

Bayesian Knowledge Tracing at Scale with `hmmsc1b1` toolkit

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Student Modeling Applications, Recent
Developments & Toolkits (SMART) at EDM 2015

hmmsc1b1 – HMM Scalable

- Toolkit for building Bayesian Knowledge Tracing (BKT) models of Big Datasets
 - And BKT is a Hidden Markov Model (HMM)
- How big is a Big Dataset?
 - 2010 Carnegie Learning Cognitive tutor data
92.5 million records, 74 000 students
- How well did hmmsc1b1 do?
 - Mac Mini late 2012, 16Gb of RAM
 - EM model finished in under 5 minutes (parallel mode)
- Tested on educational data

Agenda

- **Bayesian Knowledge Tracing**
- Getting Started with `hmmsc1bl`
- The Tinkering Points
- Advanced Topics

Bayesian Knowledge Tracing

Definition

Bayesian Knowledge Tracing Model \subseteq Hidden Markov Model \subseteq Bayes Net \subseteq Graphical Model

When modeling skill acquisition

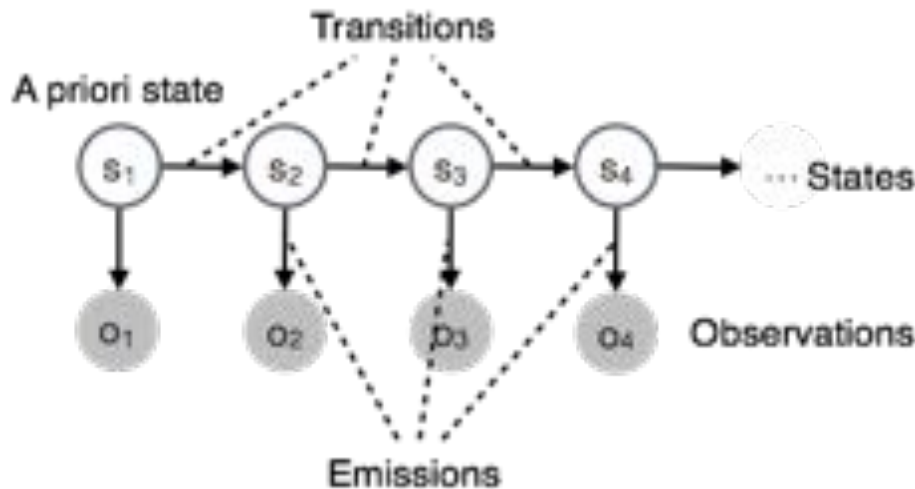
- One binary latent (unobserved) variable is represents mastery (skill is known|unknown)
- One observed binary variable represents student performance (skill is applied correctly|incorrectly)

Data

- Time-series data: correctness of skill application

Bayesian Knowledge Tracing Representation

Unrolled view of BKT



Priors – Π

Known	Unknown
pL_0	$1 - pL_0$

Transitions – A

	to Known	to Unknown
from Known	$1 - pF$	pF
from Unknown	pT	$1 - pT$

Emissions – B

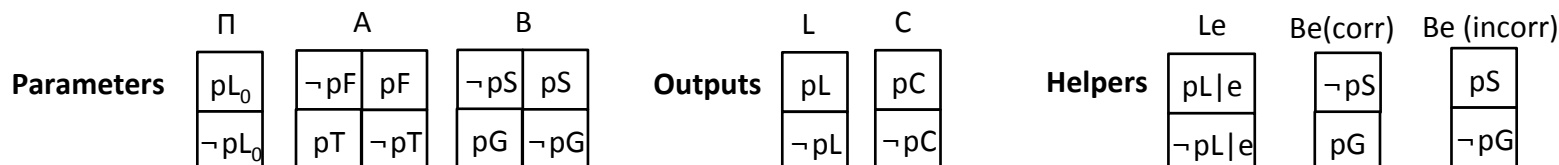
	Correct	Incorrect
Known	$1 - pS$	pS
Unknown	pG	$1 - pG$

Parameters

- Values $\in [0,1]$
- Rows sum to 1
- Forgetting (pF) = 0
- 4 parameters
 - Rows sum to 1, every last value can be omitted
 - Forgetting is 0, no 5th parameter

Bayesian Knowledge Tracing

Updating masteries, predicting



Matrix Version

1. Initialize
 $L = \Pi$
2. Correct
 - a. Predict
 $C = L^T \times B$
 - b. Pick Be
 $Be = B[:, 1]$
 - c. Compute Le
 $Le = (L \times Be) ./ (L^T \times Be)$
 - d. Update L
 $L = Le \times A$
3. Incorrect
 - b. Pick Be
 $Be = B[:, 2]$
 - c. Compute Le
4. Unknown
 - d. Update L
 $L = A^T \times L$
5. Unknown/guess
 - b. Pick Be
 $Be = B[:, \text{argmax}(C)]$
 continue as 2 or 3, depending on $\text{argmax}(C)$

Simplified Computation

$$pL = pL_0$$

$$pC = pL \cdot (1 - pS) + (1 - pL) \cdot pG$$

$$p(L|e = \text{correct}) = pL \cdot (1 - pS) / [pL \cdot (1 - pS) + (1 - pL) \cdot pG]$$

$$pL = p(L|e) \cdot (1 - pF) + (1 - p(L|e)) \cdot pT$$

$$p(L|e = \text{incorrect}) = pL \cdot pS / [pL \cdot pS + (1 - pL) \cdot (1 - pG)]$$

$$pL = pL \cdot (1 - pF) + (1 - pL) \cdot pT$$

Legend

\times – vector/matrix multiply	Xe – X given evidence
\cdot – multiply element by element	L^T – transpose
$./$ – divide element by element	$X[:, i]$ – i^{th} column of X

Bayesian Knowledge Tracing.

Special Note on Predicting

- Updating $L(pL)$ when the observation is known
 - In the tutor, the observation (correct/incorrect) is available for the update
 - When comparing models making the observation available puts BKT at an unfair advantage
 - Especially, when comparing to AFM and PFA
 - Rule 4 from previous slide is too harsh
 - Matrix B (emissions) is ignored
 - Accuracy takes a considerable hit
 - Rule 5 is a compromise
 - Accuracy is better than under Rule 4 (but worse than under Rules 2&3)

Agenda

- Bayesian Knowledge Tracing
- **Getting Started with hmmsc1b1**
- The Tinkering Points
- Advanced Topics

Getting started with hmmsc1b1

Obtaining and compiling code

Get the source code from IEDMS GitHub repository

```
$> git clone https://github.com/IEDMS/standard-bkt
```

OR download and unzip

Compile

```
$> cd standard-bkt
```

```
$> make all
```

Operating system notes

Linux – gcc with Open MP should be there by default

Mac OS X – refer to hpc.sourceforge.net

Windows – install cygwin with awk, make, wc, time, unzip commands

If you are on the Virtual Box Ubuntu 14.04 Machine we distributed

User/password – usmartedm15/usmartedm15

Files are in ~/Documents/USMARTEDM15/hmmsc1b1

Getting started with hmmsc1bl

Getting the data

KDD Cup 2010 on hosted by LearnLab's DataShop [pslcdatashop.web.cmu.edu/KDDCup] Largest freely available dataset of learner data donated by Carnegie Learning, Inc.

- Create an account

- Download Bridge to Algebra challenge set
bridge_to_algebra_2008_2009.zip (461Mb)

- ~20 000 000 rows, ~6 000 students

Data is placed in subfolder d , models will be in subfolder m , prediction files in subfolder p , code is in the root that contains these subfolders

Getting started with hmmsc1b1

Prepping the data

Instead of unzipping a 5.3Gb training file, run:

```
$> time unzip -q -c d/bridge_to_algebra_2008_2009.zip  
bridge_to_algebra_2008_2009_train.txt | awk "-F\t"  
'BEGIN{OFS="\t"} {if(NR==1)next; skill=$20; gsub("~~",  
"~", skill); skill=(skill=="")?"." : ($3"__"skill); print 2-  
$14,$2,$3"__"$4,skill;}' > d/b89_train.txt
```

Reduces rich data to tab-delimited SSSS: success, student, step, skill(s). Multiple skills are ~-delimited. No skill label is denoted by . (dot).

Resulting text file is 2.9Gb.

Note: skills are prefixed with unit and section name. Identical skill labels could mean different things in different sections.

Getting started with hmmsc1bl

Running the tool

Let's start from EM solver

```
$> ./trainhmm -s 1.1 -d ~ -m 1 -p 1 d/b89_train.txt m/  
b89_modelEM.txt p/b89_predictEM.txt
```

Here, `-s 1.1` is an EM solver, `-d ~` is setting skill delimiter, `-m 1` asks for fit metrics, and `-p 1` requires predictions to be written into a file

It takes a little under 2.5 minutes, fitting alone 1 minute

```
trained model LL=6903034.9861273 (4490671.7596719),  
AIC=13820821.972255, BIC=13930074.307203, RMSE=0.318739  
(0.352797), Acc=0.870607 (0.830841)
```

```
timing: overall 139.576931 sec, read 42.205482 sec, fit 60.997095  
sec, predict 36.144902 sec
```

Here, LL – sum of negative log-likelihoods, AIC and BIC are Akaike and Bayesian Information Criteria, RMSE - root mean squared error, Acc – accuracy.

In braces are the RMSE and Accuracy for the rows without skill labels. Otherwise the metric is given for all rows, even if skill label was omitted.

Getting started with hmmsc1b1

Switching solvers

Now let's try a Stochastic Gradient Descent solver

```
$> ./trainhmm -s 1.2 -d ~ -m 1 -p 1 d/b89_train.txt m/b89_modelGD.txt p/
b89_predictGD.txt
```

It takes a little longer

```
trained model LL=7230246.8188073 (4817883.5923708), AIC=14475245.637615,
BIC=14584497.972563, RMSE=0.328491 (0.368380), Acc=0.863745 (0.818622)
timing: overall 169.495414 sec, read 44.967394 sec, fit 100.809863 sec,
predict 23.490269 sec
```

The overall accuracy is a little worse 0.863745 (vs. 0.870607), just as the overall RMSE – 0.328491 (vs. 0.318739).

- How much is *a little* and what does it mean
 - Individualized BKT (Yudelton et al., AIED 2013)
 - Increment in the number of rows we are predicting correctly (~50K)
 - **When** the improvement happens: earlier attempts – more valuable
 - Small Improvement for the Model Accuracy Big Difference for the Students (Yudelton & Ritter, AIED 2015)
 - Small difference in accuracy could mean tangible changes in how students work
 - Larger pLearn → faster mastery → fewer problems and less time

Getting started with hmmsc1bl

Process reports

First output: there are 2 observations, 6043 students, 1844 skills, and 61848 unique steps

```
input read, nO=2, nG=6043, nK=1844, nI=61848
```

As fitting progresses, results for every skill are given

```
skill      0, seq 4027, dat 32217, iter# 1 p(O|param)= 18360.7093550 >> 18343.6728364, conv=1
skill      1, seq 3889, dat 62244, iter# 7 p(O|param)= 21660.7485382 >> 16798.8022745, conv=1
skill      2, seq 4386, dat 98701, iter# 6 p(O|param)= 33991.2924686 >> 27522.3817164, conv=1
skill      3, seq 3888, dat 62923, iter# 6 p(O|param)= 23075.1928335 >> 19305.2954508, conv=1
...
```

To fit skill #1

- We use 3889 student sequences of data comprised of 62244 data points
- It takes 7 iterations to finally converge
- The sum of negative log-likelihoods decreases from 21660.7485382 to 16798.8022745
- Average likelihood of a student-skill sequence goes up 0.0038114422 → 0.01330562239 (3.5 times)

Getting started with hmmsc1b1

Comparing to baseline

Accuracies/RMSE of the Majority Class prediction (here, always predicting “correct”)

```
$> awk -F"\t" '{N++; C+=$1==1; SSE+= (1-$1)*(1-$1); }END{print C/N"\t"sqrt(SSE/N);}' d/b89_train.txt
```

Accuracy=0.861663 and **RMSE=0.371936**. Majority class accuracy is slightly worse than BKT models we’ve seen. The majority class RMSE is tangibly worse.

Note! BKT, traditionally, uses first attempt at a problem step. Further work until student gets right is omitted.

MC Accuracy $\approx 86\%$ doesn’t mean data is severely skewed or the CL Cognitive Tutor tutor is simple or the students are incredibly smart

20 012 498 first attempts, 17 244 034 are correct = 86.1663% correct

26 798 736 all attempts, 17 244 034 are correct = 64.3464% correct

Contextual slip and guess BKT (Baker et al., 2010) uses all data.

Getting started with hmmsc1b1

Tweaking tolerance criteria

Default stopping criterion is the change in parameter values of 0.01. Here, we lower tolerance `-e 0.001` and increase the maximum number of iterations `-i 400` (from the default 200)

```
$> ./trainhmm -s 1.2 -e 0.001 -i 400 -d ~ -m 1 -p 1 d/b89_train.txt m/
b89_modelGDe3i4.txt p/b89_predictGDe3i4.txt
```

It takes a lot longer to build and the change in fit is small (compare to 0.863745 accuracy).

```
trained model LL=7045687.2615030 (4633324.0351113), AIC=14106126.523006,
BIC=14215378.857954, RMSE=0.322916 (0.359497), Acc=0.868310 (0.826750)
timing: overall 2115.538318 sec, read 44.908378 sec, fit 2041.510535 sec, predict
28.882947 sec
```

In addition, 4 skills actually failed to converge after 400 iterations.

The stopping criterion can be defined in terms of mean negative log-likelihood change per data point by using the following parameter signature `-e 0.001,1`

```
$> ./trainhmm -s 1.2 -e 0.001,1 -d ~ -m 1 -p 1 d/b89_train.txt m/b89_modelGDe2l.txt p/
b89_predictGDe2l.txt
```

Fitting results achieved show small improvement in accuracy (compare to 0.863745).

```
trained model LL=7184702.3227653 (4772339.0963109), AIC=14384156.645531,
BIC=14493408.980479, RMSE=0.327298 (0.366485), Acc=0.864662 (0.820255)
timing: overall 185.898219 sec, read 2.105894 sec, fit 159.845276 sec, predict 23.690081
sec
```


Getting started with hmmsc1bl

Let forgetting happen

Default initial parameter values are: $pL_0=0.5$, $1-pF=1$, $pT=0.4$, $1-pS=0.8$, $pG=0.2$. To set defaults, one can specify a comma separated list via `-0 0.5,1.0,0.4,0.8,0.2`

Let's set a starting version of forgetting to 0.001

```
$> ./trainhmm -s 1.2 -d ~ -m 1 -p 1
-0 0.5,0.999,0.4,0.8,0.2 d/b89_train.txt
m/b89_modelGDpf0.01.txt
p/b89_predictGDpf0.01.txt
```

Known	Unknown
pL₀	1- pL ₀

	to Known	to Unknown
from Known	1-pF	pF
from Unknown	pT	1-pT

	Correct	Incorrect
Known	1-pS	pS
Unknown	pG	1-pG

Small decrement in accuracy (rf. to 0.863745)

```
trained model LL=7747431.9193728 (5335068.6927902),
AIC=15509615.838746, BIC=15618868.173694, RMSE=0.340602
(0.387471), Acc=0.861669 (0.814925)
timing: overall 101.492081 sec, read 44.915405 sec, fit
30.866999 sec, predict 25.471139 sec
```

Getting started with hmmsc1b1

Speeding up reading the data file

Previous run – almost 1/3 of time is reading the input file

```
timing: overall 117.920293 sec, read 45.535030 sec, fit  
35.287492 sec, predict 36.842098 sec
```

There is a helper utility to convert text file to binary file

```
$> ./inputconvert -s t -t b -d ~ d/b89_train.txt d/  
b89_train.bin
```

Where, -s t means the source is text, -t b – the target is binary.

overall time running is 42.524978 seconds

If we rerun previous task, where -b 1 means binary input and specify a new .bin file

```
$> ./trainhmm -s 1.2 -d ~ -m 1 -p 1  
-0 0.5,0.999,0.4,0.8,0.2 -b 1 d/b89_train.bin  
m/b89_modelGDpf0.01.txt  
p/b89_predictGDpf0.01.txt
```

The reading time is much better

```
timing: overall 57.193008 sec, read 1.146349 sec, fit  
31.356436 sec, predict 24.474850 sec
```

Why didn't we see drastic improvements?

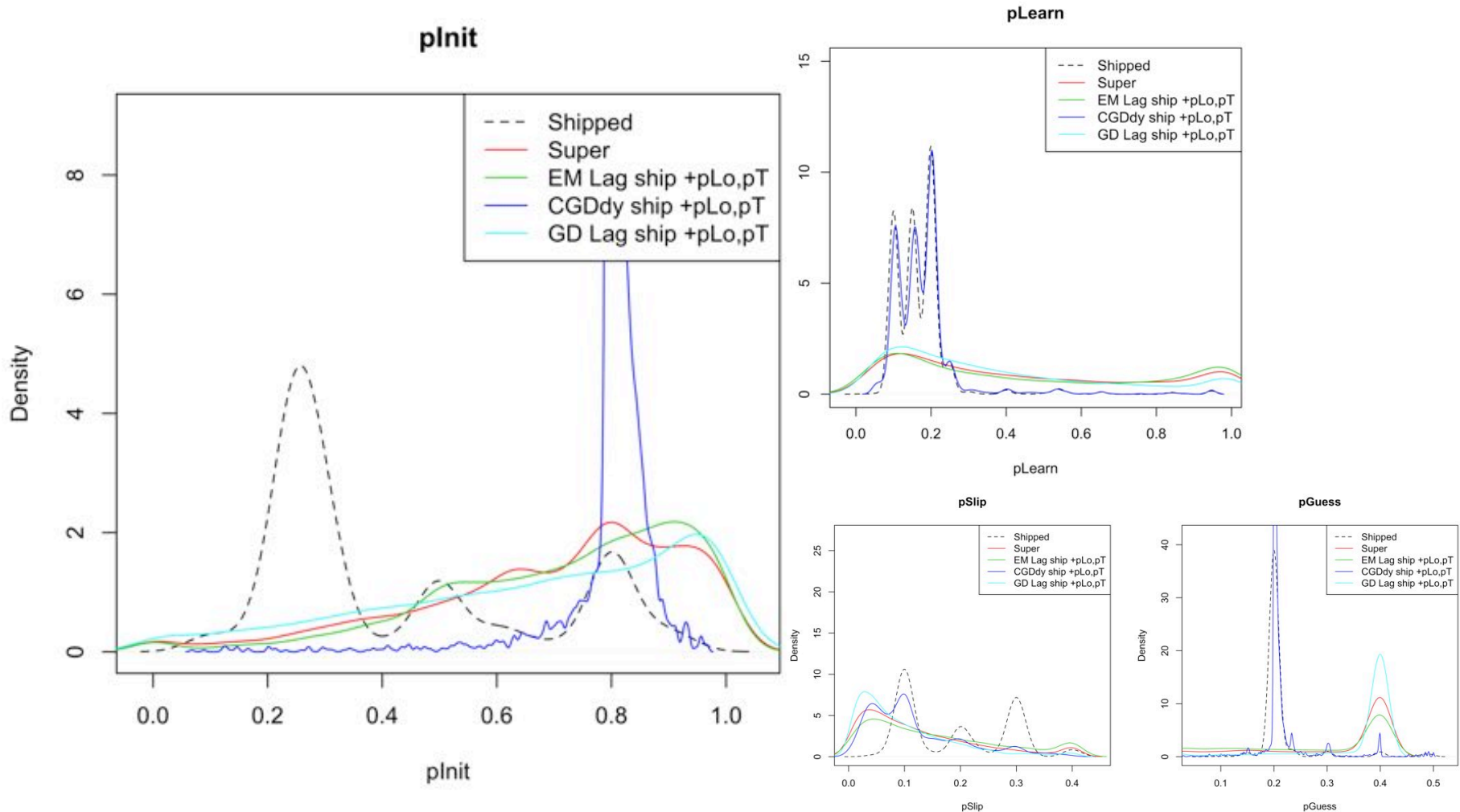
- We are using same set of starting parameters
 - Harder and easier skills start in the same place
 - Parameter space has multiple local optima
 - + Seeding pL_0 and pT using AFM – *pre-classify* skills (e.g., via modified `liblinear` in the IEDMS GitHub)
- We are not taking context into consideration
 - Learning and forgetting could be different
 - Same day, next week, after Thanksgiving break
 - + Multiplexing Transition (A) and Emission (B) matrices
- Presence of noise in the data

Tomorrow, Saturday, June 27, Room 1: Salón de Actos. 10:30-11:50

M. Falakmasir, M. Yudelson, S. Ritter and K. Koedinger

Spectral Bayesian Knowledge Tracing. (short)

How different model parameters are?



Agenda

- Bayesian Knowledge Tracing
- Getting Started with `hmmsc1bl`
- **The Tinkering Points**
- Advanced Topics

The Tinkering Points

Set Starting Parameters and Limits

- Starting parameters – 0
 - List all but the last in every row
 - Default is – 0 0.5, 1, 0.4, 0.8, 0.2

	Known	Unknown
	pL₀	1- pL ₀
	to Known	to Unknown
from Known	1-pF	pF
from Unknown	pT	1-pT
	Correct	Incorrect
Known	1-pS	pS
Unknown	pG	1-pG

- Set parameter limits
 - List all values in every row
 - Default upper – u 1, 1, 1, 0, 1, 1, 1, 0.3, 0.3, 1
 - Slip and Guess are capped at 0.3
 - Forget is capped at 0
 - Default lower – l 0, 0, 1, 0, 0, 0, 0, 0, 0, 0
 - 1-Forget is no less than 1

	Known	Unknown
	pL₀	1- pL₀
	to Known	to Unknown
from Known	1-pF	pF
from Unknown	pT	1-pT
	Correct	Incorrect
Known	1-pS	pS
Unknown	pG	1-pG

The Tinkering Points

Controlling the Process. Solvers – s

- Baum-Welch (EM) –s 1.1 – classical algorithm
- Stochastic Gradient Descent –s 1.2
 - Scales gradients so that the maximum step of 1 does not take any of them over 1 or below 0 (finds n to divide all parameters by 10^n)
 - Halves the step until either Wolfe Conditions are met (a step is far enough but not too far) or 20 attempts. If only first WC is met, us that (Nocedal & Wright, 1999).
 - Re-align parameters so that rows sum to 1 (project onto simplex)
 - Attempts to make large leaps in parameter space (Wolfe Conditions counter-balance that)
- Conjugate Gradient Descent –s 1.3. [1–4]
 - Make first step as in Stochastic Gradient Descent
 - Computes step direction on subsequent next step using formulas
 - Fletcher-Reeves –s 1.3.1
 - Polak-Ribière –s 1.3.2
 - Hestenes-Stiefel –s 1.3.3
 - Dai-Yuan –s 1.3.4
 - Makes large leaps in parameter space
- Gradient Descent using Lagrangian multipliers –s 1.4
 - Uses gradients in a manner that does not need realignment of parameters (Levinson et al., 1983)
 - Smooth[er] parameter changes, just like EM
- Temporarily excluded Barzilai-Borwein and exponentiated Gradient updates

The Tinkering Points

Controlling the Process. Iterating and Stopping

- Iterating $-i \ 200$
- Stopping
 - Mean parameter change (per parameter) $-e \ 0.01$
 - Decreasing the tolerance increases fitting time, but
 - Not necessarily improves the fit drastically
 - Mean negative log-likelihood (per sequence) $-e \ 0.01, 1$
 - On CLCT data, results in a slight improvement of the fit due to more iterations required to converge
- There is a precaution built in against parameter oscillation
 - Parameter values are stored for 2 iterations
 - If parameters at time t are close to parameters at time $t-2$ the fitting stops

The Tinkering Points

Controlling the Process. Regularization

- Regularization is often used to fight with over-fitting
- E.g., in Regularized Logistic Regression – penalize parameters for deviating from zero
 - Objective function for parameter x (neg. log-likelihood) – $f(x_i)$
 - Objective function with penalty – $f(x_i) + \lambda(\sum x_i)^{\frac{1}{2}}$
 - Gradient of a single parameter x – $g(x_i)$
 - Gradient of a single parameter with L2 penalty – $g(x_i) + \frac{1}{2}\lambda |x_i|$
 - Use center c other than 0 – $g(x_i) + \frac{1}{2}\lambda |x_i - c|$
- Specify penalty
 - All for being away from 0.5: `-c 1.0, 0.5, 0.5, 0.5`
 $\lambda=1$, centroids for π , A and B are 0.5.
 - Gravitate priors and transitions to 0.5 and emissions to 0
`-c 1.0, 0.5, 0.5, 0.0`

```
$> ./trainhmm -s 1.2 -d ~ -m 1 -p 1 -b 1 -c 1.0,0.5,0.5,0.0 d/b89_train.bin m/
b89_modelGDreg.txt p/b89_predictGDreg.txt
trained model LL=7220628.1340159 (4808264.9075270), AIC=14456008.268032,
BIC=14565260.602980, RMSE=0.327139 (0.366232), Acc=0.864803 (0.820505)
timing: overall 215.000000 seconds, read 1.441625, fit 190.277211, predict 23.271876
```

Ran longer than no-regularized Gradient Descent. Small improvement in RMSE and Accuracy: **0.327139** vs. **0.328491** and **0.864803** vs. **0.863745** respectively (additional 21 173 correct predictions)

Agenda

- Bayesian Knowledge Tracing
- Getting Started with `hmmsc1bl`
- The Tinkering Points
- **Advanced Topics**

Advanced Topics

Prediction

- Updating probability distribution of states (PDS)
 - In tutor, observation (correctness) is always known
 - When assessing the model, revealing observation is not fair (e.g., AFM and PFA always *fly blind*)
- Controlling how update of the PDS is done
 - $\cup \text{ } \mathbf{r}, \mathbf{t}$ – for known observation **reveal** them for PDS update, for unknown observation, use **transition** matrix only (Tutor)
 - $\cup \text{ } \mathbf{g}, \mathbf{g}$ – for known and unknown observations – guess most probable observation from PDO (Model Assessment)
- Probability distribution of observations (PDO) are always computed before the update and without *peeking*

Advanced Topics

Cross-Validation. Primer

- Cross-validation
 - Divide data into n pieces (folds)
 - Hide $1/n^{\text{th}}$ of the data, train on the rest
 - Predict the hidden data
 - Pick next fold, repeat
 - Combine n prediction folds, compute accuracy metric(s)
- Stratification
 - By student – $1/n^{\text{th}}$ of the students (whole student-skill sequences)
 - By-item – $1/n^{\text{th}}$ of the items (student-skill sequences are patched)
 - Un-stratified – $1/n^{\text{th}}$ of the rows (student-skill sequences are patched)
- Implementation
 - By-student stratification – simple
 - By-item/un-stratified – save $1/n^{\text{th}}$ of observations, and mark them as unknown

Advanced Topics

Cross-Validation. Practice

- 5-fold student-stratified (default) cross-validation (rf. EM Acc.=0.870607)

```
./trainhmm -s 1.1 -d ~ -p 1 -b 1 -v 5 d/b89_train.bin m/b89_modelEMv.txt p/  
b89_predictEMv.txt
```

...

```
5-fold cross-validation: LL=6932561.9857819, AIC=13879875.971564,  
BIC=13989128.306512, RMSE=0.319513 (0.354041), Acc=0.870297 (0.830288)  
timing: overall 206.000000 seconds, read 1.271167, fit 186.711080, predict  
17.153612
```

- 10-fold **item**-stratified cross-validation when predicting **incorrect observation**,
and **save** the folds into a **file**

```
./trainhmm -s 1.1 -d ~ -p 1 -b 1 -P 1 -v 10,i,2,folds.txt,o d/  
b89_train.bin m/b89_modelEMv2.txt p/b89_predictEMv2.txt
```

...

```
10-fold cross-validation: LL=19324495.2940466, AIC=38663742.588093,  
BIC=38772994.923041, RMSE=0.321796 (0.357636), Acc=0.871027 (0.831588)  
timing: overall 638.000000 seconds, read 1.312280, fit 618.701899,  
predict 17.522869
```

- Saved folding can later be reused by using the following parameter

```
-v 10,i,2,folds.txt,i
```

Advanced Topics

Parallel Execution

- Parallel code uses Open MP
 - Individual skills are fit in N (for me N=8) recycled parallel threads
 - Prediction is done in parallel on partitioned data
- Checked-in code is in sequential state
 - Sequential code is easier to debug
 - Convert to parallel state by running `./seq2par.sh`
 - Convert back to sequential state `./par2seq.sh`
- To enable parallel execution use `-P 1` switch

```
./trainhmm -s 1.1 -d ~ -p 1 -b 1 -P 1 -m 1 d/  
b89_train.bin m/b89_modelEM.txt p/b89_predictEM.txt
```

...

```
timing: overall 33.484420 sec, read 1.232642 sec, fit  
14.389842 sec, predict 17.632139 sec
```

Compare to sequential execution

```
timing: overall 139.576931 sec, read 42.205482 sec, fit  
60.997095 sec, predict 36.144902 sec
```

Advanced Topics

What if there are just too many skills/students?

- Skill/student internal index variables
 - 4 byte signed integer
 - 2,147,483,647 (2 billion)
 - Default skill/student map is non-sparse
- 2010 Big Dataset unfiltered
 - 56000 students * 3700 skills →
 $56000 * 3700 * 2 * 4 \approx 1.54\text{Gb}$ of memory for PSD alone
- `hmmSc1b1` with BOOST library's sparse 2D map
 - Compact representation
 - Currently not in public code

What I Didn't Tell You

Talk to Me If You Are Interested

- Individualized BKT (Yudelson et al., AIED 2013)
 - Parameters have a per-skill and per-student (individual) components
 - Fit using block coordinate descent (fit skills, fit students, repeat)
 - Improved model accuracy
 - More focused investigation is necessary w.r.t. interpreting parameter
- Multiplexing (under experimental investigation)
 - Using different parameters for different contexts of skill application
 - Different pLearn, pSlip, & pGuess based on response time, time since last skill was used (forgetting), etc.
- Spectral BKT (the noise fighter)

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What I'd Like to add to hmmsc1b1

Talk to Me If You Are Interested

- Multiple groupings of sequences
 - Student, Class, School – hierarchical effects
- Conjunctive BKT
 - Address multiple skill coding or other [item context] factors (in the same spirit as José and Yun did in FAST)
- Continuous observations
 - Math is already worked out
- Speed improvements
 - Optimized computation of forward-backward variables
 - NVIDIA Compute Unified Device Architecture (CUDA)
 - On this laptop: 384 GPU cores vs. 8 CPU cores

Thank You!

Project Page <http://bit.ly/hmmscblbl>

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