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Detection of weak target for MIMO radar based on Hough transform

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Abstract: An effective method of multiple input multiple output (MIMO) radar weak target detection is proposed based on the Hough transform. The detection time duration is divided into multiple coherent processing intervals (CPIs). Within each CPI, conventional methods such as fast Fourier transform (FFT) is exploit to coherent integrating in same range cell. Furthermore, noncoherent integration through several range cells can be implemented by Hough transform among all CPIs. Thus, higher integration gain can be obtained. Simulation results are also given to demonstrate that the detection performance of weak moving target can be dramatically improved.

Keywords: MIMO radar, weak target detection, Hough transform, noncoherent integration.

1. Introduction

Recently, MIMO radar has been developed. It has many advantages over conventional radar[1−2]. Two kinds of MIMO radar are introduced in Ref. [1] and references therein. One is receive and transmit diversity MIMO radar[2] (RT-MIMO). The other one is transmit diversity MIMO radar(T-MIMO). In Comparison to the latter, the former has larger space between the element of the array for the transmitter and the receiver. RT-MIMO utilizes the spatial diversity to improve the detection performance of the target by overcoming the target's RCS-fluctuation. However, the method cannot do well in practical applications due to the fact that it is difficult to implement signal processing techniques for the received data. Therefore, in this article, we will discuss the latter, i.e. T-MIMO. Although both RT-MIMO and T-MIMO emit orthogonal signals from each antenna, the advantage of T-MIMO over RT-MIMO is that T-MIMO can form a wide-beam to cover a capacious space, where the digital beam forming (DBF) is used to receive the echoes of target. T-MIMO has lower power density than conventional radar at the same range[1] so as to have shorter intercepted range. On the other hand, the integration time for the signal of T-MIMO can be longer than that of the conventional radar, which can improve detection capability and the velocity resolution. However, the target may travel from one to the other range bin of the radar during the long detection time, which is referred to as range migration. In this article, we exploit Hough transform to compensate the range migration. In a certain time, however, the target moves according to a straight line. Moreover, if only we can detect the target's track, we can detect the target. The Hough transform is an useful tool for image processing to find line in a picture[3]. Carlson is the very pioneer to apply the technique to detect target in search radar[4−6], and Li[7] use it in synthetic aperture radar(SAR) later. In this article, we exploit the Hough transform to integrate signals from CPIs. Our proposed detection procedure can be divided into two steps excluding the estimation process. First, the conventional algorithms such as DBF and MTD can be utilized to coherently integrate signal and constant false alarm rate (CFAR) be implemented to detect the signal in each CPI. Second, we can use the Hough transform to project the detection results of the first step into Hough parameter space and detect the target in the Hough parameter space then the detecting result can be projected back to data space.

2. Signal model and receive signal processing processing

The linear frequency modulated (LFM) signal is useful in practice. It has high range resolution. In this section, we use LFM as transmit signal.

The MIMO radar has M emitting antennas and N receivers. Each emitter transmits a LFM signal at a different frequency.

$$s_i(t) = \exp\left(\mathrm{j}(2\pi(f_0 + \Delta f \cdot (i-1))t + \frac{1}{2}\mu t^2)\right) \quad (1)$$

$1 \leqslant i \leqslant M$, j is imaginary unit, f_0 is the carrier frequency, Δf is the frequency interval between signals, which must be chosen large enough to satisfy the orthogonal condition described in Eq. (2). μ is the frequency modulation coefficient. T_0 is the signal duration of time.

$$\int_0^{T_0} s_m(t) s_n^*(t) \mathrm{d}t = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases} \quad (2)$$

During one detection time T, radar sends L_c CPIs. Each CPI contains L_p pulses. The target is at range R from the radar. Its radial velocity is v. In the MIMO radar of transmit-diversity model, the receive signal can be expressed as

$$r_i(t) = \sum_{l=1}^{M} \sigma_l s_l(t-\tau) \exp(\mathrm{j}2\pi f_d(t-\tau)) \quad (3)$$

$1 \leqslant i \leqslant N$, σ_l is reflecting coefficient of target, which contain the attenuation. τ is the delay. f_d is the target's Doppler frequency

$$\tau = \frac{2R}{c}$$
$$f_d = \frac{2vf_0}{c} \quad (4)$$

c is the propagation velocity of light.

At the receiver, we first use matched filter to separate each transmit signal. And we can get $M \times N$ signals. After matched filtered, the signal to noise ratio is SNR_0. We carry out the first coherent integration between different signals by conventional digital beam forming (DBF) technique. After DBF, the signal to noise ratio is SNR_1. The integration gain is[8−9]

$$SNR_1/SNR_0 = M \cdot N \quad (5)$$

In each CPI, at each range bin, MTD is utilized to detect moving target, the signal to noise ratio after MTD is SNR_2, we can obtain gain at this step[8−9]:

$$SNR_2/SNR_1 = L_p \quad (6)$$

After MTD, we perform CFAR detection. At this step, the probability of false alarm is p_{f1} the probability of detection is p_{d1}, and the detection threshold is Λ_1. This step is similar to conventional pulse integration detection[10,11]. But we chose the threshold Λ_1 differently. In conventional method, to obtain low false alarm probability, Λ_1 is usually set high. But weak target will be missed. In our method, we set Λ_1 lower to detect weak target. This will lead to high probability of false alarm in each CPI. But we can get the ultimate false alarm probability to the desired value after the noncoherent integration detection. The signal processing steps within CPI is illustrated in Fig. 1.

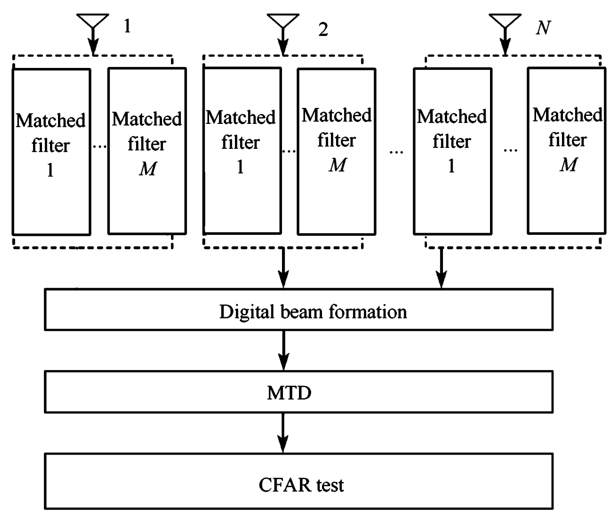

Fig. 1 Coherent integrating within CPI

All signals in every CPI are processed according to steps demonstrated in Fig. 1. And we use a tow dimension picture $t-r$ to record the detecting result of this step. The t is the number of CPI, denoting time. And the r is the number of range bin, denoting range. If a target exist at some range at a time, the value of a pixel in $t-r$ is 1 according to that time and range, otherwise its value is 0.

The main difference between the signal process in T-MIMO radar and conventional phased radar is the matched filter and beam formation. It is shown in Fig. 2.

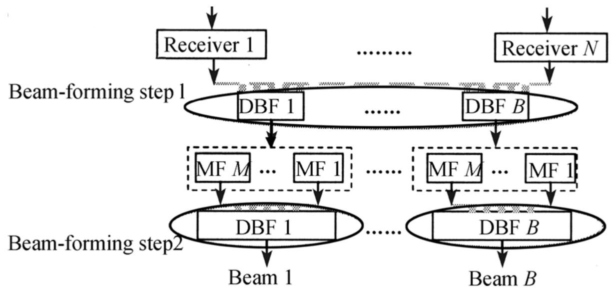

Fig. 2 Signal processing of T-MIMO radar

At the receiver, the signals from different receiver are exploited to form beams at the receiver. And then, the signal from each DBF is separated using match filter. Finally, the separated signal is used to form beams at the transmitter. Unlike the conventional phased radar, the T-MIMO radar can form multiple beams at the receiver and transmitter.

3. Theory of the Hough transform and and noncoherent integrate between CPIs

In this section, we discuss the noncoherent integration using the Hough transform. In Section 2, we exploit conventional DBF and MTD to processing the signal data of each CPI. A low primary threshold Λ_1 is set to detect the target. The detection result is a 2-D image. Because the target travels by a straight line, the target's trajectory in the image is a line. We use the Hough transform to find this line. If this line is detected, it contains all of the current information, as well as a complete history of its trajectory.

The Hough transform is a feature detection method frequently used in image processing and is well suited to locating lines in a plane of noise. Other arbitrary shapes can be detected too[12], but for now, only straight lines are considered. The Hough transform maps points in the range-time $(t-r)$ data space into curves in a $\rho-\theta$ space or Hough parameter space by

$$\rho = t\cos(\theta) + r\sin(\theta) \tag{7}$$

The mapping can be viewed as stepping through θ from 0° to 180° and calculating the corresponding ρ.It can be shown through trigonometric manipulation that Eq. (7) is equivalent to

$$\rho = \sqrt{r^2+t^2}\sin\left(\theta + \arctan\frac{r}{t}\right) \tag{8}$$

A single $\rho-\theta$ point in Hough parameter space corresponds to a single straight line in $(t-r)$ space with that ρ and θ value. Any one of the sinusoidal curves in Hough parameter space corresponds to the set of all possible lines in the data space through the corresponding data point. If a line of points does exist in $(t-r)$ data space, this line is represented in Hough parameter space as the point of intersection of all of the mapped sinusoids.

This mapping form $(t-r)$ data space into parameter space is easily implemented by a simple matrix multiplication. A data matrix D can be defined from any data crossing the primary threshold Λ_1 as

$$D = \begin{bmatrix} r_1 & \cdots & r_I \\ t_1 & \cdots & t_I \end{bmatrix} \tag{9}$$

where the columns are the range and time values of all of the I primary threshold crossings. A transformation matrix H composed of the sines and cosines from Eq. (7) can be defined as

$$H = \begin{bmatrix} \sin\theta_1 & \cdots & \cos\theta_1 \\ \vdots & \vdots & \vdots \\ \sin\theta_{N_T} & \cdots & \cos\theta_{N_T} \end{bmatrix} \tag{10}$$

where the θ values are the N_T discrete values from 0° to 180° for the cells of parameter space. The multiplication of D and H produces an N_T by I matrix, R. The subscripts of ρ values represent the index of the primary threshold crossing data point and the angle, θ, used in the discrete map

$$R = HD = \begin{bmatrix} \rho_{1,\theta_1} & \cdots & \rho_{I,\theta} \\ \vdots & \vdots & \vdots \\ \rho_{1,\theta_{N_T}} & \cdots & \rho_{I,\theta_{N_T}} \end{bmatrix} \tag{11}$$

Each column of R contains the ρ values for one of the parameter space sinusoids.

A primary threshold crossing is mapped into parameter space from the $(t-r)$ cells, and its power is added into the $\rho-\theta$ cells that intersect the corresponding sinusoidal curve in parameter space. In this way, the accumulator cell at the intersection of several sinusoids will reach a high value. A secondary threshold applied to the parameter space can now be used to declare the detections. The ρ and θ of the detected point in parameter space can now be mapped back into $(t-r)$ space to show the time history and current position of the detected target.

This is a way of noncoherent integrating the returns from a moving target over a long time period. Suppose the signal to noise ratio after Hough transform is SNR_3. Through Hough transform we get gain[8−9]

$$SNR_3/SNR_2 = \sqrt{L_c} \tag{12}$$

So, the total gain is

$$SNR_3/SNR_0 = M \cdot N \cdot L_p \cdot \sqrt{L_c} \tag{13}$$

And, the probability of false alarm and the probability of detection is[5,7]

$$P_F = \sum_{i=k}^{N} C_N^i p_{f1}^i (1-p_{f1})^{N-i}$$

$$P_D = \sum_{i=k}^{N} C_N^i p_{d1}^i (1-p_{d1})^{N-i} \tag{14}$$

where p_{f1} and p_{d1} is the false alarm probability and detection probability of first detection[5,7].

$$p_{f1} = e^{-\eta}$$

$$p_{d1} = e^{-\eta/(1+snr)} \tag{15}$$

where snr is the signal-noise ratio.

If there are many targets, and each target has a traveling track differently, the Hough transform can detect all of the straight lines simultaneously.

4. Simulation

To verify the detection performance of our method, we design the simulation as follows. MIMO radar has 16 transmitters $M = 16$ and 16 receivers $N = 16$. The pulse repetition frequency (PRF) is $f_{PRF} =$ 1 kHz. The signal bandwidth is 100 MHz. The radial velocity of target is $v = 60$ m/s and the target is approaching to radar. From these parameter, we can figure out that the range bin is 1.5 m. There are 25 pulses in each CPI.

In the first simulation, we demonstrate the detection methods. In Fig. 3, we plot the first detection result when the signal noise ratio SNR_0=2 dB. We set the threshold Λ_1 according to $p_{f1} = 10^{-3}$. We can see from Fig. 3 that there are many false targets due to the low threshold Λ_1. Then, we use the Hough transform to map the points in Fig. 3 into Hough parameter space and perform detecting in parameter space. After detecting, we map the points in Hough parameter space into $(t-r)$ space as Fig. 4.

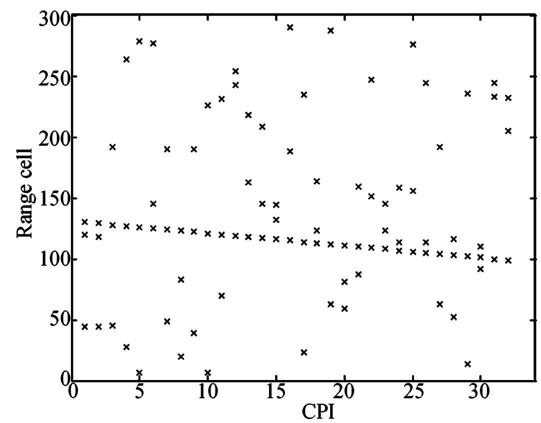

Fig. 3 First detection result

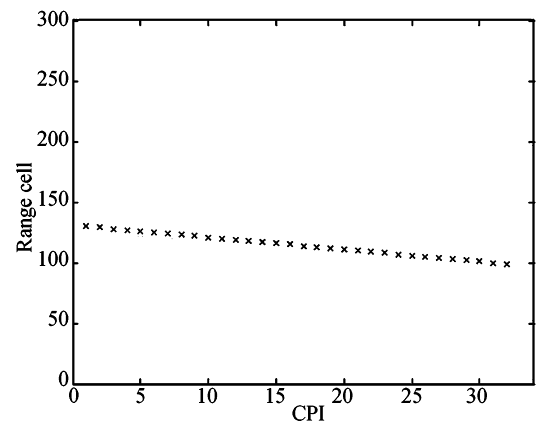

Fig. 4 Second detection result

In following simulation, we compare the detection performance when the L_c is different. After DBF and MTD in each CPI, we get the integration gain $10 \cdot \lg(16 \cdot 16 \cdot 24) = 37.9$ dB. The gain of noncoherent integration between CPIs is $10 \cdot \lg\sqrt{L_c}$. In simulation, we set the ultimate false alarm probability $P_F = 10^{-6}$, the second detection threshold is $K = \dfrac{L_c}{2}$. We plot the receiver operating curve as Fig. 5. The horizontal axis is the signal to noise ratio

SNR_2 before Hough transform. The vertical axis is the detection probability P_D. When $L_c = 1$, we do not perform noncoherent integration. The detection probability is about 0.4 for weak target $SNR_2 \leqslant 5$ dB. With L_c increasing, the detection probability P_D increases. It can been observed that when $L_c = 64$, for the weak target of $SNR_2 = 5$ dB, the detection probability P_D is 0.97. We get the noncoherent gain $10 \cdot \lg \sqrt{L_c} = 9$ dB. In the simulation condition, for the weak target $SNR_0 \geqslant -32$ dB false alarm probability $P_F = 10^{-6}$, we can get $P_D = 50\%$.

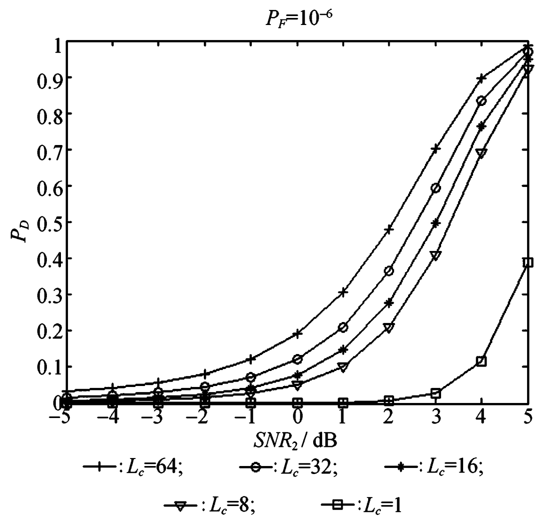

Fig. 5 Detection performance curve for different L_c

5. Conclusion

In this article, a new method based on Hough transform to realize noncoherent integration between CPIs for MIMO radar is proposed. In the long detection time of MIMO radar, the target may travel many range bins. Our proposed procedure can overcome the problem of the range migration. Within each CPI, we perform conventional DBF and MTD. Then we use Hough transform to integrate signals from different CPIs. This method can improve the detection capability for weak target.

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