PEMBAHASAN UTS ALIN 2018/2019

(16) $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ → invers matriks 3×3

$$A^{-1} = \frac{1}{\det A} \cdot \text{Adj}(A)$$

Dengan metode eliminasi gauss :

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$ lakukan operasi baris elementer

$B_1(1/2)$: $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$ B_2-B_1, B_3-B_1 $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3/2 & 1/2 & -1/2 & 1 & 0 \\ 0 & 1/2 & 3/2 & -1/2 & 0 & 1 \end{array}\right]$

$B_2(2) \sim B_3(2)$: $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 & 2 \end{array}\right]$ B_3-2B_1 $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 2 & -2 & 0 & 2 \end{array}\right]$

$B_3(1/2)$: $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 1/2 & 1/2 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$ B_2-B_3, $B_1-1/2B_3$ $\sim$ $\left[\begin{array}{ccc|ccc} 1 & 1/2 & 0 & 1 & 0 & -1/2 \\ 0 & 3 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

$B_1-1/6(B_2)$: $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/2 & -1/3 \\ 0 & 3 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$ $1/3(B_2)$: $\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1/3 & -1/3 \\ 0 & 1 & 0 & 0 & 2/3 & -1/3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$

inversnya = $\begin{bmatrix} 1 & -1/3 & -1/3 \\ 0 & 2/3 & -1/3 \\ -1 & 0 & 1 \end{bmatrix}$

(17) $x - 2y + z = 0$

$2y - 8z = 8$

$-4x + 5y + 9z = -9$

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & 8 \\ -4 & 5 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ -9 \end{bmatrix} \xrightarrow{B_3 + 4B_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 2 & 8 & 8 \\ 0 & -3 & 13 & -9 \end{array}\right]$$

$$\xrightarrow{B_2(1/2)} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 4 & 4 \\ 0 & -3 & 13 & -9 \end{array}\right] \xrightarrow{B_3 - 3/2 B_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 23/2 & -9 \end{array}\right]$$

$$\xrightarrow{B_3(2/23)} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 1 & 4 & 8 \\ 0 & 0 & 1 & -18/23 \end{array}\right] \xrightarrow[B_1 - B_3]{B_2 - 4B_3} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 18/23 \\ 0 & 1 & 0 & 164/23 \\ 0 & 0 & 1 & -18/23 \end{array}\right]$$

$$\xrightarrow{B_1 + 2B_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 346/23 \\ 0 & 1 & 0 & 164/23 \\ 0 & 0 & 1 & -18/23 \end{array}\right]$$

$x_1 = 346/23$

$x_2 = 164/23$

$x_3 = -18/23$

(18) (a) $$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 \\ 4 & -4 & 4 & 0 & 4 \\ -2 & 2 & -2 & 1 & -3 \end{array}\right] \xrightarrow[B_3 - 4B_1]{B_2 - B_1} \left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 2 & -2 \\ 0 & 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$x_1 - x_2 + x_3 - x_4 = 2$

$2x_4 = -2$

$4x_4 = -4$

$-x_4 = 1$

$\Big\rangle \; x_4 = -1$

$x_1 - x_2 + x_3 = 1$

x_1, x_2, x_3 memiliki banyak kemungkinan penyelesaian

(b.) Karena solusi persamaan diatas jamak maka SPL merupakan non-trivial solution