PEMBAHASAN

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1. Diketahui $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

   a. Tentukan eigenvalue dan eigenvektor dari matrik A tersebut

   - Menentukan eigenvalue

     $|A - \lambda I| = 0$

     $\left| \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$

     $\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$

     $((1-\lambda)(1-\lambda)(1-\lambda)+1+1)-((1-\lambda)+(1-\lambda)+(1-\lambda)) = 0$

     $(-\lambda^3+3\lambda^2-3\lambda+3)-(3-3\lambda) = 0$

     $-\lambda^3+3\lambda^2 = 0$

     $\Lambda^2(-\lambda+3) = 0$

     $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 3$

     Jadi eigenvaluenya adalah 0, atau 3

   - Menentukan eigenvector

     Eigenvector untuk $\lambda_1 = 0, \lambda_2 = 0$

     $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} H_{31(-1)} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} H_{21(-1)} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

     Misal : $u_1 = s, u_2 = t$

     $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = s \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

     Jadi eigenvector $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ dan $v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

     Eigenvector untuk $\lambda_3 = 3$

     $\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} H_{32(-1)} \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 0 & 3 & -3 \end{bmatrix} H_{21(1/2)} \begin{bmatrix} -2 & 1 & 1 \\ 0 & -\frac{3}{2} & \frac{3}{2} \\ 0 & 3 & -3 \end{bmatrix} H_{2(2)} \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} H_{32(1)}$

$$\begin{bmatrix}-2 & 1 & 1\\ 0 & -3 & 3\\ 0 & 0 & 0\end{bmatrix} H_{2(1/3)} \begin{bmatrix}-2 & 1 & 1\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix} H_{12(1)} \begin{bmatrix}-2 & 0 & 2\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix} H_{1(1/2)} \begin{bmatrix}-1 & 0 & 1\\ 0 & -1 & 1\\ 0 & 0 & 0\end{bmatrix}$$

Misal : $u_3 = s$

$$\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} = s\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$

Jadi eigenvector $v_3 = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$

b. Tentukan basis dari masing-masing eigenspacenya

Basis untuk eigenspace λ = 0 adalah $\left\{\begin{bmatrix}1\\ 0\\ -1\end{bmatrix}, \begin{bmatrix}0\\ 1\\ -1\end{bmatrix}\right\}$

Basis untuk eigenspace λ = 3 adalah $\left\{\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}\right\}$

c. Tentukan basis orthonormal dari R^3 yang berkaitan dengan eigenvector dari matrik A tersebut

$w_1 = \frac{v1}{|v1|} = \frac{(1,0,-1)}{\sqrt[2]{1^2+0^2+-1^2}} = (\frac{1}{2}\sqrt[2]{2},0,-\frac{1}{2}\sqrt[2]{2})$

$w_2 = \frac{v2-(v2\bullet w1)w1}{|v2-(v2\bullet w1)w1|} = \frac{(0,1,-1)-\left((0,1,-1)\bullet\left(\frac{1}{2}\sqrt[2]{2},0,-\frac{1}{2}\sqrt[2]{2}\right)\right)\left(\frac{1}{2}\sqrt[2]{2},0,-\frac{1}{2}\sqrt[2]{2}\right)}{|(0,1,-1)-\left((0,1,-1)\bullet\left(\frac{1}{2}\sqrt[2]{2},0,-\frac{1}{2}\sqrt[2]{2}\right)\right)\left(\frac{1}{2}\sqrt[2]{2},0,-\frac{1}{2}\sqrt[2]{2}\right)|} = (-\frac{1}{14}\sqrt[2]{14},\frac{1}{7}\sqrt[2]{14},\frac{3}{14}\sqrt[2]{14})$

$w_3 = \frac{v3}{|v3|} = \frac{(1,\ 1,\ 1)}{\sqrt[2]{1^2+1^2+1^2}} = (\frac{1}{3}\sqrt[2]{3}, \frac{1}{3}\sqrt[2]{3}, \frac{1}{3}\sqrt[2]{3})$

Jadi basis orthonormal adalah $\left\{\begin{bmatrix}\frac{1}{2}\sqrt[2]{2}\\ 0\\ -\frac{1}{2}\sqrt[2]{2}\end{bmatrix}, \begin{bmatrix}-\frac{1}{14}\sqrt[2]{140}\\ \frac{1}{7}\sqrt[2]{14}\\ \frac{3}{14}\sqrt[2]{14}\end{bmatrix}, \begin{bmatrix}\frac{1}{3}\sqrt[2]{3}\\ \frac{1}{3}\sqrt[2]{3}\\ \frac{1}{3}\sqrt[2]{3}\end{bmatrix}\right\}$

d. Tunjukkan $P^{-1}AP = D$ (D adalah matrik diagonal dengan elemen diagonal merupakan eigenvalue)

$D = P^{-1}AP$

$$= \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ -1 & -1 & 1\end{bmatrix}^{-1} \begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix} \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ -1 & -1 & 1\end{bmatrix}$$

$$= \begin{bmatrix}\frac{2}{3} & -\frac{1}{3} & -\frac{1}{3}\\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3}\\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3}\end{bmatrix} \begin{bmatrix}0 & 0 & 3\\ 0 & 0 & 3\\ 0 & 0 & 3\end{bmatrix}$$

$$= \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 3\end{bmatrix}$$

2. Diketahui: T : $R^3 \rightarrow R^3$

T[x,y,z] = [0x+y+2z, x-y-z, 2x-3y-4z]

a. Tentukan prapeta dari [4,0,-4]

$$T\left[\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\right] = \begin{pmatrix} 0x + y + 2z \\ x - y - z \\ 2x - 3y - 4z \end{pmatrix}$$

$$T(e_1) = T\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$T(e_2) = T\left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

$$T(e_3) = T\left[\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

$$[T]e = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix}$$

Misal X = Prapeta dari [4,0,-4]

$$T \bullet X = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix} \bullet X = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$$

$$X = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$$

$$X = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

b. Tentukan peta dari [3,2,1]

$$T\left[\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}\right] = \begin{pmatrix} 0x + y + 2z \\ x - y - z \\ 2x - 3y - 4z \end{pmatrix}$$

$$T(e_1) = T\left[\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$

$$T(e_2) = T\left[\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}$$

$$T(e_3) = T\left[\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

$$[T]e = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix}$$

$$T\left[\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}\right] = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & -1 \\ 2 & -3 & -4 \end{bmatrix}\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -4 \end{pmatrix}$$

c. Tentukan basis dan dimensi dari ruang peta(Image) dari transformasi tersebut

Menentukan Im(T) dengan OKE

$$\begin{bmatrix}0 & 1 & 2\\ 1 & -1 & -1\\ 2 & -3 & -4\end{bmatrix} K_{32(-2)} \begin{bmatrix}0 & 1 & 0\\ 1 & -1 & 1\\ 2 & -3 & 2\end{bmatrix} K_{12(1)} \begin{bmatrix}1 & 1 & 0\\ 0 & -1 & 1\\ -1 & -3 & 2\end{bmatrix} K_{21(-1)} \begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 1\\ -1 & -2 & -2\end{bmatrix} K_{32(1)}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ -1 & -2 & -4\end{bmatrix} K_{3(-1/4)} \begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ -1 & -2 & 1\end{bmatrix} K_{12(-1/2)} \begin{bmatrix}1 & 0 & 0\\ \frac{1}{2} & -1 & 0\\ 0 & -2 & 1\end{bmatrix} K_{23(2)} \begin{bmatrix}1 & 0 & 0\\ \frac{1}{2} & -1 & 0\\ 0 & 0 & 1\end{bmatrix} K_{12(1/2)}$$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{bmatrix} K_{2(-1)} \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Basis Im(T) = {}

Dimensi Im(T) = 0

d. Tentukan basis dan dimensi dari ruang nol(Kernel) dari transformasi tersebut

Menentukan Ker(T) dengan OBE

$$\begin{bmatrix}0 & 1 & 2\\ 1 & -1 & -1\\ 2 & -3 & -4\end{bmatrix} H_{32(-2)} \begin{bmatrix}0 & 1 & 2\\ 1 & -1 & -1\\ 0 & -1 & -2\end{bmatrix} H_{12(1)} \begin{bmatrix}1 & 0 & 1\\ 1 & -1 & -1\\ 0 & -1 & -2\end{bmatrix} H_{21(-1)} \begin{bmatrix}1 & 0 & 1\\ 0 & -1 & -2\\ 0 & -1 & -2\end{bmatrix} H_{32(-1)}$$

$$\begin{bmatrix}1 & 0 & 1\\ 0 & -1 & -2\\ 0 & 0 & 0\end{bmatrix}$$

Misal : $u_3 = \lambda$

$$\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix} = \lambda \begin{bmatrix}-1\\ -2\\ 1\end{bmatrix}$$

Basis Ker(T) = $\left\{\begin{pmatrix}-1\\ -2\\ 1\end{pmatrix}\right\}$

Dimensi Ker(T) = 1