① a) $y^3+7y-x^3=0$

$3y^2\frac{dy}{dx}+7\frac{dy}{dx}-3x^2=0$

$\frac{dy}{dx}=\frac{3x^2}{3y^2+7}$ //

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{3x^2}{3y^2+7}\right)$

$=3\frac{d}{dx}\left(\frac{x^2}{3y^2+7}\right)$

$\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$

sussitusi $\frac{dy}{dx}=\frac{3x^2}{3y^2+7}$ $\rightarrow$ $=3.\frac{(2x)(3y^2+7)-(6y\frac{dy}{dx})(x^2)}{(3y^2+7)^2}=\frac{3(2x(3y^2+7)^2-18yx^2}{(3y^2+7)^2}$

b.) $x^3y^4-1=0$

$4x^3y^3\frac{dy}{dx}+3x^2y^4=0$

$\frac{dy}{dx}=\frac{-3x^2y^4}{4x^3y^3}=-\frac{3y}{4x}$

$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(-\frac{3y}{4x}\right)$

$=-\frac{3}{4}\frac{d}{dx}\left(\frac{y}{x}\right)$

$=-\frac{3\left(\left(\frac{dy}{dx}\right)(x)-y\right)}{4x^2}$

sussitusi $\frac{dy}{dx}=-\frac{3y}{4x}$ $\rightarrow$ $=-\frac{3\left(-\frac{3y}{4x}\cdot x-y\right)}{4x^2}=\frac{21y}{16x^2}$ //

c.) $y=\sqrt{\sin(xy^2)}$ *misal $u=\sin(xy^2)$ $\rightarrow$ aturan rantai $\frac{dF(u)}{dx}=\frac{dF}{du}\cdot\frac{du}{dx}$

$\frac{dy}{dx}=\frac{1}{2\sqrt{\sin(xy^2)}}\cos(xy^2)\left(y^2+2xy\frac{dy}{dx}\right)$

$\left(\frac{dy}{dx}\right)\left(2\sqrt{\sin(xy^2)}\right)=y^2\cos(xy^2)+2xy\cos(xy^2)\frac{dy}{dx}$

$\left(\frac{dy}{dx}\right)\left(2\sqrt{\sin(xy^2)}\right)-2xy\cos(xy^2)\frac{dy}{dx}=y^2\cos(xy^2)$

$\frac{dy}{dx}=\frac{y^2\cos(xy^2)}{2\sqrt{\sin(xy^2)}-2xy\cos(xy^2)}$ //

$\frac{d^2y}{dx^2}=\ldots\ldots?$

2 a.) Titik ekstrim $f(x) = -2x^3 + 3x^2$ pada $[-\frac{1}{2}, 2]$

* $f'(x) : -6x^2 + 6x$

* titik ekstrim

$-6x^2 + 6x = 0$

$x = 1 \vee x = 0$

$f(0) = 0$ $\quad$ $f(-\frac{1}{2}) = 1$

$f(1) = 1$ $\quad$ $f(2) = -4$

maximum $\quad$ mimimum

b.) T. ekstrim $f(x) = x^{2/3}$ pada $[-1, 2]$

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

* T. ekstrim

$\frac{2}{3} x^{-\frac{1}{3}} = 0$

$x = 0$

$f(-1) = \sqrt[3]{(-1)^2} = 1$ $\quad$ $f(2) = \sqrt[3]{(2)^2} = 1{,}5874$ max $\quad$ $f(0) = \sqrt[3]{(0)^2} = 0$ min

3 $(x-4)(y-8) = 50$ $\quad$ $L = xy$

$xy - 4y - 8x + 32 = 50$

$y = \frac{18 + 8x}{x - 4}$

* masukkan y ke L

$L = (x)\left(\frac{18 + 8x}{x - 4}\right)$

$L = \frac{18x + 8x^2}{x - 4}$

* $\frac{dL}{dx} = \frac{(18 + 16x)(x-4) - (1)(18x + 8x^2)}{(x-4)^2}$

$= \frac{8x^2 - 64x - 72}{(x-4)^2}$

$= \frac{8(x^2 - 8x - 9)}{(x-4)^2} = \frac{8(x-9)(x+1)}{(x-4)^2} \Rightarrow \boxed{x = 9}$

$(9-4)(y-8) = 50$

$y - 8 = 10$

$\boxed{y = 18}$

4) a) $\int \left(\frac{4}{x^5} - \frac{3}{x^4}\right) dx$

$\int (4x^{-5} - 3x^{-4})\, dx$

$= -x^{-4} + x^{-3}$

$= \frac{1}{x^3} - \frac{1}{x^4} + C$

b.) $\int 3t^3 \sqrt{2t^2-1}\, dx$

※ Tidak ada solusi karna dx

※ Jika dt

$\int 3t^3 \sqrt{2t^2-1}\, dt$

* $u = 2t^2 - 1$

$\frac{du}{dt} = 4t$

$\frac{du}{4} = t\,dt$

$= 3 \int \frac{\sqrt[3]{u}}{4}\, du$

$= \frac{3}{4} \int u^{\frac{1}{3}} = \frac{3}{4} \cdot \frac{3}{4} u^{\frac{4}{3}}$

$= \frac{9}{16} \sqrt[3]{(2t^2-1)^4} + C$

5) a.) $y = x+6$, $y = x^3$, $2y + x$

$[(2,8), (0,0), (-4,2)] \rightarrow$ titik temu

$\int_{-4}^{2} x + 6 - x^3 + \frac{x}{2}$

$\left[\frac{x^2}{2} + 6x - \frac{x^4}{4} + \frac{x^2}{4}\right]_{-4}^{2}$

$= 87$

b.) $y = \sqrt{x}$, sumbu y, garis $y = 0$, $y = 1$

$\int_0^1 \sqrt{x}\, dx$

$= \left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^1$

$= \frac{2}{3}$