PEMBAHASAN UTS I DISKRIT 2017/2018 <IFFA ZN>

1) $\left(\frac{1}{2}x + \frac{1}{3}y\right)^5 = \sum_{k=0}^{5} C_k^5 \left(\frac{1}{2}x\right)^{5-k} \left(\frac{1}{3}y\right)^k$

>> $k=4$

$\frac{5 \times \cancel{4!}}{\cancel{4!}\, 1!} * \left(\frac{1}{2}x\right) * \left(\frac{1}{3}y\right)^4 = \frac{5}{2} * \frac{1}{81} xy^4 = \frac{5}{162} xy^4$

>> $k=3$

$\frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{3!}\, \cancel{2!}} * \left(\frac{1}{2}x\right)^2 * \left(\frac{1}{3}y\right)^3 = \frac{\cancel{10}^{5}}{\cancel{4}\,2} * \frac{1}{27} x^2y^3 = \frac{5}{54} x^2y^3$

>> $k=2$

$\frac{5 \times \overset{2}{\cancel{4}} \times \cancel{3!}}{\cancel{2!}\, \cancel{3!}} * \left(\frac{1}{2}x\right)^3 * \left(\frac{1}{3}y\right)^2 = \frac{\cancel{10}^{5}}{\cancel{8}4} * \frac{1}{9} x^3y^2 = \frac{5}{36} x^3y^2$

Jumlahan koef. pangkat $\leq 3 = \frac{5}{162} + \frac{5}{54} + \frac{5}{36} = \frac{10+30+45}{324} = \frac{85}{324}$

2) $R_1 = \{(a,b) \mid a \equiv b \pmod 3\} \rightarrow b = 3k+a,\ k \in \mathbb{Z}$

$R_2 = \{(a,b) \mid a \equiv b \pmod 4\} \rightarrow b = 4k+a,\ k \in \mathbb{Z}$

>> $R_1 \cap R_2 = \{(a,b) \mid a \equiv b \pmod 3\} \wedge \{(a,b) \mid a \equiv b \pmod 4\} = \{(a,b) \mid a \equiv b \pmod{12}\}$

>> $R_1 - R_2 = \{(a,b) \mid a \equiv b \pmod 3\} - \{(a,b) \mid a \equiv b \pmod 4\}$

$= \{(a,b) \mid b = 3k+a,\ k \in \mathbb{Z}\} - \{(a,b) \mid b = 4k+a,\ k \in \mathbb{Z}\}$

$= \{(a,b) \mid b = 3k+a,\ k \bmod 4 \neq 0,\ k \in \mathbb{Z}\}$

>> $R_1 \oplus R_2 = (R_1 \cup R_2) - (R_1 \cap R_2)$

$= \{(a,b) \mid b = 3k+a,\ b = 4k+a,\ k \in \mathbb{Z}\} - \{(a,b) \mid a \equiv b \pmod{12}\}$

$= \{(a,b) \mid b = 3k+a,\ b = 4k+a,\ k \bmod 12 \neq 0,\ k \in \mathbb{Z}\}$

3) $\sum_{j=2}^{n} \frac{1}{j^2-1} = \frac{(n-1)(3n+2)}{4n(n+1)}$

>> Basis $(n=2)$

$\frac{1}{2^2-1} = \frac{(2-1)(3.2+2)}{4.2(2+1)}$

$\frac{1}{3} = \frac{8}{24}$

$\frac{1}{3} = \frac{1}{3}$ (BENAR)

>> Langkah Induksi $(n=k)$

$\sum_{2}^{k} \frac{1}{k^2-1} = \frac{(k-1)(3k+2)}{4k(k+1)}$

Akan ditunjukkan $(n=k+1)$

$\sum_{2}^{k} = \frac{1}{(k+1)^2-1} = \frac{k(3k+5)}{4(k+1)(k+2)}$

Bukti :

$\frac{(k-1)(3k+2)}{4k(k+1)} + \frac{1}{(k+1)^2-1} = \frac{(k-1)(3k+2)k(k+2) + 4k(k+1)}{4k(k+1) \cdot k(k+2)}$

$= \frac{(3k^2-k-2)(k^2+2k) + 4k^2+4k}{4k(k+1) \cdot k(k+2)}$

$= \frac{3k^4+6k^3-k^3-2k^2-2k^2-4k+4k^2+4k}{4k(k+1) \cdot k(k+2)}$

$= \frac{k^{\cancel{3}}(3k+5)}{4\cancel{k}(k+1)\cancel{k}(k+2)} = \frac{k(3k+5)}{4(k+1)(k+2)}$ (BENAR)

Jadi, $P(k+1)$ benar

$$\qquad\qquad\qquad\quad\downarrow f(k)$$

4) $a_k = 3a_{k-1} + \boxed{2}$, $a_0 = 1$

a) >> Solusi homogen (h) dan partikulir (p)

$r^k = 3r^{k-1} \quad (: r^{k-1})$

$r = 3$

$a_k^{(h)} = C_1 \cdot 3^k$

>> karena $f(k) = 2$ maka

$a_k^{(p)} = B_1$

$= 3(B_1(k-1)) + 2$

$= B_1(3k-3) + 2$

$B_1 = -3 + 2 = -1$

$a_k = a_k^{(h)} + a_k^{(p)}$

$= C_1 \cdot 3^k - 1$

>> Mencari nilai $C_1$

$k = 0 \rightarrow 1 = C_1 \cdot 3^0 - 1$

$C_1 = 2$

Maka, $a_k = 2 \cdot 3^k - 1$ //

b) Fungsi pembangun

$a_k = 3a_{k-1} + 2$

$= 3(3a_{k-2} + 2) + 2$

$= 3^2 a_{k-2} + 3 \cdot 2 + 2$

$= 3^2(3a_{k-3} + 2) + 3 \cdot 2 + 2$

$= 3^3 a_{k-3} + 3^2 \cdot 2 + 3 \cdot 2 + 2$

$\vdots$

dst

diperoleh :

$a_k = 3^{k-1} a_1 + 3^{k-2} + \ldots + 3^2 \cdot 2 + 3 \cdot 2 + 2$

$= 3^{k-1} + 3^{k-2} + \ldots + 3^2 \cdot 2 + 3 \cdot 2 + 2$

$= 3^k \cdot 2 - 1$