TRANSFORMASI 2D

Minggu 3

Isi

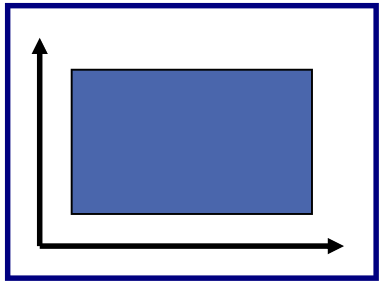
- Definisi & Motivasi
- Transformasi Geometrik 2D
  - Terjemahan
  - Rotasi
  - Scaling
- Representasi Matriks
- Koordinat Homogen
- Komposisi Matriks
- Transformasi Komposit
  - Rotasi Pivot-Point
  - Penskalaan Poin Tetap Umum
  - Refleksi dan Geser
  - Transformasi Antar Sistem Koordinasi

Transformasi Geometris

- **Motivasi** - Mengapa kita membutuhkan transformasi geometris di CG?
  - Sebagai alat bantu melihat
  - Sebagai alat pemodelan
  - Sebagai alat manipulasi gambar

Contoh: Penskalaan 2D

Pemodelan
Koordinat

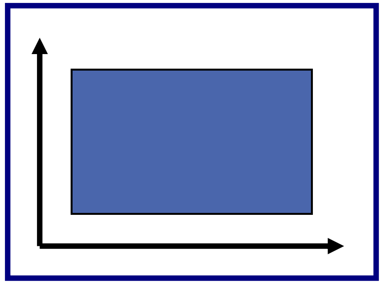


Skala (0,3, 0,3)

Koordinat Dunia

Contoh: Rotasi 2D

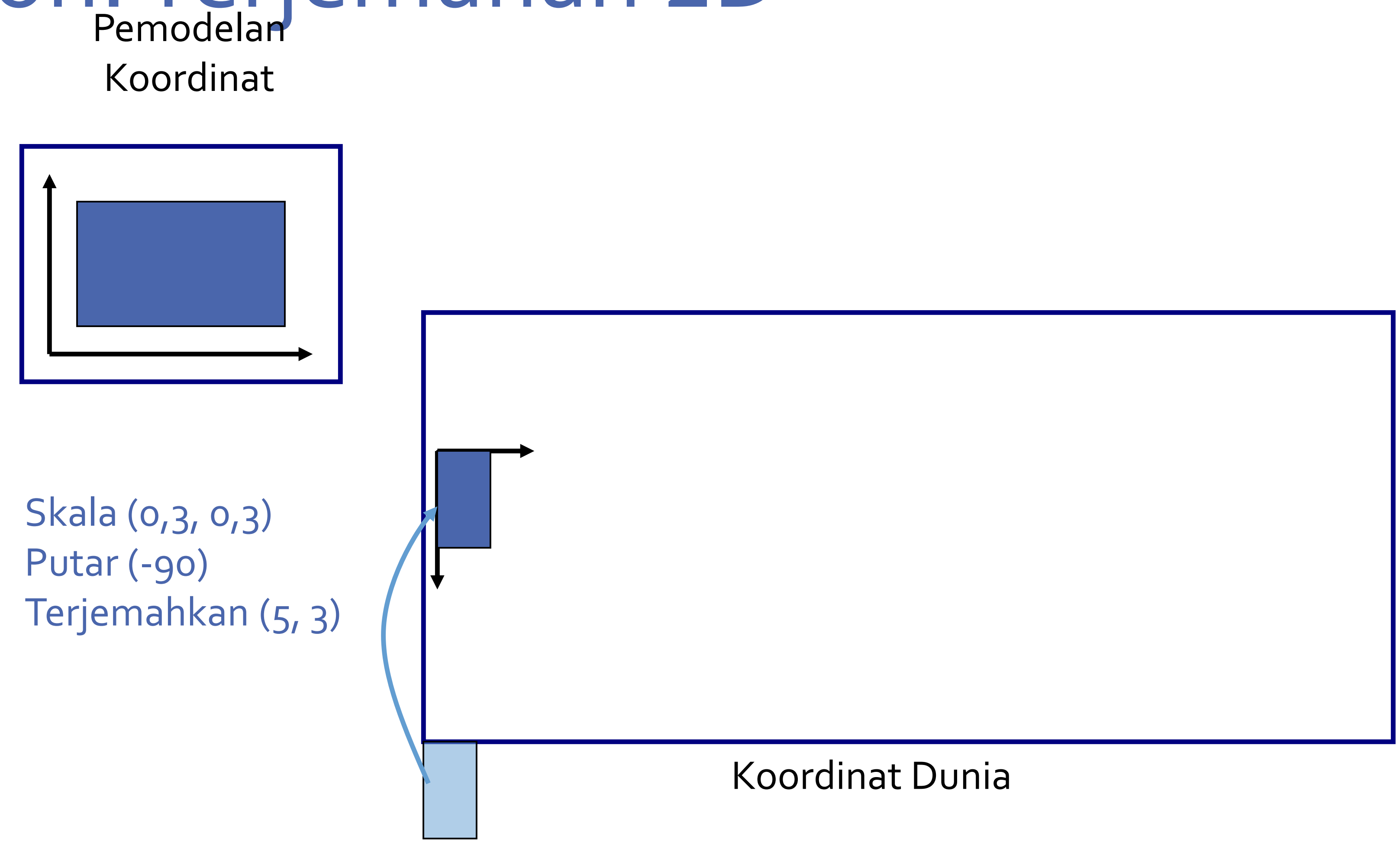
Pemodelan
Koordinat



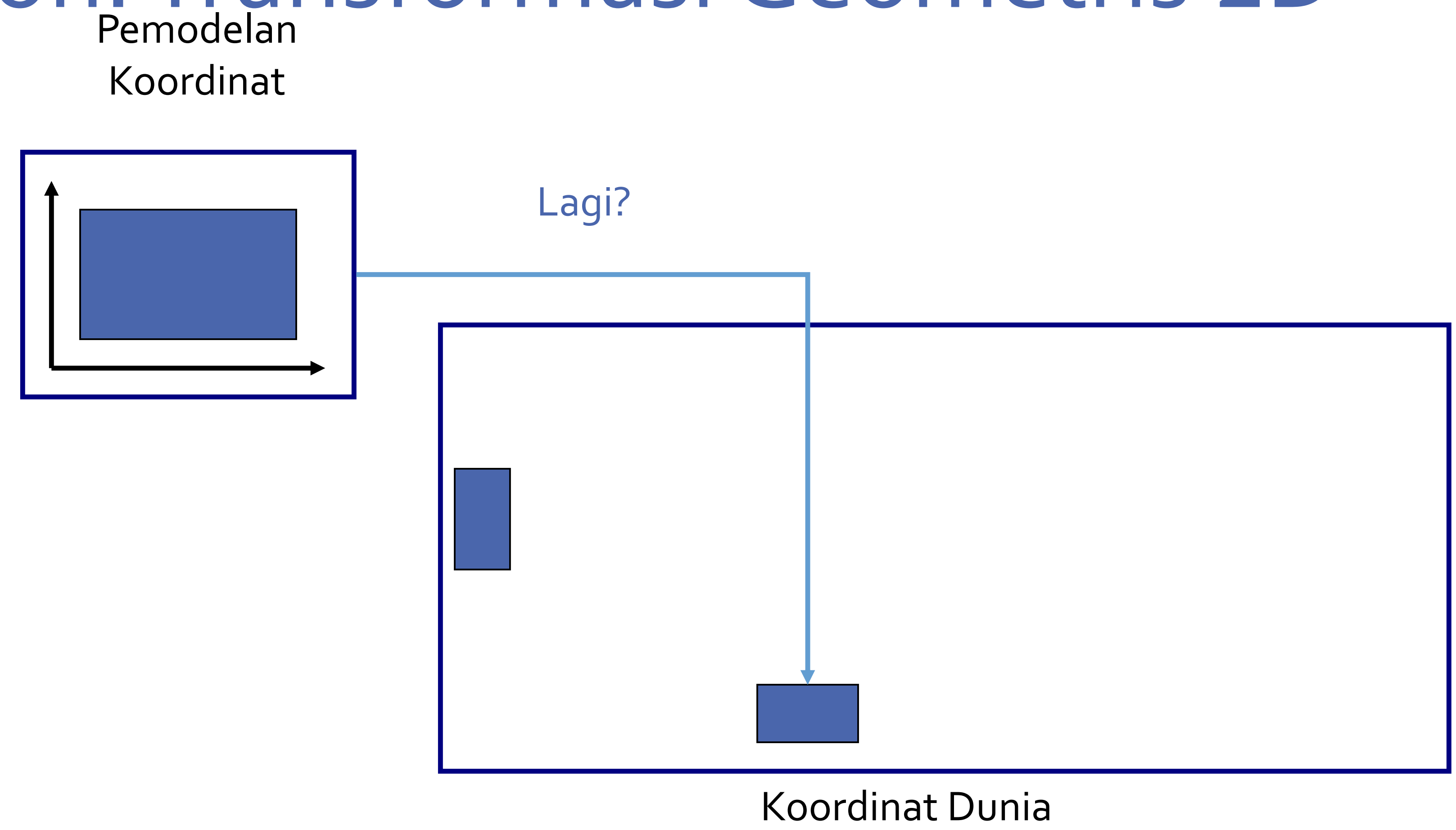
Skala (0,3, 0,3)
Putar (-90)

Koordinat Dunia

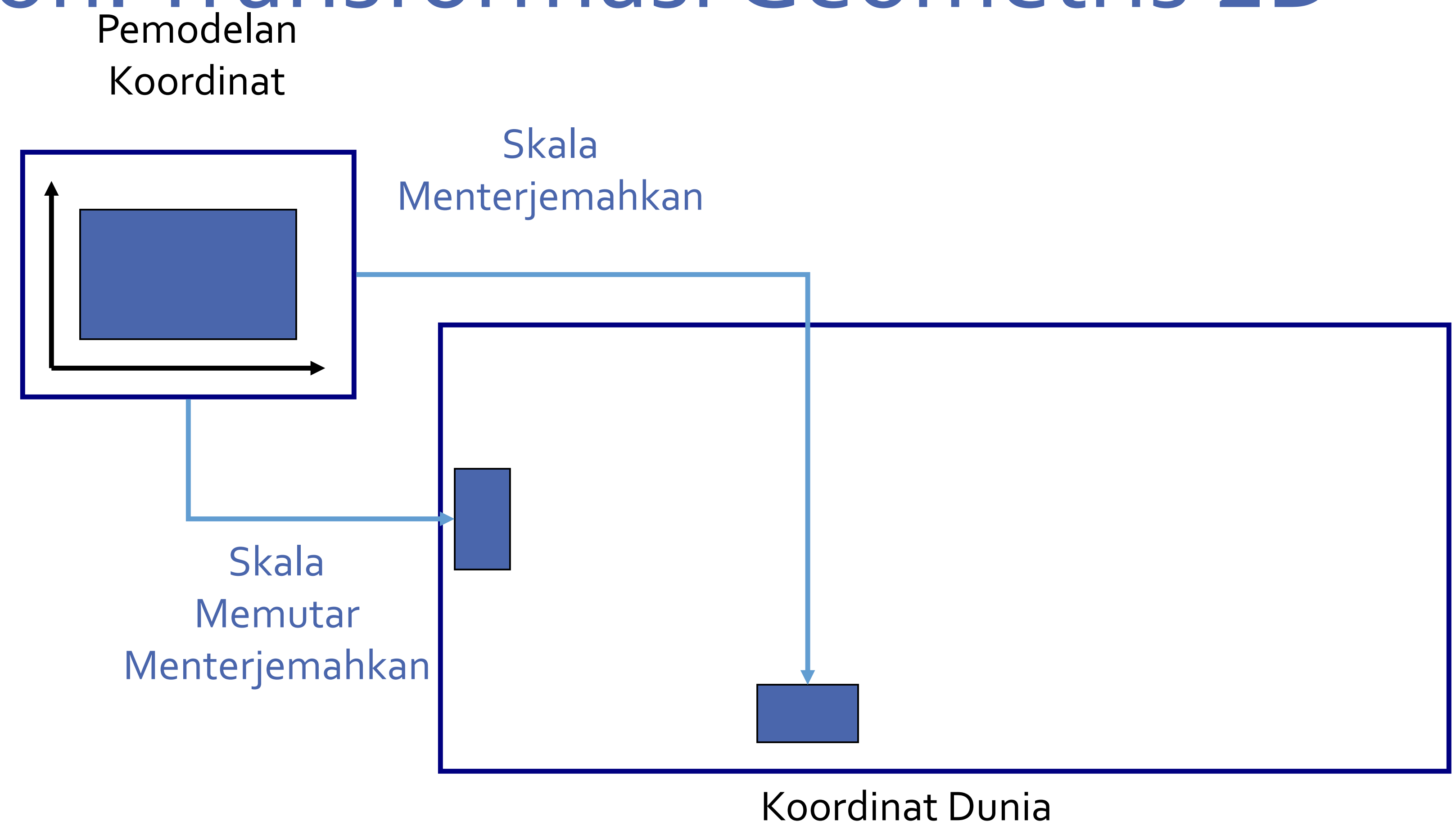
Contoh: Terjemahan 2D



Contoh: Transformasi Geometris 2D



Contoh: Transformasi Geometris 2D



Transformasi 2D Dasar

- Terjemahan
  -
  -

$$x' = x + tx$$
$$y' = y + ty$$

- Skala
  -
  -

$$x' = x \times sx$$
$$y' = y \times sy$$

- Rotasi
  -
  -

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- Mencukur
  -
  -

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$

Transformasi 2D Dasar

- **Terjemahan**

$$x' = x + tx$$
$$y' = y + ty$$

- **Skala**

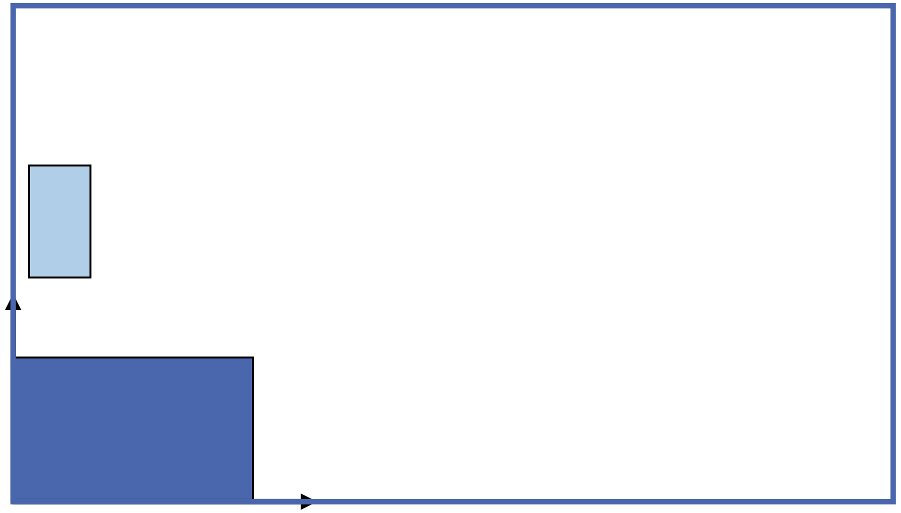
$$x' = x \times sx$$
$$y' = y \times sy$$

- **Rotasi**

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- **Mencukur**

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$



Transformasi
dapat dikombinasikan
(dengan aljabar sederhana)

Transformasi 2D Dasar

- Terjemahan

$$x' = x + tx$$
$$y' = y + ty$$

- Skala

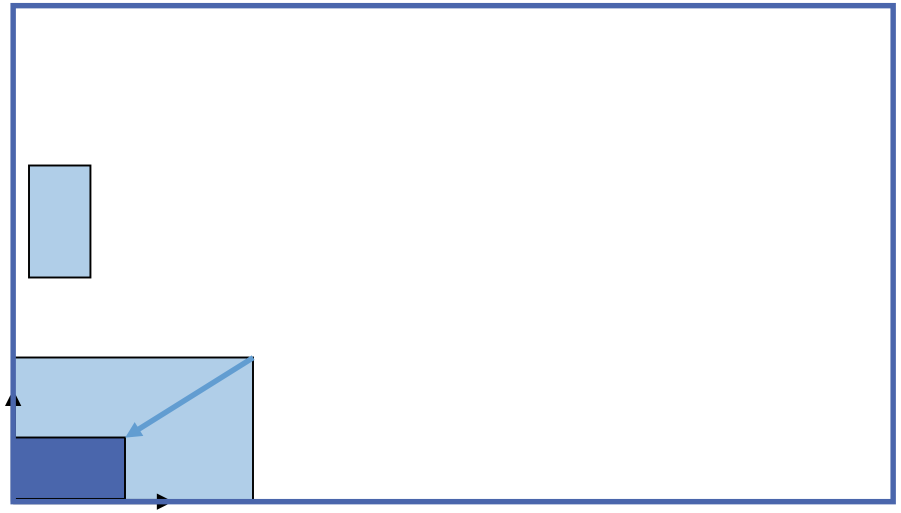
$$x' = x \times sx$$
$$y' = y \times sy$$

- Rotasi

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- Mencukur

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$



$$x' = x \times \underline{sx}$$
$$y' = y \times \underline{sy}$$

Transformasi 2D Dasar

- **Terjemahan**

$$x' = x + tx$$
$$y' = y + ty$$

- **Skala**

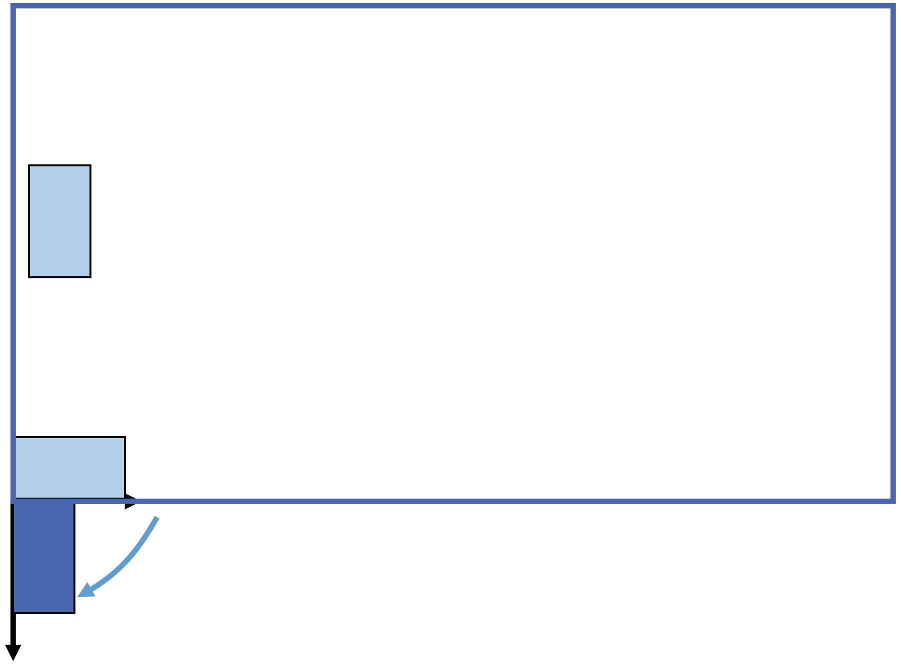
$$x' = x \times sx$$
$$y' = y \times sy$$

- **Rotasi**

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- **Mencukur**

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$



$$\boxed{\begin{aligned} x' &= ((x \times sx) \times \underline{\cos\theta} - (y \times sy) \times \underline{\sin\theta}) \\ y' &= ((x \times sx) \times \underline{\sin\theta} + (y \times sy) \times \underline{\cos\theta}) \end{aligned}}$$

Transformasi 2D Dasar

- Terjemahan

$$x' = x + tx$$
$$y' = y + ty$$

- Skala

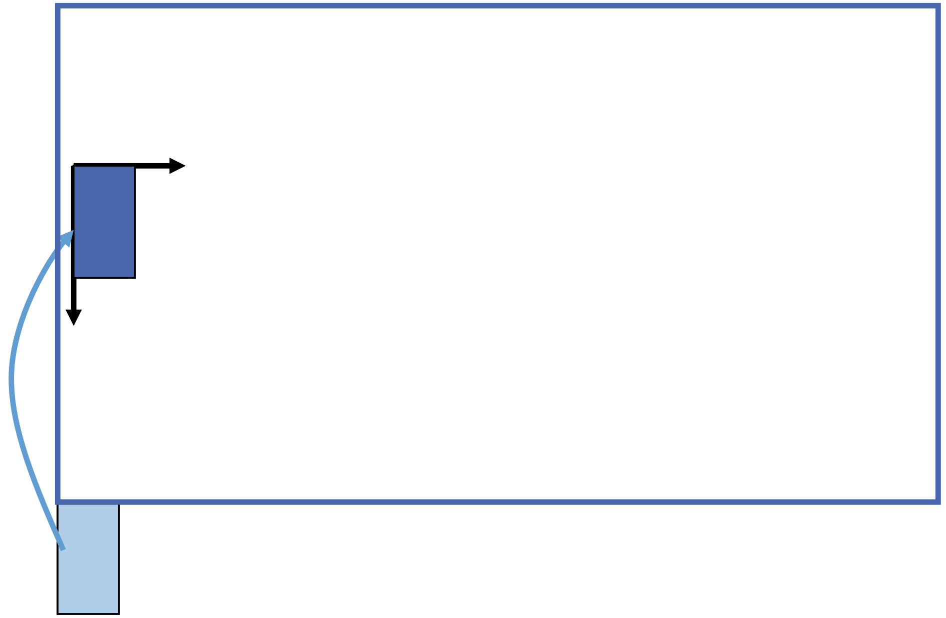
$$x' = x \times sx$$
$$y' = y \times sy$$

- Rotasi

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- Mencukur

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$



$$\boxed{\begin{aligned} x' &= ((x \times sx) \times \cos\theta - (y \times sy) \times \sin\theta) + \underline{tx} \\ y' &= ((x \times sx) \times \sin\theta + (y \times sy) \times \cos\theta) + \underline{ty} \end{aligned}}$$

Transformasi 2D Dasar

- **Terjemahan**

$$x' = x + tx$$
$$y' = y + ty$$

- **Skala**

$$x' = x \times sx$$
$$y' = y \times sy$$

- **Rotasi**

$$x' = x \times \cos\theta - y \times \sin\theta$$
$$y' = y \times \sin\theta + y \times \cos\theta$$

- **Mencukur**

$$x' = x + hx \times y$$
$$y' = y + hy \times x$$

$$\boxed{\begin{aligned} x' &= ((x \times sx) \times \cos\theta - (y \times sy) \times \sin\theta) + tx \\ y' &= ((x \times sx) \times \sin\theta + (y \times sy) \times \cos\theta) + ty \end{aligned}}$$

Representasi Matriks

- **Mewakili Transformasi 2D oleh Matriks**

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- **Terapkan Transformasi ke Titik**

$$x' = ax + by$$
$$y' = cx + dy$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Transformasi Matriks

Titik

Representasi Matriks

- **Transformasi dapat dikombinasikan dengan perkalian matriks**

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Transformasi Matriks

Matriks adalah a mudah dan efisien cara untuk mewakili urutan transformasi

2 × 2 Matriks

- **Apa jenis transformasi yang dapat direpresentasikan dengan matriks 2 × 2?**

Identitas 2D

$$x' = x \qquad y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Penskalaan 2D

$$x' = sx \times x \qquad y' = sy \times y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2 × 2 Matriks

- **Apa jenis transformasi yang dapat direpresentasikan dengan matriks 2 × 2?**

Rotasi 2D

$$x' = \cos\theta \times x - \sin\theta \times y$$
$$y' = \sin\theta \times x + \cos\theta \times y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Geser 2D

$$x' = x + shx \times y$$
$$y' = shy \times x + y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & shx \\ shy & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2 × 2 Matriks

- **Apa jenis transformasi yang dapat direpresentasikan dengan matriks 2 × 2?**

Cermin 2D atas sumbu Y

$$x' = -x$$

$$y' = y$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$x' = -x$$

$$y' = -y$$

2 × 2 Matriks

- **Apa jenis transformasi yang dapat direpresentasikan dengan matriks 2 × 2?**

Terjemahan 2D

$$x' = x + tx$$
$$y' = y + ty$$

TIDAK!!

Hanya transformasi 2D linier
dapat diwakili dengan matriks 2x2

Terjemahan 2D

- **Terjemahan 2D dapat diwakili oleh matriks 3 × 3**
  - Titik diwakili dengan koordinat homogen

$$
\begin{aligned}
x' &= x + tx \\
y' &= y + ty
\end{aligned}
\qquad
\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
$$

Transformasi 2D Dasar

- **Transformasi 2D dasar sebagai Matriks 3x3**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Menterjemahkan

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Skala

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

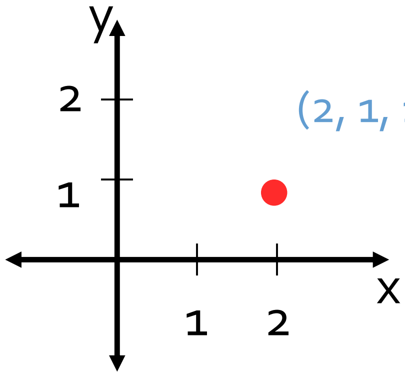
Memutar

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & shx & 0 \\ shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Mencukur

Koordinat Homogen

- **Tambahkan koordinat ke-3 untuk setiap titik 2D**
  - (x, y, w) mewakili titik di lokasi (x / w, y / w)
  - (x, y, 0) mewakili titik tak terhingga
  - (0, 0, 0) Tidak diizinkan



(2, 1, 1) atau (4, 2, 2) atau (6, 3, 3)

Sistem Koordinasi yang Nyaman untuk Mewakili Banyak Transformasi yang Berguna

Komposisi Matriks

- **Transformasi dapat dikombinasikan dengan perkalian matriks**

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{sx} & 0 & 0 \\ 0 & \mathrm{sy} & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{w} \end{bmatrix}$$

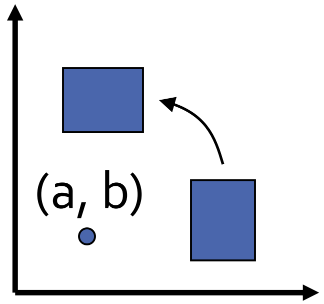
$$p' \quad = \quad \mathrm{T(tx,\ ty)} \qquad \mathrm{R}(\theta) \qquad \mathrm{S(sx,\ sy)} \qquad p$$

- **Efisiensi dengan premultiplikasi**
  - Perkalian matriks adalah asosiatif

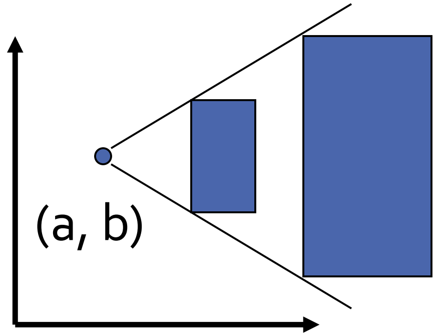
$$\mathrm{p}' = (\mathrm{T} \times (\mathrm{R} \times (\mathrm{S} \times \mathrm{p}))) \quad \Rightarrow \quad \mathrm{p}' = (\mathrm{T} \times \mathrm{R} \times \mathrm{S}) \times \mathrm{p}$$

Komposisi Matriks

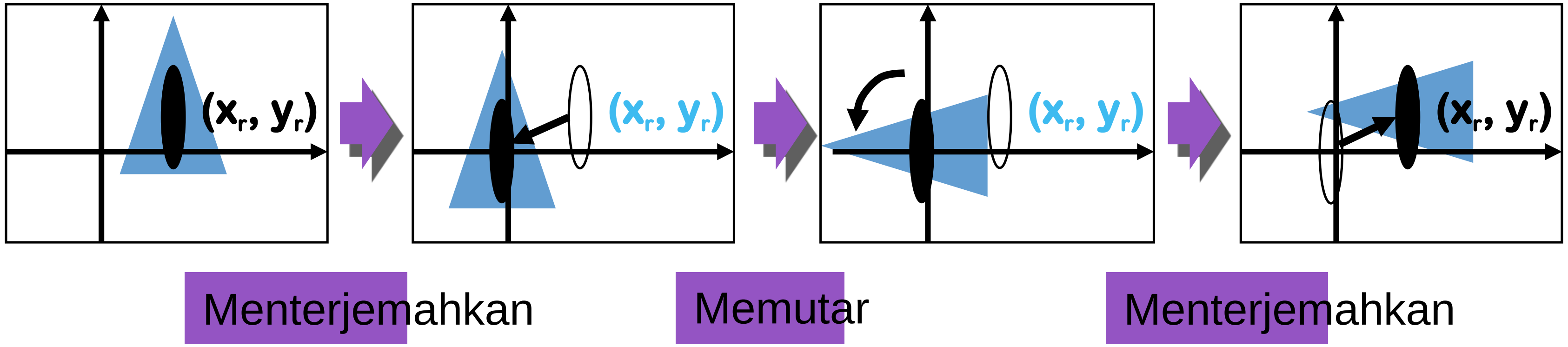
- **Putar oleh θ sekitar titik arbitrer (a, b)**
  - $$M = T(a,b) \times R(\theta) \times T(-a,-b)$$



- **Skala oleh sx, sy di sekitar titik arbitrer (a, b)**
  - $$M = T(a,b) \times S(sx,sy) \times T(-a,-b)$$



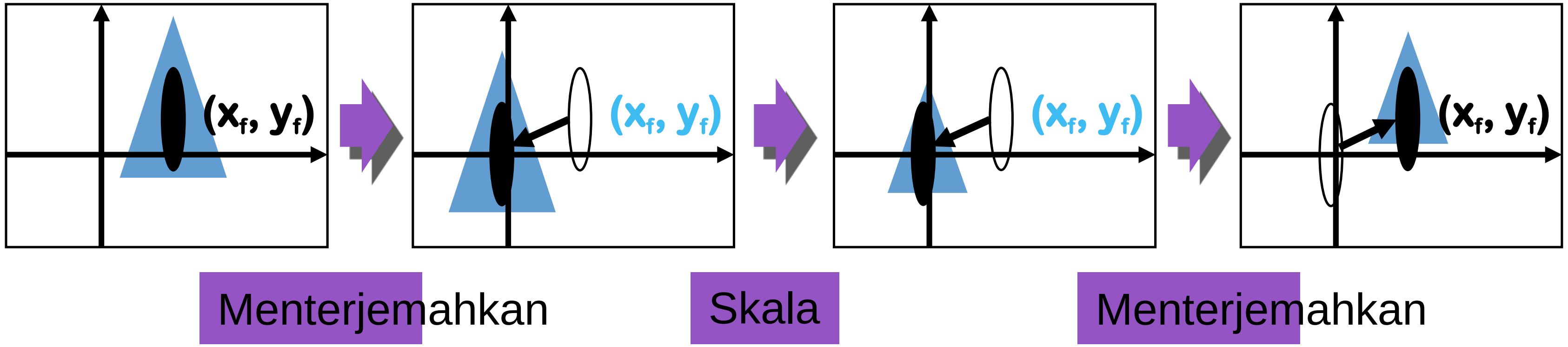
Rotasi Pivot-Point



$$T(x_r, y_r) \cdot R(\theta) \cdot T(-x_r, -y_r) = R(x_r, y_r, \theta)$$

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x_r(1-\cos\theta) + y_r \sin\theta \\ \sin\theta & \cos\theta & y_r(1-\cos\theta) - x_r \sin\theta \\ 0 & 0 & 1 \end{bmatrix}$$

Penskalaan Poin Tetap Umum



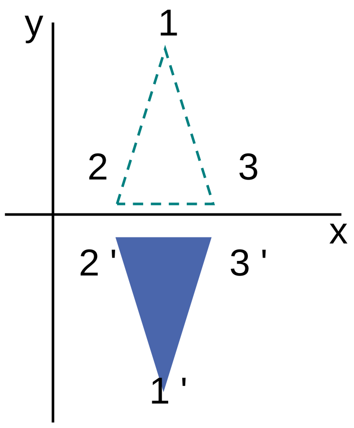
$$T(x_f, y_f) \cdot S(s_x, s_y) \cdot T(-x_f, -y_f) = S(x_f, y_f, s_x, s_y)$$

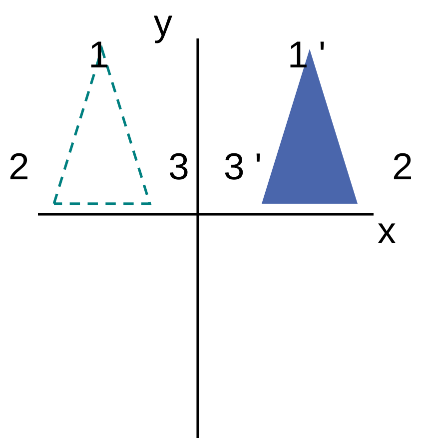
$$\begin{bmatrix} 1 & 0 & x_f \\ 0 & 1 & y_f \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -x_f \\ 0 & 1 & -y_f \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & x_f(1-s_x) \\ 0 & s_y & y_f(1-s_y) \\ 0 & 0 & 1 \end{bmatrix}$$

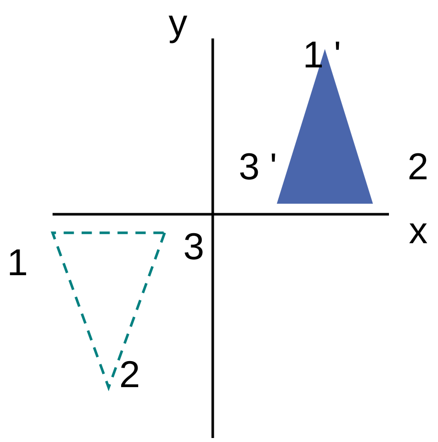
Refleksi

- Refleksi sehubungan dengan sumbu

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





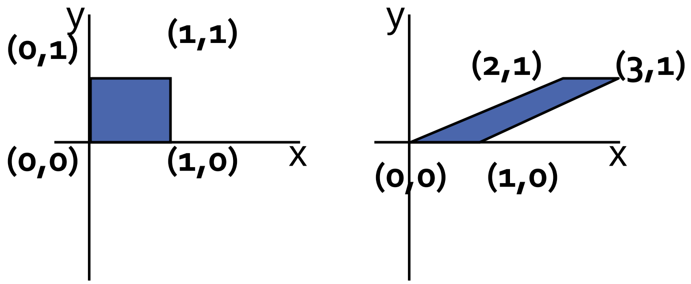


Mencukur

- Dikonversi menjadi jajar genjang

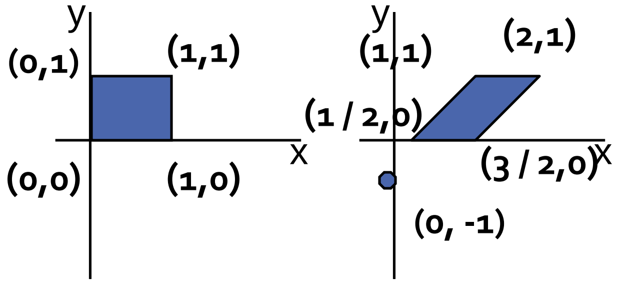
$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$x' = x + sh_x \cdot y, \; y' = y$



- Berubah menjadi jajar genjang
  (Y = Yref)

$$\begin{bmatrix} 1 & sh_x & -sh_x \cdot y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



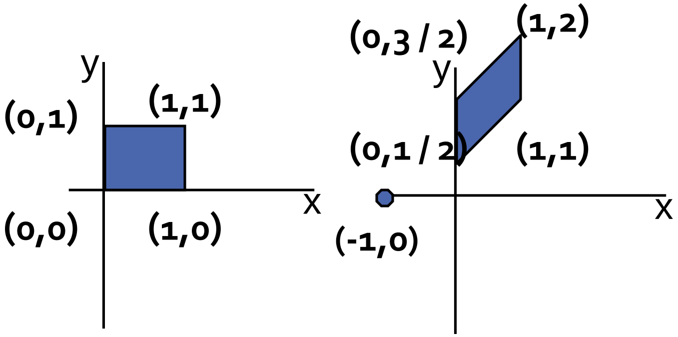
$x' = x + sh_x \cdot (Y\, y_{ref}), \; y' = y$

Mencukur

- **Berubah menjadi jajar genjang**

  (X = Xref)

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & -sh_y \cdot x_{ref} \\ 0 & 0 & 1 \end{bmatrix}$$



$x' = x,\ y' = sh_y \cdot (x x_{ref}) + y$