Pembahasan Soal UAS Matematika I
2017 / 2018

1) a.) Gradien garis terhadap $y = x^2 - 2x$ di titik (2,0)

$y = x^2 - 2x$

$y' = 2x - 2$

$y(2) = 2.2 - 2$

$= 4 - 2$

$= 2$ //

b) Sector batteri bertambang sehingga beratnya setelah t jam adalah $\frac{1}{2}t^2 + 1$ gram. Berapa laju pertambangannya pada saat $t = 2$ jam?

$y = \frac{1}{2}t^2 + 1$

$y' = t$

$y(2) = 2$

//

2.) a) $y^3 + 7y - x^3 = 0$

$\frac{dy}{dx} = -3x^2$ $\qquad$ $\frac{d^2y}{dx^2} = -6x$

b.) $x^3y^4 - 1 = 0$

$\frac{dy}{dx} = 3x^2y^4$ $\qquad$ $\frac{d^2y}{dx^2} = 6xy^4$

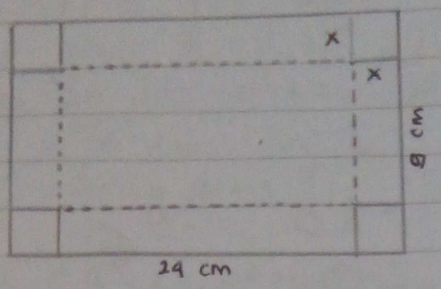
c.) $y = \sqrt{\sin(xy^2)}$

$y = (\sin(xy^2))^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(\sin(xy^2))^{-1/2} \cdot \cos(xy^2) \cdot y^2 = \frac{y^2}{2}(\sin(xy^2))^{-1/2} \cdot \cos(xy^2)$

$\frac{dy^2}{d^2x} =$

3) Sebuah kotak persegi panjang dibuat dari selembar kertas dengan memotongnya sisi-sisinya sepanjang x cm dan melipatnya. Tentukan x supaya volumenya maksimal.



Volume : Luas alas x tinggi
: $(24-2x)(9-2x) \times x$
: $(216-66x+4x^2)x$
$y = 4x^3-66x^2+216x$

Supaya max $= 0 \quad y' = 0$

$12x^2-132x+216 = 0$
$x^2-11x+18 = 0$
$(x-2)(x-9) = 0$
$x = 2 \vee x = 9$

$\rightarrow x = 2$
$y = 4(2)^3-66(2)^2+216(2)$
$= 200$

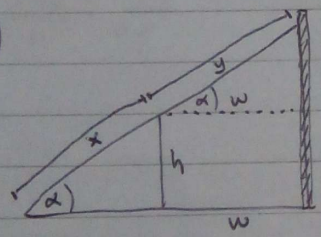
$\rightarrow x = 9$
$y = 4(9)^3-66(9)^2+216(9)$
$= -486$

Jadi nilai x supaya volumenya max $= 0$ $x = 2$

5.) a) $\int \left(\frac{4}{x^5}-\frac{3}{x^9}\right) dx = \int (4x^{-5}-3x^{-4})\,dx$
$= -\frac{4}{4}x^{-4}+\frac{3}{3}x^{-3}+C = -x^{-4}+x^{-3}+C$

b) $\int |x|\,dx \rightarrow |x| \begin{cases} -x, & x<0 \\ x, & x>0 \end{cases}$
$= \int x\,dx - \int x\,dx$
$= \frac{1}{2}x^2 - \frac{1}{2}x^2 + C$

4.)



Panjang tangga terpendek : ?

misal panjang tangga $= L$
$L = x+y$
$L(\alpha) = \frac{h}{\sin\alpha} + \frac{w}{\cos\alpha}$

$L(\alpha) = h(\sin\alpha)^{-1} + w(\cos\alpha)^{-1}$
$L'(\alpha) = -h(\sin\alpha)^{-2}\cdot\cos\theta - w((\cos\alpha)^{-2}(-\sin\alpha)$
$0 = -h\cos\alpha(\sin\alpha)^{-2} + w\sin\alpha(\cos\alpha)^{-2}$
$h\cdot\cos\alpha(\sin\alpha)^{-2} = w\sin\alpha(\cos\alpha)^{-2}$
$\frac{h\cdot\cos\alpha}{\sin^2\alpha} = \frac{w\sin\alpha}{\cos^2\alpha}$
$\frac{h}{w} = \frac{\sin^3\alpha}{\cos^3\alpha}$