We consider a pair of leaky integrate-and-fire neuron receiving correlated inputs. Their dynamics are governed by the two stochastic differential equations (SDEs)

$$\begin{aligned}\dot{V}_1(t) &= -\frac{V_1(t)}{\tau_1} + \mu_1 + \sigma_1\eta_1(t) \equiv -f_1(V_1,V_2) + \sigma_1\eta_1(t) \qquad (1)\\ \dot{V}_2(t) &= -\frac{V_2(t)}{\tau_2} + \mu_2 + \sigma_2\eta_2(t) \equiv -f_2(V_1,V_2) + \sigma_2\eta_2(t), \qquad (2)\end{aligned}$$

where $\langle\eta_1(t)\eta_2(t')\rangle = c\delta(t-t')$, $-1 \le c \le 1$. τ_i, μ_i and σ_i are constant parameters characterizing both the neuron and input properties. The associated two-dimensional Fokker-Planck equation (FPE) is given by (Risken, 1996)

$$\frac{\partial P(V_1,V_2,t)}{\partial t} = \frac{\partial f_1 P}{\partial V_1} + \frac{\partial f_2 P}{\partial V_2} + \frac{1}{2}\begin{bmatrix}\partial_{V_1} & \partial_{V_2}\end{bmatrix}\begin{bmatrix}\sigma_1^2 & c\sigma_1\sigma_2\\ c\sigma_1\sigma_2 & \sigma_2^2\end{bmatrix}\begin{bmatrix}\partial_{V_1}\\ \partial_{V_2}\end{bmatrix}P \qquad (3)$$

$P(V_1,V_2,t)$ is continuous and the above system is subject to the following boundary conditions (BC) (Brunel & Hakim, 1999): absorbing BCs at threshold θ,

$$P(V_1 \ge \theta, V_2, t) = P(V_1, V_2 \ge \theta, t) = 0, \qquad (4)$$

and BCs at $-\infty$

$$\begin{aligned}\lim_{V_1\to-\infty} P(V_1,V_2,t) &= \lim_{V_2\to-\infty} P(V_1,V_2,t) = 0, \qquad (5)\\ \lim_{V_1\to-\infty} V_1 P(V_1,V_2,t) &= \lim_{V_2\to-\infty} V_2 P(V_1,V_2,t) = 0. \qquad (6)\end{aligned}$$

In addition, whenever V_i hit the threshold θ, V_i is reset to $V_r < \theta$.

In particular, we are interested in the case $c = 1$ and its associated FPE is

$$\frac{\partial P(V_1,V_2,t)}{\partial t} = \frac{\partial f_1 P}{\partial V_1} + \frac{\partial f_2 P}{\partial V_2} + \frac{\sigma_1^2}{2}\frac{\partial^2 P}{\partial V_1^2} + \sigma_1\sigma_2\frac{\partial^2 P}{\partial V_1 \partial V_2} + \frac{\sigma_2^2}{2}\frac{\partial^2 P}{\partial V_2^2}. \qquad (7)$$

We are also interested in a simplified version called perfect integrate-and fire (PIF) model and its FPE is

$$\frac{\partial P(V_1,V_2,t)}{\partial t} = -\mu_1\frac{\partial P}{\partial V_1} - \mu_2\frac{\partial P}{\partial V_2} + \frac{\sigma_1^2}{2}\frac{\partial^2 P}{\partial V_1^2} + \sigma_1\sigma_2\frac{\partial^2 P}{\partial V_1 \partial V_2} + \frac{\sigma_2^2}{2}\frac{\partial^2 P}{\partial V_2^2}. \qquad (8)$$