

$$① T(n) = \begin{cases} a & , n=0 \\ b + nT(n-1), & n > 0 \end{cases}$$

$$\begin{aligned} T(n) &= b + nT(n-1) \\ &= b + n(b + (n-1)T(n-2)) = b + nb + (n^2 - n)(T(n-2)) \\ &= b + nb + (n^2 - n)(b + (n-2)T(n-2)) \\ &= b(1 + n - n + n^2) + n(n-1)(n-2) \cdot T(n-3) \\ &= b(1 + n^{n-1}) + n! \cdot T(0) \\ &= b(1 + n^{n-1}) + a \cdot n! \end{aligned}$$

$$T(n) = O(n!)$$

② a) ~~Algo 1~~

$$T(n) = \begin{cases} 0 & , n=1 \\ 1 & , n=2 \\ n-1 & , n > 2 \end{cases}$$

$$T(n) = O(n)$$

~~Algo 2~~

$$T(n) = \begin{cases} 0, & n=0 \\ 1 + T(\lfloor n/2 \rfloor), & n > 0 \end{cases}$$

$$\begin{aligned} T(n) &= 1 + T(\lfloor n/2 \rfloor) \\ &= 1 + (1 + T(\lfloor n/4 \rfloor)) = 2 + T(\lfloor n/4 \rfloor) \\ &= 3 + T(\lfloor n/8 \rfloor) \\ &\vdots \\ &= k + T(\lfloor n/2^k \rfloor) \end{aligned}$$

$$\begin{aligned} n/2^k = 1 &\rightarrow \log(n/2^k) = \log 1 \\ \log n - k \log 2 &= 0 \\ k &= \frac{\log n}{\log 2} = \lceil \log_2 n \rceil \end{aligned}$$

Sehingga

$$\begin{aligned} T(n) &= \lceil \log_2 n \rceil + T(1) \\ &= \lceil \log_2 n \rceil + 1 \\ T(n) &= O(\log n) \end{aligned}$$

b) Algoritma kedua yang lebih mangkus karena

$$O(2^{\log n}) < O(n)$$