2022 HiMCM

Problem A – The Need for Bees (and not just for honey)

Team: 12936

Summary sheet

The topic of inquiry involves variables from many different angles. To discover the relationship between time and population size, our team started thinking about using existing mathematical models. Considering the characteristics of different models, we finally chose the Malthus model, the Logistic model, and the multiple linear regression model. The multiple linear regression model has the advantage of finding the relationship between single variables; the Malthus model is better at dealing with the relationship between multiple data.

Before establishing the mathematical model, our group first analyzed and summarized the known conditions. Combining the information found on the Internet with the information given in the question, we can roughly identify the main variables, which are also involved in the calculation. They can be divided into three parts: external natural factors, external human factors, and the bee community's influence. From the theme itself, the influence of bee survival activities in and around the hive is the most important and should be dealt with in the first place. They give you information about the number of bees in a hive, the number of natural enemies, the number of deaths from exertion, and so on. Further study, it can be found that the fertilization rate of larvae, birth rate, age of sexual maturity, and other factors are worth considering. Due to the limited amount of work that can be completed within a certain period, we selected several variables with obvious influence as analysis and processing objects and weighted them in the process of sensitivity analysis. The same working procedure can be applied to both natural and man-made extrinsic factors. Through the study and research of previous papers, we found a lot of existing literature showing that pesticides and the number of people around the hive range influence the total population of the colony. We also analyzed them.

In the analysis of follow-up questions, we further study the existing information. Continuing our search for other papers, we find the path paved by those who came before us. In predicting pollination, we find that we can study individuals as objects and then extend to the macro perspective. Firstly, the pollinating bee demand for a single grain is studied, and then the number of grains is gradually increased, to achieve the purpose of predicting the number of bees within the unit range. All the above research can be used by humans to benefit from analysis, better environmental protection, and maximize bee productivity. After the study was completed, we planned for the future.

Keywords: Honey bee, Hive, Population, Pollination

Menu

1 Introduction

1.1 Background information

Honeybees are vital to the production of the world's food crops, but they have been decimated around the world since 2007, threatening to exacerbate an already tense global food crisis. It will threaten human survival. Such warnings may be alarmist-after all, large numbers of honeybee death can exacerbate the global food crisis and will threaten the survival of human beings. The reason why honeybees are important is that they are the most important pollinators in nature. To collect food such as nectar and pollen, honeybees form many specialized organs and special behaviors, which can not only allow them to collect food efficiently but also let bees become ideal pollinators. They can help pollinate wind-pollinated grains, such as rye, wheat, and rice, which is a good way to improve the incline of the grain and that is useful to a human.

In 2016, the IUCN Red List of Threatened species included 31 bee species. The study, published in the journal Nature Communications, shows that the number of wild bee species in England has fallen by an average of 7% since 2002. A study by the University of Vermont found that the number of wild bees in agricultural areas of California, the Pacific Northwest, and the Southwest declined by 23 percent between 2008 and 2013. The United States lost 28.1 percent of its colonies in the 2015-2016 winter, while Canada lost 16.8 percent, according to a study by Australia's Macquarie University.

1.2 Problem Restatement

The honeybee is crucial. Several factors can reduce their numbers. It can be divided into human factors and natural factors. Among human factors, pesticides and construction facilities that destroy habitat are the most influential factors. Natural factors such as viruses and predators also are considered. Among the conditions that can be exploited are the honeybee's flight distance, which can predict its home range; The number of flowers picked, one of the factors worth considering, indirectly contributes to population growth; Affected by the seasons, honeybee work longer in summer and longer in winter. By using the relevant factors, it is possible to build a model which shows the changes in the population of honeybees over time. After that, the necessary analysis of the model can make it more effective. At the same time, the model will be realistic. For example, how many beehives can support 20 acres of land, including crops that benefit from pollination.

2. Description of assumptions and the Variables' meaning

2.1 General Assumptions

In our research and our paper, we assume that the background environment of our research is a mountainous area. The species of honeybees are Chinese honeybees. It is a subspecies of Oriental honeybee, which is unique to China. They are the main pollinator in forest communities dominated by mixed trees and traditional agriculture. Chinese honeybees fly quickly and have a good sense of smell. They have a factor of the better ability to use sporadically scattered plants, strong collection ability, high utilization rate, long nectar gathering period, adaptability, strong anti-mite, and disease ability, and feed consumption. They are prone to be infected with bee vesicular larva disease, susceptible to wax moth damage, and like to migrate. They are especially prone to abandon their nests and migrate when they are short of honey or threatened by diseases and pests. They are prone to natural wasps and steal bees. So, we let the maximum number in one honeycomb be 80,000 and the initial number is 20,000.

The temperature is changing in every season and we show its change as a temperature change of the central temperature of the Chinese honeybee colony. They were (32.57 ± 0.29) ℃ and (32.78 ± 0.60) ℃ in spring and (34.03 ± 0.33) ℃ and (34.26) ℃ in summer, respectively. In autumn (32.19 ± 0.26) ℃, (32.39 ± 0.26) ℃, in winter (26.14 ± 0.39) ℃, (27.47 ± 0.31) ℃. The edge temperatures were (22.24 ± 1.77) ℃ and (22.49 ± 1.78) ℃ in spring. Respectively, (33.71 ± 0.64) ℃, (34.O g ± 0.66) ℃, autumn (19.27 ± 0.54) ℃, (19.684-0.52) ℃, (14.20 ± 0.09) ℃, (14.81 ± 0.07) ℃ in winter.

2.2 Variables and Definitions

Variable	Description
X_0	The initial population of honeybee colonies
Δt/month	Time variation
t/°C	Temperature
x (t)	The population of the honeybee colonies at time t
d (t)	Honeybee mortality
b (t)	the birth rate of honeybees
r	The population growth rate of honeybee
k	environmental carrying capacity
Φ	Imidacloprid toxicity regression to honeybee
S	Sublethal dose of imidacloprid
l	Larval mortality
c	Worker bee mortality caused by fatigue
M	Month
Δt’	Temperature change
p	Human population density (Unit: person/square kilometer)

3. Model building and discussions

In this research, we used these variables in our model-building project. The model we used in our paper, is the Malthus model and the Logistic model.

3.1 The use of established models and theories

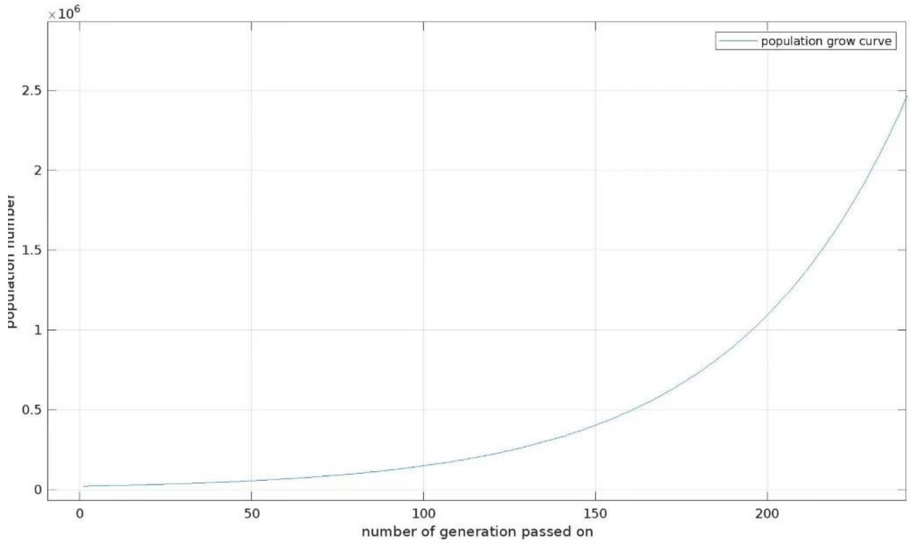
3.1.1 Malthus Model

In this research, we used these variables in our model-building site project. The model we used in our paper, are there Malthusian model and the Logistic model.

The Malthusian growth model is a mathematical equation for population growth in a short period. It holds that the growth rate is proportional to the current population. This is functionally equivalent to exponential growth, in which population size doubles in predictable time intervals. So, in our research, we used this model to simulate the population dynamics of honeybees.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = rx(t)$$

We input the initial population of the honeybee in their colonies, temperature, and the population growth rate of honeybees into this model, and here is the graph we made and the code we used to build this graph.



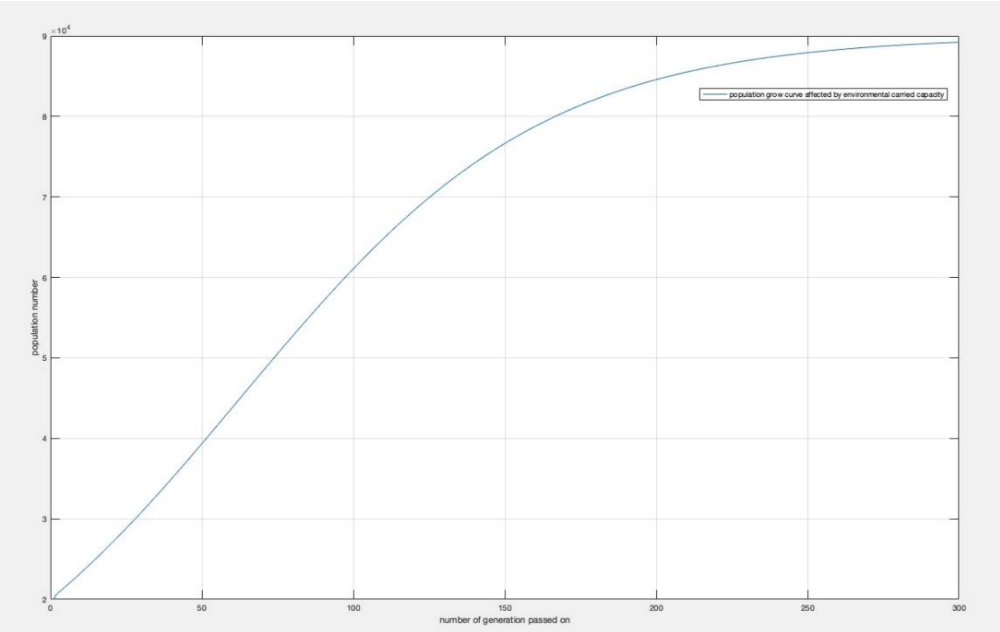
In this formula, we let the initial population of honeybee's colonies which is X_0=20,000 and the population growth rate of honeybees which is r=0.6. And as a result, we built this graph. But we find out a question is we haven't imported the environmental carrying capacity rate which is about the maximum population in a specific space. Therefore, we figure out a new model called the Logistic model.

3.1.2 Logistic Model

The Logistic model reveals that the rate of population growth is determined by its biological potential and that population size is modified by natural resistance, in other words, it means that we can solve the problem of the Malthus model and input a new variable called environmental carrying capacity into the new model. It will be able to simulate the trend of the dynamic changes in the same environment over a long period. As a result, we input the environmental carrying capacity and the other variables are the same as the last model into this formula.

$$\frac{dx}{dy} = rx\left(1 - \frac{x}{k}\right) = rx\frac{k-x}{k} = rx - \frac{r}{k}x^2$$

Later on, we input the initial population of the honeybee in their colonies, temperature, the population growth rate of the honeybee, and environmental carrying capacity into this model. Here is the graph we made and the code.



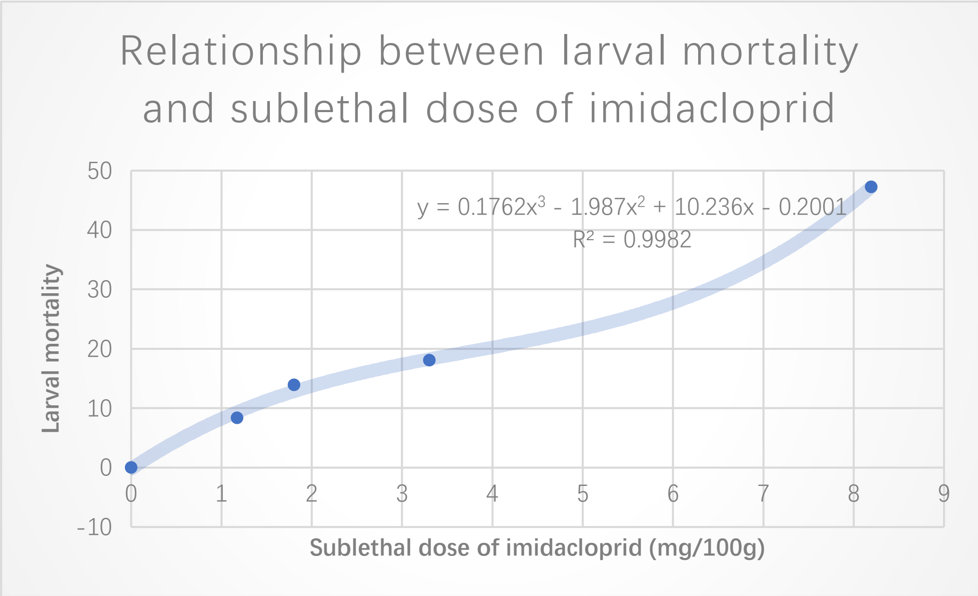
In this formula, we let the initial population of honeybee’s colonies which is X_0=20,000, and environmental carrying capacity which is K=90,000.

3.2. Demonstrating the effects of different variables on the mortality of honeybee colonies

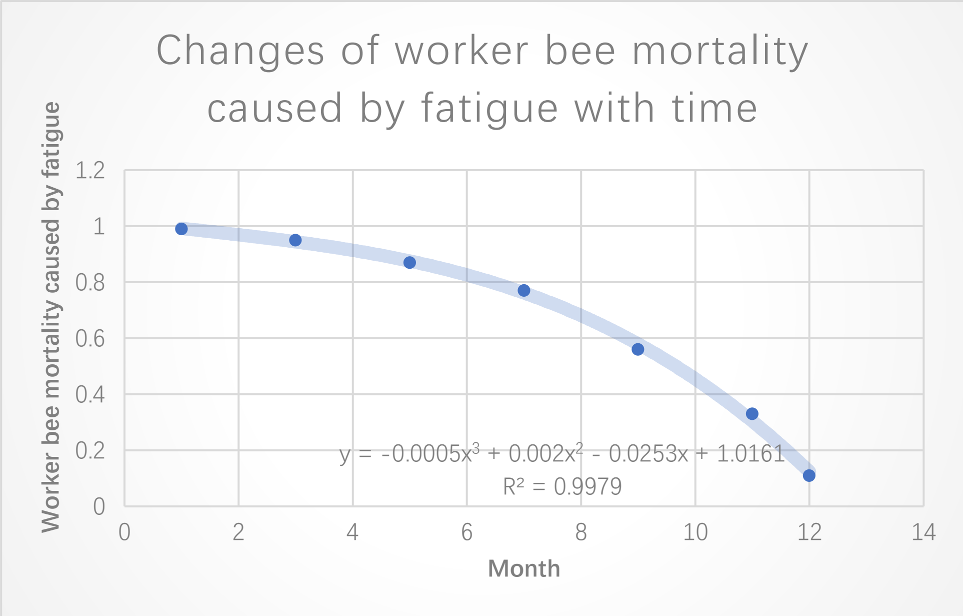
Our model is based on a particular species of honeybee which is the Apis cerana cerana Fabricius also known as the Asian honeybee, all of our databases are carried on with the data of Asian honeybees in Fujian Sichuan and Shandong provinces by agricultural universities where is also the habitat for the Asian honeybee to live.

We discover 5 different variables that affect the mortality of the honeybee’s colonies. As a result, the mortality influenced the population in the honeybee colonies.

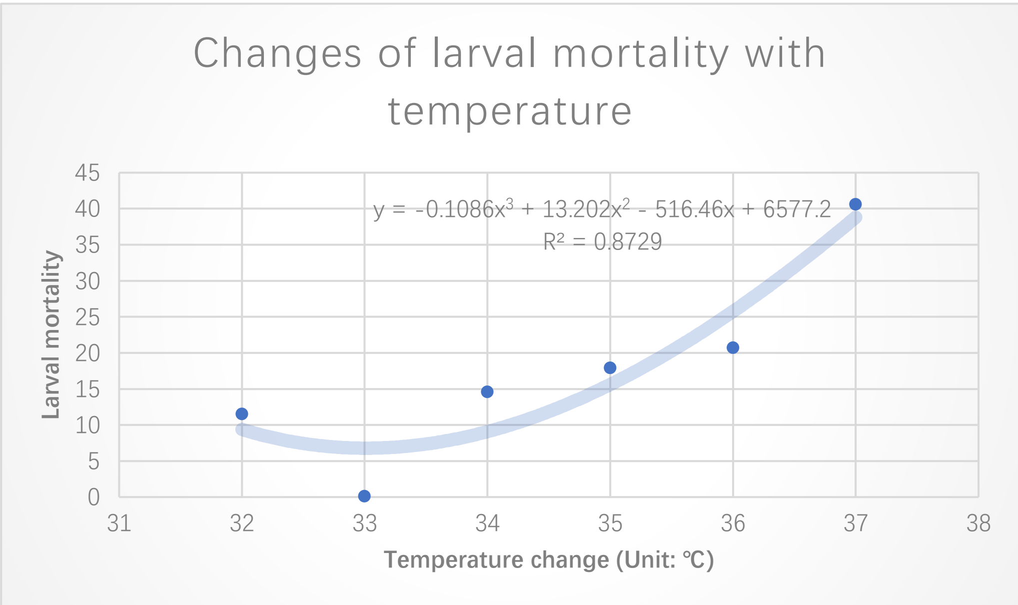
1. The first variable is the sublethal dose of imidacloprid, according to the data from research carried out by Sichuan Agricultural University. By using Trivariate regression, we plot the graph of the relationship between larval mortality and the sublethal dose of imidacloprid. From the graph, we can also calculate the coefficient of determination between these two variables.



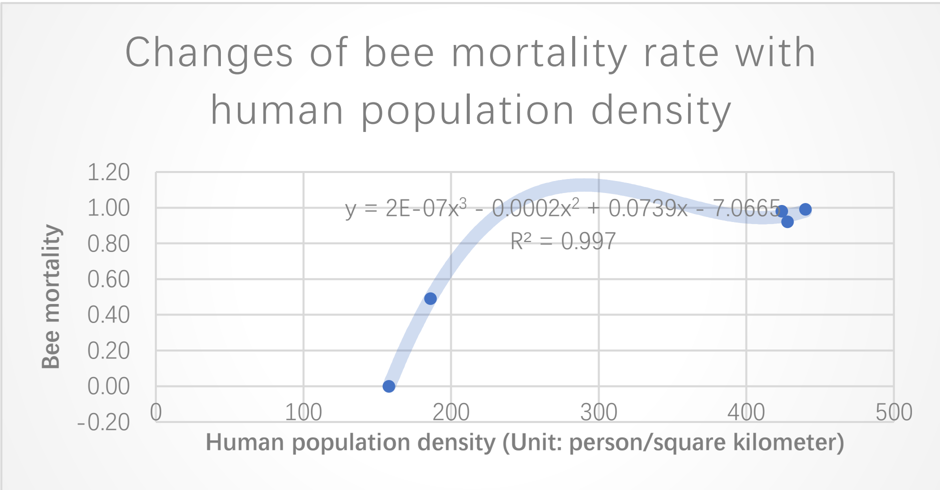
2. The second variable is the changes in worker bee mortality caused by fatigue over time. In the summer, the high temperature caused the bee to increase the rate of flapping their wings which lead to a higher cost of protein with a higher degree of strain. According to the data of "The America Bee Journal", the average age of a worker bee in the summer is only 25 to 40 days. In contrast, the average age in the winter is around 154 days. We analyze data from the research done by Fujian Agriculture and Forestry University, using polynomial regression we plot the data on the graph shown below. As a result, the mortality of worker bees rises dramatically. Based on the graph, it's obvious to see that the mortality of worker bees decreases as time went on from June to December. As the summer ends in August, the mortality declines sharply, and the population can start to increase back to the number before summer.



3. The third variable is the changes in larval mortality with temperature. From the experiment of Utah State University, we use another polynomial regression function to demonstrate the relationship between larval mortality and temperature. As the temperature increase, the mortality first decreases and then increase. The lowest mortality is at 33 Celsius which the number of larvae has the biggest survival population.



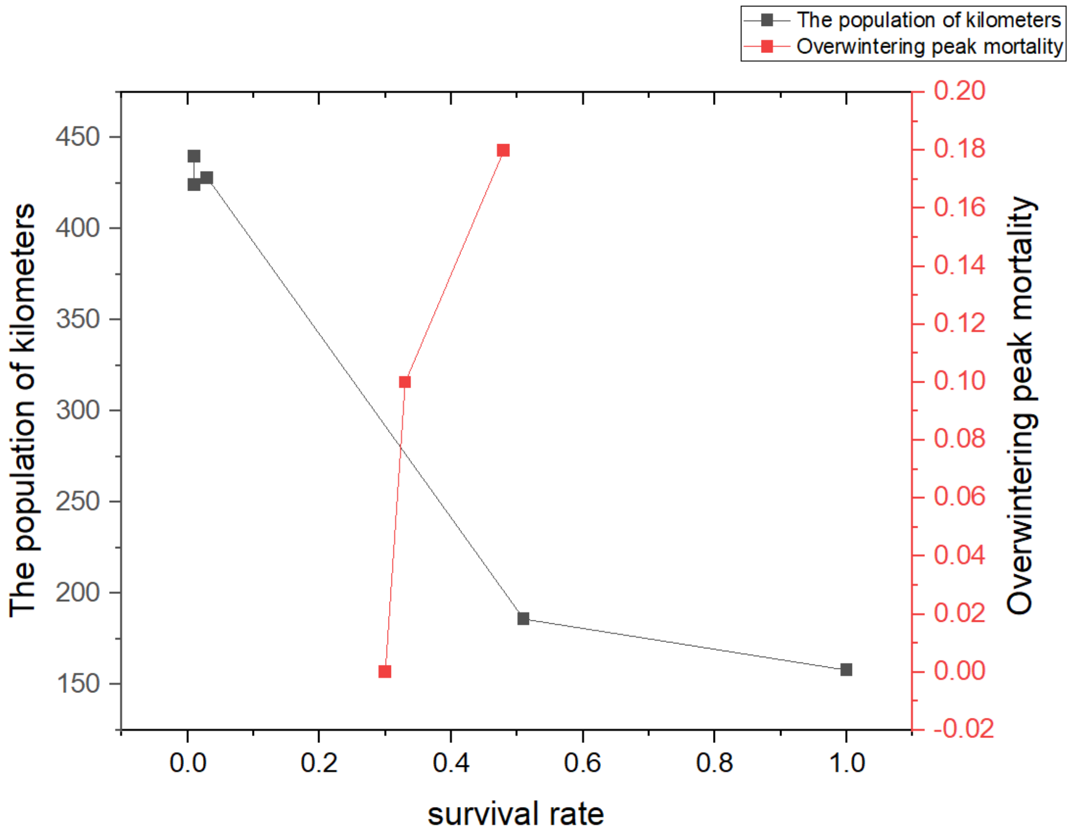
4. The fourth variable is the changes in bee mortality rate with human population density. Human actions have always been an important reason for the decline of the honeybee population. Humans destroyed forests and turn them into crops land, roads, highways, or other infrastructure. These contribute to the loss of both habitats and food sources for bees. From research carried out by the University of Shandong, we plot the graph of the changes in the bee's morality to the density of the human population. From the graph, the bee's mortality first increases and then suffered a small decline when the human density is 350 people per square kilometer.



After plotting the graph of different variables, we can use the function's coefficient of determination in each individual case. Since their dependent variables are all the mortality of the honeybee's population, we can compare the r square which is the coefficient of determination.

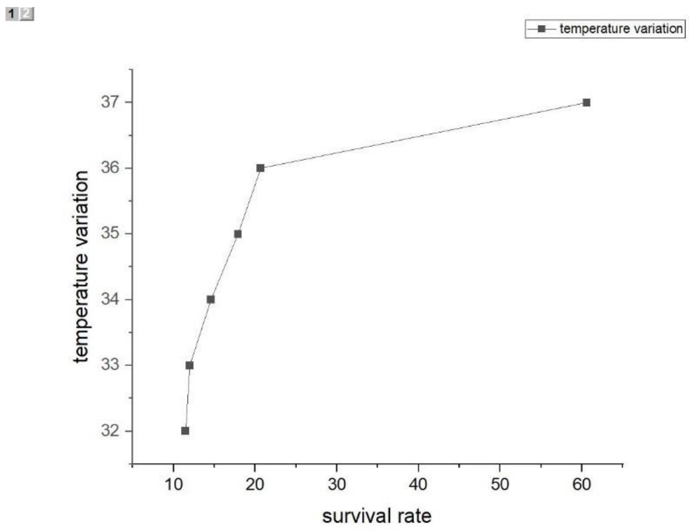
From each individual graph, we can find that the sublethal dose of imidacloprid has the greatest impact on the mortality of the honeybee's colonies since it has the greatest coefficient of determination in magnitude.

In our colonies, the mortality influenced the population in the colonies directly, so pesticides have the greatest impact on the colonies' size.



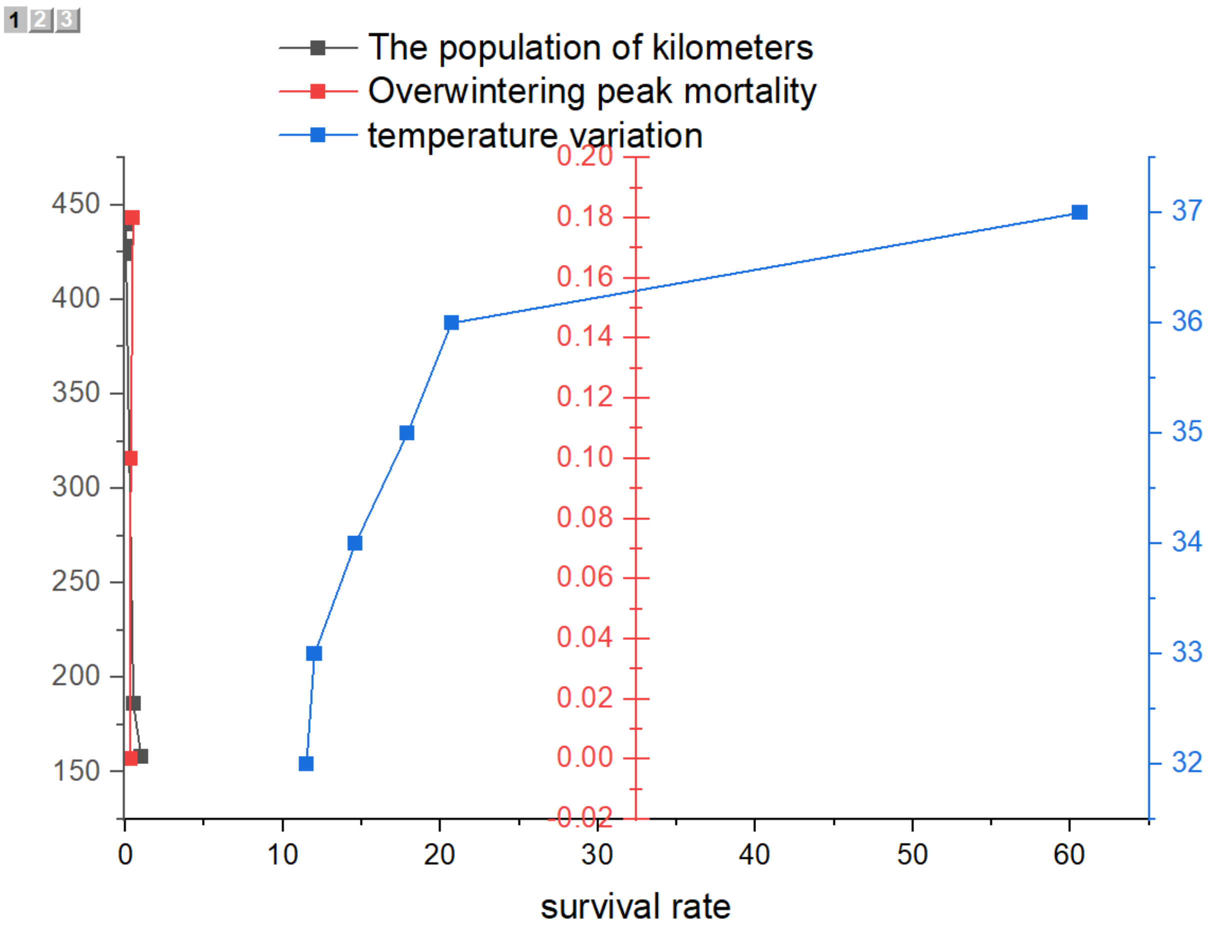
After we discover that mortality affects the larval to be born and grow up, we then decide to plot a graph with the temperature variation for other Asian honeybee colonies. According to research by Fujian Agricultural and forestry University, we drew the line between temperature with the survival rate for the rest of the Asian honeybees in the colonies.

To see the overwhelming effect of several variables that had an impact on the Asian honeybee population. We use multiple linear regression to plot the combination of survival rate, population density, and mortality. From the graph, we can see that mortality decreases when the population per square kilometer decreases, and the survival rate also decreases sharply. In the conclusion, we should be aware of how the population occurs a negative effect on Asian honeybees' survival rate. What's more, the overwintering peak mortality is another factor contributing to the survival rate of



the colonies. In the winter due to the cold temperatures, the bee's mortality also was affected since several bees were killed by the cold temperature.

Then, we use multiple linear regression to plot a combination of overwintering peak mortality, temperature variation, and population density in one graph. From the graph, we can see that the temperature varies with the greatest impact in these multiple linear graphs with the largest change in survival rate.



4. The need for hives on 81,000 km2 of farmland

According to the previous model about the change in the number of bees in all seasons, we can calculate the number of bees needed to pollinate 81,000 square meters of cultivated land.

The first step is to count the proportion of different crops in all cultivated land in China.

The geographical span of China is very vast, and it has many climatic zones, including temperate monsoon climate zone, subtropical monsoon climate zone, and tropical rainforest climate zone, which leads to some very different land types, crop planting types and maturity periods, so it is inconvenient to explore. Therefore, we choose the crop area that is ripe once a year in the temperate monsoon climate zone of China to explore. And the selected areas include plain and mountainous areas, to improve the accuracy of data and the predictability of the model as much as possible.

In this paper, we mainly discuss the proportion of herbaceous plants and use the average value of trees including fruit trees in the table. The results are as follows:

Rice %	Wheat %	Corn %	Soybean %	Rapeseed %	Cotton %	Tree %
18.52	14.96	20.67	4.71	4.53	3.10	33.51

The ratio of different crops in one hectare of cultivated land.

The second step is to look up the literature, and based on the demand of each crop for artificial pollination and the planting density and number of each different crop under normal conditions, convert it into the number of bees needed by the crop when pure bees pollinate. Also, a single honeybee can visit approximately 2,000 flowers or more in a single day. According to these, we count the number of pollinating bees needed by different crops/the value of a single crop per square kilometer, which is counted as I_N.

Rice	Wheat	Corn	Soybean	Rapeseed	Cotton	Tree
3.428	3.6	1.2	1.5457	0.857	2.7457	1.885

The number of bees' demands on different crops per square kilometer of the value of a single crop (I_n), with the unit of ten thousand times per square kilometer.

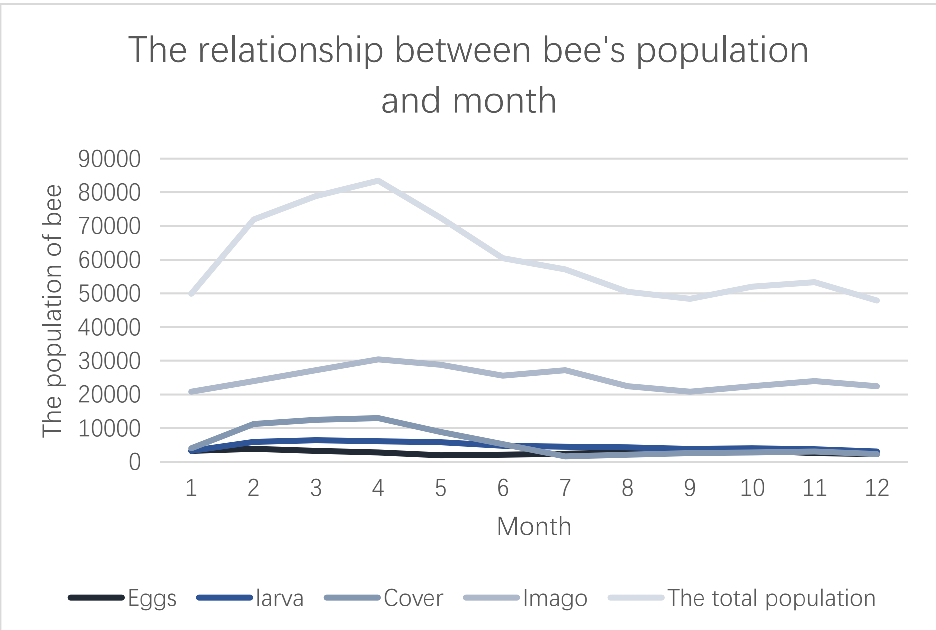
According to the formula:

$$I_N = \sum_{n=j}^{l} I_n$$

So, the $I_N = 2.25$, we have concluded that, in an ideal situation, under the condition of one crop per year and temperate monsoon climate, in an imaginary one square kilometer of cultivated land with various crops, about 13,125 bees are needed to pollinate this one square kilometer of land in a year, to ensure the normal crop production and the reproduction of plant offspring, otherwise, the production will be reduced and the number of plants will decrease year by year.

In the third step, we use the relationship between the number of the honeybee population in one hive and time (taking one year as the cycle) obtained in the first question of modeling. Get the change of the number of bees in the whole year, and the data are as follows.

Spring	Summer	Fall	Winter
56000	78000	64000	55000



And we can use the following formula to calculate how many hives we need to satisfy 81000m^2 crop land pollination:

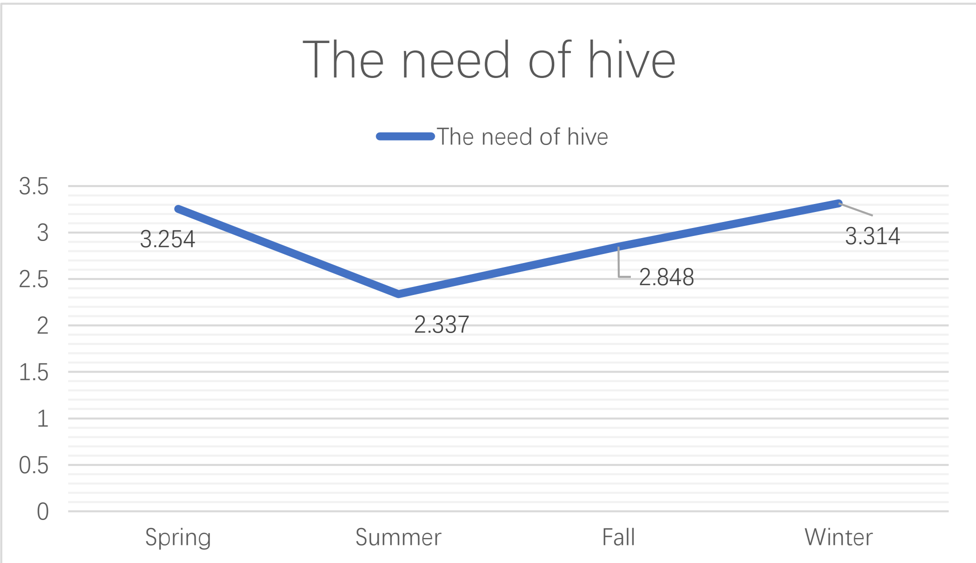
$$H = \frac{I_N \times 8.1}{Z}$$

(z is the whole number of the honey bee, and H is the hives that we need)

So, the hives we need in every season are:

Spring	Summer	Fall	Winter
3.254	2.337	2.848	3.314

And the graph is:



So, we need 4 hives in spring and winter, and 3 hives in summer and fall. This can let us pollinate 81000 m^2 of farmland.

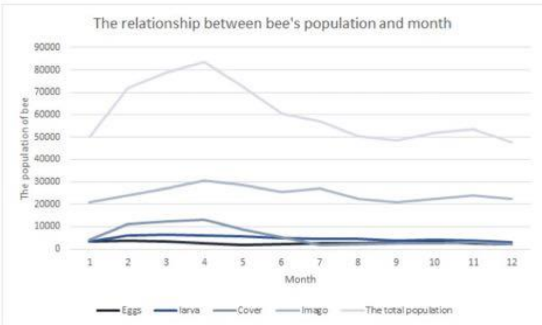
5. Conclusion and future expectancy

After all the research, we are more aware of the importance of bees. The di versity of the bee community provides tastier fruit. According to research publish ed in the Journal of Southern Agriculture, in a particular example, with honeybee pollinating the fruit of kiwi berry, the production increases by 233.8% compared t o natural pollination and increases by 27% compared to human pollination. Pollinating plants adds economic value to humans. It also increases crop yields. Bees are essential in an ecosystem, both in size and location. It promotes natural evolution and maintains ecological balance. As humans, there's not much we can do to help bees. But at the very least, by small means in our lives, we can create a stable environment for bees, directly or indirectly.

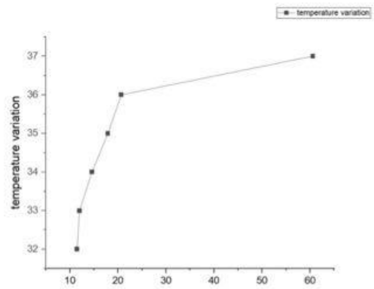
6. Appendix

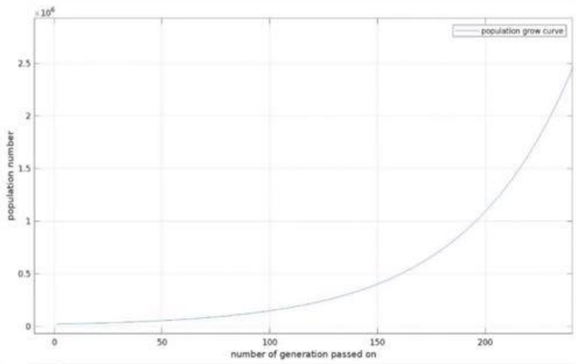
6.1 The poster

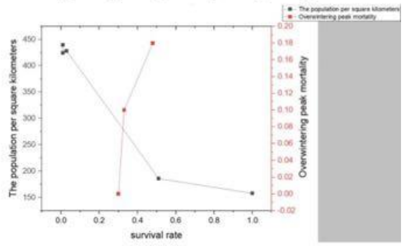
THE NEED FOR BEES



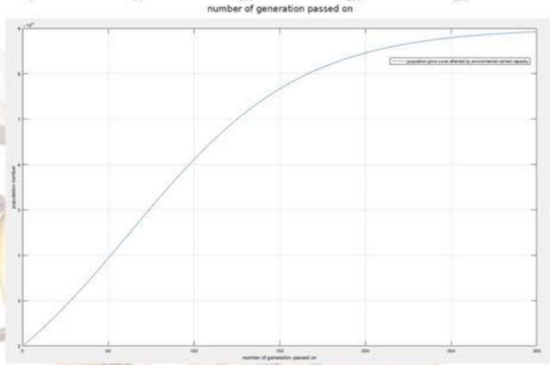
Analysis
of given information



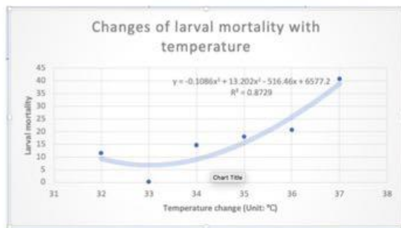




Forecast
future situation



Conclusions
and future action



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6.3 Program

Logistic Model

```
% Population growth using Logistic Model
% From Malthus Model, its obvious that population cannot grow continuously,
we decide to add environmental carried capacity which is the k in our model
% Editor: MartinLiu
n=300
x=zeros(1,n);
k=90000 % k is the environmental carried capacity
x(1,1)=20000 % We assume the initial population is 20000
% generaion starts to pass
r=0.02 %population growth rate
for t=2:n
    x(1,t)=x(1,1)*k/[x(1,1)-[x(1,1)-k]*exp(-r*t)]
end
% Drawing the figure
plot(1:1:n,x);
legend('population grow curve affected by environmental carried capacity')
xlabel('number of generation passed on')
ylabel('population number')
grid on;
```

Malthus Model

```
% Population growth using Malthus Model ′
% Editor: MartinLiu
n=250
x=zeros(1,n);
x(1,1)=20000 % We assume the initial population is 20000 ′
% generaion starts to pass
r=0.02 %population growth rate
for t=2:n
    x(1,t)=x(1,1)*exp(r*t)
end
% Drawing the figure
plot(1:1:n,x);
legend('population grow curve')
```

```
xlabel('number of generation passed on')
ylabel('population number')
grid on;
```

```
% Population growth using Malthus Model
% n is the number of generation passed on in the honeybee's colonies
n=250
x=zeros(1,n);
x(1,1)=20000 % We assume the initial population is 20000 '
% generaion starts to pass
r=0.02 %population growth rate
for t=2:n
   x(1,t)=x(1,1)*exp(r*t)
end
% Drawing the figure
plot(1:1:n,x);
legend('population grow curve')
xlabel('number of generation passed on')
ylabel('population number')
grid on;
```