Tugas Matematika
1.3 Aturan Pangkat (Teknik Pengintegralan Substitusi)

October 23, 2024

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Kerjakan 2 soal dengan ketentuan:

- Sesuai dengan 1 digit terakhir dari NIM Anda.

- (1 digit terakhir dari NIM Anda) dikali 2.

Contoh: Jika digit terakhir NIM Saudara adalah 0, maka kerjakan nomor 10 dan 8.

1.3 Aturan Pangkat (Teknik Pengintegralan Substitusi)

Latihan Soal. Selesaikan!

1. $\int (x^2 - 1)^2 x\, dx = \dots.$

2. $\int (x^3 - x)^3 (3x^2 - 1)\, dx = \dots.$

3. $\int t^2 \sqrt{4t^3 - 2}\, dt = \dots.$

4. $\int (u - 1)(u^2 - 2u + 1)^{10}\, du$

5. $\int \dfrac{(\sqrt{x} - 1)^5}{\sqrt{x}}\, dx = \dots.$

6. $\int \dfrac{3t}{\sqrt{t^2 - 5}}\, dt = \dots.$

7. $\int (4t - 2)\sqrt[3]{t^2 - t - 1}\, dx = \dots.$

8. $\int \sin(2x + 1)2\, dx = \dots.$

9. $\int \cos(x^3 + 1)x^2\, dx = \dots.$

10. $\int \sin^4 x \cos x\, dx = \dots.$

11. $\int 2\cos^6 x \sin x\, dx = \dots.$

12. $\int 2\sin x \cos x\, dx = \dots.$

Figure 1: Soal

1. SOAL NOMER 1 $\hfill$ (5 POIN)

$$\int \left(x^2 - 1\right)^2 x\,dx = \ldots$$

JAWABAN:

EKSPANSI

$$(x^2 - 1)^2 = x^4 - 2x^2 + 1.$$

$$\int (x^4 - 2x^2 + 1)x\,dx = \int (x^5 - 2x^3 + x)\,dx.$$

INTEGRAL SATU SATU

$$\int x^5\,dx = \frac{x^6}{6}, \quad \int -2x^3\,dx = -\frac{2x^4}{4} = -\frac{x^4}{2}, \quad \int x\,dx = \frac{x^2}{2}.$$

JADI

$$\int \left(x^2 - 1\right)^2 x\,dx = \frac{x^6}{6} - \frac{x^4}{2} + \frac{x^2}{2} + C.$$

2. SOAL NOMER 2 $\hfill$ (5 POIN)

$$\int \left(x^3 - x\right)^3 \left(3x^2 - 1\right)\,dx = \ldots$$

JAWABAN:

SUBSTITUSI

$$u = x^3 - x, \quad du = (3x^2 - 1)\,dx.$$

di integral jadi

$$\int u^3\,du = \frac{u^4}{4} + C.$$

MASUKIN $u = x^3 - x$

$$\frac{(x^3 - x)^4}{4} + C.$$

JADI

$$\int \left(x^3 - x\right)^3 \left(3x^2 - 1\right)\,dx = \frac{(x^3 - x)^4}{4} + C$$

3. SOAL NOMER 3   (9 POIN)

$$\int t^2 \sqrt{4t^3 - 2}\, dt = \ldots$$

JAWABAN:

SUBSTITUSI

substitusi $u = 4t^3 - 2$. turunin u terhadap t

$$du = 12t^2\, dt$$

$$dt = \frac{du}{12t^2}$$

MASUKIN KE INTEGRAL

substitusikan $u = 4t^3 - 2$ dan $dt = \frac{du}{12t^2}$ ke dalam integral

$$\int t^2 \sqrt{u} \cdot \frac{du}{12t^2}$$

$$\frac{1}{12}\int \sqrt{u}\, du$$

Integral dari $\sqrt{u}$ adalah

$$\frac{1}{12}\int u^{1/2}\, du = \frac{1}{12}\cdot\frac{2}{3}u^{3/2} = \frac{1}{18}u^{3/2} + C$$

MASUKIN $u = 4t^3 - 2$

$$\frac{1}{18}(4t^3 - 2)^{3/2} + C$$

JADI

$$\int t^2 \sqrt{4t^3 - 2}\, dt = \frac{1}{18}(4t^3 - 2)^{3/2} + C$$

4. SOAL NOMER 4 $\hfill$ (9 POIN)

$$\int (u-1)\left(u^2-2u+1\right)^{10} du = \ldots$$

JAWABAN:

NYEDERHANAIN

$u^2 - 2u + 1 = (u-1)^2$

$$\int (u-1)\left((u-1)^2\right)^{10} du = \int (u-1)^{21} du$$

INTEGRALIN

integralin $(u-1)^{21}$

$$\int (u-1)^{21} du = \frac{(u-1)^{22}}{22} + C$$

JADI

$$\int (u-1)\left(u^2-2u+1\right)^{10} du = \frac{(u-1)^{22}}{22} + C$$

5. SOAL NOMER 5 $\hfill$ (9 POIN)

$$\int \frac{(\sqrt{x}-1)^5}{\sqrt{x}} dx = \ldots$$

JAWABAN:

Substitusi

substitusi $u = \sqrt{x}$ sehingga $x = u^2$ dan $dx = 2u\,du$

$$\int \frac{(u-1)^5}{u} \cdot 2u\,du$$

$$2\int (u-1)^5 du$$

integralin

integralin $(u-1)^5$

$$2 \int (u-1)^5 \, du = 2 \cdot \frac{(u-1)^6}{6} + C = \frac{(u-1)^6}{3} + C$$

masukin $u = \sqrt{x}$

$$\frac{(\sqrt{x}-1)^6}{3} + C$$

JADI

$$\int \frac{(\sqrt{x}-1)^5}{\sqrt{x}} \, dx = \frac{(\sqrt{x}-1)^6}{3} + C$$

6. SOAL NOMER 6   (9 POIN)

$$\int \frac{3t}{\sqrt{t^2 - 5}} \, dt = \ldots$$

JAWABAN:

substitusi

substitusi $u = t^2 - 5$

$$du = 2t \, dt$$

$$\int \frac{3t}{\sqrt{t^2 - 5}} \, dt = 3 \int \frac{1}{\sqrt{u}} \cdot \frac{du}{2}$$

sederhanain integral

$$\frac{3}{2} \int u^{-1/2} \, du$$

$$\frac{3}{2} \cdot 2u^{1/2} = 3\sqrt{u} + C$$

masukin $u = t^2 - 5$

$$3\sqrt{t^2 - 5} + C$$

JADI

$$\int \frac{3t}{\sqrt{t^2 - 5}}\, dt = 3\sqrt{t^2 - 5} + C$$

7. SOAL NOMER 7 (9 POIN)

$$\int (4t - 2)\sqrt[3]{t^2 - t - 1}\, dt = \ldots$$

JAWABAN:

substitusi

substitusi $u = t^2 - t - 1$

$$du = (2t - 1)\, dt$$

ganti $4t - 2$ jadi $2(2t - 1)$

$$2\int (2t - 1)\sqrt[3]{u}\, dt$$

karena $2t - 1 = du/dt$ integral menjadi

$$2\int \sqrt[3]{u}\, du$$

integralin

Integral dari $\sqrt[3]{u}$ atau $u^{1/3}$ adalah:

$$\int u^{1/3}\, du = \frac{3}{4}u^{4/3}$$

jadi

$$2 \cdot \frac{3}{4} u^{4/3} = \frac{3}{2} u^{4/3} + C$$

masukin $u = t^2 - t - 1$

$$\frac{3}{2}(t^2 - t - 1)^{4/3} + C$$

JADI

$$\int (4t - 2) \sqrt[3]{t^2 - t - 1} \, dt = \frac{3}{2}(t^2 - t - 1)^{4/3} + C$$

8. SOAL NOMER 8   (9 POIN)

$$\int \sin(2x + 1) \cdot 2 \, dx = \ldots$$

JAWABAN:

subtitusi

substitusi $u = 2x + 1$

$$du = 2 \, dx$$

integral jadi

$$\int \sin(u) \cdot du$$

integralin

integral $\sin(u) = -\cos(u)$

$$-\cos(u) + C$$

masukin $u = 2x + 1$

$$-\cos(2x + 1) + C$$

JADI

$$\int \sin(2x+1) \cdot 2\, dx = -\cos(2x+1) + C$$

9. SOAL NOMER 9 (9 POIN)

$$\int \cos\left(x^3+1\right) x^2\, dx = \ldots$$

JAWABAN:

substitusi

substitusi $u = x^3 + 1$

$$du = 3x^2\, dx$$

$$\frac{du}{3} = x^2\, dx$$

masukin $u = x^3 + 1$ dan $x^2\, dx = \frac{du}{3}$ ke dalam integral

$$\int \cos(u) \cdot \frac{du}{3}$$

$$\frac{1}{3} \int \cos(u)\, du$$

INTEGRALIN

integral dari $\cos(u)$ adalah $\sin(u)$, jadi

$$\frac{1}{3}\sin(u) + C$$

MASUKIN $u = x^3 + 1$

$$\frac{1}{3}\sin(x^3+1) + C$$

JADI

$$\int \cos\left(x^3+1\right) x^2\, dx = \frac{1}{3}\sin(x^3+1) + C$$

10. SOAL NOMER 10

(9 POIN)

$$\int \sin^4 x \cos x \, dx = \ldots$$

JAWABAN:

SUBSTITUSI

substitusi $u = \sin x$, jadi turunan u adalah

$$du = \cos x \, dx$$

integral berubah menjadi

$$\int u^4 \, du$$

INTEGRALIN

Integral dari u^4

$$\int u^4 \, du = \frac{u^5}{5} + C$$

MASUKIN $u = \sin x$

$$\frac{\sin^5 x}{5} + C$$

JADI

$$\int \sin^4 x \cos x \, dx = \frac{\sin^5 x}{5} + C$$

11. SOAL NOMER 11   (9 POIN)

$$\int 2\cos^6 x \sin x \, dx = \dots$$

JAWABAN:

substitusi

substitusi $u = \cos x$, jadi turunan u adalah

$$du = -\sin x \, dx$$

integral menjadi

$$-2 \int u^6 \, du$$

integralin

integral dari u^6

$$-2 \cdot \frac{u^7}{7} = -\frac{2}{7} u^7 + C$$

masukin $u = \cos x$

$$-\frac{2}{7} \cos^7 x + C$$

JADI

$$\int 2\cos^6 x \sin x \, dx = -\frac{2}{7} \cos^7 x + C$$

12. SOAL NOMER 12   (9 POIN)

$$\int 2\sin x \cos x \, dx = \ldots$$

JAWABAN:

identitas trigonometri

$$\sin(2x) = 2\sin x \cos x$$

integral menjadi

$$\int 2\sin x \cos x \, dx = \int \sin(2x) \, dx$$

integralin

integralkan $\sin(2x)$

$$\int \sin(2x) \, dx = -\frac{1}{2}\cos(2x) + C$$

JADI

$$\int 2\sin x \cos x \, dx = -\frac{1}{2}\cos(2x) + C$$