ULANGAN TENGAH SEMESTER TA 2020/2021

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Kelas : B Informatika

1. Selidiki kekonvergenan improper integral

$$\int_0^\infty \frac{1}{(x+1)^2}\,dx = \lim_{a\to 0}\int_0^a \frac{1}{(x+1)^2}\,dx$$

misal $\Rightarrow t = (x+1)$

$$\int_0^a \frac{1}{t^2}\,dt = -\frac{1}{t}\Big]_0^a = \lim_{a\to\infty} \frac{-1}{x+1}\Big|_0^a$$

$$= \lim_{a\to\infty} \frac{-1}{a+1} - (-1)$$

$$= 1$$

konvergen

2. Tentukan daerah asal fungsi dan grafik sketsa

$$f(x,y) = \frac{\sqrt{1-x^2-y^2}}{\sqrt{y+x^2}}$$

syarat 1 $\sqrt{1-x^2-y^2} \geq 0$

$1-x^2-y^2 \geq 0$

$x^2+y^2 \leq 1$

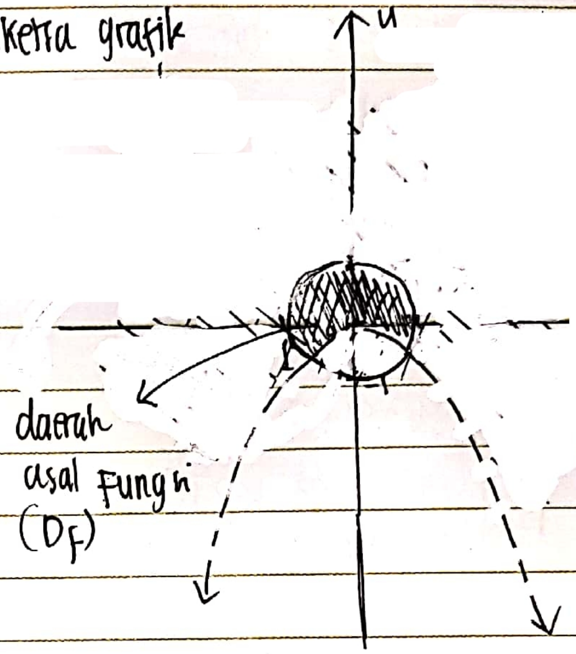
syarat 2 $\sqrt{y+x^2} > 0$

$y+x^2 > 0$

$x^2 > -y$

$D_f = \{(x,y) \in R^2 \,\|\, x^2+y^2 \leq 1,\ x^2 > -y\}$

Sketsa grafik



3. Carilah turunan parsial pertama terhadap peubah bebasnya dari fungsi:

$$f(x,y) = 3e^{2x}\cos y$$

$$\frac{\partial f(x,y)}{\partial x} = 2.3e^{2x}\cos y$$

$$= 6.e^{2x}\cos y$$

$$\frac{\partial f(x,y)}{\partial y} = -3e^{2x}.\sin y$$

4) Hitunglah volume

$$V = \iint_S \frac{x^3}{\sqrt{3x^4+y^2}}\, dy\,dx$$ dengan S

adalah daerah yang dibatasi oleh

$y = x^2$, $y = 6$ dan sumbu-y

$y = y$ $\quad 0 \leq x \leq \sqrt{6}$

$x^2 = 6$ $\quad 0 \leq y \leq 6$

$x = \sqrt{6}$

Jawab:

$$\int_0^6 \int_0^{\sqrt{y}} \frac{x^3}{\sqrt{3x^4+y^2}}\, dx\,dy$$

$$\int_0^6 \left[\int_0^{\sqrt{y}} \frac{x^3}{\sqrt{3x^4+y^2}}\, dx \right] dy$$

misal $u = 3x^4 + y^2$

$du = 12x^3 \quad dx$

$$\to = \int_0^{\sqrt{y}} \frac{\cancel{x^3}}{\sqrt{u}}\, du \cdot \frac{1}{12\cancel{x^3}}$$

$$= \frac{1}{12} \int_0^{\sqrt{y}} u^{-1/2}\, du$$

$$= \frac{1}{12}\, 2\sqrt{u} \Big]_0^{\sqrt{y}}$$

$$= \frac{1}{12}\, 2\sqrt{3x^4+y^2} \Big]_0^{\sqrt{y}}$$

$$= \frac{1}{12}(4y - 2y) = \frac{1}{12} \cdot 2y = \frac{1}{6} y$$

$$\to \int_0^6 \frac{1}{6} y\, dy$$

$$= \frac{1}{6}\left(\frac{1}{2} y^2\right]_0^6$$

$$= \frac{1}{6} \cdot \left(\frac{1}{2} \cdot 36 - 0\right) = 3$$ satuan volume

5) Tunjukkan bahwa bola volume

$x^2 + y^2 + z^2 = R^2$

dan $V = \frac{4}{3}\pi R^3$ dengan R adl jari²

Hitunglah volume di oktan pertama

$$V = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{R} 1\, \rho^2 \sin\phi\, d\rho)\, d\theta\, d\phi$$

$$\to = \frac{1}{3} \rho^3 \sin\phi \Big]_0^R$$

$$= \frac{1}{3} R^3 \sin\phi$$

$$\int_0^{\pi/2}\int_0^{\pi/2} \frac{1}{3} R^3 \sin\phi\, d\theta\, d\phi$$

$$\to = \left(\frac{1}{3} R^3 \sin\phi\right) \theta \Big]_0^{\pi/2}$$

$$= \frac{\pi}{6} R^3 \sin\phi$$

$$= \int_0^{\pi/2} \frac{\pi}{6} R^3 \sin\phi\, d\phi$$

$$= \frac{\pi}{6} R^3 \int_0^{\pi/2} \sin\phi\, d\phi$$

$$= \frac{\pi}{6} R^3 \left(-\cos\phi \Big|_0^{\pi/2}\right)$$

$$= -\frac{\pi}{6} R^3 (0 - 1)$$

$$= \frac{\pi}{6} R^3$$

Perbandingan dengan volume awal

$$V_2\ \frac{\cancel{\pi}}{6} \cancel{R^3} = \frac{4}{3} \cancel{\pi R^3}\, V_1$$

$$\frac{1}{6} V_2 = \frac{4}{3} V_1$$

$$V_2 = 8 V_1$$

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