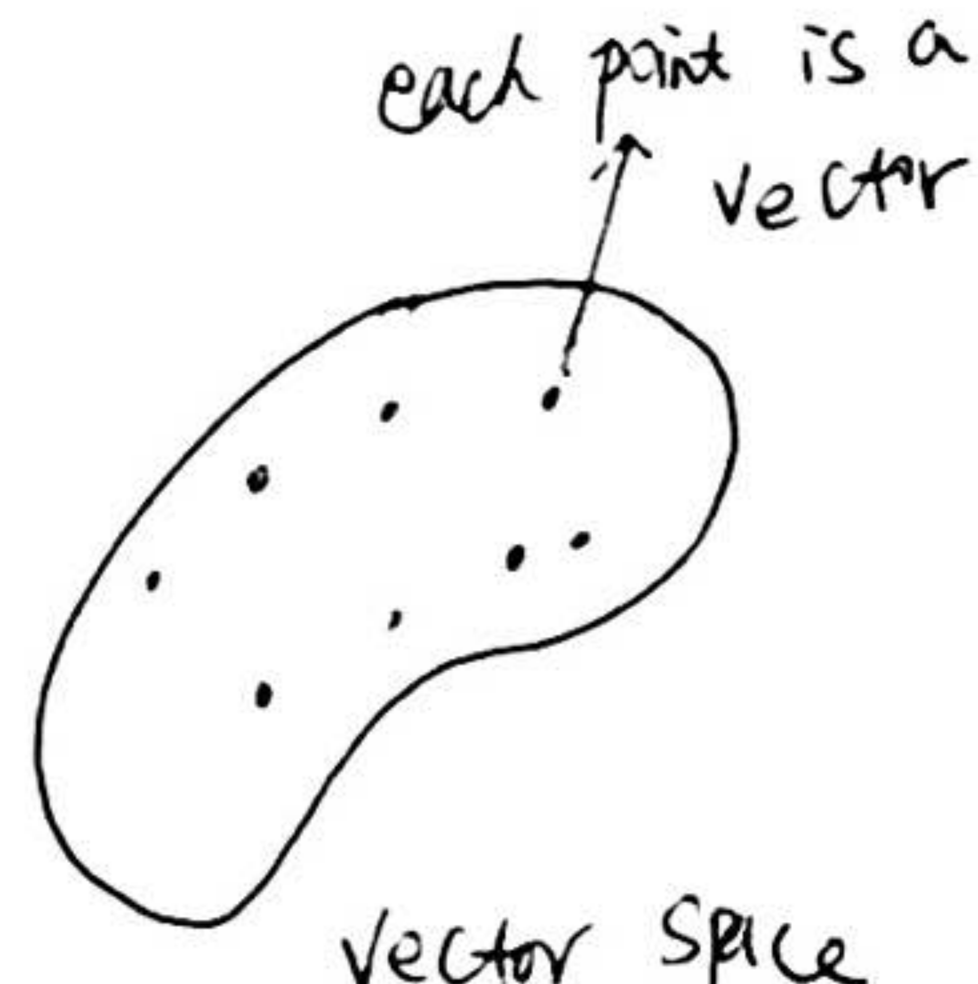
Vector Space Review

$V \in \mathbb{R}^n$, vector norm $\|v\|$, length size of vector V (e.g., feature representation)

↳ n-tuple of values (usually real numbers)

$$V = \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix}, \quad V^T = (V_1, V_2, \cdots V_n)$$



vector space (Compute length of a vector)

l^2 norm $\Rightarrow \|V\|_2 = \left(|V_1|^2 + |V_2|^2 + \ldots |V_n|^2\right)^{1/2}$

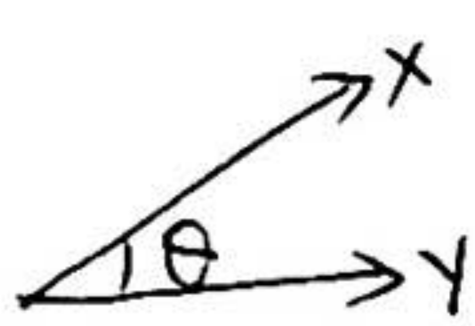
vector operations:

- Addition $\quad Z = X + Y = (X_1 + Y_1, \ldots X_n + Y_n)$
- scalar multiplication $\quad Y = aX = (aX_1, aX_2, \ldots aX_n)$
- Dot product $\quad \langle x, y \rangle = X^T Y = a$ (scalar)

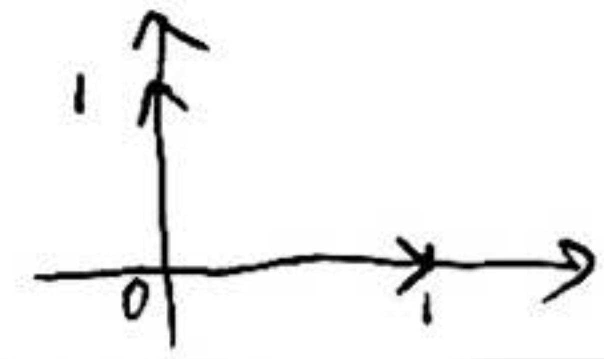
$$\Downarrow$$

$$\sum_{i=1}^{n} x_i y_i \qquad \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \leftrightarrow \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

Alternatively, geometric interpretation



$$\theta = \arccos\left(\frac{\langle X, Y \rangle}{\|X\|_2 \|Y\|_2}\right)$$

Example: 

$X = (0, 1)$, $Y = (1, 0)$

$\theta = 90°$

$$\| X - Y \|_2 = \left(\sum_{i=1}^{n} (X_n - Y_n)^2 \right)^{1/2}$$