Classification and Naïve Bayes

Foundations of Data Analysis

February 18, 2021

Irises

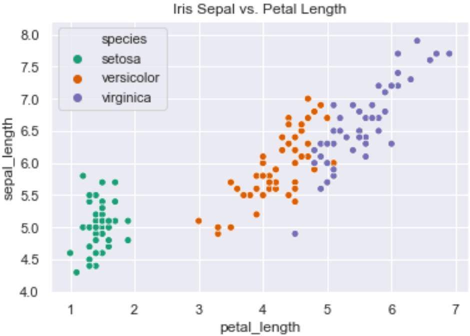
iris virginica

iris versicolor

iris setosa

Classification

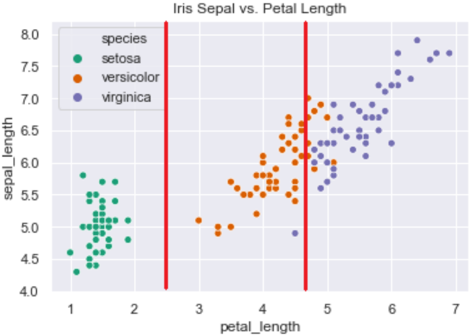
Say we want to automatically identify an iris species based on its petal and sepal length measurements.



This is a famous data set in machine learning / statistics, from Ronald Fisher in 1936!

A Classifier is a *Decision Rule*

`x` = “petal length”, `c` = “species”




```
if x < 2.5 :  c = ’setosa’
if 2.5 < x < 4.7 :  c = ’versicolor’
if x > 4.7 :  c = ’virginica’
```

Classification Task

Training:
Learn a decision rule, based on training data, to predict a class C from features X.

Testing:
Use trained classifier to predict unknown class C^* from features of new testing data, X^*.

Important! Training and testing data should be completely separate!

Probabilistic Classifier

Features X and class C are random variables.

Learn a probability distribution from the training data:

$$P(C \mid X)$$

Imaginary Example:

An iris test point X^* might give something like this:

C^*	setosa	versicolor	virginica
$P(C^* \mid X^*)$	0.80	0.15	0.05

Bayes' Rule for Classification

$$P(C \mid X) = \frac{P(X \mid C)P(C)}{P(X)}$$

$P(X \mid C)$ **Likelihood** - learned from data

$P(C)$ **Prior** - determined beforehand

$P(X)$ **Evidence** - not needed for decision

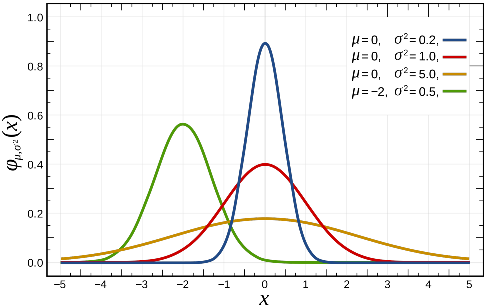
Naïve Bayes

Multidimensional features $X = (X_1, X_2, \ldots, X_d)$

"Naïve" Assumption:
Assume features X_i are independent, given the class C:

$$P(X \mid C) = P(X_1 \mid C) \times P(X_2 \mid C) \times \cdots \times P(X_d \mid C)$$

Gaussian or Normal Distribution



Probability density function:

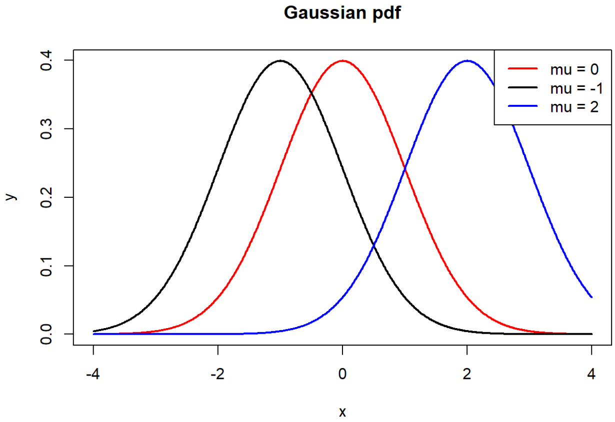
$$p(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Notation: $x \sim N(\mu, \sigma^2)$

Mean, μ, and variance, σ^2, are parameters.

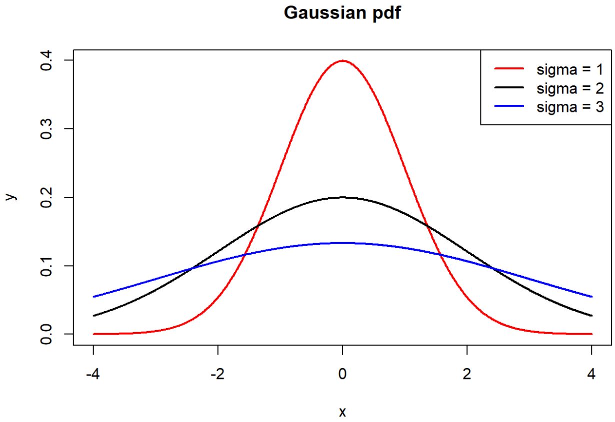
See `https://en.wikipedia.org/wiki/Normal_distribution`

Gaussian μ Parameter



Shifts the pdf, shape stays the same

Gaussian σ Parameter



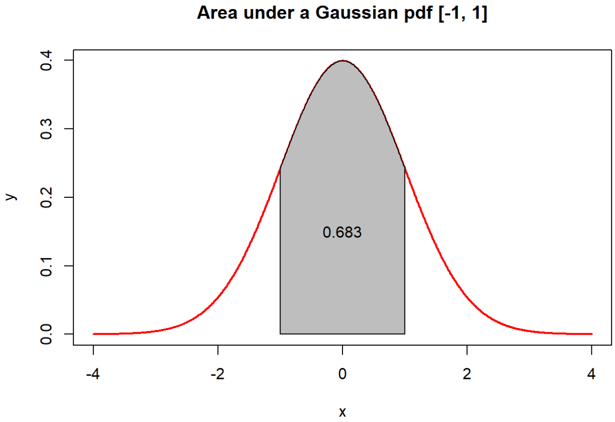
Stretches/shrinks the pdf, position stays the same

Probabilities of Continuous Random Variables

Probability is given by area under the pdf:

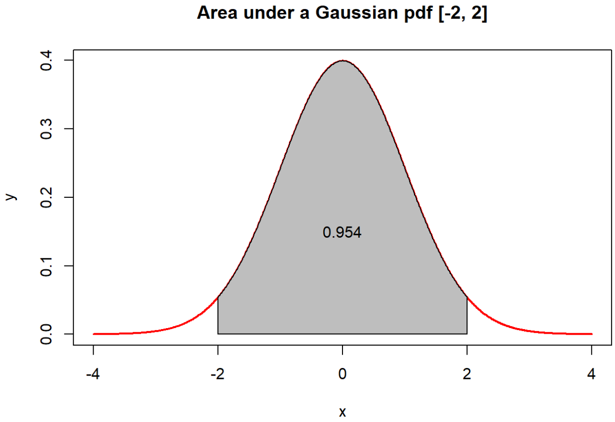
$$P(a < X < b) = \int_a^b p(x)dx$$

Gaussian Area



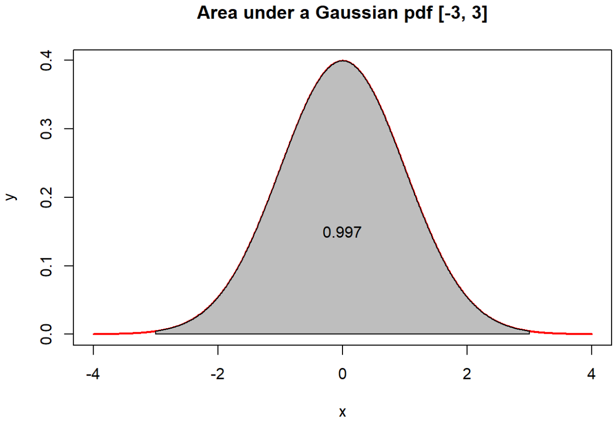
Units of horizontal axis are σ

Gaussian Area



Units of horizontal axis are σ

Gaussian Area



Units of horizontal axis are σ

How to "Train" a Normal Distribution

Given training data: $x_1, x_2, \ldots, x_n$

Set parameters:
Mean: $\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$
Variance: $\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \hat{\mu})^2$

Evidence Calculation

If we have K classes $C \in \{c_1, c_2, \ldots, c_K\}$:

$$P(X) = P(X \mid C = c_1)P(C = c_1) + P(X \mid C = c_2)P(C = c_2$$

using Total Probability.