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1. Selidiki kekonvergenan dari improper integral:

$$\int_0^\infty \frac{1}{(x+K+1)^2}\,dx \qquad \text{Nim akhir} = 4.$$

Solusi:

$$\int_0^\infty \frac{1}{(x+5)^2}\,dx = \lim_{b\to\infty}\int_0^b \frac{1}{(x+5)^2}\,dx$$

#1 Mencari nilai integral.

misal: $x+5 = u$

$du = 1 \cdot dx$

$dx = du$

sehingga,

$$\int \frac{1}{(x+5)^2}\,dx$$

$$\int \frac{1}{u^2}\cdot du = -\frac{1}{u} = -\frac{1}{x+5}.$$

#2 Menyelesaikan limit.

$$\lim_{b\to\infty}\int_0^b \frac{1}{(x+5)^2} = \lim_{b\to\infty}\left(-\frac{1}{x+5}\right)\Big|_0^b$$

$$= \lim_{b\to\infty}\left(-\frac{1}{b+5} - \left(-\frac{1}{0+5}\right)\right)$$

$$= \lim_{b\to\infty}\left(-\frac{1}{b+5} + \frac{1}{5}\right)$$

$$= -\frac{1}{\infty+5} + \frac{1}{5}$$

$$= -\frac{1}{\infty} + \frac{1}{5}$$

$$= 0 + \frac{1}{5}$$

$$= \frac{1}{5}$$

Karena memiliki nilai, maka **Konvergen** terhadap nilai limitnya.

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2. Tentukan nilai integral dari $\int_0^{k+1} \frac{k+1}{\sqrt{x(k+1-x)}}\,dx$ { k = num Akhir = 4 }

melalui fungsi beta dan gamma!

Solusi :

$\int_0^5 \frac{5}{\sqrt{x(5-x)}}\,dx$, Karena batasnya : $0 \le x \le 5$, maka kita perlu ubah agar bisa digunakan fungsi betanya, Oleh karena itu, $x = 5y$, sehingga batasnya menjadi $0 \le y \le 1$ dan $dx = 5dy$. maka Persamaan menjadi :

$$\int_0^1 \frac{5}{\sqrt{5y(5-5y)}}\,5dy$$

$$= \int_0^1 \frac{25}{\sqrt{25y-25y^2}}\,dy$$

$$= \int_0^1 \frac{\cancel{25}\,5}{\cancel{5}\cdot\sqrt{y-y^2}}\,dy$$

$$= 5\int_0^1 1\cdot(y-y^2)^{-\frac{1}{2}}\,dy$$

$$= 5\int_0^1 1\cdot y^{-\frac{1}{2}}\cdot(1-y)^{-\frac{1}{2}}\,dy$$

$$= 5\int_0^1 y^{\overset{m}{\frac{1}{2}}-1}\cdot(1-y)^{\overset{n}{\frac{1}{2}}-1}\,dy$$ (Kita gunakan fungsi beta)

$$= 5\cdot B\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$ (Kita gunakan fungsi gamma)

$$= 5\cdot\frac{\Gamma(\frac{1}{2})\cdot\Gamma(\frac{1}{2})}{\Gamma(1)}$$

$$= 5\cdot\Gamma(\tfrac{1}{2})\cdot\Gamma(\tfrac{1}{2}) \quad \left(\Gamma(\tfrac{1}{2}) = \sqrt{\pi}\right)$$

$$= 5\cdot\sqrt{\pi}\cdot\sqrt{\pi}$$

$$= 5\pi$$

$$\Gamma(\tfrac{1}{2}) = 2\int_0^\infty e^{-x}x^{-\frac{1}{2}}\,dx$$

$$= 2\int_0^\infty e^{-x}\frac{1}{2\sqrt{x}}\,dx$$ misal: $u=\sqrt{x}$, $du = \frac{1}{2\sqrt{x}}dx$

$$= 2\int_0^\infty e^{-u^2}\,du$$

$$= \int_R e^{-u^2}\,du$$

$$= \sqrt{\pi}$$

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3. Tentukan deret cosinus Fourier fungsi $f(x) = (K+1) - x$, $0 \leq x < (K+1)$!

Solusi:

maka fungsi menjadi : $f(x) = (4+1) - x$

$f(x) = 5 - x$

Batas : $0 \leq x < 5$

Karena mengandung deret cosinus maka merupakan fungsi genap. Juga karena batas $0 \leq x < 5$ dimana $f(x)$ selalu positif, maka fungsi genap. Periodik dengan periode $2 \times 5 = 10$, maka koefisien Fouriernya:

$\rightarrow \boxed{L=5}$

$$a_0 = \frac{1}{L} \int_0^L f(x)\, dx$$

$$a_0 = \frac{1}{5} \int_0^5 (5-x)\, dx$$

$$= \frac{1}{5} \cdot \left(5x - \frac{1}{2}x^2\right) \Big|_0^5$$

$$= \frac{1}{5} \cdot \left(5(5) - \frac{1}{2}(25) - 0\right)$$

$$= \frac{1}{5} \cdot \left(25 - \frac{25}{2}\right) = \frac{1}{5} \cdot \frac{25}{2} = \frac{25}{10} = \frac{5}{2}$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi}{L} x \, dx, \quad n = 1, 2, 3$$

$$= \frac{2}{5} \int_0^5 (5-x) \cos \frac{n\pi}{5}\, dx$$

$$= \frac{2}{5} \left(\int_0^5 5 \cos \frac{n\pi}{5} - \int_0^5 x \cos \frac{n\pi}{5} \right) dx$$

$$= \frac{2}{5} \left(\frac{5}{n\pi} \sin \frac{n\pi x}{5} \Big|_0^5 - \left[\frac{x}{n\pi} \sin n\pi x + \left(\frac{1}{n\pi}\right)^2 \cos n\pi x \right]_0^5 \right.$$

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$$a_n = \frac{2}{5}\left(\frac{5}{n\pi} \cdot \sin n\pi(5) - \frac{5}{n\pi} \sin n\pi x\right)$$

$$= 2\left(\frac{5}{n\pi}\right)^2 \left(\cos n\pi - 5\right) = \frac{4}{5n^2\pi^2}$$

$$= \begin{cases} n \text{ genap}, \text{ nilai} = 0 \\ n \text{ ganjil}, \dfrac{4}{5n^2\pi^2} \end{cases}$$

Jadi deret cosinus fourier :

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos \frac{n\pi}{L} x$$

$$= \frac{5}{2} + \frac{4}{25\pi^2} \cos x\pi + \frac{1}{5}\pi^2 \cos\left(\frac{2\pi x}{5}\right)$$

maka deret cosinus fourier nya:

$$f(x) = \frac{5}{2} + \frac{4}{25\pi^2} \cos x\pi + \frac{1}{5}\pi^2 \cos \frac{2\pi x}{5}$$