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Matriks
~ Invers ~

3. Dengan Transformasi Elementer

   Dengan mencari matriks bujur sangkar P dan Q sedemikian sehingga PAQ = I_n

   P didapat dengan sederetan transformasi elementer baris, Q didapat dengan sederetan transformasi elementer kolom terhadap I_n

   P : Tulis I dimuka A, jadikan nol elemen-elemen di bawah diagonal utama dengan transformasi elementer baris

   Q : Tulis I dibawah matriks segi atas tersebut. Jadikan nol elemen-elemen diatas diagonal utama dengan transformasi kolom elementer

   $PAQ = I$

   $P^{-1}PAQQ^{-1} = P^{-1} I Q^{-1}$

   $A = P^{-1}Q^{-1} = (QP)^{-1}$

   $A^{-1} = QP$

Contoh :

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 4 & 6 \\ 2 & 5 & 7 \end{pmatrix}$$

Tentukan A^{-1}

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 & 5 & 7 \end{array}\right) \begin{array}{c} H_{21(-1)} \\ \sim \\ H_{31(-2)} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -2 & 0 & 1 & 0 & -1 & 3 \end{array}\right) \begin{array}{c} H_{32(1)} \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -3 & 1 & 1 & 0 & 0 & 7 \end{array}\right) \begin{array}{c} H_{3(1/7)} \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -3/7 & 1/7 & 1/7 & 0 & 0 & 1 \end{array}\right)$$

P

$$\left(\begin{array}{ccc} 1 & 3 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array}\right) \xrightarrow[K31(-2)]{K21(-3)} \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{array}\right) \xrightarrow{K32(-4)} \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc} 1 & -3 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc} 1 & -3 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{array}\right)$$

Q

$$A^{-1} = QP = \left(\begin{array}{ccc} 1 & -3 & 10 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{array}\right) \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3/7 & 1/7 & 1/7 \end{array}\right) = \left(\begin{array}{ccc} -2/7 & -11/7 & 10/7 \\ 5/7 & 3/7 & -4/7 \\ -3/7 & 1/7 & 1/7 \end{array}\right)$$

4. Metode Gauss-Jordan

Steps:

1. Set up the given matrix with the identity matrix as the form of "augmented matrix"
2. Transforming the left matrix into the identical matrix follow the rules of row operations

Contoh :

$$A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 4 & 6 \\ 2 & 5 & 7 \end{pmatrix}$$

Tentukan A^{-1}

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ 0 & 1 & 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 & 5 & 7 \end{array}\right) \begin{array}{c} H_{21(-1)} \\ \sim \\ H_{31(-2)} \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -2 & 0 & 1 & 0 & -1 & 3 \end{array}\right) \begin{array}{c} H_{32(1)} \\ \sim \end{array}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -3 & 1 & 1 & 0 & 0 & 7 \end{array}\right) \begin{array}{c} H_{3(1/7)} \\ \sim \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -3/7 & 1/7 & 1/7 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 2 \\ -1 & 1 & 0 & 0 & 1 & 4 \\ -\frac{3}{7} & \frac{1}{7} & \frac{1}{7} & 0 & 0 & 1 \end{array}\right) \begin{array}{l} H_{23}(-4) \\ \sim \\ H_{13}(-2) \end{array} \left(\begin{array}{ccc|ccc} \frac{13}{7} & -\frac{2}{7} & -\frac{3}{7} & 1 & 3 & 0 \\ \frac{5}{7} & \frac{3}{7} & -\frac{4}{7} & 0 & 1 & 0 \\ -\frac{3}{7} & \frac{1}{7} & \frac{1}{7} & 0 & 0 & 1 \end{array}\right) \begin{array}{l} H_{12}(-3) \\ \sim \end{array}$$

$$-1 + \frac{12}{7} = -\frac{7}{7} + \frac{12}{7} = \frac{5}{7} \qquad 1 + \frac{6}{7} = \frac{13}{7}$$

$$1 - \frac{4}{7} = \frac{7}{7} - \frac{4}{7} = \frac{3}{7} \qquad 0 - \frac{2}{7} = -\frac{2}{7}$$

$$0 - \frac{4}{7} = -\frac{4}{7} \qquad 0 - \frac{2}{7} = -\frac{2}{7}$$

Hasil Invers

$$\left(\begin{array}{ccc|ccc} -\frac{2}{7} & -\frac{11}{7} & \frac{10}{7} & 1 & 0 & 0 \\ \frac{5}{7} & \frac{3}{7} & -\frac{4}{7} & 0 & 1 & 0 \\ -\frac{3}{7} & \frac{1}{7} & \frac{1}{7} & 0 & 0 & 1 \end{array}\right)$$

$$\frac{13}{7} - \frac{15}{7} = \frac{-2}{7}$$

$$-\frac{2}{7} + \frac{9}{7} = -\frac{11}{7}$$

$$-\frac{2}{7} + \frac{12}{7} = \frac{10}{7}$$