Ujian Tengah Semester Genap 22/23 $n \to$ digit akhir nim $= 0$

Tugas Pengerjaan Ulang - Alfonso Clement Sutantio - 24060122130080

1. $\displaystyle\int_0^{n+1} \frac{x}{\sqrt{(n+1)^2 - x^2}}\,dx = \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx = \lim_{a\to 1^-} \int_0^a \frac{x}{\sqrt{1-x^2}}\,dx$

Tentukan integral dari $\frac{x}{\sqrt{1-x^2}}$ dahulu

$\displaystyle\int \frac{x}{\sqrt{1-x^2}}\,dx$

misal:

$u = 1-x^2$

$du = -2x\,dx \to dx = \frac{du}{-2x}$

$\displaystyle(=) \int \frac{x}{\sqrt{u}} \cdot \frac{du}{-2x}$

$\displaystyle(=) -\frac{1}{2} \int u^{-\frac{1}{2}}\,du$

$(=) -\frac{1}{\cancel{2}} \cdot \cancel{2} \cdot u^{\frac{1}{2}}$

$(=) -(1-x^2)^{\frac{1}{2}} = -\sqrt{1-x^2}$

$\displaystyle(=) \lim_{a\to 1^-} \left[-\sqrt{1-x^2}\right]_0^a$

$(=) -\sqrt{1-1^2} - (-\sqrt{1-0^2})$

$(=) -\sqrt{0} + \sqrt{1}$

$(=) \boxed{1}$

2. $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x}}\,dx = \int_0^1 x \cdot (1-x)^{-\frac{1}{2}}\,dx$

Maka: (gunakan Fungsi Beta)

$1 = m-1$ $\quad -\frac{1}{2} = n-1$

$\underline{m = 2}$ $\quad \underline{n = \frac{1}{2}}$

dengan rumus Beta

$\displaystyle\to B(2,\tfrac{1}{2}) = \frac{\Gamma(2)\,\Gamma(\frac{1}{2})}{\Gamma(\frac{5}{2})} = \frac{(2-1)!\,\sqrt{\pi}}{\frac{3}{2}\cdot\Gamma(\frac{3}{2})} = \frac{\sqrt{\pi}}{\frac{3}{2}\cdot\frac{1}{2}\cdot\Gamma(\frac{1}{2})}$

$\displaystyle= \frac{\sqrt{\pi}}{\frac{3}{4}\cdot\sqrt{\pi}} = \frac{1}{3/4} = \boxed{\frac{4}{3}}$

3. $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases} \rightarrow T = 2\pi$

$$a_0 = \frac{1}{T} \int_{-\pi}^{\pi} f(x)\, dx$$

$$= \frac{1}{2\pi} \left(\int_{-\pi}^{0} -1\, dx + \int_{0}^{\pi} 1\, dx \right) = \frac{1}{2\pi} \left(-x \Big|_{-\pi}^{0} + x \Big|_{0}^{\pi} \right)$$

$$= \frac{1}{2\pi} \cdot (-\pi + \pi) = \underbrace{\frac{1}{2\pi} \cdot 0}_{=0}, \text{ maka } \boxed{a_0 = 0}$$

$$a_n = \frac{2}{T} \int_{-\pi}^{\pi} f(x) \cos nx\, dx$$

$$= \frac{2}{2\pi} \left(\int_{-\pi}^{0} -\cos nx\, dx + \int_{0}^{\pi} \cos nx\, dx \right) = \frac{1}{\pi} \left(\left(-\frac{1}{n} \sin nx\right) \Big|_{-\pi}^{0} + \left(\frac{1}{n} \sin nx\right) \Big|_{0}^{\pi} \right)$$

$$= \frac{1}{\pi} \cdot \left(-\frac{1}{n} \sin n\pi + \frac{1}{n} \sin n\pi \right) = \frac{1}{\pi} \cdot 0 = 0$$

$$\boxed{\text{maka } a_n = 0}$$

$$b_n = \frac{2}{T} \int_{-\pi}^{\pi} f(x) \sin nx\, dx$$

$$= \frac{2}{2\pi} \left(\int_{-\pi}^{0} -\sin nx\, dx + \int_{0}^{\pi} \sin nx\, dx \right) = \frac{1}{\pi} \left(\frac{1}{n} \cos nx \Big|_{-\pi}^{0} + -\frac{1}{n} \cos nx \Big|_{0}^{\pi} \right.$$

$$= \frac{1}{\pi} \left(\frac{1 - \cos(\pi n)}{n} + \frac{1 - \cos(\pi n)}{n} \right)$$

$$= \frac{2}{\pi} \left(\frac{1 - \cos(n\pi)}{n} \right)$$

Maka deret Fouriernya adalah $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin n$

$$= \sum^{\infty} \frac{2}{n\pi} (1 - \cos(n\pi)) . \sin n$$

dengan nilai b_n untuk n gonjil $= \frac{4}{n\pi}$ dan genap $= 0$

$$\therefore f(x) = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right)$$

Gambar grafik fungsinya: