```
// William E. Boyce/Richard C. DiPrima, "Elementary Differential
// Equations and Boundary Value Problems 10th Ed." 第413頁-416頁

using System;
using Matrix_0;

namespace ConsoleApp40
{
    internal class Program
    {
        static void Main(string[] args)
        {

// 初始值 [y1|y2|y3|y4] = [y1, y2, y3, y4]t
double[,] y0Start = { { -1 }, { 4 }, { 1 }, { 1 } };

// 系統矩陣A0，並轉換為實數矩陣物件 A。
double[,] A0 =  { {0, 0, 1, 0}, {0, 0, 0, 1},
    {-2, 1.5, 0, 0}, {1.33333, -3, 0, 0 } };
ReMatrix A = new ReMatrix(A0);

// 系統特徵值V和特徵向量Q。
EIG eig = new EIG(A);
CxMatrix D = eig.CxMatrixD;
CxMatrix V = eig.CxVector;
CxMatrix Q = eig.CxMatrixQ;

// 狀態[空間]變數響應函數 CxToHexp ，  係數向量 d ，兩者均爲複數。
CxToHexp Hexp = new CxToHexp(D, Q, 0);
CxMatrix MatTemp = Hexp.GetCxMatrix;
CxMatrix d = ~MatTemp * y0Start;

// 列印系統與狀態狀態參數。
Console.Write("\n****{0,7}系{0,5}統{0,5}與{0,5}狀{0,5}態{0,5}參{0,5}數{0,7}****\n", "");

Console.Write("\n***{0,5}系統特徵值V{0,5}***\n{1}\n", "", new PR(V));
Console.Write("\n***{0,5}系統特徵向量Q{0,5}***\n{1}\n", "", new PR(Q));
Console.Write("\n***{0,5}係數向量d{0,5}***\n{1}\n", "", new PR(d));

double step = 0.5;
int iRow = (int)(20 / step + 1);
int iCol = 4 + 1;
ReMatrix Disp = new ReMatrix(iRow, iCol);
ReMatrix Vel = new ReMatrix(iRow, iCol);
```

```
45
46  for (int i = 0; i != iRow; i++)
47  {
48      double t = step * i;
49
50      Hexp = new CxToHexp(D, Q, t);
51      MatTemp = Hexp.GetCxMatrix;
52      CxMatrix yh_Cx = MatTemp * d;
53      ReMatrix yh_Re = (ReMatrix)yh_Cx;
54      ReMatrix yhDot_Re = A * yh_Re;
55
56      Vel.Matrix[i, 0] = t;
57      Vel.Matrix[i, 1] = yhDot_Re.Matrix[0, 0];
58      Vel.Matrix[i, 2] = yhDot_Re.Matrix[1, 0];
59      Vel.Matrix[i, 3] = yhDot_Re.Matrix[2, 0];
60      Vel.Matrix[i, 4] = yhDot_Re.Matrix[3, 0];
61
62      Disp.Matrix[i, 0] = t;
63      Disp.Matrix[i, 1] = yh_Re.Matrix[0, 0];
64      Disp.Matrix[i, 2] = yh_Re.Matrix[1, 0];
65      Disp.Matrix[i, 3] = yh_Re.Matrix[2, 0];
66      Disp.Matrix[i, 4] = yh_Re.Matrix[3, 0];
67  }
68
69  // 列印狀態響應，節點之變位與速度。
70  Console.Write("\n****{0,10}狀{0,7}態{0,7}響{0,7}應{0,10}****\n",
      "");
71
72  Console.Write("\n{0,5}***位移反應量***{0,5}\n{0,8}時間(秒)" +
73  "{0,8}第0點位移{0,8}第1點位移{0,8}第2點位移{0,8}第3點位移\n\n{1}",
74  "", new PR(Disp));
75  Console.Write("\n{0,5}***速度反應量***{0,5}\n{0,8}時間(秒)" +
76  "{0,8}第0點速度{0,8}第1點速度{0,8}第2點速度{0,8}第3點速度\n\n{1}",
77  "", new PR(Vel));
78
79  //列印時間、節點變位、和速度等序列。
80  Console.Write("\n時間序列 : \n{0}\n",
81      new PR4(Disp, 0, "{0,8:F2}"));
82
83  Console.Write("\n第0點變位序列 : \n{0}\n", new PR4(Disp, 1));
84  Console.Write("\n第1點變位序列 : \n{0}\n", new PR4(Disp, 2));
85  Console.Write("\n第2點變位序列 : \n{0}\n", new PR4(Disp, 3));
86  Console.Write("\n第3點變位序列 : \n{0}\n", new PR4(Disp, 4));
87
88  Console.Write("\n第0點速度序列 : \n{0}\n", new PR4(Vel, 1));
89  Console.Write("\n第1點速度序列 : \n{0}\n", new PR4(Vel, 2));
90  Console.Write("\n第2點速度序列 : \n{0}\n", new PR4(Vel, 3));
```

```
 91  Console.Write("\n第3點速度序列 : \n{0}\n", new PR4(Vel, 4));
 92          }
 93      }
 94  }
 95
 96  /*
 97  ****        系      統      與      狀      態      參      數        ****
 98
 99  ***      系統特徵值V      ***
100      0.00000  +     2.00000i
101      0.00000  -     2.00000i
102      0.00000  +     1.00000i
103      0.00000  -     1.00000i
104
105  ***      系統特徵向量Q      ***
106     0.26833 +    0.00000i,    0.26833 +    0.00000i,
107     0.58835 +    0.00000i,    0.58835 +    0.00000i
108
109    -0.35777 +    0.00000i,   -0.35777 +    0.00000i,
110     0.39223 +    0.00000i,    0.39223 +    0.00000i
111
112     0.00000 +    0.53666i,    0.00000 -    0.53666i,
113     0.00000 +    0.58835i,    0.00000 -    0.58835i
114
115     0.00000 -    0.71554i,    0.00000 +    0.71554i,
116     0.00000 +    0.39223i,    0.00000 -    0.39223i
117
118  ***      係數向量d      ***
119    -4.34791 +    0.15528i
120    -4.34791 -    0.15528i
121     1.13312 -    0.99148i
122     1.13312 +    0.99148i
123
124  ****          狀       態       響       應          ****
125
126   ***位移反應量***
127     時間(秒)      第0點位移     第1點位移     第2點位移     第3點位移
128     0.00000     -1.00000      4.00000      1.00000      1.00000
129     0.50000      0.39861      2.92740      4.22143     -4.85934
130     1.00000      2.59735     -0.05890      3.82114     -6.07807
131     1.50000      3.55628     -2.22559     -0.42391     -1.92973
132     2.00000      2.09422     -1.78033     -5.12071      3.43180
133     2.50000     -0.95195      0.52930     -6.25489      4.87459
134     3.00000     -3.37247      2.18591     -2.80712      1.05653
135     3.50000     -3.47172      1.31324      2.31547     -4.33692
136     4.00000     -1.49741     -1.51237      4.88775     -6.02401
137     4.50000      0.67012     -3.73650      3.13254     -2.06181
```

138	5.00000	1.26265	-3.16458	-0.78940	4.27155
139	5.50000	0.19478	-0.01617	-2.89986	7.40148
140	6.00000	-0.97002	3.20187	-1.15191	4.52138
141	6.50000	-0.59930	3.90525	2.66204	-1.84436
142	7.00000	1.37008	1.71661	4.60360	-6.13102
143	7.50000	3.27492	-1.25354	2.31503	-4.77925
144	8.00000	3.21878	-2.37122	-2.67283	0.58596
145	8.50000	0.85105	-0.87698	-6.20766	4.74287
146	9.00000	-2.21220	1.48150	-5.22715	3.74458
147	9.50000	-3.73672	2.14779	-0.52853	-1.42160
148	10.00000	-2.78174	0.20207	3.93885	-5.75888
149	10.50000	-0.45205	-2.71797	4.61385	-4.91549
150	11.00000	1.17324	-3.88580	1.46388	0.72516
151	11.50000	0.93687	-2.00303	-2.12908	6.30103
152	12.00000	-0.41512	1.55180	-2.59682	6.86227
153	12.50000	-1.04874	3.90439	0.46965	1.87883
154	13.00000	0.12709	3.23080	3.94922	-4.26873
155	13.50000	2.33292	0.35139	4.13413	-6.26749
156	14.00000	3.56151	-2.07266	0.26339	-2.67379
157	14.50000	2.41824	-1.98940	-4.63288	2.85593
158	15.00000	-0.53184	0.20058	-6.38988	5.01313
159	15.50000	-3.16447	2.09180	-3.45480	1.77279
160	16.00000	-3.60526	1.58146	1.70088	-3.73459
161	16.50000	-1.81931	-1.10814	4.79808	-6.13822
162	17.00000	0.44740	-3.57353	3.57138	-2.84011
163	17.50000	1.29869	-3.42282	-0.29067	3.50129
164	18.00000	0.38555	-0.50547	-2.83528	7.32375
165	18.50000	-0.87916	2.87854	-1.57804	5.20953
166	19.00000	-0.76006	3.99971	2.17747	-0.99603
167	19.50000	1.06505	2.11504	4.57430	-5.85699
168	20.00000	3.10328	-0.91931	2.84719	-5.27863
169					
170	***速度反應量***				
171	時間(秒)	第0點速度	第1點速度	第2點速度	第3點速度
172	0.00000	1.00000	1.00000	8.00000	-13.33333
173	0.50000	4.22143	-4.85934	3.59387	-8.25071
174	1.00000	3.82114	-6.07807	-5.28306	3.63983
175	1.50000	-0.42391	-1.92973	-10.45095	11.41847
176	2.00000	-5.12071	3.43180	-6.85893	8.13327
177	2.50000	-6.25489	4.87459	2.69784	-2.85716
178	3.00000	-2.80712	1.05653	10.02381	-11.05435
179	3.50000	2.31547	-4.33692	8.91329	-8.56866
180	4.00000	4.88775	-6.02401	0.72627	2.54057
181	4.50000	3.13254	-2.06181	-6.94499	12.10300
182	5.00000	-0.78940	4.27155	-7.27217	11.17726
183	5.50000	-2.89986	7.40148	-0.41382	0.30822
184	6.00000	-1.15191	4.52138	6.74285	-10.89896

```
185     6.50000     2.66204    -1.84436     7.05647   -12.51482
186     7.00000     4.60360    -6.13102    -0.16524    -3.32305
187     7.50000     2.31503    -4.77925    -8.43015     8.12717
188     8.00000    -2.67283     0.58596    -9.99438    11.40534
189     8.50000    -6.20766     4.74287    -3.01757     3.76567
190     9.00000    -5.22715     3.74458     6.64665    -7.39409
191     9.50000    -0.52853    -1.42160    10.69513   -11.42566
192    10.00000     3.93885    -5.75888     5.86658    -4.31519
193    10.50000     4.61385    -4.91549    -3.17286     7.55119
194    11.00000     1.46388     0.72516    -8.17518    13.22171
195    11.50000    -2.12908     6.30103    -4.87829     7.25825
196    12.00000    -2.59682     6.86227     3.15794    -5.20890
197    12.50000     0.46965     1.87883     7.95406   -13.11149
198    13.00000     3.94922    -4.26873     4.59201    -9.52294
199    13.50000     4.13413    -6.26749    -4.13876     2.05639
200    14.00000     0.26339    -2.67379   -10.23202    10.96666
201    14.50000    -4.63288     2.85593    -7.82058     9.19252
202    15.00000    -6.38988     5.01313     1.36455    -1.31085
203    15.50000    -3.45480     1.77279     9.46664   -10.49468
204    16.00000     1.70088    -3.73459     9.58271    -9.55138
205    16.50000     4.79808    -6.13822     1.97641     0.89869
206    17.00000     3.57138    -2.84011    -6.25509    11.31711
207    17.50000    -0.29067     3.50129    -7.73160    12.00003
208    18.00000    -2.83528     7.32375    -1.52930     2.03046
209    18.50000    -1.57804     5.20953     6.07614    -9.80784
210    19.00000     2.17747    -0.99603     7.51969   -13.01254
211    19.50000     4.57430    -5.85699     1.04246    -4.92505
212    20.00000     2.84719    -5.27863    -7.58552     6.89562
213
214 時間序列 :
215      0.00,     0.50,     1.00,     1.50,     2.00,
216      2.50,     3.00,     3.50,     4.00,     4.50,
217      5.00,     5.50,     6.00,     6.50,     7.00,
218      7.50,     8.00,     8.50,     9.00,     9.50,
219     10.00,    10.50,    11.00,    11.50,    12.00,
220     12.50,    13.00,    13.50,    14.00,    14.50,
221     15.00,    15.50,    16.00,    16.50,    17.00,
222     17.50,    18.00,    18.50,    19.00,    19.50,
223     20.00,
224
225 第0點變位序列 :
226    -1.0000,     0.3986,     2.5974,     3.5563,     2.0942,
227    -0.9519,    -3.3725,    -3.4717,    -1.4974,     0.6701,
228     1.2627,     0.1948,    -0.9700,    -0.5993,     1.3701,
229     3.2749,     3.2188,     0.8510,    -2.2122,    -3.7367,
230    -2.7817,    -0.4520,     1.1732,     0.9369,    -0.4151,
231    -1.0487,     0.1271,     2.3329,     3.5615,     2.4182,
```

```
232    -0.5318,    -3.1645,    -3.6053,    -1.8193,     0.4474,
233     1.2987,     0.3855,    -0.8792,    -0.7601,     1.0651,
234     3.1033,
235
236  第1點變位序列 :
237     4.0000,     2.9274,    -0.0589,    -2.2256,    -1.7803,
238     0.5293,     2.1859,     1.3132,    -1.5124,    -3.7365,
239    -3.1646,    -0.0162,     3.2019,     3.9053,     1.7166,
240    -1.2535,    -2.3712,    -0.8770,     1.4815,     2.1478,
241     0.2021,    -2.7180,    -3.8858,    -2.0030,     1.5518,
242     3.9044,     3.2308,     0.3514,    -2.0727,    -1.9894,
243     0.2006,     2.0918,     1.5815,    -1.1081,    -3.5735,
244    -3.4228,    -0.5055,     2.8785,     3.9997,     2.1150,
245    -0.9193,
246
247  第2點變位序列 :
248     1.0000,     4.2214,     3.8211,    -0.4239,    -5.1207,
249    -6.2549,    -2.8071,     2.3155,     4.8878,     3.1325,
250    -0.7894,    -2.8999,    -1.1519,     2.6620,     4.6036,
251     2.3150,    -2.6728,    -6.2077,    -5.2271,    -0.5285,
252     3.9389,     4.6138,     1.4639,    -2.1291,    -2.5968,
253     0.4697,     3.9492,     4.1341,     0.2634,    -4.6329,
254    -6.3899,    -3.4548,     1.7009,     4.7981,     3.5714,
255    -0.2907,    -2.8353,    -1.5780,     2.1775,     4.5743,
256     2.8472,
257
258  第3點變位序列 :
259     1.0000,    -4.8593,    -6.0781,    -1.9297,     3.4318,
260     4.8746,     1.0565,    -4.3369,    -6.0240,    -2.0618,
261     4.2716,     7.4015,     4.5214,    -1.8444,    -6.1310,
262    -4.7792,     0.5860,     4.7429,     3.7446,    -1.4216,
263    -5.7589,    -4.9155,     0.7252,     6.3010,     6.8623,
264     1.8788,    -4.2687,    -6.2675,    -2.6738,     2.8559,
265     5.0131,     1.7728,    -3.7346,    -6.1382,    -2.8401,
266     3.5013,     7.3237,     5.2095,    -0.9960,    -5.8570,
267    -5.2786,
268
269  第0點速度序列 :
270     1.0000,     4.2214,     3.8211,    -0.4239,    -5.1207,
271    -6.2549,    -2.8071,     2.3155,     4.8878,     3.1325,
272    -0.7894,    -2.8999,    -1.1519,     2.6620,     4.6036,
273     2.3150,    -2.6728,    -6.2077,    -5.2271,    -0.5285,
274     3.9389,     4.6138,     1.4639,    -2.1291,    -2.5968,
275     0.4697,     3.9492,     4.1341,     0.2634,    -4.6329,
276    -6.3899,    -3.4548,     1.7009,     4.7981,     3.5714,
277    -0.2907,    -2.8353,    -1.5780,     2.1775,     4.5743,
278     2.8472,
```

```
279
280 第1點速度序列 :
281     1.0000,     -4.8593,     -6.0781,     -1.9297,      3.4318,
282     4.8746,      1.0565,     -4.3369,     -6.0240,     -2.0618,
283     4.2716,      7.4015,      4.5214,     -1.8444,     -6.1310,
284    -4.7792,      0.5860,      4.7429,      3.7446,     -1.4216,
285    -5.7589,     -4.9155,      0.7252,      6.3010,      6.8623,
286     1.8788,     -4.2687,     -6.2675,     -2.6738,      2.8559,
287     5.0131,      1.7728,     -3.7346,     -6.1382,     -2.8401,
288     3.5013,      7.3237,      5.2095,     -0.9960,     -5.8570,
289    -5.2786,
290
291 第2點速度序列 :
292     8.0000,      3.5939,     -5.2831,    -10.4510,     -6.8589,
293     2.6978,     10.0238,      8.9133,      0.7263,     -6.9450,
294    -7.2722,     -0.4138,      6.7429,      7.0565,     -0.1652,
295    -8.4302,     -9.9944,     -3.0176,      6.6467,     10.6951,
296     5.8666,     -3.1729,     -8.1752,     -4.8783,      3.1579,
297     7.9541,      4.5920,     -4.1388,    -10.2320,     -7.8206,
298     1.3646,      9.4666,      9.5827,      1.9764,     -6.2551,
299    -7.7316,     -1.5293,      6.0761,      7.5197,      1.0424,
300    -7.5855,
301
302 第3點速度序列 :
303   -13.3333,     -8.2507,      3.6398,     11.4185,      8.1333,
304    -2.8572,    -11.0544,     -8.5687,      2.5406,     12.1030,
305    11.1773,      0.3082,    -10.8990,    -12.5148,     -3.3230,
306     8.1272,     11.4053,      3.7657,     -7.3941,    -11.4257,
307    -4.3152,      7.5512,     13.2217,      7.2582,     -5.2089,
308   -13.1115,     -9.5229,      2.0564,     10.9667,      9.1925,
309    -1.3109,    -10.4947,     -9.5514,      0.8987,     11.3171,
310    12.0000,      2.0304,     -9.8078,    -13.0125,     -4.9250,
311     6.8956,
312
313 請按任意鍵繼續 . . .
314 */
```