Hw2_Stats

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Web Analytics

1

1.1 $P(4 \leq X \leq 6) = P(X = 4) + P(X = 5) + P(X = 6) = 0.37$
1.2
$\mu_X = \sum_{i=0}^{6} P(X = i)x_i = 0 + 0.09 + 2 \cdot 0.24 + 3 \cdot 0.27 + 4 \cdot 0.25 + 5 \cdot .09 + 6 \cdot 0.03 = 3.01$
$\sigma_X^2 = \frac{1}{n} \sum_{i=0}^{n} (P(X = x_i) - \mu)^2 \cdot x_i \approx 1.75$
$\sigma_X = \sqrt{\sigma_X^2} \approx 1.32$

2

2.1 A visitor will only visit the site once during each hour and each visitor is unrelated to others, so every visitor (trail) is independent.
2.2 n is the total number of visitor we examed for the test, for the last question, $n = 11$. p is the probability that the visitor is a student, here we are given $p = 0.68$.
2.3 $P(7) = \frac{11!}{(11-7)! \cdot 7!} \cdot 0.68^7 \cdot (1 - .68)^{11-7} \approx 0.23$
2.4
$\mu_X = np = 11 \cdot 0.68 = 7.48$
$\sigma_X = \sqrt{np(1 - p)} = \sqrt{11 \cdot 0.68 \cdot (1 - .68)} \approx 1.55$

3

3.1 $P(8) = \frac{20!}{(20-8)! \cdot 8!} \cdot 0.38^8 \cdot (1 - .38)^{20-8} \approx 0.18$
3.2 Assume X is the event when the customer is "desktop" customer, then $p = 1 - .38 = 0.62$. In this case,

$$P(12 \leq X \leq 14) = P(X = 14) + P(X = 13) + P(X = 14) \approx 0.50$$

3.3 Keep the assumption in 2), then we have

$$P(X \leq 3) = \sum_{i=0}^{3} P(X = i) \approx 2.16 \cdot 10^{-5}$$

A game of chance

4.1 Let A be the event that the correct answer is (a) and B be the event that the correct answer is (b). Also, let a be the event that the friend will suggest (a) to be the answer and b be the friend suggesting (b). Knowing that $P(A) = 0.6$, $P(B) = 1 - P(A) = 0.4$, and $P(a|A) = P(b|B) = 0.8$, we can calculate $P(a|B) = 1 - P(b|B) = 0.2$. Therefore,

$$P(a) = P(A) \cdot P(a|A) + P(B) \cdot P(a|B) = 0.56$$

4.2 $P(b) = 1 - P(a) = 0.44$
4.3 Choose to call a friend for help, $E(prize)_{max} = \$806,400$.
Reason:

- Givcen the facts, when answering question by self, since we know (a) has a higher posibility to be the right answer, we will choose to answer (a). Therefore, we have $P(correct) = P(A) = 0.6$, and $E(prize, self) = 0.6 \cdot \$1,000,000 + (1 - 0.6) \cdot \$32,000 = \$612,800$.

- When asking firend for help, $P(correct) = P(a \cap A) + P(b \cap B) = P(a|A) \cdot P(A) + P(b|B) \cdot P(B) = 0.8$.Then, $E(prize, call) = 0.8 \cdot \$1,000,000 + (1 - 0.8) \cdot \$32,000 = \$806,400$.
- The last case, if I just walk away, $E(prize, leave) = \$500,00$
- Since $E(prize, call) > E(prize, self) > E(prize, leave)$, we choose to call friend.

Healthcare analytics

5.1 Assume that A represents that the person has the disease, and B represents that the person is diagnoses as he/she has the disease. We are given $P(\bar{B}|A) = 0.06$, $P(\bar{B}|\bar{A}) = 0.91$
Then we have

$$P(B|A) = 1 - P(\bar{B}|A) = 0.94$$

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5.2 This is a binomial distribution question with $n = 3$, $p = P(B|\bar{A}) = 1 - P(\bar{B}|\bar{A}) = 0.09$. Thus,

$$P(X \geq 1) = 1 - P(X = 0) \approx 0.25$$

5.3 Here we are given $P(A) = .17$, so $P(\bar{A}) = 1 - P(A) = 0.83$. We can also calculate $P(B|\bar{A}) = 1 - P(\bar{B}|\bar{A}) = 0.09$.
Thus,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = \frac{0.94 \cdot 0.17}{0.94 \cdot 0.17 + 0.09 \cdot 0.83} \approx 0.68$$

Airline analytics

6.1 $p = 1 - .2 = .8$, so $P(X \leq 5) = Binomcdf(11, .8, 5) \approx 0.01$
6.2 $P(X = 10) = Binompdf(11, .8, 10) \approx 0.24$
6.3 $E_{profit} = \sum_{i=0}^{10} P(X = x_i) \cdot x_i \cdot \$1,200 + P(X = 11) \cdot \$9,000 \approx \$10,199.22$
6.4 $E_{profit} = \sum_{i=0}^{10} P(X = x_i) \cdot x_i \cdot \$1,200 \approx \$9,600$
6.5 Yes, it will. Usually people will plan the trip in unit of groups, so each person is not individual doing the decision whether to show up or not.