Pembahasan Soal UTS Metode Numerik 2017/2018

1.) a) Metode Bisection untuk mencari akar persamaan $6x^3-13x^2+9=0$ pada interval $[1,2]$ sampai iterasi ke-4

Iterasi 1 :

$a = 1$

$b = 2$

$c = \frac{2+1}{2} = 1,5$

$f(a) = f(1) = 6.1^3 - 13.1^2 + 9 = 2$

$f(b) = f(2) = 6.2^3 - 13.2^2 + 9 = 5$

$f(c) = f(1,5) = 6.(1,5)^3 - 13.(1,5)^2 + 9 = 0$

Dari perhitungan di atas $f(c)$ bernilai 0 dengan $c = 1,5$. Hal itu menunjukkan bahwa $c = 1,5$ adalah akar dari persamaan $6x^3-13x^2+9=0$.

b) Metode Newton untuk mencari akar persamaan $6x^3-13x^2+9=0$ dengan nilai awal $x_0 = 1$ sampai iterasi ke-4.

$f(x) = 6x^3 - 13x^2 + 9 = 0$

$f'(x) = 18x^2 - 26x$

Iterasi 1 :

$x_0 = 1$

$f(1) = 6.1^3 - 13.1^2 + 9 = 2$

$f'(1) = 18.1^2 - 26.1 = -8$

$x_1 = 1 - \left(-\frac{2}{8}\right) = 1,25$

Iterasi 2 :

$f(1,25) = 6.(1,25)^3 - 13.(1,25)^2 + 9 = 0,40625$

$f'(1,25) = 18.(1,28)^2 - 26.(1,25)$

$= -10$

$x_2 = 1,25 - \left(-\frac{0,40625}{10}\right) = 1,290625$

Iterasi 3 :

$f(1,290625) = 6.(1,290625)^3 - 13(1,290625)^2 + 9$

$= 0,24459$

$f'(1,290625) = 18(1,290625)^2 - 26.(1,290625)$

$= -3,57341$

$x_3 = 1,290625 - \left(-\frac{0,24459}{3,57341}\right)$

$= 1,35907$

Iterasi 4 :

$f(1,35907) = 6(1,35907)^3 - 13(1,35907)^2 + 9$

$= 0,04986$

$f'(1,35907) = 18(1,35907)^2 - 26(1,35907)$

$= -2,08853$

$x_4 = 1,35907 - \left(-\frac{0,04986}{2,08853}\right)$

$= 1,38294.$

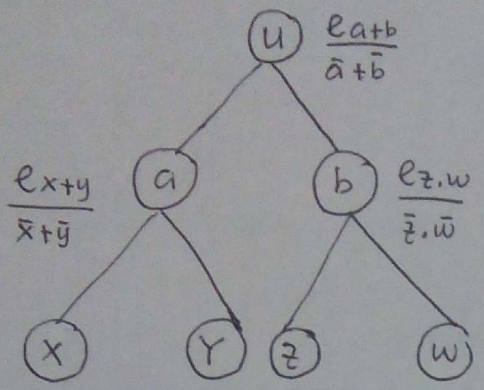
$\therefore$ Setelah iterasi ke-4 didapatkan nilai $x = 1,38294$

2.) a) Buatlah Diagram proses untuk ekspresi $u = (X + Y) + (Z * W)$

~~A~~ misal $X + Y = a$

$Z * W = b$, maka

$u = a + b$



b.) Tentukan batas atas kesalahan relative dengan aturan pemangkasan (relatif) dari ekspresi $u = (X+Y) + (Z*W)$ dengan X, Y, Z, dan W tidak eksak.

* menggunakan aturan pemangkasan (relatif)

$$|a| = \left|\frac{e_{x+y}}{\bar{x}+\bar{y}}\right| = \left|\frac{\bar{x}}{\bar{x}+\bar{y}} \cdot \frac{e_x}{\bar{x}} + \frac{\bar{y}}{\bar{x}+\bar{y}} \cdot \frac{e_y}{\bar{y}} + r_1\right|$$

$$\leq \left|\frac{\bar{x}}{\bar{x}+\bar{y}} \cdot 10.10^{-t}\right| + \left|\frac{\bar{y}}{\bar{x}+\bar{y}} \cdot 10.10^{-t}\right| + \left|10.10^{-t}\right|$$

$$\leq \left|\frac{\bar{x}+\bar{y}}{\bar{x}+\bar{y}} \cdot 10.10^{-t}\right| + \left|10.10^{-t}\right|$$

$$\leq 10.10^{-t} + 10.10^{-t}$$

$$\leq 20.10^{-t}$$

$$|b| = \left|\frac{e_{z.w}}{\bar{z}.\bar{w}}\right| = \left|\frac{e_z}{\bar{z}} + \frac{e_w}{\bar{w}} + r_2\right|$$

$$\leq \left|10.10^{-t}\right| + \left|10.10^{-t}\right| + \left|10.10^{-t}\right|$$

$$\leq 30.10^{-t}$$

$$|u| = \left|\frac{e_{a+b}}{\bar{a}+\bar{b}}\right| = \left|\frac{\bar{a}}{\bar{a}+\bar{b}} \cdot \frac{e_a}{\bar{a}} + \frac{\bar{b}}{\bar{a}+\bar{b}} \cdot \frac{e_b}{\bar{b}} + r_3\right|$$

$$\leq \left|\frac{\bar{a}}{\bar{a}+\bar{b}} \cdot 20.10^{-t}\right| + \left|\frac{\bar{b}}{\bar{a}+\bar{b}} \cdot 30.10^{-t}\right| + \left|10.10^{-t}\right|$$

$$\leq 10.10^{-t}\left(\frac{2\bar{a}}{\bar{a}+\bar{b}} + \frac{3\bar{b}}{\bar{a}+\bar{b}} + \frac{\bar{a}+\bar{b}}{\bar{a}+\bar{b}}\right)$$

$$\leq 10.10^{-t} \cdot \left(\frac{3\bar{a}+4\bar{b}}{\bar{a}+\bar{b}}\right)$$

3.) Diberikan nilai-nilai x dan f(x)

x	0,2	0,4	0,5	0,7	0,9
f(x)	1,8534	1,9057	2,1374	2,2448	2,3649

a.) Tentukan p(x) dengan interpolasi Newton order 3.

$a_0 = f[x_0] = f(x_0) = y_0 = 1{,}8534$

$a_1 = f[x_0,x_1] = \dfrac{f[x_1]-f[x_0]}{x_1-x_0} = \dfrac{f(x_1)-f(x_0)}{x_1-x_0} = \dfrac{y_1-y_0}{x_1-x_0} = \dfrac{1{,}9057-1{,}8534}{0{,}4-0{,}2}$

$= \dfrac{0{,}0523}{0{,}2} = 0{,}2615$

$f[x_1,x_2] = \dfrac{f[x_2]-f[x_1]}{x_2-x_1} = \dfrac{f(x_2)-f(x_1)}{x_2-x_1} = \dfrac{2{,}1374-1{,}9057}{0{,}5-0{,}4}$

$= \dfrac{0{,}2317}{0{,}1} = 2{,}317$

$a_2 = f[x_0,x_1,x_2] = \dfrac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \dfrac{2{,}317-0{,}2615}{0{,}5-0{,}2}$

$= \dfrac{2{,}0555}{0{,}3} = 6{,}85167$

$f[x_2,x_3] = \dfrac{f[x_3]-f[x_2]}{x_3-x_2} = \dfrac{f(x_3)-f(x_2)}{x_3-x_2} = \dfrac{2{,}2448-2{,}1374}{0{,}7-0{,}5}$

$= \dfrac{0{,}1074}{0{,}2} = 0{,}537$

$f[x_1,x_2,x_3] = \dfrac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_1} = \dfrac{0{,}537-2{,}317}{0{,}7-0{,}4}$

$= \dfrac{-1{,}78}{0{,}3} = -5{,}933$

$a_3 = f[x_0,x_1,x_2,x_3] = \dfrac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0} = \dfrac{-5{,}933-6{,}85167}{0{,}7-0{,}2}$

$= \dfrac{-12{,}78467}{0{,}5} = -25{,}56934$

$P(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2)$

$= 1{,}8534 + 0{,}2615(x-0{,}2) + 6{,}85167(x-0{,}2)(x-0{,}4) + (-25{,}56934)(x-0{,}2)$
$(x-0{,}4)(x-0{,}5)$

$= 1{,}8534 + 0{,}2615(x-0{,}2) + 6{,}85167(x^2-0{,}6x+0{,}08) + (-25{,}56934)(x^3-1{,}1x^2-0{,}38x-0{,}04)$

b.) Tentukan nilai dari p(0,25)

$p(0{,}25) = 1{,}8534 + 0{,}2615(0{,}25-0{,}2) + 6{,}85167((0{,}25)^2-0{,}6.(0{,}25)+0{,}08) - 25{,}56934$
$((0{,}25)^3 - 1{,}1(0{,}25)^2 - 0{,}38.(0{,}25) - 0{,}04)$

$= 1{,}8534 + 0{,}2615 \cdot 0{,}05 + 6{,}85167 \cdot (-0{,}0075) - 25{,}56934 \cdot (-0{,}188125)$

$= 1{,}8534 + 0{,}013075 - 0{,}05138 + 4{,}81023$

$= 6{,}62532$

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