```
1 // .NET 8, Project Name @ C:\2302\Misc 10\App 40
 2 // William E. Boyce/Richard C. DiPrima, "Elementary Differential
 3 // Equations and Boundary Value Problems 10th Ed." 第413頁-416頁
 4
 5 using Matrix_0;
 6
 7 // 已知數據(m=4, r=1), 包含初始值和系統矩陣A
 8 double[,] y0Start = { { -1 }, { 4 }, { 1 }, { 1 } };
9 double[,] A0 = \{ \{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \}
       \{-2, 1.5, 0, 0\}, \{1.33333, -3, 0, 0\}\};
10
11 ReMatrix A = new ReMatrix(A0);
12
13 // 複數特徵矩陣D和複數模態矩陣Q
14 EIG eig = new EIG(A);
15 CxMatrix D = eig. CxMatrixD;
16 CxMatrix Q = eig. CxMatrixQ;
17
18 // 複數向量係數 d
19 CxHexp Hexp = new CxHexp(D, Q, 0);
20 CxMatrix MatTemp = Hexp. GetCxMatrix;
21 CxMatrix d = ~MatTemp * yOStart;
22
23 // 列印系統特徵矩陣、模態矩陣、和係數向量。
24 Console. Write ("\n****{0,7} 系 {0,5} 統 {0,5} 與 {0,5} 狀 {0,5} 態 {0,5} 參 {0,5} ⊋
     數 {0,7}***\n", "");
25 Console. Write ("\n***{0,5}特徵矩陣 D{0,5}***\n{1}\n", "", new PR
26 Console. Write ("\n***{0,5} 模態矩陣 Q{0,5}***\n{1}\n", "", new PR
27 Console. Write ("\n***{0,5}向量係數 d{0,5}***\n{1}\n", "", new PR
     (d));
28
29 // 建構Disp、Vel、和Acc等三個空的儲存矩陣
30 double step = 0.5;
31 int iRow = (int)(20 / step + 1);
32 int iCo1 = 5; // 4 + 1
33 ReMatrix Disp = new ReMatrix(iRow, iCol);
34 ReMatrix Vel = new ReMatrix(iRow, iCol);
35
36 //將計算的結果存入Vel、Dis、和Acc矩陣中
   for (int i = 0; i != iRow; i++)
37
38
   {
39
       double t = step * i;
40
       Hexp = new CxHexp(D, Q, t);
41
       MatTemp = Hexp. GetCxMatrix;
42
       ReMatrix yh Re = (ReMatrix)(MatTemp * d);
43
```

```
44
       // 位移
45
       Disp. Matrix[i, 0] = t;
       Disp. Matrix[i, 1] = yh Re. Matrix[0, 0];
46
       Disp. Matrix[i, 2] = yh_Re. Matrix[1, 0];
47
48
       Disp. Matrix[i, 3] = yh_Re. Matrix[2, 0];
49
       Disp. Matrix[i, 4] = yh_Re. Matrix[3, 0];
50
51
       // 速度
52
       ReMatrix yhDot_Re = A * yh_Re;
       Vel.Matrix[i, 0] = t;
53
       Vel. Matrix[i, 1] = yhDot Re. Matrix[0, 0];
54
       Vel. Matrix[i, 2] = yhDot Re. Matrix[1, 0];
55
       Vel. Matrix[i, 3] = yhDot Re. Matrix[2, 0];
56
       Vel. Matrix[i, 4] = yhDot Re. Matrix[3, 0];
57
58 }
59
60 // 列印節點之狀態響應(變位,速度、和加速度),適用於Excel繪圖
   Console. Write ("\n****\{0,10\}\mbox{ } m\{0,7\}\mbox{ } m\{0,7\}\mbox{ } m\{0,7\}\mbox{ } m\{0,10\}\mbox{ } m***\n",
62 Console. Write ("\n{0,5}***位移反應量***{0,5}\n{0,8} 時間(秒)"+
   "{0,8}第0點位移{0,8}第1點位移{0,8}第2點位移{0,8}第3點位移\n\n{1}",
64 "", new PR(Disp));
65 Console. Write ("\n{0,5}***速度反應量***{0,5}\n{0,8} 時間(秒)"+
   "{0,8}第0點速度{0,8}第1點速度{0,8}第2點速度{0,8}第3點速度\n\n{1}",
67 "", new PR(Ve1));
68
69 //列印時間、變位、和速度等序列,使用於Matplotlib.pyplot套件繪圖
70 Console. Write ("\n時間序列: \n{0}\n", new PR4(Disp, 0,
     "\{0,8:F2\}"));
71 Console. Write ("\n第0點變位序列: \n{0}\n", new PR4(Disp, 1));
72 Console. Write("\n第1點變位序列: \n{0}\n", new PR4(Disp, 2));
73 Console. Write("\n第2點變位序列: \n{0}\n", new PR4(Disp, 3));
74 Console. Write("\n第3點變位序列: \n{0}\n", new PR4(Disp, 4));
75 Console. Write ("\n第0點速度序列 : \n{0}\n", new PR4(Vel, 1));
76 Console. Write("\n第1點速度序列 : \n{0}\n", new PR4(Vel, 2));
77 Console. Write("\n第2點速度序列 : \n{0}\n", new PR4(Vel, 3));
78 Console. Write ("\n第3點速度序列 : \n{0}\n", new PR4(Vel, 4));
79
80
   /* 輸出結果如下:
              系
                                  狀
                                         熊
   ****
                    統
                           與
                                                參
                                                      數
81
                                                               ***
82
83
   ***
           系統特徵值V
                    2.00000i
84
       0.00000 +
       0.00000 -
                    2.00000i
85
86
       0.00000 +
                    1.00000i
87
       0.00000 -
                    1.00000i
88
```

```
89
     ***
              系統特徵向量Q
                                 ***
                                  0.26833 +
 90
        0.26833 +
                     0.00000i.
                                               0.00000i,
                     0.00000i,
                                               0.00000i
 91
        0.58835 +
                                  0.58835 +
 92
 93
       -0.35777 +
                     0.00000i.
                                 -0.35777 +
                                               0.00000i,
 94
        0.39223 +
                     0.00000i,
                                  0.39223 +
                                               0.00000i
 95
                                  0.00000 -
 96
        0.00000 +
                     0.53666i,
                                               0.53666i,
 97
        0.00000 +
                     0.58835i,
                                  0.00000 -
                                               0.58835i
 98
 99
        0.00000 -
                     0.71554i,
                                  0.00000 +
                                               0.71554i,
100
        0.00000 +
                     0.39223i,
                                  0.00000 -
                                               0.39223i
101
              係數向量d
102
     ***
                             ***
       -4.34791 +
                     0.15528i
103
104
       -4.34791 -
                     0.15528i
105
        1.13312 -
                     0.99148i
        1.13312 +
                     0.99148i
106
107
                    狀
                              態
                                        鑾
                                                 應
108
     ***
                                                              ***
109
110
      ***位移反應量***
       時間(秒)
                     第0點位移
                                                第2點位移
111
                                   第1點位移
                                                              第3點位移
                    -1.00000
112
        0.00000
                                  4.00000
                                               1.00000
                                                            1.00000
                     0.39861
                                               4. 22143
113
        0.50000
                                  2. 92740
                                                           -4. 85934
114
        1.00000
                     2.59735
                                 -0.05890
                                               3.82114
                                                           -6.07807
115
        1.50000
                     3.55628
                                 -2.22559
                                              -0.42391
                                                           -1.92973
                     2.09422
                                 -1.78033
                                              -5.12071
116
        2.00000
                                                            3.43180
117
        2.50000
                    -0.95195
                                  0.52930
                                              -6.25489
                                                            4.87459
118
        3.00000
                    -3.37247
                                  2.18591
                                              -2.80712
                                                            1.05653
119
        3.50000
                    -3.47172
                                  1.31324
                                               2. 31547
                                                           -4.33692
                                                           -6.02401
120
        4.00000
                    -1.49741
                                 -1.51237
                                               4.88775
121
        4.50000
                     0.67012
                                 -3.73650
                                               3. 13254
                                                           -2.06181
122
        5.00000
                     1.26265
                                 -3.16458
                                              -0.78940
                                                            4. 27155
123
                                              -2.89986
        5.50000
                     0.19478
                                 -0.01617
                                                            7.40148
124
        6.00000
                    -0.97002
                                  3.20187
                                              -1.15191
                                                            4.52138
125
        6.50000
                    -0.59930
                                  3.90525
                                               2.66204
                                                           -1.84436
126
        7.00000
                     1.37008
                                  1.71661
                                               4.60360
                                                           -6. 13102
127
        7.50000
                     3.27492
                                 -1.25354
                                               2.31503
                                                           -4.77925
                                 -2.37122
128
        8.00000
                     3. 21878
                                              -2.67283
                                                            0.58596
                                              -6.20766
129
        8.50000
                     0.85105
                                 -0.87698
                                                            4.74287
130
        9.00000
                    -2. 21220
                                  1.48150
                                              -5. 22715
                                                            3. 74458
131
        9.50000
                    -3.73672
                                  2.14779
                                              -0.52853
                                                           -1.42160
                    -2.78174
                                               3.93885
132
       10.00000
                                  0.20207
                                                           -5.75888
133
       10.50000
                    -0.45205
                                 -2.71797
                                               4.61385
                                                           -4.91549
134
       11.00000
                     1.17324
                                 -3.88580
                                               1.46388
                                                            0.72516
135
       11.50000
                     0.93687
                                 -2.00303
                                              -2.12908
                                                            6.30103
```

C:\2302\Misc_10\App_40\Program.cs						
136	12.00000	-0.41512	1.55180	-2.59682	6.86227	
137	12.50000	-1.04874	3.90439	0.46965	1.87883	
138	13.00000	0.12709	3. 23080	3.94922	-4. 26873	
139	13.50000	2.33292	0.35139	4. 13413	-6.26749	
140	14.00000	3.56151	-2.07266	0. 26339	-2.67379	
141	14.50000	2.41824	-1.98940	-4.63288	2.85593	
142	15.00000	-0.53184	0. 20058	-6. 38988	5.01313	
143	15. 50000	-3. 16447	2.09180	-3. 45480	1.77279	
144	16.00000	-3.60526	1.58146	1.70088	-3.73459	
145	16.50000	-1.81931	-1.10814	4. 79808	-6. 13822	
146	17.00000	0.44740	-3. 57353	3. 57138	-2.84011	
147	17.50000	1.29869	-3. 42282	-0.29067	3.50129	
148	18.00000	0.38555	-0.50547	-2.83528	7. 32375	
149	18.50000	-0.87916	2.87854	-1.57804	5. 20953	
150	19.00000	-0.76006	3.99971	2. 17747	-0.99603	
151	19.50000	1.06505	2.11504	4. 57430	-5.85699	
152	20.00000	3. 10328	-0. 91931	2.84719	-5. 27863	
153						
154						
155	時間(秒)	第0點速度	第1點速度	第2點速	度 第3點速度	
156	0.00000	1.00000	1.00000	8.00000	-13.33333	
157	0.50000	4. 22143	-4 . 85934	3.59387	-8.25071	
158	1.00000	3.82114	-6.07807	-5.28306	3.63983	
159	1.50000	-0. 42391	-1.92973	-10.45095	11.41847	
160	2.00000	-5. 12071	3. 43180	-6 . 85893	8. 13327	
161	2.50000	-6.25489	4.87459	2.69784	-2.85716	
162	3.00000	-2.80712	1.05653	10.02381	-11.05435	
163	3.50000	2. 31547	-4. 33692	8. 91329	-8.56866	
164	4.00000	4.88775	-6.02401	0.72627	2.54057	
165	4.50000	3. 13254	-2.06181	-6.94499	12. 10300	
166	5.00000	-0.78940	4. 27155	<i>−</i> 7. 27217	11. 17726	
167	5.50000	-2.89986	7. 40148	-0.41382	0.30822	
168	6.00000	-1.15191	4. 52138	6. 74285	-10.89896	
169	6.50000	2.66204	-1.84436	7.05647	-12 . 51482	
170	7.00000	4.60360	-6 . 13102	-0.16524	-3. 32305	
171	7.50000	2. 31503	-4.77925		8. 12717	
172	8.00000	-2.67283	0. 58596	-9.99438		
173	8.50000	-6. 20766	4. 74287	-3.01757	3. 76567	
174	9.00000	−5 . 22715	3. 74458	6.64665	-7. 39409	
175	9.50000	-0 . 52853	-1.42160	10.69513	-11.42566	
176	10.00000	3. 93885	-5.75888	5.86658	-4 . 31519	
177	10. 50000	4. 61385	-4. 91549	-3. 17286	7. 55119	
178	11. 00000	1. 46388	0. 72516	-8. 17518	13. 22171	
179	11. 50000	-2. 12908	6. 30103	-4. 87829	7. 25825	
180	12.00000	-2.59682	6.86227	3. 15794	-5.20890	
181 182	12. 50000 13. 00000	0. 46965 3. 94922	1. 87883 -4. 26873	7. 95406 4. 59201	-13. 11149 -9. 52294	

```
183
       13.50000
                      4.13413
                                   -6.26749
                                                 -4.13876
                                                                2.05639
184
       14.00000
                      0.26339
                                   -2.67379
                                               -10.23202
                                                              10.96666
185
       14.50000
                     -4.63288
                                    2.85593
                                                 -7.82058
                                                               9.19252
186
       15.00000
                     -6.38988
                                    5.01313
                                                  1.36455
                                                              -1.31085
                     -3.45480
                                                  9.46664
187
       15. 50000
                                    1.77279
                                                             -10.49468
188
       16.00000
                      1.70088
                                   -3.73459
                                                  9.58271
                                                              -9.55138
189
       16.50000
                      4.79808
                                   -6.13822
                                                  1.97641
                                                               0.89869
       17.00000
                      3.57138
                                   -2.84011
                                                -6.25509
190
                                                              11. 31711
                     -0.29067
                                    3.50129
                                                -7.73160
191
       17.50000
                                                              12.00003
       18.00000
                     -2.83528
                                    7.32375
                                                -1.52930
                                                               2.03046
192
       18.50000
                     -1.57804
                                                  6.07614
193
                                    5. 20953
                                                              -9.80784
194
       19.00000
                      2.17747
                                   -0.99603
                                                  7. 51969
                                                             -13.01254
                                                  1.04246
195
       19.50000
                      4.57430
                                   -5.85699
                                                              -4.92505
       20.00000
                                   -5.27863
                                                 -7.58552
                                                               6.89562
196
                      2.84719
197
198
     時間序列:
199
          0.00,
                    0.50,
                              1.00,
                                        1.50,
                                                   2.00,
          2.50,
                    3.00,
                              3.50,
                                        4.00,
                                                   4.50,
200
201
          5.00,
                    5.50,
                              6.00,
                                        6.50,
                                                   7.00,
202
          7.50,
                    8.00,
                              8.50,
                                        9.00,
                                                   9.50,
         10.00,
                   10.50,
                             11.00,
                                                  12.00,
203
                                        11.50,
204
         12.50,
                   13.00,
                             13.50,
                                       14.00,
                                                  14.50,
         15.00,
                   15.50,
                             16.00,
                                        16.50,
                                                  17.00,
205
         17.50,
                                       19.00,
                                                  19.50,
206
                   18.00,
                             18. 50,
207
         20.00,
208
209
     第0點變位序列 :
       -1.0000,
                      0.3986,
                                    2.5974,
                                                                2.0942,
210
                                                  3. 5563,
211
       -0.9519,
                     -3.3725,
                                   -3.4717,
                                                -1.4974,
                                                               0.6701,
212
         1.2627,
                      0.1948,
                                   -0.9700,
                                                -0.5993
                                                                1.3701,
        3.2749,
                      3.2188,
                                    0.8510,
                                                -2.2122,
                                                              -3.7367,
213
214
       -2.7817,
                     -0.4520,
                                    1. 1732,
                                                  0.9369,
                                                              -0.4151,
       -1.0487,
                      0.1271,
                                    2.3329,
                                                  3.5615,
                                                               2.4182,
215
       -0.5318,
                     -3.1645,
                                   -3.6053,
                                                -1.8193,
                                                               0.4474,
216
217
         1.2987,
                      0.3855,
                                   -0.8792,
                                                -0.7601,
                                                                1.0651,
         3.1033,
218
219
220
     第1點變位序列
221
         4.0000,
                      2.9274,
                                   -0.0589,
                                                -2.2256,
                                                              -1.7803,
222
         0.5293,
                      2.1859,
                                    1.3132,
                                                -1.5124,
                                                              -3.7365,
223
       -3.1646,
                     -0.0162,
                                    3.2019,
                                                  3.9053,
                                                                1.7166,
224
       -1.2535,
                     -2.3712,
                                   -0.8770,
                                                  1.4815,
                                                               2. 1478,
225
                     -2.7180,
                                   -3.8858,
                                                -2.0030,
                                                               1.5518,
        0.2021,
                      3.2308,
                                                -2.0727,
226
         3.9044,
                                    0.3514,
                                                              -1.9894,
227
        0.2006,
                      2.0918,
                                    1.5815,
                                                -1.1081,
                                                              -3.5735,
228
       -3.4228,
                     -0.5055,
                                    2.8785,
                                                  3.9997,
                                                               2.1150,
229
       -0.9193,
```

```
230
231
     第2點變位序列 :
                      4. 2214,
232
         1.0000,
                                    3.8211,
                                                -0.4239,
                                                              -5.1207,
233
       -6.2549,
                     -2.8071,
                                    2.3155,
                                                  4.8878,
                                                               3. 1325,
       -0.7894,
                     -2.8999,
                                                  2.6620,
                                                               4.6036,
234
                                   -1.1519,
235
        2.3150,
                     -2.6728,
                                   -6.2077,
                                                 -5.2271,
                                                              -0.5285,
236
         3.9389,
                      4.6138,
                                    1.4639,
                                                -2.1291,
                                                              -2.5968,
                      3.9492,
                                    4. 1341,
                                                  0.2634,
                                                              -4.6329,
237
        0.4697,
                                                  4.7981,
238
       -6.3899,
                     -3.4548,
                                    1.7009,
                                                               3. 5714,
239
       -0.2907,
                     -2.8353,
                                   -1.5780,
                                                  2. 1775,
                                                               4.5743,
         2.8472,
240
241
     第3點變位序列
242
                                   -6.0781,
                                                -1.9297,
243
         1.0000,
                     -4.8593,
                                                               3.4318,
244
        4.8746,
                      1.0565,
                                   -4. 3369,
                                                -6.0240,
                                                              -2.0618,
245
        4. 2716,
                      7.4015,
                                    4. 5214,
                                                -1.8444,
                                                              -6.1310,
246
       -4.7792,
                      0.5860,
                                    4.7429,
                                                  3.7446,
                                                              -1.4216,
       -5.7589,
                                    0.7252,
247
                     -4.9155,
                                                  6.3010,
                                                               6.8623,
248
         1.8788,
                     -4.2687,
                                   -6.2675,
                                                -2.6738,
                                                               2.8559,
         5.0131,
                      1.7728,
                                   -3.7346,
                                                -6.1382,
                                                              -2.8401,
249
250
         3.5013,
                      7.3237,
                                    5. 2095,
                                                -0.9960,
                                                              -5.8570,
       -5.2786,
251
252
     第0點速度序列 :
253
                      4. 2214,
254
         1.0000,
                                    3.8211,
                                                -0.4239,
                                                              -5.1207,
       -6.2549,
                     -2.8071,
                                    2.3155,
                                                  4.8878,
                                                               3. 1325,
255
       -0.7894,
256
                     -2.8999,
                                   -1.1519,
                                                  2.6620,
                                                               4.6036,
257
        2.3150,
                     -2.6728,
                                   -6.2077,
                                                 -5.2271,
                                                              -0.5285,
         3.9389,
                      4.6138,
                                    1.4639,
                                                -2.1291,
                                                              -2.5968,
258
        0.4697,
                      3.9492,
                                    4. 1341,
                                                  0.2634,
                                                              -4.6329,
259
       -6.3899,
                     -3.4548,
                                                  4.7981,
                                                               3.5714,
260
                                    1.7009,
261
       -0.2907,
                     -2.8353,
                                   -1.5780,
                                                  2. 1775,
                                                               4. 5743,
         2.8472,
262
263
     第1點速度序列
264
                     -4.8593,
                                   -6.0781,
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