

$$1. a) \frac{x-7}{x} \leq 1$$

$$\frac{x-7}{x} - 1 \leq 0$$

$$x^2 - x - 7 \leq 0$$

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

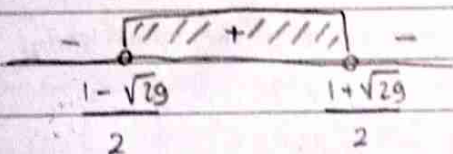
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+28}}{2}$$

$$= \frac{1 \pm \sqrt{29}}{2}$$

$$x_1 = \frac{1 + \sqrt{29}}{2} \quad \vee \quad x_2 = \frac{1 - \sqrt{29}}{2}$$

\* uji titik



$$HP = \left\{ x \in \mathbb{R}, \frac{1 - \sqrt{29}}{2} < x < \frac{1 + \sqrt{29}}{2} \right\}$$

$$b) \frac{x+1}{x^2} \leq \frac{x-2}{x+3}$$

$$\frac{x+1}{x^2} - \frac{x-2}{x+3} \leq 0$$

$$\frac{(x+1)(x+3) - (x-2)(x^2)}{x^2(x+3)} \leq 0$$

$$\frac{x^3 + 4x + 3 - (x^3 - 2x^2)}{x^2(x+3)} \leq 0$$

$$\frac{-x^3 + 3x^2 + 4x + 3}{x^2(x+3)} \leq 0$$

\* pembuat

$$x \neq 0 \quad \vee \quad x \neq -3$$

\* pembilang

$$-x^3 + 3x^2 + 4x + 3 \leq 0, -1$$

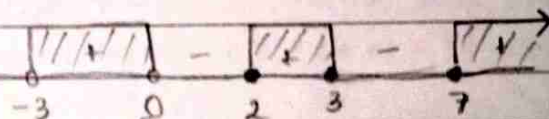
$$x^3 - 3x^2 - 4x - 3 \geq 0$$

$$x^3 - 3x^2 - 4x \geq 3$$

$$x(x-4)(x+1) \geq 3$$

$$x = 3 \quad \vee \quad x = 7 \quad \vee \quad x = -2$$

\* uji titik



$$HP = \{ x \in \mathbb{R}, -3 < x < 0 \text{ atau } 2 \leq x \leq 3 \text{ atau } x \geq 7 \}$$

$$c) x|x-1| \leq |x-2|$$

$$x|x-1| - |x-2| \leq 0$$

\* pembuat nol

$$\rightarrow |x-2| \geq 0$$

$x \geq 2$ , maka

$$x^2 - x + 2 \leq 0 \rightarrow D = b^2 - 4ac$$

$$\text{karena } a > 0 \text{ dan } D < 0 = (-1)^2 - 4(1)(2)$$

$$\text{maka definit } (+) = 1 - 8 = -7$$

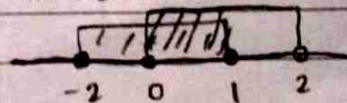
tidak ada nilai  $x$  yang memenuhi

$$\rightarrow 0 \leq x < 2$$

$$x^2 - (2-x) \leq 0$$

$$x^2 + x - 2 \leq 0$$

$$(x+2)(x-1) \leq 0 \rightarrow x = -2 \vee x = 1$$



$$HP = \{ 0 \leq x \leq 1 \}$$

$$\rightarrow x < 0$$

$$-x(x) - (2-x) \leq 0$$

$$-x^2 + x - 2 \leq 0, x = -1$$

$$x^2 - x + 2 \geq 0$$

Definit positif



maka semua  $x \geq 0$  merupakan  
himpunan penyelesaian untuk

$$x^2 - x + 2 > 0$$

$$HP = \{x \in \mathbb{R}, x \geq 0\}$$

$\Rightarrow$  maka himpunan penyelesaian  
dari pertidaksamaan

$$x|x| - |x-2| \leq 0 \text{ adalah}$$

$$HP = \{x | -\infty \leq x \leq -1\}$$

$$d) |2x-3| \leq |x+2|$$

$$|2x-3| - |x+2| \leq 0$$

$$(2x-3) - (x+2) \leq 0$$

$$x-5 \leq 0$$

$$x \leq 5$$

$$-(2x-3) - (x+2) \leq 0$$

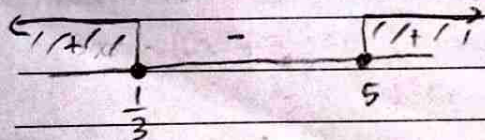
$$-2x+3-x-2 \leq 0$$

$$-3x+1 \leq 0$$

$$-3x \leq -1$$

$$x \geq \frac{1}{3}$$

\*uji titik



$$HP = \{x \in \mathbb{R}, x \geq \frac{1}{3} \text{ atau } x \leq 5\}$$

$$2. a) r = 5$$

$$p = (1, -5)$$

$$x = 2, y = ?$$

\* Persamaan lingkaran

$$(x-1)^2 + (y+5)^2 = 5^2$$

$$x^2 - 2x + 1 + y^2 + 10y + 25 - 25 = 0$$

$$x^2 + y^2 - 2x + 10y + 1 = 0$$

\* untuk  $x = 2$

$$2^2 + y^2 - 2(2) + 10y + 1 = 0$$

$$y^2 + 10y + 1$$

$$y_1, y_2 = -10 \pm \sqrt{100-4}$$

$$2$$

$$= -10 \pm \sqrt{96}$$

$$2$$

$$= -10 \pm 4\sqrt{6}$$

$$2$$

$$= -5 \pm 2\sqrt{6}$$

$$y_1 = -5 + 2\sqrt{6}, y_2 = -5 - 2\sqrt{6}$$

b) Pers. lingkaran yg mempunyai

potongan garis dari  $(1, 3)$  ke  $(7, 11)$

sbg garis tengahnya?

\* Titik pusat lingkaran  $(h, k)$

$$\left(\frac{1+7}{2}, \frac{3+11}{2}\right) = \text{Titik pusat } (4, 7)$$

$$d = 2r = \sqrt{(7-1)^2 + (11-3)^2}$$

$$= \sqrt{6^2 + 8^2}$$

$$= \sqrt{10^2}$$

$$2r = 10$$

$$r = 5$$

\* Pers. lingkaran

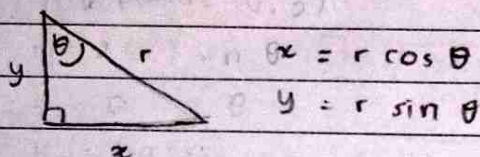
$$(x-4)^2 + (y-7)^2 = 5^2$$

$$(x-4)^2 + (y-7)^2 = 25$$



c)  $x^2 + (y-3)^2 = 9$

$(x-0)^2 + (y-3)^2 = 9$



$\Rightarrow (r \cos \theta)^2 + (r \sin \theta - 3)^2 = 9$

$r^2 \cos^2 \theta + r^2 \sin^2 \theta - 6r \sin \theta + 9 = 9$

$r^2 \cos^2 \theta + r^2 \sin^2 \theta - 6r \sin \theta = 0$

$r^2 (\sin^2 \theta + \cos^2 \theta) - 6r \sin \theta = 0$

$r^2 (1) - 6r \sin \theta = 0$

$r(r - 6 \sin \theta) = 0$

$r - 6 \sin \theta = 0$

$r = 6 \sin \theta$

atau

Lingkaran dgn titik pusat  $(0,3)$

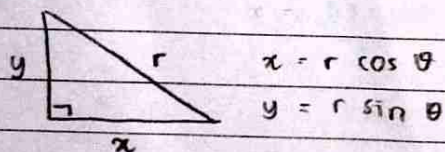
$r = 2a \sin \theta$

titik pusat  $(0,3)$

$r = 2(3) \sin \theta$

$r = 6 \sin \theta$

d)  $y^2 - 4x = 0$



$\Rightarrow (r \sin \theta)^2 - 4(r \cos \theta) = 0$

$r^2 \sin^2 \theta - 4r \cos \theta = 0$

$r(r \sin^2 \theta - 4 \cos \theta) = 0$

$r \sin^2 \theta - 4 \cos \theta = 0$

$r \sin^2 \theta = 4 \cos \theta$

$r = \frac{4 \cos \theta}{\sin^2 \theta}$

$r = 4 \cdot \frac{\cos \theta}{\sin^2 \theta} = 4 \cot \theta \cdot \csc \theta$

3. Fungsi genap atau ganjil

a)  $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$= -x^3 - x$

$= - (x^3 + x)$

$f(-x) = -f(x)$

Fungsi ini merupakan fungsi ganjil

b)  $f(x) = |x|$

$f(-x) = |-x|$

$= |-1||x|$

$= 1 \cdot |x| = |x|$

$f(-x) = f(x)$

Fungsi ini merupakan fungsi genap

c)  $f(x) = x + \cos x$

$f(-x) = (-x) + \cos(-x)$

$= -x + \cos x$

$f(-x) \neq f(x)$

Fungsi ini bukan merupakan fungsi genap maupun fungsi ganjil

3. Definisi turunan

a) Kemiringan garis singgung terhadap

$y = x^2 - 2x$  di titik  $(2,0)$

$\Rightarrow \frac{d}{dx} y = \frac{d}{dx} x^2 + \frac{d}{dx} (-2x)$

$= 2x - 2$

$x = 2 \rightarrow y = m = 2x - 2$

$= 2(2) - 2$

$m = 2$

b)  $f(t) = \frac{1}{2}t^2 + 1$  gram

merupakan berat bakteri saat  $t$  jam

Laju perkembangan saat  $t = 2$  jam?



$$\frac{d}{dt} f(t) = \frac{d}{dt} \frac{1}{2} t^2 + \frac{d}{dt} 1$$

$$= \frac{1}{2} \cdot 2t = t$$

$$t=2 \rightarrow \frac{1}{2} t^2 = 2 \text{ gram/jam}$$

c) cari  $\frac{dy}{dx}$  dan  $\frac{d^2y}{dx^2}$  dari

$$(1) y^3 + 7y - x^3 = 0$$

$$\Rightarrow \frac{dy}{dx} (y^3 + 7y - x^3) = 0$$

$$= -3x^2$$

$$\Rightarrow \frac{d^2y}{dx^2} (y^3 + 7y - x^3)$$

$$\Rightarrow \frac{dy}{dx} (y^3 + 7y - x^3) = \frac{dy}{dx} - 3x^2$$

$$\frac{dy}{dx} - 3x^2$$

$$= -6x$$

$$(2) x^3 y^4 - 1 = 0$$

$$\Rightarrow \frac{dy}{dx} x^3 y^4 - 1$$

$$= 3x^2 y^4$$

$$\Rightarrow \frac{d^2y}{dx^2} x^3 y^4 - 1$$

$$\Rightarrow \frac{dy}{dx} x^3 y^4 - 1 = \frac{dy}{dx} 3x^2 y^4$$

$$\frac{dy}{dx} 3x^2 y^4$$

$$= 6x y^4$$

$$(3) y = \sqrt{\sin(xy^2)}$$

$$y = (\sin(xy^2))^{\frac{1}{2}}$$

$$\Rightarrow \frac{dy}{dx} (\sin(xy^2))^{\frac{1}{2}}$$

$$= \frac{1}{2} \cdot y^2 (\cos(xy^2)) \cdot (\sin(xy^2))^{-\frac{1}{2}}$$

$$= \frac{\cos(xy^2) \cdot y^2}{2\sqrt{\sin(xy^2)}}$$

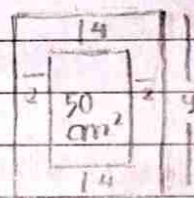
$$\Rightarrow \frac{d^2y}{dx^2} \sqrt{\sin(xy^2)}$$

$$\frac{dy}{dx} \sqrt{\sin(xy^2)} = \frac{dy}{dx} \frac{\cos(xy^2) \cdot y^2}{2\sqrt{\sin(xy^2)}}$$

$$\frac{dy}{dx} \frac{\cos(xy^2) \cdot y^2}{2\sqrt{\sin(xy^2)}}$$

$$= \frac{-2y^4 \cdot \sin(xy^2)^2 + \cos(xy^2)^2 \cdot y^4}{4\sqrt{\sin(xy^2)} \cdot \sin(xy^2)}$$

4.



L. tulisan = 50 cm

L. kertas =  $\pi y$

\* Luas tulisan

$$(x-4) \cdot (y-8) = 50$$

$$\pi y - 8x - 4y + 32 = 50 \Rightarrow \pi y - 8x - 4y = 18$$

$$\pi y - 8x - 4y - 18 = 0$$

$$\pi(y-8) - 4y - 18 = 0$$

$$\pi = \frac{4y+18}{y-8}$$

\* Luas kertas

$$f(y) = \pi y = \frac{4y+18}{y-8} \cdot y$$

$$= \frac{4y^2+18y}{y-8} \rightarrow u; u' = 8y+18$$

$$y-8 \rightarrow v; v' = 1$$

\* L kertas semimumimum mungkin

$$\frac{d}{dy} f(y) = 0 \rightarrow \frac{u'v - uv'}{v^2}$$

$$\frac{(8y+18)(y-8) - (4y^2+18y)(1)}{(y-8)^2} = 0$$

$$8y^2 - 46y - 144 - (4y^2 + 18y) = 0$$

$$y^2 - 16y + 64$$



$$4y^2 - 64y - 144 = 0$$

$$y^2 - 16y + 36$$

$$4(y^2 - 16y - 36) = 0$$

$$y^2 - 16y + 36$$

$$4(y - 18)(y + 2) = 0$$

$$(y - 8)^2$$

$$y = 18 \quad \vee \quad y = -2 \text{ (TM)}$$

$$\Rightarrow x = \frac{4y + 18}{y - 8}$$

$$y - 8$$

$$= \frac{4(18) + 18}{18 - 8}$$

$$18 - 8$$

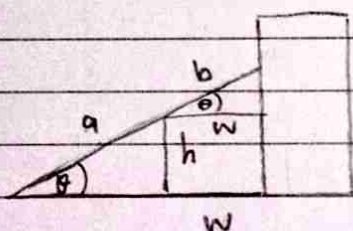
$$= \frac{90}{10} = 9$$

$$10$$

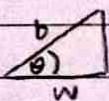
$$x = 9$$

5.

Panjang tangga terpendek?



\*



$$\cos \theta = \frac{w}{b}$$

$$b = \frac{w}{\cos \theta}$$

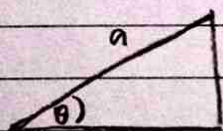
$$\text{Panjang tangga} = L = a + b$$

$$L(\theta) = \frac{h}{\sin \theta} + \frac{w}{\cos \theta}$$

$$\sin \theta \cos \theta$$

$$= h (\sin \theta)^{-1} + w (\cos \theta)^{-1}$$

\*



$$\sin \theta = \frac{h}{a}$$

$$a = \frac{h}{\sin \theta}$$

$$\Rightarrow L'(\theta) = 0$$

$$0 = -h (\sin \theta)^{-2} \cdot \cos \theta - w (\cos \theta)^{-2} (-\sin \theta)$$

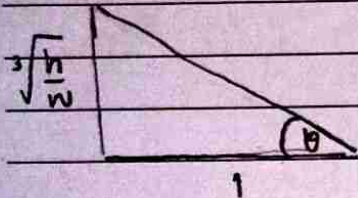
$$h (\sin \theta)^{-2} \cdot \cos \theta = w (\cos \theta)^{-2} \cdot \sin \theta$$

$$h \cos^3 \theta = w \sin^3 \theta$$

$$\frac{h}{w} = \frac{\sin^3 \theta}{\cos^3 \theta} = \tan^3 \theta$$



$$\theta = \cot^{-1} \sqrt[3]{\frac{h}{w}}$$



$$\begin{aligned} \text{side miring} &= \sqrt{\left(\sqrt[3]{\frac{h}{w}}\right)^2 + 1^2} \\ &= \sqrt{\left(\frac{h}{w}\right)^{2/3} + 1} \end{aligned}$$

$$\therefore \sin \theta = \frac{\sqrt[3]{h/w}}{\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}}$$

$$\therefore \cos \theta = \frac{1}{\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}}$$

$$\begin{aligned} \Rightarrow L(\theta) &= \frac{h}{\sin \theta} + \frac{w}{\cos \theta} \\ &= \frac{h}{\frac{\sqrt[3]{h/w}}{\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}}} + \frac{w}{\frac{1}{\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}}} \\ &= \frac{h \left(\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}\right)}{\sqrt[3]{h/w}} + w \left(\sqrt{\left(\frac{h}{w}\right)^{2/3} + 1}\right) \end{aligned}$$