# Classification and Naïve Bayes

Foundations of Data Analysis

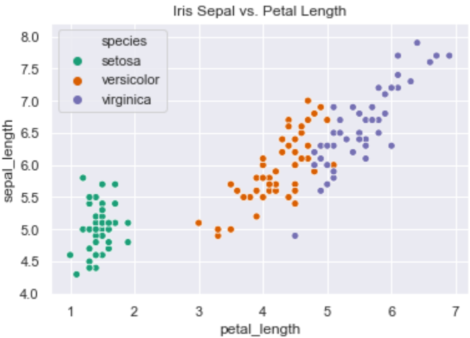
February 18, 2021

# Irises

*iris virginica* *iris versicolor* *iris setosa*

# Classification

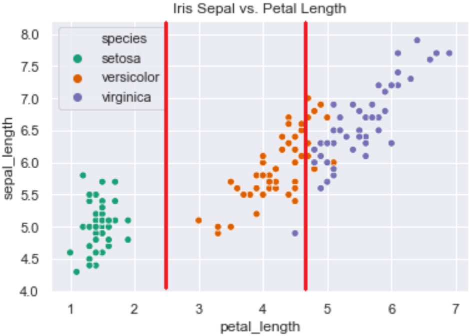
Say we want to automatically identify an iris species based on its petal and sepal length measurements.




This is a famous data set in machine learning / statistics, from Ronald Fisher in 1936!

# A Classifier is a *Decision Rule*

`x` = "petal length", `c` = "species"




```
if x < 2.5 :  c = 'setosa'
if 2.5 < x < 4.7 :  c = 'versicolor'
if x > 4.7 :  c = 'virginica'
```

# Classification Task

**Training:**
Learn a decision rule, based on training data, to predict a class $C$ from features $X$.

**Testing:**
Use trained classifier to predict unknown class $C^*$ from features of new testing data, $X^*$.

**Important!** Training and testing data should be completely separate!

# Probabilistic Classifier

Features $X$ and class $C$ are random variables.

Learn a probability distribution from the training data:

$$P(C \mid X)$$

**Imaginary Example:**

An iris test point $X^*$ might give something like this:

| $C^*$ | setosa | versicolor | virginica |
|---|---|---|---|
| $P(C^* \mid X^*)$ | 0.80 | 0.15 | 0.05 |

# Bayes' Rule for Classification

$$P(C \mid X) = \frac{P(X \mid C)P(C)}{P(X)}$$

$P(X \mid C)$ **Likelihood** - learned from data

$P(C)$ **Prior** - determined beforehand

$P(X)$ **Evidence** - not needed for decision

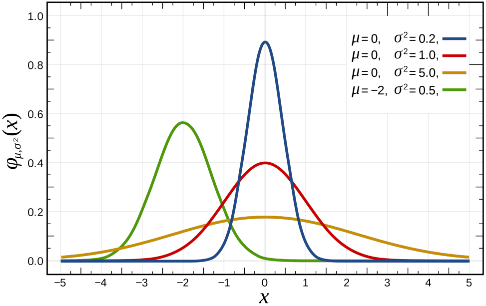
# Naïve Bayes

Multidimensional features $X = (X_1, X_2, \ldots, X_d)$

**"Naïve" Assumption:**
Assume features $X_i$ are independent, given the class $C$:

$$P(X \mid C) = P(X_1 \mid C) \times P(X_2 \mid C) \times \cdots \times P(X_d \mid C)$$

# Gaussian or Normal Distribution




Probability density function (pdf):

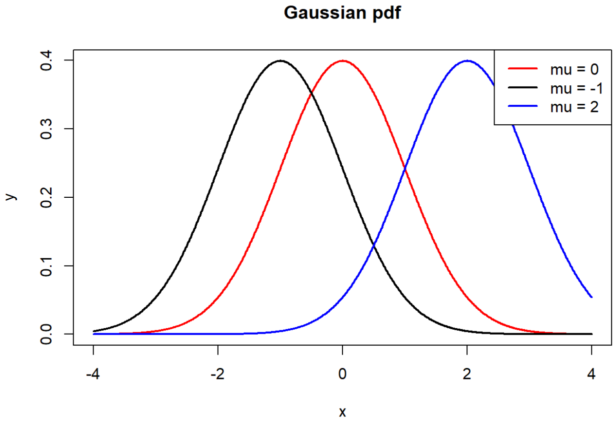
$$p(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

**Notation:** $x \sim N(\mu, \sigma^2)$

Mean, $\mu$, and variance, $\sigma^2$, are parameters.

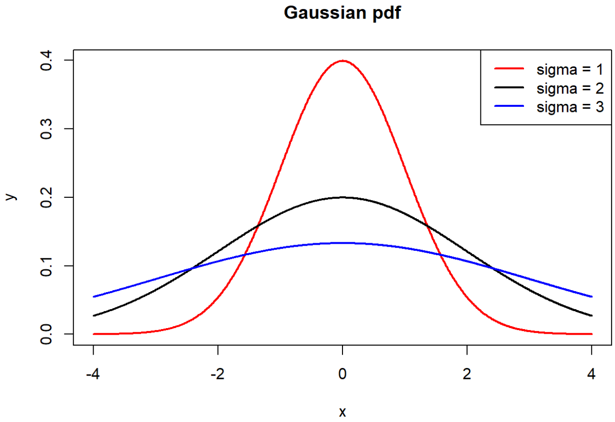
See `https://en.wikipedia.org/wiki/Normal_distribution`

# Gaussian $\mu$ Parameter




Shifts the pdf, shape stays the same

# Gaussian $\sigma$ Parameter



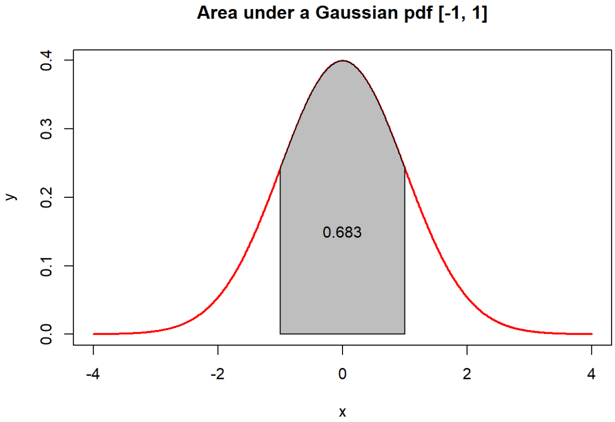

Stretches/shrinks the pdf, position stays the same

# Probabilities of Continuous Random Variables

Probability is given by area under the pdf:

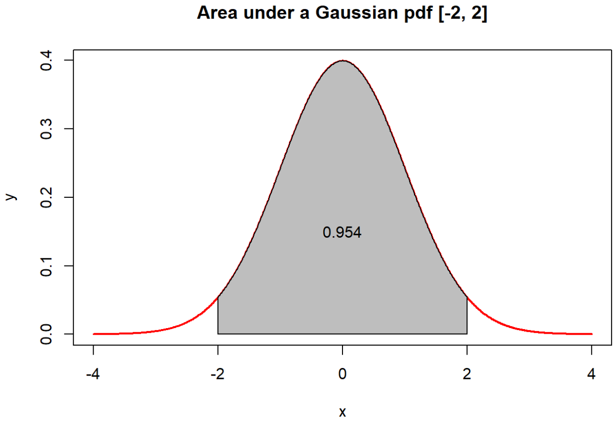
$$P(a < X < b) = \int_a^b p(x)dx$$

# Gaussian Area



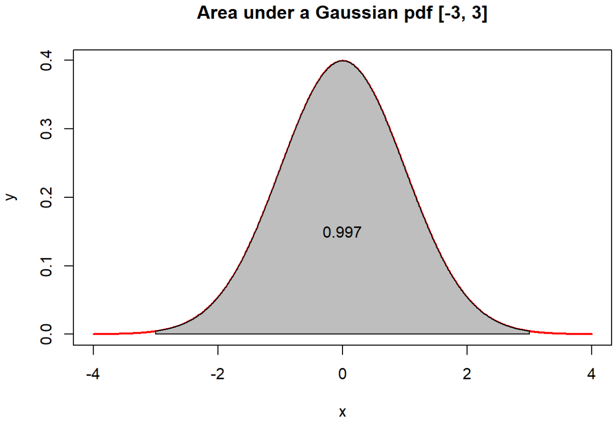

Units of horizontal axis are $\sigma$

# Gaussian Area




Units of horizontal axis are $\sigma$

# Gaussian Area




Units of horizontal axis are $\sigma$

# Gaussian Naïve Bayes

Likelihood is Gaussian pdf:

$$p(x \mid C = c_k) = \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left(-\frac{(x-\mu_k)^2}{2\sigma_k^2}\right)$$

- The Gaussian depends on the class $C \in \{c_1, c_2, \ldots, c_K\}$
- Each class needs a mean, $\mu_k$, and a variance, $\sigma_k^2$

# How to “Train” a Gaussian Distribution

For each feature in your data, given training data:
$x_1, x_2, \ldots, x_n$ all from the $k$th class

Set parameters:
Mean: $\hat{\mu}_k = \frac{1}{n} \sum_{i=1}^{n} x_i$
Variance: $\hat{\sigma}_k^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu}_k)^2$

# Evidence Calculation

If we have $K$ classes $C \in \{c_1, c_2, \ldots, c_K\}$:

$$p(x) = \sum_{k=1}^{K} p(X \mid C = c_k) P(C = c_k),$$

using Total Probability.

For the case that we have two classes:

$$p(x) = p(x \mid C = c_1)P(C = c_1) + p(x \mid C = c_2)P(C = c_2)$$

# Choosing a Prior

How to set $P(C = c_k)$?

- Equally likely: $P(C = c_k) = \frac{1}{K}$
- Frequency of classes in training data
- Derive from previous experiments or knowledge

# Putting It All Together

- Pick a prior: $P(C = c_k)$
- Train your Gaussians on training data: $\mu_k, \sigma_k^2$
- For each test data point, $x^* = (x_1^*, x_2^*, \ldots, x_d^*)$, compute the likelihood:

$$p(x^* \mid C = c_k) = p(x_1^* \mid C = c_k) \times p(x_2^* \mid C = c_k) \times \cdots \times p(x_d^* \mid C = c_k)$$

- Compute the class probabilities:

$$P(C = c_k \mid x^*) = \frac{p(x^* \mid C = c_k) P(C = c_k)}{p(x^*)}$$

- Classify $x^*$ as the class $c_k$ with highest probability