[H03H1A] Musculoskeletal Biomechanics
Assignment 2: Biomechanical (over-)loading

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Contents

1 Situation

A man is driving a SUV in the village center when suddenly a pedestrian crosses the road. The man immediately brakes but unfortunately he is unable to stop the car before it hits the pedestrian. On the bumper of the car a bull bar is mounted (Figure 2) which, in first instance, hits the upper leg of the pedestrian. In this paper we will investigate what the forces exerted on the femur by the bull bar are, what the internal stresses in the femur are and whether the femur will break or not.

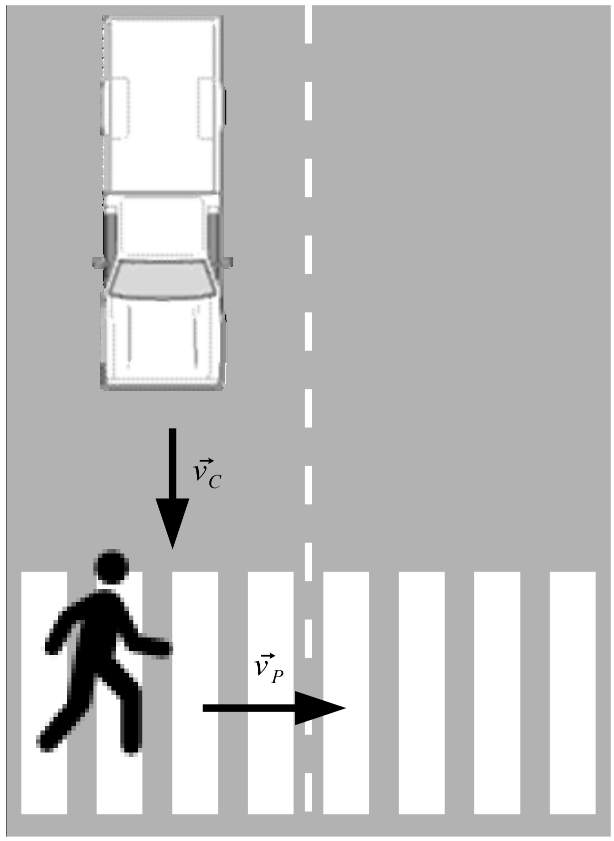

Figure 1: Overview of the situation.

In Figure 3 the initial events in a car-pedestrian collision are visualized. Although the pictures give the reader a good impression of what happens during a crash, it is not entirely conforming to the situation described above. In Figure 3 a normal passenger car hits the person in front of it. The bumper of a passenger car is at a much lower height than the bumper of a SUV. Also, there is no bull bar present.

Now that the situation is properly defined, we will discuss the assumptions we made in the next section. These will be the basis of our mechanical analysis in section 3

2 Assumptions

Figure 3 shows how complex even the initial events during a collision are. As it would be too difficult to analyse these subsequent situations without the proper software, only the first event (A) is taken into account

Figure 2: A bullbar mounted on a car. The protuding part is circled in red.

to reduce the complexity. To decrease the complexity even more, a number of assumptions are made. In the following paragraphs these assumptions are explained in detail.

Our victim is a healthy thirty year old male with a height of 1,80m and a mass of 80kg. Before the collision he is located right in front of the car. He is walking and at the moment of the crash the leg closest to the car is in stand phase. The other leg is in swing phase. We examine the femur of the former leg, because it is most likely to fail. The weight of both his upper and lower leg segments are also calculated. It is assumed the weight is equally distributed over the length of both parts, so the center of gravity is situated in the middle of both parts. Using Figure 4, we estimate the distance from respectively the angle, knee and hip to the ground to be 0,07m, 0,51m and 0,95m. The lower leg has a mass of 3,72kg and the upper leg has a mass of 8kg[BDK06]. [HS77] suggest E_{femur} = 20 GPa and ν_{femur} = 0,37.

The car weighs 2400kg, the bull bar is made of aluminium (E_{Al}=69 GPa and ν_{Al}=0,32) and its most protuding part sits at 0,80m heigh. At the time of collision the car is decelerating, but it still has a speed of 36 km/h (10 m/s). Due to the bullbar that is mounted on the bumper, the initial contact between the body of the pedestrian and the car is limited to the bar touching the upper leg of the pedestrian. The femur is the long bone that provides support in the upper leg. Around the femur the hamstring muscles, the quadriceps muscles and a layer of fat act as shock absorbers during the impact. The literature suggests using a damping factor of roughly 15% [KPP99].

As only the initial phase of the impact is considered in this paper, it is assumed that the horizontal velocity of the hip is zero at the time of impact due to the inertia of the body. This boundary condition is also used in the literature [SWM$^+$05]. The foot also remains in place, but rotates inwards at the ankle. Because of this the bones in the lower leg do not deform. Visual analysis of crash dummy tests [1] confirms this. Further it is assumed that the knee does not bend initially. So the deformation induced by the bull bar is only reflected in the femur shaft. In reality the knee will also be affected by the forces exerted by the bull bar. Depending on the situation and the relative strength of both, the femur will break or the knee will be distorted. In the following analysis however, it is assumed that the femur will absorb the initial impact. This means we

[1] https://www.youtube.com/watch?v=tNRHB75NiIc

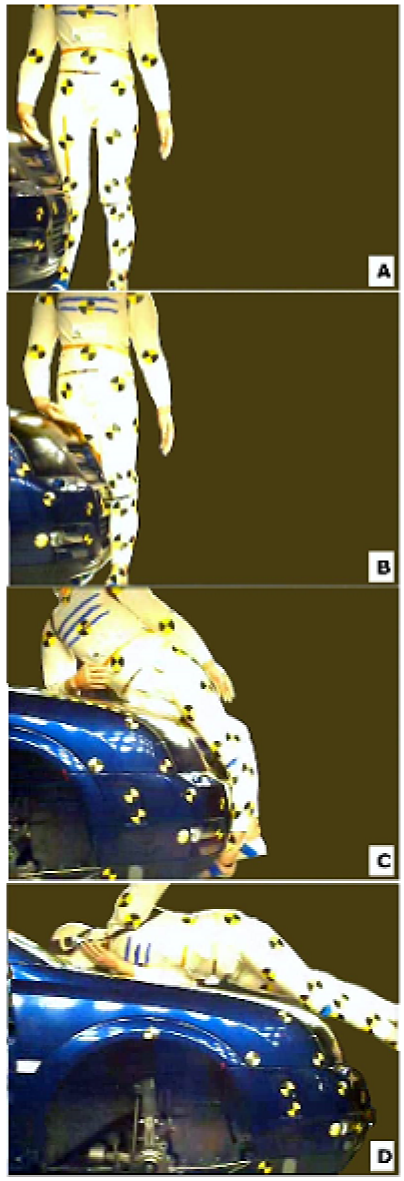

Figure 3: Subsequent events in car-pedestrian collision.

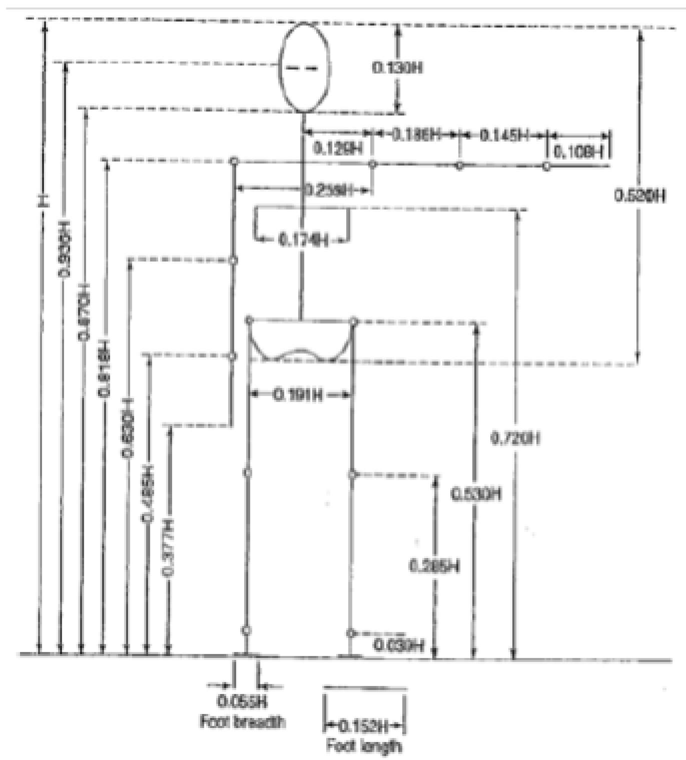

Figure 4: Various lengths of human body segments in proportion to total height.

assume that the knee will not become distorted (so it will continue to properly connect the upper and lower leg) and that the other bones – from pelvis to the little bones in the foot – are not affected initially.

Because of these (rather rough) assumptions the whole leg can be treated as one structure which is clamped at the top (pelvis) and which hinges at the level of the ankle. The force exerted on this structure by the bull bar is treated as a force concentrated in one point at a height of 80 cm. It is assumed that the bull bar will not deform during the collision, nor that the crumple zone of the car is activated. The complete system is visualised in Figure 5.

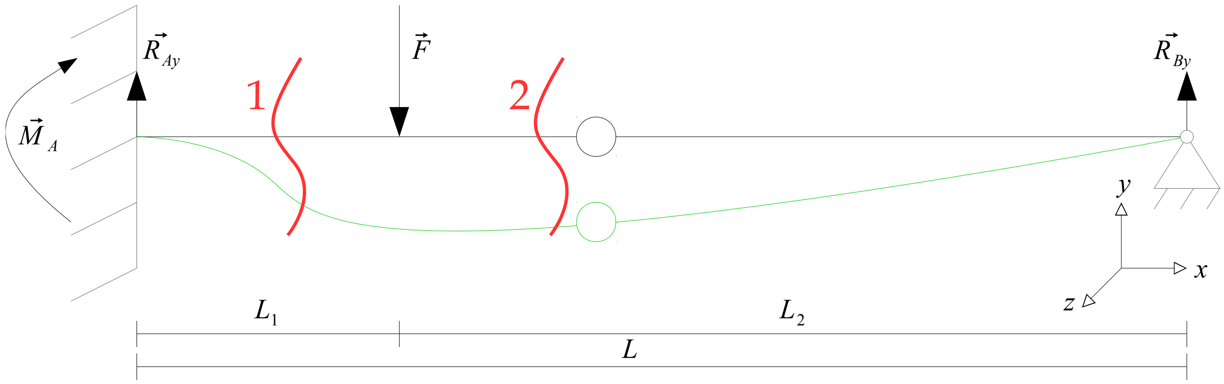

Figure 5: Schematic of the system under consideration. The pelvis (left) is modeled as a clamp, while the angle (right) is replaced by a hinge. The knee (circle) is merely added for visual guidance, it does not play any significant part in this analysis. The total length L is split up in L_1 and L_2 at the point of impact.

As the pedestrian is a young male it is assumed he has normal bones, not affected by any disease. The femoral bone has an average length of 48 cm, a shaft diameter of 2.43 cm and the femoral canal has a diameter of 13 mm.[2]

3 Mechanical Analysis

In the following paragraphs the effects of the impact of the car on the femur of the pedestrian are investigated. First an estimation of the forces exerted on the femur is made. Then a force diagram and bending moment diagram are calculated. Based on these diagrams the stresses in the bone are estimated and by comparing the stresses with experimental data of previous research the probability that the bone will fail is estimated.

3.1 Calculating impact force

We start from the conservation of momentum law in the direction the car is traveling. This assumes a perfect ellastic collision, which is a very crude approximation of reality. Because the velocity of the pedestrian in this direction is zero, so is his momentum. Figure 3 shows that the car and the pedestrian cling together during the first few seconds after time of impact, so we consider them as one system with one momentum.

$$p_{C_0} + 0 = p_1 \tag{1}$$

[2] `http://www.orthopaedicsone.com`

$$m_C \cdot v_{C_0} + 0 = (m_C + m_P) \cdot v_1 \tag{2}$$

Solve for v_1.

$$v_1 = \frac{m_C}{m_C + m_P} \cdot v_{C_0} = \frac{2400 \text{ kg}}{2480 \text{ kg}} \cdot 10 \text{ m/s} \approx 9.68 \text{ m/s} \tag{3}$$

From this, we can calculate the force acting on the pedestrian.

$$F' = \frac{\Delta p}{\Delta t} \tag{4}$$

The equation shows that the force F is also dependent on the time interval Δt in which the collision occurs. When two hard materials hit each other, e.g. a club hitting a golfbal, this interval is approximately $400\mu s$[3]. On the other hand, the typical impact time - from full speed to standstill - of a car crashing into a wall is about 100ms [Hua02] [4]. This interval is larger because of crumple zones. However, because our car is equipped with a bullbar, and because we only take into account the initial event, we estimate $\Delta t \approx$ 10ms.

Plugging this value into the equation gives:

$$F' = \frac{80 \text{ kg} \cdot 9.68 \text{ m/s}}{0.01 \text{ s}} = 77440 \text{ N} \tag{5}$$

We assumed a soft tissue damping factor of roughly 15%, which allows us to calculate the resulting foce on the femur:

$$F = 0.85F' = 65824 \text{ N} \tag{6}$$

3.2 Discussion on force and bending moment diagrams

After the mechanical system in section 2 was decided upon, the force and bending moment equilibrium equations were calculated in the y-direction.

$$\Sigma F_y = 0 : R_{Ay} + R_{By} - F = 0 \tag{7}$$

$$\Sigma M_z \overset{w.r.t.B}{=} 0 : L_2 F - (L_1 + L_2) R_{Ay} - M_A = 0 \tag{8}$$

We did not calculate the forces in the x-direction, as these were less relevant for our analysis.

From these equations it is clear we have a hyperstatic sytem of level 1, which means we have three unknowns and only two equations. Therefore, this system has to be solved using the method of the chord. The first step in applying this technique is choosing two points a and b that are fixed in your system. Then a point c is determined which is situated between points a and b. Point c indicates where you want to calculate the deformation in the system due to external forces and bending moments. In this case point c coincides with point a. Next the bending moment diagram M, taking into account all external forces, is calculated. From this bending moment diagram M the reduced bending moment M_{red} is deduced by dividing M by the multiplication of the Youngs modulus and the moment of inertia of the system (EI). In this way the

[3] `http://www.golfswing.com.au/139`
[4] `http://auto.howstuffworks.com/car-driving-safety/accidents-hazardous-conditions/crash-test.htm`

stiffness of the system is also taken into account. This allows us to get an exact solution for the hyperstatic mechanical system.

In the next step the reduced bending moment diagram is treated as a distributed force acting on the entire length of the beam. To calculate the equivalent force of these distributed forces the reduced bending moment diagram is divided into triangular parts. For each of these triangular parts an F_e is calculated. By applying a force and bending moment equilibrium on each part of the beam the reaction force in point c is determined. The method of the chord then states that the reaction force in this point is equal to the angle of deformation in that point.

In order to solve the hyperstatic system it has to be divided into two subsystems: a main system (Figure 6) and a recovery system (Figure 7). The main system is the hyperstatic system made static again by cancelling one force or bending moment. In this case we chose to cancel the bending moment of the clamped pelvis. The task of the recovery system is to restore the distortion, due to cancelling the bending moment, of the line of deformation of the main system. Therefore, a bending moment m_a is added in the recovery system. The method of the chord is applied on both systems and then the results of the two systems are superimposed to obtain the results for the initial hyperstatic system.

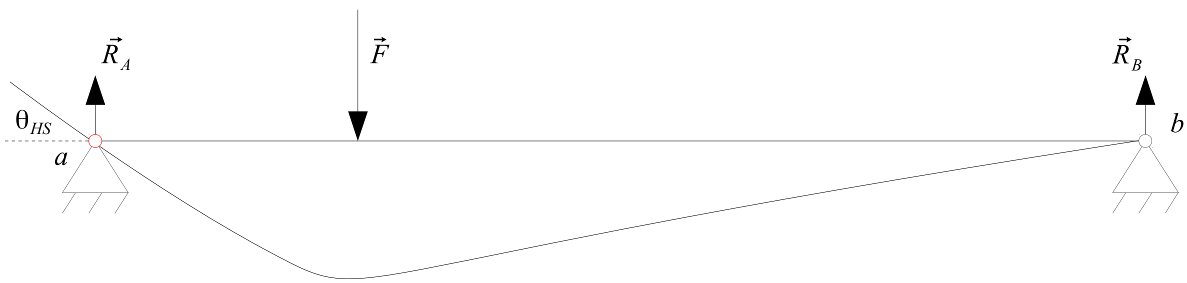

Figure 6: The main system (HS).

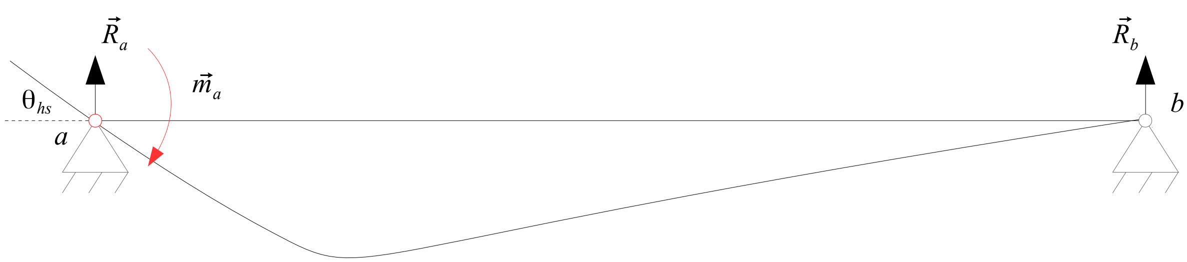

Figure 7: The recovery system (hs).

3.2.1 Analysis of the main system

First, we calculate the bending moment M for the main system. From this bending moment M the reduced bending moment M_{red} is deduced in Figure 8.

$$M_{HS} = R_A \cdot L_1 = \frac{-L_1 \cdot L_2}{L_1 + L_2} \cdot F \tag{9}$$

$$M_{red} = \frac{-L_1 \cdot L_2}{L_1 + L_2} \cdot \frac{F}{EI} \tag{10}$$

Based on M_{red} the equivalent forces F_e are calculated. Then R_A (and thus θ_{HS}) is calculated.

$$F_{e_1} = \frac{-L_1 L_2}{L_1 + L_2} \frac{F}{EI} \frac{L_1}{2} \tag{11}$$

$$F_{e_2} = \frac{-L_1 L_2}{L_1 + L_2} \frac{F}{EI} \frac{L_2}{2} \tag{12}$$

$$\Sigma M_z \overset{w.r.t.b}{=} 0 : \frac{2}{3} L_2 F_{e_2} + (L_2 + \frac{1}{3} L_1) F_{e_1} - R_A (L_1 + L_2) = 0 \tag{13}$$

$$R_A = \frac{\frac{2}{3} L_2 F_{e_2} + L_2 F_{e_1} + \frac{1}{3} L_1 F_{e_1}}{L_1 + L_2} = \theta_{HS} \tag{14}$$

$$R_A = \frac{-1}{3} \frac{L_1 L_2^3 F}{(L_1 + L_2)^2 EI} - \frac{L_1^2 L_2^2 F}{(L_1 + L_2)^2 2EI} - \frac{1}{6} \frac{L_1^3 L_2 F}{(L_1 + L_2)^2 EI} = \theta_{HS} \tag{15}$$

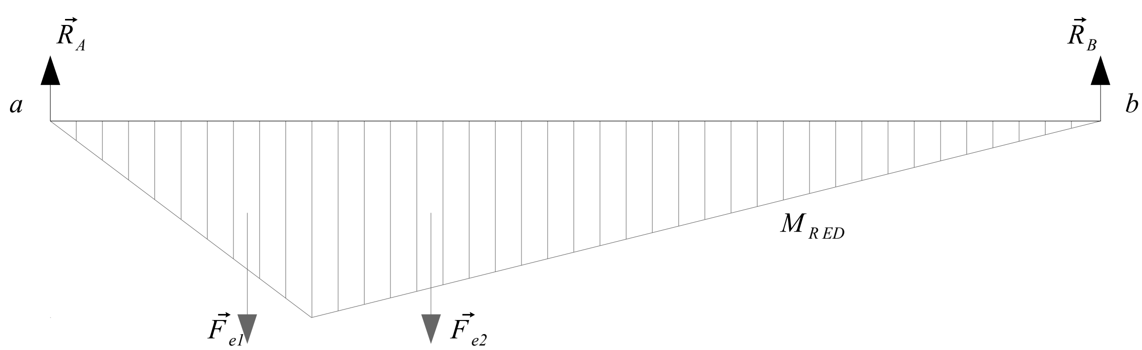

Figure 8: Reduced bending moment diagram M_{red} of the main system.

3.2.2 Analysis of the recovery system

We repeat this exercise for the recovery system in Figure 9.

$$M_{hs} = -m = -m_a \tag{16}$$

$$M_{red} = \frac{-m}{EI} \tag{17}$$

$$\Sigma M_z \overset{w.r.t.b}{=} 0 : \frac{2}{3}(L_1 + L_2)F_e - (L_1 + L_2)R_a = 0 \tag{18}$$

Solve for R_a:

$$R_a = \frac{\frac{2}{3}(L_1 + L_2)F_e}{L_1 + L_2} = \frac{2}{3}F_e \tag{19}$$

We also know F_e:

$$F_e = \frac{-m}{EI}\frac{L_1 + L_2}{2} \tag{20}$$

So we can rewrite R_a as:

$$R_a = \frac{-2}{3}\frac{m}{EI}\frac{L_1 + L_2}{2} = \theta_{hs} \tag{21}$$

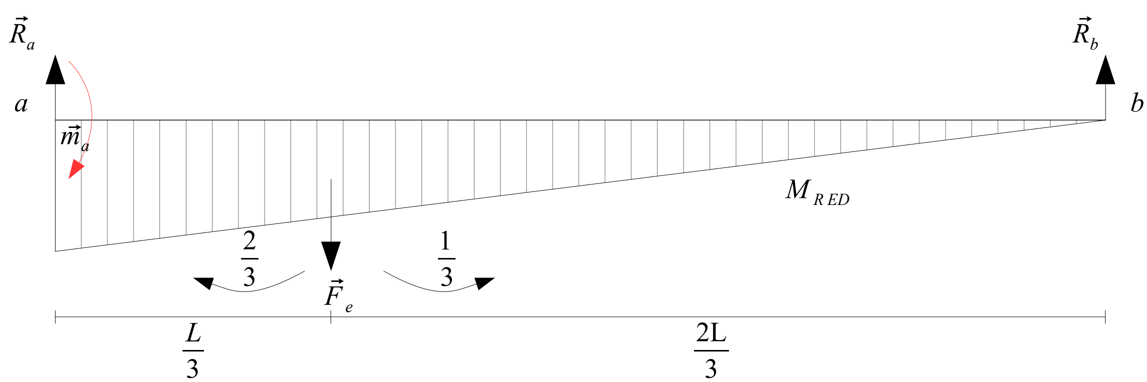

Figure 9: Reduced bending moment diagram of the recovery system.

3.2.3 Combining both systems

To combine the two systems, the following equation must hold:

$$\theta_{HS} + \theta_{hs} = 0 \tag{22}$$

$$\frac{-1}{3}\frac{L_1 L_2^3 F}{(L_1 + L_2)^2 EI} - \frac{L_1^2 L_2^2 F}{(L_1 + L_2)^2 2EI} - \frac{1}{6}\frac{L_1^3 L_2 F}{(L_1 + L_2)^2 EI} - \frac{2}{3}\frac{m}{EI}\frac{L_1 + L_2}{2} = 0 \tag{23}$$

Solve for m:

$$m = \frac{-2L_1L_2^3F - 3L_1^2L_2^2 - L_1^3L_2F}{2(L_1+L_2)^3} \tag{24}$$

Using $L_1 = 0.154$m, $L_2 = 0.8$m and $F = 65824$N, we can calculate M_A by applying the superposition principle in point c:

$$M_A = 0 - m = 7814.44\text{Nm} \tag{25}$$

3.2.4 Solving the original system

With this knowledge, we can finally solve the original system and construct the accompanying force (Figure 10) and bending moment (Figure 11) diagrams by using the principle of superposition.

$$R_{Ay} = \frac{L_2}{L_1+L_2}F = 55198.32\text{N} \tag{26}$$

$$R_{By} = \frac{L_1}{L_1+L_2}F = 10625.68\text{N} \tag{27}$$

$$M_A = 7814.44\text{Nm} \tag{28}$$

$$M_{HS,\text{ at }F} = -\frac{L_1L_2}{L_1+L_2}F = -8500.54\text{Nm} \tag{29}$$

$$M_{hs,\text{ at }F} = \frac{L_2M_A}{L_1+L_2} = 6552.98\text{Nm} \tag{30}$$

$$M_{\text{at }F} = M_{HS,\text{ at }F} + M_{hs,\text{ at }F} = -1947,56\text{Nm} \tag{31}$$

Figure 10: Force diagram of the original system.

The forces applied on the upper part of the upper leg are definitely the largest: they are five times larger than the forces applied on the lower parts. Nevertheless, both distributed forces are of significant magnitude:

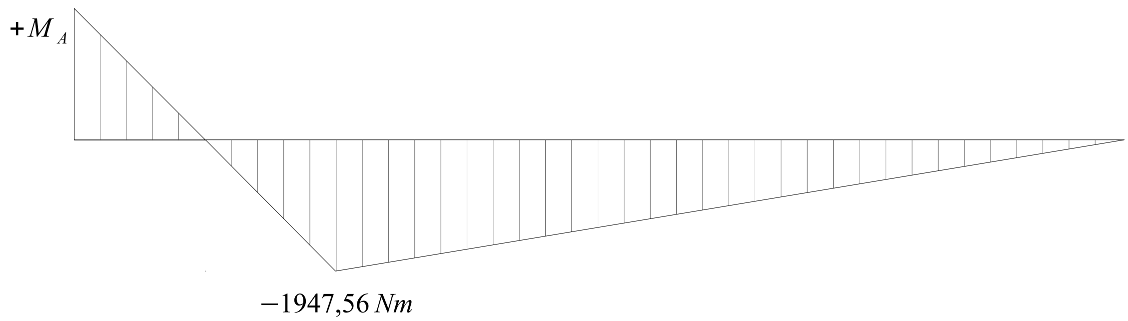

Figure 11: Bending moment diagram of the original system.

55,2 kN and 10,6 kN. As they are exerted in the transverse direction on the femur, it is very likely they will cause a fracture of the bone. All long bones have anisotropic structure characteristics. This causes them to be weaker in the transverse direction than in the longitudinal direction, which makes a fracture in our scenario very likely.

3.3 Calculating stresses in the femur

In this section, we calculate the stresses σ in the femur, assuming it has not yet broken. For this, we need the applied force F and the contact area A. We already know F, so this section will focus on estimating A. To do so, we make use of Hertz theory. We model both the femur and the bull bar as cylinders, and assume they are in direct contact. This theory requires three main assumptions to hold true:

1. both materials must have a similar Young's modulus
2. both surfaces must deform
3. the contact area must be relatively small compared to the two bodies in contact

The first assumption will hold true in our case because the Young's modulus of the femur bone is 20 GPa and of the aluminium bull bar is 69 GPa, which is only a factor 3.5 difference. For the second assumption we can expect that the deformation will mostly affect the femur rather than the bull bar. The third assumption is likely to be met as well.

Our goal is to calculate the area A of the circular contact area. This requires a contact radius a:

$$a = \sqrt{Rd} \tag{32}$$

We estimate the radius of the femur and bull bar as respectively $R_{bone} = 12.15\text{mm}$ and $R_{bar} = 30.00\text{mm}$. The diameters D_{bone} and D_{bar} are obviously double those values. Combining these two radii with the formule below yields R:

$$\frac{1}{R} = \frac{1}{R_{bone}} + \frac{1}{R_{bar}} = 115.31/\text{m} \Leftrightarrow R = 0.0087\text{m} \tag{33}$$

We can also calculate d – the total elastic compression at the contact surface, measured along the line of the applied force F – using a formule proposed by [PT69]. Note that we do not explicitely take into account that the femur is actually composed of both cortical (shaft) and trabecular (canal) bone. However, the Young's modulus used is that of the femur in general. As a consequence, we use the entire diameter of the femur.

$$d = \frac{(3\pi)^{\frac{2}{3}}}{2} \cdot F^{\frac{2}{3}} \cdot (v_{bone} + v_{bar})^{\frac{2}{3}} \cdot \left(\frac{1}{D_{bone}}\right)^{\frac{1}{3}} \tag{34}$$

Herein, v_{bone} and v_{bar} are given by:

$$v_{bone} = \frac{1-\nu_{femur}^2}{\pi E_{femur}} = \frac{1-0,37^2}{\pi \cdot 20 \text{ GPa}} = 1,3737 \cdot 10^{-11} \frac{1}{\text{Pa}} \tag{35}$$

$$v_{bar} = \frac{1-\nu_{Al}^2}{\pi E_{Al}} = \frac{1-0.32^2}{\pi \cdot 69 \text{ GPa}} = 4.1408 \cdot 10^{-12} \frac{1}{\text{Pa}} \tag{36}$$

Plug in all known variables, and we get this result:

$$d = \frac{(3\pi)^{\frac{2}{3}}}{2} \cdot (65824 \text{ N})^{\frac{2}{3}} \cdot (1,3737 \cdot 10^{-11} \frac{1}{\text{Pa}} + 4,1408 \cdot 10^{-12} \frac{1}{\text{Pa}})^{\frac{2}{3}} \cdot \left(\frac{1}{0,0243 \text{ m}}\right)^{\frac{1}{3}} = 0,0008585 \text{ m} \tag{37}$$

At last, we can find A and thus σ:

$$A = \pi a^2 = \pi R d = 0,000023465 \text{ m}^2 \tag{38}$$

$$\sigma = \frac{F}{A} = \frac{65824 \text{ N}}{0,000023465 \text{ m}^2} = 28052 \cdot 10^5 \text{ Pa} \approx 2,8 \text{ GPa} \tag{39}$$

3.4 Will the femur break?

According to our calculations above, the transverse stresses at the contact area are approximately 2,8 GPa. We compare this with the values in Figure 12. As 2,8 GPa is much higher than 133 MPa, the maximum compression stresses are exceeded, and thus the femur will fail. Note that we made no distinction between cortical and trabecular bone. Because the table only contains data for the relatively stronger cortical bone, it is safe to assume that it will fracture regardless.

As soon as the femur fractures, it can no longer absorb the stresses. Instead, they are redirected to the soft tissue. However, the damage calculations in this case are out of the scope of this report.

4 Conclusions

In this report we presented a simplified biomechanical analysis of the situation in which a pedestrian collides with a SUV that has a bull bar. More specifically, we researched whether the femur shaft will break under the assumptions made in section 2, the most important of which is the speed of the SUV (10 m/s). We conclude that under these conditions, the femur shaft will most definitely break.

TABLE 3.2

Average Anisotropic and Asymmetric Ultimate Stress Properties of Human Femoral Cortical Bone[1]

Longitudinal (MPa)	*Tension*	133
	Compression	193
Transverse (MPa)	*Tension*	51
	Compression	133
Shear (MPa)		68

[1]Note that these properties are referred to the principal material coordinate system.
Source: Reilly and Burstein (1975) *J Biomechanics* 8:393–405.

Figure 12: Properties of the femur. [BDK06]

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