1. a. $\int \frac{x}{4-x^2}\,dx$

$u = 4-x^2$

$\frac{du}{dx} = -2x \rightarrow \frac{du}{-2x} = dx$

$= \int \frac{x}{u} \cdot \frac{du}{-2x} = -\frac{1}{2}\int \frac{1}{u}\,du$

$= -\frac{1}{2}(\ln(u)) + C$

$= -\frac{1}{2}\ln|u| + C$

$= -\frac{1}{2}\ln|4-x^2| + C$

b. $\int_0^5 \frac{1}{\sqrt{x+4}}\,dx$

$u = x+4$

$\frac{du}{dx} = 1 \rightarrow du = dx$

$= \int_0^5 \frac{1}{\sqrt{u}}\,du$

$= \int_0^5 u^{-\frac{1}{2}}\,du$

$= \int_0^5 2u^{\frac{1}{2}} = \int_0^5 2\sqrt{x+4}$

$= 2\sqrt{x+4}\,\Big]_0^5 = 2\sqrt{5+4} - 2\sqrt{0+4}$

$= 2\sqrt{9} - 2\sqrt{4}$

$= 6 - 4 = 2$ //

2. a. $\int \frac{\sin^2 x \cos^4 x}{\tan x}\,dx$

$= \int \frac{\sin^2 x \cos^4 x}{\frac{\sin x}{\cos x}}\,dx$

$= \int \sin^2 x \cos^4 x \cdot \frac{\cos x}{\sin x}\,dx$

$= \int \sin x \cos^5 x\,dx$

$= \int \sin x \cos^5 x \cdot \frac{1}{-\sin x}\,dx$

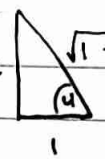
$= \int -\cos^5 x\,dx \rightarrow u = \cos x$

$= \int -u^5\,du$

$= -\frac{1}{6}u^6 + C = -\frac{\cos^6 x}{6} + C$ //

b. $\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

$x = \tan u$



$dx = \frac{1}{\cos^2 u}\,du$

$= \int \frac{1}{\tan^2 u\sqrt{1+\tan^2 u}} \cdot \frac{1}{\cos^2 u}\,du$

$= \int \frac{1}{\frac{\sin^2 u}{\cos^2 u}\sqrt{\frac{1}{\cos^2 u}}} \cdot \frac{1}{\cos^2 u}\,du$

$= \int \frac{\cos u}{\sin^2 u}\,du \rightarrow t = \sin u$

$dt = \cos u\,du$

$= \int \frac{1}{t^2}\,dt = \int t^{-2}\,dt$

$= -t^{-1} + C = -\frac{1}{\sin u} + C$

$= -\frac{1}{\sin(\arctan(x))} + C = -\frac{1}{\frac{x}{\sqrt{1+x^2}}} + C$

$= -\frac{\sqrt{1+x^2}}{x} + C$ //

c. $\int x\,e^{x^2}\,dx$ $\quad t = e^{x^2}$

$= \int x \cdot e^{x^2} \cdot \frac{1}{e^{x^2} \cdot 2x}\,dt$

$= \int e^{x^2} \cdot \frac{1}{e^{x^2} \cdot 2}\,dt$

$= \int \frac{1}{2}\,dt = \frac{1}{2}t + C = \frac{1}{2}e^{x^2} + C$ //

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d. $\int \frac{x}{x^2+x-2}\,dx$

$\int \frac{x}{(x+2)(x-1)}\,dx$

$\frac{x}{x^2+x-2} = \frac{A}{(x+2)} + \frac{B}{(x-1)}$

$x = A(x-1) + B(x+2)$

$x = Ax - A + Bx + 2B$

$x = x(A+B) + (-A+2B)$

$A + B = 1$

$-A + 2B = 0$

$3B = 1$

$B = \frac{1}{3}$, $A = \frac{2}{3}$

$\Rightarrow \int \frac{x}{x^2+x-2}\,dx = \int \frac{2/3}{(x+2)}\,dx + \int \frac{1/3}{(x-1)}\,dx$

$= \frac{2}{3}\ln|x+2| + \frac{1}{3}\ln|x-1| + C$ //

3. Daerah D dibatasi kurva $y = \sqrt{x}$, garis $y = x-6$, dan garis $y = 0$. Sketsalah daerah D dan cari luasnya!

$\Rightarrow y_1 = \sqrt{x}$

$y_2 = x-6$

$y = 0$

$\Rightarrow y_1 = y_2$

$(\sqrt{x})^2 = (x-6)^2$

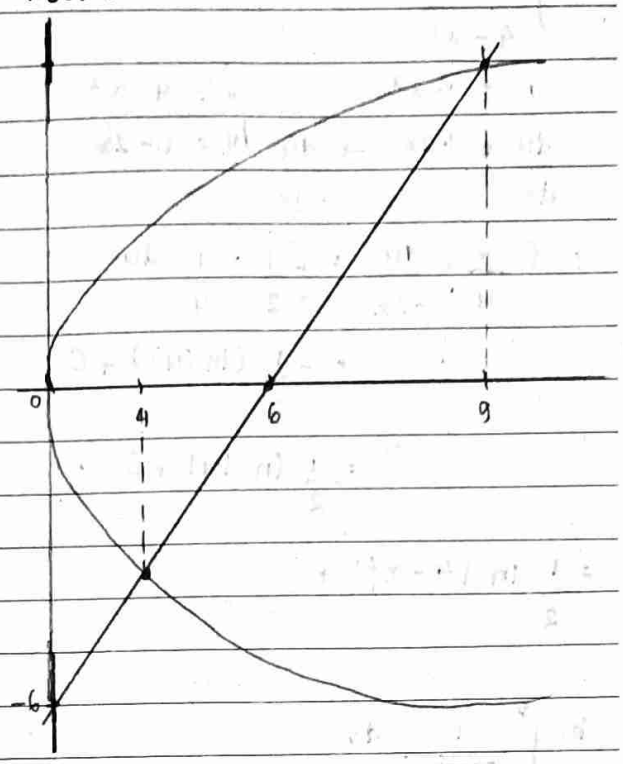
$x = x^2 - 12x + 36$

$0 = x^2 - 13x + 36$

$0 = (x-4)(x-9)$

$x_1 = 4 \vee x_2 = 9$

• Sketsa



• Luas daerah:

$L = \int_0^6 y_1\,dx + \int_6^9 (y_1 - y_2)\,dx$

$= \int_0^6 \sqrt{x}\,dx + \int_6^9 (\sqrt{x} - x + 6)\,dx$

$= \frac{2}{3}x\sqrt{x}\Big]_0^6 + \frac{2}{3}x\sqrt{x} - \frac{x^2}{2} + 6x\Big]_6^9$

$= (4\sqrt{6}) + \frac{2}{3}(27 - 6\sqrt{6}) - \frac{1}{2}(81-36) +$

$6(9-6)$

$= 4\sqrt{6} + 18 - 4\sqrt{6} - \frac{1}{2}.45 + 18$

$= 13,5$ satuan luas //

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4. Pada soal 3, jika daerah D diputar terhadap sumbu y satu kali putaran, tentukan volume benda putarnya!

=) $\Delta V_1 = \pi . y_1^2 \Delta y$

$= \pi (y+6)^2 . \Delta y$

$\Delta V_2 = \pi . y_2^2 . \Delta y$

$= \pi y^2 . \Delta y$

$V = \pi \int_0^3 (y+6)^2 - (y^2)^2 dy$

$= \pi \int_0^3 y^2 + 12y + 36 - y^4 \, dy$

$= \pi \int_0^3 (\frac{1}{3}y^3 + 6y^2 + 36y - \frac{1}{5}y^5)$

$= \pi (\frac{1}{3}y^3 + 6y^2 + 36y - \frac{1}{5}y^5) \Big]_0^3$

$=$

$= \pi (9 + 54 + 108 - \frac{243}{5}) - 0$

$= \pi (171 - 48,6)$

$= 122,4$ satuan volume //