## TKS 4007
## Matematika III

# Deret Fourier

(Pertemuan XIII)

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## Fungsi Periodik 2L

- Jika fungsi ***f***(***x***) tertentu dalam interval **(-*L*, *L*)** dan di luar interval tersebut oleh ***f***(***x*** + 2***L***) = ***f***(***x***).
- Periode dari fungsi tersebut adalah ***T*** = 2***L***.
- Deret Fourier yang berkaitan dengan fungsi tersebut dapat dinyatakan sebagai :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right)$$

## Fungsi Periodik 2L *(lanjutan)*

Untuk menentukan $a_0$ :

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{0\,\pi\,x}{L}\,dx$$

$$= \frac{1}{L}\int_{-L}^{L} f(x)\cos 0\,dx$$

$$= \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

## Fungsi Periodik 2L *(lanjutan)*

Jika f(x) mempunyai periode 2L, maka koefisien Fourier $a_n$ dan $b_n$ dapat ditentukan dengan cara yang sama menggunakan persamaan :

$$a_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\cos\frac{n\,\pi\,x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{c}^{c+2L} f(x)\sin\frac{n\,\pi\,x}{L}\,dx$$

dengan $c$ sembarang bilangan riil ($c \in \Re$)

# Contoh *(lanjutan)*

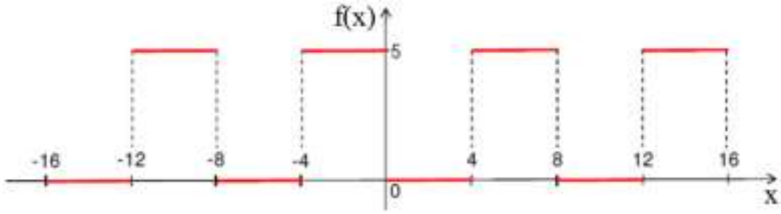
Gambar fungsi berikut dan berikan deret Fourier-nya :

$$f(x) = \begin{cases} 0, & 0 \le x \le 4 \\ 5, & -4 < x < 0 \end{cases}$$

Jawab :

Periode, T = 8 $\rightarrow$ 2L = 8 $\rightarrow$ L = 4

Gambar fungsi yang dimaksud :




# Contoh *(lanjutan)*

Deret Fourier dengan L = 4 :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos\frac{n\pi x}{4} + b_n \sin\frac{n\pi x}{4} \right)$$

Koefisien Fourier :

$$a_0 = \frac{1}{4}\int_{-4}^{4} f(x)\,dx = \frac{1}{4}\left( \int_{-4}^{0} 5\,dx + \int_{0}^{4} 0\,dx \right) = \frac{1}{4}\left( \int_{-4}^{0} 5\,dx \right)$$

$$= \frac{1}{4}[5x]_{-4}^{0} = \frac{5}{4}\big(0-(-4)\big) = \frac{5}{4}(4)$$

$$\boxed{a_0 = 5}$$

## Contoh *(lanjutan)*

$$a_n = \frac{1}{4}\int_{-4}^{4} f(x)\ \cos\frac{n\,\pi\,x}{4}\ dx$$

$$= \frac{1}{4}\left(\int_{-4}^{0} 5\ \cos\frac{n\,\pi\,x}{4}\ dx + \underbrace{\int_{0}^{4} 0\ \cos\frac{n\,\pi\,x}{4}\ dx}_{0}\right)$$

$$= \frac{1}{4}\left(\int_{-4}^{0} 5\ \cos\frac{n\,\pi\,x}{4}\ dx\right)$$

$$= \frac{5}{4}\int_{-4}^{0} \cos\frac{n\,\pi\,x}{4}\ dx = \frac{5}{4}\left(\frac{4}{n\,\pi}\ \sin\frac{n\,\pi\,x}{4}\right)_{-4}^{0}$$

## Contoh *(lanjutan)*

$$a_n = \frac{5}{4}\left(\frac{4}{n\,\pi}\ \sin\frac{n\,\pi\,x}{4}\right)_{-4}^{0}$$

$$= \frac{5}{n\,\pi}\left(\sin\frac{n\,\pi\,x}{4}\right)_{-4}^{0}$$

$$= \frac{5}{n\,\pi}\left(\sin\frac{n\,\pi\,0}{4} - \sin\frac{n\,\pi\,(-4)}{4}\right)$$

$$= \frac{5}{n\,\pi}\left(\sin 0 - \sin(-n\,\pi)\right)$$

## Contoh (*lanjutan*)

$$a_n = \frac{5}{n\pi}\left(0 - \sin(-n\pi)\right) = \frac{5}{n\pi}\left(-\sin(-n\pi)\right)$$

oleh karena $\sin(-n\pi) = -\sin(n\pi)$,

maka $a_n = \frac{5}{n\pi}\sin(n\pi)$

untuk sembarang $n$, $\sin n\pi = 0$,

jadi $a_n = \frac{5}{n\pi}\,0 = 0$ $\boxed{a_1 = a_2 = a_2 = \ldots = 0}$

## Contoh (*lanjutan*)

$$b_n = \frac{1}{4}\int_{-4}^{4} f(x)\sin\frac{n\pi x}{4}\,dx$$

$$= \frac{1}{4}\left(\int_{-4}^{0} 5\sin\frac{n\pi x}{4}\,dx + \int_{0}^{4} 0\sin\frac{n\pi x}{4}\,dx\right)$$

$$= \frac{1}{4}\left(\int_{-4}^{0} 5\sin\frac{n\pi x}{4}\,dx\right)$$

$$= \frac{5}{4}\int_{-4}^{0}\sin\frac{n\pi x}{4}\,dx = \frac{5}{4}\left(\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)\right)_{-4}^{0}$$

## Contoh *(lanjutan)*

$$b_n = \frac{5}{4}\left(\frac{4}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)\right)_{-4}^{0}$$

$$= \frac{5}{n\pi}\left(-\cos\frac{n\pi x}{4}\right)_{-4}^{0}$$

$$= \frac{5}{n\pi}\left(-\cos\frac{n\pi 0}{4} - \left(-\cos\frac{n\pi(-4)}{4}\right)\right)$$

$$= \frac{5}{n\pi}\left(-\cos 0 + \cos(-n\pi)\right)$$

## Contoh *(lanjutan)*

$$b_n = \frac{5}{n\pi}\left(-1 + \cos(-n\pi)\right)$$

oleh karena $\cos(-n\pi) = \cos(n\pi)$,

maka $\boxed{b_n = \frac{5}{n\pi}\left(-1 + \cos(n\pi)\right)}$

bila n genap, $\cos n\pi = 1$

bila n ganjil, $\cos n\pi = (-1)$

# Contoh *(lanjutan)*

jadi $b_n = 0$, untuk n genap

dan $b_n = \dfrac{-10}{n\pi}$, untuk n ganjil

$$b_2 = b_4 = b_6 = \ldots = 0 \;;\; b_1 = \frac{-10}{\pi},\; b_3 = \frac{-10}{3\pi}, \ldots$$

# Contoh *(lanjutan)*

Jadi deret Fourier dari fungsi :

$$f(x) = \begin{cases} 0, & 0 \le x \le 4 \\ 5, & -4 < x < 0 \end{cases}, \qquad T = 8$$

adalah :

$$f(x) = \frac{5}{2} + \sum_{n=1}^{\infty} \frac{5}{n\pi}(-1 + \cos n\pi) \sin \frac{n\pi x}{4}$$

# Latihan

Tentukan deret Fourier dari fungsi berikut :

1. $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & -\pi < x < 0 \end{cases}$
2. $f(x) = \begin{cases} \cos x, & 0 \le x \le \pi \\ 1, & -\pi < x < 0 \end{cases}$
3. $f(x) = \begin{cases} x, & 0 \le x \le 4 \\ 4, & 4 < x < 0 \end{cases}$
4. $f(x) = \begin{cases} x^2, & 0 \le x \le 5 \\ 25, & 5 < x < 0 \end{cases}$

*Terima kasih*
*dan*
*Semoga Lancar Studinya!*