

Pembahasan soal UAS Matematika II 2016/2017

1.) $(x^3 + \sin y) dx + (x \cos y + y^2) dy = 0$
 misal : $M(x, y) = x^3 + \sin y$ dan $N(x, y) = x \cos y + y^2$, maka
 $\frac{\partial M}{\partial y} = \cos y$ $\frac{\partial N}{\partial x} = \cos y$

solusi

$$\begin{aligned} F &= \int M dx + w(y) \\ &= \int (x^3 + \sin y) dx + w(y) \\ &= \frac{1}{4} x^4 + x \sin y + w(y) \end{aligned}$$

$\frac{\partial F}{\partial y} = x \cos y + w'(y)$, karena $\frac{\partial F}{\partial y} = N(x, y)$, maka

$$\begin{aligned} x \cos y + w'(y) &= x \cos y + y^2 \\ w'(y) &= y^2 \\ w(y) &= \int y^2 dy \\ w(y) &= \frac{1}{3} y^3 \end{aligned}$$

solusi umumnya $\Rightarrow F = \frac{1}{4} x^4 + x \sin y + \frac{1}{3} y^3 = c$

2.) PD eksak dengan FI (Faktor integral) $\mu = x$ adalah

$$x(2y - 2xy^3) dx + x(x - 2x^2y^2) dy = 0$$

$$(2xy - 2x^2y^3) dx + (x^2 - 2x^3y^2) dy = 0$$

misal $M(x, y) = 2xy - 2x^2y^3$ dan $N(x, y) = x^2 - 2x^3y^2$, maka
 $\frac{\partial M}{\partial y} = 2x - 6x^2y^2$ $\frac{\partial N}{\partial x} = 2x - 6x^2y^2$

solusi :

$$\begin{aligned} f &= \int M dx + w(y) \\ &= \int (2xy - 2x^2y^3) dx + w(y) \\ &= x^2y - \frac{2}{3} x^3y^2 + w(y) \end{aligned}$$

$$\frac{\partial F}{\partial y} = x^2 - \frac{4}{3} x^2y + w'(y)$$

Karena $\frac{\partial f}{\partial y} = N(x, y)$, maka

$$x^2 - \frac{4}{3} x^2 y + w'(y) = x^2 - 2x^3 y^2$$

$$w'(y) = -2x^3 y^2 + \frac{4}{3} x^2 y$$

$$w(y) = \int (-2x^3 y^2 + \frac{4}{3} x^2 y) dy$$

$$w(y) = -\frac{2}{3} x^3 y^3 + \frac{2}{3} x^2 y^2$$

$$w(y) = \frac{2}{3} x^2 y^2 - \frac{2}{3} x^3 y^3$$

$$\text{Solusi umumnya} \Rightarrow F = x^2 y - \frac{2}{3} x^3 y^2 + \frac{2}{3} x^2 y^2 - \frac{2}{3} x^3 y^3 = C$$