

Lecture 3

Quantitative Variables

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We have analyzed a quantitative variable already. Where?

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In the Colombia COVID data!

```
df_CO = pd.read_csv(url + "colombia_2020-05-28.csv")  
df_CO
```

	Departamento	Edad	Sexo	Tipo	Ubicación	Estado	Fecha de inicio de síntomas
0	Bogotá D.C.	19	F	Importado	Recuperado	Leve	2020-02-27
1	Valle del Cauca	34	M	Importado	Recuperado	Leve	2020-03-04
2	Antioquia	50	F	Importado	Recuperado	Leve	2020-02-29
3	Antioquia	55	M	Relacionado	Recuperado	Leve	2020-03-06
4	Antioquia	25	M	Relacionado	Recuperado	Leve	2020-03-08
...
25361	Buenaventura D.E.	48	M	En estudio	Hospital	Moderado	2020-05-12
25362	Valle del Cauca	55	F	En estudio	Casa	Leve	2020-05-21
25363	Buenaventura D.E.	39	F	En estudio	Casa	Leve	2020-05-23
25364	Valle del Cauca	13	F	En estudio	Casa	Leve	2020-05-13
25365	Córdoba	0	F	En estudio	Hospital	Moderado	2020-05-11

25366 rows x 10 columns

departamento → department / region

edad → age

sexo → sex / gender

tipo → case_type

ubicacion → location / status

fecha de inicio de síntomas → date_of_symptom_onset

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25366 rows × 10 columns

This example will motivate our discussion of quantitative variables today!

- ① Visualizing One Quantitative Variable
- ② Summarizing One Quantitative Variable
- ③ Recap

① Visualizing One Quantitative Variable

② Summarizing One Quantitative Variable

③ Recap

Visualizing One Quantitative Variable

Visualizing One Quantitative Variable

To visualize the age variable, we did the following:

quantitative
variable

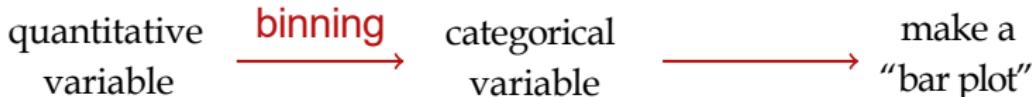
Visualizing One Quantitative Variable

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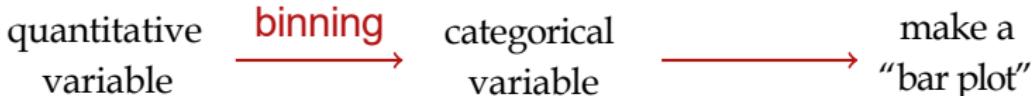
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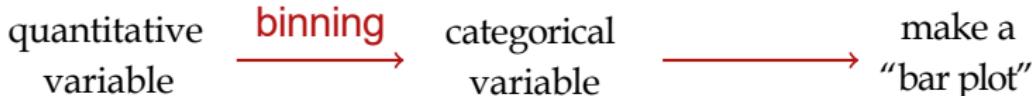
To visualize the age variable, we did the following:



```
df_CO["age"] = pd.cut(  
    df_CO["Edad"],  
    bins=[0, 10, 20, 30, 40, 50, 60, 70, 80, 120],  
    labels=["0-9", "10-19", "20-29", "30-39", "40-49", "50-59", "60-69",  
    right=False)
```

Visualizing One Quantitative Variable

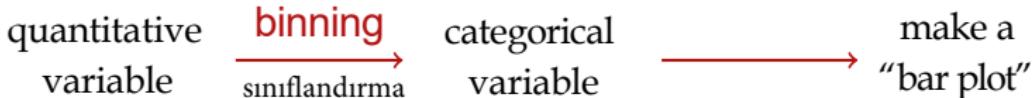
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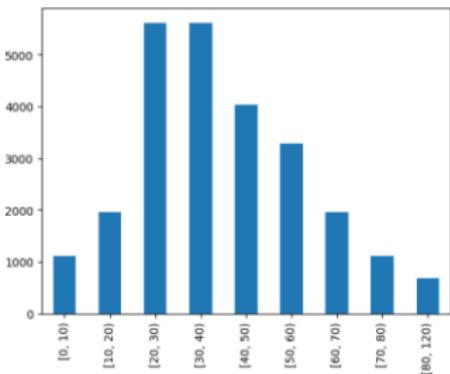
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df_CO["age"].value_counts(sort=False).plot.bar()
```

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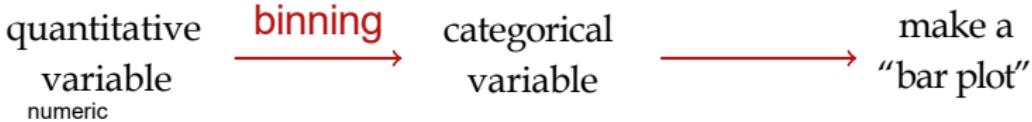
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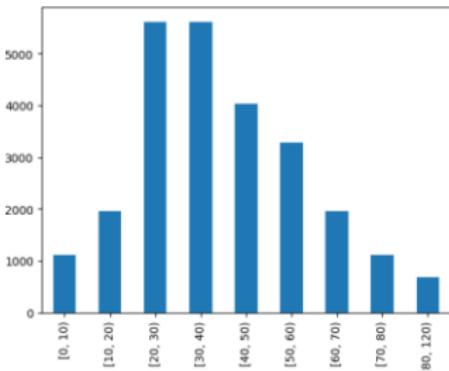
value_counts()
by default sorts results from highest to lowest frequency.
sort=False from lowest to highest

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```



This is the idea behind a visualization called the **histogram**.

Histograms

Pandas provides a built-in method for constructing histograms:
`Series.plot.hist()`.

Histogram

Used for quantitative (numerical) data.

Shows the distribution of continuous values.

The x-axis represents numeric ranges (bins).

The y-axis shows the frequency (count) of values within each range.

Bars touch each other, because the numeric intervals are continuous.

Bar Plot

Used for categorical data.

Shows the frequency or comparison of categories.

The x-axis represents categories (e.g., "0–9", "10–19", "20–29").

The y-axis shows the count (or mean, etc.) for each category.

Bars are separated, because categories are independent.

Histograms

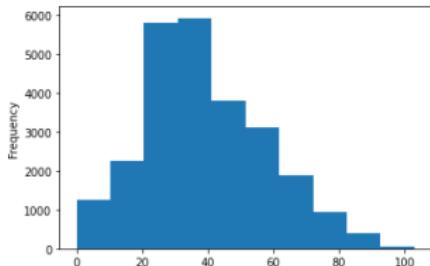
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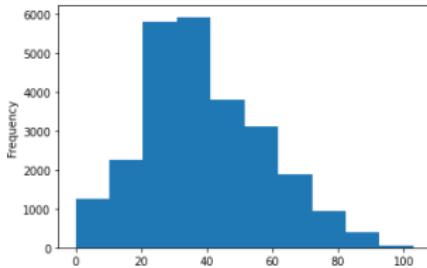
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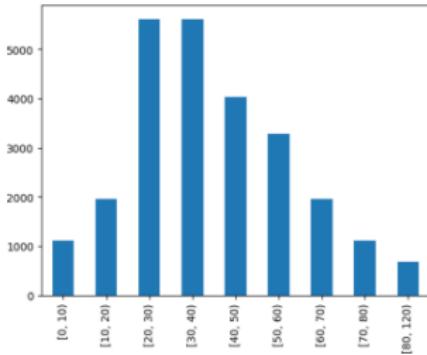
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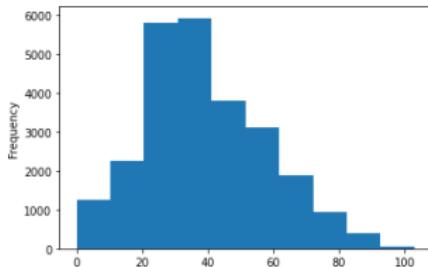
How does this differ from the manual histogram from earlier?



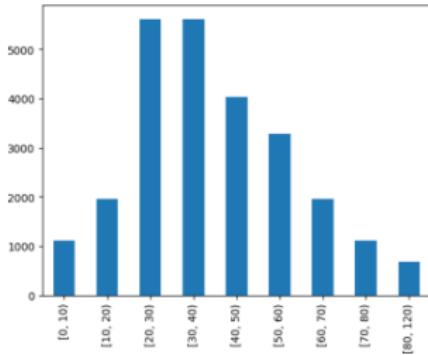
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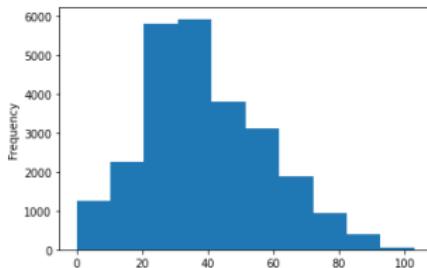


- There are no spaces between the bars.

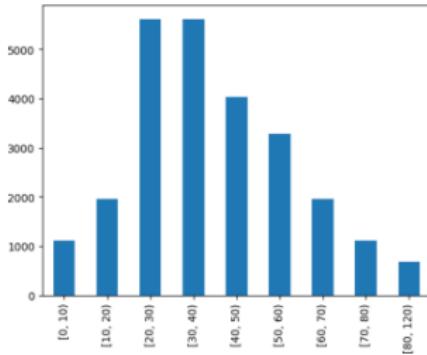
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df_CO["Edad"].plot.hist()
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How does this differ from the manual histogram from earlier?



- There are no spaces between the bars.
- The *x*-axis is just numbers, rather than bins.

Distributions

Recall the distribution of a categorical variable.

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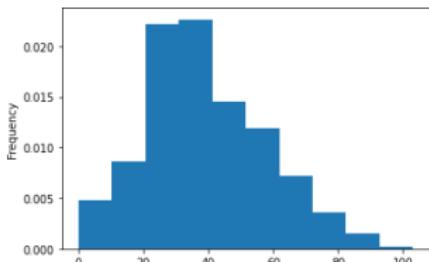
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df_CO["Edad"].plot.hist(density=True)
```

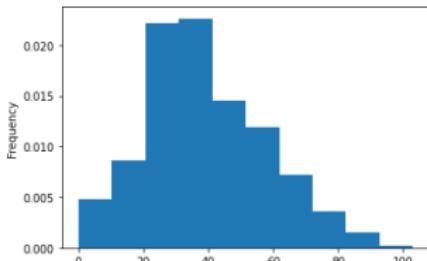


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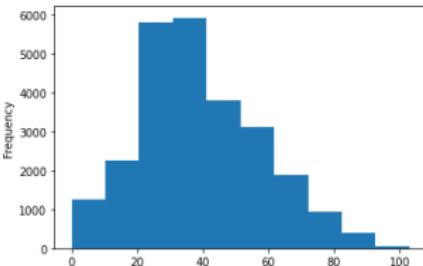


density =True changes y-axis
y-axis shows pdf (probability density function)

density = False

y-axis shows Actual frequency (number of people)

How does this differ from the (counts) histogram from earlier?

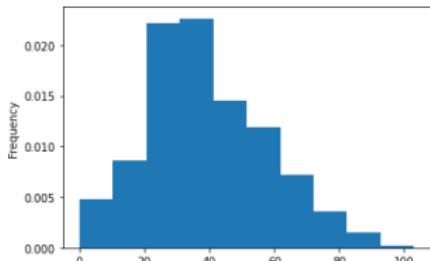


Distributions

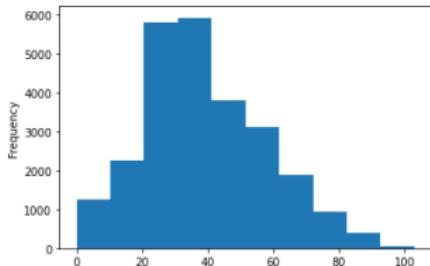
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How does this differ from the (counts) histogram from earlier?



- Only the y-axis changes.
- The shape is the same!

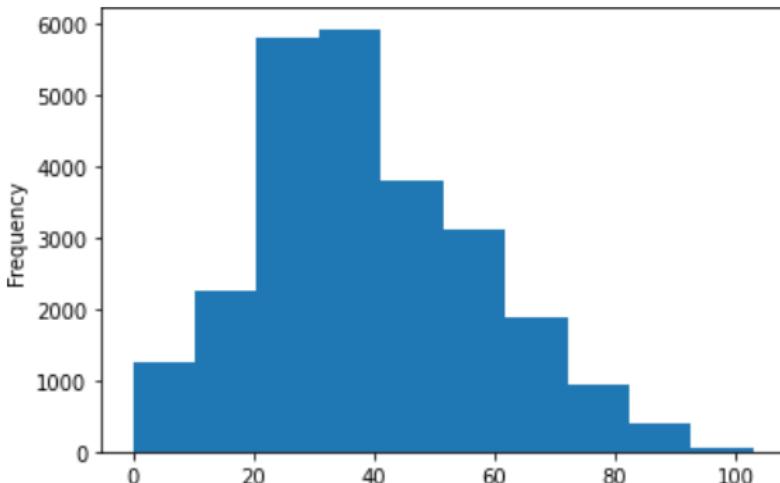
1 Visualizing One Quantitative Variable

2 Summarizing One Quantitative Variable

3 Recap

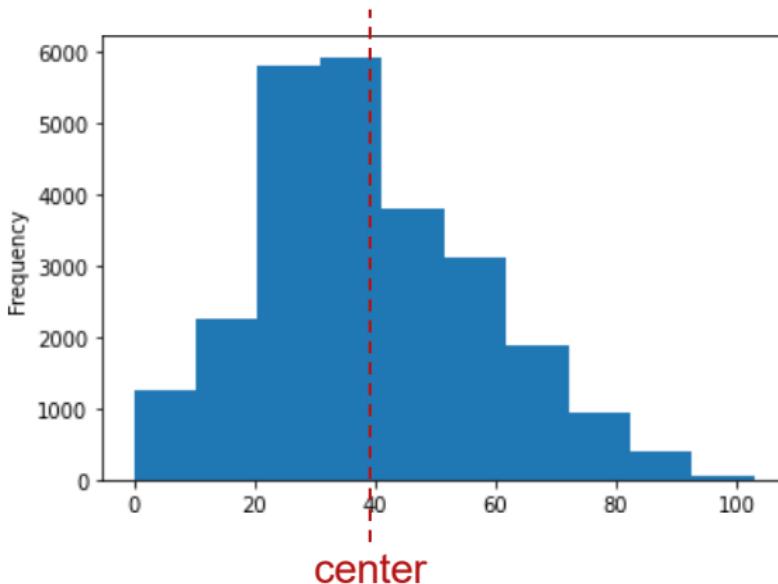
Summarizing a Quantitative Variable

If you had to summarize this data using a single number, what number would you pick?



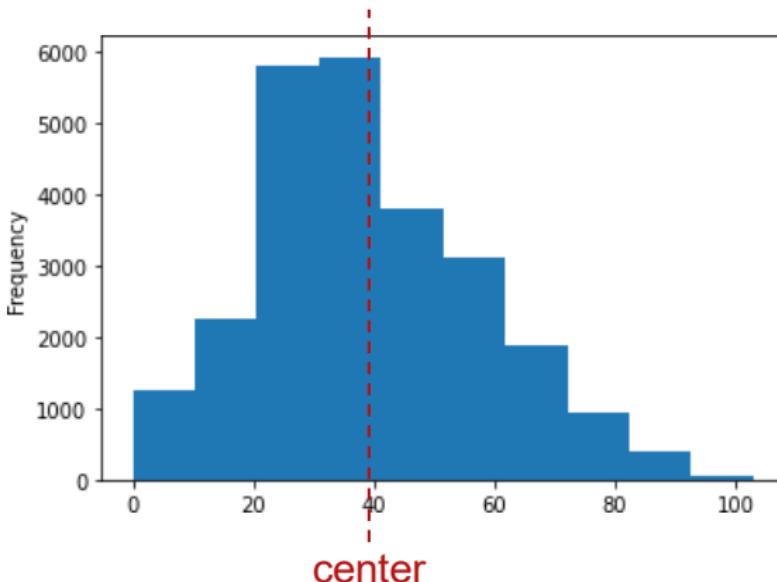
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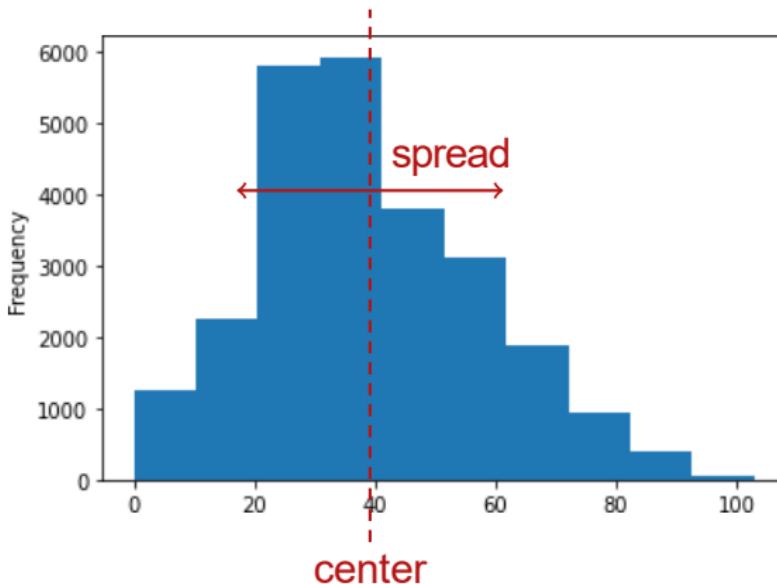
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To calculate the mean of a quantitative variable \mathbf{x} with values $x_1, x_2, x_3, \dots, x_n$, we use the formula:

$$\text{mean}(\mathbf{x}) = \frac{\sum_{i=1}^n x_i}{n}$$

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You can calculate it manually...

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...or using a built-in Python function.

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df_CO["Edad"].mean()
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Summaries of Center: Mean

Don't be fooled by the humble mean,

$$\bar{x} = \text{mean}(\mathbf{x}) = \frac{\sum_{i=1}^n x_i}{n}$$

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Let's investigate one reason in a notebook.



*If I have seen further [than others],
it is by standing on the shoulders of
giants.*

– Isaac Newton

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 - If n is odd, then the middle value is $x_{(\frac{n+1}{2})}$.
 - If n is even, then there are two middle values, $x_{(\frac{n}{2})}$ and $x_{(\frac{n}{2}+1)}$. It is conventional to report the mean of the two values (but you can actually pick any value between them).

Summaries of Center: Median

We can implement these steps in Python code manually. I asked ChatGPT, and it generated this code:

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But it's easier to use the built-in Python function.

```
df_CO["Edad"].median()
```

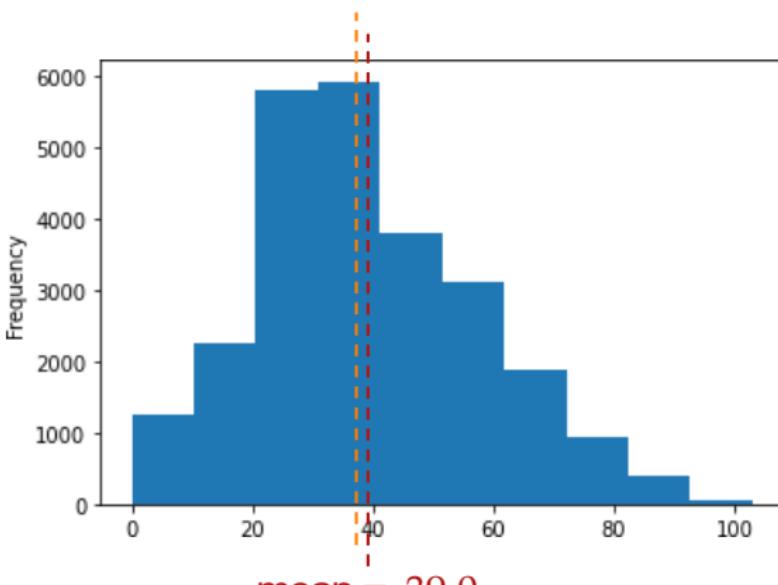
37.0

Summaries of Center: Mean vs. Median

We now have two summaries of center. How do they compare?

A mean around 39 indicates the data mostly represents young to middle-aged adults.

median = 37.0



mean = 39.0

The data might be right-skewed (a few larger values pulling the mean upward).

That means a few larger values (older ages) pull the mean upward.

Most observations are likely clustered around the 30–40 age range.

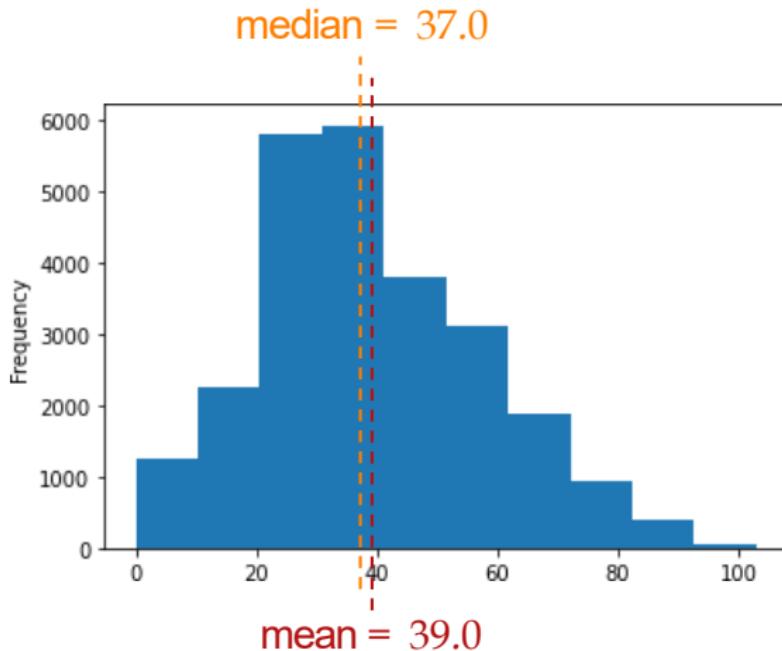
The median is 37, meaning half of the individuals are younger than 37 and half are older.

The difference between mean and median suggests possible outliers,

For example, a few very high ages (80–100) affecting the mean.

Summaries of Center: Mean vs. Median

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How would we summarize spread now?

Summaries of Spread: Variance

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One measure of spread is the **variance**.

Variance is a statistic that measures how much the values in a dataset spread around the mean.

Deviation from the mean

If the variance is large → the data is spread out far from the mean.

If the variance is small → the data is concentrated around the mean.

Homogeneity or heterogeneity of the data

Small variance → the data is more consistent/homogeneous.

Large variance → the data is more variable/heterogeneous.

Effect of outliers

Variance is affected by outliers.

Very large deviations increase the variance.

Summaries of Spread: Variance

One measure of spread is the **variance**.

The variance of a variable \mathbf{x} whose values are $x_1, x_2, x_3, \dots, x_n$ is calculated using the formula

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You can implement this formula manually...

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((df_CO["Edad"] - df_CO["Edad"].mean() ** 2).sum() /  
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df_CO["Edad"].var()
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348.0870469898451

...or using a built-in Python function.

```
df_CO["Edad"].var()
```

348.0870469898451

What are the units?

Summaries of Spread: Variance

One measure of spread is the **variance**.

The variance of a variable **x** whose values are $x_1, x_2, x_3, \dots, x_n$ is calculated using the formula

$$\text{var}(\mathbf{x}) = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1}$$

You can implement this formula manually...

```
((df_CO["Edad"] - df_CO["Edad"].mean()) ** 2).sum() /  
(len(df_CO) - 1))
```

348.0870469898451

...or using a built-in Python function.

```
df_CO["Edad"].var()
```

348.0870469898451

What are the units? **years²**

Summaries of Spread: Standard Deviation

To fix the units, we take the square root to get the **standard deviation**:

$$sd(\mathbf{x}) = \sqrt{var(\mathbf{x})}$$

Standard Deviation (SD) is a statistic that measures how much the values in a dataset deviate from the mean, just like variance, but in the same units as the original data.

Small SD → data points are close to the mean, less spread out.

Large SD → data points are more spread out from the mean.

Like variance, SD is sensitive to outliers.

Extreme values increase the SD.

Summaries of Spread: Standard Deviation

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We can calculate it using the built-in Pandas method `Series.std`:

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```
18.65709106452142
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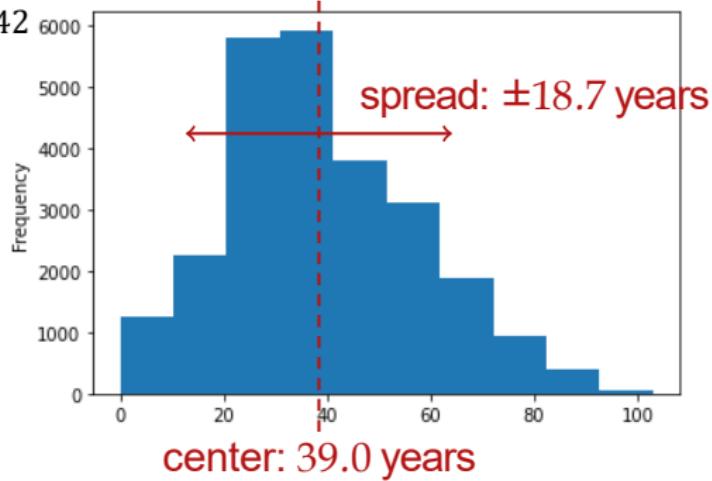
$$\text{sd}(\mathbf{x}) = \sqrt{\text{var}(\mathbf{x})}$$

years *years²*

We can calculate it using the built-in Pandas method `Series.std`:

```
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18.65709106452142



1 Visualizing One Quantitative Variable

2 Summarizing One Quantitative Variable

3 Recap

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What We Learned Today

- visualizing a quantitative variable using a histogram

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 - We've now seen several plots that can be made within Pandas: `.plot.bar()`, `.plot.hist()`, and `.plot.line()`.

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 - summarizing the spread **by the standard deviation**
- some new Python tricks