Conditional Probability

Foundations of Data Analysis

February 11, 2021

Brain Teaser

Say I have two children.

1. If I tell you the oldest child is a boy, what is the probability that the youngest is a boy?

2. If I tell you at least one of the children is a boy, what is the probability the other is a boy?

3. If I tell you one of the children is a boy and born on Tuesday, what is the probability the other is a boy?

Conditional Probability

$$P(A \mid B) = \text{"the probability of event } A$$
$$\text{given that we know } B \text{ happened"}$$

Formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule

Rearranging the definition of conditional probability:
$P(A \mid B) = P(A \cap B)/P(B)$

$$P(A \cap B) = P(A \mid B)P(B)$$

Tree Diagrams

Think of conditional probability, $P(A \mid B)$, as two stages:

1. Compute probability of first event B:

$$P(B)$$

2. Compute conditional probability of second event, A, given the first, B:

$$P(A \mid B)$$

3. Multiply probabilities of each stage to get joint probability:

$$P(A \cap B) = P(A \mid B)P(B)$$

Example

You are given two boxes with balls numbered 1 - 5. One box contains balls 1, 3, 5, and the other contains balls 2 and 4. You first pick a box at random, then pick a ball from that box at random. What is the probability that you pick a 2?

Tree Diagram Solution

Picking a Box   Picking a Ball   Probability
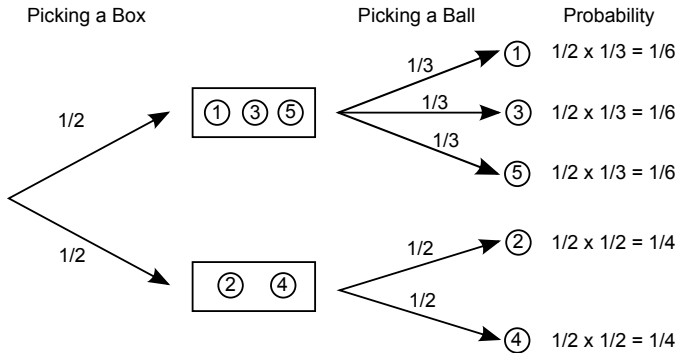
Example

You are analyzing the effectiveness of online advertising for a company that sells widgets. The company finds that 50% of traffic to their website comes from clicks of online ads. In addition, 20% of visitors to their website both had clicked an online ad and purchased a widget. If a person clicks on the company's ad, what is the probability that they will purchase a widget?

Example

In Charlottesville the sky is overcast on about 40% of days. If it is overcast, there is a 25% chance that it will also be windy. What is the probability that it is both overcast and windy?