

UAS ALIN 2019/2020 by Udin (Maaf kalau salah)

1) Himpunan Vektor A: $\{(2,1,4),(1,2,1),(5,4,9)\}$

Himpunan Vektor B: $\{(5,1,3),(2,0,1),(2,2,5)\}$

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 4 \\ 4 & 1 & 9 \end{bmatrix}$$

$$\det A = 36 + 16 + 5 - (40 + 8 + 9)$$

$$\det A = 0$$

Himpunan vektor A

Linear independen ($\det A = 0$)

$$B = \begin{bmatrix} 5 & 2 & 2 \\ 1 & 0 & 2 \\ 3 & 1 & 5 \end{bmatrix}$$

$$\det B = 0 + 12 + 2 - (0 + 10 + 10)$$

$$\det B = -6$$

Himpunan vektor B

Linear dependen (bukan 0)

2)

$$\begin{aligned} x_1 + x_2 + 2x_3 + 2x_4 &= 0 \\ 6x_1 + 2x_2 + 6x_3 + 6x_4 &= 0 \\ x_1 + 0x_2 + 2x_3 + x_4 &= 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & 2 & 2 \\ 6 & 2 & 6 & 6 \\ 1 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = 0$$

Handwritten row reduction steps for the system of equations:

$$\begin{aligned} &\begin{bmatrix} 1 & 1 & 2 & 2 \\ 6 & 2 & 6 & 6 \\ 1 & 0 & 2 & 1 \end{bmatrix} \xrightarrow{H_{21}(-6)} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -4 & -6 & -6 \\ 1 & 0 & 2 & 1 \end{bmatrix} \xrightarrow{H_{31}(-1)} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -4 & -6 & -6 \\ 0 & -1 & 0 & -1 \end{bmatrix} \xrightarrow{H_2(-\frac{1}{4})} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{3}{2} \\ 0 & -1 & 0 & -1 \end{bmatrix} \\ &\xrightarrow{H_{32}(1)} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & \frac{3}{2} & \frac{1}{2} \end{bmatrix} \xrightarrow{H_2(\frac{2}{3})} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & \frac{3}{2} & \frac{3}{2} \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} \xrightarrow{H_{23}(-\frac{3}{2})} \begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} \\ &\xrightarrow{H_{13}(-2)} \begin{bmatrix} 1 & 1 & 0 & \frac{4}{3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} \xrightarrow{H_{12}(-1)} \begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & \frac{1}{3} \end{bmatrix} \\ &\begin{aligned} 1x_1 + \frac{1}{3}x_4 &= 0 \Rightarrow x_1 = -\frac{1}{3}x_4 \\ 1x_2 + 1x_4 &= 0 \Rightarrow x_2 = -1x_4 \\ 1x_3 + \frac{1}{3}x_4 &= 0 \Rightarrow x_3 = -\frac{1}{3}x_4 \end{aligned} \\ &\text{maka basisnya } \begin{bmatrix} -1/3 \\ -1 \\ -1/3 \\ 1 \end{bmatrix} \quad \text{Dimensi} = 1 \end{aligned}$$

3) $u_1 = (1,2,3)$

$u_2 = (1,0,1)$

$u_3 = (2,1,2)$

$$u = v$$

$$\mathbf{u}_1 = \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \mathbf{u}_2 = \mathbf{v}_2 - \frac{\mathbf{u}_1 \cdot \mathbf{v}_2}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1 = \begin{bmatrix} \frac{5}{7} \\ -\frac{4}{7} \\ \frac{1}{7} \end{bmatrix}$$

$$\mathbf{e}_1 = \frac{\mathbf{u}_1}{\sqrt{\mathbf{u}_1 \cdot \mathbf{u}_1}} = \begin{bmatrix} \frac{\sqrt{14}}{14} \\ \frac{\sqrt{14}}{7} \\ \frac{3\sqrt{14}}{14} \end{bmatrix} \qquad \mathbf{e}_2 = \frac{\mathbf{u}_2}{\sqrt{\mathbf{u}_2 \cdot \mathbf{u}_2}} = \begin{bmatrix} \frac{5\sqrt{42}}{42} \\ -\frac{2\sqrt{42}}{21} \\ \frac{\sqrt{42}}{42} \end{bmatrix}$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \frac{\mathbf{u}_1 \cdot \mathbf{v}_3}{\mathbf{u}_1 \cdot \mathbf{u}_1} \mathbf{u}_1 - \frac{\mathbf{u}_2 \cdot \mathbf{v}_3}{\mathbf{u}_2 \cdot \mathbf{u}_2} \mathbf{u}_2 = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \end{bmatrix}$$

$$\mathbf{e}_3 = \frac{\mathbf{u}_3}{\sqrt{\mathbf{u}_3 \cdot \mathbf{u}_3}} = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{bmatrix}$$

$$\mathbf{e}_1 = \begin{bmatrix} \frac{\sqrt{14}}{14} \\ \frac{\sqrt{14}}{7} \\ \frac{3\sqrt{14}}{14} \end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix} \frac{5\sqrt{42}}{42} \\ -\frac{2\sqrt{42}}{21} \\ \frac{\sqrt{42}}{42} \end{bmatrix}, \mathbf{e}_3 = \begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} \end{bmatrix}$$

$$4) A = \begin{bmatrix} 2 & 0 & 0 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{vmatrix} 2-\lambda & 0 & 0 \\ 5 & 1-\lambda & 0 \\ 0 & 1 & 3-\lambda \end{vmatrix} = -\lambda^3 + 6\lambda^2 - 11\lambda + 6$$

$$\lambda_1 = 2;$$

$$\lambda_2 = 1;$$

$$\lambda_3 = 3$$

$$\text{Eigenvalue: 2, eigenvector: } \begin{bmatrix} -\frac{1}{5} \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -1 \\ 1 \end{bmatrix}$$

$$\text{Eigenvalue: 1, eigenvector: } \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{Eigenvalue: 3, eigenvector: } \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$5) u = \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotasi 30°

$$T = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$T = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Pencerminan

$$T = \begin{bmatrix} \frac{\sqrt{3}}{2}x & -\frac{1}{2}y \\ \frac{1}{2}x & \frac{\sqrt{3}}{2}y \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Proyeksi orthogonal thd sumbu y

$$T = \begin{bmatrix} \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\text{Maka } T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$