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Research Statement

My research investigates how ideas from algebraic topology reveal the structural relationships between algebra and arithmetic. In particular, I use methods from stable homotopy theory to understand how duality and equivariant phenomena organize and connect seemingly distinct areas of mathematics. My doctoral work contributed to leveraging arithmetic duality at the homotopical level, within the framework of algebraic K-theory. Beyond these arithmetic and equivariant settings, I also explore how the homotopical viewpoint extends to analytic geometry illustrating the broader reach of topology across modern mathematics. More broadly, my research aims to develop conceptual and computational tools that show how homotopical structures govern both arithmetic and equivariant phenomena.

1 Background

One of the central aims of topology is to classify spaces according to appropriate notions of equivalence. The most rigid notion of sameness is that of a *homeomorphism*, a continuous map with its inverse. While this captures topological equivalence precisely, it is often too strict for understanding spaces up to deformation. *Homotopy theory* offers a more flexible perspective by identifying spaces that can be continuously deformed into one another, leading to the notion of a *homotopy type*. The resulting invariants, *homotopy groups* carry the deepest information: they completely determine the homotopy type, but are notoriously difficult to compute.

A guiding idea in *stable homotopy theory* is to study properties of spaces that are preserved under iterated suspension (Σ):

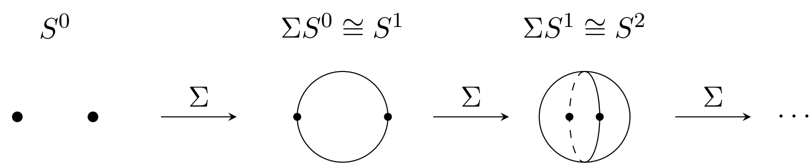

Figure 1: Iterated suspensions $\Sigma^n S^0 \cong S^n$ of spheres.

In particular, the *Freudenthal suspension theorem* shows that the homotopy classes of maps $[X, Y]$ stabilize after finitely many suspensions, providing a linear approximation to homotopy theory itself.

$$\begin{tikzcd}
[S^1,S^0] \arrow[r, "\Sigma"] \arrow[d, phantom, "\wr\|"] & [S^2,S^1] \arrow[r, "\Sigma"] \arrow[d, phantom, "\wr\|"] & [S^3,S^2] \arrow[r, "\Sigma"] \arrow[d, phantom, "\wr\|"] & [S^4,S^3] \arrow[r, "\Sigma"] \arrow[d, phantom, "\wr\|"] & [S^5,S^4] \arrow[r] \arrow[d, phantom, "\wr\|"] & \cdots \arrow[d, phantom, "\wr\|"] \\
0 \arrow[r] & 0 \arrow[r] & \mathbb{Z} \arrow[r] & \mathbb{Z}/2 \arrow[r] & \mathbb{Z}/2 \arrow[r, dashed] & \mathbb{Z}/2
\end{tikzcd}$$

Figure 2: The sequence $0 \to 0 \to \mathbb{Z} \to \mathbb{Z}/2 \to \cdots$ stabilizes at $\mathbb{Z}/2$, illustrating the suspension theorem in a concrete case.

Suspensions of homotopy groups eventually stabilize after finitely many iterations, and this stabilization provides the linear approximation. In this stabilized setting, new patterns and algebraic structures emerge, making it possible to study homotopical phenomena that were previously too complex to handle directly. This shift from spaces to stable homotopy types establishes a framework in which algebraic and homotopical ideas can be applied in a unified way, forming the foundation of modern algebraic topology.

Much of my work takes place within this stable framework, where duality and equivariant structures reveal deep connections between algebra and topology. In particular, *higher algebraic K-theory* arises within this framework: for a suitable ring or category, it is defined as the sequence of stable homotopy groups of its *algebraic K-theory spectrum*. This viewpoint places algebraic K-theory firmly within stable homotopy theory, bridging algebraic and arithmetic structures with topological methods.

2 Current research and direction

Building on the framework of stable homotopy theory, my current research explores how algebraic and homotopical ideas interact across several fronts, particularly through *algebraic K-theory*, *equivariant algebra*, and their connections to number theory and combinatorics. Rather than treating these areas in isolation, I study how organizing principles such as duality, symmetry, and operadic compatibility arise in parallel forms across them. The following sections describe how these themes manifest in my recent and ongoing work.

2.1 Algebraic K-theory and Arithmetic Duality. My dissertation investigates how duality phenomena from number theory manifest in algebraic K-theory. In arithmetic geometry, a central duality known as *Tate–Poitou duality* describes relationships between the cohomology of number rings and their completions. A natural question is whether an analogous duality exists in algebraic K-theory.

The case at the prime 2 had remained open since the work of Blumberg and Mandell [BM20]. They proved the duality for all odd primes but left the even-prime case conjectural due to the essential role of real embeddings and the resulting technical complications. My dissertation resolves this case, establishing a K-theoretic version of Tate–Poitou duality at the prime 2.

Theorem 1 ([Cho25])**.** *Let F be a number field. There is a canonical weak equivalence between the homotopy fiber of the completion map κ in $K(1)$-local algebraic K-theory:*

$$\kappa\colon L_{K(1)}K(\mathcal{O}_F[\tfrac{1}{2}]) \longrightarrow \prod_{\nu|2} L_{K(1)}K(F_\nu^\wedge)$$

and the $\mathbb{Z}_2$-Anderson dual of the algebraic K-theory of $\mathcal{O}_F[\frac{1}{2}]$:

$$\mathrm{Fib}(\kappa) \simeq \Sigma^{-1} I_{\mathbb{Z}_2} L_{K(1)}K(\mathcal{O}_F[\tfrac{1}{2}]).$$

This equivalence can be viewed as a homotopical counterpart of Artin-Verdier duality of number rings. Figure 3 illustrates how the arithmetic and homotopical dualities align, connected through the Thomason descent spectral sequence.

$$\begin{tikzcd}
\textbf{Arithmetic duality:} \arrow[d, squiggly, "\text{Thomason SS}"] & H^s_{\text{ét}}\big(\mathcal{O}_F[\tfrac{1}{p}];\, \mathbb{Z}/p^k(\tfrac{t}{2})\big) \arrow[r, leftrightarrow, "\text{Pontryagin dual}"] \arrow[d, Rightarrow] & H^{3-s}_c\big(\mathcal{O}_F[\tfrac{1}{p}];\, \mathbb{Z}/p^k(-\tfrac{t}{2})\big) \arrow[d, Rightarrow] \\
\textbf{Homotopical duality:} & L_{K(1)}K(\mathcal{O}_F[\tfrac{1}{p}]) \arrow[r, leftrightarrow, "\text{Anderson dual}"] & \mathrm{Fib}(\kappa)
\end{tikzcd}$$

Figure 3: Arithmetic and homotopical dualities in parallel, linked by the Thomason descent spectral sequence.

An immediate consequence of this theorem gives new insight into the algebraic K-theory of the *sphere spectrum* $\mathbb{S}$. Using the case $F = \mathbb{Q}$ together with *trace methods*, one obtains the following result.

Corollary 2 ([Cho25])**.** *Let $\tau_{\mathbb{S}}\colon K(\mathbb{S})^\wedge_2 \to TC(\mathbb{S})^\wedge_2$ be the 2-completed cyclotomic trace map for the sphere spectrum. After taking connective covers, the homotopy fiber $\mathrm{Fib}(\tau_{\mathbb{S}})$ is canonically weakly equivalent to the $\mathbb{Z}_2$-Anderson dual of $L_{K(1)}K(\mathbb{Z})$:*

$$\mathrm{Fib}(\tau_{\mathbb{S}})[0,\infty) \simeq \Sigma^{-1} I_{\mathbb{Z}_2}\, L_{K(1)}K(\mathbb{Z})[0,\infty).$$

Together, these results provide concrete computational tools for the 2-primary algebraic K-theory of $\mathbb{S}$, clarifying how the homological behavior of the integers is reflected in stable homotopy theory through algebraic K-theory.

Beyond computation, this work relates to several active areas in arithmetic and homotopy theory. The homotopy fiber $\mathrm{Fib}(\kappa)$ appearing in the duality theorem connects to objects such as the p-adic Langlands correspondence [CE12] and compactly supported K-theory [Cla13]. By giving a precise homotopical description of $\mathrm{Fib}(\kappa)$, this work provides a foundation for exploring how these structures interact across arithmetic geometry and higher algebra.

Overall, this research develops new methods in 2-primary algebraic K-theory that clarify the structure of number rings and point toward broader extensions in arithmetic topology.

2.2 Equivariant algebras and homotopical combinatorics. My research in equivariant algebras investigates how combinatorial structures capture and control operations arising in *equivariant stable homotopy theory*. A central organizing idea is the notion of a *transfer system* [Rub21], which encodes which "transfer" maps between subgroups are allowed, subject to natural coherence conditions such as compatibility with conjugation and restriction. For example, in the case of the cyclic group C_{p^2}, the possible transfer systems are shown in Figure 4.

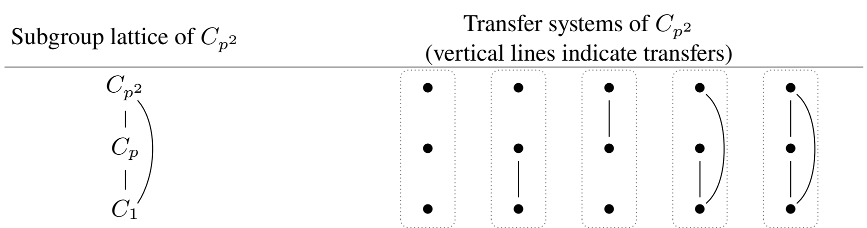

Figure 4: Subgroup lattice of the cyclic group C_{p^2} of order p^2 (left) and its five distinct transfer systems (right). These five systems classify, up to homotopy, the N_∞-operads for C_{p^2}.

Transfer systems provide a combinatorial model for the algebraic structure underlying genuine equivariant commutative ring spectra, encoded by *Tambara functors* [Tam93]. Their additive (transfer) and multiplicative (norm) operations are governed by a pair of transfer systems satisfying a compatibility condition, forming a *compatible pair*. In collaboration with David Chan, David Mehrle, Pablo Sanchez Ocal, Angelica Osorno, Ben Szczesny, and Paula Verdugo, we study the correspondence between such combinatorial data and the *pairings of N_∞-operads* [May09] that model these operations. Constructing an operad pairing is subtle even non-equivariantly, and our results give the first systematic framework for realizing them in the genuine equivariant setting through explicit combinatorial data.

Theorem 3 ([CCM+25])**.** *If two N_∞-operads admit a pairing, then their associated transfer systems form a compatible pair. If a compatible pair of transfer systems has a complete additive component, then there exists a pairing of N_∞-operads that realizes it.*

Motivated by this result, we conjecture that the correspondence between compatible pairs and operadic pairings holds in full generality:

Conjecture. *For every finite group G, every compatible pair of G-transfer systems should be realizable by some pairing of N_∞-operads.*

Establishing this equivalence would complete the conceptual bridge between the combinatorial and homotopical descriptions of incomplete Tambara functors and provide a combinatorial classification of admissible equivariant multiplicative structures.

Building on this framework, we have proved several further realizability results that support this conjecture. For instance, the transfer systems associated to the equivariant linear isometries and Steiner operads are realized by the corresponding pairing of operads, and J-local transfer systems $(\mathcal{T}_J, \mathcal{T}_J)$ are likewise realizable for any subgroup $J \leq G$. Moreover, we introduce a general construction principle showing that whenever two *intersection monoids* M and N admit a pairing $\xi\colon M \times N \to N$, one obtains a corresponding pairing of operads $(\mathcal{O}^\vee(M), \mathcal{O}^\vee(N))$.

Together, these results form the first systematic framework for classifying compatible transfer system pairs and understanding how combinatorial and homotopical perspectives on equivariant algebra align. They provide a computable foundation for studying incomplete Tambara functors and genuine equivariant ring spectra, and illustrate how ideas from combinatorics and operad theory drive new advances in equivariant homotopy theory.

2.3 Real algebraic K-theory and homological trace methods. *Real algebraic K-theory* offers a framework for studying algebraic structures equipped with involutive symmetries, combining topological and arithmetic ideas. Introduced by Hesselholt and Madsen [HM13], it assigns genuine C_2-equivariant spectra to rings with an *anti-involution*. This theory refines and unifies classical algebraic K-theory, Hermitian K-theory, and L-theory by incorporating this additional symmetry.

In collaboration with Teena Gerhardt, Liam Keenan, Juan Moreno, and J.D. Quigley, we are extending the *homological trace methods* developed by Bruner and Rognes [BR05] to the C_2-equivariant setting. Our work constructs a new spectral sequence based on $RO(C_2)$-graded homology, providing computational access to invariants in real algebraic K-theory.

Theorem 4 (Gerhardt–Cho–Keenan–Moreno–Quigley)**.** *For a C_2-equivariant E_∞-ring spectrum R with twisted $\mathbb{T}$-action, there is a natural $\mathcal{A}_\star^{C_2}$-comodule algebra spectral sequence of Mackey functors*:

$$E^2_{*,\star} = H^{-*}(B_{C_2}\mathbb{T}; H\underline{\mathbb{F}_2}_\star(R)) \Rightarrow H\underline{\mathbb{F}_2}^c_\star(R^{hC_2\mathbb{T}})$$

This construction provides the first homological tool for accessing real algebraic K-theory, extending classical trace methods while incorporating genuine C_2-equivariance.

Our approach builds on recent advances in *real trace methods*, which adapt topological Hochschild and cyclic homology to the C_2-equivariant context [Dot12, Høg16]. Extending the homological approach of Bruner and Rognes to this setting yields new computational methods for studying equivariant invariants and new approximations for important spectra such as the *real bordism spectrum* $MU_\mathbb{R}$.

The long-term goal of this project is to apply these techniques to analyze the real algebraic K-theory of fundamental spectra including $MU_\mathbb{R}$ and the *real Brown–Peterson spectrum* $BP_\mathbb{R}$. In particular, we aim to establish a *real analogue of the Segal conjecture* for these spectra, which would parallel one of the most powerful computational results in classical topology and provide new insights into how symmetry shapes algebraic K-theory at chromatic height 1 and beyond.

2.4 Hyperbolicity and Homotopical Perspective The framework of homotopy theory provides a remarkably general language for studying invariance and deformation, extending beyond topological spaces through simplicial presheaves and higher stacks. In complex geometry, the notion of *hyperbolicity* reflects a closely related idea: for instance, a compact Riemann surface of genus $g \geq 2$ is hyperbolic, admitting no nonconstant holomorphic maps from $\mathbb{C}$, which serves as a geometric analogue of homotopical rigidity.

Demailly's approach to hyperbolicity translates curvature conditions into cohomological language. Building on this perspective, Borghesi and Tomassini [BT17] extended the notions of Brody and Kobayashi hyperbolicity to simplicial presheaves and established their equivalence for compact analytic Deligne–Mumford stacks, generalizing the classical manifold case. My recent joint work with Gunhee Cho [CC25] develops this further by proving that the Green–Griffiths–Demailly thresholds governing invariant jet differentials remain unchanged when passing from a compact complex manifold to a smooth analytic Deligne–Mumford stack. This invariance provides an analytic manifestation of homotopical stability in a stack-theoretic setting.

Theorem 5 ([CC25])**.** *Let $\mathcal{X}$ be a compact smooth analytic Deligne–Mumford stack satisfying suitable cohomological regularity assumptions. Then every nonconstant entire map $f\colon \mathbb{C} \to \mathcal{X}$ factors through a proper closed analytic substack of $\mathcal{X}$.*

This result offers a concrete model for how invariance principles from homotopy theory can appear in analytic geometry and forms the foundation for ongoing work extending hyperbolicity and deformation theory to higher and derived stacks.

3 Future research direction

My future research builds directly on my current work, extending results on duality phenomena in algebraic K-theory and advancing the foundations of equivariant algebra. A central aim is to further develop the connections between homotopical methods, arithmetic dualities, and symmetry in algebraic and geometric contexts.

In parallel, I plan to develop projects that emphasize concrete computations and examples, especially in areas such as homotopical combinatorics and topological data analysis. These topics are well suited for undergraduate involvement and provide natural entry points into algebraic topology. I view cultivating such accessible research directions as an essential part of advancing the field and training the next generation of mathematicians.

Duality and Symmetry in Algebraic K-theory The emerging framework of *K-theoretic duality* reveals deep structural parallels between arithmetic geometry and stable homotopy theory. Its development points naturally toward several new directions, including higher-dimensional analogues, chromatic refinements, and real equivariant formulations that extend the reach of duality across algebraic and homotopical contexts.

A natural first step is to understand how K-theoretic duality behaves in higher dimensions. While substantial progress has been made for odd primes [Bra25], the higher-dimensional case at the prime 2 remains unresolved. The techniques developed in my thesis for addressing the subtleties of the 2-adic setting provide the tools needed to approach this open case and to clarify how duality extends across arithmetic dimension.

At the same time, these arithmetic phenomena suggest parallel structures in the *chromatic* layers of stable homotopy theory. Recent results [HRW22] indicate that the connective Adams summand l exhibits arithmetic-like behavior analogous to that of local rings in number theory. This analogy points toward a chromatic form of arithmetic duality, and I aim to formulate a corresponding K-theoretic duality at higher chromatic heights.

A third direction arises in the *real* or C_2-equivariant context, where algebraic K-theory interacts with symmetry in a genuinely equivariant way. An ultimate goal is the construction of a *real Thomason spectral sequence*, serving as a computational bridge between equivariant cohomology and real algebraic K-theory. Achieving this requires developing an equivariant form of 'etale cohomology for commutative ring spectra and adapting descent methods to the C_2-equivariant setting. Such a framework would parallel Thomason's classical spectral sequence while opening a systematic approach to the computation of real algebraic K-theory.

Foundations for Equivariant Algebra A key direction of my future work is to develop the basic algebraic framework underlying equivariant stable homotopy theory. While advanced constructions such as Tambara functors capture much of the structure, many of the elementary notions familiar from commutative algebra remain to be formulated in the equivariant setting.

A natural entry point is the study of commutative G-rings, a simpler type of equivariant algebraic object that Tambara functors generalize. Even in this case, elementary concepts such as algebraic extensions and algebraic closures exhibit subtle equivariant pathologies. Investigating these questions offers an accessible entry point that also exposes conceptual obstructions in the more general theory of Tambara functors. Beginning with these manageable cases allows one to develop the algebraic intuition and categorical techniques needed to approach the full equivariant framework.

My goal is to establish such foundational concepts and structures within the equivariant setting. This foundational program would provide the analogue of commutative algebra for equivariant algebraic geometry, bridging the gap between algebraic and homotopical formulations of equivariant descent.

Undergraduate Research Directions Several aspects of my work give rise to problems and projects that are accessible at an undergraduate level yet remain closely connected to broader research questions. Two areas in particular, combinatorial transfer systems and topological data analysis (TDA), offer tractable problems that both engage students and contribute to the development of my research programs.

One direction of my work focuses on the study of *transfer systems* from a combinatorial and educational perspective. Because transfer systems are defined directly in terms of subgroup lattices, they are easy to state and visualize, making them particularly suitable for undergraduate research projects. At the same time, their enumeration already reveals rich combinatorial structure: for example, the number of transfer systems for cyclic groups coincides with the *Catalan numbers* [BBR21], and many related patterns remain to be understood. Exploring these systems provides an accessible way to connect discrete mathematics with deeper questions in equivariant topology. Counting and classifying finite posets associated with transfer systems not only offer combinatorial insight but also inform the structural classification of N_∞-operads and Tambara functors, clarifying how discrete data govern admissible algebraic operations. In this way, the topic serves both as an entry point for students and as a bridge between combinatorics and the homotopical study of equivariant algebra.

Another area is *topological data analysis* (TDA), which applies algebraic topology to study the structure of high-dimensional data sets. Methods such as *persistent homology* can be introduced in a visual and computationally accessible way, yet they lead naturally into deeper questions about stability, functoriality, and the role of cohomology operations. These connections remain an active area of research, and concrete case studies in TDA provide a natural testing ground for exploring how homotopical invariants can yield new insights into the shape and geometry of data.

In this way, projects in combinatorics and data science serve not only as entry points for undergraduate participation but also as stepping stones toward more advanced problems aligned with my core research agenda.

4 Conclusion

At the heart of my research lies the perspective of *stable homotopy theory*, a framework in which topology, algebra, and geometry meet on equal footing. Within this setting, ideas such as duality and equivariance reveal how seemingly distinct structures reflect the same underlying homotopical patterns. My work develops this viewpoint to expose the shared logic that links arithmetic, combinatorics, and topology.

Looking ahead, I aim to expand this program by using stable homotopy theory as a unifying language for understanding duality and symmetry across mathematical contexts. A central goal is to build conceptual and computational tools—within algebraic K-theory, equivariant homotopy theory, and beyond—that clarify how these structures behave and interact. Equally important is keeping this program accessible: developing problems that invite undergraduate participation and creating avenues for collaboration across levels of expertise.

In this sense, my research advances not only the technical reach of stable homotopy theory but also its role as a common ground where diverse mathematical ideas can meet, evolve, and inspire new connections.

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