UAS ALIN 2015/2016

[1] $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{bmatrix}$

$Au = \lambda u$

$[A - \lambda I] u = 0, \; u \neq 0$

$|A - \lambda I| = 0$

$\left[ \begin{pmatrix} 0 & 2 & -1 \\ 2 & 3 & -2 \\ -1 & -2 & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right] u = 0$

$\begin{pmatrix} -\lambda & 2 & -1 \\ 2 & 3-\lambda & -2 \\ -1 & -2 & -\lambda \end{pmatrix} u = 0, \; u \neq 0$

$\begin{vmatrix} -\lambda & 2 & -1 \\ 2 & 3-\lambda & -2 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$

$\left| \begin{array}{ccc|cc} -\lambda & 2 & -1 & -\lambda & 2 \\ 2 & 3-\lambda & -2 & 2 & 3-\lambda \\ -1 & -2 & -\lambda & -1 & -2 \end{array} \right| = 0$

$\Rightarrow [(-\lambda)(3-\lambda)(-\lambda) + (2)(-2)(-1) + (-1)(2)(-1)] -$
$[(-1)(3-\lambda)(-1) + (-\lambda)(-2)(-2) + (2)(2)(-\lambda)] = 0$

$\Rightarrow [3\lambda^2 - \lambda^3 + 4 + 4] - [(3-\lambda) - (4\lambda) - (4\lambda)] = 0$

$\Rightarrow -\lambda^3 + 3\lambda^2 + 8 - [3 - 9\lambda] = 0$

$\Rightarrow -\lambda^3 + 3\lambda^2 + 8 - 3 + 9\lambda = 0$

$\Rightarrow -\lambda^3 + 3\lambda^2 + 9\lambda + 5 = 0$

$(\lambda + 1)(\lambda - 5)(-\lambda - 1)$

$\lambda = -1 \vee \lambda = 5 \vee \lambda = -1$ {eigen value}.

u/ $\lambda = -1$

$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ -1 & -2 & 0 \end{bmatrix} \xrightarrow{\text{*OBE*}} \begin{array}{c} \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array}$

misal $x_1 = \alpha$, $x_2 = \beta$

$\alpha + 2\beta - x_3 = 0$

$x_3 = \alpha + 2\beta$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \alpha + 2\beta \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

u/ $\lambda = 5$

$\begin{bmatrix} -5 & 2 & -1 \\ 2 & -2 & -2 \\ -1 & -2 & -5 \end{bmatrix} \xrightarrow{\text{*OBE*}} \begin{array}{c} \begin{bmatrix} 1 & -2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array}$

misal $x_2 = \lambda$

$\lambda + 2x_3 = 0$

$x_3 = -\frac{1}{2}\lambda$

$x_1 - 2\lambda + 5(-\frac{1}{2}\lambda) = 0$

$x_1 = 2\lambda + \frac{5}{2}\lambda = 9/2\,\lambda$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9/2\,\lambda \\ \lambda \\ -1/2\,\lambda \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \alpha \begin{bmatrix} 9/2 \\ 1 \\ -1/2 \end{bmatrix}$

$\boxed{2}$ $p_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rightarrow |p_1| = \sqrt{1+1+1} = \sqrt{3} \rightarrow u_1 = \dfrac{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \{w_1\}$

$p_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \rightarrow |p_2| = \sqrt{1+1+0} = \sqrt{2} \rightarrow u_2 = \dfrac{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \quad \{w_2\}$

$p_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \rightarrow |p_3| = \sqrt{1+0+1} = \sqrt{2} \rightarrow u_3 = \dfrac{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \{w_3\}$

buktikan

$$\left.\begin{aligned} w_1 \cdot w_1 &= 1 \\ w_1 \cdot w_2 &= 0 \\ w_1 \cdot w_3 &= 0 \\ w_2 \cdot w_3 &= 0 \\ w_2 \cdot w_2 &= 1 \\ w_3 \cdot w_3 &= 1 \end{aligned}\right\} \checkmark$$

$$P = \left\{ \begin{bmatrix} \sqrt{3}/3 \\ \sqrt{3}/3 \\ \sqrt{3}/3 \end{bmatrix}, \begin{bmatrix} \sqrt{2}/2 \\ -\sqrt{2}/2 \\ 0 \end{bmatrix}, \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ -\sqrt{2}/2 \end{bmatrix} \right\}$$

$\boxed{3}$ $\bar{V}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \bar{V}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \bar{V}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(a) Tent A

(b) Tent $T\begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$

$\bar{p}_1 = (1,-1)$ ; $\bar{p}_2 = (1,0)$ ; $\bar{p}_3 = (0,2)$

$T(v_i) = A v_i = P_i$

rincian $\rightarrow$

$$\left.\begin{aligned} A \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} &= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ A \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ A \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} &= \begin{bmatrix} 0 \\ 2 \end{bmatrix} \end{aligned}\right\}$$

$$\boxed{\begin{aligned} AB &= C \\ ABB^{-1} &= CB^{-1} \\ \boxed{A} &= \boxed{CB^{-1}} \end{aligned}}$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}^{-1}$$

misal $B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} \rightarrow [B\ I] \xrightarrow{\text{*OBE*}} [I\ B^{-1}]$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[H_{31}(1)]{H_{21}(-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \end{array}\right] \xrightarrow{H_{32}(1)}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix}$$

(b) $T\left(\begin{bmatrix}1\\-1\\2\end{bmatrix}\right) = \begin{bmatrix}0 & 1 & 0\\-1 & 2 & 2\end{bmatrix}\begin{bmatrix}1\\-1\\2\end{bmatrix}$

$T\left(\begin{bmatrix}1\\-1\\2\end{bmatrix}\right) = \begin{pmatrix}-1\\1\end{pmatrix} \Rightarrow (-1,1)$ ; $P_4 = (-1,1)$

[4] $T: R^3 \to R^3$ $T[x,y,z] = [0x - 2y - z, 2x + 4y + 1z, 1x + 3y + z]$

tent basis & dimensi dr ruang image & kernel!

$T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = (0x - 2y - z, 2x + 4y + 1z, 1x + 3y + z)$

(i) Image (T)

$T(e_1) = T\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = (0, 2, 1)$

$T(e_2) = T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = (-2, 4, 3)$

$T(e_3) = T\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = (-1, 1, 1)$

u/ menentukan basis & dimensi, dg OKE

$$A = \begin{bmatrix}0 & 2 & 1\\-2 & 4 & 3\\-1 & 1 & 1\end{bmatrix} \xrightarrow[K_{31}(1)]{K_{21}(1)} \begin{bmatrix}0 & 2 & 1\\-2 & 2 & 1\\-1 & 0 & 0\end{bmatrix} \xrightarrow{K_{23}(-2)} \begin{bmatrix}0 & 0 & 1\\-2 & 0 & 1\\-1 & 0 & 0\end{bmatrix}$$

dimensi Image(T) = rank(A) = 2

basis Image(T) = $\left\{\begin{bmatrix}0\\-2\\-1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}\right\}$

(ii) kernel (T)

ambil $u \in \ker(T)$

$u = \begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} \Rightarrow T(u) = Au = 0$

$$\begin{bmatrix}0 & 2 & 1\\-2 & 4 & 3\\-1 & 1 & 1\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = 0 \quad \text{(SPL Homogen)}$$

diselesaikan dg OBE

$$\begin{bmatrix}0 & 2 & 1\\-2 & 4 & 3\\-1 & 1 & 1\end{bmatrix} \xrightarrow{H_{23}(-2)} \begin{bmatrix}0 & 2 & 1\\0 & 2 & 1\\-1 & 1 & 1\end{bmatrix} \xrightarrow{H_{12}(-1)} \begin{bmatrix}0 & 0 & 0\\0 & 2 & 1\\-1 & 1 & 1\end{bmatrix} \xrightarrow{H_{32}(-1)} \begin{array}{c}\begin{bmatrix}0 & 0 & 0\\0 & 2 & 1\\-1 & -1 & 0\end{bmatrix}\\ \;u_1 \;\; u_2 \;\; u_3\end{array}$$

misal. $u_2 = \lambda \to u_3 = -\lambda$

$u_2 + u_3 = 0$

$u_3 = -u_2$

$-u_1 - u_2 = 0 \to u_1 = -\lambda$

$u_1 = -u_2$

$$\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \lambda\begin{bmatrix}-1\\1\\-1\end{bmatrix}$$