① $f(x) = x^2$, interval $[-\pi, \pi]$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} x^2 \, dx$$

$$= \frac{1}{2\pi} \left[\frac{1}{3} x^3 \right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi} \cdot \frac{2\pi^3}{3}$$

$$= \frac{1}{3} \pi^2 = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos nx \, dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{n} \sin nx + \frac{2x}{n^2} \cos nx - \frac{2x}{n^3} \sin nx \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left(\frac{2\pi}{n^2} \cos n\pi + \frac{2\pi}{n^2} \cos n\pi \right)$$

$$= \frac{4}{n^2} \cos n\pi \quad \begin{cases} \text{Genap} : \frac{4}{n^2} \\ \text{Ganjil} : -\frac{4}{n^2} \end{cases}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin nx \, dx$$

$$= \frac{1}{\pi} \left[-\frac{x^2}{n} \cos nx + \frac{2x}{n^2} \sin nx + \frac{2}{n^3} \cos nx \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left(-\frac{\pi^2}{n} \cos n\pi + \frac{2}{n^3} \cos n\pi - \frac{\pi^2}{n} \cos nx - \frac{2}{n^3} \cos n\pi \right)$$

$$= \frac{1}{\pi} \left(-\frac{2\pi^2}{n} \cos n\pi \right)$$

$$= -\frac{2\pi}{n} \cos n\pi \quad \begin{cases} \text{Genap} : -\frac{2\pi}{n} \\ \text{Ganjil} : \frac{2\pi}{n} \end{cases}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos nx + \sum_{n=1}^{\infty} (-1)^n \left(-\frac{2\pi}{n} \right) \sin nx$$

② a.) $\int_0^\infty x^2 e^{-2x^2}\,dx$

※ Misal $2x^2 = u$

$4x = \frac{du}{dx}$

$x\,dx = \frac{1}{4}\cdot du$

$= \int_0^\infty \frac{u^{\frac{1}{2}}}{\sqrt{2}} \frac{1}{4} e^{-u}\,du$

$= \frac{1}{4\sqrt{2}} \cdot \int_0^\infty u^{\frac{1}{2}} e^{-u}\,du$

$= \frac{1}{4\sqrt{2}}\, \Gamma\left(\frac{3}{2}\right)$

$= \frac{1}{4\sqrt{2}}\ \frac{1}{2}\cdot \Gamma\left(\frac{1}{2}\right)$

$= \frac{1}{8\sqrt{2}} \cdot \sqrt{\pi} = \frac{\sqrt{\pi}}{8\sqrt{2}}$ //

b.) $\Gamma\left(-\frac{7}{2}\right) = \frac{\Gamma\left(-\frac{5}{2}\right)}{-\frac{7}{2}} = \frac{\Gamma\left(-\frac{3}{2}\right)}{-\frac{7}{2}\cdot -\frac{5}{2}} = \frac{\Gamma\left(-\frac{1}{2}\right)}{-\frac{7}{2}\cdot -\frac{5}{2}\cdot -\frac{3}{2}}$

$= \frac{\Gamma\left(\frac{1}{2}\right)}{-\frac{7}{2}\cdot -\frac{5}{2}\cdot -\frac{3}{2}\cdot -\frac{1}{2}} = \frac{8\sqrt{\pi}}{105}$ //

③ $(2x+3y)\,dx + (x-y)\,dy = 0$

$\frac{dy}{dx} = \frac{2x+3y}{y-x}$

※ misal $y = vx$

$\frac{dy}{dx} = v + \frac{x\,dv}{dx}$

$v + \frac{x\,dv}{dx} = \frac{2x+3vx}{vx-x}$

$v + \frac{x\,dv}{dx} = \frac{\cancel{x}(2+3v)}{\cancel{x}(v-1)}$

$\frac{x\,dv}{dx} = \frac{2+3v}{v-1} - \frac{v(v-1)}{v-1}$

$\frac{x\,dv}{dx} = \frac{2+4v-v^2}{v-1}$

$\int \frac{v-1}{2+4v-v^2}\,dv = \int \frac{1}{x}\,dx$