1. invers Matriks menggunakan Metode Gauss-Jordan

$$A = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{-2}{5} \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{10} \\ \frac{1}{5} & \frac{-4}{5} & \frac{1}{10} \end{bmatrix}$$

$$\rightarrow \quad A^{-1} = \left[\begin{array}{ccc|ccc} 1/5 & 1/5 & -2/5 & 1 & 0 & 0 \\ 1/5 & 1/5 & 1/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{H_1(5)} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 1/5 & 1/5 & 1/10 & 0 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right] \xrightarrow{H_{21}(-1/5)} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \\ 1/5 & -4/5 & 1/10 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{H_{31}(-1/5)} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \\ 0 & -1 & 1/2 & -1 & 0 & 1 \end{array}\right] \xrightarrow{H_{23}} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & -1 & 1/2 & -1 & 0 & 1 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{H_2(-1)} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 5 & 0 & 0 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \end{array}\right] \xrightarrow{H_{12}(-1)} \left[\begin{array}{ccc|ccc} 1 & 0 & -3/2 & 4 & 0 & 1 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{H_3(2)} \left[\begin{array}{ccc|ccc} 1 & 0 & -3/2 & 4 & 0 & 1 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right] \xrightarrow{H_{13}(3/2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & -1/2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{H_{23}(1/2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 3 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 2 & 0 \end{array}\right]$$

$\therefore$ Jadi hasilnya $\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & -1 \\ -2 & 2 & 0 \end{bmatrix}$

(2) $x + 3y + z = 4$

$2x + 2y + z = -1$

$2x + 3y + z = 3$

SPL Menggunakan Gauss-Jordan

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 2 & 2 & 1 & -1 \\ 2 & 3 & 1 & 3 \end{array}\right] \xrightarrow{H_{21}(-2)} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & -4 & -1 & -9 \\ 2 & 3 & 1 & 3 \end{array}\right] \xrightarrow{H_{31}(-2)} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & -4 & -1 & -9 \\ 0 & -3 & -1 & -5 \end{array}\right]$$

$$\xrightarrow{H_2(-1/4)} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 4 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & -3 & -1 & -5 \end{array}\right] \xrightarrow[H_{32}(3)]{H_{12}(-3)} \left[\begin{array}{ccc|c} 1 & 0 & 1/4 & -11/4 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & 0 & -1/4 & 7/4 \end{array}\right]$$

$$\xrightarrow{H_3(-4)} \left[\begin{array}{ccc|c} 1 & 0 & 1/4 & -11/4 \\ 0 & 1 & 1/4 & 9/4 \\ 0 & 0 & 1 & -7 \end{array}\right] \xrightarrow[H_{23}(-1/4)]{H_{13}(-1/4)} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

∴ didapat $r(A) = 3$, $r(A,B) = 3$, dan $n=3$

karena $r(A) = r(A,B) = n$, maka SPL diatas solusi tunggal.

SPL memiliki solusi konsisten tunggal $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$