Pembahasan UTS Matematika I 2023

1. a. $\lim_{x \to 1^-} \frac{x^2+1}{x-1}$

$\lim_{x \to 1^-} x^2+1 = 2 > 0$

$g(x) = x-1$ akan menuju 0 dari arah bawah karena x mendekati 1 dari kiri berarti x lebih kecil dari 1, akibatnya $x-1$ akan bernilai negatif sehingga

$$\lim_{x \to 1^-} \frac{x^2+1}{x-1} = -\infty$$

b. $\lim_{x \to 1^-} \frac{x^2+1}{x^2-1}$

$\lim_{x \to 1^-} \frac{x^2+1}{(x-1)(x+1)}$

$\lim_{x \to 1^-} \frac{2}{(x-1)2} = \frac{1}{x-1}$

saat $x \to 1^-$, $x-1 \to 0^-$ sehingga

$$\lim_{x \to 1^-} \frac{x^2+1}{x^2-1} = -\infty$$

c. $\lim_{x \to \pi^+} \frac{x}{\sin x}$

$\lim_{x \to \pi^+} x = \pi > 0$

Jika x menuju π dari arah kanan maka nilai $\sin x$ menuju 0 dari arah bawah (arah nilai $\sin x$ negatif)

Sehingga

$$\lim_{x \to \pi^+} \frac{x}{\sin x} = -\infty$$

2. a. $f(x) = \dfrac{x^2-4}{x-2}$

$f(x)$ tidak kontinu di $x=2$

karena fungsi tidak terdefinisi di $x=2$

$\dfrac{2^2-4}{2-2} = \dfrac{0}{0}$

b. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ 3, & x = 2 \end{cases}$

$f(2) = 3$

$\lim\limits_{x \to 2} \dfrac{x^2-4}{x-2} = \lim\limits_{x \to 2} \dfrac{(x-2)(x+2)}{(x-2)} = \lim\limits_{x \to 2} x+2 = 4$

$\lim\limits_{x \to 2} f(x) \neq f(2)$

limit tidak sama dengan nilai fungsi

maka $f(x)$ tidak kontinu di $x=2$

c. $f(x) = \begin{cases} x+1, & x < 2 \\ x^2-1, & x \geq 2 \end{cases}$

$f(2) = 2^2 - 1 = 3$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2} x+1 = 3$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2} x^2-1 = 3$

$\Big\} \lim\limits_{x \to 2} f(x) = 3$

$\lim\limits_{x \to 2} f(x) = f(2)$

memenuhi semua syarat, $f(x)$ kontinu di $x=2$

3. $f(x) = \begin{cases} x^2 + b, & x < 1 \\ ax, & x \geq 1 \end{cases}$

- f kontinu di $x = 1$ jika f kontinu kiri dan kontinu kanan di $x = 1$

$$f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$$

$$a = \lim_{x \to 1} x^2 + b = \lim_{x \to 1} ax \leftrightarrow a = 1 + b = a \leftrightarrow b = a - 1$$

$$f_-(1) = \lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x^2 + b - a}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^2 + (a-1) - a}{x - 1}$$

$$= \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x-1)(x+1)}{x - 1}$$

$$= \lim_{x \to 1} x + 1 = 2$$

$$f_+(1) = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{ax - a}{x - 1}$$

$$= a \lim_{x \to 1} \frac{x - 1}{x - 1} = a$$

$f_-(1) = f_+(1) \rightarrow a = 2$

$b = a - 1 = 2 - 1 = 1$

Maka diperoleh : $a = 2$ dan $b = 1$

4. Tentukan asimtot

a. $f(x) = \frac{x^2+2x}{x^2-1}$

- asimtot tegak $x = 1$

$\lim_{x \to 1^-} \frac{x^2+2x}{x^2-1} = -\infty$ $\lim_{x \to 1^+} \frac{x^2+2x}{x^2-1} = \infty$

- asimtot datar

$\lim_{x \to \infty} \frac{x^2+2x}{x^2-1} = \lim_{x \to \infty} \frac{1+\frac{2}{x}}{1-\frac{1}{x^2}} = \frac{1+0}{1-0} = 1$

asimtot datar adalah $y = 1$

- asimtot miring

tidak ada

b. $f(x) = \frac{x^2+2x}{x^2+1}$

- asimtot tegak

penyebut sama dengan nol

$x^2 + 1 = 0 \rightarrow x^2 = -1$ (tidak ada solusi)

asimtot tegak tidak ada

- asimtot datar

$\lim_{x \to \infty} \frac{x^2+2x}{x^2+1} = \lim_{x \to \infty} \frac{1+\frac{2}{x}}{1-\frac{1}{x^2}} = \frac{1+0}{1+0} = 1$

asimtot datar adalah $y = 1$

- asimtot miring

tidak ada.

5. a. $f(x) = \dfrac{x^2-3x+1}{x-3}$

· Selang kemonotonan

$$f'(x) = \frac{(2x-3)(x-3) - (x^2-3x+1)(1)}{(x-3)^2}$$

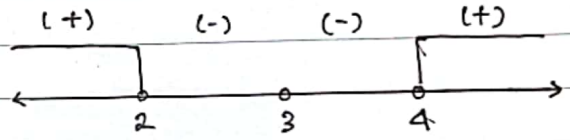
$$= \frac{2x^2-9x+9 - x^2+3x-1}{x^2-6x+9}$$

$$= \frac{x^2-6x+8}{x^2-6x+9}$$

$$= \frac{(x-4)(x-2)}{(x-3)^2}$$

- monoton naik jika $f'(x) > 0$

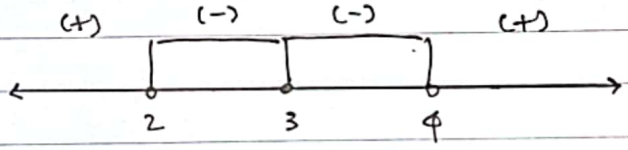
$$\frac{(x-4)(x-2)}{(x-3)^2} > 0$$



$f(x)$ monoton naik pada selang $(-\infty, 2)$ dan $(4, \infty)$

- monoton turun jika $f'(x) < 0$

$$\frac{(x-4)(x-2)}{(x-3)^2} < 0$$



$f(x)$ monoton turun pada selang $(2,3)$ dan $(3,4)$

· ekstrim fungsi

$f'(x) = 0$

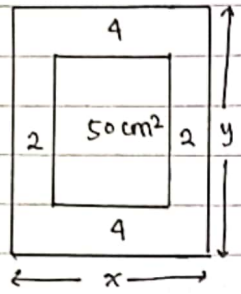
$$\frac{(x-4)(x-2)}{(x-3)^2} = 0$$

$x_1 = 4$ dan $x_2 = 2$

$f(4) = \dfrac{4^2-3(4)+1}{4-3} = 5 \rightarrow$ minimum lokal

$f(2) = \dfrac{2^2-3(2)+1}{2-3} = 1 \rightarrow$ maksimum lokal

6. berapa ukuran x dan y supaya luas kertas seminimum mungkin



luas tulisan

$(x-4)(y-8) = 50\ cm^2$

$xy - 8x - 4y + 32 = 50$

$xy - 8x - 4y - 18 = 0$

$x(y-8) - 4y - 18 = 0$

$$x = \frac{4y+18}{y-8}$$

luas kertas

$$f(y) = x.y = \frac{4y+18}{y-8} \cdot y$$

$$= \frac{4y^2+18y}{y-8} \qquad u' = 8y+18 \qquad v' = 1$$

$$f'(y) = \frac{u'v - uv'}{v^2}$$

$$\Rightarrow \frac{(8y+18)(y-8) - (4y+18y)(1)}{(y-8)^2} = 0$$

$$\Rightarrow \frac{8y^2 - 46y - 144 - 4y^2 - 18y}{(y-8)^2} = 0$$

$$\Rightarrow \frac{4y^2 - 64y - 144}{(y-8)^2} = 0$$

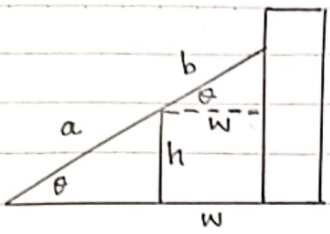
$$\Rightarrow \frac{4(y+2)(y-18)}{(y-8)^2} = 0$$

$\boxed{y = 18}$ V $y = -2$ (TM)

$$\Rightarrow x = \frac{4(18)+18}{18-8} = \frac{90}{10} = 9$$

Jadi $x = 9$ dan $y = 18$

7.



$\sin\theta = \frac{h}{a}$ $\quad \cos\theta = \frac{w}{b}$ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$a = \frac{h}{\sin\theta}$ $\quad b = \frac{w}{\cos\theta}$

$\cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$

$= \frac{1}{\sqrt{1+\left(\sqrt[3]{\frac{h}{w}}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{h}{w}\right)^{2/3}}}$

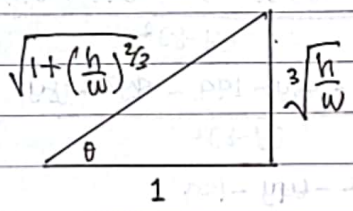
total panjang tangga :

$P(\theta) = \frac{h}{\sin\theta} + \frac{w}{\cos\theta}$

$\sin\theta = \tan\theta \cdot \cos\theta$

$= \sqrt[3]{\frac{h}{w}} \cdot \frac{1}{\sqrt{1+\left(\frac{h}{w}\right)^{2/3}}}$

$= \frac{\sqrt[3]{\frac{h}{w}}}{\sqrt{1+\left(\frac{h}{w}\right)^{2/3}}}$



Panjang minimum $P'(\theta) = 0$

$P'(\theta) = -\frac{h\cos\theta}{\sin^2\theta} + \frac{w\sin\theta}{\cos^2\theta} = 0$

$\frac{w\sin\theta}{\cos^2\theta} = \frac{h\cos\theta}{\sin^2\theta}$

$\frac{h}{w} = \frac{\sin^3\theta}{\cos^3\theta}$

$\frac{h}{w} = \tan^3\theta$

$\tan\theta = \sqrt[3]{\frac{h}{w}}$

$\theta = \tan^{-1}\left(\sqrt[3]{\frac{h}{w}}\right)$

Substitusi ke $P(\theta)$

$P(\theta) = \frac{h}{\sin\theta} + \frac{w}{\cos\theta}$

$P\left(\tan^{-1}\left(\sqrt[3]{\frac{h}{w}}\right)\right) = h \cdot \frac{\sqrt{1+\left(\frac{h}{w}\right)^{2/3}}}{\sqrt[3]{\frac{h}{w}}} + w \cdot \sqrt{1+\left(\frac{h}{w}\right)^{2/3}}$

$= \sqrt{1+\left(\frac{h}{w}\right)^{2/3}}\left(\frac{h}{\sqrt[3]{\frac{h}{w}}} + w\right)$