Table 2 Algorithm parameter Settings

|  |  |
| --- | --- |
| Algorithm | parameter settings |
| MTLBO | g=20000,=5 |
| CSA | *AP* = 0.1, *fl* = 2 |
| BOA | *p* = 0.8, c = 0.01, α = 0.1 |
| GWO | *a*max=2, *a*min=0, *b*=1 |
| WOA | *a*max=2, *a*min=0 |

Table 3 Unimodal benchmark functions

|  |  |  |  |
| --- | --- | --- | --- |
| Function | Dim | Range | *f*min |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-1,1] | 0 |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-10,10] | 0 |
|  | 30 | [-10,10] | 0 |
|  | 30 | [-1,4] | 0 |
|  | 30 | [-10,10] | 0 |
|  | 30 | [-4,5] | 0 |
|  | 30 | [-5.12,5.12] | 0 |

Table 4 Statistical results of the algorithms

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Functions | Index | MTLBO | TLBO | SLTLBO | BOA | CSA | GWO | WOA |
| F1 | best | 0.00E+00 | 6.73E-04 | 1.34E-63 | 1.30E-25 | 0.030312 | 6.77E-62 | 1.93E-167 |
| worst | 0.00E+00 | 1.18E+01 | 3.03E-57 | 1.09E-24 | 0.19873 | 3.39E-58 | 4.15E-151 |
| mean | 0.00E+00 | 5.33E-01 | 1.20E-58 | 3.70E-25 | 0.088894 | 4.57E-59 | 2.04E-152 |
| std | 0.00E+00 | 2.17E+00 | 5.51E-58 | 2.57E-25 | 0.042956 | 7.98E-59 | 7.96E-152 |
| F2 | best | 0.00E+00 | 1.68E-15 | 9.98E-122 | 1.01E-25 | 7.06E-09 | 4.34E-198 | 3.11E-233 |
| worst | 0.00E+00 | 2.40E-13 | 3.66E-115 | 1.52E-25 | 7.20E-08 | 1.43E-192 | 3.66E-209 |
| mean | 0.00E+00 | 2.09E-14 | 1.51E-116 | 1.06E-25 | 1.33E-08 | 4.81E-194 | 1.22E-210 |
| std | 0.00E+00 | 5.20E-14 | 6.71E-116 | 1.67E-26 | 1.61E-08 | 0.00E+00 | 0.00E+00 |
| F3 | best | 3.21E-244 | 3.89E-03 | 1.56E-37 | 1.11E-22 | 5.35E+00 | 2.33E-35 | 2.57E-115 |
| worst | 1.58E-239 | 1.91E+01 | 9.40E-33 | 5.62E-22 | 3.69E+01 | 3.08E-33 | 1.82E-100 |
| mean | 1.86E-240 | 1.69E+00 | 6.04E-34 | 2.16E-22 | 1.63E+01 | 8.00E-34 | 7.66E-102 |
| std | 0.00E+00 | 4.28E+00 | 1.74E-33 | 1.06E-22 | 6.41E+00 | 7.72E-34 | 3.41E-101 |
| F4 | best | 1.55E-226 | 2.03E+01 | 1.14E+00 | 9.34E-23 | 1.94E+00 | 2.83E-16 | 1.23E-01 |
| worst | 1.19E-221 | 3.78E+01 | 7.64E+00 | 2.30E-22 | 7.11E+00 | 2.29E-13 | 8.36E+01 |
| mean | 1.11E-222 | 2.51E+01 | 3.97E+00 | 1.40E-22 | 4.19E+00 | 1.94E-14 | 2.91E+01 |
| std | 0.00E+00 | 3.63E+00 | 1.89E+00 | 2.93E-23 | 1.25E+00 | 4.22E-14 | 2.50E+01 |
| F5 | best | 4.10E+00 | 4.01E-04 | 0.00E+00 | 5.61E+00 | 3.49E-02 | 1.74E-05 | 4.20E-03 |
| worst | 6.01E+00 | 2.37E+00 | 2.09E-30 | 7.50E+00 | 1.77E-01 | 1.98E+00 | 5.23E-01 |
| mean | 5.28E+00 | 1.82E-01 | 1.16E-31 | 7.22E+00 | 8.88E-02 | 7.25E-01 | 8.93E-02 |
| std | 5.15E-01 | 4.98E-01 | 3.90E-31 | 4.21E-01 | 3.86E-02 | 4.47E-01 | 1.26E-01 |
| F6 | best | 2.92E-243 | 3.19E-03 | 7.24E-37 | 6.67E+38 | 1.74E+02 | 9.00E-35 | 8.88E-113 |
| worst | 6.41E-239 | 1.14E+01 | 2.14E-31 | 2.59E+44 | 3.67E+02 | 5.06E-33 | 3.87E-102 |
| mean | 3.56E-240 | 6.97E-01 | 1.89E-32 | 2.09E+43 | 2.44E+02 | 1.17E-33 | 2.45E-103 |
| std | 0.00E+00 | 2.09E+00 | 4.47E-32 | 6.16E+43 | 4.77E+01 | 1.14E-33 | 7.79E-103 |
| F7 | best | 0.00E+00 | 3.22E-05 | 3.34E-123 | 1.05E-32 | 2.31E-12 | 5.54E-213 | 0.00E+00 |
| worst | 0.00E+00 | 3.74E-01 | 5.94E-102 | 4.03E+09 | 7.25E-07 | 6.30E-184 | 0.00E+00 |
| mean | 0.00E+00 | 2.57E-02 | 1.98E-103 | 2.27E+08 | 3.52E-08 | 2.16E-185 | 0.00E+00 |
| std | 0.00E+00 | 7.97E-02 | 1.08E-102 | 8.01E+08 | 1.33E-07 | 0.00E+00 | 0.00E+00 |
| F8 | best | 0.00E+00 | 2.36E-05 | 5.81E-65 | 8.49E-26 | 1.66E-02 | 8.19E-64 | 3.72E-167 |
| worst | 0.00E+00 | 1.54E-02 | 2.00E+00 | 2.02E-25 | 1.97E-01 | 8.34E-61 | 3.41E-146 |
| mean | 0.00E+00 | 1.68E-03 | 1.33E-01 | 1.22E-25 | 6.00E-02 | 1.26E-61 | 1.14E-147 |
| std | 0.00E+00 | 3.09E-03 | 5.07E-01 | 2.63E-26 | 3.98E-02 | 2.04E-61 | 6.23E-147 |
| F9 | best | 6.67E-01 | 1.19E+00 | 6.67E-01 | 9.88E-01 | 8.54E-01 | 6.67E-01 | 6.67E-01 |
| worst | 6.77E-01 | 1.87E+01 | 6.67E-01 | 1.00E+00 | 4.52E+00 | 6.67E-01 | 6.68E-01 |
| mean | 6.69E-01 | 6.29E+00 | 6.67E-01 | 9.98E-01 | 1.72E+00 | 6.67E-01 | 6.67E-01 |
| std | 2.72E-03 | 3.63E+00 | 1.22E-16 | 2.86E-03 | 9.84E-01 | 3.51E-06 | 2.07E-04 |
| F10 | best | 0.00E+00 | 2.92E-02 | 9.57E-06 | 9.61E-26 | 3.97E-01 | 6.58E-08 | 1.96E-152 |
| worst | 6.54E-316 | 2.64E+00 | 1.24E-04 | 5.45E-25 | 3.06E+00 | 2.92E-05 | 1.50E-05 |
| mean | 2.18E-317 | 4.11E-01 | 4.42E-05 | 2.12E-25 | 1.20E+00 | 6.37E-06 | 2.14E-06 |
| std | 0.00E+00 | 5.94E-01 | 2.79E-05 | 1.18E-25 | 6.29E-01 | 7.01E-06 | 4.00E-06 |
| F11 | best | 0.00E+00 | 5.31E-04 | 7.01E-65 | 1.11E-25 | 1.34E-01 | 1.17E-62 | 2.53E-171 |
| worst | 0.00E+00 | 2.31E+00 | 1.84E-57 | 5.45E-25 | 2.15E+00 | 9.69E-59 | 5.63E-150 |
| mean | 0.00E+00 | 1.00E-01 | 1.12E-58 | 2.06E-01 | 9.10E-01 | 7.69E-60 | 1.96E-151 |
| std | 0.00E+00 | 4.18E-01 | 3.52E-58 | 1.06E-25 | 5.30E-01 | 1.91E-59 | 1.03E-150 |

Table 5 Multimodal benchmark functions

|  |  |  |  |
| --- | --- | --- | --- |
| Function | Dim | Range | *f*min |
|  | 30 | [-5.12,5.12] | 0 |
|  | 30 | [-10,10] | 0 |
|  | 30 | [-5,5] | 0 |
|  | 30 | [-600,600] | 0 |
|  | 30 | [-1.28,1.28] | 0 |
|  | 30 | [-10,10] | 0.9 |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-100,100] | 0 |
|  | 30 | [-100,100] | 0 |

Table 6 Computational results in multimodal functions

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Function | Index | MTLBO | TLBO | SLTLBO | BOA | CSA | GWO | WOA |
| F12 | best | 0.00E+00 | 1.49E+01 | 1.19E+01 | 0.00E+00 | 9.96E+00 | 0.00E+00 | 0.00E+00 |
| worst | 0.00E+00 | 5.08E+01 | 5.57E+01 | 2.27E-13 | 5.18E+01 | 5.68E+00 | 0.00E+00 |
| mean | 0.00E+00 | 3.24E+01 | 2.98E+01 | 1.08E-13 | 2.78E+01 | 4.81E-01 | 0.00E+00 |
| std | 0.00E+00 | 8.32E+00 | 8.19E+00 | 6.22E-14 | 1.22E+01 | 1.32E+00 | 0.00E+00 |
| F13 | best | 2.92E-245 | 1.40E-04 | 2.27E-57 | 8.50E-23 | 3.99E-02 | 3.27E-35 | 1.61E-113 |
| worst | 7.59E-242 | 4.05E-01 | 4.39E-15 | 1.71E-22 | 4.41E+00 | 8.77E-04 | 8.53E-98 |
| mean | 1.14E-242 | 2.48E-02 | 2.25E-15 | 1.20E-22 | 1.04E+00 | 6.26E-05 | 2.84E-99 |
| std | 0.00E+00 | 8.66E-02 | 1.10E-15 | 2.38E-23 | 1.08E+00 | 1.94E-04 | 1.56E-98 |
| F14 | best | 1.69E-167 | 1.29E-01 | 2.97E-43 | 1.54E-16 | 4.78E-05 | 1.27E-78 | 3.29E-26 |
| worst | 1.11E-141 | 2.73E+02 | 1.61E-07 | 3.23E-10 | 6.73E-02 | 1.19E-50 | 1.65E-02 |
| mean | 3.70E-143 | 2.91E+01 | 5.37E-09 | 3.82E-11 | 6.48E-03 | 3.96E-52 | 1.19E-03 |
| std | 2.03E-142 | 6.75E+01 | 2.94E-08 | 7.18E-11 | 1.54E-02 | 2.17E-51 | 3.75E-03 |
| F15 | best | 0.00E+00 | 4.38E-05 | 0.00E+00 | 2.22E-16 | 1.71E-02 | 0.00E+00 | 0.00E+00 |
| worst | 0.00E+00 | 1.39E-01 | 9.51E-02 | 1.22E-15 | 7.87E-02 | 1.84E-02 | 6.03E-02 |
| mean | 0.00E+00 | 3.25E-02 | 1.47E-02 | 6.44E-16 | 4.15E-02 | 1.78E-03 | 2.01E-03 |
| std | 0.00E+00 | 3.34E-02 | 1.96E-02 | 2.45E-16 | 1.68E-02 | 4.88E-03 | 1.10E-02 |
| F16 | best | 2.71E-06 | 1.15E-01 | 6.83E-03 | 1.01E-05 | 1.67E-02 | 3.11E-04 | 6.10E-05 |
| worst | 8.16E-05 | 6.47E-01 | 4.09E-02 | 3.78E-03 | 5.40E-02 | 2.86E-03 | 9.69E-03 |
| mean | 2.64E-05 | 3.15E-01 | 1.72E-02 | 1.01E-03 | 3.44E-02 | 1.01E-03 | 1.55E-03 |
| std | 1.82E-05 | 1.38E-01 | 7.40E-03 | 8.60E-04 | 9.63E-03 | 5.45E-04 | 2.18E-03 |
| F17 | best | 9.00E-01 | 1.02E+00 | 1.00E+00 | 9.00E-01 | 1.00E+00 | 1.05E+00 | 1.00E+00 |
| worst | 9.00E-01 | 6.59E+00 | 1.30E+00 | 9.00E-01 | 1.06E+00 | 6.64E+00 | 2.26E+00 |
| mean | 9.00E-01 | 3.29E+00 | 1.01E+00 | 9.00E-01 | 1.01E+00 | 1.44E+00 | 1.05E+00 |
| std | 4.52E-16 | 1.66E+00 | 5.40E-02 | 4.52E-16 | 1.29E-02 | 1.07E+00 | 3.01E-01 |
| F18 | best | 8.88E-16 | 5.32E+00 | 6.22E-15 | 2.04E-14 | 1.35E+00 | 9.77E-15 | 8.88E-16 |
| worst | 2.66E-15 | 1.16E+01 | 3.16E+00 | 3.11E-14 | 6.54E+00 | 2.04E-14 | 6.22E-15 |
| mean | 1.78E-16 | 7.56E+00 | 1.53E+00 | 2.71E-14 | 3.48E+00 | 1.46E-14 | 1.72E-15 |
| std | 1.66E-15 | 1.45E+00 | 8.66E-01 | 3.70E-15 | 9.01E-01 | 2.87E-15 | 2.27E-15 |
| F19 | best | 0.00E+00 | 4.78E+00 | 2.06E-60 | 1.00E-24 | 1.70E+05 | 3.41E-58 | 6.52E-167 |
| worst | 0.00E+00 | 9.32E+04 | 7.39E-52 | 1.67E+09 | 1.60E+06 | 5.45E-55 | 2.63E-142 |
| mean | 0.00E+00 | 4.90E+03 | 2.69E-53 | 1.52E+08 | 6.29E+05 | 7.34E-56 | 9.09E-144 |
| std | 0.00E+00 | 1.77E+04 | 1.35E-52 | 4.32E+08 | 4.13E+05 | 1.16E-55 | 4.81E-143 |
| F20 | best | 0.00E+00 | 5.19E+01 | 3.47E-60 | 5.18E-25 | 3.81E+03 | 1.92E-58 | 3.78E-170 |
| worst | 0.00E+00 | 2.69E+06 | 5.86E-53 | 1.50E+12 | 1.68E+05 | 8.87E-55 | 1.79E-149 |
| mean | 0.00E+00 | 9.45E+04 | 2.39E-54 | 3.42E+11 | 2.78E+04 | 5.89E-56 | 1.24E-150 |
| std | 0.00E+00 | 4.91E+05 | 1.07E-53 | 4.84E+11 | 3.22E+04 | 1.65E-55 | 4.20E-150 |

Table 7 Fixed-dimensional benchmark functions

|  |  |  |  |
| --- | --- | --- | --- |
| Function | Dim | Range | *f*min |
|  | 2 | [-5,5] | 0 |
|  | 2 | [-100,100] | 0 |
|  | 2 | [-100,100] | 0 |
|  | 2 | [0,10] | 0 |

Table 8 Statistical results of the algorithms in Fixed-dimensional benchmark functions

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Function | Index | MTLBO | TLBO | SLTLBO | BOA | CSA | GWO | WOA |
| F21 | best | 0.00E+00 | 6.7081e-310 | 0.00E+00 | 4.50E-26 | 8.02E-07 | 0.00E+00 | 3.36E-257 |
| worst | 0.00E+00 | 7.01E-291 | 0.00E+00 | 2.21E-25 | 1.24E-03 | 0.00E+00 | 2.32E-219 |
| mean | 0.00E+00 | 3.09E-292 | 0.00E+00 | 1.20E-25 | 2.03E-04 | 0.00E+00 | 7.75E-221 |
| std | 0.00E+00 | 0.00E+00 | 0.00E+00 | 4.04E-26 | 2.97E-04 | 0.00E+00 | 0.00E+00 |
| F22 | best | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.67E-16 | 2.59E-03 | 0.00E+00 | 0.00E+00 |
| worst | 0.00E+00 | 0.00E+00 | 0.00E+00 | 5.55E-16 | 3.07E-01 | 0.00E+00 | 0.00E+00 |
| mean | 0.00E+00 | 0.00E+00 | 0.00E+00 | 3.18E-16 | 7.61E-02 | 0.00E+00 | 0.00E+00 |
| std | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.03E-16 | 6.83E-02 | 0.00E+00 | 0.00E+00 |
| F23 | best | 0.00E+00 | 4.77E-219 | 0.00E+00 | 3.29E-35 | 7.81E-09 | 2.02E-239 | 0.00E+00 |
| worst | 0.00E+00 | 4.75E-202 | 0.00E+00 | 4.44E-31 | 1.67E-05 | 4.44E-209 | 0.00E+00 |
| mean | 0.00E+00 | 1.72E-203 | 0.00E+00 | 6.05E-32 | 3.91E-06 | 2.05E-210 | 0.00E+00 |
| std | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.21E-31 | 4.28E-06 | 0.00E+00 | 0.00E+00 |
| F24 | best | 0.00E+00 | 4.33E-308 | 0.00E+00 | 1.32E-26 | 1.15E-07 | 0.00E+00 | 2.44E-275 |
| worst | 0.00E+00 | 1.95E-291 | 0.00E+00 | 1.96E-25 | 9.77E-04 | 0.00E+00 | 1.64E-219 |
| mean | 0.00E+00 | 6.98E-293 | 0.00E+00 | 1.19E-25 | 2.49E-04 | 0.00E+00 | 6.51E-221 |
| std | 0.00E+00 | 0.00E+00 | 0.00E+00 | 3.49E-26 | 2.54E-04 | 0.00E+00 | 0.00E+00 |

Table 9 MAE ranking of algorithms

|  |  |  |
| --- | --- | --- |
| Algorithm | MAE | Rank |
| GWO | 7.86E-02 | 1 |
| MTLBO | 1.49E+00 | 2 |
| WOA | 4.46E+01 | 3 |
| SLTLBO | 4.77E+01 | 4 |
| TLBO | 4.71E+08 | 5 |
| CSA | 2.09E+10 | 6 |
| BOA | 2.30E+85 | 7 |

Table 10 P values in Wilcoxon test

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Function | TLBO | SLTLBO | BOA | CSA | GWO | WOA |
| F1 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F2 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F3 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F4 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F5 | 3.02E-11 | 2.88E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F6 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F7 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 0.3337 |
| F8 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F9 | 3.02E-11 | 1.72E-12 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 6.70E-11 |
| F10 | 1.72E-12 | 1.72E-12 | 1.72E-12 | 1.72E-12 | 1.72E-12 | 1.72E-12 |
| f11 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F12 | 1.21E-12 | 1.21E-12 | 3.51E-12 | 1.21E-12 | 0.0055 | 0.3337 |
| F13 | 3.02E-11 | 3.00E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F14 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 | 3.02E-11 |
| F15 | 1.21E-12 | 1.30E-07 | 1.08E-12 | 1.21E-12 | 0.0419 | 0.1608 |
| F16 | 3.02E-11 | 3.02E-11 | 1.07E-09 | 3.02E-11 | 3.02E-11 | 8.15E-11 |
| F17 | 1.21E-12 | 5.31E-13 | NaN | 1.21E-12 | 1.21E-12 | 2.21E-06 |
| F18 | 1.14E-11 | 1.11E-11 | 5.48E-12 | 1.14E-11 | 4.30E-12 | 0.1271 |
| F19 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F20 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 | 1.21E-12 |
| F21 | 1.21E-12 | NaN | 1.21E-12 | 1.21E-12 | NaN | 1.21E-12 |
| F22 | NaN | NaN | 9.14E-13 | 1.21E-12 | NaN | NaN |
| F23 | 1.21E-12 | NaN | 1.21E-12 | 1.21E-12 | 1.21E-12 | NaN |
| F24 | 1.21E-12 | NaN | 1.21E-12 | 1.21E-12 | NaN | 1.21E-12 |

Table 11 Comparison of objective function values obtained by five methods

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Image | m | MTLBO | TLBO | SLTLBO | CSA | GWO |
| Lena | 3 | 14.8270 | 14.8234 | 14.8324 | 14.8324 | 14.8324 |
| 4 | 17.4697 | 17.4608 | 17.4686 | 17.4705 | 17.4705 |
| 5 | 19.9745 | 19.6933 | 19.9727 | 19.9751 | 19.9668 |
| Foxes | 3 | 16.0864 | 16.0858 | 16.0864 | 16.0864 | 16.0864 |
| 4 | 19.0907 | 19.0907 | 19.0907 | 19.0907 | 19.0907 |
| 5 | 21.8909 | 21.8817 | 21.8915 | 21.8921 | 21.8921 |
| Bridge | 3 | 15.6481 | 15.6481 | 15.6481 | 15.6481 | 15.6481 |
| 4 | 18.6647 | 18.6578 | 18.66377 | 18.6647 | 18.6647 |
| 5 | 21.5288 | 21.5240 | 21.5237 | 21.5319 | 21.5082 |