

# Notes: Conditional Probability

## Foundations of Data Analysis

January 24, 2019

### Review of “English translation” for events:

- $A \cap B$  = “both events  $A$  and  $B$  happen”
- $A \cup B$  = “either event  $A$  or  $B$  (or both) happens”
- $A^c$  = “event  $A$  does not happen”

### Set Theory Rules: (try drawing Venn diagrams of these)

- Definition of set difference:  $A - B = A \cap B^c$  “event  $A$  happens, but  $B$  does not”
- Associative Law:
$$(A \cup B) \cup C = A \cup (B \cup C)$$
- Commutative Law:
$$(A \cap B) \cap C = A \cap (B \cap C)$$
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$
- Distributive Law:
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$
- DeMorgan’s Law:
$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

### Counting:

- **Number of permutations of  $n$  items:**  $n! = n \times (n - 1) \times (n - 2) \times \cdots \times 2$   
(a.k.a. number of unique orderings)
- **Number of ways to select  $k$  items out of  $n$  choices:**  $\binom{n}{k} = \frac{n!}{k!(n-k)!}$   
(here order does not matter, just which  $k$  items you select)

### Probability Rules:

- **Equally likely outcomes:**  $P(A) = \frac{|A|}{|\Omega|}$
- **Inclusion-Exclusion Rule:**  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Complement Rule:**  $P(A^c) = 1 - P(A)$
- **Difference Rule:**  $P(A - B) = P(A) - P(A \cap B)$

Exercise: Try deriving these rules from the definition of a probability function. Draw a Venn diagram to convince yourself they work.

### Conditional Probability:

$P(A|B)$  = “the probability of event  $A$  given that we know  $B$  happened”

Formula:  $P(A|B) = P(A \cap B)/P(B)$

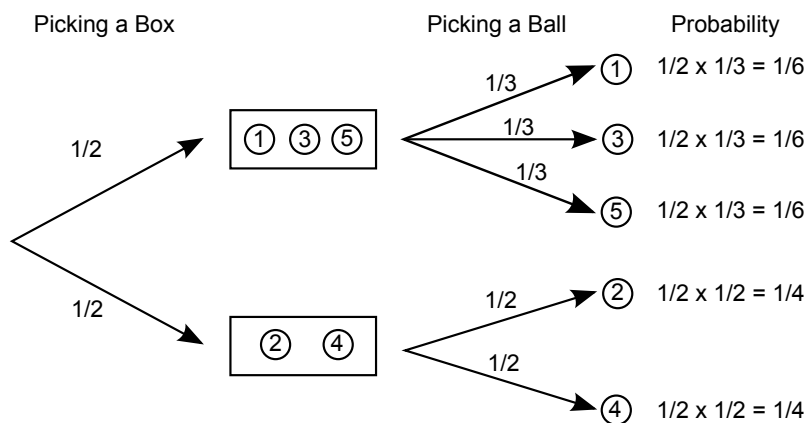
### Multiplication Rule:

$$P(A \cap B) = P(A|B)P(B)$$

**Tree diagrams** to compute “two stage” probabilities ( $B$  = first stage,  $A$  = second stage):

1. First branch computes probability of first stage:  $P(B)$
2. Second branch computes probability of second stage, given the first:  $P(A|B)$
3. Multiply probabilities along a path to get final probabilities  $P(A \cap B)$

Example: You are given two boxes with balls numbered 1 - 5. One box contains balls 1, 3, 5, and the other contains balls 2 and 4. You first pick a box at random, then pick a ball from that box at random. What is the probability that you pick a 2?



**Exercise:** You are analyzing the effectiveness of online advertising for a company that sells widgets. The company finds that 50% of traffic to their website comes from clicks of online ads. In addition, 20% of visitors to their website both had clicked an online ad and purchased a widget. If a person clicks on the company’s ad, what is the probability that they will purchase a widget?

**Exercise:** In Charlottesville the sky is overcast on about 40% of days. If it is overcast, there is a 25% chance that it will also be windy. What is the probability that it is both overcast and windy?