Advanced Digital Communications (EQ2411)
Period 3, 2022/23

Homework Project 2
Due: Friday, February 17, 2023, 18:00 R. Thobaben

Introduction

This homework assignment is the second of three continuous examination assignments and will examine the course goals:

1. use mathematical models for describing advanced communication channels and systems such as communication systems with dispersion, interference, multiple users, multipath propagation, multiple carriers and multiple antennas
2. use mathematical models for characterising properties for advanced communication channels and systems and identify properties that limit the communication
4. summarise advantages and disadvantages with different advanced communication technologies and be able to discuss their optimality and complexity
6. for a given combination of channel model and communication technique use mathematical models for analysing the expected performance (e.g., error probabilities, speed) and compare the performance for different solutions.

Grading In the following you are given three problems that are to be solved individually. The total number of points that can be obtained in this homework assignment is **12.5 points**, and the grades are determined as follows:

Grade	**Requirement**
C	10 or more points
E	7 or more points
F	less than 7 points

- Before finalizing the grades, the examiner may call randomly selected students to a short interview in order to discuss the solutions.

Submission Solutions in PDF format (e.g., scans or photographs of handwritten solutions) are submitted in Canvas, and the submission closes on Feb. 17, 2023 18:00.

- Solutions that are uploaded later than 6 hours after the deadline can at most achieve grade E.
- Solutions uploaded later than 30 hours after the deadline will be graded with F.

Plagiarism This is an individual assignment, and the submitted solutions have to be prepared by the students individually. Students are allowed to discuss general concepts that are relevant for solving the problem with other students, and students are also allowed to solve related old exam problems together. However, proof reading each others solutions or helping in solving each others assignments is not allowed.

If plagiarism is detected, the case will be reported following KTH rules and policies.

Assignment

The second homework assignment is centered around a matlab implementation of a simple LDPC coded transmission system with iterative decoding. The matlab script `main.m` contains the implementation of the system, and the scripts `decoder_1.m` and `decoder_2.m` contain the implementation of the variable and check node decoders for a specific decoding algorithm. Note that in this implementation, the messages exchanged between the component decoders are BPSK modulated hard decision bit estimates.

Problem 1: [3 points]

Make yourself familiar with the code, have a look at some variables, and answer the following questions:

(a) Why is transmission of the all-zero codeword (mapped to the all-one codeword after BPSK modulation) sufficient to evaluate the performance of the code? (0.5 point)

(b) Which channel model is implemented? (0.5 point)

(c) Which decoder is implemented? (0.5 point)

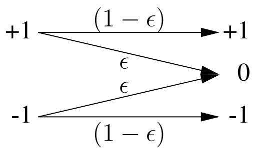
(d) Which degree distributions are given by the distributions `Distr_1A`, `Distr_1B`, `Distr_2A`, and `Distr_2A`? (0.5 point)

(e) Which problems can occur when generating the check matrix? (0.5 point)
(Hint: Verify how the check matrix was constructed in `main.m` , and consider a regular length 1000 (3,7) LDPC code. How many edges and how many check nodes do you get?).

(f) How can the current choice of a random interleaver in some cases (i.e., for specific realizations) limit the decoder performance? (0.5 point)

Problem 2: [7 point]

In the following, we focus on belief-propagation decoding for the binary erasure channel (BEC) with erasure probability ϵ:



(a) Change the channel model in the implementation to the BEC, and implement the belief-propagation decoder for the BEC case. (1 point)

Hint: Note that the bits at the output of the channel are either erased or correct and that therefore the messages exchanged by the component decoders can be represented either by the correct bits (-1 or $+1$) or by the erasure symbol 0.

(b) Explain why it is sufficient in the BEC case to track the erasure probability for the messages which are exchanged by the component decoders in order to perform density evolution. (1 point)

(c) Let in the following $p(l)$ and $q(l)$ denote the erasure probabilities at the output for the variable-node decoder and the check-node decoder, respectively, at iteration l.

1. For a variable node with degree d_v, show and explain that (1 point)

$$p(l) = \epsilon \cdot q(l-1)^{d_v - 1}.$$

2. For a check node with degree d_c, show and explain that (1 point)

$$q(l) = 1 - (1 - p(l))^{d_c - 1}.$$

(d) Verify your implementation in the following way:

1. Choose Gallagher's (3,6) LDPC code, and run computer simulations for $\epsilon \in \{0.1, 0.15, \ldots, 0.8\}$ and a block length of $n = 10000$ code symbols. Set the maximum number of iterations to 20.

   Plot the erasure probability after decoding over ϵ, and check for which value of ϵ the decoder starts to converge. (1 point)
2. Let in the following $f(.)$ and $g(.)$ denote the functions $p(l) = f(q(l-1))$ and $q(l) = g(p(l))$.

   Plot the function $f(q)$ and the inverse function $g^{-1}(q)$ for $q \in [0,1]$ for $\epsilon \in \{0.1, 0.15, \ldots, 0.8\}$ into one diagram. (1 point)
3. Explain why convergence is only possible if $g^{-1}(q) > f(q)$. Determine the decoding threshold ϵ_T for the (3,6) LDPC code, and compare it to the result of your simulation. (1 point)

Problem 3: **[2.5 point]**

Now, try to design a good regular LDPC code with $d_c \geq 3$ for the BEC for the decoding thresholds ϵ_T given in the following table.

	ϵ_T
Student A	0.3
Student B	0.4
Student C	0.5
Student D	0.6

Try to maximize the code rate R under the constraint that the decoder converges for $\epsilon \leq \epsilon_T$. Explain how you have optimized your code and show simulation results which verify that your design fulfils the requirements.