# SOAL A

1.) $a = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$, $c = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$

$$k_1 a + k_2 b + k_3 c = \bar{0}$$

$$\begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

dengan OBE diperoleh :

$$\left(\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 \end{array}\right) \xrightarrow{B1 \leftarrow B2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 \\ 2 & 3 & 5 & 0 \end{array}\right) \xrightarrow[B3+B1(-2)]{B2+B1(-2)}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right) \xrightarrow{B_3+B_2(-3)} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

didapat :

$k_1 + k_3 = 0 \rightarrow k_1 = -3k_3$

$k_2 + k_3 = 0 \rightarrow k_2 = -3k_3$

menunjukkan bahwa :

$k_1, k_2, k_3$ memiliki solusi non-trivial .

Jadi a, b, c adalah vektor-vektor <u>bergantung linear</u>.

Dipindai dengan CamScanner

SOAL B

1.) diket $A = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{pmatrix}$

ditanya: eigen value dan eigen vektor matriks A?

dijawab:

• eigen value

$$A - \lambda I = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}$$

$$= \begin{pmatrix} -1-\lambda & 1 & 1 \\ 2 & -\lambda & 2 \\ 3 & 3 & 1-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\begin{vmatrix} -1-\lambda & 1 & 1 \\ 2 & -\lambda & 2 \\ 3 & 3 & 1-\lambda \end{vmatrix} = 0$$

Ekspansi baris 2:

$$-2\begin{vmatrix} 1 & 1 \\ 3 & 1-\lambda \end{vmatrix} - \lambda \begin{vmatrix} -1-\lambda & 1 \\ 3 & 1-\lambda \end{vmatrix} - 2 \begin{vmatrix} -1-\lambda & 1 \\ 3 & 3 \end{vmatrix} = 0$$

$$-2[(1.(1-\lambda)) - 3] - \lambda[(-1-\lambda)(1-\lambda) - 3] - 2[(-1-\lambda)(3) - 3] = 0$$

$$-2(1-\lambda-3) - \lambda(-1+\lambda-\lambda+\lambda^2-3) - 2(-3-3\lambda-3) = 0$$

$$-2(-\lambda-2) - \lambda(-4+\lambda^2) - 2(-3\lambda-6) = 0$$

$$2\lambda + 4 + 4\lambda - \lambda^3 + 6\lambda + 12 = 0$$

$$-\lambda^3 + 12\lambda + 16 = 0$$

$$(\lambda-4)(\lambda+2)(\lambda+2)$$

$$\lambda_1 = 4, \lambda_2 = -2, \lambda_3 = -2$$

• eigen vector

$$A - \lambda I = \begin{pmatrix} -1 & 1 & 1 \\ 2 & 0 & 2 \\ 3 & 3 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} -1-\lambda & 1 & 1 \\ 2 & -\lambda & 2 \\ 3 & 3 & 1-\lambda \end{pmatrix}$$

• untuk $\lambda = 4$

$$(A - I)\bar{v} = \bar{0}$$

$$\begin{pmatrix} -5 & 1 & 1 \\ 2 & -4 & 2 \\ 3 & 3 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

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Cari OBE :

$$\begin{pmatrix} -5 & 1 & 1 \\ 2 & -4 & 2 \\ 3 & 3 & -3 \end{pmatrix} \xrightarrow{B_1 \leftarrow B_2} \begin{pmatrix} 2 & -4 & 2 \\ -5 & 1 & 1 \\ 3 & 3 & -3 \end{pmatrix} \xrightarrow[B_3 \times (1/3)]{B_1 \times (1/2)} \begin{pmatrix} 1 & -2 & 1 \\ -5 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\xrightarrow[B_3 + B_1(-1)]{B_2 + B_1(5)} \begin{pmatrix} 1 & -2 & 1 \\ 0 & -9 & 6 \\ 0 & 3 & -2 \end{pmatrix} \xrightarrow{B_2 + B_3(3)} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 0 & 0 \\ 0 & 3 & -2 \end{pmatrix} \xrightarrow{B_2 \leftarrow B_3} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\xrightarrow{B_1 + B_2\left(\frac{2}{3}\right)} \begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 3 & -2 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{B_2 \times \left(\frac{1}{3}\right)} \begin{pmatrix} 1 & 0 & -1/3 \\ 0 & 1 & -2/3 \\ 0 & 0 & 0 \end{pmatrix}$$

Misal $x_3 = s$ , $s \in \mathbb{R}$

$x_1 - 1/3 x_3 = 0 \rightarrow x_1 = -1/3\, x_3$

$x_1 = -1/3\, s$

$x_2 - 2/3 x_3 = 0 \rightarrow x_2 = -2/3\, x_3$

$x_2 = -2/3\, s$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -1/3\, s \\ -2/3\, s \\ 1 \end{pmatrix} = \begin{pmatrix} -1/3 \\ -2/3 \\ 1 \end{pmatrix} s, s \in \mathbb{R}$$

Vektor eigen untuk $\lambda = 4 = \begin{pmatrix} -1/3 \\ -2/3 \\ 1 \end{pmatrix}$

$$BRE_{\lambda = 4} = \left\{ \begin{pmatrix} -1/3 \\ -2/3 \\ 1 \end{pmatrix} \right\}$$

0

untuk $\lambda = -2$

$(A - I)\bar{v} = \bar{0}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Cari OBE : $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix} \xrightarrow[B_3 + B_1(-3)]{B_2 + B_1(-2)} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\begin{matrix} 1 \\ 2 \\ 1-\lambda \end{matrix}$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

. misal $x_2 = s$, $x_3 = t$ , $s, t \in \mathbb{R}$

$x_1 + x_2 + x_3 = 0$

$x_1 = -x_2 - x_3 \rightarrow x_1 = -s - t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -s-t \\ s \\ t \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} t$$

$$BRE_{\lambda = -2} = \left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

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Dipindai dengan CamScanner

2.) diket: $V_1 = \begin{pmatrix}-1\\1\\0\end{pmatrix}, V_2 = \begin{pmatrix}1\\0\\1\end{pmatrix}, V_3 = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$, $S = \{V_1, V_2, V_3\}$, $S \in \mathbb{R}^3$

ditanya : dimensi dan basis?

dijawab:

$$S = \left\{ \begin{pmatrix}-1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\}$$

$$\begin{pmatrix}-1 & 1 & -1\\1 & 0 & -1\\0 & 1 & 1\end{pmatrix} \xrightarrow{B_1 \times (-1)} \begin{pmatrix}1 & -1 & 1\\1 & 0 & -1\\0 & 1 & 1\end{pmatrix} \xrightarrow{B_2 + B_1(-1)} \begin{pmatrix}1 & -1 & 1\\0 & 1 & -2\\0 & 1 & 1\end{pmatrix}$$

$$\xrightarrow[B_3 + B_2(-1)]{B_1 + B_2} \begin{pmatrix}1 & 0 & -1\\0 & 1 & -2\\0 & 0 & 3\end{pmatrix} \xrightarrow{B_3 \times (1/3)} \begin{pmatrix}1 & 0 & -1\\0 & 1 & -2\\0 & 0 & 1\end{pmatrix} \xrightarrow[B_2 + B_3(2)]{B_1 + B_3} \begin{pmatrix}\boxed{1} & 0 & 0\\0 & \boxed{1} & 0\\0 & 0 & \boxed{1}\end{pmatrix}$$

$$\text{Basis} = \left\{ \begin{pmatrix}-1\\1\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}-1\\-1\\1\end{pmatrix} \right\}$$

Dimensi = 3

3.) $v_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $v_3 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$, $S = \{v_1, v_2, v_3\}$, $S \in \mathbb{R}^3$

→ • Cek ortonormal

$\langle v_1, v_2 \rangle = v_1 \cdot v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = -1 \neq 0$

bukan ortogonal, jadi bukan ortonormal

• Gram-Schmidt

$u_1 = v_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ $\qquad \|u_1\| = \sqrt{(-1)^2 + 1^2 + 0^2} = \sqrt{2}$

$u_2 = v_2 - \frac{\langle v_2, u_1 \rangle}{\|u_1\|^2} \cdot u_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{(\sqrt{2})^2} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \frac{(-1)}{2} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 1/2 \\ -1/2 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}$

$\|u_2\| = \sqrt{(1/2)^2 + (1/2)^2 + 1^2}$
$= \sqrt{(1/4) + (1/4) + 1}$
$= \sqrt{3/2} = \frac{\sqrt{6}}{2}$

$u_3 = v_3 - \frac{\langle v_3, u_1 \rangle}{\|u_1\|^2} \cdot u_1 - \frac{\langle v_3, u_2 \rangle}{\|u_2\|^2} \cdot u_2$

$= \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\rangle}{(\sqrt{2})^2} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix} \right\rangle}{\left(\frac{\sqrt{6}}{2}\right)^2} \cdot \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} - \frac{1 - 1 + 0}{2} \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} - \frac{(-1/2) + (-1/2) + 1}{6/4} \begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + 0 - 0 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$

$\|u_3\| = \sqrt{(-1)^2 + (-1)^2 + 1^2}$
$= \sqrt{3}$

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$$w = \left\{ \frac{u_1}{\|u_1\|}, \frac{u_2}{\|u_2\|}, \frac{u_3}{\|u_3\|} \right\}$$

$$= \left\{ \frac{\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}{\sqrt{2}}, \frac{\begin{pmatrix} 1/2 \\ 1/2 \\ 1 \end{pmatrix}}{\sqrt{6}/2}, \frac{\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}}{\sqrt{3}} \right\}$$

$$= \left\{ \begin{pmatrix} -2/\sqrt{2} \\ 2/\sqrt{2} \\ 0 \end{pmatrix}, \begin{pmatrix} \sqrt{6}/6 \\ \sqrt{6}/6 \\ \sqrt{6}/3 \end{pmatrix}, \begin{pmatrix} -\sqrt{3}/3 \\ -\sqrt{3}/3 \\ \sqrt{3}/3 \end{pmatrix} \right\}$$

w merupakan basis ortonormal

CS Dipindai dengan CamScanner