Solution to "Boy Born On Tuesday" Brain Teaser

Foundations of Data Analysis

Problem: If I have two children, and one is a boy born on a Tuesday, what is the probability that the other child is a boy?

Solution:
We want to phrase this as a conditional probability $P(A|B)$. Remember: A is always the outcome you are seeking (in this case A = "both children are boys"), and B is the information that you are given to be true (in this case B = "one child is a boy born on Tuesday"). So, assuming equally likely outcomes, we have the formula

$$P(A|B) = P(A \cap B)/P(B) = |A \cap B|/|B|$$

$A \cap B$ = "both children are boys, and one was born on Tuesday". Let's enumerate all the possibilities in the set $A \cap B$. If the oldest child is the boy born on Tuesday, then there are 7 options for the second child (boy born on any day of the week). Similarly, if the youngest child is the boy born on Tuesday, then there are 7 options for the oldest child. Both of these scenarios include the possibility that *both* children are boys born on Tuesday, and we have to be careful not to double count this case. So, the size of this set is $|A \cap B| = 7 + 7 - 1 = 13$.

Now, let's compute the denominator $|B|$. Again, we'll split it into two cases of whether the boy on Tuesday is the first or second child. For this case the other child may be a girl or boy and born on any day of the week. This gives us 14 cases for each scenario, but again we must not double count the case where both children are boys born on Tuesday. So, we get $|B| = 14 + 14 - 1 = 27$.

Combining our two results we get $P(A|B) = |A \cap B|/|B| = 13/27 \approx 48\%$.