

$$1. \int \frac{x^2}{\sqrt{1-x}} dx$$

misal $1-x=u$, maka $x=1-u$ dan $dx=-du$

sehingga

$$\int \frac{x^2}{\sqrt{1-x}} dx = \int \frac{(1-u)^2}{\sqrt{u}} (-du)$$

$$= - \int \frac{u^2 - 2u + 1}{\sqrt{u}} du$$

$$= - \int (u^{3/2} - 2u^{1/2} + u^{-1/2}) du$$

$$= - \left(\frac{2}{5} u^{5/2} - \frac{4}{3} u^{3/2} + 2u^{1/2} + C \right)$$

Kembalikan nilai u dalam x

$$= - \frac{2 \sqrt{(1-x)^5}}{5} + \frac{4 \sqrt{(1-x)^3}}{3} - 2 \sqrt{(1-x)} + C$$

, $C \in \mathbb{R}$

$$2. \int \frac{x}{\sqrt{1-x^2}} dx$$

misal $1-x^2=u$, maka $-2x dx = du$

$x dx = -du/2$, sehingga

$$\int \frac{x}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{u}} \left(-\frac{du}{2} \right)$$

$$= -\frac{1}{2} (2\sqrt{u} + C)$$

Kembalikan nilai u kedalam x

$$= -\sqrt{1-x^2} + C, C \in \mathbb{R}$$

$$3. \int \frac{x}{1-x} dx$$

misal $1-x=u$, maka $x=1-u$ dan $dx=-du$, sehingga

$$\begin{aligned} \int \frac{x}{1-x} dx &= \int \frac{1-u}{u} (-du) \\ &= -\int \left(\frac{1}{u} - 1 \right) du \\ &= -\left(\ln|u| - u \right) + C \end{aligned}$$

Kembalikan u ke dalam x

$$\begin{aligned} &= (1-x) - \ln|1-x| + C \\ &= -x - \ln|1-x| + C, \quad C \in \mathbb{R} \end{aligned}$$

$$4. \int x e^{x^2} dx$$

misal $x^2 = u$, maka $x dx = du/2$, sehingga

$$\begin{aligned} \int x e^{x^2} dx &= \int e^u \frac{du}{2} \\ &= \frac{1}{2} \int e^u du \\ &= \frac{1}{2} \frac{e^u}{\ln(e)} + C \\ &= \frac{e^u}{2} + C \end{aligned}$$

Kembalikan nilai u ke dalam x

$$= \frac{e^{x^2}}{2} + C, C \in \mathbb{R}$$

$$\begin{aligned} 5. \int x \ln x^2 dx &= \int 2x \ln x dx \\ &= 2 \int x \ln x dx \end{aligned}$$

misal $\ln x = u$ dan $x dx = dv$, maka

$du = \frac{1}{x} dx$ dan $V = \frac{1}{2} x^2$, sehingga dengan integral parsial didapat

$$\begin{aligned} \int x \ln x dx &= \ln x \cdot \frac{1}{2} x^2 - \int \frac{1}{2} x^2 \frac{1}{x} dx + C \\ &= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int (\frac{1}{2} x^2 + C) + C \end{aligned}$$

$$2 \int x \ln x dx = x^2 \ln x - \frac{1}{2} x^2 + C, C \in \mathbb{R}$$

$$6. \int x \sqrt{1+x} dx$$

misal $1+x=u$ maka $x=u-1$ dan $dx=du$, sehingga

$$\int x \sqrt{1+x} dx = \int (u-1) \cdot \sqrt{u} du$$

$$= \int (u^{3/2} - u^{1/2}) du$$

$$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C, C \in \mathbb{R}$$

Kembalikan nilai u dalam x

$$= \frac{2}{5} \sqrt{(x+1)^5} - \frac{2}{3} \sqrt{(x+1)^3} + C$$

$$7. \int \sin^2 x \cos^3 x dx = \int \sin^2 x (1 - \sin^2 x) \cos x dx$$

misal $\sin x = u$ maka $\cos x dx = du$

$$\int \sin^2 x \cos^3 x dx = \int u^2 (1 - u^2) du$$

$$= \int u^2 - u^4 du$$

$$= \frac{1}{3} u^3 - \frac{1}{5} u^5 + C$$

Kembalikan u dalam x

$$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C, C \in \mathbb{R}$$