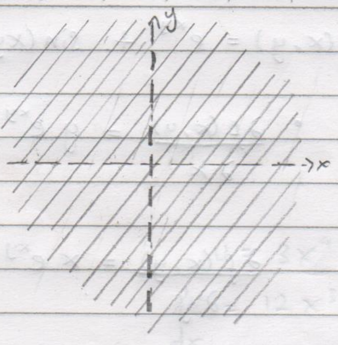
Mat II

VAS 2015 / 2016

1\. a. $f(x,y) = \frac{1}{x} + \frac{1}{y}$

Syarat : $x \neq 0$ dan $y \neq 0$

$D = \{(x,y) \in \mathbb{R}^2 \mid x \neq 0 \text{ dan } y \neq 0\}$



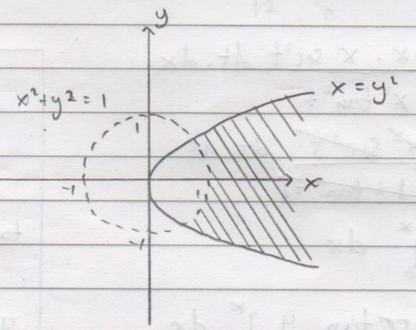
b. $g(x,y) = \sqrt{\frac{x - y^2}{1 - x^2 - y^2}}$

Syarat : $\cdot\ x - y^2 \geq 0$
$\cdot\ 1 - x^2 - y^2 > 0$ ①

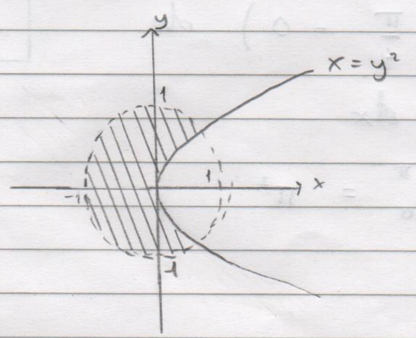
atau

$\cdot\ x - y^2 \leq 0$
$1 - x^2 - y^2 < 0$ ②

1\. $D = \{(x,y) \in \mathbb{R}^2 \mid x \geq y^2,\ 1 > x^2 + y^2\}$



2\. $D = \{(x,y) \in \mathbb{R}^2 \mid x \leq y^2,\ 1 \leq x^2 + y^2\}$



2. $f(x,y) = e^{xy} + \sin(xy) + xy + 2$

- $\dfrac{\partial f(x,y)}{\partial x} = y\,e^{xy} + y\sin(xy) + y$

- $\dfrac{\partial f(x,y)}{\partial y} = x\,e^{xy} + x\sin(xy) + x$

3. a. $\displaystyle\int_1^\pi\int_1^e \left(\frac{1}{x} + \frac{1}{y}\right) dx\,dy = \int_1^\pi \left(\ln(x) + \frac{x}{y}\right)\Big]_1^e dy$

$\displaystyle = \int_1^\pi \left(1 + \frac{e}{y}\right) - \left(0 + \frac{1}{y}\right) dy$

$\displaystyle = \int_1^\pi 1 + \frac{e}{y} - \frac{1}{y}\, dy$

$= y + e\ln(y) - \ln(y)\Big]_1^\pi$

$= \pi - 1 + e\ln(\pi) - \ln(\pi) = \pi - 1 + \ln(\pi)(e-1)$

b. $\displaystyle\int_0^\pi\int_0^x \frac{4x}{x^2+y^2}\,dy\,dx = \int_0^\pi\int_0^x \frac{4x \cdot x\sec^2 t\,dt \cdot dx}{x^2 + x^2\tan^2 t}$

$\displaystyle = \int_0^\pi\int_0^x \frac{4\cancel{x^2}\,\cancel{\sec^2 t}}{\cancel{x^2}\,\cancel{(1+\tan^2 t)}}\,dt\,dx$

$\displaystyle = \int_0^\pi 4t\Big]_0^x dx$

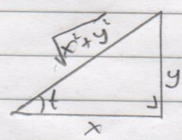
$\displaystyle = \int_0^\pi 4.\arctan\frac{y}{x}\Big]_0^x dx$

$\displaystyle = \int_0^\pi 4.(\arctan 1 - \arctan 0)\,dx$

$\displaystyle = \int_0^\pi 4\left(\frac{\pi}{4} - 0\right) dx$

$\displaystyle = \int_0^\pi \pi\,dx$

$= \pi x\Big]_0^\pi = \pi^2$



$\tan t = \dfrac{y}{x} \Rightarrow t = \arctan\dfrac{y}{x}$

$y = x\tan t$

$\dfrac{dy}{dt} = x\sec^2 t$

$dy = x\sec^2 t\,dt$

4. batas $x = \sqrt{y}$

$y = 6$

$x = 0$

$$V = \int_0^6 \int_0^{\sqrt{y}} \frac{x^3}{\sqrt{3x^4 + y^2}} \, dx \, dy$$

$$U = 3x^4 + y^2$$

$$\frac{du}{dx} = 12x^3$$

$$dx = \frac{du}{12x^3}$$

$$= \int_0^6 \int_0^{\sqrt{y}} \frac{\cancel{x^3}}{\sqrt{u}} \frac{du}{12\cancel{x^3}} \, dy$$

$$= \int_0^6 \sqrt{u} \, \frac{\cancel{2}}{\cancel{12}\,6} \Big]_0^{\sqrt{y}} \, dy$$

$$= \int_0^6 \frac{\sqrt{3x^4 + y^2}}{6} \Big]_0^{\sqrt{y}} \, dy = \int_0^6 \frac{2y}{6} - \frac{y}{6} \, dy$$

$$= \int_0^6 \frac{y}{6} \, dy$$

$$= \frac{y^2}{12} \Big]_0^6 = \frac{36}{12} = 3$$