

# Exercises

Check whether the following statements are true or false.  
(Hint: you might use Venn diagrams.)

- ▶  $A - B \subset A$
- ▶  $(A - B)^c = A^c \cup B$
- ▶  $A \cup B \subset B$
- ▶  $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

# Permutations

A **permutation** is an ordering of an  $n$ -tuple. For instance, the  $n$ -tuple  $(1, 2, 3)$  has the following permutations:

$$(1,2,3), (1,3,2), (2,1,3) \\ (2,3,1), (3,1,2), (3,2,1)$$

The number of unique orderings of an  $n$ -tuple is  $n$  **factorial**:

$$n! = n \times (n - 1) \times (n - 2) \times \cdots \times 2$$

How many ways can you rearrange  $(1, 2, 3, 4)$ ?

# Binomial Coefficient or “ $n$ choose $k$ ”

The **binomial coefficient**, written as  $\binom{n}{k}$  and spoken as “ $n$  choose  $k$ ”, is the number of ways you can select  $k$  items out of a list of  $n$  choices.

**Formula:**

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

# Binomial Coefficient or “ $n$ choose $k$ ”

**Example:** You have cards numbered 1 through 10. If you pick five cards at random, what is the probability that you selected the cards 1, 2, 3, 4, 5 (not necessarily in that order)?

## Answer

We'll use the formula  $P(A) = \frac{|A|}{|\Omega|}$ .

There is only one combination that gives us cards 1,2,3,4,5, so  $|A| = 1$ .

The total number of possible 5 card selections is

$$|\Omega| = \binom{10}{5} = \frac{10!}{5!(10-5)!} = 252$$

So, finally the probability is

$$P(A) = \frac{|A|}{|\Omega|} = \frac{1}{252} \approx 0.00397 = 0.397 \%$$

# Conditional Probability

Foundations of Data Analysis

January 25, 2023

# Conditional Probability

$P(A \mid B)$  = “the probability of event  $A$  given that we know  $B$  happened”

Formula: 
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

# Brain Teaser

Say I have two children.

1. If I tell you the oldest child is a boy, what is the probability that the youngest is a boy?
2. If I tell you at least one of the children is a boy, what is the probability the other is a boy?
3. If I tell you one of the children is a boy and born on Monday, what is the probability the other is a boy?



# Multiplication Rule

Rearranging the definition of conditional probability:

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A | B)P(B)$$

# Tree Diagrams

Think of conditional probability,  $P(A | B)$ , as two stages:

1. Compute probability of first event B:

$$P(B)$$

2. Compute conditional probability of second event, A, given the first, B:

$$P(A | B)$$

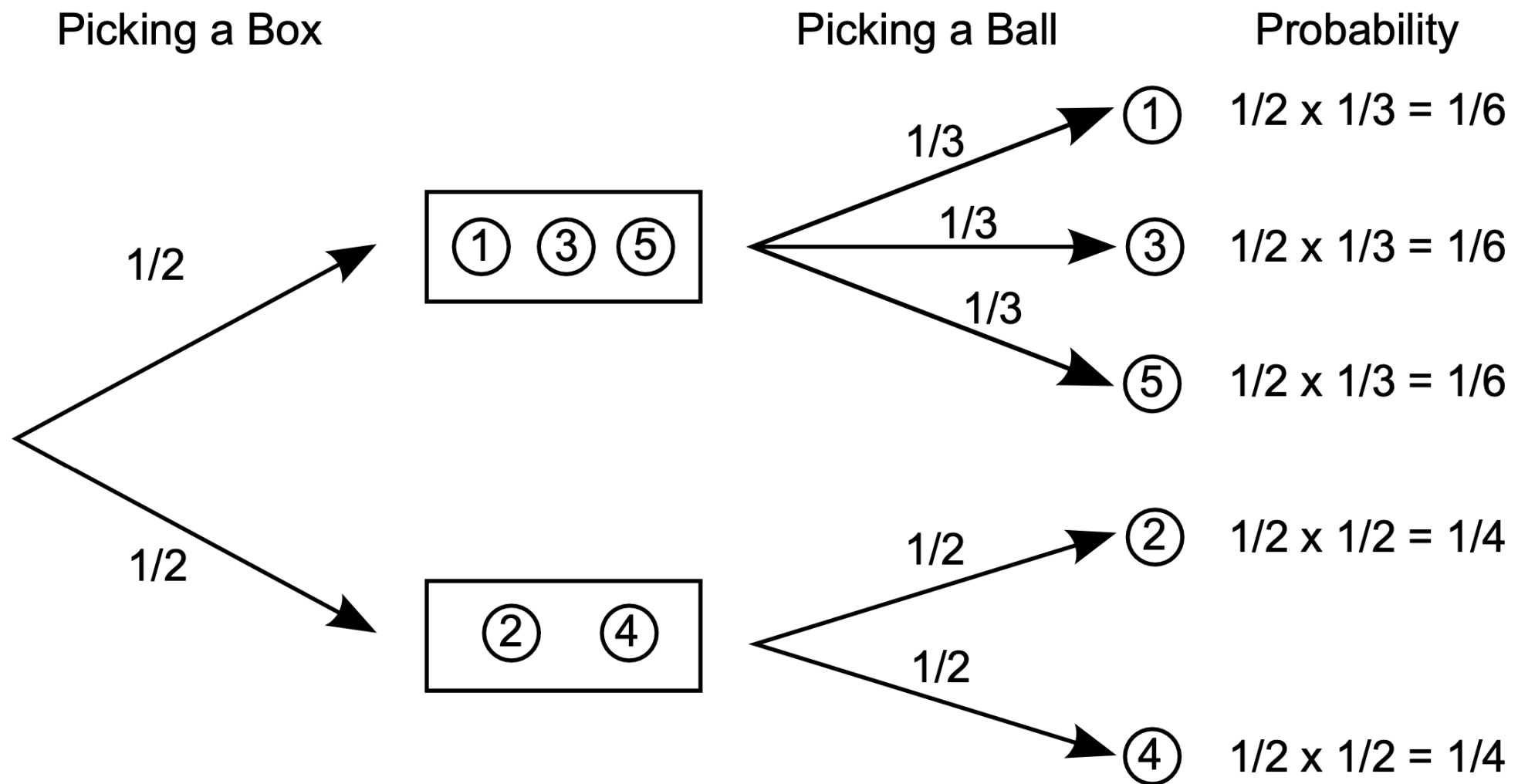
3. Multiply probabilities of each stage to get joint probability:

$$P(A \cap B) = P(A | B)P(B)$$

## Example

You are given two boxes with balls numbered 1-5. One box contains balls 1, 3, 5, and the other contains balls 2 and 4. You first pick a box at random, then pick a ball from that box at random. What is the probability that you pick a 2?

# Tree Diagram Solution



## Example

You are analyzing the effectiveness of online advertising for a company that sells widgets. The company finds that 50% of traffic to their website comes from clicks of online ads. In addition, 20% of visitors to their website both had clicked an online ad and purchased a widget. If a person clicks on the company's ad, what is the probability that they will purchase a widget?

## Example

In Charlottesville the sky is overcast on about 40% of days. If it is overcast, there is a 25% chance that it will also be windy. What is the probability that it is both overcast and windy?