

## UAS MATEMATIKA II.

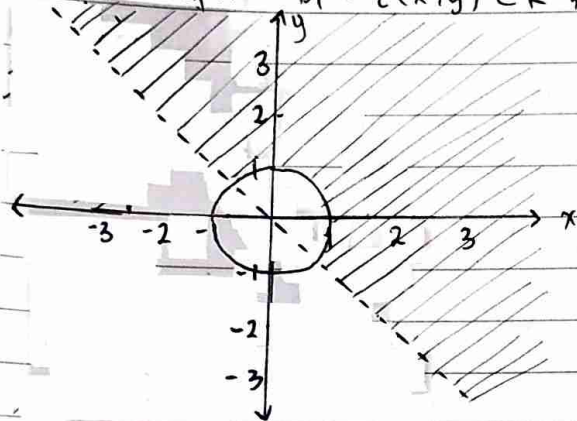
$$1) f(x, y) = \frac{\sqrt{x^2 + y^2 - 1}}{\sqrt{x + y}}$$

$$\bullet x^2 + y^2 - 1 \geq 0$$

$$x^2 + y^2 \geq 1, D_F = \{(x, y) \in \mathbb{R}^2, x^2 + y^2 \geq 1, x < -y\}$$

$$\bullet x + y > 0, D_F = \{(x, y) \in \mathbb{R}^2, x^2 + y^2 \leq 1, x > -y\}$$

$$\bullet \text{Grafik } D_F = \{(x, y) \in \mathbb{R}^2, x^2 + y^2 \geq 1, x > -y\}$$



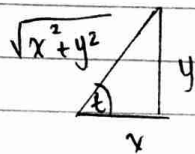
$$2) f(x, y) = xy + e^y \sin x$$

$$a. \frac{\partial f(x, y)}{\partial x} = y + e^y \cos x$$

$$b. \frac{\partial f(x, y)}{\partial y} = x + \sin x \cdot e^y$$

$$\begin{aligned} 3) a. \int_{-3}^7 \int_0^{2x} \int_y^{x-1} 2 \, dz \, dy \, dx &= \int_{-3}^7 \int_0^{2x} 2z \Big|_y^{x-1} dy \, dx \\ &= \int_{-3}^7 (2x-2)y - 2 \cdot \frac{1}{2} y^2 \Big|_0^{2x} dx \\ &= \int_{-3}^7 (2x-2)(2x) - (2x)^2 dx \\ &= \int_{-3}^7 4x^2 - 4x - 4x^2 dx \\ &= \int_{-3}^7 -4x dx \\ &= -2x^2 \Big|_{-3}^7 \\ &= -98 + 18 \\ &= -80 \end{aligned}$$

$$\begin{aligned}
 \text{b.) } \int_1^5 \int_0^x \frac{3}{x^2 + y^2} dy dx &= \int_1^5 \int_0^x \frac{3 \cdot x \sec^2 t \cdot dt}{x^2 + x^2 \tan^2 t} dx \\
 &= \int_1^5 \int_0^x \frac{3 \cdot x \sec^2 t \cdot dt}{x^2 (1 + \tan^2 t)} dx \\
 &= \int_1^5 \frac{3}{x} t \Big|_0^x dx
 \end{aligned}$$



$$\begin{aligned}
 y &= x \tan t \\
 dy &= x \sec^2 t dt \\
 \tan t &= \frac{y}{x}
 \end{aligned}$$

$$t = \arctan y/x$$

$$\begin{aligned}
 &\int_1^5 \frac{3}{x} \arctan \frac{y}{x} \Big|_0^x dx \\
 &= \int_1^5 \frac{3}{x} (\arctan(1) - \arctan(0)) dx \\
 &= \int_1^5 \frac{3}{x} \cdot \frac{\pi}{4} \cdot dx \\
 &= \frac{3\pi}{4} \int_1^5 \frac{1}{x} dx \\
 &= \frac{3\pi}{4} (\ln(x)) \Big|_1^5 dx \\
 &= \frac{3\pi}{4} (\ln(5) - \ln(1)) \\
 &= \frac{3\pi}{4} \ln(5)
 \end{aligned}$$

4) Volume bola padat  $x^2 + y^2 + z^2 = 9$

$$\rho^2 = x^2 + y^2 + z^2$$

$$\rho^2 = 9$$

$$\rho = 3, \text{ batas } 0 \leq \rho \leq 3$$

Jika bola terdapat pada oktan pertama, maka

$$0 \leq \theta \leq \frac{\pi}{2} \text{ dan } 0 \leq \phi \leq \frac{\pi}{2}$$

$$dV = \rho^2 \sin \theta \, d\rho \, d\theta \, d\phi$$

$$V_{\text{bola}} = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^3 \rho^2 \sin \theta \, d\rho \, d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} \left[ \frac{1}{3} \rho^3 \sin \theta \right]_0^3 d\theta \, d\phi$$

$$= \int_0^{\pi/2} \int_0^{\pi/2} (9 \sin \theta) d\theta \, d\phi$$

$$= \int_0^{\pi/2} (9 \sin \theta) \theta \Big|_0^{\pi/2} d\phi$$

$$= \int_0^{\pi/2} \frac{9}{2} \pi \sin \theta \, d\theta$$

$$= \frac{9}{2} \pi (-\cos \theta) \Big|_0^{\pi/2}$$

$$= -\frac{9}{2} \pi \left( \cos \frac{\pi}{2} - \cos 0 \right)$$

$$= -\frac{9}{2} \pi (0 - 1)$$

$$= \frac{9}{2} \pi$$

Jadi, volume bola padat adalah  $\frac{9}{2} \pi$