Advanced Digital Communications (EQ2411)
Period 3, 2022/23

Homework Project 3
Due: Friday, March 03, 2023, 18:00 R. Thobaben

Introduction

This homework assignment is the third of three continuous examination assignments and will examine the course goals:

1. use mathematical models for describing advanced communication channels and systems such as communication systems with dispersion, interference, multiple users, multipath propagation, multiple carriers and multiple antennas
2. use mathematical models for characterising properties for advanced communication channels and systems and identify properties that limit the communication
4. summarise advantages and disadvantages with different advanced communication technologies and be able to discuss their optimality and complexity
6. for a given combination of channel model and communication technique use mathematical models for analysing the expected performance (e.g., error probabilities, speed) and compare the performance for different solutions.

Grading In the following you are given two problems that are to be solved individually. The total number of points that can be obtained in this homework assignment is **16 points**, and the grades are determined as follows:

Grade	Requirement
C	12 or more points
E	9 or more points
F	less than 9 points

- Before finalizing the grades, the examiner may call randomly selected students to a short interview in order to discuss the solutions.

Submission Solutions in PDF format (e.g., scans or photographs of handwritten solutions) are submitted in Canvas, and the submission closes on March 3, 2023 18:00.

- Solutions that are uploaded later than 6 hours after the deadline can at most achieve grade E.
- Solutions uploaded later than 30 hours after the deadline will be graded with F.

Plagiarism This is an individual assignment, and the submitted solutions have to be prepared by the students individually. Students are allowed to discuss general concepts that are relevant for solving the problem with other students, and students are also allowed to solve related old exam problems together. However, proof reading each others solutions or helping in solving each others assignments is not allowed.

If plagiarism is detected, the case will be reported following KTH rules and policies.

Assignment

Problem 1 **[8 points]**

In this problem, we study receive diversity and diversity combining techniques for a system with N parallel diversity branches (e.g., N parallel receive antennas). The matlab code corresponding to this problem can be found in the script `ReceiveDiversity.m`.

(a) Give an expression for the PDF $p(W)$ of the noise W and the PDF $p(\boldsymbol{H})$ of the channel vector $\boldsymbol{H}$. **(1 point)**

(b) Complete the code by implementing the decision statistics for the following combining schemes (please submit the updated matlab code as part of your solution, e.g., as an appendix):

1. **Maximum-Ratio Combining:** This combining method is equivalent to doing matched filtering with the vector $\boldsymbol{H}$:
$$Z_{MRC} = \boldsymbol{H}^H \boldsymbol{Y}.$$
(1 point)

2. **Selection Combining:** This combining method selects the element Y_i of the received signal vector that carries the maximum signal energy:
$$Z_{SC} = H_{\hat{\imath}}^* Y_{\hat{\imath}}, \quad \text{with } \hat{\imath} = \arg\max_j |H_j|^2.$$
Note that the multiplication with $H_{\hat{\imath}}^*$ is done in order to allow for coherent detection (i.e., the phase rotation due to the channel gain is undone). **(1 point)**

3. **Equal-Gain Combining:** This combining method correlates the receive vector with an all-one vector:
$$Z_{EQ} = \mathbf{1}^T \boldsymbol{Y}, \quad \text{with } \mathbf{1} = [1, \dots, 1]^T.$$
(1 point)

4. **Equal-Gain Combining with Phase Compensation:** This combining method only corrects the phase offsets in order to enable coherent combining but does not take into account the SNRs on the diversity branches:
$$Z_{EQ-PC} = \sum_{i=1}^{N_T} e^{-j\phi_i} Y_i, \quad \text{with } \phi_i = \arg(H_i).$$
(1 point)

5. **Switched-Diversity Combining:** This combining method selects an element Y_i of the received signal vector at random:
$$Z_{SD} = H_{\hat{\imath}}^* Y_{\hat{\imath}},$$
where $\hat{\imath}$ is a random variable drawn from a uniform distribution over the set $\{1, \dots, N\}$. **(1 point)**

(c) Simulate the system for $N = 2$ and for $N = 3$ and choose the number of simulation runs and the block length appropriately in order to get smooth result plots.

Which combining methods achieve the full diversity gain N? Can you explain what goes wrong in the cases where the full diversity gain N is not reached? Based on these results, summarize advantages and disadvantages of the different schemes (e.g., complexity, required channel knowledge, achieved diversity gain). **(2 point)**

Problem 2 **[8 points]**

In this problem, we continue to study receive diversity and diversity combining techniques for a system with N parallel diversity branches and specialize to the case $N = 2$.

(a) In order to calculate the average bit-error probability for the switched diversity scheme, derive first the PDF of the instantaneous SNR. Use this PDF to calculate the average error probability. Verify your result by comparing it to the simulation results obtained in Problem 1. **(3.5 point)**

(b) In order to calculate the average bit-error probability for the selection combining scheme, derive first the PDF of the instantaneous SNR. Use this PDF to calculate the average error probability. Verify your result by comparing it to the simulation results obtained in Problem 1. **(4.5 point)**

Hints:

1. In the calculation of the average error probability, it may be convenient to replace the Q-function by the underlying integral and to proceed as demonstrated in Lecture 9 (see slide 18 in the meeting material).

2. Instead of calculating the exact average error probability, you can use an upper bound on the Q function which simplifies the calculations. You will lose 0.5 points.

3. In part (b), it may be convenient to first calculate the CDF of the instantaneous SNR and derive the PDF in a second step.