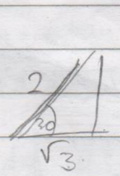
1\. a. $A(-3, \sqrt{3})$; $x = r\cos\theta$ $\quad \tan\theta = \frac{y}{x}$

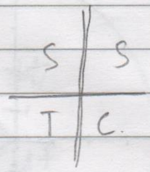
; $y = r\sin\theta$

$r = \sqrt{y^2 + x^2}$

$r = \sqrt{9+3} = \sqrt{12}$

$= 2\sqrt{3}$ //





$\tan\theta = \frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}} \Rightarrow \theta = 150°$ // $\vee$ $\theta = 330°$

$= \frac{5\pi}{6}$

$A(-3, \sqrt{3}) \Rightarrow A\left(2\sqrt{3}, \frac{5}{6}\pi\right)$

$A(x, y) \Rightarrow A(r, \theta)$

b. $f(x,y) = \frac{x-y}{x^2+y^2} \Rightarrow$

$r^2 = x^2 + y^2$

$x = r\cos\theta$

$y = r\sin\theta$

$$\frac{x-y}{x^2+y^2} = \frac{r\cos\theta - r\sin\theta}{r^2} = \frac{\cancel{r}(\cos\theta - \sin\theta)}{r^{\cancel{2}}}$$

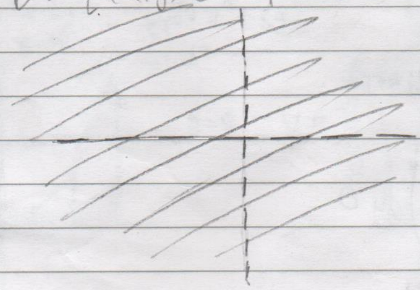
$$f(r,\theta) = \frac{\cos\theta - \sin\theta}{r}$$

2\. a. $f(x,y) = 2xy + \frac{1}{xy}$

Syarat : $x \cdot y \neq 0$; $x \neq 0 \vee y \neq 0$.

Domain : $D = \{(x,y) \in \mathbb{R}^2 \mid x \neq 0 \text{ atau } y \neq 0\}$

Grafik :



$\Rightarrow$ [hatched box] $\Rightarrow$ daerah hasil.

$\Rightarrow$ Sumbu x & y putus-putus karena bukan merupakan daerah hasil.

b. $g(x,y) = \sqrt{\frac{4-y^2-x^2}{y^2-x}} + 2xy$.

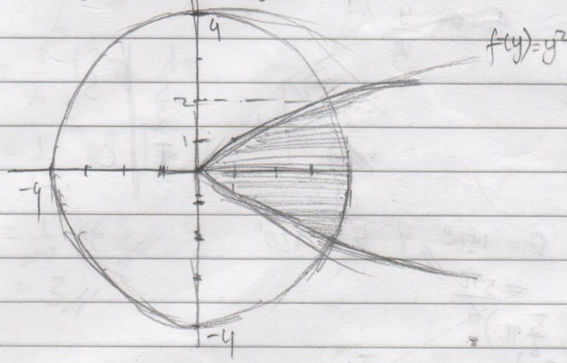
Syarat : $\frac{4-y^2-x^2}{y^2-x} \geq 0$ < (a) $4-y^2-x^2 \geq 0$ dan $y^2-x \geq 0$; (b) $4-y^2-x^2 \leq 0$ dan $y^2-x \leq 0$

(a). $x^2+y^2 \leq 4$ dan $x \leq y^2$

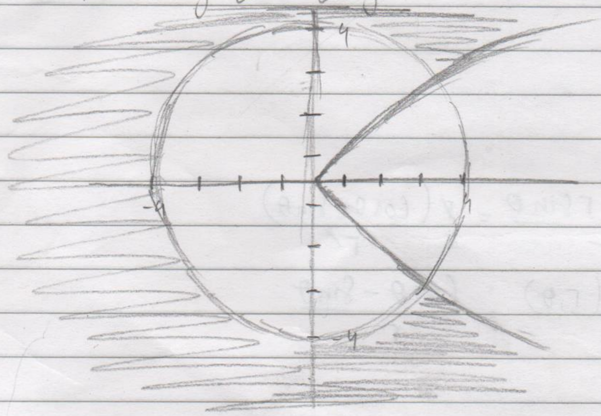
(b) $x^2+y^2 \geq 4$ dan $x \geq y^2$

Domain : $D = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 4 \text{ dan } x \leq y^2 \text{ atau } x^2+y^2 \geq 4 \text{ dan } x \geq y^2\}$.

Grafik : a. $4 \geq x^2+y^2$ dan $x \leq y^2$



b. $4 \leq x^2+y^2$ dan $x \geq y^2$



3. $f(x,y) = e^{xy} + \cos(2x+3y) + xy^2$

a. $\frac{\partial f(x,y)}{\partial x} = ye^{xy} - \sin(2x+3y).2 + y^2$

$= y\,e^{xy} - 2\sin(2x+3y) + y^2$

b. $\frac{\partial f(x,y)}{\partial y} = x\,e^{xy} - 3\sin(2x+3y) + 2xy$.

4. $\int_1^{\pi}\int_1^{2\pi} \left(\frac{2}{x}+\frac{1}{y}\right) dx\,dy = \int_1^{\pi}\left(\int_1^{2\pi} \frac{2}{x}\,dx + \int_1^{2\pi} \frac{1}{y}\,dx\right) dy$

$= \int_1^{\pi} 2\ln x + \frac{x}{y}\Big]_1^{2\pi} dy$

$= \int_1^{\pi} 2(\ln 2\pi - 0) + \frac{2\pi}{y} - \frac{1}{y}\,dy$

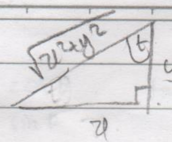
$= \int_1^{\pi} 2\ln 2\pi + \frac{2\pi-1}{y}\,dy$

$= 2y\ln 2\pi + (2\pi-1)\ln y\Big]_1^{\pi}$

$= (2\pi-1)\ln 2\pi + \cancel{2\pi}(2\pi-1)\ln\pi$

$= (2\pi-1)(\ln 2\pi + \ln\pi)$

b. $\int_0^{\pi}\int_0^{y} \frac{4y}{x^2+y^2}\,dx\,dy$



$\tan t = \frac{x}{y} \rightarrow t = \arctan\left(\frac{x}{y}\right)$

$x = y \tan t$

$\frac{dx}{dt} = y \sec^2 t$

$dx = y \sec^2 t \, dt$

$\int_0^{\pi}\int_0^{y} \frac{4y}{x^2+y^2}\,dx\,dy$

$\int_0^{\pi}\int_0^{y} \frac{4y}{y^2+y^2\tan^2 t}\; y\sec^2 t \cdot dt\,dx$;

$\int_0^{\pi}\int_0^{y} \frac{4\cancel{y}\;\cancel{y}\,\cancel{\sec^2 t}}{\cancel{y^2}\,\cancel{(1+\tan^2 t)}} \cdot dt\,dx$; $1+\tan^2 t = \sec^2 t$

$\int_0^{\pi}\int_0^{y} 4 \cdot dt\,dx = \int_0^{\pi} 4t \Big]_0^{y} \cdot dy$

$= \int_0^{\pi} 4\left(\arctan \frac{x}{y}\right)\Big]_0^{y} dy$

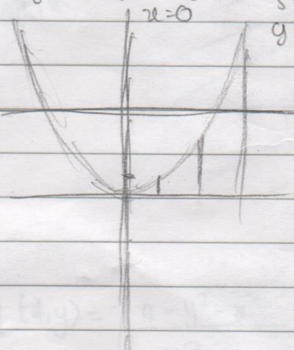
$= 4\int_0^{\pi} \arctan 1 - \arctan 0 \; dy$

$= \cancel{4}\int_0^{\pi} \frac{\pi}{\cancel{4}}\, dy$

$\pi \int_0^{\pi} 1\,dy = \pi \; y\Big]_0^{\pi}$

$= \pi^2$

5 $V = \iint_S \frac{y^4}{\sqrt{x^2y^2+3y^6}}\,dy\,dx = \iint_S \frac{y^{\cancel{4}\,3}}{\cancel{y}\sqrt{x^2+3y^4}}\,dy\,dx$ $\quad y = x^2$, $y = 6$, $x = 0$



$V = \int_0^{\sqrt{6}}\int_0^{\sqrt{x}} \frac{y^3}{\sqrt{3y^4+x^2}}\,dy\,dx$;

$u = 3y^4+x^2$

$\frac{du}{dy} = 12y^3$

$\frac{du}{12y^3} = dy$

$\int_0^{6}\int_0^{\sqrt{x}} u^{-1/2}\,\frac{du}{12}\,dx$

$\int_0^{6} \frac{2\sqrt{u}}{12\,6}\Big]_0^{\sqrt{x}} dx$

$\int_0^{6} \frac{\sqrt{3y^4+x^2}}{6}\Big]_0^{\sqrt{x}} dx$

$\int_0^{6} \frac{\sqrt{4x^2}-\sqrt{x^2}}{6}\,dx$

$$\int_0^6 \frac{2x-x}{6}\,dx = \int_0^6 \frac{x}{6}\,dx$$

$$= \frac{1}{6}\int_0^6 x\,dx$$

$$= \frac{1}{12}\Big[x^2\Big]_0^6$$

$$= \frac{1}{\cancel{12}}\,\cancel{36}\,3$$

$$= 3$$