1. $u = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}$, $v = \begin{bmatrix} 6 & 4 & 2 \end{bmatrix}$

a) $w = a(1, 2, -1) + b(6, 4, 2)$ $\longrightarrow$ a = bebas, b = bebas

• $a = 1$, $b = 1$

$W_1 = 1(1, 2, -1) + 1(6, 4, 2)$

$= (7, 6, 1)$

• $a = 1$, $b = 2$

$W_2 = 1(1, 2, -1) + 2(6, 4, 2)$

$= (1, 2, -1) + (12, 8, 4)$

$= (13, 10, 3)$

• $a = 2$, $b = 1$

$W_3 = 2(1, 2, -1) + 1(6, 4, 2)$

$= (2, 4, -2) + (6, 4, 2)$

$= (8, 8, 0)$

• $a = 1$, $b = 3$

$W_4 = 1(1, 2, -1) + 3(6, 4, 2)$

$= (1, 2, -1) + (18, 12, 6)$

$= (19, 14, 5)$

• $a = 3$, $b = 1$

$W_5 = 3(1, 2, -1) + 1(6, 4, 2)$

$= (3, 6, -3) + (6, 4, 2)$

$= (9, 10, -1)$

b) $w = (8, 8, 0)$ dgn (u, v)

$$\begin{bmatrix} 6 & 4 & 2 \\ 8 & 8 & 0 \\ 1 & 2 & -1 \end{bmatrix} \begin{matrix} 6 & 4 \\ 8 & 8 \\ 1 & 2 \end{matrix}$$

$\det = (-48 + 0 + 32) - (16 - 32)$

$= 0$

$\det = 0 \longrightarrow$ Dependent linear (karena homogeneus equation)

(3)

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \qquad \lambda I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

a) Character equation

$$\det(\lambda I - A) = 0$$

$$\det \begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix} = 0$$

$$\begin{bmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{bmatrix} \begin{matrix} \lambda-4 & 0 \\ 2 & \lambda-1 \\ 2 & 0 \end{matrix}$$

$$(\lambda-4)(\lambda-1)^2 - (-2(\lambda-1)) = 0$$
$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 + 2\lambda - 2 = 0$$
$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$
$$(\lambda-3)(\lambda-2)(\lambda-1)$$

b) eigen value = $\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_3 = 1$

c) eigen vektor

•) $\lambda_1 = 3$

$$\begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \iff \begin{pmatrix} -1 & 0 & -1 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 & -1 & | & 0 \\ 2 & 2 & 0 & | & 0 \\ 2 & 0 & 2 & | & 0 \end{pmatrix} \xrightarrow{B_3 + 2B_1} \begin{bmatrix} -1 & 0 & -1 & | & 0 \\ 2 & 2 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$$-x_1 - x_3 = 0$$
$$x_1 = -x_3$$
$$2x_1 + 2x_2 = 0$$
$$x_1 = -x_2$$
$$x_2 = x_3$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_3 \\ x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} -s \\ s \\ s \end{pmatrix} = s \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

misal $x_3 = s$

•) $\lambda_2 = 2$

$$\begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -2 & 0 & -1 \\ 2 & 1 & 0 \\ 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 0 & -1 & | & 0 \\ 2 & 1 & 0 & | & 0 \\ 2 & 0 & 1 & | & 0 \end{pmatrix} \underset{B_3+B_1}{\Rightarrow} \begin{pmatrix} -2 & 0 & -1 & | & 0 \\ 2 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$-2x_1 - x_3 = 0$

$x_3 = -2x_1$

$2x_1 + x_2 = 0$

$x_2 = -2x_1$

misal $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2}t \\ t \\ t \end{pmatrix} = t \begin{pmatrix} -\frac{1}{2} \\ 1 \\ 1 \end{pmatrix}$$

•) $\lambda = 1$

$$\begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} -3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 & -1 & | & 0 \\ 2 & 0 & 0 & | & 0 \\ 2 & 0 & 0 & | & 0 \end{pmatrix} \underset{B_3-B_2}{\Rightarrow} \begin{pmatrix} -3 & 0 & -1 & | & 0 \\ 2 & 0 & 0 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}$$

$-3x_1 - x_3 = 0$

$x_3 = -3x_1$

$2x_1 = 0$

$x_1 = 0$

$x_2 = v$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} = v \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

maka eigen vektor :

$$\begin{pmatrix} -s \\ s \\ s \end{pmatrix}, \begin{pmatrix} -\frac{1}{2}t \\ t \\ t \end{pmatrix}, \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix} \quad dgn \quad \begin{matrix} s = x_3 & (\lambda=3) \\ t = x_3 & (\lambda=2) \\ v = x_2 & (\lambda=1) \end{matrix}$$