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Jawaban Matematika diskrit

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2 $R = \{(0,1), (0,2), (1,1), (1,3), (2,2), (3,0)\}$
$A = \{0, 1, 2, 3\}$.

$$M_R = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_R^{[2]} = M_R \cdot M_R$$
$$= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$M_R^{[3]} = M_r^{[2]} \cdot M_r$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$M_R^{[4]} = M_R^{[3]} \cdot M_R$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$R = \{(0,1), (0,2), (0,3), (0,4), (1,1), (1,2), (1,3), (1,4)$
$(2,2), (3,0), (3,1), (3,2), (3,3)\}$

3. Misalkan $P(n)$ adalah proposisi yang menyatakan bahwa untuk semua $n \geq 1$,

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

(1) Basis Induksi : $P(1)$ benar karena :

$$\frac{1}{(3-2)(3+1)} = \frac{1}{3+1}$$

$$\frac{1}{1\cdot 4} = \frac{1}{4}$$

$$\frac{1}{4} = \frac{1}{4} \text{ terbukti.}$$

(2) Langkah Induksi : Misalkan $p(n)$ benar yaitu untuk preposisi :

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

diasumsikan benar . maka $p(n+1)$ juga benar yaitu :

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)} + \frac{1}{(3(n+1)-2)(3(n+1)+1)} = \frac{n+1}{3n+4}$$

Hal ini ditunjukan sbb :

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots + \frac{1}{(3n-2)(3n+1)} + \frac{1}{(3(n+1)-2)(3(n+1)+1)} =$$

$$= \frac{1}{(3(n+1)-2)(3(n+1)+1)} + \frac{n}{3n+1}$$

$$= \frac{1}{(3n+1)(3n+4)} + \frac{n}{3n+1}$$

$$= \frac{1 + n(3n+4)}{(3n+1)\ (3n+4)}$$

$$= \frac{1 + 3n^2 + 4n}{(3n+1)\ (3n+4)}$$

$$= \frac{\cancel{(3n+1)}(n+1)}{\cancel{(3n+1)}\ (3n+4)}$$

$$= \frac{n+1}{3n+4} \text{ (terbukti)}$$

karena langkah (1) dan (2) telah diperlihatkan benar maka untuk $n \geq 1$ memenuhi :

$$\frac{1}{1\cdot 4} + \frac{1}{4\cdot 7} + \frac{1}{7\cdot 10} + \cdots \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

(4). a. Inverse 19 modulo 141

PBB(141, 19)

$141 = 7 \cdot 19 + 8$
$19 = 2 \cdot 8 + 3$
$8 = 2 \cdot 3 + 2$
$3 = 1 \cdot 2 + 1$
$2 = 2 \cdot 1 + 0$

Invers :

$1 = 3 - 1 \cdot 2$ $\quad 3 = 19 - 2 \cdot 8$
$2 = 8 - 2 \cdot 3$ $\quad 8 = 141 - 7 \cdot 19$

※ $1 = 3 - 1 \cdot 2$
$1 = 3 - 1(8 - 2 \cdot 3)$
$1 = 3 - 8 + 2 \cdot 3$
$1 = 3 \cdot 3 - 8$
※ $1 = 3(19 - 2 \cdot 8) - 8$
$1 = 3 \cdot 19 - 6 \cdot 8 - 8$
$1 = 3 \cdot 19 - 7 \cdot 8$
※ $1 = 3 \cdot 19 - 7(141 - 7 \cdot 19)$
$1 = 3 \cdot 19 - 7 \cdot 141 + 49 \cdot 19$
$1 = 47 \cdot 19 - 7 \cdot 141$

Invers $= \underline{\underline{-7}}$

(4). b. Selesaikan kongruen $4x \equiv 5 \pmod 9$.

$$x = \frac{5 + 9k}{4}$$

u/ $k = 3 \rightarrow x = 8$
u/ $k = -1 \rightarrow x = -1$
u/ $k = -5 \rightarrow x = -10$
u/ $k = 7 \rightarrow x = 17$

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Jadi nilai x yang memenuhi adalah
(3, 7 ..... dan -1, -5 ....)