```
1
2  // ***  時頻數值計算(Precisely Time-Frequency Numerical Computations)  ***
3  // 微分方程式 : M(t) * yh''(t) + C(t) * yh'(t) + K(t) * yh(t) = dh
4  //(多空間多階之齊次微分方程式，求得系統或狀態矩陣 A，再求得 A * Q = Q * D)
5  // 稱此法為 : 實數與複數矩陣轉換( Real And Complex Matrix Transform )
6  // 本求解法可對應於 Laplace、 Fourier、 Z Transform 或是 捲積積分法等等。
7
8  // 假設已知 M(t), K(t), C(t)，再求得隨時間變化的頻率(*** 即系統矩陣 A 之
9  // 複數特徵值中的虛數值  ***)。
10 // M(t) = { {19, -1.5, -2+13.3*sin(0.85*t)}, {-1, 15, 0},
11 // {-10-2.7*cos(1.3*t), -3, 27}      }
12 // K(t) = { {60, -8, -2-332*sin(1.37*t)},    {-16, 180, -120},
13 // {-20, -100+579*cos(0.24*t), 300}  }
14 // C(t) = { {35, -1-13.2*sin(0.35t), -0.5},  {-1.5, 40, -1.5},
15 // {-1.2+22.5*cos(1.95*t), -1.5, 75} }
16 // 系統矩陣 A 的特徵值與特徵向量，為系統的潛在特性，並在系統受到
17 // 外力時，才會顯現出來。
18
19 // 若要求得系統的訊號響應值[Signal Response]，應由實際量測的初始值或
20 // 是邊界值，求得複數係數向量dh。
21 // 再依據如下推導的公式求得。(初始值和邊界值分別參見App_6J和App_6M儲存庫)
22 // [yh''(t) | yh'(t)] = A * [yh'(t) | yh(t)]。  "|"是垂直合併運算子。
23 // [yh'(t) | yh(t)] = Hexp(D, Q, t) *  dh 。
24 // A * Q = Q * D  => A = Q * D * Qi 。
25 // D為複數特徵值矩陣，Q為複數特徵向量矩陣，Qi為Q之逆矩陣，Hexp(D, Q, t)和dh分別
26 // 爲複數矩陣和向量。
27 // [y'(t) | y(t)] = [yh'(t) | yh(t)] + [yp'(t) | yp(t) ]
28
29
30 using System;
31 using Matrix_0;
32
33 namespace ConsoleApp48
34 {
35     internal class Program
36     {
37         static void Main(string[] args)
38         {
39
```

```
40 // 建構初始矩陣 M、K、C。
41 double[,] M0 =
42     { {19, -1.5, -2}, {-1, 15, 0},       {-10, -3, 27}    };
43 double[,] K0 =
44     { {60, -8, -2},    {-16, 180, -120}, {-20, -100, 300} };
45 double[,] C0 =
46     { {35, -1, -0.5}, {-1.5, 40, -1.5}, {-1.2, -1.5, 75} };
47
48 // 轉爲SMS型態之矩陣(好處是可使用矩陣的運算子)。
49 ReMatrix M = new ReMatrix(M0);
50 ReMatrix K = new ReMatrix(K0);
51 ReMatrix C = new ReMatrix(C0);
52
53 // 狀態響應。速度，變位，加速度。(t = 20秒)
54 double step = 1.0;
55 int iRow = (int)(20 / step + 1);
56
57 int m = M.Row;
58 int r = 2;
59 int iColD = m * r + 1;
60
61 CxMatrix CxVal = new CxMatrix(iRow, iColD);
62 ReMatrix ReVal = new ReMatrix(iRow, iColD);
63
64 // 時間軸(t-axis)之計算 :
65 for (int i = 0; i != iRow; i++)
66 {
67     double t = step * i;
68
69     // 建構 M、k、C 為變數 t 矩陣。
70     double[,] M1 = { {0, 0, 13.3*Math.Sin(0.85*t)},  {0, 0, 0},
71         {-2.7*Math.Cos(1.3*t), 0, 0}  };
72     double[,] K1 = { {0, 0, -332*Math.Sin(1.37*t)},  {0, 0, 0},
73         {0, 579*Math.Cos(0.24*t), 0}  };
74     double[,] C1 = { {0, -13.2*Math.Sin(0.35*t), 0}, {0, 0, 0},
75         {22.5*Math.Cos(1.95*t), 0, 0} };
76
77     // M(t)=M+(ReMatrix)M1; K(t)=K+(ReMatrix)K1; C(t)=C+(ReMatrix)
       C1;
78     M += (ReMatrix)M1;
79     K += (ReMatrix)K1;
80     C += (ReMatrix)C1;
81
82     // 隨時間變化的系統(狀態)矩陣 A。
83     MKCMatrix mkc = new MKCMatrix(M, K, C);
84     ReMatrix A = mkc.Matrix;
85
```

```
86      // 隨時間變化的系統特徵值矩陣 D
87      EIG eig = new EIG(A);
88      CxMatrix D = eig.CxMatrixD;
89
90      // 將時間轉爲單一的複數值(Complex Scalar),
91      // 再轉爲C1X1的複數矩陣(Complex Matrix)。
92      CxScalar cxScalar = new CxScalar(t, 0);
93      CxMatrix cxMatrix = new CxMatrix(cxScalar);
94
95      // 隨時間變化的特徵值矩陣。
96      CxVal[i, 0] =  cxMatrix;
97      CxVal[i, 1] = D[0, 0];
98      CxVal[i, 2] = D[1, 1];
99      CxVal[i, 3] = D[2, 2];
100     CxVal[i, 4] = D[3, 3];
101     CxVal[i, 5] = D[4, 4];
102     CxVal[i, 6] = D[5, 5];
103
104     // 隨時間變化的角矩陣。
105     double[,] tMatrix = { {t} };
106     ReVal[i, 0] = (ReMatrix)tMatrix;
107     ReVal[i, 1] = D[0, 0].Im;
108     ReVal[i, 2] = D[1, 1].Im;
109     ReVal[i, 3] = D[2, 2].Im;
110     ReVal[i, 4] = D[3, 3].Im;
111     ReVal[i, 5] = D[4, 4].Im;
112     ReVal[i, 6] = D[5, 5].Im;
113 }
114
115 Console.WriteLine("\n***  時間(設為複數值，計有一組)，" +
116     "特徵值(計有六組)，合計共有七組複數值  ***");
117 Console.WriteLine("\n{0}\n", new PR(CxVal));
118
119 Console.WriteLine("\n***  時間(計有一組)，特徵值的虛" +
120     "數值(計有六組)，合計共有七組實數值 ***");
121 Console.WriteLine("  (特徵值的虛數值即角頻率，係依據特徵值" +
122     "的絕對值[模數 Modulus]，由大致小排序而成)");
123 Console.WriteLine("\n{0}\n", new PR(ReVal));
124
125 Console.WriteLine("\n時間序列:  t\n{0}\n", new PR4(ReVal, 0));
126 Console.WriteLine("\n角頻率序列w0\n{0}\n", new PR4(ReVal, 1));
127 Console.WriteLine("\n角頻率序列w1\n{0}\n", new PR4(ReVal, 2));
128 Console.WriteLine("\n角頻率序列w2\n{0}\n", new PR4(ReVal, 3));
129 Console.WriteLine("\n角頻率序列w3\n{0}\n", new PR4(ReVal, 4));
130 Console.WriteLine("\n角頻率序列w4\n{0}\n", new PR4(ReVal, 5));
131 Console.WriteLine("\n角頻率序列w5\n{0}\n", new PR4(ReVal, 6));
132
```

```
133             }
134         }
135 }
136
137 /*輸出結果如下 :
138 ***  時間(設為複數值，計有一組)，特徵值(計有六組)，合計共有七組複數值  ***
139   0.00000 +   0.00000i,   -3.21988 +   3.37745i,   -3.21988 -   3.37745i,
140   0.34355 +   3.61083i,    0.34355 -   3.61083i,   -0.92621 +   1.52681i,
141  -0.92621 -   1.52681i
142   1.00000 +   0.00000i,   -3.49561 +   2.87514i,   -3.49561 -   2.87514i,
143   1.52214 +   3.58851i,    1.52214 -   3.58851i,   -1.16587 +   1.95302i,
144  -1.16587 -   1.95302i
145   2.00000 +   0.00000i,   -3.77991 +   2.98608i,   -3.77991 -   2.98608i,
146   1.91092 +   3.42431i,    1.91092 -   3.42431i,   -1.06189 +   2.20875i,
147  -1.06189 -   2.20875i
148   3.00000 +   0.00000i,   -3.47421 +   3.34401i,   -3.47421 -   3.34401i,
149   2.06396 +   3.78510i,    2.06396 -   3.78510i,   -1.09093 +   1.81984i,
150  -1.09093 -   1.81984i
151   4.00000 +   0.00000i,   -3.48623 +   3.70145i,   -3.48623 -   3.70145i,
152   1.98893 +   3.90537i,    1.98893 -   3.90537i,   -1.00603 +   1.45004i,
153  -1.00603 -   1.45004i
154   5.00000 +   0.00000i,   -4.11146 +   3.77624i,   -4.11146 -   3.77624i,
155   2.09078 +   3.97058i,    2.09078 -   3.97058i,   -0.98601 +   1.74462i,
156  -0.98601 -   1.74462i
157   6.00000 +   0.00000i,   -4.96305 +   3.81551i,   -4.96305 -   3.81551i,
158   2.47131 +   4.26442i,    2.47131 -   4.26442i,   -0.96842 +   2.05116i,
159  -0.96842 -   2.05116i
160   7.00000 +   0.00000i,   -5.09309  +   4.02288i,   -5.09309 -   4.02288i,
161   2.27333 +   4.49039i,    2.27333  -   4.49039i,   -0.93997 +   1.95020i,
162  -0.93997 -   1.95020i
```

```
163   8.00000 + 0.00000i,  -4.34736 + 4.21289i,  -4.34736 - 4.21289i,
164   1.87158 + 4.25208i,   1.87158 - 4.25208i,  -0.91078 + 1.59671i,
165  -0.91078 - 1.59671i
166   9.00000 + 0.00000i,  -3.69977 + 3.86498i,  -3.69977 - 3.86498i,
167   1.77905 + 3.90293i,   1.77905 - 3.90293i,  -0.95020 + 1.48003i,
168  -0.95020 - 1.48003i
169  10.00000 + 0.00000i,  -3.20907 + 3.26497i,  -3.20907 - 3.26497i,
170   2.08552 + 3.50364i,   2.08552 - 3.50364i,  -1.11760 + 1.80711i,
171  -1.11760 - 1.80711i
172  11.00000 + 0.00000i,  -3.12689 + 2.81192i,  -3.12689 - 2.81192i,
173   1.80788 + 3.06483i,   1.80788 - 3.06483i,  -1.22332 + 2.11202i,
174  -1.22332 - 2.11202i
175  12.00000 + 0.00000i,  -2.77481 + 3.06109i,  -2.77481 - 3.06109i,
176   0.94221 + 3.09674i,   0.94221 - 3.09674i,  -1.09811 + 1.77316i,
177  -1.09811 - 1.77316i
178  13.00000 + 0.00000i,  -2.08277 + 3.29191i,  -2.08277 - 3.29191i,
179  -0.60032 + 3.27430i,  -0.60032 - 3.27430i,  -0.58513 + 1.33906i,
180  -0.58513 - 1.33906i
181   4.00000 + 0.00000i,  -1.42937 + 4.61481i,  -1.42937 - 4.61481i,
182  -3.83242 + 0.00000i,  -0.88546 + 1.72871i,  -0.88546 - 1.72871i,
183   0.88418 + 0.00000i
184  15.00000 + 0.00000i,  -5.78077 + 0.00000i,  -1.28854 + 4.81809i,
185  -1.28854 - 4.81809i,   2.58466 + 0.00000i,  -0.75862 + 2.06551i,
186  -0.75862 - 2.06551i
187  16.00000 + 0.00000i,  -5.29346 + 0.00000i,  -0.94129 + 5.04363i,
188  -0.94129 - 5.04363i,   2.71209 + 0.00000i,  -0.74314 + 2.22660i,
189  -0.74314 - 2.22660i
190  17.00000 + 0.00000i,  -0.96325 + 5.43042i,  -0.96325 - 5.43042i,
```

```
-3.83949 +  0.00000i,    2.41368 +  0.00000i,   -0.98277 +  1.86790i,
-0.98277 -  1.86790i
18.00000 +  0.00000i,   -1.15844 +  5.52255i,   -1.15844 -  5.52255i,
-3.36773 +  0.00000i,    2.20706 +  0.00000i,   -1.15196 +  1.53500i,
-1.15196 -  1.53500i
19.00000 +  0.00000i,   -0.93302 +  5.43235i,   -0.93302 -  5.43235i,
-4.28644 +  0.00000i,    2.78350 +  0.00000i,   -0.92041 +  2.01410i,
-0.92041 -  2.01410i
20.00000 +  0.00000i,   -5.72653 +  0.00000i,   -0.95713 +  5.33069i,
-0.95713 -  5.33069i,    3.26973 +  0.00000i,   -0.76582 +  2.24583i,
-0.76582 -  2.24583i

***  時間(計有一組)，特徵值的虛數值(計有六組)，合計共有七組實數值  ***
(特徵值的虛數值即角頻率，係依據特徵值的絕對值[模數 Modulus]，由大致小排序而成)
 0.00000   3.37745   -3.37745   3.61083   -3.61083   1.52681   -1.52681
 1.00000   2.87514   -2.87514   3.58851   -3.58851   1.95302   -1.95302
 2.00000   2.98608   -2.98608   3.42431   -3.42431   2.20875   -2.20875
 3.00000   3.34401   -3.34401   3.78510   -3.78510   1.81984   -1.81984
 4.00000   3.70145   -3.70145   3.90537   -3.90537   1.45004   -1.45004
 5.00000   3.77624   -3.77624   3.97058   -3.97058   1.74462   -1.74462
 6.00000   3.81551   -3.81551   4.26442   -4.26442   2.05116   -2.05116
 7.00000   4.02288   -4.02288   4.49039   -4.49039   1.95020   -1.95020
 8.00000   4.21289   -4.21289   4.25208   -4.25208   1.59671   -1.59671
 9.00000   3.86498   -3.86498   3.90293   -3.90293   1.48003   -1.48003
10.00000   3.26497   -3.26497   3.50364   -3.50364   1.80711   -1.80711
11.00000   2.81192   -2.81192   3.06483   -3.06483   2.11202   -2.11202
```

```
217   12.00000     3.06109    -3.06109     3.09674    -3.09674     1.77316
        -1.77316
218   13.00000     3.29191    -3.29191     3.27430    -3.27430     1.33906
        -1.33906
219   14.00000     4.61481    -4.61481     0.00000     1.72871    -1.72871
        0.00000
220   15.00000     0.00000     4.81809    -4.81809     0.00000     2.06551
        -2.06551
221   16.00000     0.00000     5.04363    -5.04363     0.00000     2.22660
        -2.22660
222   17.00000     5.43042    -5.43042     0.00000     0.00000     1.86790
        -1.86790
223   18.00000     5.52255    -5.52255     0.00000     0.00000     1.53500
        -1.53500
224   19.00000     5.43235    -5.43235     0.00000     0.00000     2.01410
        -2.01410
225   20.00000     0.00000     5.33069    -5.33069     0.00000     2.24583
        -2.24583
226
227  時間序列:  t
228     0.0000,     1.0000,     2.0000,     3.0000,     4.0000,
229     5.0000,     6.0000,     7.0000,     8.0000,     9.0000,
230    10.0000,    11.0000,    12.0000,    13.0000,    14.0000,
231    15.0000,    16.0000,    17.0000,    18.0000,    19.0000,
232    20.0000,
233
234  角頻率序列w0
235     3.3774,     2.8751,     2.9861,     3.3440,     3.7015,
236     3.7762,     3.8155,     4.0229,     4.2129,     3.8650,
237     3.2650,     2.8119,     3.0611,     3.2919,     4.6148,
238     0.0000,     0.0000,     5.4304,     5.5225,     5.4323,
239     0.0000,
240
241  角頻率序列w1
242    -3.3774,    -2.8751,    -2.9861,    -3.3440,    -3.7015,
243    -3.7762,    -3.8155,    -4.0229,    -4.2129,    -3.8650,
244    -3.2650,    -2.8119,    -3.0611,    -3.2919,    -4.6148,
245     4.8181,     5.0436,    -5.4304,    -5.5225,    -5.4323,
246     5.3307,
247
248  角頻率序列w2
249     3.6108,     3.5885,     3.4243,     3.7851,     3.9054,
250     3.9706,     4.2644,     4.4904,     4.2521,     3.9029,
251     3.5036,     3.0648,     3.0967,     3.2743,     0.0000,
252    -4.8181,    -5.0436,     0.0000,     0.0000,     0.0000,
253    -5.3307,
254
```

```
255 角頻率序列w3
256   -3.6108,    -3.5885,    -3.4243,    -3.7851,    -3.9054,
257   -3.9706,    -4.2644,    -4.4904,    -4.2521,    -3.9029,
258   -3.5036,    -3.0648,    -3.0967,    -3.2743,     1.7287,
259    0.0000,     0.0000,     0.0000,     0.0000,     0.0000,
260    0.0000,
261
262 角頻率序列w4
263    1.5268,     1.9530,     2.2088,     1.8198,     1.4500,
264    1.7446,     2.0512,     1.9502,     1.5967,     1.4800,
265    1.8071,     2.1120,     1.7732,     1.3391,    -1.7287,
266    2.0655,     2.2266,     1.8679,     1.5350,     2.0141,
267    2.2458,
268
269 角頻率序列w5
270   -1.5268,    -1.9530,    -2.2088,    -1.8198,    -1.4500,
271   -1.7446,    -2.0512,    -1.9502,    -1.5967,    -1.4800,
272   -1.8071,    -2.1120,    -1.7732,    -1.3391,     0.0000,
273   -2.0655,    -2.2266,    -1.8679,    -1.5350,    -2.0141,
274   -2.2458,
275
276 請按任意鍵繼續 . . .
277 */
278
```