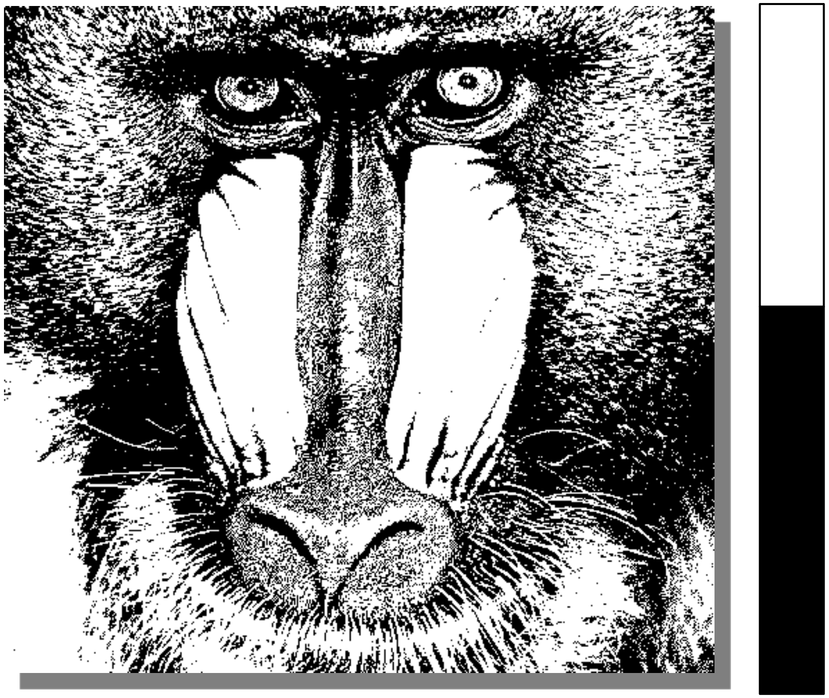
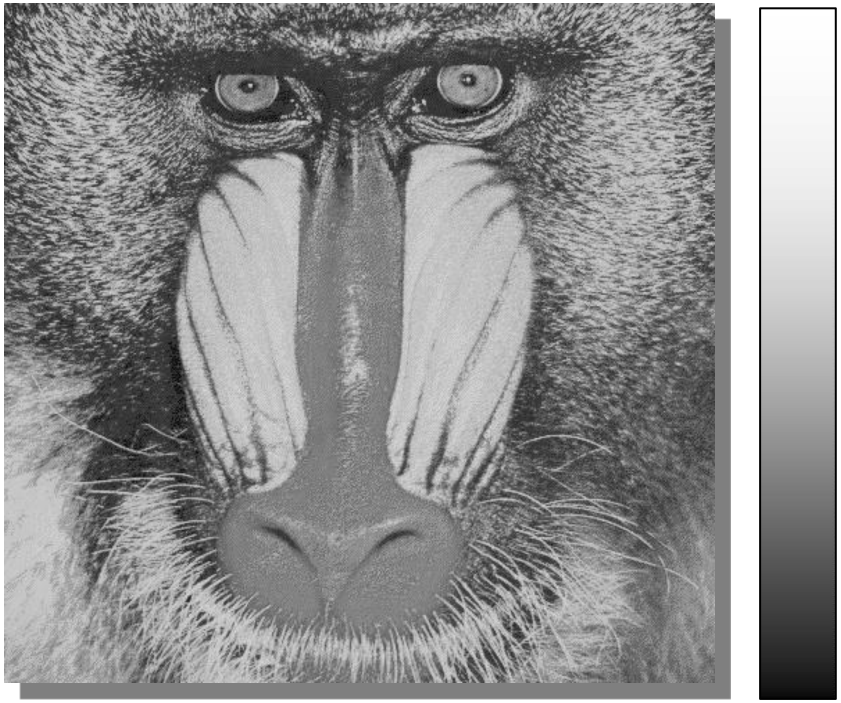
Histogram

Peningkatan Kualitas Citra

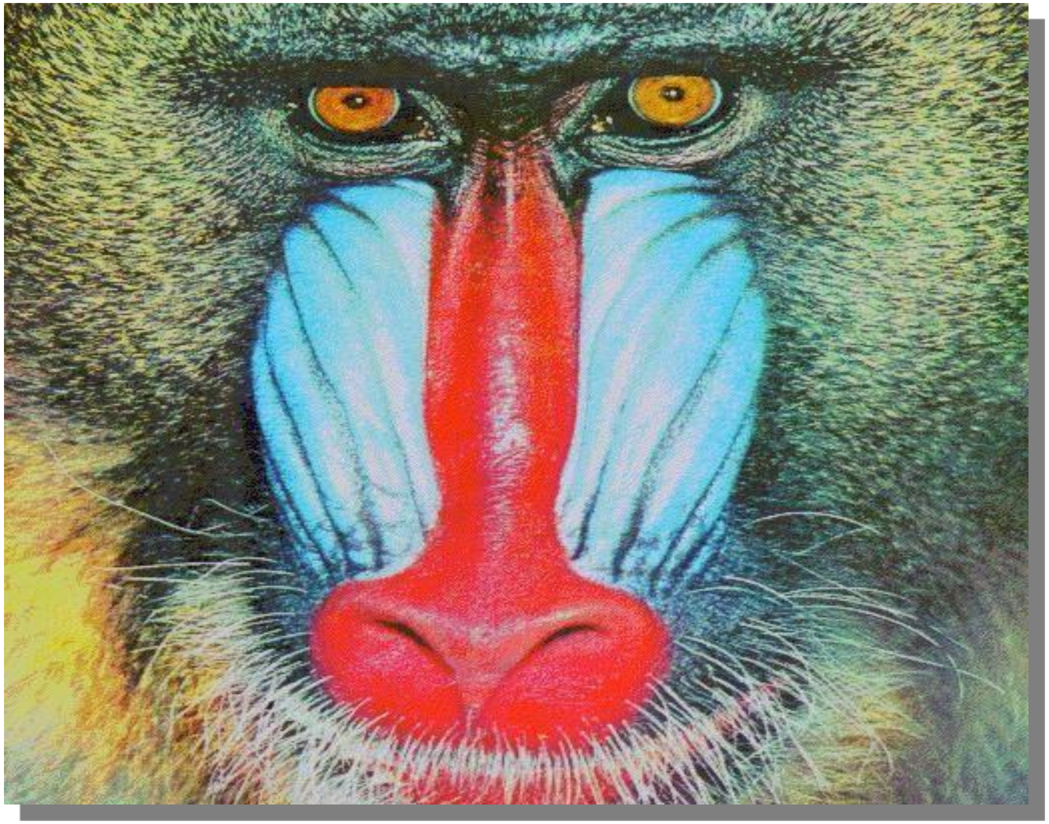
Representasi Image





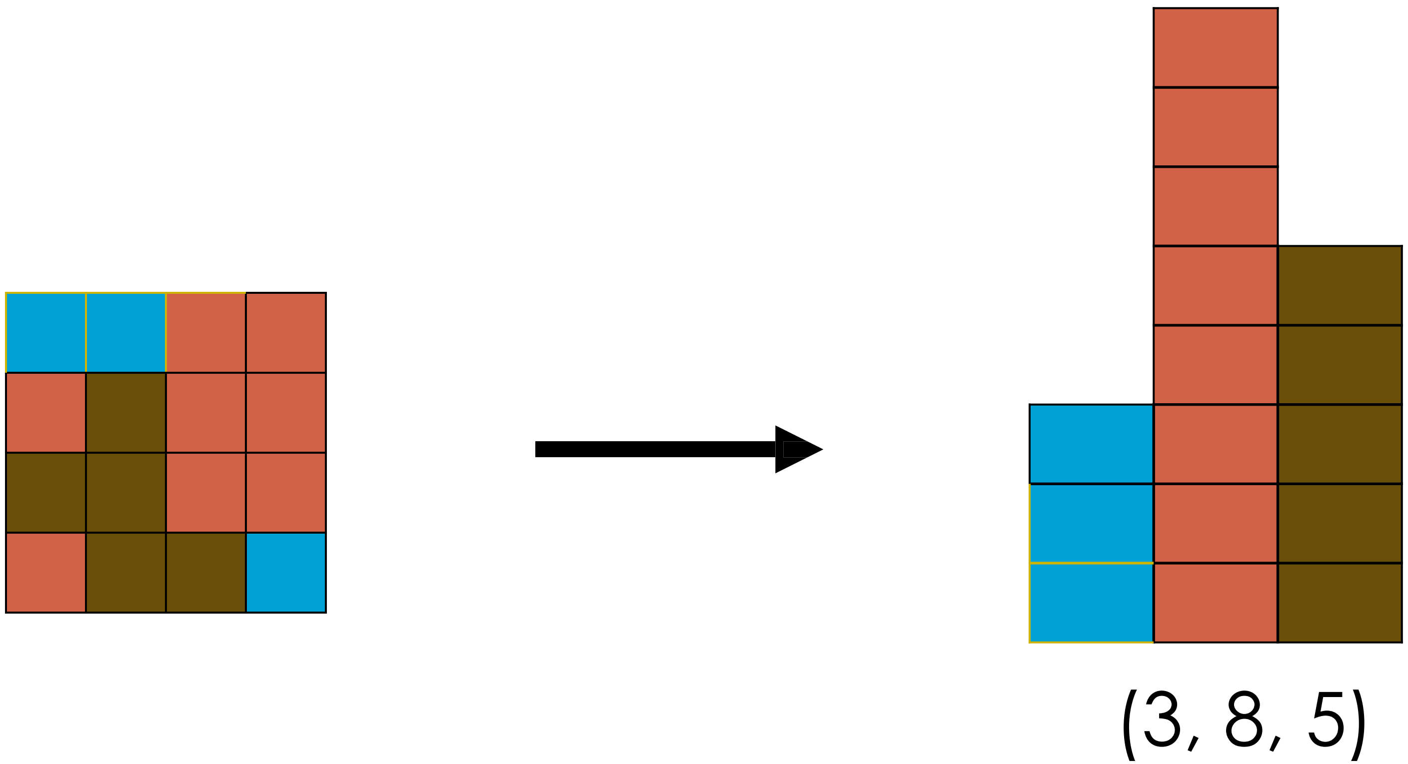
1 bit

8 bits



24 bits

Apakah itu histogram?



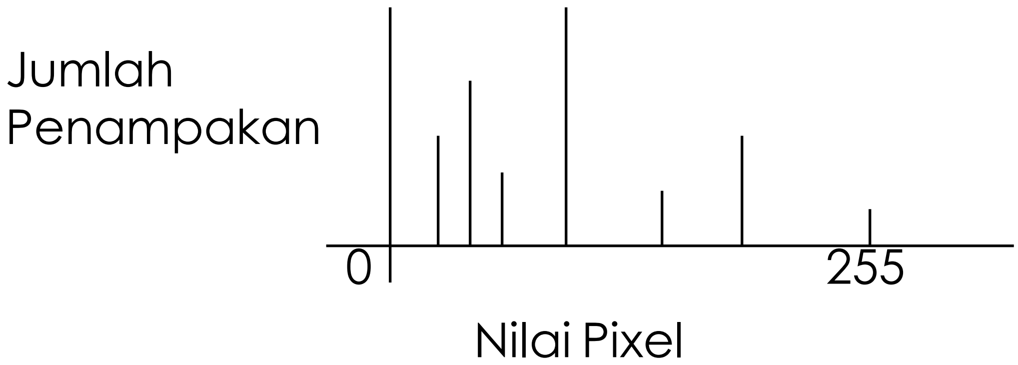
Histogram memberikan deskripsi global dari penampakan sebuah image.

- Histogram dari image digital dengan gray levels dari 0 sampai L-1 adalah fungsi diskrit $h(r_k)=n_k$, dimana:
  - r_k adalah nilai gray level ke k
  - n_k adalah jumlah pixels dalam image yang memiliki gray level k
  - n adalah jumlah keseluruhan pixel pada image
  - $k = 0, 1, 2, \ldots, L-1$

- Histogram dari image digital dengan gray level yang berada dalam range [0, L-1] adalah sebuah fungsi diskrit

$$h(rk) = nk$$

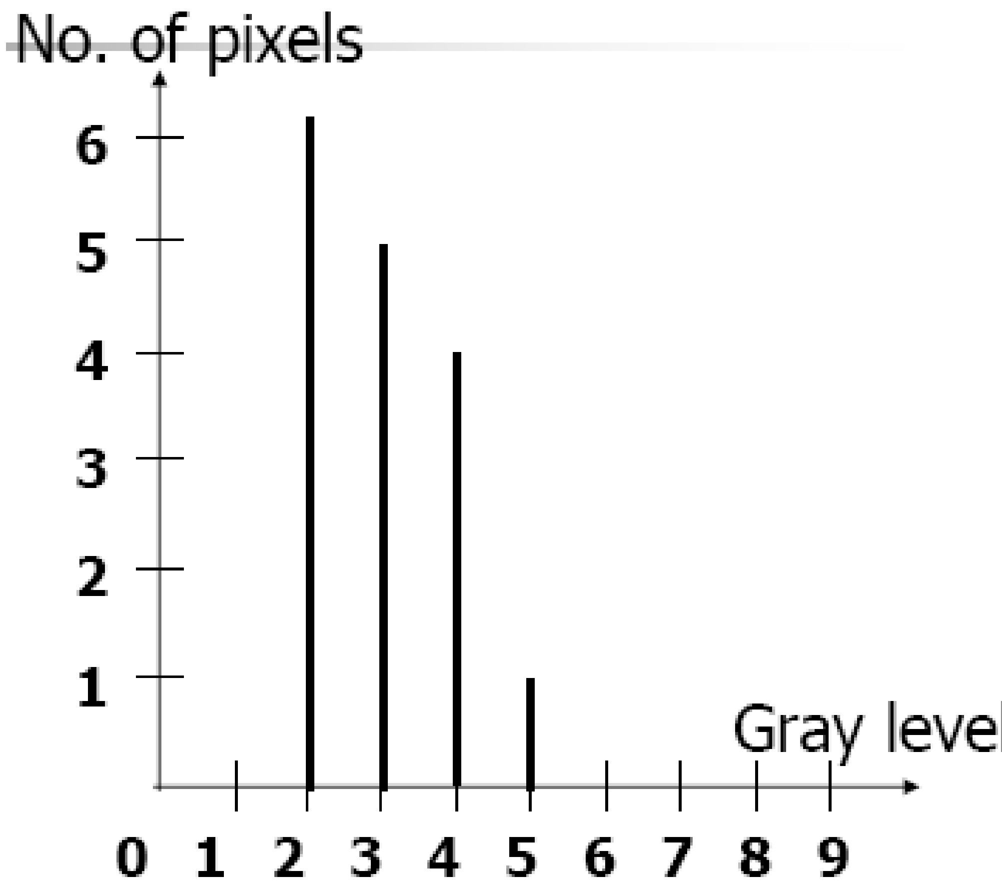
dimana *rk* adalah nilai gray level ke k dan *nk* adalah jumlah pixel yang memiliki nilai gray level *rk*.



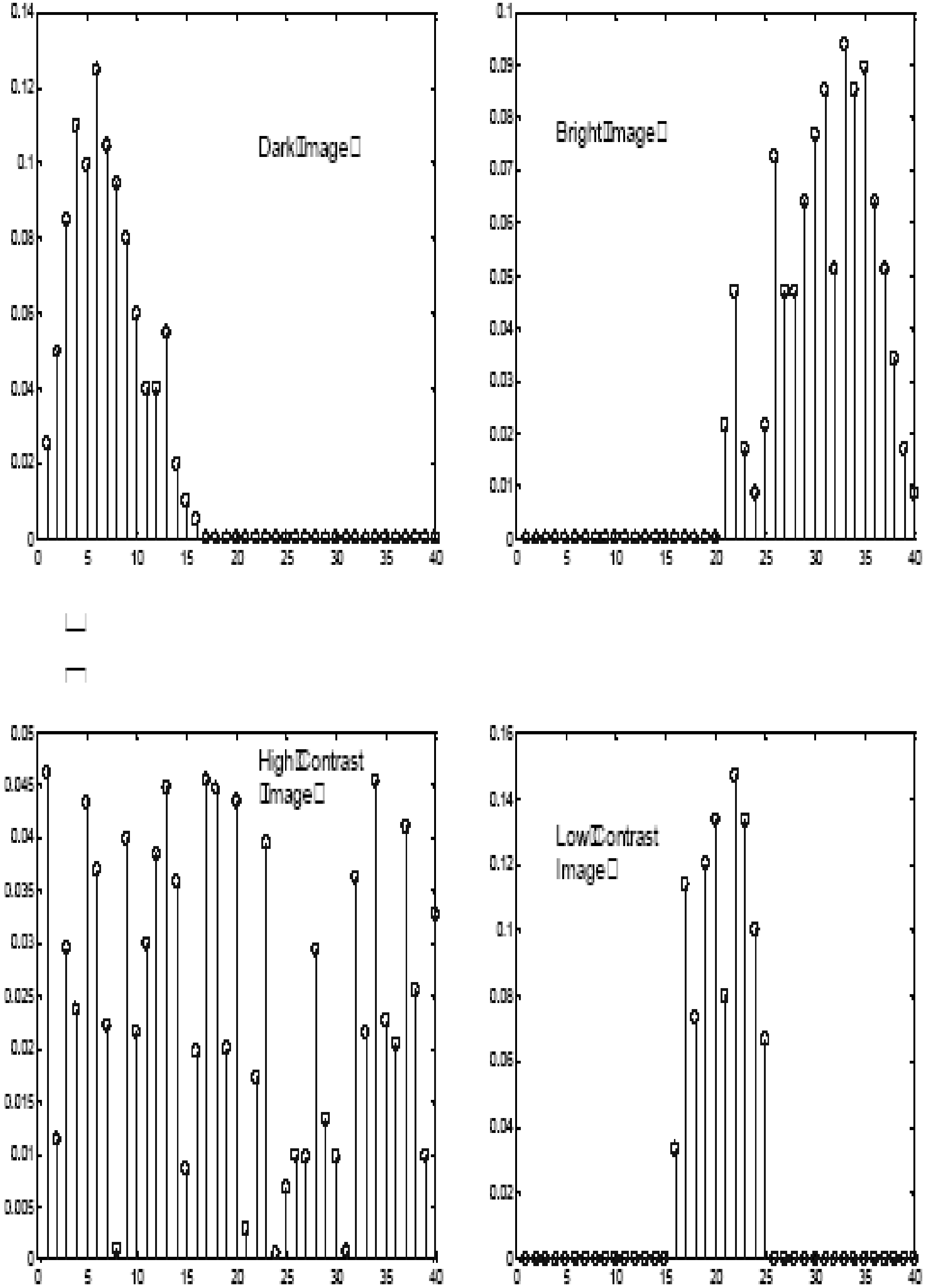
2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

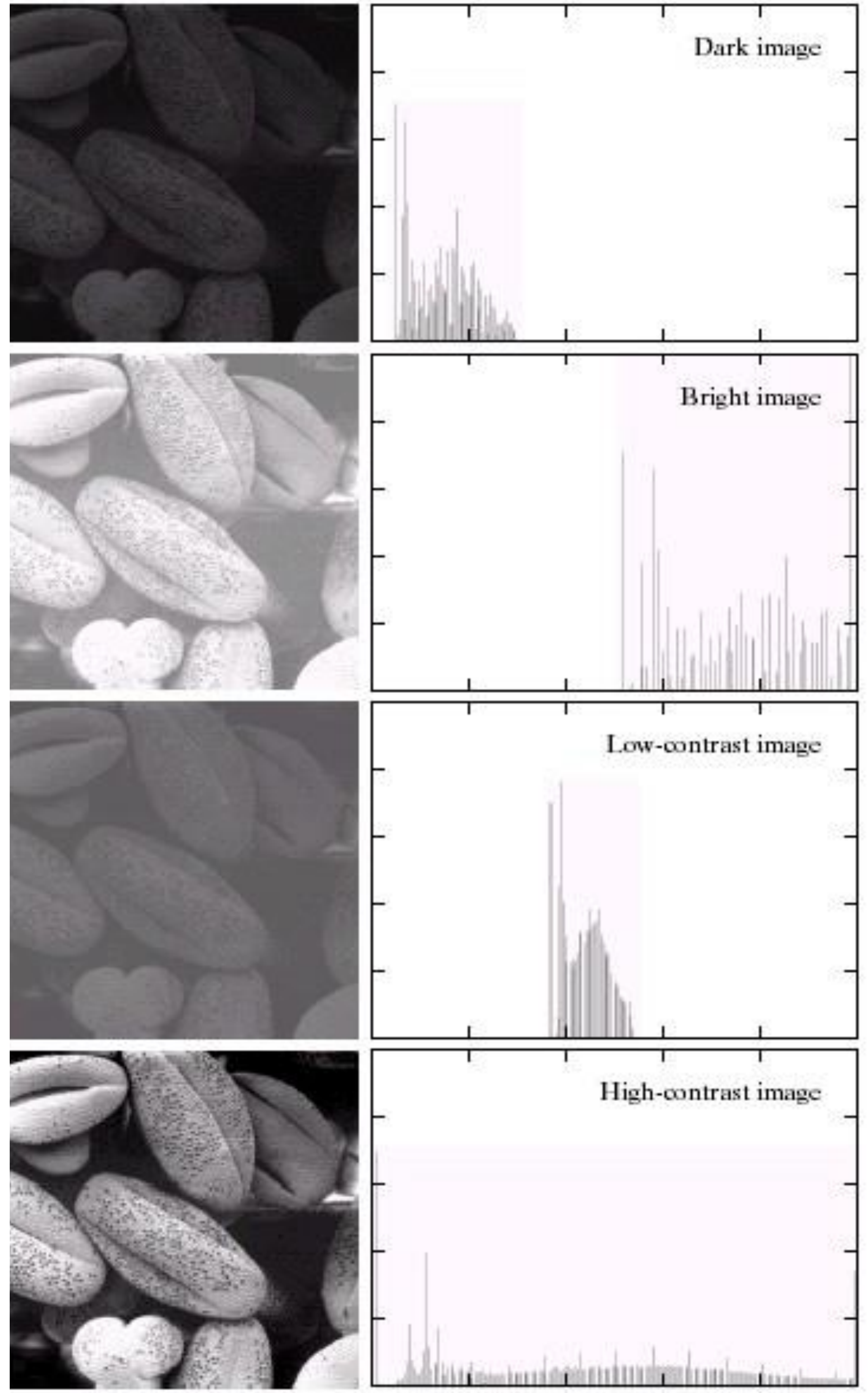
4x4 image

Gray scale = [0,9]



histogram


Dark Image
Bright Image
High Contrast Image
Low Contrast Image


Dark image
Bright image
Low-contrast image
High-contrast image

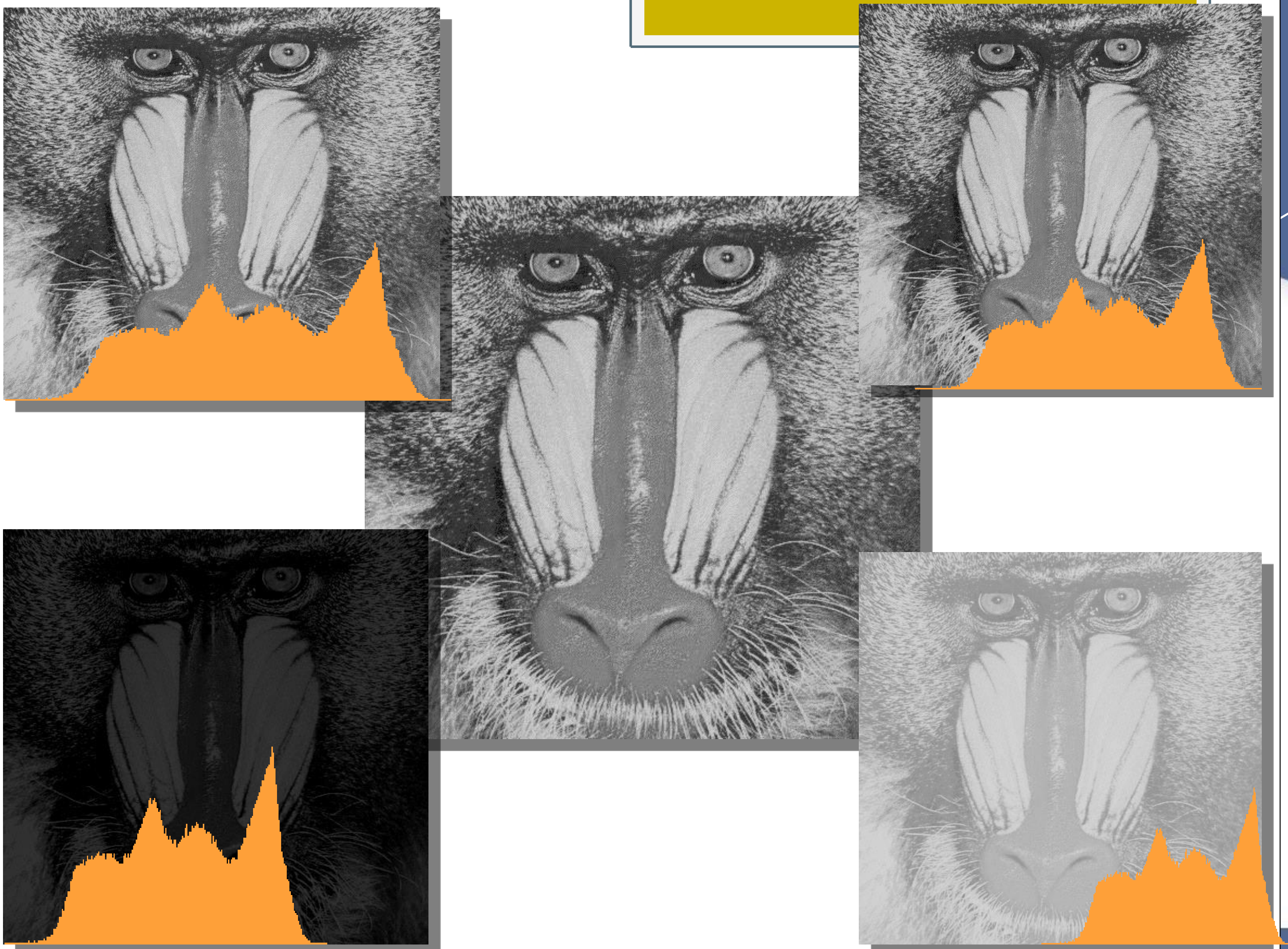
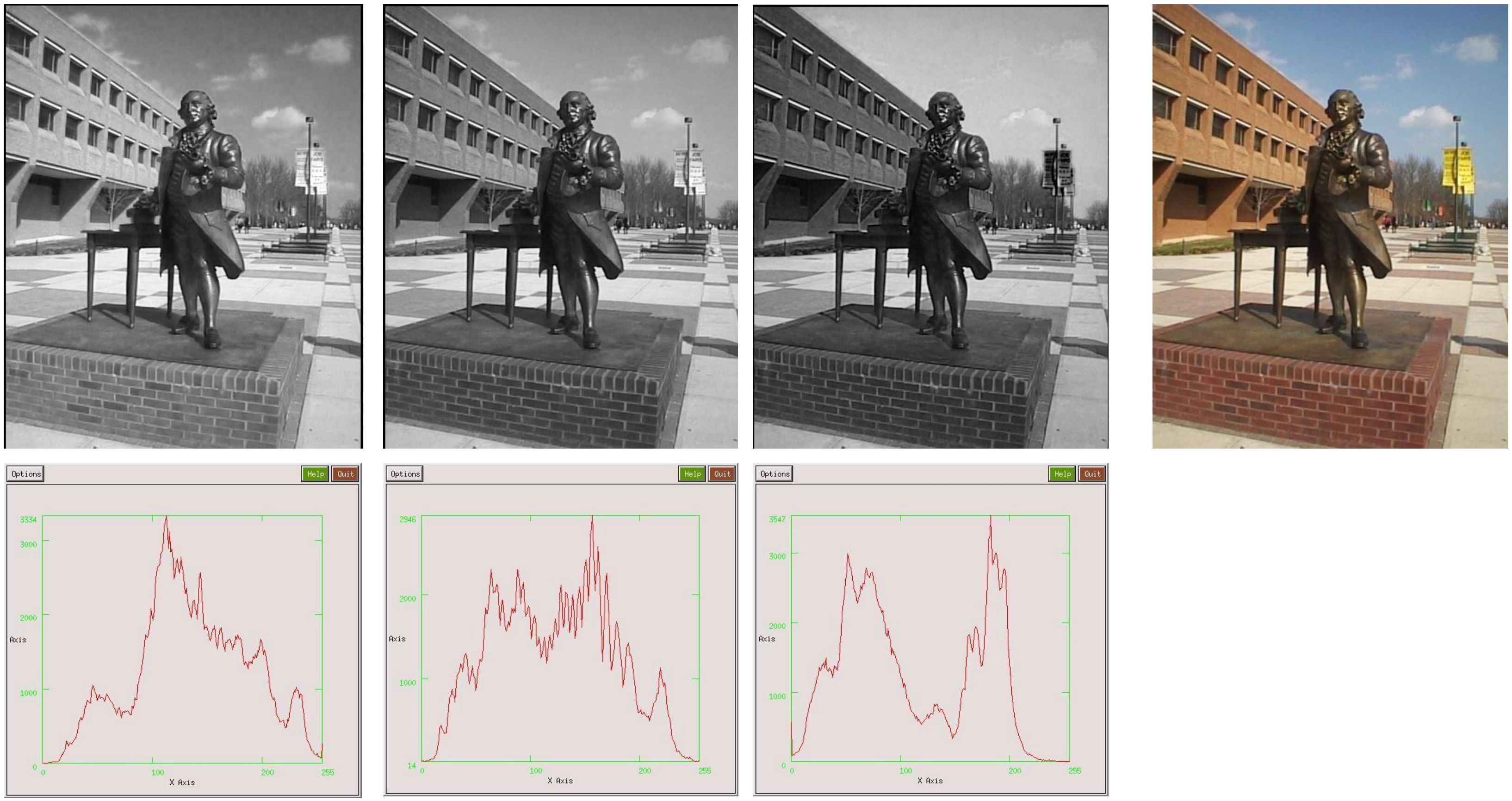


Image colors



red green blue

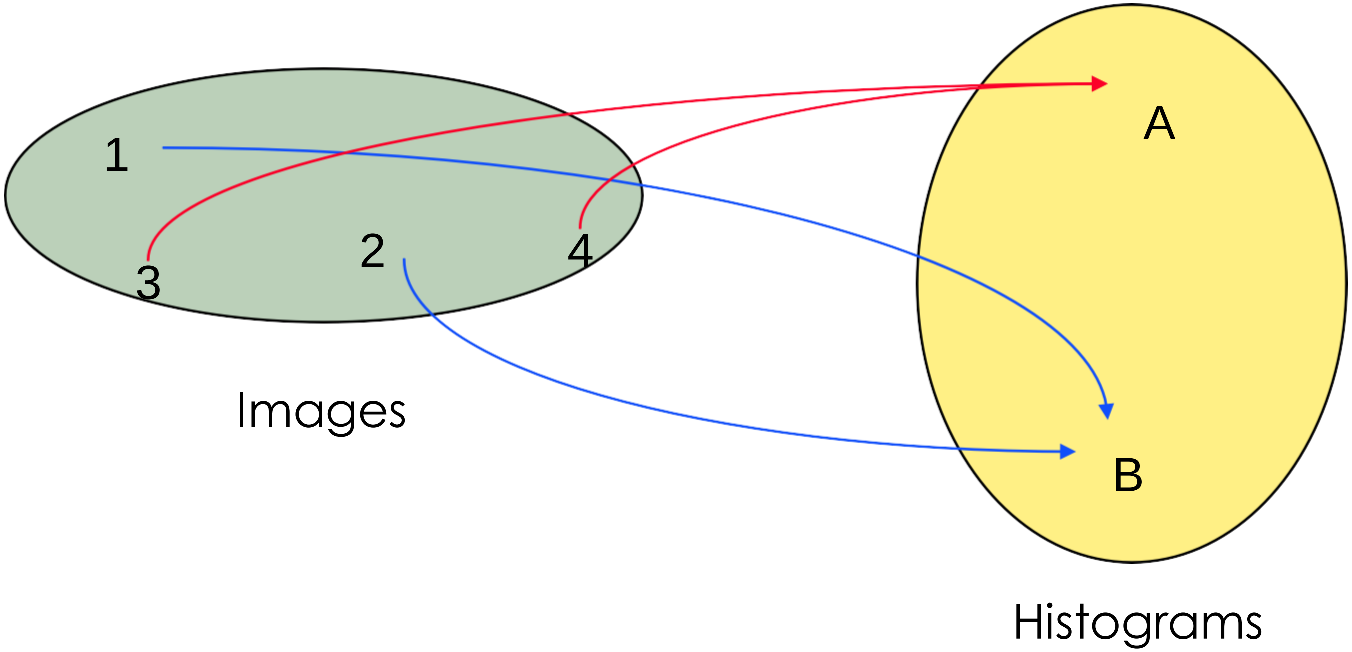
Dengan Histogram informasi spasial dari image diabaikan dan hanya mempertimbangkan frekuensi relatif penampilan gray level.

	0	3	3	2	5	5	
	1	1	0	3	4	5	
	2	2	2	4	4	4	
	3	3	4	4	5	5	
	3	4	5	5	6	6	
	7	6	6	6	6	5	

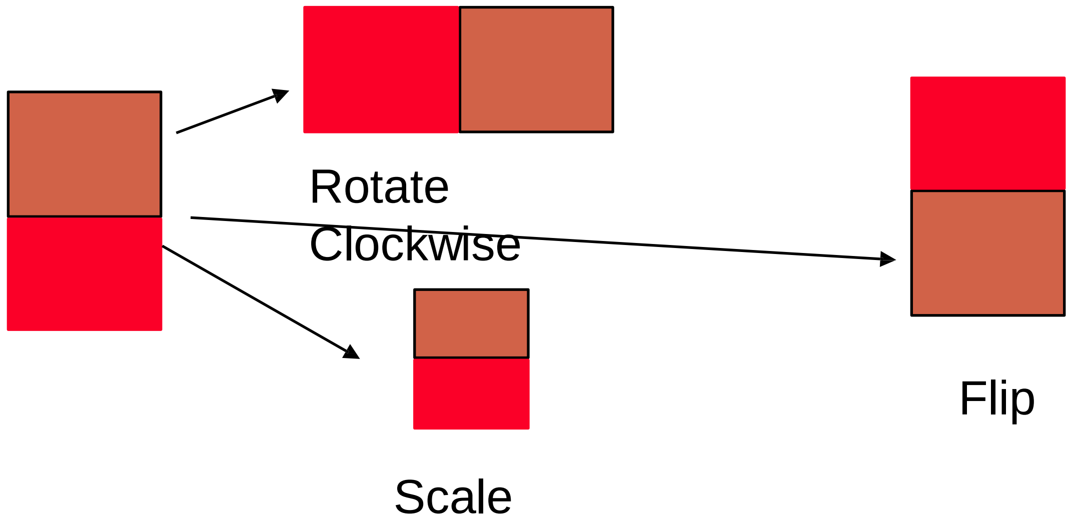
Gray Value	Count	Rel. Freq.
0	2	.05
1	2	.05
2	4	.11
3	6	.17
4	7	.20
5	8	.22
6	6	.17
7	1	.03

Sifat – Sifat Histogram

- Histogram adalah pemetaan Many-to-One
- Image yang berbeda dimungkinkan untuk memiliki histogram yang sama.

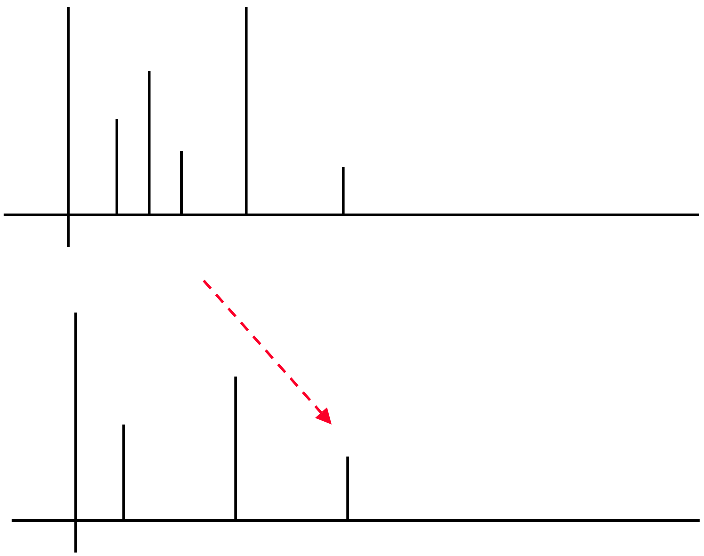


- Histogram sebuah image tidak berubah bila image dikenakan operasi tertentu seperti : Rotation, scaling, flip.

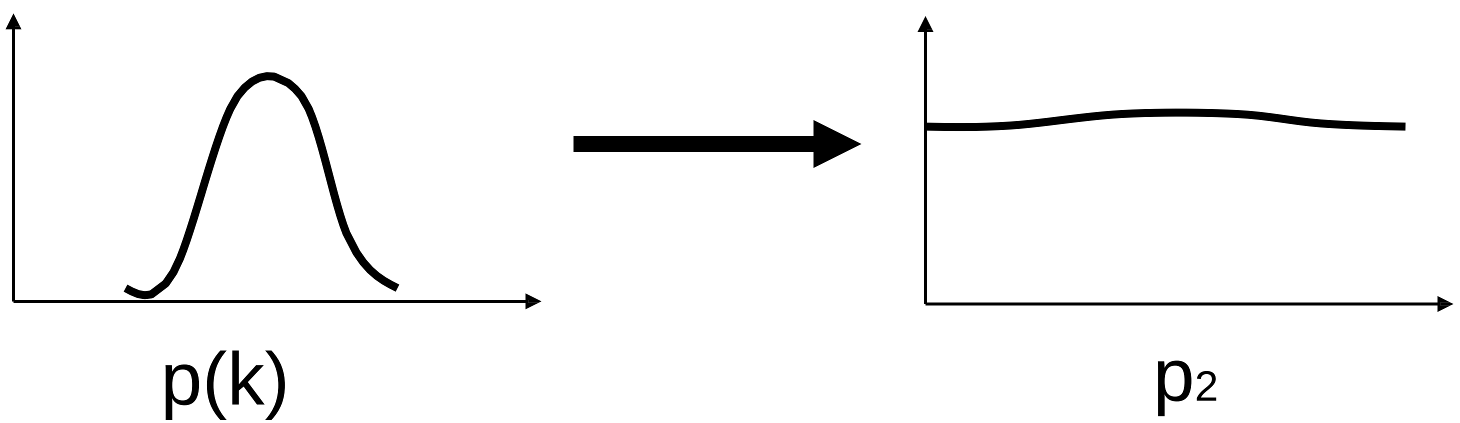


Ekualisasi Histogram

- Adalah proses Mapping dari Grey Levels "p" menjadi Grey Levels "q" sedemikian sehingga distribusi dari Grey Levels pada "q" mendekati bentuk **Uniform**



- Bila p(k) = image histogram pada
  k = [0..1]
- Tujuan: mencari transformasi contrast stretching T(k) sedemikian sehingga
  I_2 = T(I) and p_2 = 1 (uniform)



Normalisasi Histogram

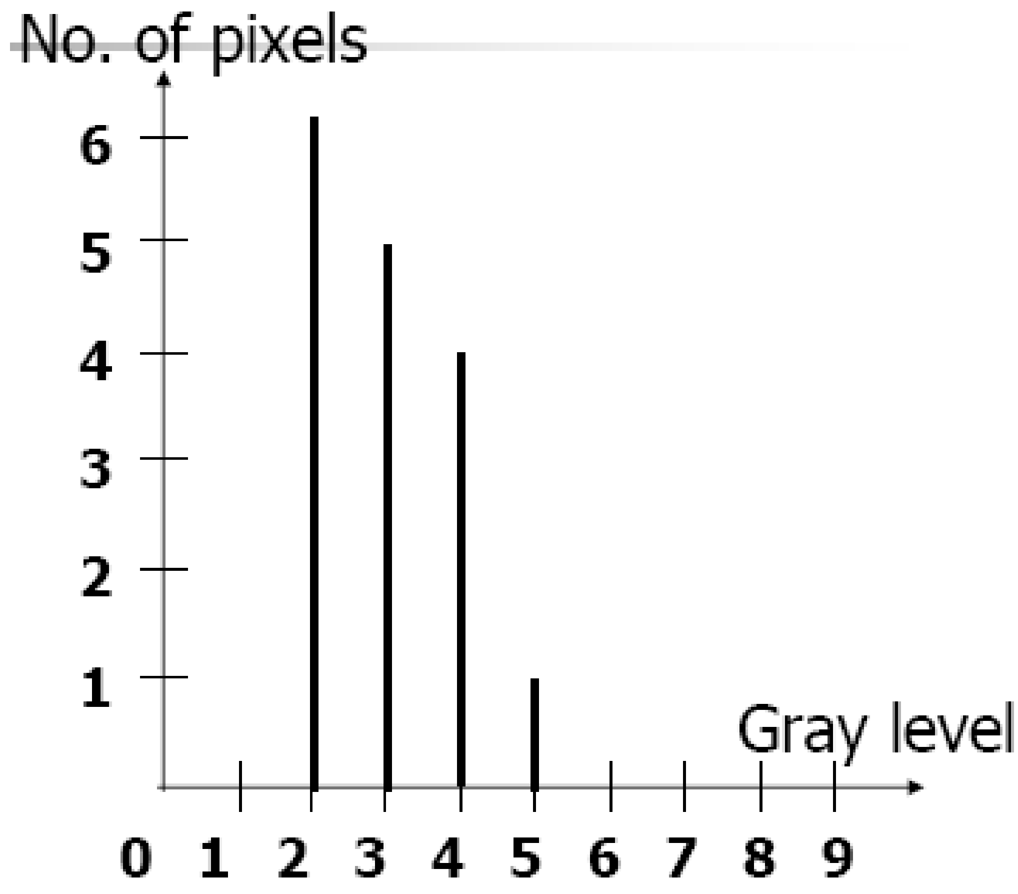
- Normalisasi Histogram berguna untuk melihat statistika dari image.
- Normalized histogram: $p(r_k)=n_k/n$
  - Jumlah keseluruhan komponen = 1
- Adalah membagi setiap nilai dari histogram dengan jumlah pixel dari image (n),

$$p(rk) = nk /n.$$

2	3	3	2
4	2	4	3
3	2	3	5
2	4	2	4

4x4 image

Gray scale = [0,9]



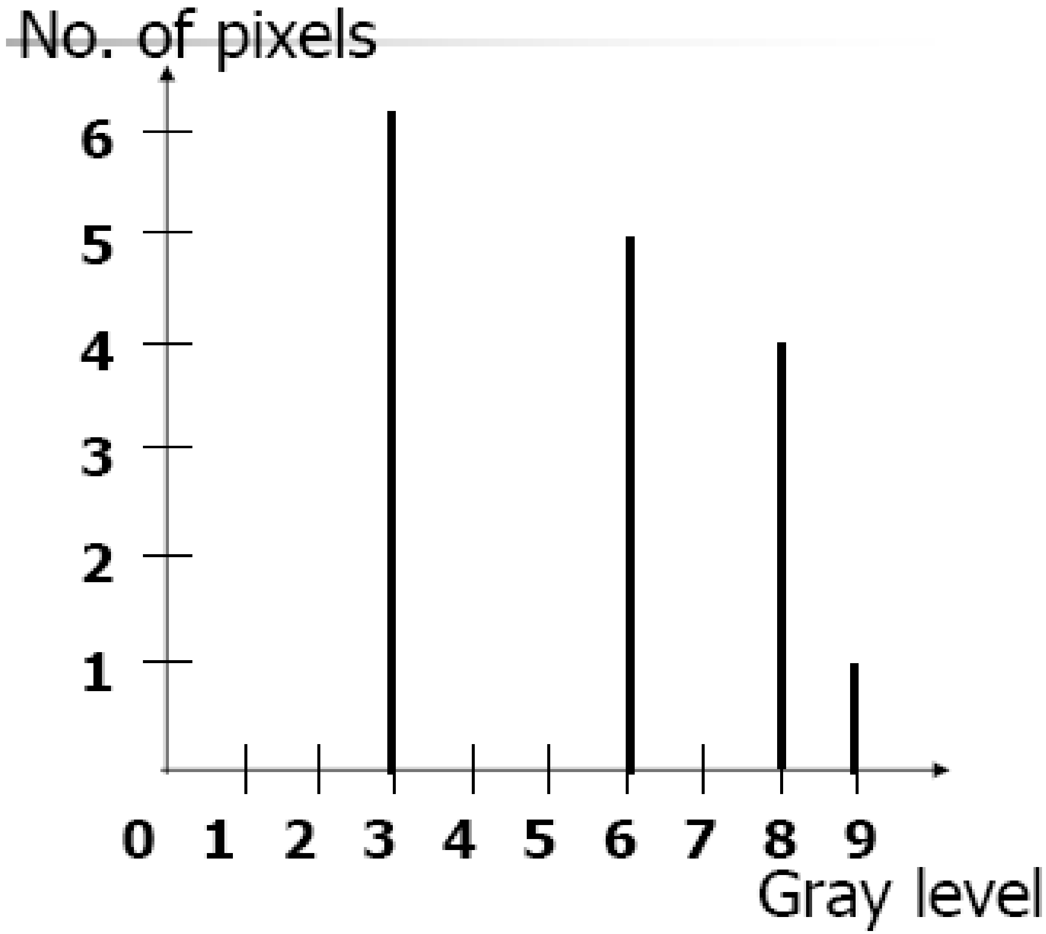
histogram

Gray Level(j)	0	1	2	3	4	5	6	7	8	9
No. of pixels	0	0	6	5	4	1	0	0	0	0
$\sum_{j=0}^{k} n_j$	0	0	6	11	15	16	16	16	16	16
$s = \sum_{j=0}^{k} \frac{n_j}{n}$	0	0	6 / 16	11 / 16	15 / 16	16 / 16	16 / 16	16 / 16	16 / 16	16 / 16
s x 9	0	0	3.3 $\approx$3	6.1 $\approx$6	8.4 $\approx$8	9	9	9	9	9

3	6	6	3
8	3	8	6
6	3	6	9
3	8	3	8

Output image

Gray scale = [0,9]



Histogram equalization

Contoh

- Diberikan sebuah image 8-level berukuran 64 x 64 dengan nilai gray value (0, 1, …, 7). Nilai normalisasi dari gray value adalah (0, 1 / 7 , 2 / 7 , … , … , … . , … . , 1) .

k	r_k	n_k	$p(r_k) = n_k/n$
0	0	790	0.19
1	1/7	1023	0.25
2	2/7	850	0.21
3	3/7	656	0.16
4	4/7	329	0.08
5	5/7	245	0.06
6	6/7	122	0.03
7	1	81	0.02

$$s_0 = T(r_0) = \sum_{j=0}^{0} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) = 0.19 \rightarrow {}^{1}\!/_{7}$$

$$s_1 = T(r_1) = \sum_{j=0}^{1} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) = 0.44 \rightarrow {}^{3}\!/_{7}$$

$$s_2 = T(r_2) = \sum_{j=0}^{2} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + p_{\text{in}}(r_2) = 0.65 \rightarrow {}^{5}\!/_{7}$$

$$s_3 = T(r_3) = \sum_{j=0}^{3} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + \cdots + p_{\text{in}}(r_3) = 0.81 \rightarrow {}^{6}\!/_{7}$$

$$s_4 = T(r_4) = \sum_{j=0}^{4} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + \cdots + p_{\text{in}}(r_4) = 0.89 \rightarrow {}^{6}\!/_{7}$$

$$s_5 = T(r_5) = \sum_{j=0}^{5} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + \cdots + p_{\text{in}}(r_5) = 0.95 \rightarrow 1$$

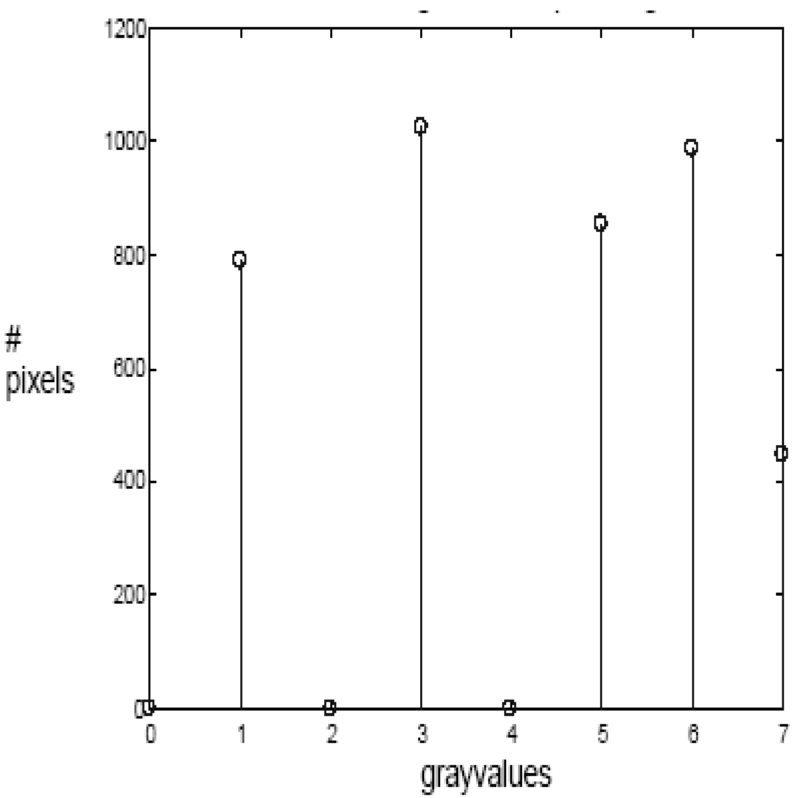
$$s_6 = T(r_6) = \sum_{j=0}^{6} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + \cdots + p_{\text{in}}(r_6) = 0.98 \rightarrow 1$$

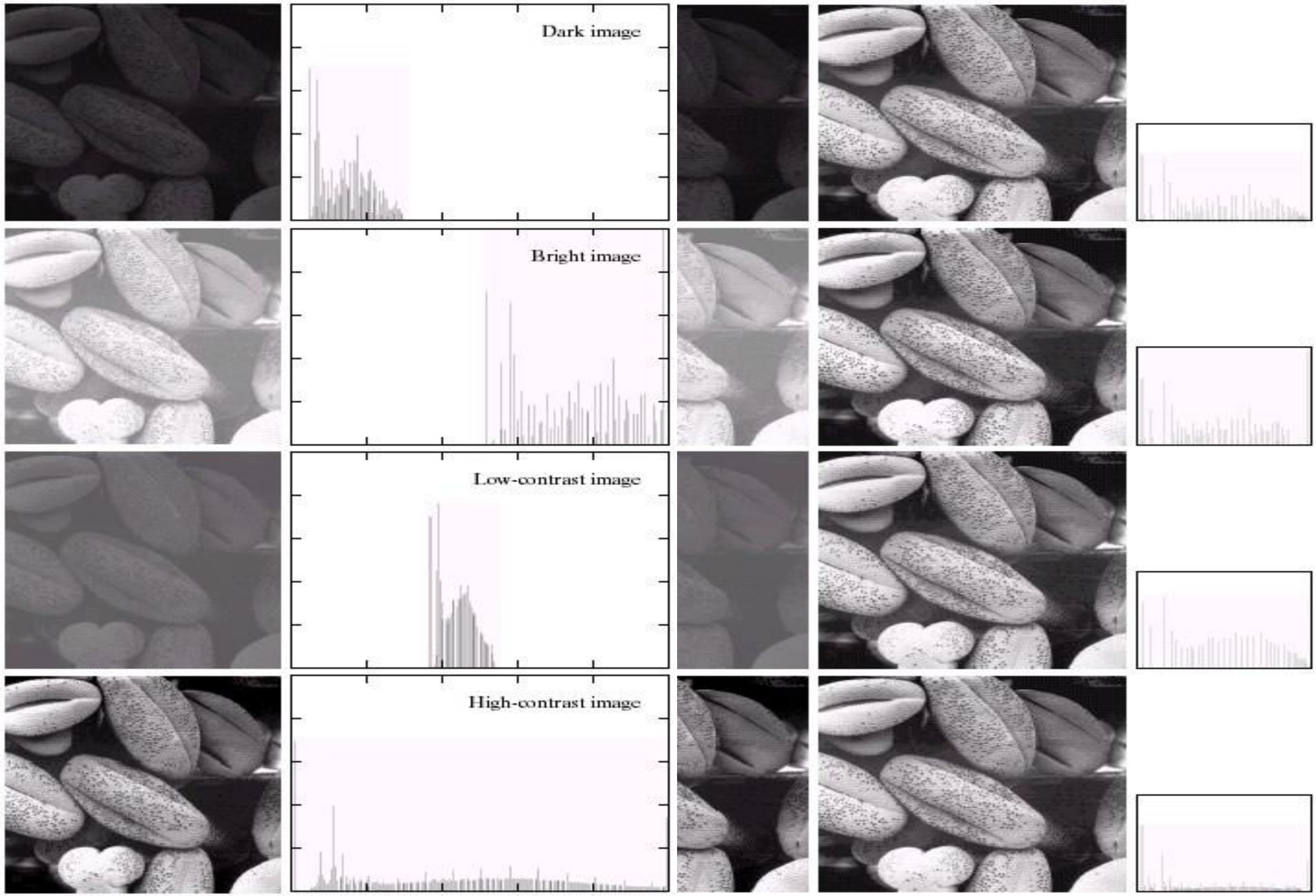
$$s_7 = T(r_7) = \sum_{j=0}^{7} p_{\text{in}}(r_j) = p_{\text{in}}(r_0) + p_{\text{in}}(r_1) + \cdots + p_{\text{in}}(r_7) = 1.00 \rightarrow 1$$

- Hanya ada 5 nilai gray level yang berbeda yang berpengaruh dalam image tsb.

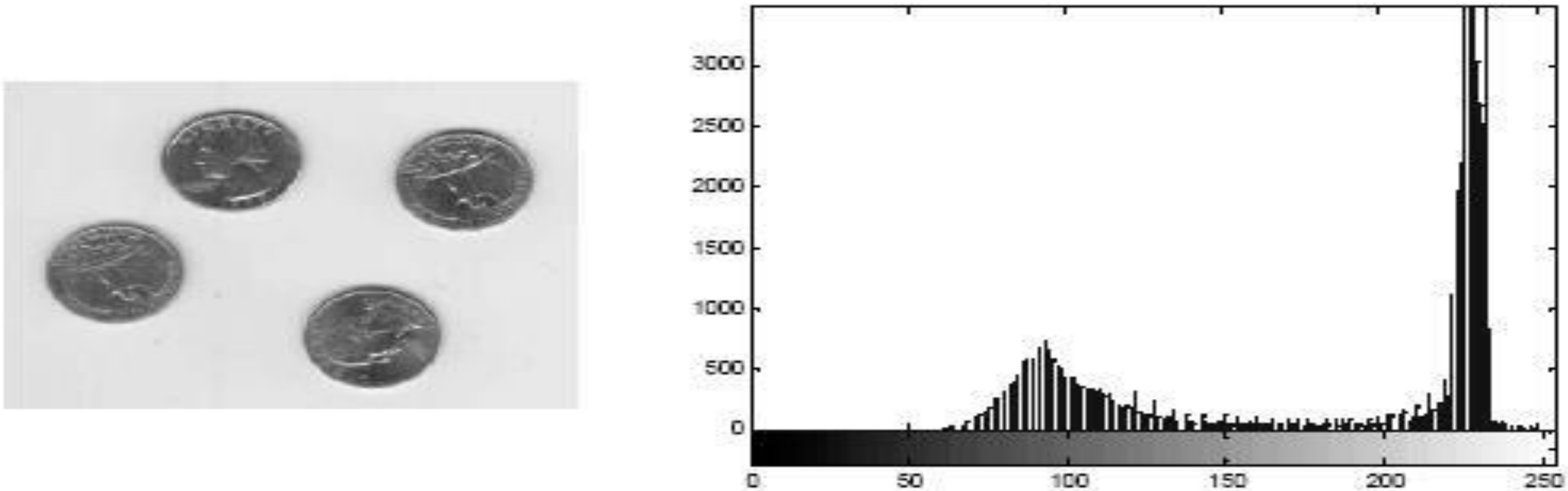
k	s_k	n_k	$p(s_k) = n_k/n$
0	1/7	790	0.19
1	3/7	1023	0.25
2	5/7	850	0.21
3	6/7	985	0.24
4	1	448	0.11

Hasil ekualisasi adalah pendekatan terhadap bentuk histogram yang uniform

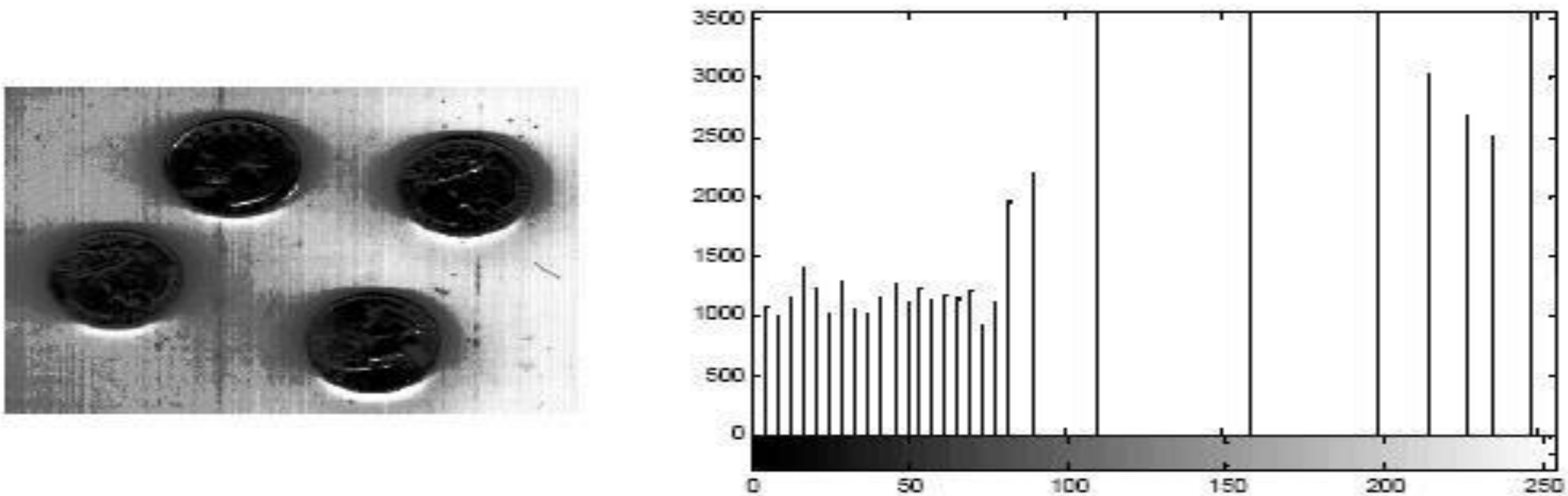




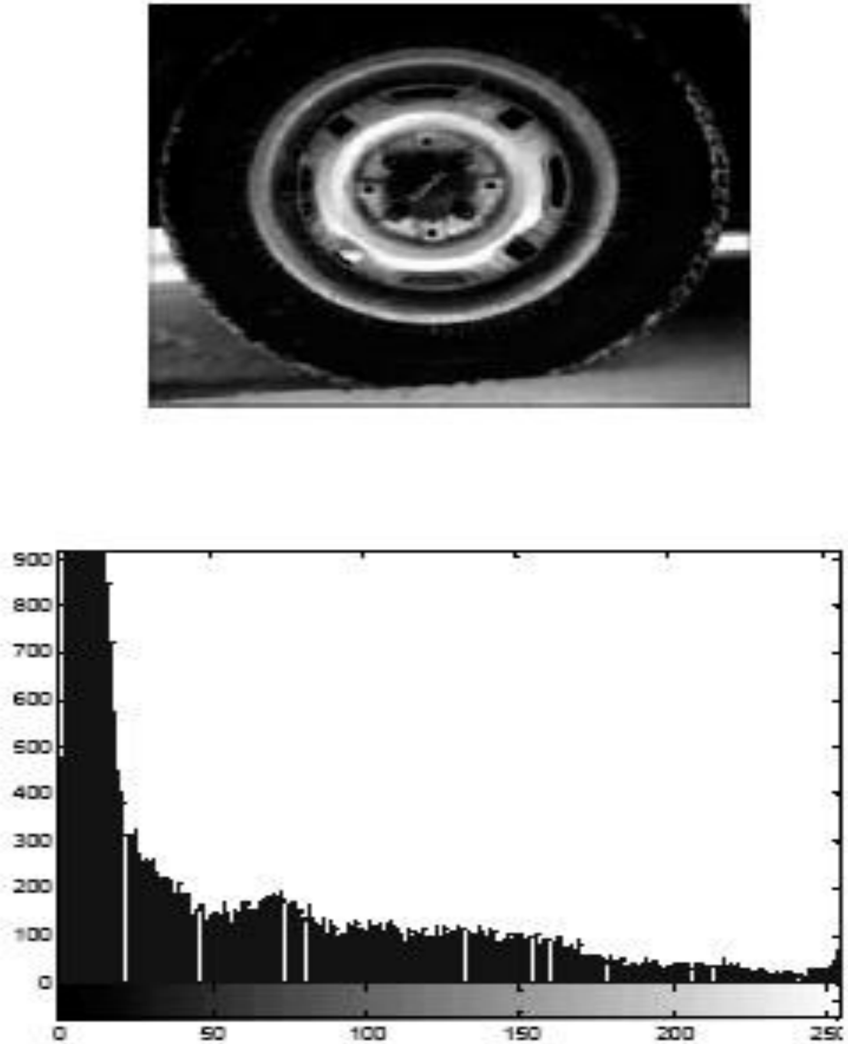
Dark image
Bright image
Low-contrast image
High-contrast image



Original Image and its histogram

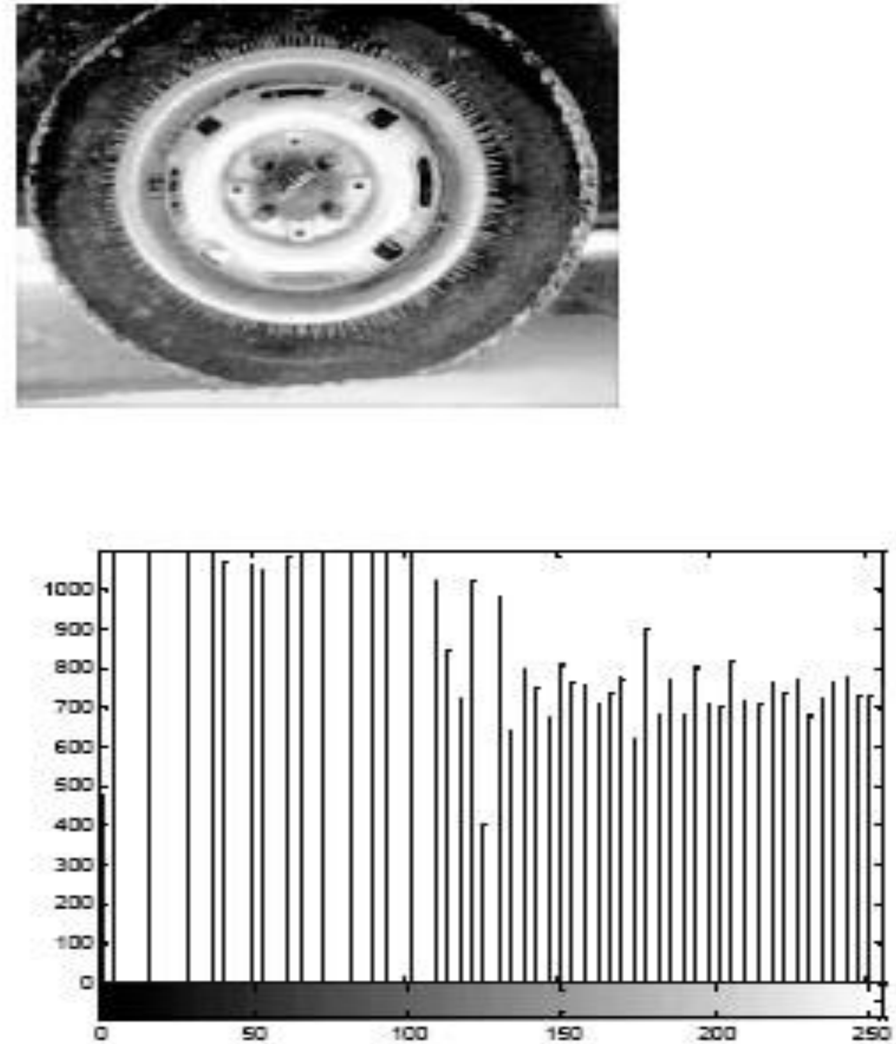


Histogram equalized image and its histogram



Original image and its histogram

```
a=imread('tire.tif');⊠
imhist(a);⊠
```



Histogram equalized image and its histogram

```
b=histeq(a);⊠
imhist(b);⊠
```

Spatial Filtering

Peningkatan Kualitas Citra

Mask Processing

- Jika pada point processing kita hanya melakukan operasi terhadap masing-masing piksel, maka pada mask processing kita melakukan operasi terhadap suatu jendela ketetanggaan pada citra.
- Kemudian kita menerapkan (mengkonvolusikan) suatu *mask* terhadap jendela tersebut.
- *Mask* sering juga disebut *filter, window, kernel.*

Jenis-jenis filter spasial

- Smoothing filters:
  - Lowpass filter (linear filter, mengambil nilai rata-rata)
  - Median filter (non-linear filter, mengambil median dari setiap jendela ketetanggan)
- Sharpening filters:
  - Highpass filter
  - Roberts
  - Prewitt
  - Sobel

Contoh penerapan filter spasial

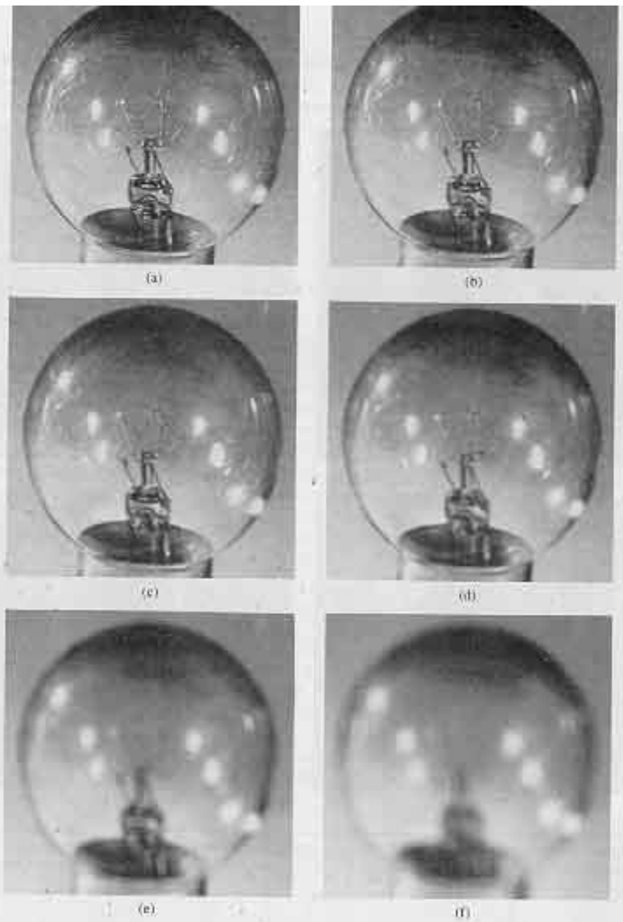


Figure 4.22 (a) Original image; (b)–(f) results of spatial lowpass filtering with a mask of size $n \times n$, $n = 3, 5, 7, 15, 25$.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Average lowpass filter

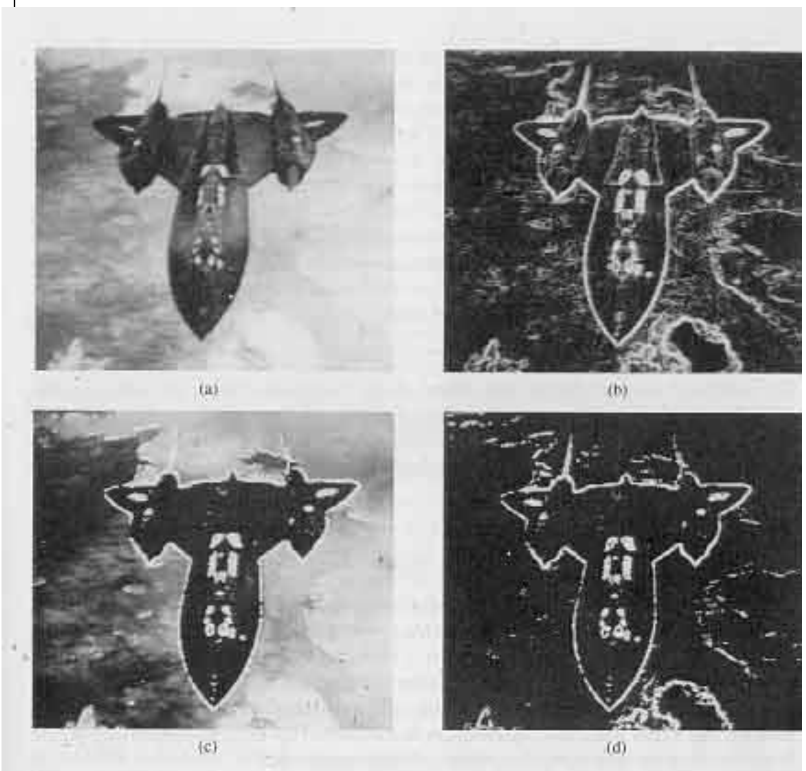
(a) Gambar Asli
(b)-(f) hasil dari spatial lowpass filtering dengan ukuran mask 3, 5, 7, 15, 25

Contoh penerapan filter low pass dan median

(a) Gambar asli
(b) Gambar yang diberi noise
(c) Hasil dari 5x5 lowpass average filtering
(d) Hasil dari 5x5 median filtering

Figure 4.23 (a) Original image; (b) image corrupted by impulse noise; (c) result of 5×5 neighborhood averaging; (d) result of 5×5 median filtering. (Courtesy of Martin Connor, Texas Instruments, Inc., Lewisville, Tex.)

Contoh Highpass Filtering



-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Sobel

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Prewitt

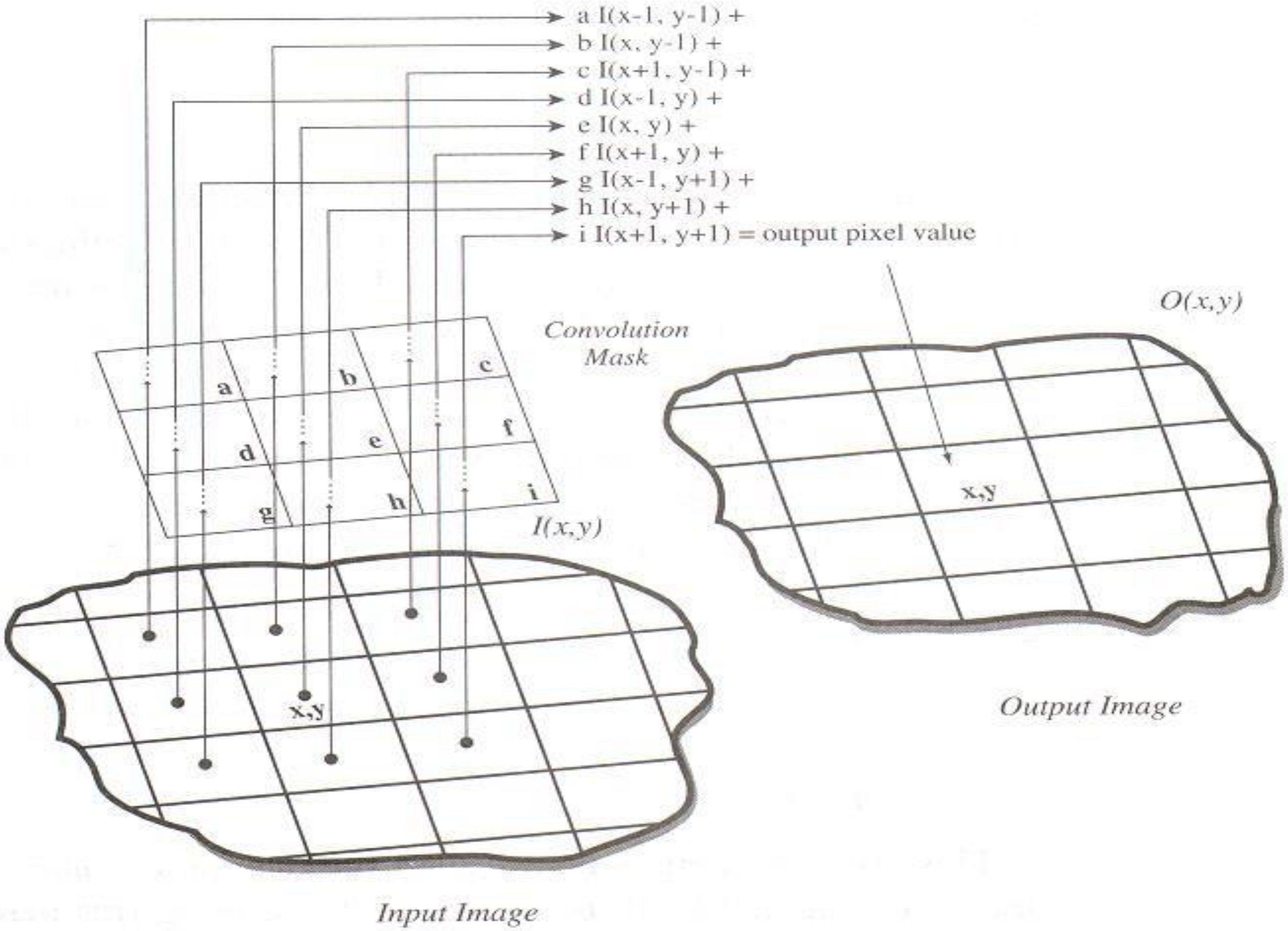
(a) Gambar awal, (b) hasil dari Prewitt Mask, (c) thresholding dari (b) pada nilai > 25 (d) thresholding dari (b) pada nilai >25 dan < 25 (black)

Pixel Group Processing

1	2	3
8	x	4
7	6	5

Contoh:
Jendela ketetanggan 3x3,
Nilai piksel pada posisi x dipengaruhi oleh nilai 8 tetangganya

→ Perbedaan dengan point processing: pada point processing, nilai suatu piksel tidak dipengaruhi oleh nilai tetangga-tetangganya



$$\begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$$

$$\begin{aligned} O(x,y) = {} & aI(x-1,y-1) + bI(x,y-1) + cI(x+1,y-1) \\ & + dI(x-1,y) + eI(x,y) + fI(x+1,y) \\ & + gI(x-1,y+1) + hI(x,y+1) + iI(x+1,y+1) \end{aligned}$$

W_1	W_2	W_3
W_4	W_5	W_6
W_7	W_8	W_9

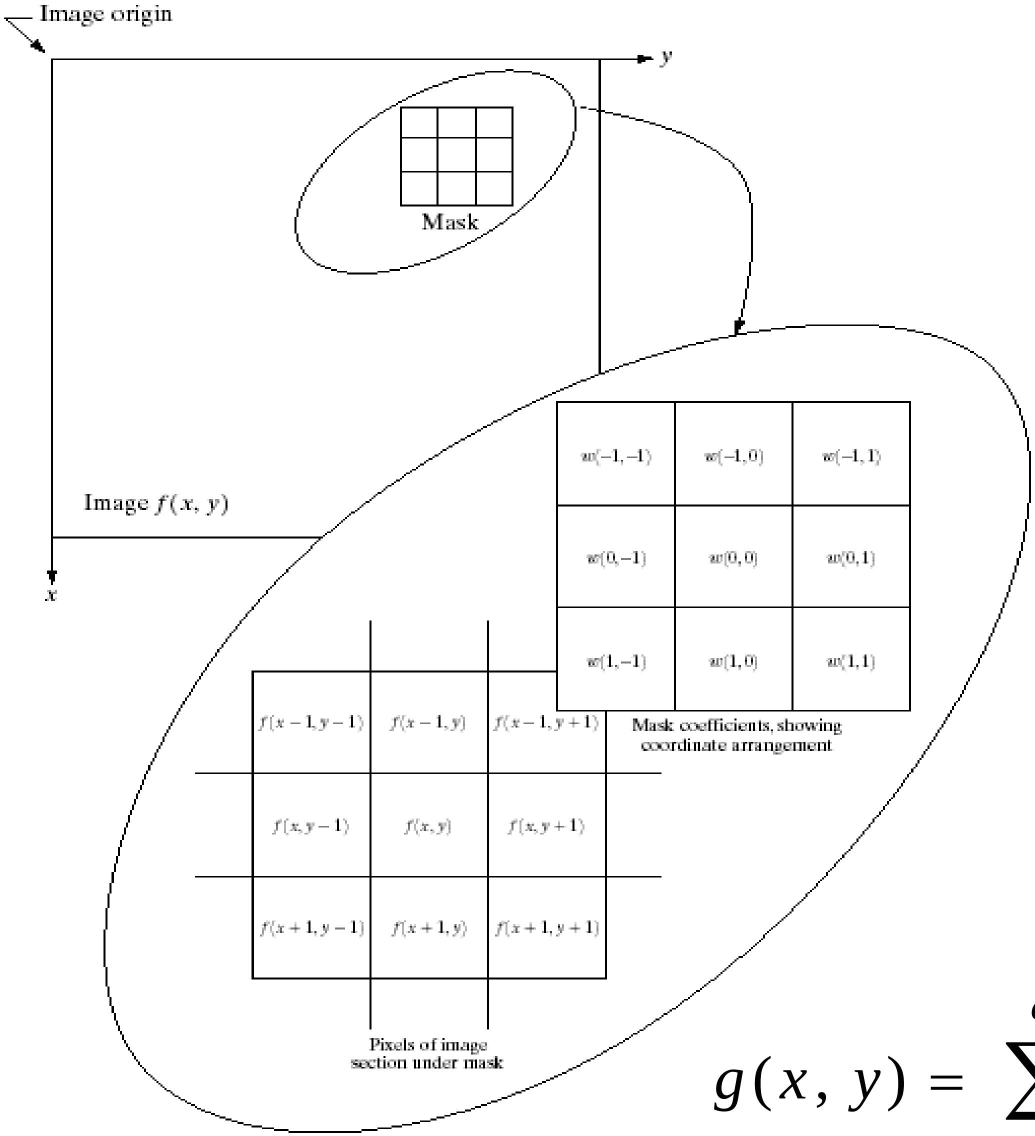
Contoh sebuah mask berukuran 3x3. Filter ini akan diterapkan / dikonvolusikan pada setiap jendela ketetanggaan 3x3 pada citra

G_{11}	G_{12}	G_{13}	G_{14}	G_{15}
G21	G_{22}	G_{23}	G_{24}	G_{25}
G_{31}	G_{32}	G_{33}	G_{34}	G_{35}
G_{41}	G_{42}	G_{43}	G_{44}	G_{45}
G_{51}	G_{52}	G_{53}	G_{54}	G_{55}

$$
\begin{aligned}
\mathbf{G_{22}'} = \; & \mathbf{w_1 G_{11} + w_2 G_{12} + w_3 G_{13} +} \\
& \mathbf{w_4 G_{21} + w_5 G_{22} + w_6 G_{23} +} \\
& \mathbf{w_7 G_{31} + w_8 G_{32} + w_9 G_{33}}
\end{aligned}
$$

Spatial Filtering

- 2D Finite Impulse Response (FIR) filtering
  - Mask filtering: operasi konvolusi image dengan 2 D masking
  - Aplikasinya antara lain untuk image enhancement:
    - Smoothing: low pass
    - Sharpening: high pass
- Data-dependent nonlinear filters
  - Local histogram
  - Order statistic filters
    - Medium filter



Spatial filtering adalah operasi yang dilakukan terhadap intensitas pixel dari suatu image dan bukan terhadap komponen frekuensi dari image

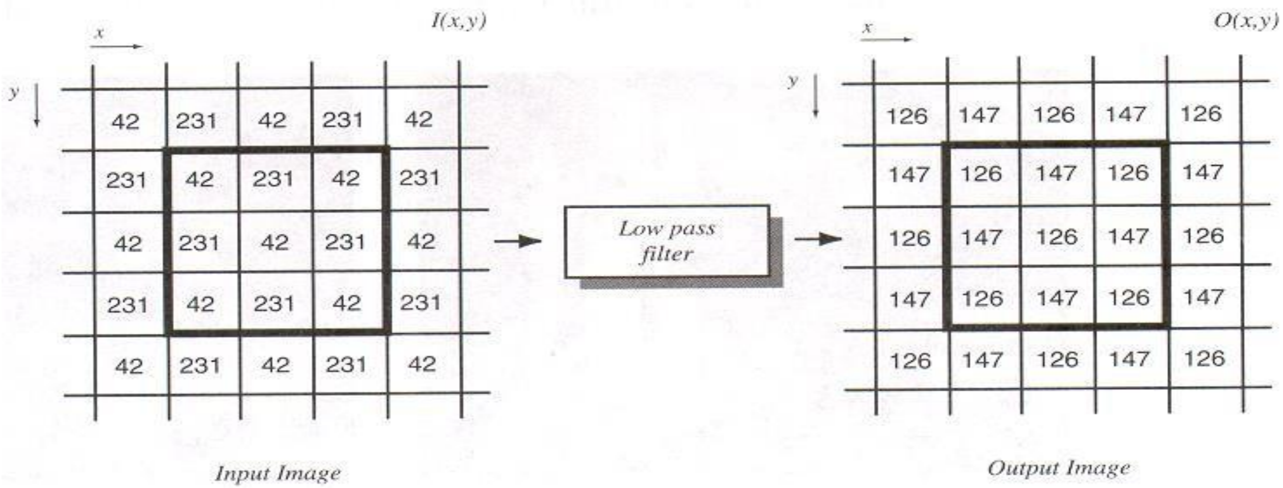
$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

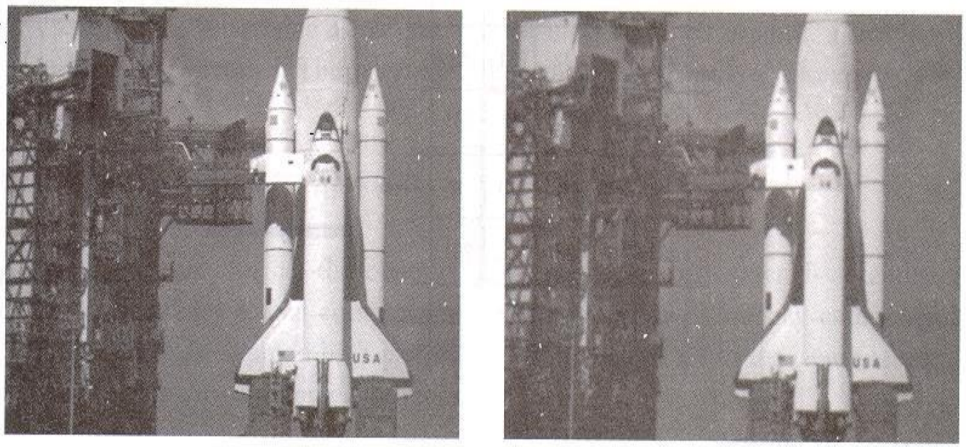
$$a = (m - 1) / 2 \qquad b = (n - 1) / 2$$

Spatial Filtering

Low-Pass Spatial Filter

$$
\begin{matrix}
\frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\
\frac{1}{9} & \frac{1}{9} & \frac{1}{9}
\end{matrix}
$$

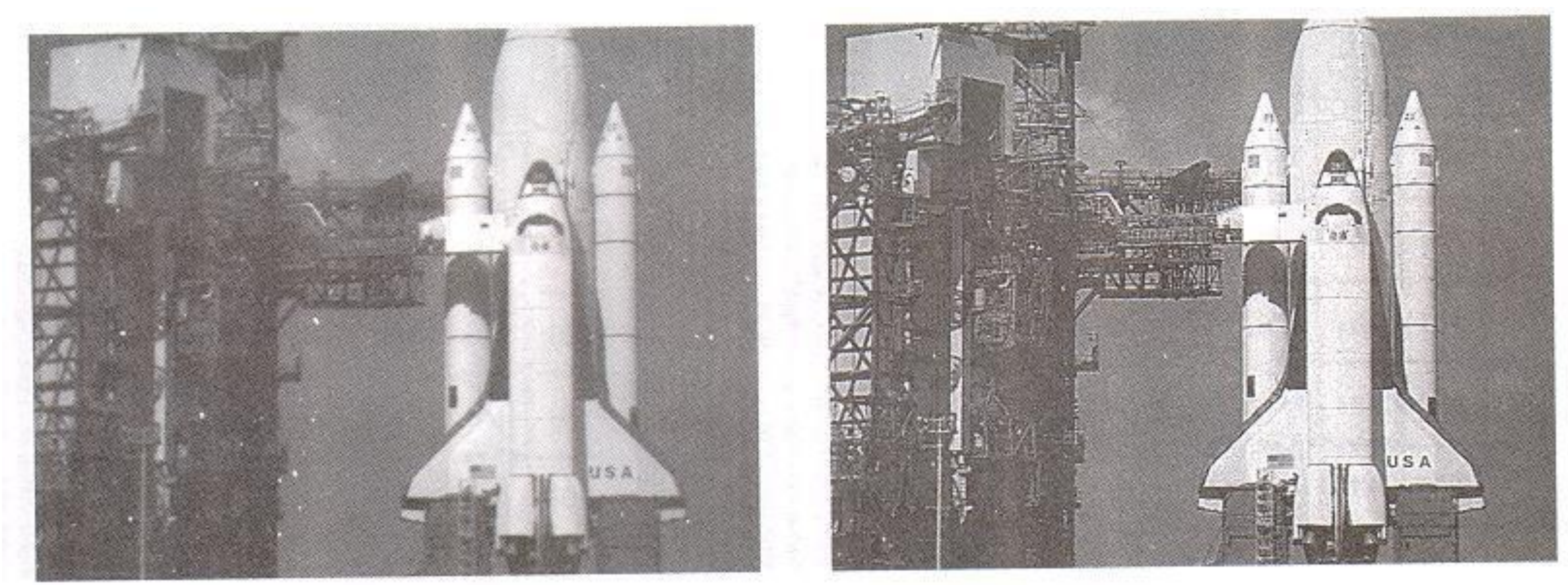




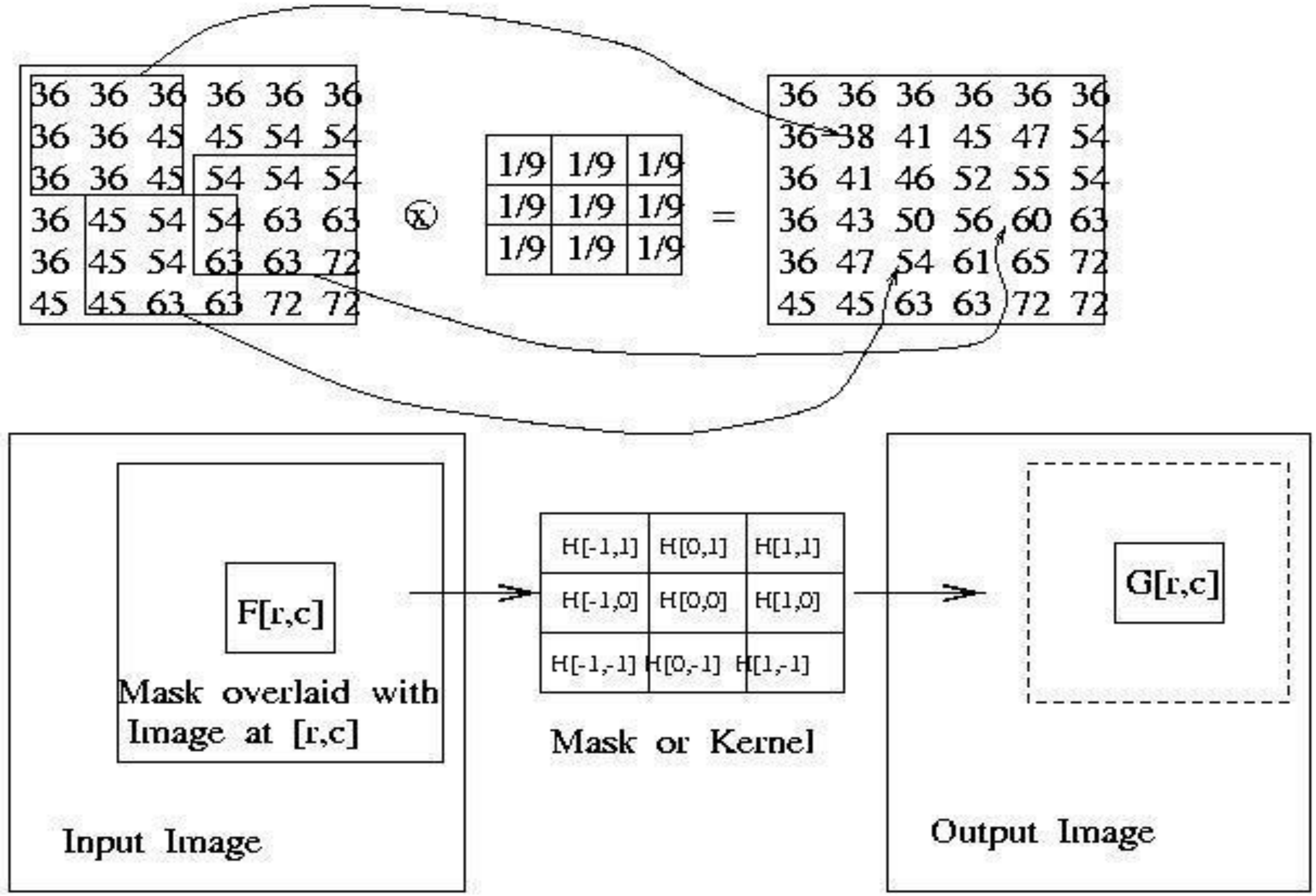
Spatial Filtering

High-Pass Spatial Filter

$$\begin{matrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{matrix}$$



Konvolusi Citra



Smoothing Spatial Filters
Linear averaging (lowpass) filters

Smoothing filters digunakan untuk kepentingan :
- Reduksi Noise
- Smoothing of false contours
- Reduksi dari detail yang irrelevant

Efek lain yang tidak diharapkan dari penggunaan smoothing filters
- Blur edges

Penggunaan Weighted average filter Akan mereduksi efek blurring dalam smoothing process.

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Box filter | **Weighted average**

a b

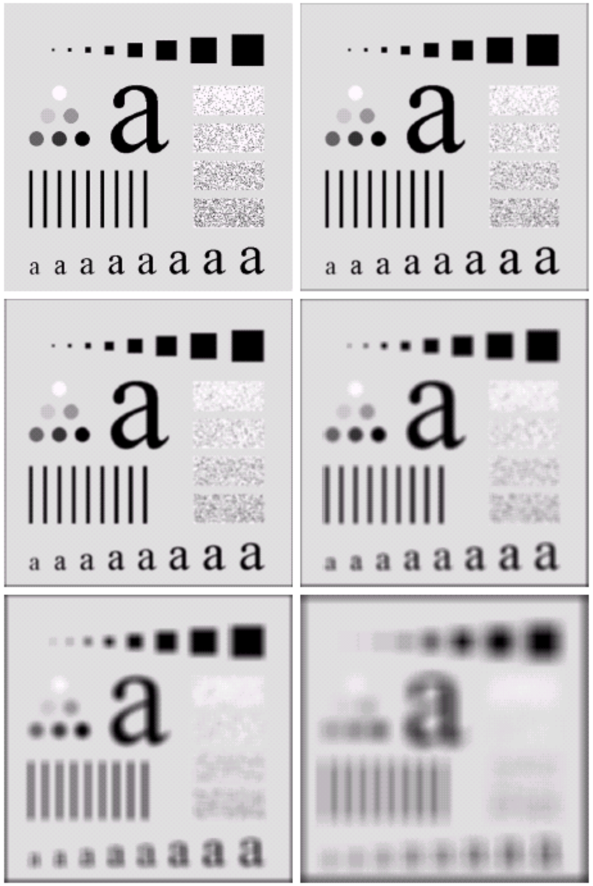
FIGURE 3.34 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to the sum of the values of its coefficients, as is required to compute an average.

Smoothing Linear Filters

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \qquad \frac{1}{16} \times \begin{array}{|c|c|c|} \hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

$$g(m,n) = \frac{\sum_{i=-I}^{I} \sum_{j=-J}^{J} w(i,j)\, f(m-i, n-j)}{\sum_{i=-I}^{I} \sum_{j=-J}^{J} w(i,j)}$$

Normalization of coefficient to ensure
$0 \leq g(m,n) \leq L\text{-}1$



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15,$ and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

Sharpening Linear Filters

- High boosting filter:

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

  - $A \geq 1$

- Derivative filter:
  - Use derivatives to approximate high pass filters. Usually 2nd derivatives are preferred. The most common one is the Laplacian operator.

- Laplacian operator:

$$\nabla^2 f(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

$$= f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1) - 4f(x,y)$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

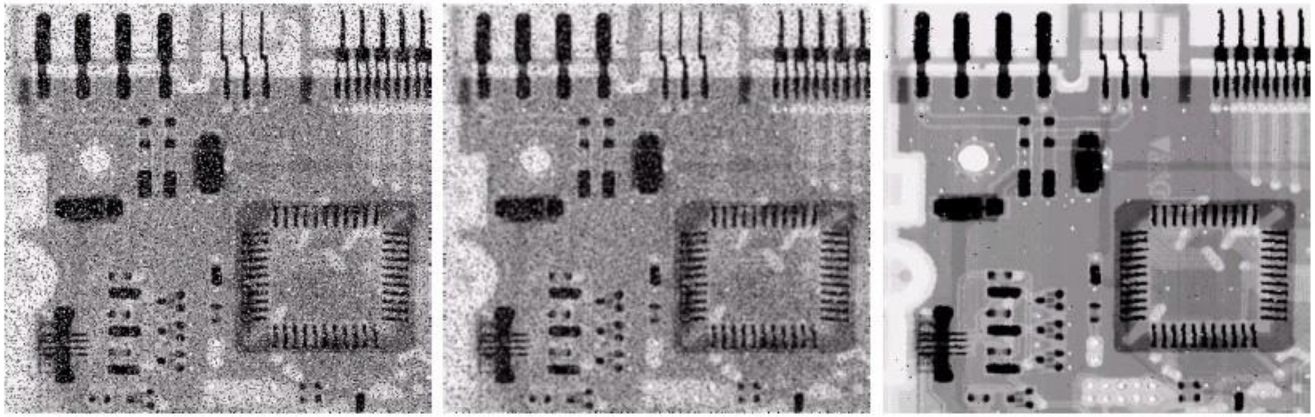
Order Statistics Filters

Order-statistics filters adalah filter nonlinear spatial dengan response didasarkan pada urutan / ranking dari pixels yang termuat dalam area image yang dicover oleh filter, kemudian mengganti nilai tengah pixel dengan nilai yang ditentukan oleh urutan tersebut.

3×3 Median filter	[10 125 125 135 141 141 144 230 240] = 141
3×3 Max filter	[10 125 125 135 141 141 144 230 240] = 240
3×3 Min filter	[10 125 125 135 141 141 144 230 240] = 10

$n = 3$
Average
filter

$n = 3$
Median
filter



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

High-boost Filtering

- Unsharp masking:

$$f_s(x,y) = f(x,y) - \overline{f}(x,y)$$

- Highpass filtered image =

  Original – lowpass filtered image.

- If A is an amplification factor then:

  - High-boost = A · original – lowpass (blurred)

    = (A-1) · original + original – lowpass

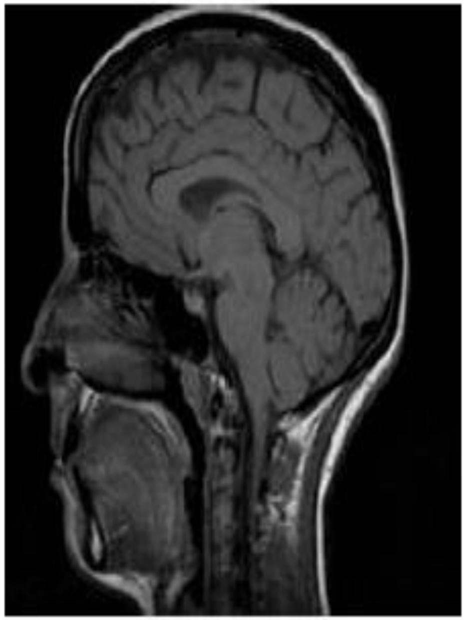
    = (A-1) · original + highpass

High-boost Filtering

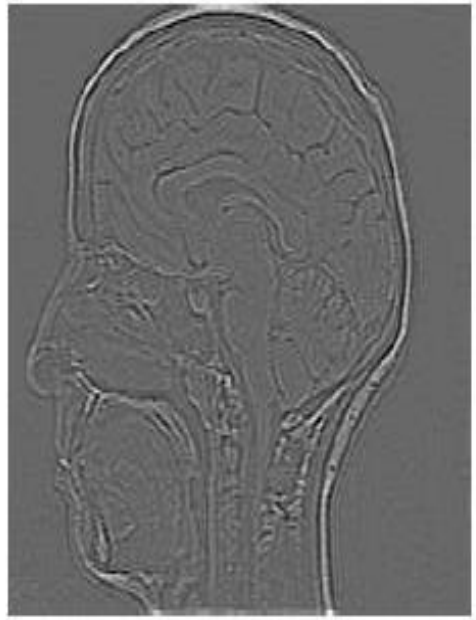
- A=1 : standard highpass result
- A>1 : the high-boost image looks more like the original with a degree of edge enhancement, depending on the value of A.

$$\frac{1}{9} \times \begin{array}{|c|c|c|} \hline -1 & -1 & -1 \\ \hline -1 & w & -1 \\ \hline -1 & -1 & -1 \\ \hline \end{array}$$

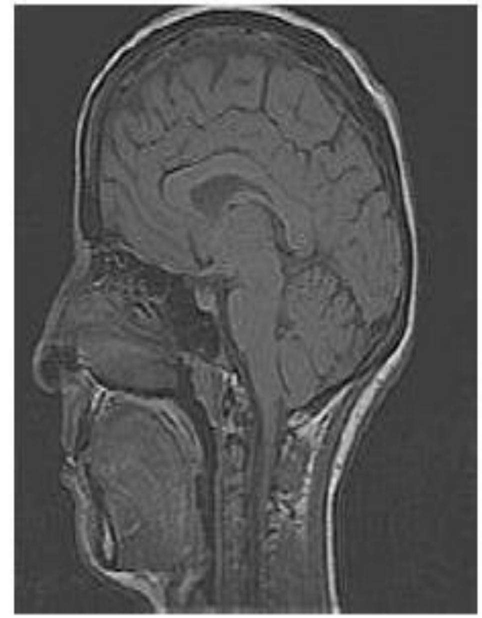
$w=9A-1,\ A\geq 1$



Original Image



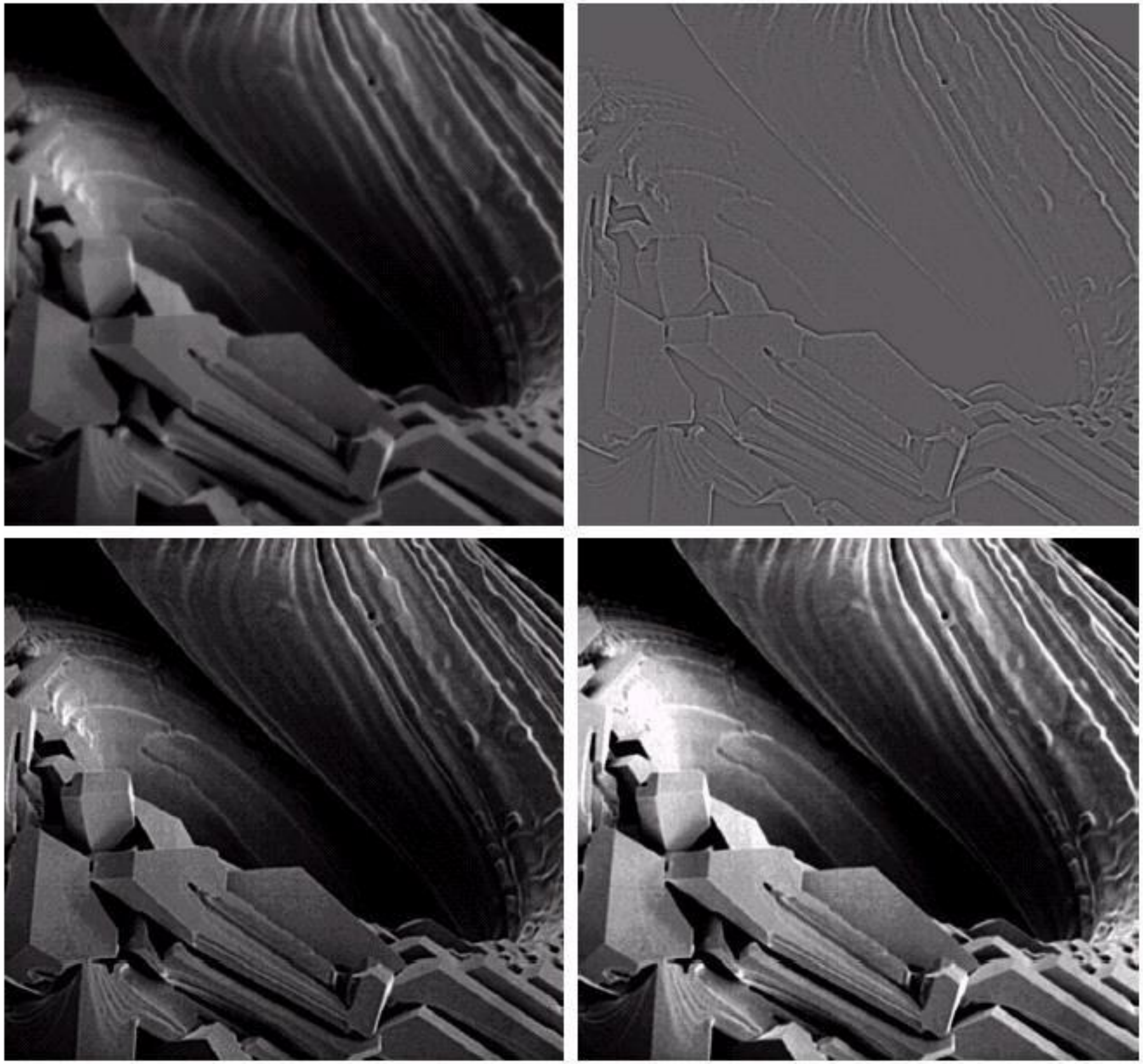
Highpass filtering



High-boost filtering

a b
c d

FIGURE 3.43
(a) Same as Fig. 3.41(c), but darker.
(a) Laplacian of (a) computed with the mask in Fig. 3.42(b) using $A = 0$.
(c) Laplacian enhanced image using the mask in Fig. 3.42(b) with $A = 1$. (d) Same as (c), but using $A = 1.7$.



1st Derivatives

- The most common method of differentiation in Image Processing is the *gradient*:

$$\nabla F = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix} \quad \text{at } (x,y)$$

- The magnitude of this vector is:

$$\nabla f = mag(\nabla f) = [G_x^2 + G_y^2]^{\frac{1}{2}} = \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

The Gradient

- Non-isotropic (regardless direction)
- Its magnitude (often call the gradient) is rotation invariant
- Computations: $$\nabla f \approx |G_x| + |G_y|$$
- Roberts uses: $$G_x = (z_9 - z_5)$$ $$G_y = (z_8 - z_6)$$
- Approximation (Roberts Cross-Gradient Operators):

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

a
b c
d e

FIGURE 3.44
A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

(a)

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

(b)

−1	0
0	1

(c)

0	−1
1	0

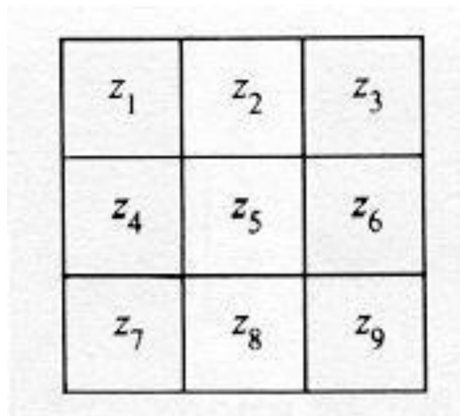
(d)

−1	−2	−1
0	0	0
1	2	1

(e)

−1	0	1
−2	0	2
−1	0	1

Derivative Filters



At z_5, the magnitude can be approximated as:

$$\nabla f \approx [(z_5 - z_8)^2 + (z_5 - z_6)^2]^{1/2}$$

$$\nabla f \approx |z_5 - z_8| + |z_5 - z_6|$$

Derivative Filters

- Another approach is:

$$\nabla f \approx [(z_5 - z_9)_2 + (z_6 - z_8)_2]_{1/2}$$

$$\nabla f \approx | z_5 - z_9 | + | z_6 - z_8 |$$

- One last approach is (Sobel Operators):

$$\nabla f = |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$

a
b c
d e

FIGURE 3.44 A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum to zero, as expected of a derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Robert operator

−1	0
0	1

0	−1
1	0

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Sobel operators

Example : Robert Operator

$$\begin{vmatrix} 4 & 5 & 7 & 5 & 1 \\ 2 & 1 & 3 & 4 & 5 \\ 4 & 3 & 2 & 6 & 9 \\ 4 & 2 & 5 & 7 & 1 \\ 2 & 4 & 8 & 6 & 3 \end{vmatrix} \qquad \begin{vmatrix} 6 & 8 & 5 & 3 & 1 \\ 4 & 1 & 5 & 6 & 5 \\ 3 & 2 & 6 & 7 & 9 \\ 0 & 7 & 2 & 5 & 1 \\ 2 & 4 & 8 & 6 & 3 \end{vmatrix}$$

citra awal

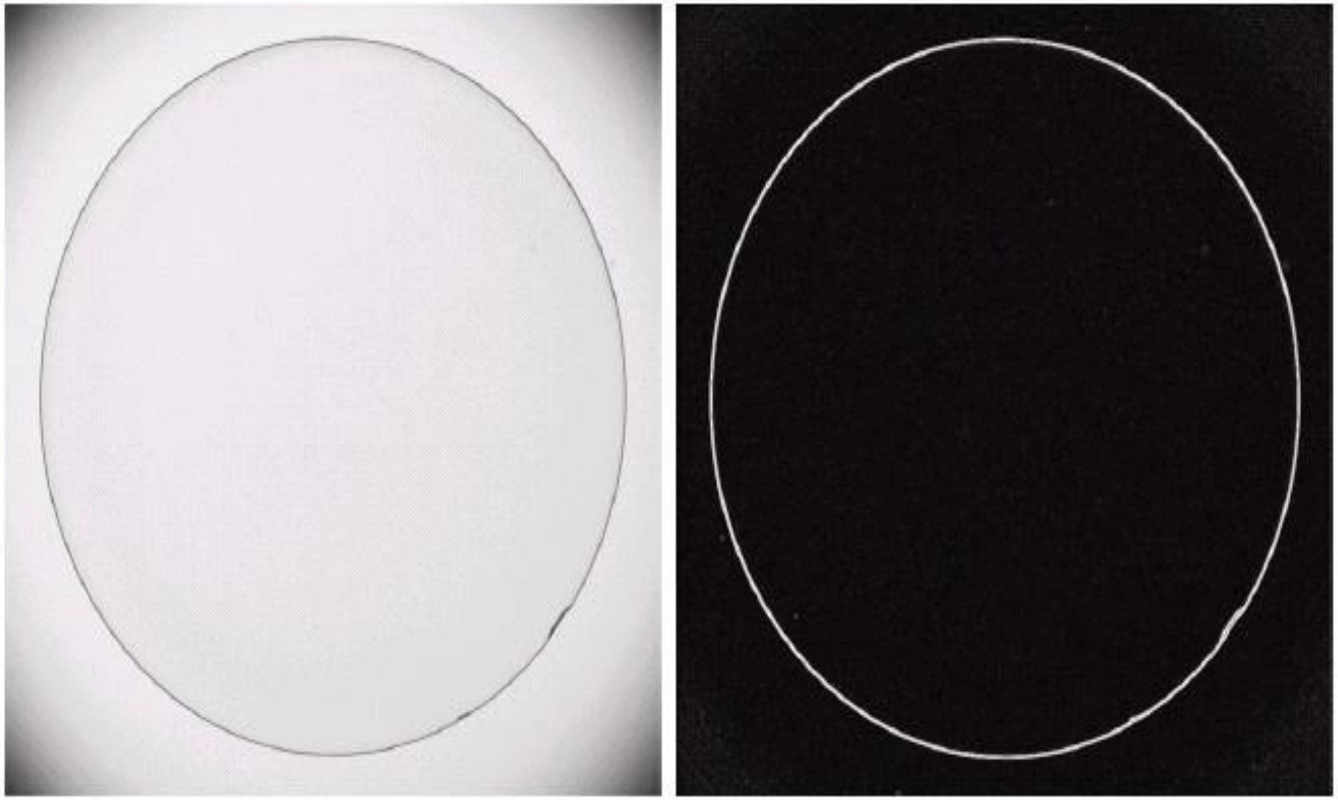
citra hasil deteksi tepi

$f'[0,0] = |4-1| + |5-2| = 6$

Robert operator

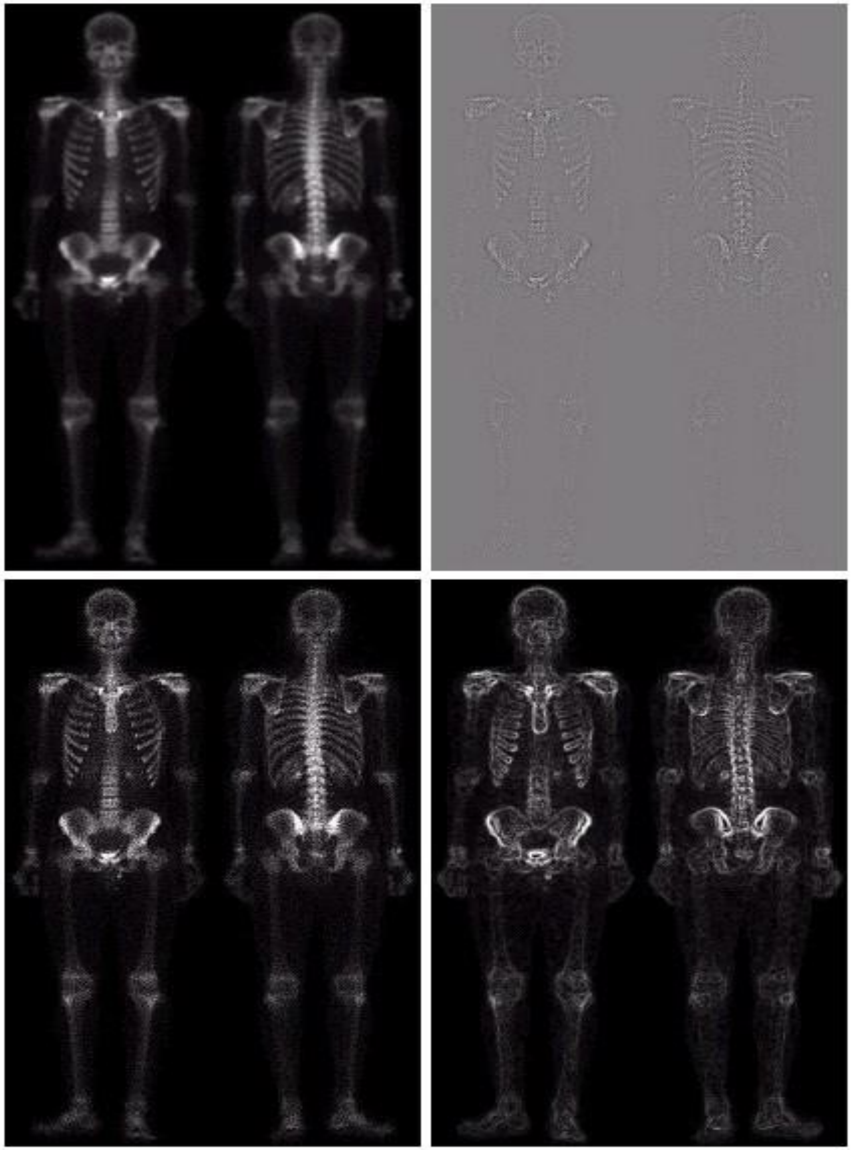
−1	0
0	1

0	−1
1	0



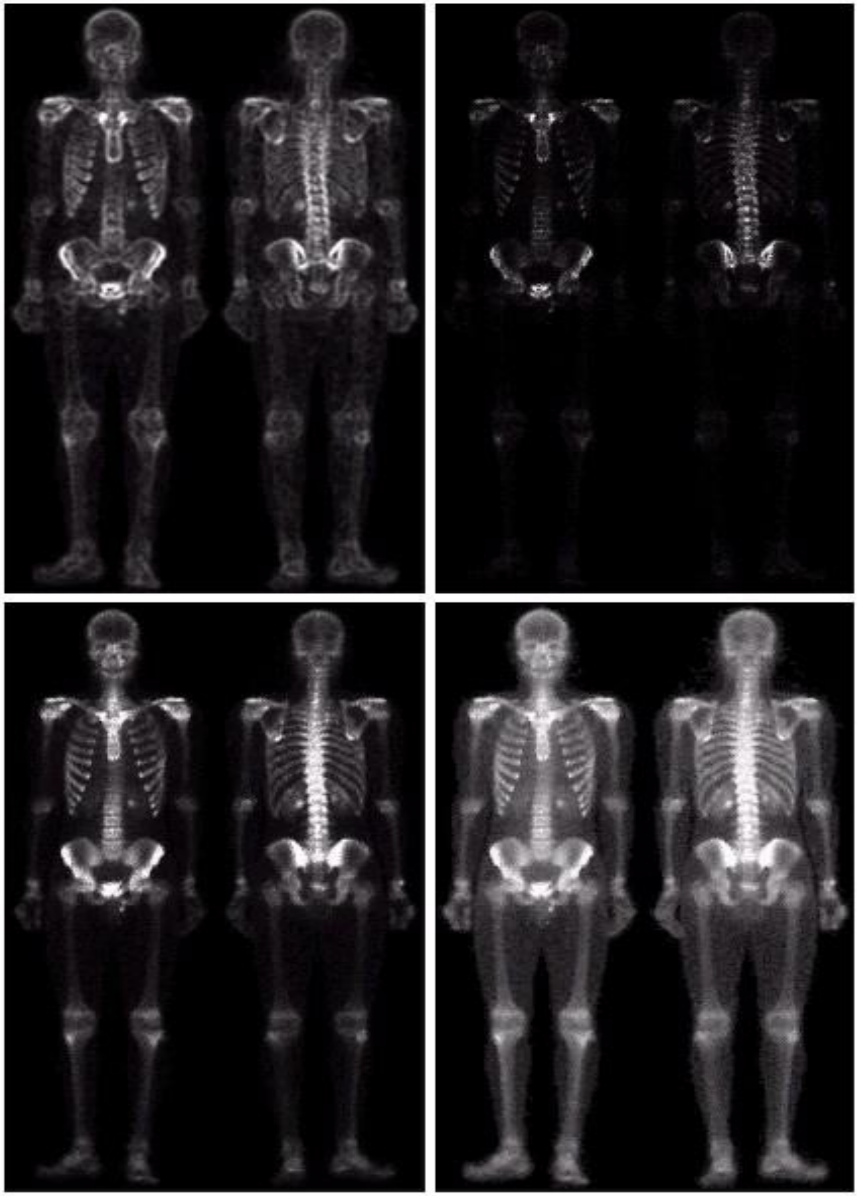
a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)



a	b
c	d

FIGURE 3.46
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).



e f
g h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)

(b)

Penggunaan Bentuk Turunan ke 2

- Isotropic filters: rotation invariant
- Laplacian (linear operator):

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

- Discrete version:

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Laplacian

- Digital implementation:

$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

- Two definitions of Laplacian: one is the negative of the other
- Accordingly, to recover background features:

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & (I) \\ f(x,y) + \nabla^2 f(x,y) & (II) \end{cases}$$

I: if the center of the mask is negative

II: if the center of the mask is positive

Simplification

- Filter and recover original part in one step:

$$g(x,y)=f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]+4f(x,y)$$

$$g(x,y)=5f(x,y)-[f(x+1,y)+f(x-1,y)+f(x,y+1)+f(x,y-1)]$$

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

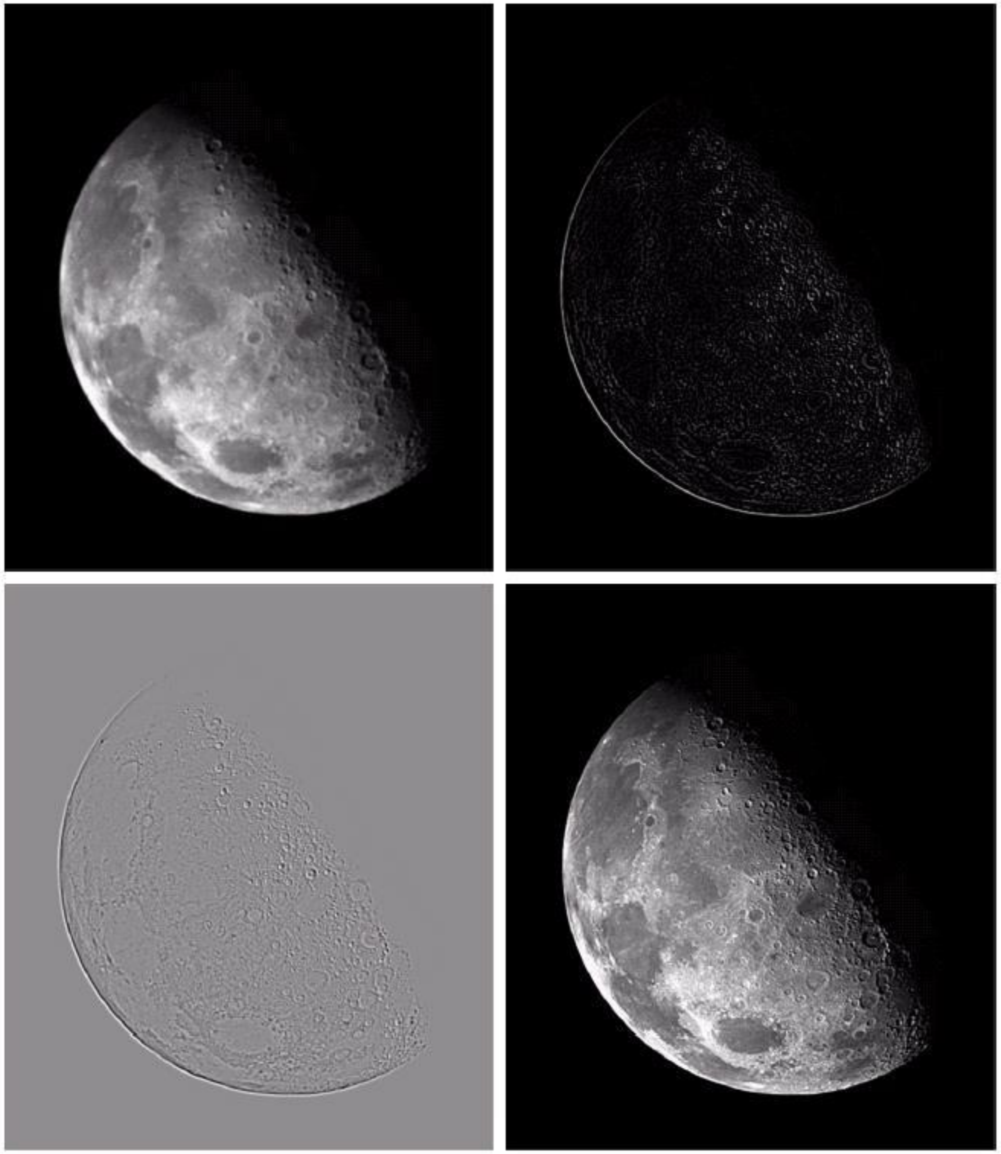
−1	−1	−1
−1	8	−1
−1	−1	−1

a	b
c	d

FIGURE 3.39
(a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

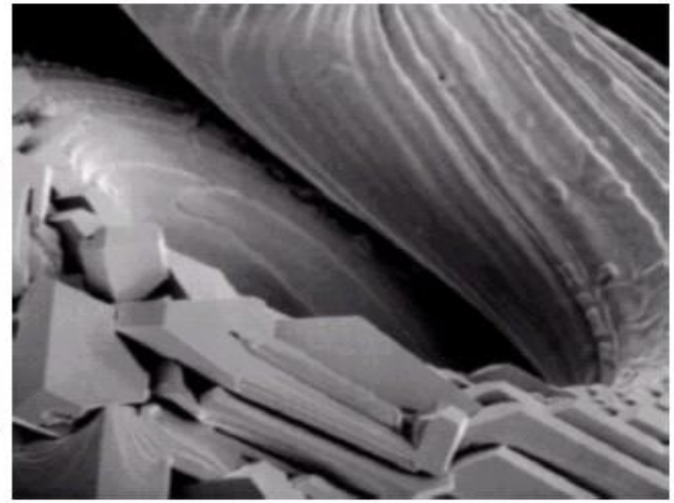
a b
c d

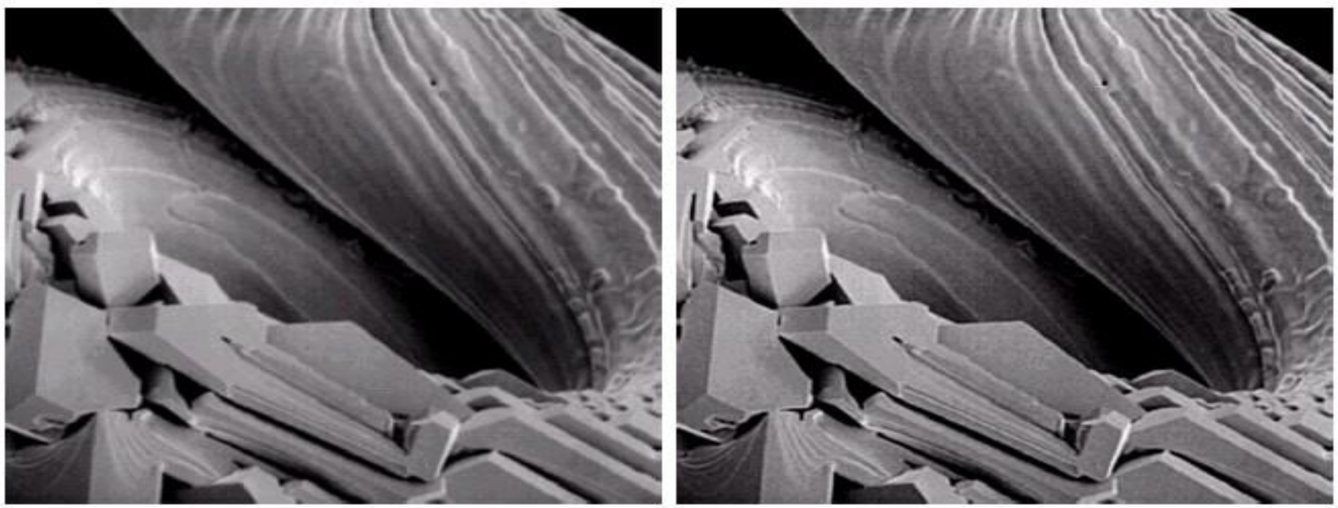
FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1





a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Latihan :

1. Bagaimana hasil yang diperoleh jika pada citra tersebut dilewatkan filter lowpass

29	10	12	13
34	12	13	13
31	10	11	12
30	11	14	14
31	12	12	11