \angle D = \angle B = 115^\circ$$

Hence, $\angle B = 115^\circ$, $\angle C = 65^\circ$ and $\angle D = 115^\circ$

Example 4.3

Suppose $ABCD$ is a rectangle whose diagonals AC and BD intersect at O. If $\angle OAB = 62^\circ$, find $\angle OBC$.

Solution The diagonals of a rectangle are equal and bisect each other. So,

$OA = OB$ and $\angle OBA = \angle OAB = 62^\circ$

Since the measure of each angle of rectangle is 90°

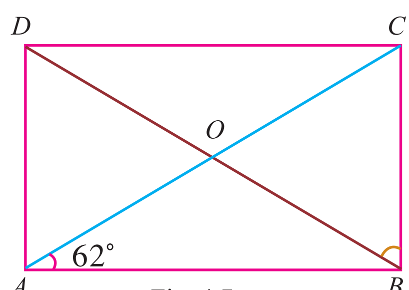


Fig. 4.7

$$\angle ABC = 90^\circ$$
$$\angle ABO + \angle OBC = 90^\circ$$
$$62^\circ + \angle OBC = 90^\circ$$
$$\angle OBC = 90^\circ - 62^\circ$$
$$= 28^\circ$$

Example 4.4

If $ABCD$ is a rhombus and if $\angle A = 76^\circ$, find $\angle CDB$.

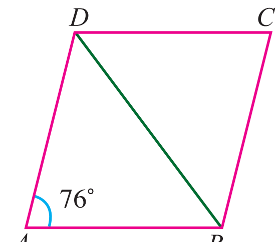


Fig. 4.8

Solution $\angle A = \angle C = 76^\circ$ (Opposite angles of a rhombus)

Let $\angle CDB = x^\circ$. In $\triangle\ CDB$, $CD = CB$

$$\angle CDB = \angle CBD = x^\circ$$

$\angle CDB + \angle CBD + \angle DCB = 180^\circ$ (Angles of a triangle)

$$2x^\circ + 76^\circ = 180^\circ \implies 2x = 104^\circ$$

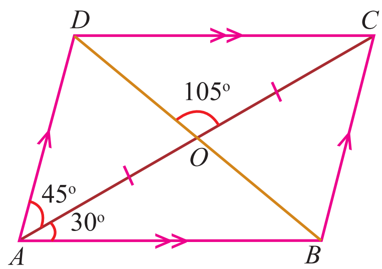
$$x^\circ = 52^\circ$$

$$\therefore \angle CDB = 52^\circ$$

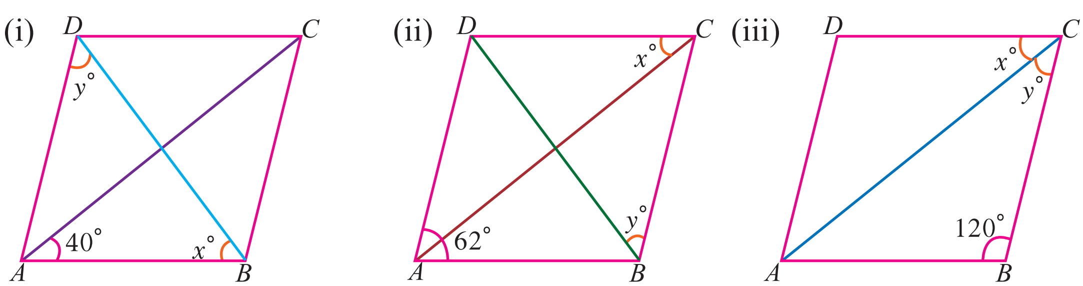
Exercise 4.2

1. In a quadrilateral $ABCD$, the angles $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are in the ratio 2:3:4:6. Find the measure of each angle of the quadrilateral.
2. Suppose $ABCD$ is a parallelogram in which $\angle A = 108^\circ$. Calculate $\angle B$, $\angle C$ and $\angle D$.
3. In the figure at right, $ABCD$ is a parallelogram $\angle BAO = 30^\circ$, $\angle DAO = 45^\circ$ and $\angle COD = 105^\circ$. Calculate

   (i) $\angle ABO$ (ii) $\angle ODC$ (iii) $\angle ACB$ (iv) $\angle CBD$

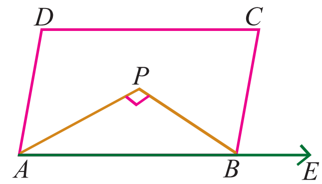


4. Find the measure of each angle of a parallelogram, if larger angle is 30° less than twice the smaller angle.
5. Suppose $ABCD$ is a parallelogram in which $AB = 9$ cm and its perimeter is 30 cm. Find the length of each side of the parallelogram.
6. The length of the diagonals of a rhombus are 24 cm and 18 cm. Find the length of each side of the rhombus.
7. In the following figures, $ABCD$ is a rhombus. Find the values of x and y.



8. The side of a rhombus is 10 cm and the length of one of the diagonals is 12 cm. Find the length of the other diagonal.

9. In the figure at the right, $ABCD$ is a parallelogram in which the bisectors of $\angle A$ and $\angle B$ intersect at the point P. Prove that $\angle APB = 90^\circ$.



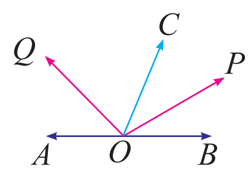
Exercise 4.3

Multiple Choice Questions.

1. If an angle is equal to one third of its supplement, its measure is equal to

   (A) 40° (B) 50° (C) 45° (D) 55°

2. In the given figure, OP bisect $\angle BOC$ and OQ bisect $\angle AOC$. Then $\angle POQ$ is equal to



   (A) 90° (B) 120°

   (C) 60° (D) 100°

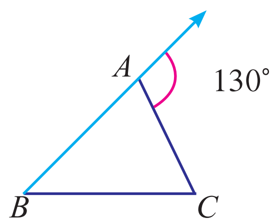
3. The complement of an angle exceeds the angle by 60°. Then the angle is equal to

   (A) 25° (B) 30° (C) 15° (D) 35°

4. Find the measure of an angle, if six times of its complement is 12° less than twice of its supplement.

   (A) 48° (B) 96° (C) 24° (D) 58°

5. In the given figure, $\angle B : \angle C = 2:3$, Find $\angle B$



   (A) 120° (B) 52°

   (C) 78° (D) 130°

6. $ABCD$ is a parllellogram, E is the mid-point of AB and CE bisects $\angle BCD$. Then $\angle DEC$ is

   (A) 60° (B) 90° (C) 100° (D) 120°

Points to Remember

- In a parallelogram the opposite sides are equal.
- In a parallelogram, the opposite angles are equal.
- The diagonals of a parallelogram bisect each other.
- A rectangle is an equiangular parallelogram.
- A rhombus is an equilateral parallelogram.
- A square is an equilateral and equiangular parallelogram. Thus a square is a rectangle, a rhombus and a parallelogram.
- Each diagonal divides the parallelogram into two congruent triangles.
- A parallelogram is rhombus if its diagonals are perpendicular.
- Parallelograms on the same base and between the same parallels are equal in area.
- A diagonal of a parallelogram divides it into two triangles of equal area.

Activity 1

Draw a quadrilateral and find the mid-point of each side. Connect the mid-points of the adjacent sides.

1. What is the resulting figure?
2. Is it true for every quadrilateral?
3. How does the inner quadrilateral change with respect to the changes of the outer quadrilateral?

Activity 2

Make a parallelogram in a card and cut it along a diagonal to obtain two triangles.

Superimpose one triangle on the other. What do you observe?

Repeat the same activity with the other diagonal.

Activity 3

Draw the diagonals of a parallelogram and cut the four triangles formed.

Observe that there are two pairs of congruent triangles by superimposing them one on another.

Project 1

Objective: To explore the sum of interior angles of quadrilateral.

Material Required: Drawing sheet, scale, protractor and scissors.

Instructions:

1. Draw different types of quadrilaterals in a chart paper or a graph sheet.
2. Cut the quadrilateral into four pieces so that each piece has one of the angles of the quadrilateral as the vertex angle.
3. Arrange the angles in such a manner that can help to find the sum of all angles of quadrilateral without measuring them.
4. Verify the result by measuring the angles and completing the table.

Name of quadrilateral	Angle 1	Angle 2	Angle 3	Angle 4	Sum of all angles
Parallelogram					
Rectangle					
Square					
Rhombus					
Trapezium					

Project 2

Objective: To explore the properties of parallelogram.

Material Required: Drawing sheet, scale, protractor and scissors.

Instructions:

1. Cut-out different types of quadrilaterals in a graph sheet.
2. Using the cut-outs, complete the given table by putting ✓ or ×.

Property	Square	Rhombus	Rectangle	Parallelogram
Opposite sides are equal				
Opposite sides are parallel				
Adjacent sides are equal				
All the angles are of 90°				
Opposite angles are equal				
Diagonals bisect each other				
Diagonals bisect at 90°				
Diagonals divide it into two congruent triangles				
Diagonals are equal in length				

Exercise 4.1

1. (i) 27° (ii) 66° (iii) 42° (iv) 55° (v) 70° **2.** (i) 122° (ii) 32° (iii) 60° (iv) 140
(v) 80° **3.** (i) 80° (ii) 35° **4.** (i) 60° (ii) 144° (iii) 60° (iv) 72° (v) $80^\circ, 100^\circ$
(vi) $54^\circ, 36^\circ$ **5.** (i) 36° (ii) 40° (iii) $40^\circ, 50^\circ$

6. (i) $\angle A = \angle C = \angle E = \angle G = 115^\circ$, $\angle B = \angle D = \angle H = 65^\circ$ **7.** (i) 30° (ii) 32°

8. $30^\circ, 60^\circ, 90^\circ$ **9.** $45^\circ, 25^\circ, 110^\circ$ **10.** $80^\circ, 60^\circ$

Exercise 4.2

1. $48^\circ, 72^\circ, 96^\circ, 144^\circ$ **2.** $72^\circ, 108^\circ, 72^\circ$ **3.** (i) 45° (ii) 45° (iii) 45° (iv) 60°

4. $70^\circ, 110^\circ, 70^\circ, 110^\circ$ **5.** $l = 9, b = 6$ **6.** 15

7. (i) $50^\circ, 50^\circ$ (ii) $31^\circ, 59^\circ$ (iii) $30^\circ, 30^\circ$ **8.** 16

Exercise 4.3

1. C **2.** A **3.** C **4.** A **5.** B **6.** B

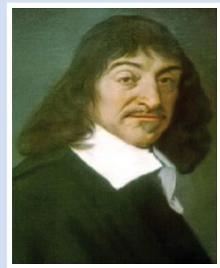
5 COORDINATE GEOMETRY

I hope that posterity will judge me kindly, not only as to the things which I have explained, but also as to those which I have intentionally omitted so as to leave to others the pleasure of discovery

- RENE DESCARTES

Main Targets

- To understand Cartesian coordinate system
- To identify abscissa, ordinate and coordinates of a point
- To plot the points on the plane
- To find the distance between two points



DESCARTES
(1596-1650)

Descartes (1596-1650) has been called the father of modern philosophy, perhaps because he attempted to build a new system of thought from the ground up, emphasized the use of logic and scientific method, and was "profoundly affected in his outlook by the new physics and astronomy." Descartes went far past Fermat in the use of symbols, in 'Arithmetizing' analytic geometry, in extending it to equations of higher degree. The fixing of a point position in the plane by assigning two numbers - coordinates - giving its distance from two lines perpendicular to each other, was entirely Descartes' invention.

5.1 Introduction

Coordinate Geometry or Analytical Geometry is a system of geometry where the position of points on the plane is described using an ordered pair of numbers called coordinates. This method of describing the location of points was introduced by the French mathematician René Descartes (Pronounced "day CART"). He proposed further that curves and lines could be described by equations using this technique, thus being the first to link algebra and geometry. In honour of his work, the coordinates of a point are often referred to as its Cartesian coordinates, and the coordinate plane as the Cartesian Coordinate Plane. The invention of analytical geometry was the beginning of modern mathematics.

In this chapter we learn how to represent points using cartesian coordinate system and derive formula to find distance between two points in terms of their coordinates.

5.2 Cartesian Coordinate System

In the chapter on *Real Number System*, you have learnt` how to represent real numbers on the *number line*. Every real number, whether rational or irrational, has a unique location on the number line. Conversely, a point P on a number line can be specified by a real number x called its *coordinate*. Similarly, by using a Cartesian coordinate system we can specify a point P in the plane with two real numbers, called its coordinates.

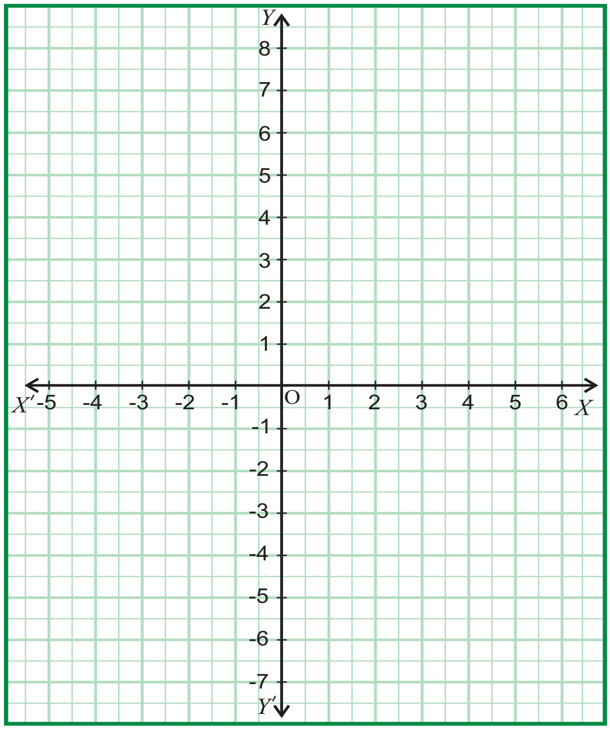


Fig. 5.1

A Cartesian coordinate system or rectangle coordinate system consists of two perpendicular number lines, called *coordinate axes*. The two number lines intersect at the zero point of each as shown in the Fig. 5.1 and this point is called *origin 'O'*. Generally the horizontal number line is called the x-axis and the vertical number line is called the y-axis. The x-coordinate of a point to the right of the y-axis is positive and to the left of y-axis is negative. Similarly, the y-coordinate of a point above the x-axis is positive and below the x-axis is negative. We use the same scale (that is, the same unit distance) on both the axes.

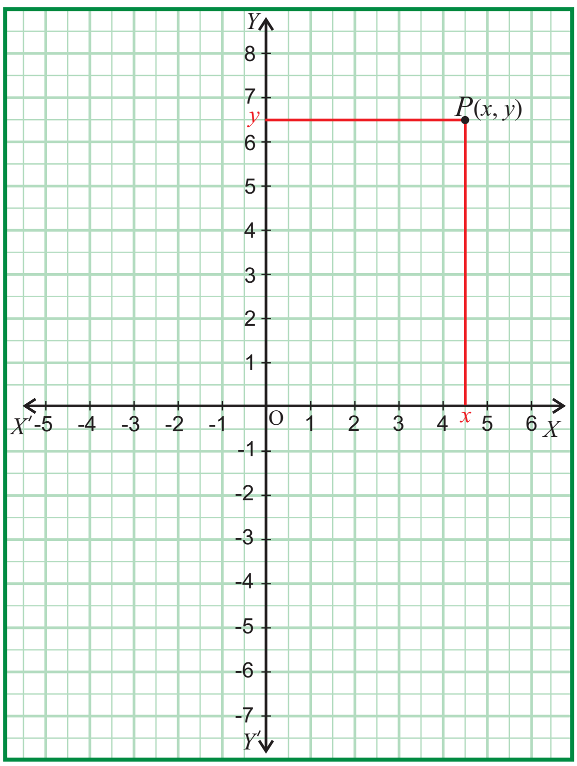


Fig. 5.2

5.2.1 Coordinates of a Point

In Cartesian system, any point P in the plane is associated with an ordered pair of real numbers. To obtain these number, we draw two lines through the point P parallel (and hence perpendicular) to the axes. We are interested in the coordinates of the points of intersection of the two lines with the axes. There are two coordinates: x-coordinate on the x-axis and y-coordinate on the y-axis. The x-coordinate is called the *abscissa* and the y-coordinate is called the *ordinate* of the point at hand. These two numbers associated with the point P are called *coordinates* of P. They are usually written as (x, y), the abscissa coming first, the ordinate second.

Remarks

1. In an ordered pair (a, b), the two elements a and b are listed in a specific order. So the ordered pairs (a, b) and (b, a) are not equal, i.e., $(a,b) \neq (b,a)$.
2. Also $(a_1,b_1) = (a_2,b_2)$ is equivalent to $a_1 = a_2$ and $b_1 = b_2$.
3. The terms *point* and *coordinates of a point* are used interchangeably.

5.2.2 Identifying the *x*-coordinate

The *x*-coordinate or abscissa, of a point is the value which indicates the distance and direction of the point to the right or left of the *y*-axis. To find the *x*-coordinate of a point *P*:

(i) Drop a perpendicular from the point *P* to the *x*-axis.

(ii) The number where the line meets the *x*-axis is the value of the *x*-coordinate.

In Fig. 5.3., the *x*-coordinate of *P* is 1 and the *x*-coordinate of *Q* is 5.

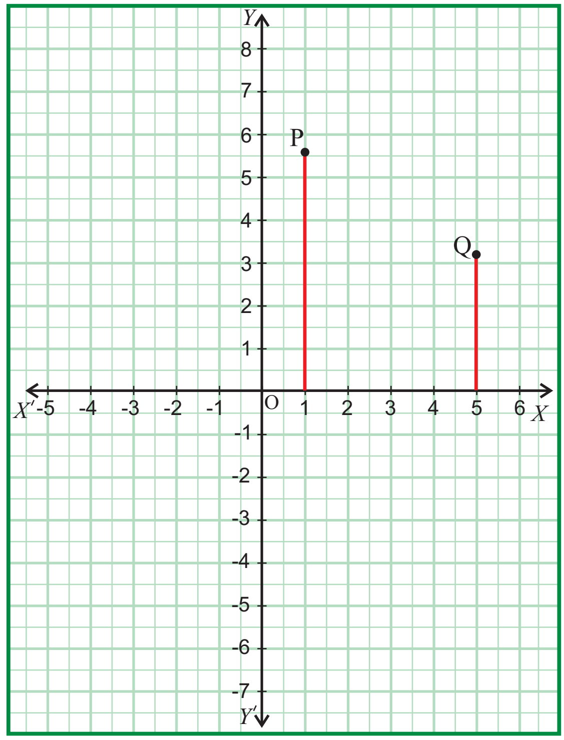


Fig. 5.3

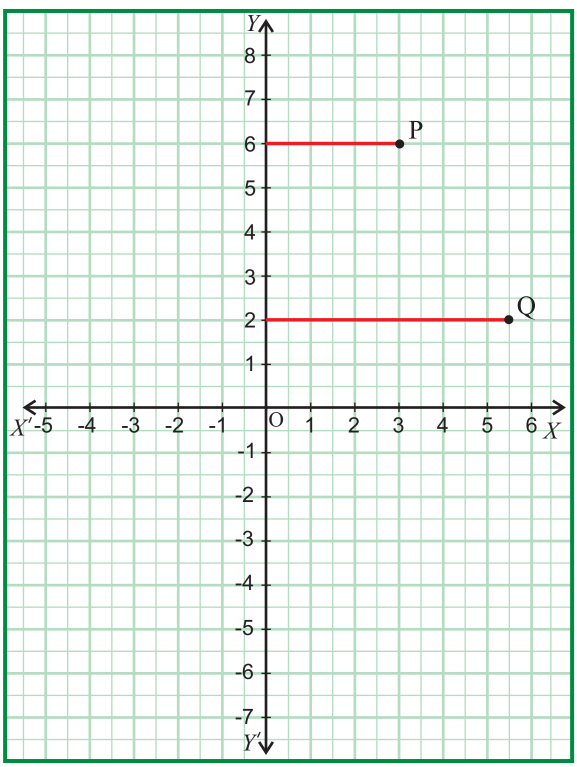


Fig. 5.4

5.2.3 Identifying the *y*-coordinate

The *y*-coordinate or ordinate, of a point is the value which indicates the distance and direction of the point above or below the *x*-axis. To find the *y*-coordinate of a point *P*:

(i) Drop a perpendicular from the point *P* to the *y*-axis.

(ii) The number where the line meets the *y*-axis is the value of the *y*-coordinate.

In Fig. 5.4 ., the *y*-coordinate of *P* is 6 and the *y*-coordinate of *Q* is 2.

Note (i) For any point on the x-axis, the value of y-coordinate (ordinate) is zero.

(ii) For any point on the y-axis, the value of x-coordinate (abscissa) is zero.

5.2.4 Quadrants

A plane with the rectangular coordinate system is called the cartesian plane. The coordinate axes divide the plane into four parts called quadrants, numbered counter-clockwise for reference as shown in Fig. 5.5. The x-coordinate is positive in the I and IV quadrants and negative in II and III quadrants. The y-coordinate is positive in I and II quadrants and negative in III and IV quadrants. The signs of the coordinates are shown in parentheses in Fig. 5.5.

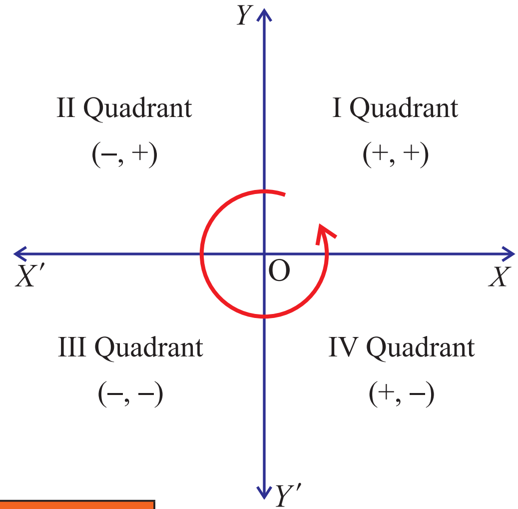


Fig. 5.5

Region	Quadrant	Nature of x, y	Signs of the coordinates
XOY	I	$x > 0, y > 0$	+, +
X′OY	II	$x < 0, y > 0$	–, +
X′OY′	III	$x < 0, y < 0$	–, –
XOY′	IV	$x > 0, y < 0$	+, –

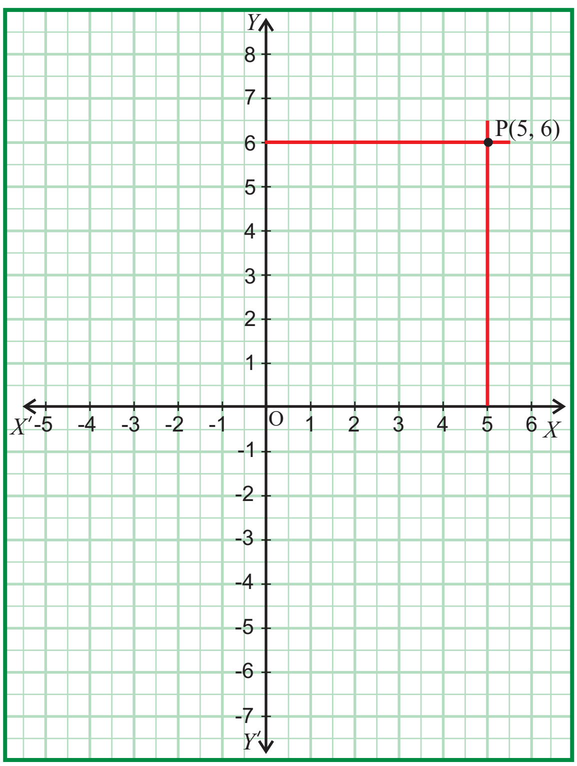


Fig. 5.6

5.2.5 Plotting Points in Cartesian Coordinate System

Let us now illustrate through an example how to plot a point in Cartesian coordinate system. To plot the point (5, 6) in cartesian coordinate system we follow the x-axis until we reach 5 and draw a vertical line at $x = 5$. Similarly, we follow the y-axis until we reach 6 and draw a horizontal line at $y = 6$. The intersection of these two lines is the position of (5, 6) in the cartesian plane.

That is we count from the origin 5 units along the positive direction of x-axis and move along the positive direction of y-axis through 6 units and mark the corresponding point. This point is at a distance of 5 units from the y-axis and 6 units from the x-axis. Thus the position of (5, 6) is located in the cartesian plane.

Example 5.1

Plot the following points in the rectangular coordinate system.

(i) $A\ (5, 4)$ (ii) $B\ (-4, 3)$ (iii) $C\ (-2, -3)$ (iv) $D\ (3, -2)$

Solution (i) To plot (5,4), draw a vertical line at $x = 5$ and draw a horizontal line at $y = 4$.

The intersection of these two lines is the position of (5, 4) in the Cartesian plane. Thus, the point A (5, 4) is located in the Cartesian plane.

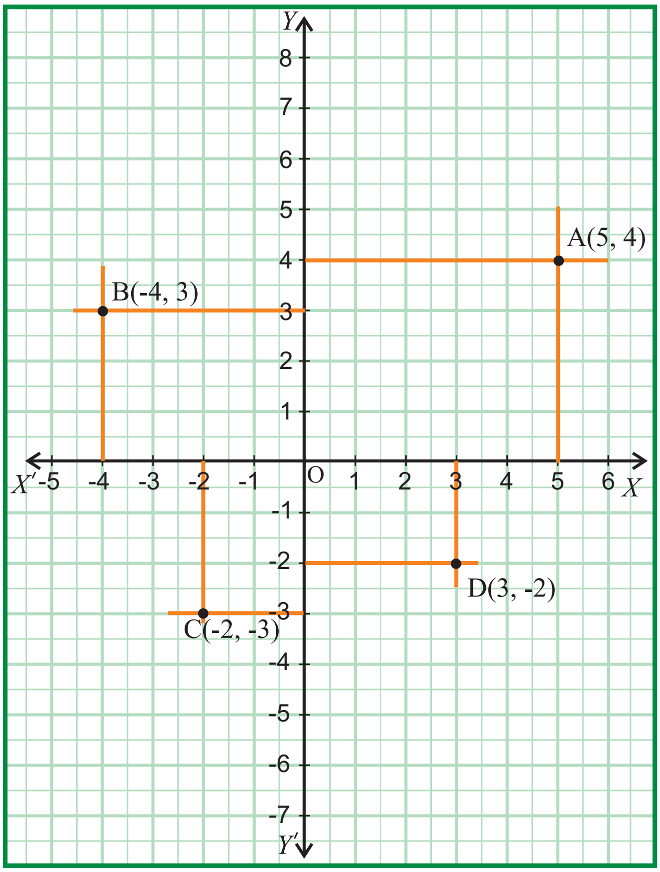


Fig. 5.7

(ii) To plot $(-4, 3)$, draw a vertical line at $x = -4$ and draw a horizontal line at $y = 3$. The intersection of these two lines is the position of $(-4, 3)$ in the Cartesian plane. Thus, the point $B(-4, 3)$ is located in the Cartesian plane.

(iii) To plot $(-2, -3)$, draw a vertical line at $x = -2$ and draw a horizontal line at $y = -3$. The intersection of these two lines is the position of $(-2, -3)$ in the Cartesian plane. Thus, the point $C(-2, -3)$ is located in the Cartesian plane.

(iv) To plot $(3, -2)$, draw a vertical line at $x = 3$ and draw a horizontal line at $y = -2$. The intersection of these two lines is the position of $(3, -2)$ in the Cartesian plane Thus, the point $D\ (3, -2)$ is located in the Cartesian plane.

Example 5.2

Locate the points (i) (3, 5) and (5, 3) (ii) $(-2, -5)$ and $(-5, -2)$ in the rectangular coordinate system.

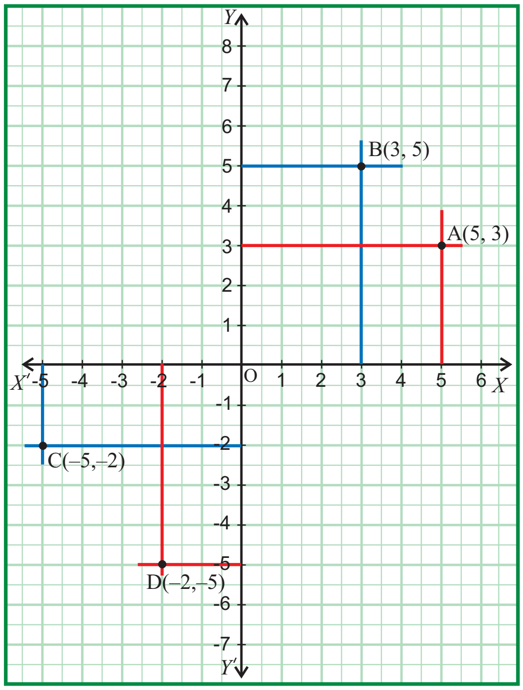


Fig. 5.8

Remark Observe that if we interchange the abscissa and ordinate of a point, then it may represent a different point in the Cartesian plane.

Example 5.3

Plot the points $(-1, 0)$, $(2, 0)$, $(-5, 0)$ and $(4, 0)$ in the cartesian plane.

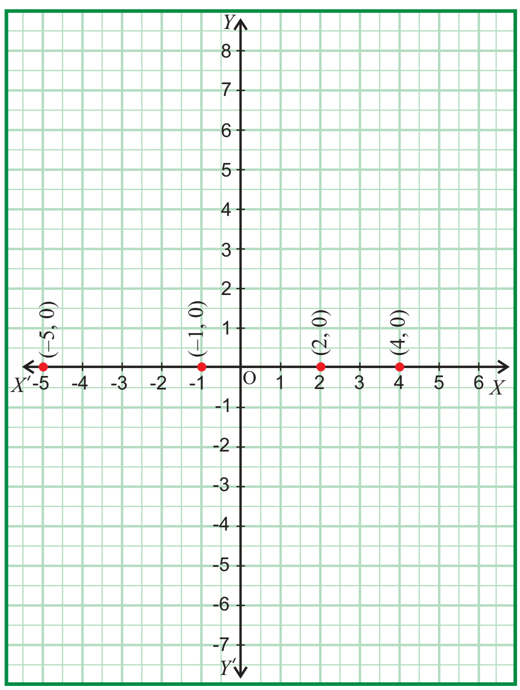


Fig. 5.9

Example 5.4

Plot the points (0, 4), (0, − 2), (0, 5) and (0, − 4) in the cartesian plane.

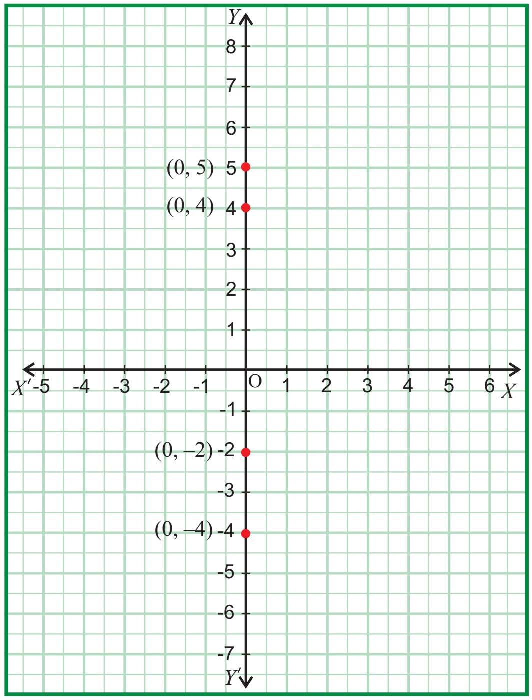


Fig. 5.10

Example 5.5

Plot the points (i) (− 1, 2), (ii) (− 4, 2), (iii) (4, 2) and (iv) (0, 2). What can you say about the position of these points?

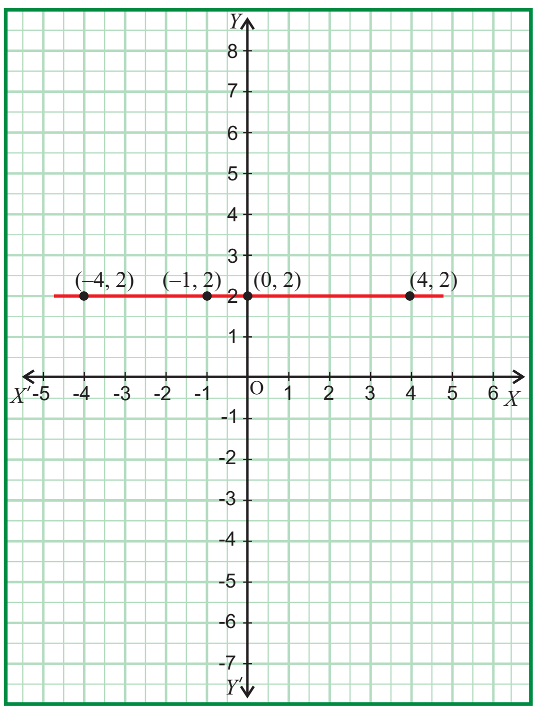


Fig. 5.11

When you join these points, you see that they lie on a line which is parallel to x-axis.

Note For points on a line parallel to x-axis, the y-coordinates are equal.

Example 5.6

Identify the quadrants of the points A (2, 3), B (−2, 3), C (−2, −3) and D (2, −3). Discuss the type of the diagram by joining all the points.

Solution

Point	A	B	C	D
Quadrant	I	II	III	IV

$ABCD$ is a rectangle

Can you find the length, breadth and area of the rectangle?

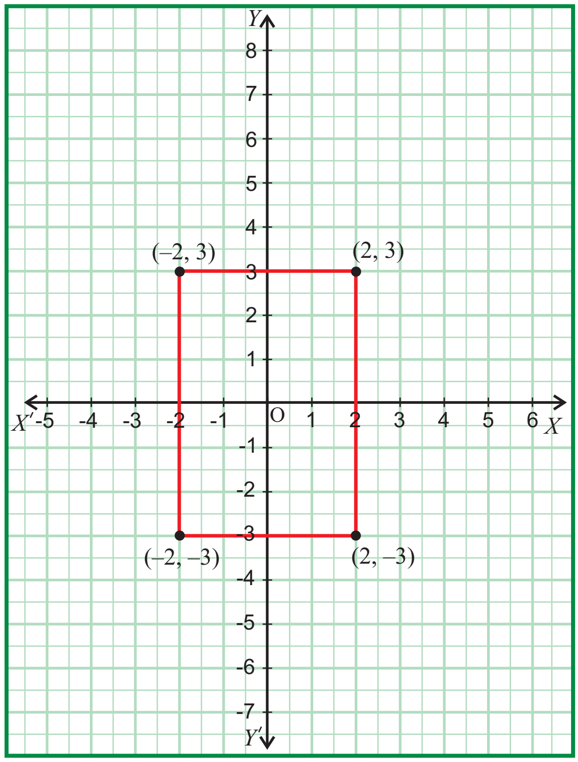


Fig. 5.12

Example 5.7

Find the coordinates of the points shown in the Fig. 5.13., where each square is a unit square.

Solution Consider the point A. A is at a distance of 3 units from the origin along the positive direction of x-axis and 2 units distance from the origin along the positive direction of y-axis. Hence the coordinates of A are (3, 2).

Similarly, B is (−3, 2), C is (−2, −2), D is (2, −1), E is (5, −3), F is (3, 4) and G is (−3, 1).

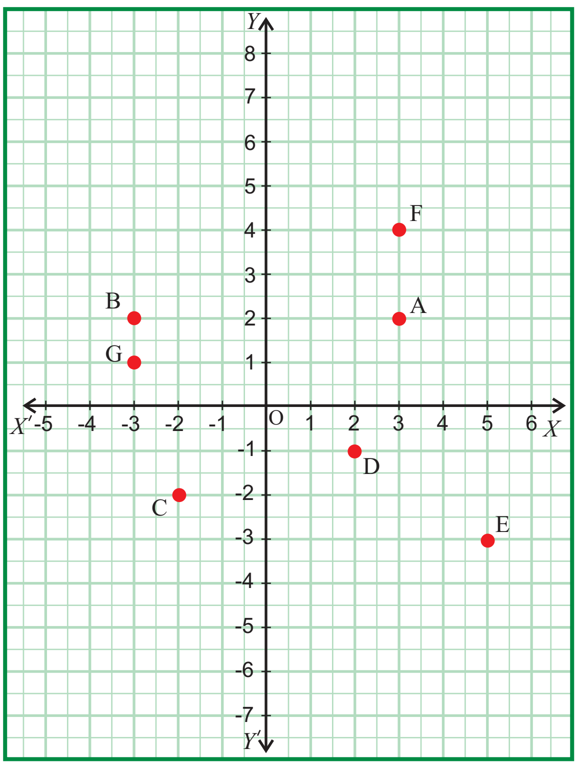


Fig. 5.13

Exercise 5.1

1. State whether the following statements are true / false .
   - (i) (5, 7) is a point in the IV quadrant.
   - (ii) $(-2, -7)$ is a point in the III quadrant.
   - (iii) $(8, -7)$ lies below the x-axis.
   - (iv) (5, 2) and $(-7, 2)$ are points on the line parallel to y-axis.
   - (v) $(-5, 2)$ lies to the left of y-axis.
   - (vi) (0, 3) is a point on x-axis.
   - (vii) $(-2, 3)$ lies in the II quadrant.
   - (viii) $(-10, 0)$ is a point on x-axis.
   - (ix) $(-2, -4)$ lies above x-axis.
   - (x) For any point on the x-axis its y-coordinate is zero.
2. Plot the following points in the coordinate system and specify their quadrant.

   (i) (5, 2) (ii) $(-1, -1)$ (iii) (7, 0) (iv) $(-8, -1)$ (v) $(0, -5)$

   (vi) (0, 3) (vii) $(4, -5)$ (viii) (0, 0) (ix) (1, 4) (x) $(-5, 7)$
3. Write down the abscissa for the following points.

   (i) $(-7, 2)$ (ii) (3, 5) (iii) $(8, -7)$ (iv) $(-5, -3)$
4. Write down the ordinate of the following points.

   (i) (7, 5) (ii) (2, 9) (iii) $(-5, 8)$ (iv) $(7, -4)$
5. Plot the following points in the coordinate plane.

   (i) (4, 2) (ii) $(4, -5)$ (iii) (4, 0) (iv) $(4, -2)$

   How is the line joining them situated?
6. The ordinates of two points are each -6. How is the line joining them related with reference to x-axis?
7. The abscissa of two points is 0. How is the line joining situated?
8. Mark the points A (2, 4), B $(-3, 4)$, C $(-3, -1)$ and D $(2, -1)$ in the cartesian plane. State the figure obtained by joining *A and B*, *B and C*, *C and D* and *D and A*.
9. With rectangular axes plot the points O (0, 0), A (5, 0), B (5, 4). Find the coordinate of point C such that $OABC$ forms a rectangle.
10. In a rectangle $ABCD$, the coordinates of A, B and D are (0, 0) (4, 0) (0, 3). What are the coordinates of C?

5.3 Distance between any Two Points

One of the simplest things that can be done with analytical geometry is to calculate the distance between two points. The distance between two points A and B is usually denoted by AB.

5.3.1 Distance between two points on coordinate axes

If two points lie on the x-axis, then it is easy to find the distance between them because the distance is equal to the difference between x-coordinates. Consider the two points $A\ (x_1, 0)$ and $B\ (x_2, 0)$ on the x-axis.

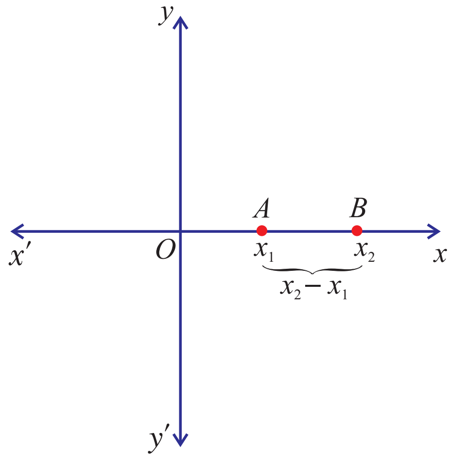


Fig. 5.14

$$\begin{aligned}\text{The distance of } B \text{ from } A &= AB = OB - OA \\ &= x_2 - x_1 \quad \text{if } x_2 > x_1 \\ &= x_1 - x_2 \quad \text{if } x_1 > x_2 \\ \therefore\ AB &= |x_2 - x_1|\end{aligned}$$

Similarly, if two points lie on y-axis, then the distance between them is equal to the difference between the y-coordinates. Consider two points $A(0, y_1)$ and $B(0, y_2)$. These two points lie on the y-axis.

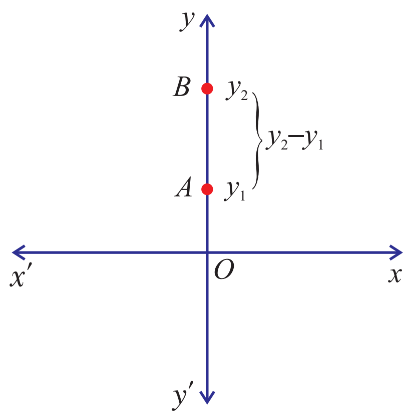


Fig. 5.15

$$\begin{aligned}\text{The distance of } B \text{ from } A &= AB = OB - OA \\ &= y_2 - y_1 \quad \text{if } y_2 > y_1 \\ &= y_1 - y_2 \quad \text{if } y_1 > y_2 \\ \therefore\ AB &= |y_2 - y_1|\end{aligned}$$

5.3.2 Distance between two points on a line parallel to coordinate axes

Consider the points $A\ (x_1, y_1)$ and $B(x_2, y_1)$. Since the y-cordinates are equal, the two points lie on a line parallel to x-axis. Draw AP and BQ perpendicular to x-axis. Distance between A and B is equal to distance between P and Q. Hence

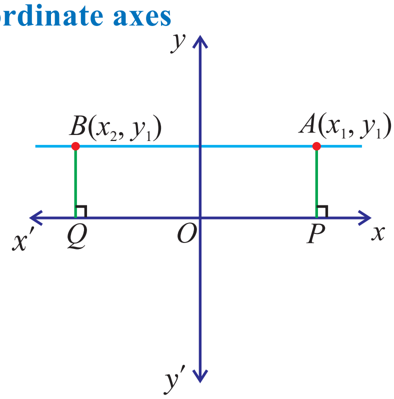


Fig. 5.16

Distance AB = Distance PQ $= |x_1 - x_2|$.

Now consider the points $A(x_1, y_1)$ and $B(x_1, y_2)$ that lie on a line parallel to y-axis. Draw AP and BQ perpendicular to y-axis. The distance between A and B is equal to the distance between P and Q. Hence

Distance AB = Distance $PQ = |y_1 - y_2|$

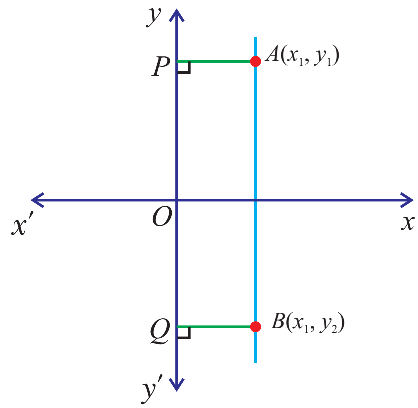


Fig. 5.17

Remark The distance between two points on a line parallel to the coordinate axes is the absolute value of the difference between respective coordinates.

5.3.3 Distance between two points:

Let $A(x_{1,}\ y_1)$ and $B(x_2\ y_2)$ be any two points in the plane. We shall now find the distance between these two points.

Let P and Q be the foot of the perpendiculars from A and B to the x-axis respectively. AR is drawn perpendicular to BQ. From the diagram,

$AR = PQ = OQ - OP = x_2 - x_1$ and

$BR = BQ - RQ = y_2 - y_1$

From right triangle ARB

$$AB^2 = AR^2 + RB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

(By Pythagoras theorem)

i.e., $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

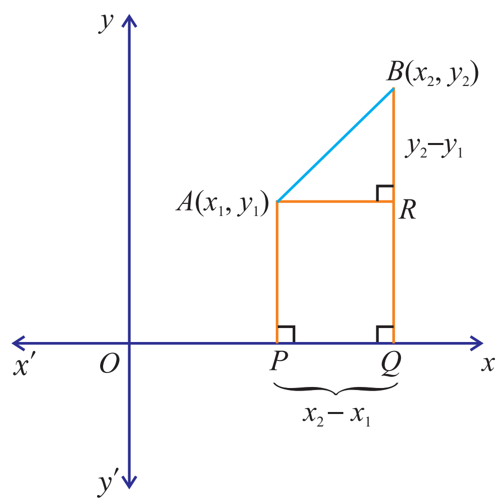


Fig. 5.18

Hence the distance between the points A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Key Concept **Distance Between Two Points**

Given the two points (x_1, y_1) and (x_2, y_2), the distance between these points is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note (i) This formula holds good for all the above cases.

(ii) The distance of the point $P(x_1, y_1)$ from the origin O is $OP = \sqrt{x_1^2 + y_1^2}$

Example 5.8

Find the distance between the points $(-4, 0)$ and $(3, 0)$

Solution The points $(-4, 0)$ and $(3, 0)$ lie on the x-axis. Hence

$$d = |x_1 - x_2| = |3 - (-4)| = |3 + 4| = 7$$

Aliter :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3+4)^2 + 0^2} = \sqrt{49} = 7$$

Example 5.9

Find the distance between the points $(-7, 2)$ and $(5, 2)$

Solution The line joining $(5, 2)$ and $(-7, 2)$ is parallel to x-axis. Hence, the distance

$$d = |x_1 - x_2| = |-7 - 5| = |-12| = 12$$

Aliter :

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5+7)^2 + (2-2)^2} = \sqrt{12^2} = \sqrt{144} = 12$$

Example 5.10

Find the distance between the points $(-5, -6)$ and $(-4, 2)$

Solution Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we find

$$d = \sqrt{(-4+5)^2 + (2+6)^2} = \sqrt{1^2 + 8^2} = \sqrt{1 + 64} = \sqrt{65}$$

Example 5.11

Find the distance between the points $(0, 8)$ and $(6, 0)$

Solution The distance between the points $(0, 8)$ and $(6, 0)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6-0)^2 + (0-8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$$

Aliter :

Let A and B denote the points $(6, 0)$ and $(0, 8)$ and let O be the origin. The point $(6, 0)$ lies on the x-axis and the point $(0, 8)$ lies on the y-axis. Since the angle between coordinate axes is right angle, the points A, O and B form a right triangle. Now $OA = 6$ and $OB = 8$. Hence, using Pythagorean Theorem

$$AB^2 = OA^2 + OB^2 = 36 + 64 = 100.$$

$$\therefore\ AB = \sqrt{100} = 10$$

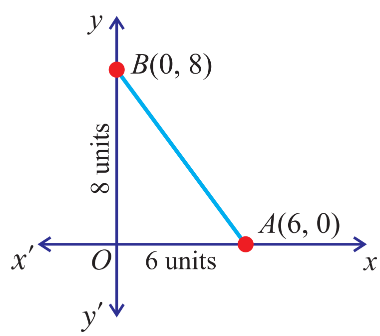


Fig. 5.19

Example 5.12

Find the distance between the points (–3, –4), (5, –7)

Solution The distance between the points (–3, – 4), (5, – 7) is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5+3)^2 + (-7+4)^2} = \sqrt{8^2 + (-3)^2} = \sqrt{64+9} = \sqrt{73}$$

Example 5.13

Show that the three points (4, 2), (7, 5) and (9, 7) lie on a straight line.

Solution Let the points be A (4, 2), B (7, 5) and C (9, 7). By the distance formula

$$AB^2 = (4-7)^2 + (2-5)^2 = (-3)^2 + (-3)^2 = 9 + 9 = 18$$

$$BC^2 = (9-7)^2 + (7-5)^2 = 2^2 + 2^2 = 4 + 4 = 8$$

$$CA^2 = (9-4)^2 + (7-2)^2 = 5^2 + 5^2 = 25 + 25 = 50$$

So, $AB = \sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$; $BC = \sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$;

$CA = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$.

This gives $AB + BC = 3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2} = AC$. Hence the points A, B and C are collinear.

Example 5.14

Determine whether the points are vertices of a right triangle
A (–3, –4), B (2, 6) and C (–6, 10)

Solution Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$$AB^2 = (2+3)^2 + (6+4)^2 = 5^2 + 10^2 = 25 + 100 = 125$$

$$BC^2 = (-6-2)^2 + (10-6)^2 = (-8)^2 + 4^2 = 64 + 16 = 80$$

$$CA^2 = (-6+3)^2 + (10+4)^2 = (-3)^2 + (14)^2 = 9 + 196 = 205$$

i. e., $AB^2 + BC^2 = 125 + 80 = 205 = CA^2$

Hence ABC is a right angled triangle since the square of one side is equal to sum of the squares of the other two sides.

Example 5.15

Show that the points $(a, a), (-a, -a)$ and $(-a\sqrt{3}, a\sqrt{3})$ form an equilateral triangle.

Solution Let the points be represented by $A\,(a, a)$, $B\,(-a, -a)$ and $C(-a\sqrt{3}, a\sqrt{3})$. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we have

$$
\begin{aligned}
AB &= \sqrt{(a+a)^2+(a+a)^2}\\
&= \sqrt{(2a)^2+(2a)^2} = \sqrt{4a^2+4a^2} = \sqrt{8a^2} = 2\sqrt{2}\,a\\
BC &= \sqrt{(-a\sqrt{3}+a)^2+(a\sqrt{3}+a)^2} = \sqrt{3a^2+a^2-2a^2\sqrt{3}+3a^2+a^2+2a^2\sqrt{3}}\\
&= \sqrt{8a^2} = \sqrt{4\times 2a^2} = 2\sqrt{2}\,a\\
CA &= \sqrt{(a+a\sqrt{3})^2+(a-a\sqrt{3})^2} = \sqrt{a^2+2a^2\sqrt{3}+3a^2+a^2-2a^2\sqrt{3}+3a^2}\\
&= \sqrt{8a^2} = 2\sqrt{2}\,a\\
\therefore\ AB &= BC = CA = 2\sqrt{2}\,a.
\end{aligned}
$$

Since all the sides are equal the points form an equilateral triangle.

Example 5.16

Prove that the points (–7, –3), (5, 10), (15, 8) and (3, –5) taken in order are the corners of a parallelogram.

Solution Let A, B, C and D represent the points $(-7, -3)$, $(5, 10)$, $(15, 8)$ and $(3, -5)$ respectively. Using the distance formula $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, we find

$$
\begin{aligned}
AB^2 &= (5+7)^2+(10+3)^2 = 12^2+13^2 = 144+169 = 313\\
BC^2 &= (15-5)^2+(8-10)^2 = 10^2+(-2)^2 = 100+4 = 104\\
CD^2 &= (3-15)^2+(-5-8)^2 = (-12)^2+(-13)^2 = 144+169 = 313\\
DA^2 &= (3+7)^2+(-5+3)^2 = 10^2+(-2)^2 = 100+4 = 104
\end{aligned}
$$

So, $AB = CD = \sqrt{313}$ and $BC = DA = \sqrt{104}$

i.e,. The opposite sides are equal. Hence $ABCD$ is a parallelogram.

Example 5.17

Show that the following points (3, –2), (3, 2), (–1, 2) and (–1, –2) taken in order are vertices of a square.

Solution Let the vertices be taken as $A\,(3, -2)$, $B\,(3, 2)$, $C\,(-1, 2)$ and $D\,(-1, -2)$.

$$
\begin{aligned}
AB^2 &= (3-3)^2+(2+2)^2 = 4^2 = 16\\
BC^2 &= (3+1)^2+(2-2)^2 = 4^2 = 16\\
CD^2 &= (-1+1)^2+(2+2)^2 = 4^2 = 16\\
DA^2 &= (-1-3)^2+(-2+2)^2 = (-4)^2 = 16
\end{aligned}
$$

$$AC^2 = (3+1)^2 + (-2-2)^2 = 4^2 + (-4)^2 = 16 + 16 = 32$$

$$BD^2 = (3+1)^2 + (2+2)^2 = 4^2 + 4^2 = 16 + 16 = 32$$

$AB = BC = CD = DA = \sqrt{16} = 4$. (That is, all the sides are equal.)

$AC = BD = \sqrt{32} = 4\sqrt{2}$. (That is, the diagonals are equal.)

Hence the points A, B, C and D form a square.

Example 5.18

Let P be a point on the perpendicular bisector of the segment joining (2, 3) and (6, 5). If the abscissa and the ordinate of P are equal, find the coordinates of P.

Solution Let the point be $P(x, y)$. Since the abscissa of P is equal to its ordinate, we have $y = x$. Therefore, the coordinates of P are (x, x). Let A and B denote the points (2, 3) and (6, 5). Since P is equidistant from A and B, we get $PA = PB$. Squaring on both sides, we get $PA^2 = PB^2$.

$$\text{i.e.,} \quad (x-2)^2 + (x-3)^2 = (x-6)^2 + (x-5)^2$$

$$x^2 - 4x + 4 + x^2 - 6x + 9 = x^2 - 12x + 36 + x^2 - 10x + 25$$

$$2x^2 - 10x + 13 = 2x^2 - 22x + 61$$

$$22x - 10x = 61 - 13$$

$$12x = 48$$

$$x = \frac{48}{12} = 4$$

Therefore, the coordinates of P are (4, 4).

Example 5.19

Show that (4, 3) is the centre of the circle which passes through the points (9, 3), (7, –1) and (1, –1). Find also its radius.

Solution Suppose C represents the point (4, 3). Let P, Q and R denote the points (9, 3), (7, -1) and (1, -1) respectively. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we get

$$CP^2 = (9-4)^2 + (3-3)^2 = 5^2 = 25$$

$$CQ^2 = (7-4)^2 + (-1-3)^2 = 3^2 + (-4)^2 = 9 + 16 = 25$$

$$CR^2 = (4-1)^2 + (3+1)^2 = 3^2 + 4^2 = 9 + 16 = 25$$

So, $CP^2 = CQ^2 = CR^2 = 25$ or $CP = CQ = CR = 5$. Hence the points P, Q, R are on the circle with centre at (4, 3) and its radius is 5 units.

Example 5.20

If the point (α,β) is equidistant from (3, –4) and (8, –5), show that $5\alpha - \beta - 32 = 0$.

Solution Let P deonte the point (α,β). Let A and B represent the points $(3, -4)$ and $(8, -5)$ respectively. Since P is equidistant from A and B, we have $PA = PB$ and hence $PA^2 = PB^2$. Using the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, we have

$$(\alpha - 3)^2 + (\beta + 4)^2 = (\alpha - 8)^2 + (\beta + 5)^2$$

$$\alpha^2 - 6\alpha + 9 + \beta^2 + 8\beta + 16 = \alpha^2 - 16\alpha + 64 + \beta^2 + 10\beta + 25$$

$$-6\alpha + 8\beta + 25 + 16\alpha - 10\beta - 89 = 0$$

$$10\alpha - 2\beta - 64 = 0$$

Dividing throughout by 2, we get $5\alpha - \beta - 32 = 0$

Example 5.21

Show that S (4, 3) is the circum-centre of the triangle joining the points A (9, 3), B (7, –1) and C (1, –1).

Solution $SA = \sqrt{(9-4)^2 + (3-3)^2} = \sqrt{25} = 5$

$$SB = \sqrt{(7-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$$SC = \sqrt{(1-4)^2 + (-1-3)^2} = \sqrt{25} = 5$$

$$\therefore\ SA = SB = SC.$$

It is known that the circum-centre is equidistant from all the vertices of a triangle. Since S is equidistant from all the three vertices, it is the circum-centre of the triangle ABC.

Exercise 5.2

1. Find the distance between the following pairs of points.

(i) $(7, 8)$ and $(-2, -3)$ (ii) $(6, 0)$ and $(-2, 4)$

(iii) $(-3, 2)$ and $(2, 0)$ (iv) $(-2, -8)$ and $(-4, -6)$

(v) $(-2, -3)$ and $(3, 2)$ (vi) $(2, 2)$ and $(3, 2)$

(vii) $(-2, 2)$ and $(3, 2)$ (viii) $(7, 0)$ and $(-8, 0)$

(ix) $(0, 17)$ and $(0, -1)$ (x) $(5, 7)$ and the origin

2. Show that the following points are collinear.

(i) $(3, 7)$, $(6, 5)$ and $(15, -1)$ (ii) $(3, -2)$, $(-2, 8)$ and $(0, 4)$

(iii) $(1, 4)$, $(3, -2)$ and $(-1, 10)$ (iv) $(6, 2)$, $(2, -3)$ and $(-2, -8)$

(v) $(4, 1)$, $(5, -2)$ and $(6, -5)$

3. Show that the following points form an isosceles triangle.

(i) $(-2, 0)$, $(4, 0)$ and $(1, 3)$ (ii) $(1, -2)$, $(-5, 1)$ and $(1, 4)$

(iii) $(-1, -3)$, $(2, -1)$ and $(-1, 1)$ (iv) $(1, 3)$, $(-3, -5)$ and $(-3, 0)$

(v) $(2, 3)$, $(5, 7)$ and $(1, 4)$

4. Show that the following points form a right angled triangle.

(i) $(2, -3)$, $(-6, -7)$ and $(-8, -3)$ (ii) $(-11, 13)$, $(-3, -1)$ and $(4, 3)$

(iii) $(0, 0)$, $(a, 0)$ and $(0, b)$ (iv) $(10, 0)$, $(18, 0)$ and $(10, 15)$

(v) $(5, 9)$, $(5, 16)$ and $(29, 9)$

5. Show that the following points form an equilateral triangle.

(i) $(0, 0)$, $(10, 0)$ and $(5, 5\sqrt{3})$ (ii) $(a, 0)$, $(-a, 0)$ and $(0, a\sqrt{3})$

(iii) $(2, 2)$, $(-2, -2)$ and $(-2\sqrt{3}, 2\sqrt{3})$ (iv) $(\sqrt{3}, 2)$, $(0, 1)$ and $(0, 3)$

(v) $(-\sqrt{3}, 1)$, $(2\sqrt{3}, -2)$ and $(2\sqrt{3}, 4)$

6. Show that the following points taken in order form the vertices of a parallelogram.

(i) $(-7, -5)$, $(-4, 3)$, $(5, 6)$ and $(2, -2)$ (ii) $(9, 5)$, $(6, 0)$, $(-2, -3)$ and $(1, 2)$

(iii) $(0, 0)$, $(7, 3)$, $(10, 6)$ and $(3, 3)$ (iv) $(-2, 5)$, $(7, 1)$, $(-2, -4)$ and $(7, 0)$

(v) $(3, -5)$, $(-5, -4)$, $(7, 10)$ and $(15, 9)$

7. Show that the following points taken in order form the vertices of a rhombus.

(i) $(0, 0)$, $(3, 4)$, $(0, 8)$ and $(-3, 4)$ (ii) $(-4, -7)$, $(-1, 2)$, $(8, 5)$ and $(5, -4)$

(iii) $(1, 0)$, $(5, 3)$, $(2, 7)$ and $(-2, 4)$ (iv) $(2, -3)$, $(6, 5)$, $(-2, 1)$ and $(-6, -7)$

(v) $(15, 20)$, $(-3, 12)$, $(-11, -6)$ and $(7, 2)$

8. Examine whether the following points taken in order form a square.

(i) $(0, -1)$, $(2, 1)$, $(0, 3)$ and $(-2, 1)$ (ii) $(5, 2)$, $(1, 5)$, $(-2, 1)$ and $(2, -2)$

(iii) $(3, 2)$, $(0, 5)$, $(-3, 2)$ and $(0, -1)$ (iv) $(12, 9)$, $(20, -6)$, $(5, -14)$ and $(-3, 1)$

(v) $(-1, 2)$, $(1, 0)$, $(3, 2)$ and $(1, 4)$

9. Examine whether the following points taken in order form a rectangle.

(i) $(8, 3)$, $(0, -1)$, $(-2, 3)$ and $(6, 7)$ (ii) $(-1, 1)$, $(0, 0)$, $(3, 3)$ and $(2, 4)$

(iii) $(-3, 0)$, $(1, -2)$, $(5, 6)$ and $(1, 8)$

10. If the distance between two points $(x, 7)$ and $(1, 15)$ is 10, find x.

11. Show that $(4, 1)$ is equidistant from the points $(-10, 6)$ and $(9, -13)$.

12. If two points $(2, 3)$ and $(-6, -5)$ are equidistant from the point (x, y), show that $x + y + 3 = 0$.

13. If the length of the line segment with end points $(2, -6)$ and $(2, y)$ is 4, find y.

14. Find the perimeter of the triangle with vertices (i) $(0, 8)$, $(6, 0)$ and origin ; (ii) $(9, 3)$, $(1, -3)$ and origin.

15. Find the point on the y-axis equidistant from $(-5, 2)$ and $(9, -2)$ (Hint: A point on the y-axis will have its x-coordinate as zero).

16. Find the radius of the circle whose centre is $(3, 2)$ and passes through $(-5, 6)$.

17. Prove that the points $(0, -5)$, $(4, 3)$ and $(-4, -3)$ lie on the circle centred at the origin with radius 5.

18. In the Fig. 5.20, PB is perpendicular segment from the point A (4,3). If $PA = PB$ then find the coordinates of B.

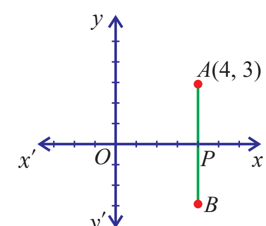


Fig. 5.20

19. Find the area of the rhombus $ABCD$ with vertices A $(2, 0)$, B $(5, -5)$, C $(8, 0)$ and D $(5, 5)$. [Hint: Area of the rhombus $ABCD = \frac{1}{2} d_1 d_2$]

20. Can you draw a triangle with vertices $(1, 5)$, $(5, 8)$ and $(13, 14)$? Give reason.

21. If origin is the centre of a circle with radius 17 units, find the coordinates of any four points on the circle which are not on the axes. (Use the Pythagorean triplets)

22. Show that (2, 1) is the circum-centre of the triangle formed by the vertices (3, 1), (2, 2) and (1, 1).

23. Show that the origin is the circum-centre of the triangle formed by the vertices (1, 0), (0, – 1) and $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$.

24. If the points $A(6, 1)$, $B(8, 2)$, $C(9, 4)$ and $D(p, 3)$ taken in order are the vertices of a parallelogram, find the value of p using distance formula.

25. The radius of the circle with centre at the origin is 10 units. Write the coordinates of the point where the circle intersects the axes. Find the distance between any two of such points.

Exercise 5.3

Multiple Choice Questions.

1. The point (–2,7) lies is the quadrant

   (A) I (B) II (C) III (D) IV

2. The point $(x,0)$ where $x < 0$ lies on

   (A) OX (B) OY (C) OX' (D) OY'

3. For a point A (a,b) lying in quadrant III

   A) $a > 0, b < 0$ (B) $a < 0, b < 0$ C) $a > 0, b > 0$ D) $a < 0, b > 0$

4. The diagonal of a square formed by the points (1,0) (0,1) (–1,0) and (0,–1) is

   A) 2 B) 4 C) $\sqrt{2}$ D) 8

5. The triangle obtained by joining the points A (–5,0) B (5,0) and C (0,6) is

   A) an isosceles triangle B) right triangle
   C) scalene triangle D) an equilateral triangle

6. The distance between the points (0,8) and (0,–2) is

   A) 6 B) 100 C) 36 D) 10

7. (4,1), (–2,1), (7,1) and (10,1) are points

   (A) on x-axis (B) on a line parallel to x-axis
   (C) on aline paralled to y-axis (D) on y-axis

8. The distance between the points (a, b) and $(-a, -b)$ is

   (A) 2a (B) 2b (C) 2a + 2b (D) $2\sqrt{a^2 + b^2}$

9. The relation between p and q such that the point (p,q) is equidistant from $(-4, 0)$ and $(4,0)$ is

(A) $p = 0$ (B) $q = 0$ (C) $p + q = 0$ (D) $p + q = 8$

10. The point which is on y-axis with ordinate -5 is

(A) $(0, -5)$ (B) $(-5, 0)$ (C) $(5, 0)$ (D) $(0, 5)$

Points to Remember

- Two perpendicular lines are needed to locate the position of a point in a plane.
- In rectangular coordinate systems one of them is horizontal and the other is vertical.
- These two horizontal and vertical lines are called the coordinate axes (x-axis and y-axis).
- The point of intersection of x-axis and y-axis is called the origin with coordinates $(0, 0)$.
- The distance of a point from y-axis is x coordinate or abscissa and the distance of the point from x-axis is called y-coordinate or ordinate.
- y-coordinate of the points on x-axis is zero.
- x-coordinate of the points on y-axis is zero.
- y-coordinate of the points on the horizontal lines are equal.
- x-coordinate of the points on the vertical lines are equal.
- If x_1 and x_2 are the x coordinates of two points on the x-axis, then the distance between them is $|x_1 - x_2|$.
- If y_1 and y_2 are the y coordinates of two points on the y-axis, then the distance between the point is $|y_1 - y_2|$.
- Distance between (x_1, y_1) and the origin is $\sqrt{x_1^2 + y_1^2}$.
- Distance between the two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Activity 1

Plot the points (1, 1), (3, 1) and (3, 4) in a graph sheet and join them to form a triangle. Draw the mirror image of the diagram

(i) about x-axis (ii) about y-axis.

Tabulate the changes you could observe in the coordinates of the mirror images.

Activity 2

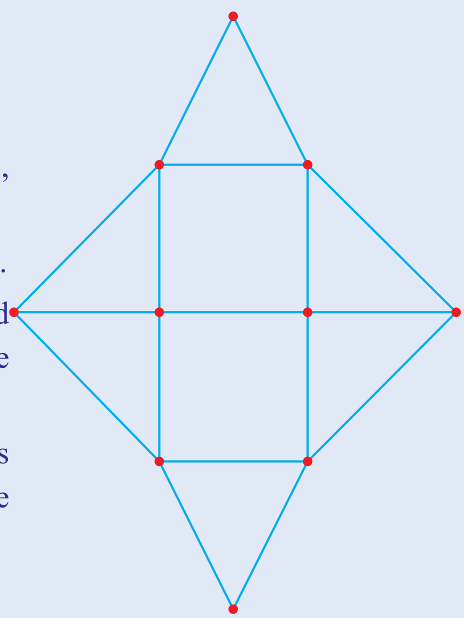
Plot the points $A(4, 5)$, $B(4, 2)$ and $C(-2, 2)$ in a graph sheet and join them to form a triangle. Plot the points $D(1, 2)$ and $E(1, 3.5)$. Join AD and BE. Check whether the two lines intersect at $G(2, 3)$. This point $G(2, 3)$ is the centroid of the triangle and the lines AD, BE are the medians of the triangle. Using the distance formula find the length of AG and GD. See that AG:GD = 2:1.

Activity 3

Plot the given points on a graph sheet :

(2, 0), (–2, 0), (–6, 0), (6, 0), (2, 4), (2,–4), (–2, 4), (–2, –4), (0, 8) and (0, –8).

Join the points to get the following diagram.



Find how many triangles, right angled triangles, squares and rectangles are there in the diagram.

Can you draw this diagram as a continuous curve that passes through every segment of the diagram exactly once.

Try this with your own examples.

Activity 4

Plot the points given below and join them in the given order on a graph sheet:

(4, 4), (0, 0), (–3, 0), (–2, 2), (4, 4), (–3, 7), (–4, 11), (0, 10), (4, 4), (3, 9), (5, 11), (6, 9), (4,4), (8, 9), (11, 8), (10, 6), (4, 4), (12, 4), (14, 2), (11, 0), (4, 4), (8, –2), (7, –5), (4, –3), (4, 4), (–1, –11), (–4, –6), (–7, –5), (–6, –8), (–1, –11), (1, –16), (4, –11), (8, –10), (7, –13), (1, –16), (3, –21).

What do you see?

Exercise 5.1

1. (i) False (ii) True (iii) True (iv) False (v) True (vi) False (vii) True (viii) True (ix) False (x) True **2.** (i) I (ii) III (iii) on x axis (iv) III (v) on y axis (vi) on y axis (vii) IV (viii) origin (ix) I (x) II **3.**(i) –7 (ii) 3 (iii) 8 (iv) –5

4. (i) 5 (ii) 9 (iii) 8 (iv) –4 **5.** parallel to y axis **6.** parallel to x axis **7.** y axis

8. $ABCD$ is a square **9.** (0,4) **11.** (4,3)

Exercise 5.2

1. (i) $\sqrt{202}$ (ii) $4\sqrt{5}$ (iii) $\sqrt{29}$ (iv) $2\sqrt{2}$ (v) $5\sqrt{2}$ (vi) 1 (vii) 5 (viii) 15 (ix) 18 (x) $\sqrt{74}$ **10.** 7, –5 **13.** –10, –2 **14.** (i) 24 (ii) $10 + 4\sqrt{10}$

15. (0,–7) **16.** $4\sqrt{5}$ **18.** (4,–3) **19.** 30 **20.** No, collinear points

21. (8,–15) (–8,–15) (–8,15)(8, 15) **24.**11, 7 **25.** 20

Exercise 5.3

1. B **2.** C **3.** B **4.** A **5.** A **6.** D **7.** B **8.** D **9.** A **10.** A

6 PRACTICAL GEOMETRY

Main Targets

- To construct the Circumcentre
- To construct the Orthocentre

6.1 Introduction

The fundamental principles of geometry deal with the properties of points, lines, and other figures. Practical Geometry is the method of applying the rules of geometry to construct geometric figures. "Construction" in Geometry means to draw shapes, angles or lines accurately. The geometric constructions have been discussed in detail in Euclid's book 'Elements'. Hence these constructions are also known as Euclidean constructions. These constructions use only a compass and a straightedge (i.e. ruler). The compass establishes equidistance and the straightedge establishes collinearity. All geometric constructions are based on those two concepts.

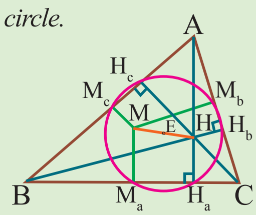
It is possible to construct rational and irrational numbers using a straightedge and a compass as seen in chapter II. In 1913 the Indian Mathematical Genius, Ramanujan gave a geometrical construction for $^{355}/_{113} = \pi$. Today with all our accumulated skill in exact measurements, it is a noteworthy feature when lines driven through a mountain meet and make a tunnel. How much more wonderful is it that lines, starting at the corner of a perfect square, should be raised at a certain angle and successfully brought to a point, hundreds of feet aloft! For this, and more, is what is meant by the building of a pyramid:

LEONHARD EULER
1707 - 1783

The Swiss mathematician Leonhard Euler lived during the 18th century. Euler wrote more scientific papers than any mathematician before or after him. For Euler, mathematics was a tool to decipher God's design of our world. With every new discovery, he felt a step nearer to understanding nature and by this understanding God.

Euler even found a new theorem in Euclidean geometry, a field which had been looked at as completed. Here's a short explanation of this theorem:

The three altitudes of a triangle meet in point H, and the three perpendicular bisectors in point M. Point E in the middle of the line between H and M is the center of a circle on which are all the inter-sections of the altitudes and the perpendicular bisectors with the triangle. This circle known as 9 points circle.



In class VIII we have learnt the construction of triangles with the given measurements. In this chapter we learn to construct ortho-centre and circum-centre of a triangle.

6.2 Special line segments within Triangles

First let us learn to identify and to construct

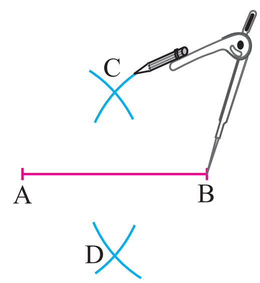
(i) Perpendicular bisector to a given line segment

(ii) Perpendicular from an external point to a given line

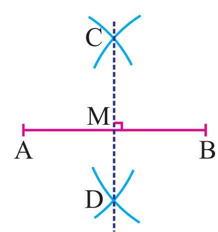
6.2.1 Construction of the Perpendicular Bisector of a given line segment

Step1 : Draw the given line segment *AB*.

Step 2 : With the two end points *A* and *B* of the line segment as centres and more than half the length of the line segment as radius draw arcs to intersect on both sides of the line segment at *C* and *D*.



Step 3 : Join *C* and *D* to get the perpendicular bisector of the given line segment *AB*.

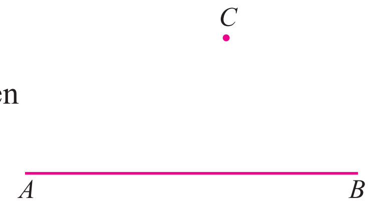


Key Concept **Perpendicular Bisector**

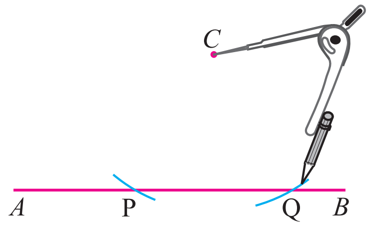
The line drawn perpendicular through the midpoint of a given line segment is called the perpendicular bisector of the line segment.

6.2.2 Construction of Perpendicular from an External Point to a given line

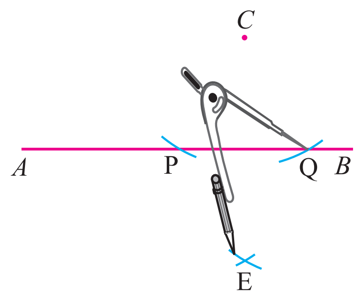
Step 1 : Draw the given line AB and mark the given external point C.



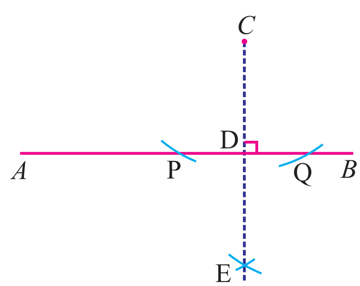
Step 2 : With C as centre and any convenient radius draw arcs to cut the given line at two points P and Q.



Step 3 : With P and Q as centres and more than half the distance between these points as radius draw two arcs to intersect each other at E.

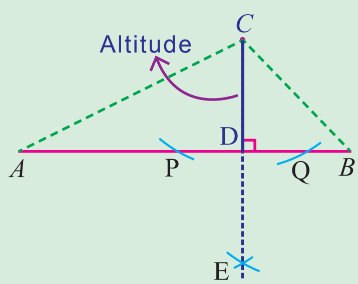


Step 4 : Join C and E to get the required perpendicular line.



Key Concept **Altitude**

In a triangle, an altitude is the line segment drawn from a vertex of the triangle perpendicular to its opposite side.



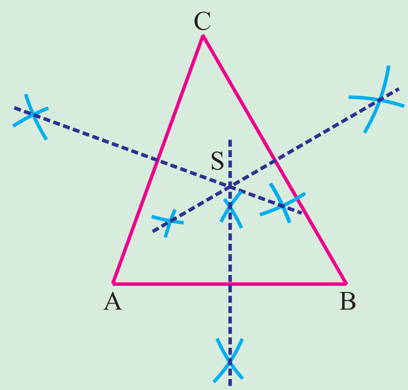
6.3 The Points of Concurrency of a Triangle

As we have already learnt how to draw the Perpendicular Bisector and Altitude, now let us learn to locate the Circumcentre and Othocentre of a given triangle.

6.3.1 Construction of the Circumcentre of a Triangle

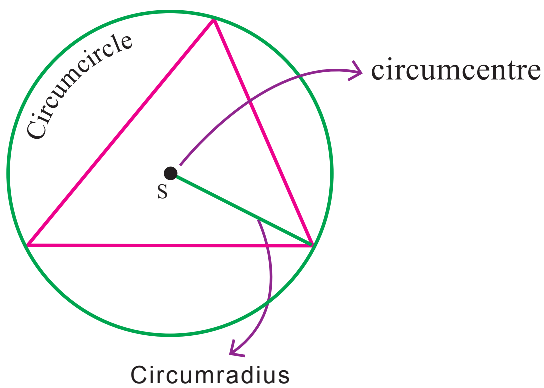
Key Concept — **Circumcentre**

The point of concurrency of the perpendicular bisectors of the sides of a triangle is called the circumcentre and is usually denoted by *S*.



Circumcircle

The circle drawn with *S* (circumcentre) as centre and passing through all the three vertices of the triangle is called the circumcircle.

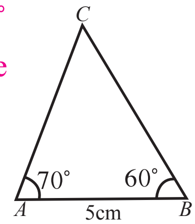


Circumradius

The radius of a circumcircle is called circumradius of the triangle. In other words, the distance between the circumcentre *S* and any vertex of the triangle is the circumradius.

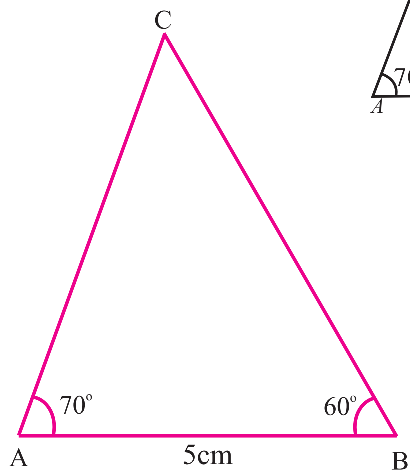
Example 6.1

Construct the circumcentre of the ΔABC with $AB = 5\text{cm}$, $\angle A = 70^\circ$ and $\angle B = 60^\circ$. Also draw the circumcircle and find the circumradius of the ΔABC.

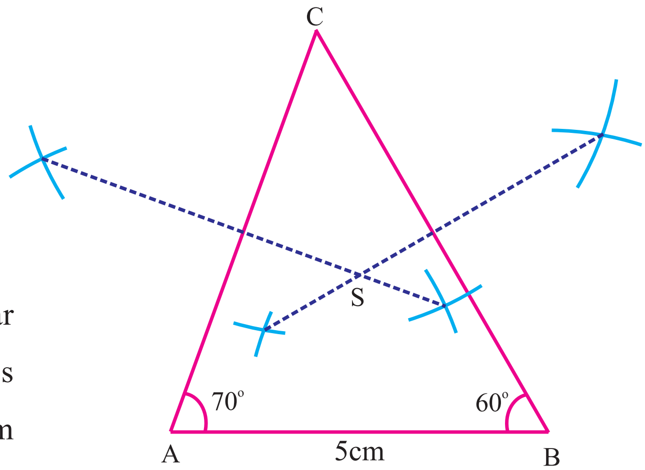


Rough Diagram

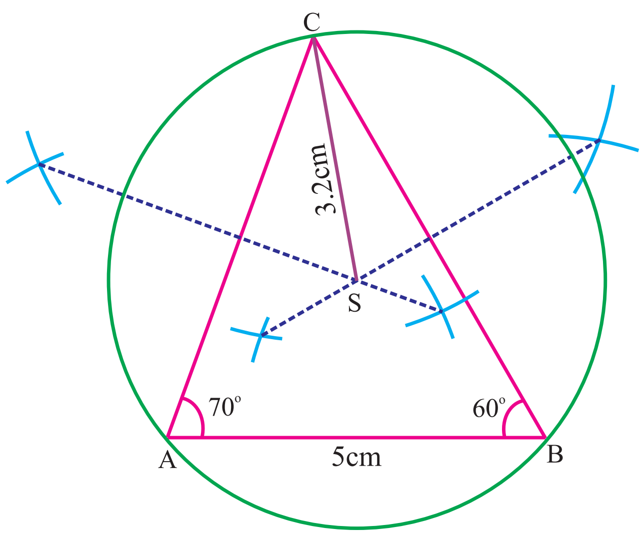
Solution



Step 1 : Draw the ΔABC with the given measurements.



Step2 : Construct the perpendicular bisectors of any two sides (AC and BC) and let them meet at S which is the circumcentre.



Step 3 : With S as centre and $SA = SB = SC$ as radius draw the circumcircle to pass through A, B and C.

Circumradius = 3. 2cm.

Remark

1. The circumcentre of an acute angled triangle lies inside the triangle.
2. The circumcentre of a right triangle is at the midpoint of its hypotenuse.
3. The circumcentre of an obtuse angled triangle lies outside the triangle.

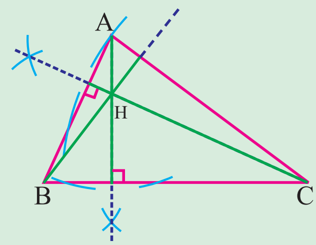
Exercise 6.1

1. Construct ΔPQR with $PQ = 5$cm, $\angle P = 100^\circ$ and $PR = 5$cm and draw its circumcircle.
2. Draw the circumcircle for
   (i) an equilateral triangle of side 6cm.
   (ii) an isosceles right triangle having 5cm as the length of the equal sides.
3. Draw ΔABC, where $AB = 7$cm, $BC = 8$cm and $\angle B = 60^\circ$ and locate its circumcentre.
4. Construct the right triangle whose sides are 4.5cm, 6cm and 7.5cm. Also locate its circumcentre.

6.3.2 Construction of the Orthocentre of a Triangle

Key Concept — **Orthocentre**

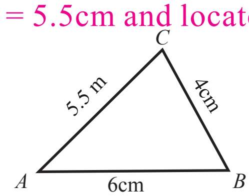
The point of concurrency of the altitudes of a triangle is called the orthocentre of the triangle and is usually denoted by H.



Example 6.2

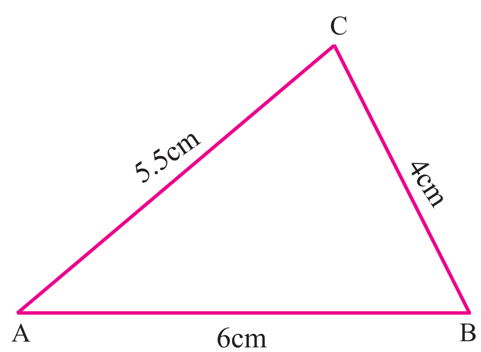
Construct ΔABC whose sides are $AB = 6$cm, $BC = 4$cm and $AC = 5.5$cm and locate its orthocentre.

Solution

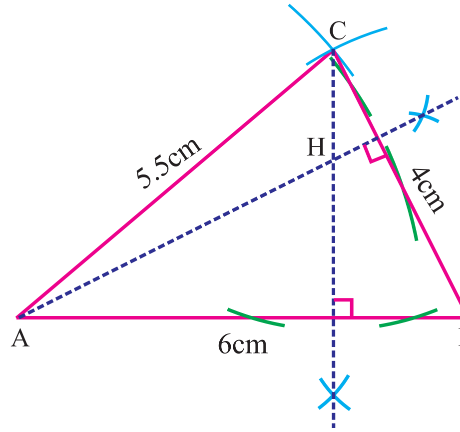


Rough Diagram

Step 1 : Draw the ΔABC with the given measurements.



Step 2 : Construct altitudes from any two vertices (A and C) to their opposite sides (BC and AB respectively).



The point of intersection of the altitudes H is the orthocentre of the given ΔABC.

Remark

1. Three altitudes can be drawn in a triangle.
2. The orthocentre of an acute angled triangle lies inside the triangle.
3. The orthocentre of a right triangle is the vertex of the right angle.
4. The orthocentre of an obtuse angled triangle lies outside the triangle.

Exercise 6.2

1. Draw ΔABC with sides $AB = 8$cm, $BC = 7$cm and $AC = 5$cm and construct its orthocentre.
2. Construct the orthocentre of ΔLMN, where $LM = 7$cm, $\angle M = 130^\circ$ and $MN = 6$cm.
3. Construct an equilateral triangle of sides 6cm and locate its orthocentre.
4. Draw and locate the orthocentre of a right triangle PQR right angled at Q, with $PQ = 4.5$cm and $QR = 6$cm.
5. Construct an isosceles triangle ABC with sides $AB = BC = 6$cm and $\angle B = 80^\circ$ and locate its orthocentre.

Activity 1

Objective : To find the mid-point of a line segment using paper folding

Procedure : Make a line segment on a paper by folding it and name it PQ. Fold the line segment PQ in such a way that P falls on Q and mark the point of intersection of the line segment and the crease formed by folding the paper, as M. M is the mid-point of PQ.

Activity 2

Objective : To construct the perpendicular bisector of a line segment using paper folding.

Procedure : Make a line segment on a paper by folding it and name it PQ. Fold PQ in such a way that P falls on Q and thereby creating a crease RS. This crease RS is the perpendicular bisector of PQ.

Activity 3

Objective : To construct the perpendicular to a line segment through an external point using paper folding.

Procedure : Draw a line segment AB and mark an external point P. Move B along BA till the fold passes through P and crease it along that line. The crease thus formed is the perpendicular to AB through the external point P.

Activity 4

Objective : To locate the circumcentre of a triangle using paper folding

Procedure : Using Activity 2 find the perpendicular bisectors for any two sides of the given triangle. The meeting point of these is the circumcentre of the given triangle.

Activity 5

Objective : To locate the orthocentre of a triangle using paper folding

Procedure : Using Activity 3 with any two vertices of the triangle as external points, construct the perpendiculars to opposite sides. The point of intersection of these perpendiculars is the orthocentre of the given triangle.

‘I can, I did’

Student's Activity Record

Subject :

Sl. No.	Date	Lesson No.	Topic of the Lesson	Activities	Remarks