Gunadarma University

KALKULUS 4

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SARMAG TEKNIK MESIN

KALKULUS 4 - SILABUS

1. **Deret Fourier**
    1.1. Fungsi Periodik
    1.2. Fungsi Genap dan Ganjil,
    1.3. Deret Trigonometri,
    1.4. Bentuk umum Deret Fourier,
    1.5. Kondisi Dirichlet,
    1.6. Deret Fourier sinus atau cosinus separuh jangkauan.
2. **Integral Fourier**
3. **Transformasi Laplace**
    3.1. Definisi dan sifat Transformasi Laplace
    3.2. Invers dari transformasi Laplace
    3.3. Teorema Konvolusi
    3.4. Penerapan transformasi Laplace dalam penyelesaian P. D. dengan syarat batas.
4. **Fungsi Gamma dan Fungsi Beta**
    4.1. Fungsi Gamma
    4.2. Fungsi Beta
    4.3. Penerapan fungsi Gamma dan fungsi Beta

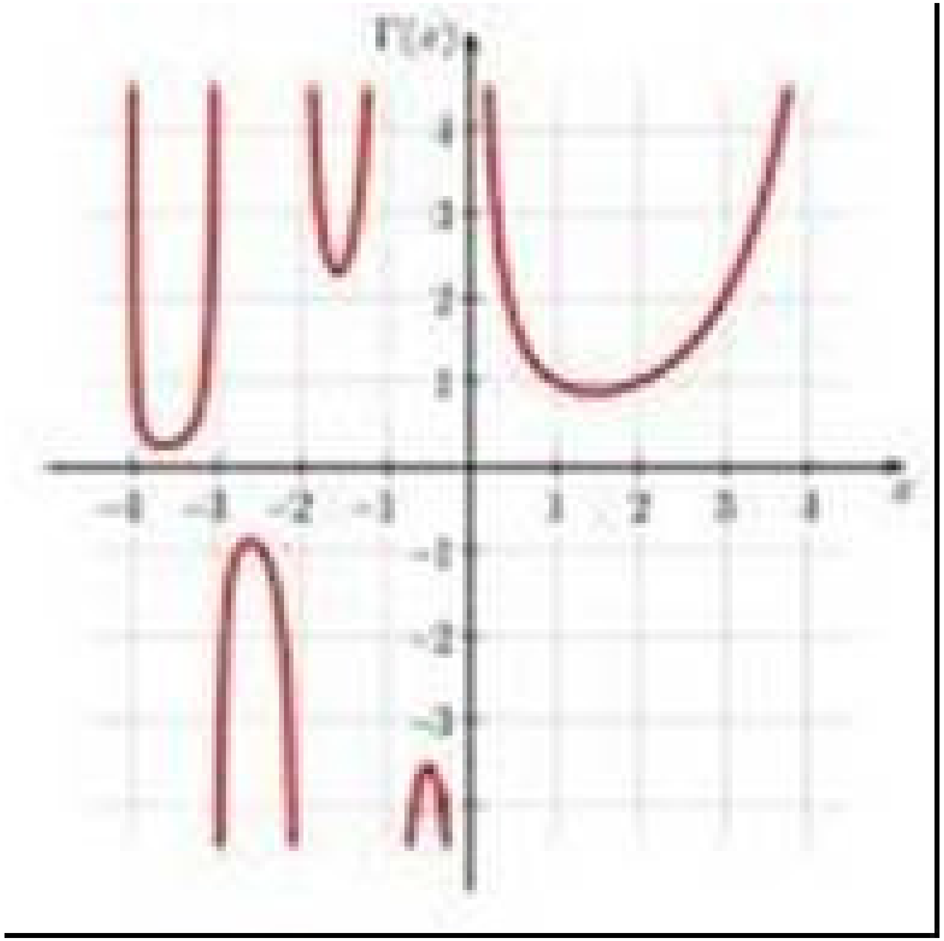
4.1. Fungsi Gamma

Tabel Nilai Fungsi Gamma

n	$\Gamma(n)$
1,00	1,0000
1,10	0,9514
1,20	0,9182
1,30	0,8975
1,40	0,8873
1,50	0,8862
1,60	0,8935
1,70	0,9086
1,80	0,9314
1,90	0,9618
2,00	1,0000

4.1. Fungsi Gamma

Grafik Fungsi Gamma



4.1. FUNGSI GAMMA

FUNGSI GAMMA : $\Gamma(n)$

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx = \lim_{b \to \infty} \int_0^b x^{n-1} e^{-x}\, dx$$

konvergen untuk n>0

4.1. Fungsi Gamma

Contoh:

$$\Gamma(1) = \int_0^\infty x^{1-1} e^{-x}\, dx$$

$$= \lim_{b \to \infty} \int_0^b x^{1-1} e^{-x}\, dx$$

$$= \lim_{b \to \infty} \int_0^b e^{-x}\, dx$$

$$= \lim_{b \to \infty} \left[-e^{-x} \right]_0^b = \lim_{b \to \infty} \left[-e^{-b} + e^0 \right] = 1$$

4.1. Fungsi Gamma

Contoh:

$$\Gamma(2) = \int_0^\infty x^{2-1} e^{-x}\, dx$$

$$= \lim_{b \to \infty} \int_0^b x^1 e^{-x}\, dx$$

$$= \ldots\ldots\ldots\ldots$$

4.1. Fungsi Gamma

Rumus Rekursi dari Fungsi Gamma

$$\Gamma(n+1) = n\,\Gamma(n)$$

dimana $\Gamma(1) = 1$

Contoh:

1. $\Gamma(2) = \Gamma(1+1) = 1\,\Gamma(1) = 1.$
2. $\Gamma(3) = \Gamma(2+1) = 2\,\Gamma(2) = 2.$
3. $\Gamma(3/2) = \Gamma(\tfrac{1}{2}+1) = \tfrac{1}{2}\,\Gamma(\tfrac{1}{2}).$

4.1. Fungsi Gamma

Bila n bilangan bulat positif

$$\Gamma(n+1) = n!$$

dimana $\Gamma(1) = 1$

Contoh:

1. $\Gamma(2) = \Gamma(1+1) = 1! = 1.$
2. $\Gamma(3) = \Gamma(2+1) = 2! = 2.$
3. $\Gamma(4) = \Gamma(3+1) = 3! = 6.$

4.1. Fungsi Gamma

Contoh:

Hitunglah

4. $\Gamma(6)$
5. $\Gamma(5)/_{\Gamma(3)}$
6. $\Gamma(6)/_{2\Gamma(3)}$

4.1. Fungsi Gamma

Bila n bilangan pecahan positif

$$\Gamma(n) = (n-1) \,.\, (n-2) \,.\, \ldots\, \alpha\, \Gamma(\alpha)$$

dimana $0 < \alpha < 1$

Contoh:

1. $\Gamma(3/2) = (1/2)\,\Gamma\,(1/2)$
2. $\Gamma(7/2) = (5/2)(3/2)(1/2)\Gamma(1/2)$
3. $\Gamma(5/3) = (2/3)\Gamma(2/3)$.

4.1. Fungsi Gamma

Bila n bilangan pecahan negatif

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

atau

$$\Gamma(n) = \frac{\Gamma(n+m)}{\underbrace{n(n-1)\ldots}_{\text{m bilangan}}}$$

4.1. Fungsi Gamma

Contoh:

$$\Gamma\left(-\frac{3}{2}\right)=\frac{\Gamma\left(-\frac{3}{2}+1\right)}{-\frac{3}{2}}=\frac{\Gamma\left(-\frac{1}{2}\right)}{-\frac{3}{2}}=\frac{\Gamma\left(-\frac{1}{2}+1\right)}{\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)}$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)}{\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)}=\frac{\Gamma\left(\frac{1}{2}\right)}{\left(\frac{3}{4}\right)}$$

4.1. Fungsi Gamma

Contoh:

$$\Gamma\left(-\frac{5}{2}\right)=\frac{\Gamma\left(-\frac{5}{2}+1\right)}{-\frac{5}{2}}=\frac{\Gamma\left(-\frac{3}{2}\right)}{-\frac{5}{2}}=\frac{\Gamma\left(-\frac{3}{2}+1\right)}{\left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)}$$

$$=\frac{\Gamma\left(-\frac{1}{2}\right)}{\left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)}=\frac{\Gamma\left(-\frac{1}{2}+1\right)}{\left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)}$$

$$=\frac{\Gamma\left(\frac{1}{2}\right)}{\left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)}=\frac{\Gamma\left(\frac{1}{2}\right)}{\left(-\frac{15}{8}\right)}$$

4.1. Fungsi Gamma

Beberapa hubungan dalam fungsi gamma

$$\Gamma(\tfrac{1}{2}) = \sqrt{\pi}$$

$$\Gamma(n) = (n-1)!$$

$$\Gamma(n) = \frac{\Gamma(n+1)}{n}$$

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$$

4.1. Fungsi Gamma

Contoh Soal:

1. $\Gamma\left(\frac{5}{2}\right)$

2. $\Gamma\left(-\frac{1}{2}\right)$

3. $\dfrac{\Gamma\left(-\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$

3. $\dfrac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$

4. $\dfrac{\Gamma(3)\Gamma(2{,}5)}{\Gamma(5{,}5)}$

5. $\dfrac{6\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{2}{3}\right)}$

4.1. Fungsi Gamma

Penggunaan Fungsi Gamma

1. Hitung $\int_0^\infty x^6 e^{-2x}\,dx$

Jawab :

Misalkan $2x = y \rightarrow dx = \tfrac{1}{2}\,dy$

bila $x = 0$, maka $y = 0$

bila $x = \infty$, maka $y = \infty$

$$\int_0^\infty x^6 e^{-2x}\,dx = \int_0^\infty (\tfrac{1}{2}y)^6 e^{-y}\,\tfrac{1}{2}\,dy = \int_0^\infty (\tfrac{1}{2})^7 y^6 e^{-y}\,dy$$

$$= (\tfrac{1}{2})^7 \int_0^\infty y^6 e^{-y}\,dy = (\tfrac{1}{2})^7 \int_0^\infty y^{7-1} e^{-y}\,dy$$

$$= (\tfrac{1}{2})^7\,\Gamma(7) = \frac{6!}{2^7} = \frac{45}{8}$$

4.1. Fungsi Gamma

2. Hitung $\int_0^\infty \sqrt{y}\ e^{-y^3}\,dy$ dengan substitusi $y^3 = x$

Jawab : Misalkan $y^3 = x \rightarrow dx = 3y^2\,dy$

bila $x = 0$, maka $y = 0$

bila $x = \infty$, maka $y = \infty$

$$\int_0^\infty \sqrt{y}\ e^{-y^3}\,dy = \int_0^\infty \sqrt{x^{1/3}}\ e^{-x}\,\frac{1}{3\left(x^{1/3}\right)^2}\,dx = \int_0^\infty x^{1/6}\ e^{-x}\,\frac{1}{3\,x^{2/3}}\,dx$$

$$= \frac{1}{3}\int_0^\infty x^{1/6-2/3}\ e^{-x}\,dx = \frac{1}{3}\int_0^\infty x^{-1/2}\ e^{-x}\,dx$$

$$= \frac{1}{3}\int_0^\infty x^{1/2-1}\ e^{-x}\,dx = \frac{1}{3}\Gamma(1/2) = \frac{1}{3}\sqrt{\pi}$$

4.1. Fungsi Gamma

3. Hitung $\int_0^1 \frac{dx}{\sqrt{-\ln x}}$ dengan substitusi $-\ln x = u$

Jawab :

Misalkan $-\ln x = u \rightarrow x = e^{-u} \rightarrow dx = -e^{-u} du$

Bila $x = 0$, maka $u = \infty$ dan bila $x = 1$, maka $u = 0$

$$\int_0^1 \frac{dx}{\sqrt{-\ln x}} = \int_\infty^0 \frac{-e^{-u}\,du}{\sqrt{u}} = \int_\infty^0 u^{-1/2}(-e^{-u})\,du$$

$$= \int_0^\infty u^{-1/2} e^{-u}\,du = \Gamma(1/2) = \sqrt{\pi}$$

4.2. FUNGSI BETA

FUNGSI BETA dinyatakan sebagai

$$B(m,n)=\int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

konvergen untuk $m > 0$ dan $n > 0$.

Sifat: $B(m,n) = B(n,m)$

Bukti: … … …

4.2. Fungsi Beta

Bukti:

$$B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

$$= \int_0^1 (1-y)^{m-1}(y)^{n-1}\,dx$$

$$= \int_0^1 (y)^{n-1}(1-y)^{m-1}\,dx$$

$$= B(n,m)$$

$\therefore$ Terbukti

HUBUNGAN
Fungsi Beta dengan Fungsi Gamma

$$B(m,n)=\frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$$

4.2. Fungsi Beta

Contoh:

1. Hitung B(3,5).

   Jawab:

$$B(3,5) = \frac{\Gamma(3)\,\Gamma(5)}{\Gamma(3+5)} = \frac{\Gamma(3)\,\Gamma(5)}{\Gamma(8)} = \ldots\ldots$$

4.2. Fungsi Beta

Contoh:

2. Hitung $B(5\ ,\ 2)$.

   Jawab:

3. Hitung $B(^3/_2\ ,\ 2)$.

   Jawab:

4. Hitung $B(^1/_3\ ,\ ^2/_3)$.

   Jawab:

4.2. Fungsi Beta

Penggunaan Fungsi Beta

1. Hitung $\int_0^1 x^4 (1-x)^3\, dx$

Jawab :

$$\int_0^1 x^4 (1-x)^3\, dx = \int_0^1 x^{5-1} (1-x)^{4-1}\, dx$$

$$= B(5,4)$$

$$= \frac{\Gamma(5)\ \Gamma(4)}{\Gamma(5+4)} = \frac{4!\ 3!}{8!}$$

$$= \frac{1}{280}$$

4.2. Fungsi Beta

2. Hitung $\displaystyle\int_0^2 \frac{x^2\,dx}{\sqrt{2-x}}$

Jawab : Misalkan $x = 2u \rightarrow dx = 2\,du$

$$\int_0^2 \frac{x^2\,dx}{\sqrt{2-x}} = \int_0^1 \frac{(2u)^2\,2du}{\sqrt{2-2u}} = \int_0^1 \frac{8u^2\,du}{\sqrt{2}\sqrt{1-u}}$$

$$= \frac{8}{\sqrt{2}}\int_0^1 \frac{u^2\,du}{\sqrt{1-u}} = 4\sqrt{2}\int_0^1 u^2(1-u)^{-\frac{1}{2}}\,du$$

$$= 4\sqrt{2}\;B(3, \tfrac{1}{2}) = \ldots$$

4.2. Fungsi Beta

Penggunaan Fungsi Beta

3. Hitung $\displaystyle\int_0^a y^4 \sqrt{a^2 - y^2}\; dy$

LATHAN

1. Hitung a). $\dfrac{\Gamma(7)}{2\Gamma(4)\Gamma(3)}$ b). $\dfrac{\Gamma(3)\Gamma(3/2)}{\Gamma(9/2)}$

2. Hitung $\int_0^\infty x^4 e^{-x}\, dx$

3. Hitung $\int_0^\infty \sqrt[4]{x}\; e^{-\sqrt{x}}\, dx$

4. Hitung a). $B(3/2, 2)$ b). $B(1/3, 2/3)$

5. Hitung $\int_0^1 x^2 (1-x)^3\, dx$

6. Hitung $\int_0^4 u^{3/2}\,(4-u)^{5/2}\, du$