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2. a) $r=5$, pusat $(1,5)$, cari y absis 2.

↳ $(x-1)^2 + (y-5)^2 = 25$

$x^2 - 2x + 1 + y^2 + 10y + 25 = 25$

$x^2 - 2x + y^2 + 10y + 1 = 0$

* Untuk $x = 2$

$4 - 4 + y^2 + 10y + 1 = 0$

$y^2 + 10y + 1 = 0$

$y_{1,2} = \frac{-10 \pm \sqrt{100 - 4(1)(1)}}{2(1)}$

$= \frac{-10 \pm \sqrt{96}}{2}$

$= \frac{-10 \pm 2\sqrt{24}}{2}$

$= -5 \pm \sqrt{24}$

$= -5 \pm 2\sqrt{6}$

$y_1 = -5 + 2\sqrt{6}$, $y_2 = -5 - 2\sqrt{6}$

b) $x^2 - 2x + y^2 + 6y = -6$

$x^2 - 2x + y^2 + 6y + 6 = 0$

Bentuk Umum $x^2 + y^2 + Ax + By + C = 0 \Rightarrow A = -2, B = 6, C = 6$

→ Syarat Lingkaran, dimana persamaan membentuk bentuk umum terpenuhi

⇒ Titik Pusat

$P = \left(\frac{-A}{2}, \frac{-B}{2}\right)$

$P = \left(\frac{-(-2)}{2}, \frac{-6}{2}\right)$

$P = (1, -3)$

⇒ Jari - Jari

$r = \sqrt{\frac{A^2}{4} + \frac{B^2}{4} - C}$

$r = \sqrt{\frac{4}{4} + \frac{36}{4} - 6}$

$r = \sqrt{1 + 9 - 6}$

$r = \sqrt{4}$

$r = 2$

3. Cari daerah asal

a) $f(x) = \frac{1}{x-3}$

Syarat : $x - 3 \neq 0$

$x \neq 3$.

$D(f) = \{x \mid x \neq 3\}$

b) $g(t) = \sqrt{9 - t^2}$

Syarat : $9 - t^2 \geq 0$.

$t^2 - 9 \leq 0$

$(t-3)(t+3) \leq 0$.

$D(g) = \{x \mid -3 \leq x \leq 3\}$

4. Ganjil, genap

a) $f(x) = \frac{x^3 + 3x}{x^4 - 3x^2 + 4}$

$f(-x) = \frac{(-x)^3 + 3(-x)}{(-x)^4 - 3(-x)^2 + 4}$

$f(-x) = \frac{-x^3 - 3x}{x^4 - 3x^2 + 4}$

$f(-x) = -\left(\frac{x^3 + 3x}{x^4 - 3x^2 + 4}\right)$

$f(-x) = -f(x)$

Fungsi ini merupakan fungsi ganjil.

b) $\theta(z) = \frac{2z + 1}{z - 1}$

$\theta(-z) = \frac{2(-z) + 1}{(-z) - 1}$

$\theta(-z) = \frac{-2z + 1}{-z - 1}$.

$\theta(-z) \neq \theta(z)$

⇒ Fungsi ini bukan merupakan fungsi ganjil maupun fungsi genap.

5a. $\lim_{x \to 2} \frac{2x^2 - 3x - 2}{x - 2} = 5$.

↳ Karena $x \to 2$, berarti $x \neq 2$, sehingga $x - 2 \neq 0$, akibatnya $\frac{2x^2 - 3x - 2}{x - 2} = \frac{(x-2)(2x+1)}{x-2} = 2x + 1$. Jadi $\lim_{x \to 2} \frac{2x^2 - 3x - 2}{x - 2}$

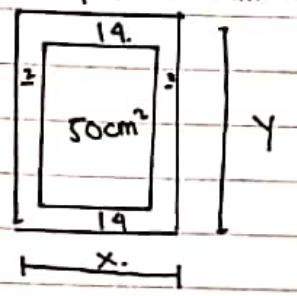
$= \lim_{x \to 2} 2x + 1 = 5$ dimana terbukti sama dengan diawal

5b. $\lim_{x \to 4} \frac{\sqrt{x^2 + 9}}{x}$

↳ Limit ini dapat langsung dieksekusi krn tidak menghasilkan $\frac{0}{0}$

⇒ $\lim_{x \to 4} \frac{\sqrt{x^2 + 9}}{x} = \frac{\sqrt{4^2 + 9}}{4} = \frac{\sqrt{25}}{4} = \frac{5}{4} = 1,25$ //.

8) Berapa ukuran x dan y supaya luas kertas seminim mungkin ?



1) Luas tulisan.

$$(x-4)(y-8) = 50\ cm^2$$
$$xy - 8x - 4y + 32 = 50\ cm^2.$$
$$xy - 8x - 4y - 18 = 0.$$
$$x(y-8) - 4y - 18 = 0$$
$$x = \frac{4y+18}{y-8}$$

2) Luas kertas

$$f(y) = xy = \frac{4y+18}{y-8} \cdot y.$$
$$= \frac{4y^2+18y}{y-8} \quad \rightarrow u' = 8y+18,\ v' = 1$$

3) L kertas minimum.

$$\frac{d}{dy} f(y) = 0 \rightarrow \frac{u'v - uv'}{v^2}$$
$$\Rightarrow \frac{(8y+18)(y-8) - (4y^2+18y)(1)}{(y-8)^2} = 0$$
$$\Rightarrow \frac{8y^2 - 46y - 144 - 4y^2 - 18y}{(y-8)^2} = 0$$
$$\Rightarrow \frac{4y^2 - 64y - 144}{(y-8)^2} = 0$$
$$\Rightarrow \frac{4(y+2)(y-18)}{(y-8)^2} = 0$$

$\boxed{y = 18}$ V $y = -2$ (TM)

$$\Rightarrow x = \frac{4(18)+18}{18-8.}$$
$$x = \frac{90}{10}$$
$$x. = 9$$

Jadi $x = 9$ dan $y = 18$