

Ex 4.2

$$1. \mathcal{L}^{-1} \left\{ \frac{1}{s^3} \right\}$$

$$\text{Sol.) } \mathcal{L}^{-1} \left\{ \frac{1}{s^3} \times \frac{2!}{2!} \right\}$$

$$\boxed{\frac{1}{2} t^2}$$

$$\text{Sol.b) } \mathcal{L}^{-1} \left\{ \frac{1}{s^4} \right\}$$

$$\frac{1}{3!} t^3 = \boxed{\frac{1}{6} t^3}$$

$$\text{Sol.3) } \mathcal{L}^{-1} \left\{ \frac{1}{s^2} - \frac{48}{s^5} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} - \mathcal{L}^{-1} \left\{ \frac{48}{s^5} \right\}$$

$$\cancel{\frac{1}{2}} t - \frac{48}{4!} t^4$$

$$= \boxed{t - 12t^4}$$

$$\text{Sol. 4)} \quad \mathcal{L}^{-1} \left\{ \left(\frac{2}{s} - \frac{1}{s^3} \right)^2 \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{4}{s^2} - \frac{4}{s^4} - \frac{1}{s^6} \right\}$$

$$\mathcal{L}^{-1} \left(\frac{4}{s^2} \right) - \mathcal{L}^{-1} \left(\frac{4}{s^4} \right) - \mathcal{L}^{-1} \left(\frac{1}{s^6} \right)$$

$$\mathcal{L}^{-1} \left\{ \frac{4t - \frac{4t^3}{3!} - \frac{1}{5!} t^5}{s^2} \right\}$$

$$\text{Sol. 5)} \quad \mathcal{L}^{-1} \left\{ \frac{(s^2 + 2s + 1)(s+1)}{s^4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s^3 + 2s^2 + s + s^2 + 2s + 1}{s^4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s^3 + 3s^2 + 3s + 1}{s^4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} + \frac{3}{s^2} + \frac{3}{s^3} + \frac{1}{s^4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + 3 \mathcal{L}^{-1} \left(\frac{1}{s^2} \right) + 3 \mathcal{L}^{-1} \left\{ \frac{2!}{s^3} \right\} + \frac{1}{3!} \left\{ \frac{3!}{s^4} \right\}$$

$$\left[1 + 3t + \frac{3}{2}t^2 + \frac{1}{6}t^3 \right]$$

$$\text{Ans(6)} \quad \mathcal{L}^{-1} \left\{ \frac{(s+2)^3}{s^3} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s^2 + 4s + 4}{s^3} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} + \frac{4}{s^2} + \frac{4}{s^3} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + 4 \mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} + \frac{4}{2!} \mathcal{L}^{-1} \left\{ \frac{2!}{s^3} \right\}$$

$$[1 + 4t + 2t^2]$$

$$\text{Ans(7)} \quad \mathcal{L}^{-1} \left\{ \frac{1}{s^2} - \frac{1}{s} + \frac{1}{s-2} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\}$$

$$[t - 1 + e^{2t}]$$

$$\text{Ans(8)} \quad \mathcal{L}^{-1} \left\{ \frac{4}{s} + \frac{6}{s^5} - \frac{1}{s+8} \right\}$$

$$4 \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + \frac{6}{4!} \mathcal{L}^{-1} \left\{ \frac{4!}{s^5} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s+8} \right\}$$

$$[4 + \frac{1+t^4}{4} - e^{-8t}]$$

$$\text{Ans 9) } \mathcal{L}^{-1} \left\{ \frac{1}{4s+1} \right\}$$

$$\frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s + 1/4} \right\}$$

$$\boxed{\frac{1}{4} e^{-1/4 t}}$$

$$\text{Ans 10) } \mathcal{L}^{-1} \left\{ \frac{1}{5s-2} \right\}$$

$$\frac{1}{5} \mathcal{L}^{-1} \left\{ \frac{1}{s - 2/5} \right\}$$

$$\boxed{\frac{1}{5} e^{2/5 t}}$$

$$\text{Ans 11) } \mathcal{L}^{-1} \left\{ \frac{5}{s^2 + 49} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{5}{s^2 + 49} \cdot \frac{1}{F} \right\}$$

$$\boxed{\frac{5}{F} \sin 7t}$$

$$\text{Ans 12) } \mathcal{L}^{-1} \left\{ \frac{10s}{s^2 + 16} \right\}$$

$$T \underbrace{10 \cos 4t}_{}$$

$$\text{Ans 13) } \mathcal{L}^{-1} \left\{ \frac{4s}{4s^2 + 1} \right\}$$

$$\frac{4}{4} \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 1} \right\}$$

$$\cancel{\mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 1} \right\}} \boxed{\cos \frac{1}{2}t}$$

$$\text{Ans 14) } \mathcal{L}^{-1} \left\{ \frac{1}{4s^2 + 1} \right\}$$

$$\frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 1} \right\}$$

$$\cancel{\frac{1}{4}} \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + 1} \cdot \frac{1}{4} \right\}$$

$$\cancel{\frac{1}{4} \mathcal{L}^{-1}} \boxed{\frac{1}{2} \sin \frac{1}{2}t}$$

$$\text{Ans 15) } \mathcal{L}^{-1} \left\{ \frac{2s-6}{s^2+9} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{2s}{s^2+9} - \frac{6}{s^2+9} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{2s}{s^2+9} \right\} - \mathcal{L}^{-1} \left\{ \frac{6}{s^2+9} \right\}$$

$$2\cos 3t - \frac{6}{3} \sin 3t$$

$$\boxed{2\cos 3t - 2\sin 3t}$$

$$\text{Ans 16) } \mathcal{L}^{-1} \left\{ \frac{s+1}{s^2+2} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s}{s^2+2} \right\} + \mathcal{L}^{-1} \left\{ \frac{1}{s^2+2} \right\}$$

$$\boxed{\cos \sqrt{2}t + \frac{1}{\sqrt{2}} \sin \sqrt{2}t}$$

$$\text{Ans 17) } \mathcal{L}^{-1} \left\{ \frac{1}{s^2+3s} \right\} = \mathcal{L}^{-1} \left\{ \frac{1}{s(s+3)} \right\}$$

$$\frac{1}{s(s+3)} = \frac{A}{s} + \frac{B}{s+3}$$

$$1 = A(s+3) + sB$$

$$\text{let } s = -3$$

$$1 = -3B$$

$$\boxed{B = -1/3}$$

$$\text{let } s = 0$$

$$1 = 3A$$

$$A = 1/2$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{3s} - \frac{1}{3(s+3)} \right\}$$

$$\frac{1}{3} \mathcal{L}^{-1} \left(\frac{1}{s} \right) + \frac{1}{3} \mathcal{L}^{-1} \left(\frac{1}{s+3} \right)$$

$$\boxed{\frac{1}{3} + \frac{1}{3} e^{-3t}}$$

$$\text{Ans 18}) \quad \mathcal{L}^{-1} \left\{ \frac{s+1}{s^2-4s} \right\}$$

$$\frac{s+1}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$s+1 = A(s-4) + Bs$$

$$\text{let } s=4$$

$$\text{let } s=0$$

$$5 = 4B$$

$$\boxed{B = 5/4}$$

$$1 = -4A$$

$$\boxed{A = -1/4}$$

$$\mathcal{L}^{-1} \left\{ -\frac{1}{4s} \right\} + \mathcal{L}^{-1} \left\{ \frac{4}{5} - \frac{1}{s-4} \right\}$$

$$-\frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + \frac{4}{5} \mathcal{L}^{-1} \left\{ \frac{1}{s-4} \right\}$$

$$\boxed{-\frac{1}{4} + \frac{5}{4} e^{4t}}$$

$$\text{Ans} 19) \mathcal{L}^{-1} \left\{ \frac{s}{s^2 + 2s - 3} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{s}{(s-1)(s+3)} \right\}$$

$$s = A + B$$
$$\frac{1}{(s-1)} \quad \frac{1}{(s+3)}$$

$$s = A(s+3) + B(s-1)$$

$$\text{let } s = -3$$

$$\text{let } s = 1$$

$$1 = 4A$$

$$1 = 1/4B$$

$$-3 = -4B$$

$$B = 3/4$$

$$\frac{1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\} + \frac{3}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s+3} \right\}$$

$$\frac{1}{4} e^t + \frac{3}{4} e^{-3t}$$

$$\text{Ans 20) } \mathcal{L}^{-1} \left\{ \frac{1}{s^2 + s - 20} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s-4)(s+5)} \right\}$$

$$1 = \frac{A}{s-4} + \frac{B}{s+5}$$

$$1 = A(s+5) + B(s-4)$$

$$\text{let } s = -5$$

$$\text{let } s = 4$$

$$1 = -9B$$

$$\boxed{B = -1/9}$$

$$\cancel{A = 1} \quad 1 = 9A$$

$$\boxed{A = 1/9}$$

$$\frac{1}{9} \mathcal{L}^{-1} \left\{ \frac{1}{s-4} \right\} - \frac{1}{9} \mathcal{L}^{-1} \left\{ \frac{1}{s+5} \right\}$$

$$\boxed{\frac{1}{9} e^{4t} - \frac{1}{9} e^{-5t}}$$

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$$\text{Ans 20)} \quad f^{-1} \left\{ \frac{0.9s}{(s-0.1)(s+0.2)} \right\}$$

$$\frac{0.9s}{(s-0.1)(s+0.2)} = \frac{A}{s-0.1} + \frac{B}{s+0.2}$$

$$0.9s = A(s+0.2) + B(s-0.1)$$

or

$$\text{let } s=0.1$$

$$\text{let } s=-0.2$$

$$0.09 = 0.3A$$

$$0.18 = -0.3B$$

$$B = \frac{3}{5}$$

$$A = \frac{3}{10}$$

f

$$f^{-1} \left\{ \frac{3}{10(s-0.1)} \right\} + f^{-1} \left\{ \frac{3}{5(s+0.2)} \right\}$$
$$\frac{3}{10} \frac{1}{s-0.1} + \frac{3}{5} \frac{1}{s+0.2}$$

$$\boxed{\frac{3}{10} e^{0.1t} + \frac{3}{5} e^{-0.2t}}$$

$$\text{Ans 22) } \mathcal{L}^{-1}(S-3) \\ ((S-\sqrt{3})(S+\sqrt{3}))$$

$$\mathcal{L}^{-1}\left(\frac{S-3}{S^2-3}\right)$$

$$\mathcal{L}^{-1}\left(\frac{S}{S^2-3} - \frac{3}{S^2-3}\right)$$

$$\mathcal{L}^{-1}\left(\frac{S}{S^2-3} - \sqrt{3}, \frac{\sqrt{3}}{S^2-3}\right)$$

$$\boxed{\cosh \sqrt{3}t = \sqrt{3} \sinh \sqrt{3}t}$$

$$\text{Ans 23) } \mathcal{L}^{-1} \left\{ \frac{S}{(S-2)(S-3)(S+6)} \right\}$$

$$S = A + B + C$$

$$S = A(S-3)(S+6) + B(S-2)(S+6) + C(S-2)(S-3)$$

$$\text{let } B=3$$

$$S=6$$

$$S=2$$

$$3 = -3B$$

$$\boxed{B=-1}$$

$$6 = 12C$$

$$\boxed{C=\frac{1}{2}}$$

$$2 = 8A$$

$$\boxed{A=\frac{1}{4}}$$

$$\frac{1}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s-3} \right\} + \frac{1}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s-6} \right\}$$

$$\boxed{\frac{1}{2} e^{2t} - e^{3t} + \frac{1}{2} e^{6t}}$$

Ans 24) $\mathcal{L}^{-1} \left\{ \frac{s^2+1}{s(s-1)(s+1)(s-2)} \right\}$

$$s^2+1 = \frac{A}{s} + \frac{B}{s-1} + \frac{C}{s+1} + \frac{D}{s-2}$$

$$s^2+1 = A(s-1)(s+1)(s-2) + B(s)(s+1)(s-2) \\ + C(s)(s-1)(s-2) + D(s)(s-1)(s+1)$$

let s=0 let s=1 let s=2 let s=-1

$$1 = 2A$$

$$\boxed{A = \frac{1}{2}}$$

$$2 = -2B$$

$$\boxed{B = -1}$$

$$5 = 6D$$

$$\boxed{D = \frac{5}{6}}$$

$$-6C = 2$$

$$\boxed{C = -\frac{1}{3}}$$

$$\frac{1}{2} \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\} - \frac{1}{3} \mathcal{L}^{-1} \left\{ \frac{1}{s+1} \right\} + \frac{5}{6} \mathcal{L}^{-1} \left\{ \frac{1}{s-2} \right\}$$

$$\boxed{\frac{1}{2} - e^t - \frac{1}{3} e^{-t} + \frac{5}{6} e^{2t}}$$

$$\text{Ans 25) } \mathcal{F}^{-1} \left\{ \frac{1}{s(s^2+5)} \right\}$$

$$I = \frac{A}{s} + \frac{Bs+C}{s^2+5}$$

$$I = A(s^2+5) + (Bs+C)(s)$$

$$\text{let } s=0$$

$$\text{let } s=1$$

$$I = 5A$$

$$\boxed{A = 1/5}$$

$$I = 6A + B + C$$

$$5 = 6 + 5B - 5C$$

$$\boxed{\cancel{6s} - 1 = B + C}$$

$$\text{let } s = -1$$

$$\cancel{6s} \quad I = 6A + B - C$$

$$1 - \frac{6}{5} = B - C$$

$$\frac{-1}{5} = B - C$$

$$\boxed{B = -1/5}$$

$$B - C = B + C$$

$$\boxed{C=0}$$

$$\frac{1}{5} \mathcal{F}^{-1} \left\{ \frac{1}{s} \right\} - \frac{1}{5} \mathcal{F}^{-1} \left\{ \frac{5}{s^2+5} \right\} = \frac{1}{5} - \frac{1}{5} \cos 5t$$

$$= \boxed{\frac{1}{5} - \frac{1}{5} \cos 5t}$$

$$= \boxed{\frac{1}{5} - \frac{\sqrt{5}}{25} \cos 5t}$$

$$\text{Ans 26) } \mathcal{L}^{-1} \left\{ \frac{s}{(s+2)(s^2+4)} \right\}$$

$$S = \frac{A}{s+2} + \frac{Bs+C}{s^2+4}$$

$$S = A(s^2+4) + (Bs+C)(s+2)$$

$$\text{let } S = -2 \quad S = 0$$

$$-2 = 8A$$

$$\boxed{A = -1/4}$$

$$0 = -1 + 2C$$

$$\boxed{C = 1/2}$$

$$\text{let } S = 1$$

$$1 = -5 + 3B + 3C$$

$$\frac{9-3C}{4} = 3B$$

$$3B = \frac{9}{4} - \frac{3}{2} ; \quad 3B = \frac{9-6}{4} = \frac{1}{2}$$

$$\boxed{B = 1/6}$$

$$\frac{-1}{4} \mathcal{L}^{-1} \left\{ \frac{1}{s+2} \right\} + \mathcal{L}^{-1} \left\{ \frac{1/4s + 1/2}{s^2+4} \right\}$$

$$\boxed{\frac{-1}{4} e^{-2t} + \frac{1}{4} \cos 2t + \frac{1}{4} \sin 2t}$$

$$\text{Ans 2)} \quad \mathcal{L}^{-1} \left\{ \frac{2s-4}{(s^2+s)(s^2+1)} \right\}$$

$$2\mathcal{L}^{-1} \left\{ \frac{s-2}{s(s+1)(s^2+1)} \right\}$$

$$s-2 = \frac{A}{s} + \frac{B}{s+1} + \frac{Cs+D}{s^2+1}$$

$$s-2 = A(s+1)(s^2+1) + B(s)(s^2+1) + (Cs+D)(s)(s+1)$$

$$\text{let } s = -1$$

$$\text{let } s = 0$$

$$\text{let } s = 1$$

$$-3 = -2B$$

$$-2 = A$$

$$-1 = -8 + 3 + 2C + 2D$$

$$B = \frac{3}{2}$$

$$4 = 2C + 2D$$

$$C = 2 - D$$

$$\text{let } s = 2$$

$$0 = 15A + 10B + 12C + 6D$$

$$-30 + 15 + 24 - 12D + 6D = 0$$

$$9 = 6D$$

$$D = \frac{3}{2}$$

$$C = \frac{1}{2}$$

$$-2\mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + 3\frac{1}{2}\mathcal{L}^{-1} \left\{ \frac{1}{s+1} \right\} + \frac{1}{2}\mathcal{L}^{-1} \left\{ s + 3 \right\} \frac{1}{s^2+1}$$

$$\boxed{-2 + 3e^{-t} + \frac{1}{2} \cos \sqrt{3}t + \frac{3}{2} \sin t}$$

$$3\sqrt{3} - T_7 = (-39 + 21\sqrt{3})B \rightarrow 6\sqrt{3}C$$

$-6 + 3\sqrt{3}$

$$\text{Ans 28) } \mathcal{L}^{-1} \left\{ \frac{1}{s^4 - 9} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s^2-3)(s^2+3)} \right\}$$

$$1 = \frac{A}{s^2-3} + \frac{B}{s^2+3}$$

$$1 = A(s^2+3) + B(s^2-3)$$

$$\text{let } s = \sqrt{3} \quad | \text{ let } s = 1$$

$$1 = 6A$$

$$1 = 4A + B(-2)$$

$$\boxed{A = 1/6}$$

$$1 = \frac{2}{3} - 6B$$

$$1 = -6B$$

$$\boxed{B = -1/6}$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{6\sqrt{3}s} \right\} = \frac{1}{6\sqrt{3}} \mathcal{L}^{-1} \left\{ \frac{1}{s^2-3} \right\}$$

$$\boxed{\frac{\sinh \sqrt{3}t}{6\sqrt{3}} \rightarrow \frac{\sin \sqrt{3}t}{6\sqrt{3}}}$$

$$\text{Ans 29) } f^{-1}\left\{ \frac{1}{(s^2+1)(s^2+4)} \right\}$$

$$1 = \frac{A}{s^2+1} + \frac{B}{s^2+4}$$

$$1 = A(s^2+4) + B(s^2+1)$$

$$\text{let } s=1$$

$$\text{let } s=0$$

$$1 = 5A + 2B$$

$$1 = 4A + B$$

$$\begin{cases} 1 - 2B = A \\ 5 \end{cases}$$

$$5 = 4 - 8B + 5B$$

$$1 = -3B$$

$$B = -\frac{1}{3}$$

$$A = \frac{1}{3}$$

$$\frac{1}{3} f^{-1}\left\{ \frac{1}{s^2+1} \right\} + -\frac{1}{3} f^{-1}\left\{ \frac{1}{s^2+4} \right\}$$

~~$$\frac{1}{3\sqrt{3}} f^{-1}\left\{ \frac{\sqrt{3}}{s^2+1} \right\} - \frac{1}{3} f^{-1}\left\{ \frac{2}{s^2+4} \right\}$$~~

$$\frac{\sinh \sqrt{3}t}{3\sqrt{3}} = \frac{\sinh 2t}{6}$$

$$\frac{1}{3} \sinh t - \frac{1}{6} \sin 2t$$

$$\text{Ans 30) } \mathcal{L}^{-1} \left\{ \frac{6s+3}{(s^2+1)(s^2+4)} \right\}$$

$$6s+3 = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

$$6s+3 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$$\text{let } s=0$$

$$\text{let } s=1$$

$$9 = 5A + 5B \rightarrow 2C + 2D$$

$$3 = 4B + D$$

$$9 = 5A + 5B + 2C + 6 - 8B$$

$$\boxed{D = 3 - 4B}$$

$$\cancel{9 = 5A + 3B}$$

$$\boxed{3 = 5A - 3B + 2C}$$

$$\text{let } s = -1$$

$$-3 = -5A + 5B - 2C + 2D$$

$$-3 = -5A + 5B - 2C + 6 - 8B$$

$$-9 = -5A - 3B - 2C$$

$$9 = 5A + 3B + 2C$$

$$\boxed{\frac{9 - 5A - 3B}{2} = C}$$

$$3 = 5A - 3B + (9 - 5A - 3B)$$

$$-6 = -6B$$

$$\boxed{B = 1}$$

$$\boxed{D = -1}$$

let $s=2$

$$15 = 16A + 8B + 10C + 5D$$

$$15 = 16A + 8 + 10C - 5$$

$$12 = 16A + \cancel{18}(\cancel{9+10C-5})$$

$$12 = 16A + 45 - 25A - 15B$$

$$12 = -9A + 30$$

$$-18 = -9A \quad C = 9 - 10 - 3 = -2$$

$$\boxed{A=2}$$

$$\boxed{C=-2}$$

$$\mathcal{L}^{-1} \left\{ \frac{2s+1}{s^2+1} = \frac{2s+1}{s^2+4} \right\}$$

$$\mathcal{L}^{-1} \left\{ \frac{2s}{s^2+1} + \frac{1}{s^2+1} - \frac{2s}{s^2+4} - \frac{1}{s^2+4} \right\}$$

~~2cost + sint~~

$$2\text{cost} + \text{sint} - 2\text{cos}2t - \frac{1}{2}\text{sin}2t$$

Ans 31) $\frac{dy}{dt} - y = 1 ; y(0) = 0$

$$\mathcal{L}\{y'\} - \mathcal{L}(y) = 1$$

$$sY(s) + Y(0) - Y(s) = \frac{1}{s}$$

where $Y(s) = \mathcal{L}\{y(t)\}$

$$(s-1)(Y(s)) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s(s-1)}$$

$$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s(s-1)}\right\}$$

$$\frac{1}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1}$$

$$1 = A(s-1) + Bs$$

$$\text{let } s=0 \quad \text{let } s=1$$

$$\frac{1 = -A}{1A = -1}$$

$$\boxed{1 = B}$$

$$y(t) = \mathcal{L}^{-1}\left\{-1\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}$$

$$y(t) = -1 + e^t$$

Ans 32) $2 \frac{dy}{dt} + y = 0, y(0) = -3$

$$2\mathcal{L}\{y'\} + \mathcal{L}\{y\} = 0$$

$$2\{sy(s) + y(0)\} + Y(s) = 0$$

$$2sy(s) - 6 + Y(s) = 0$$

$$(2s+1)(Y(s)) = 6$$

$$\mathcal{L}\{y(t)\} = \frac{6}{2s+1}$$

$$y(t) = 6\mathcal{L}^{-1}\left\{\frac{1}{2s+1}\right\}$$

$$y(t) = 3\mathcal{L}^{-1}\left\{\frac{1}{s+0.5}\right\}$$

$$y(t) = 3e^{-0.5t}$$

$$\text{Ans33) } \mathcal{D}\{y'\} + 6\mathcal{D}\{y\} = \mathcal{D}\{e^{4t}\}, \quad y(0) = 2$$

$$sY(s) - y(0) + 6Y(s) = \frac{1}{s-4}$$

$$(s+6)(Y(s)) - 2 = \frac{1}{s-4}$$

$$Y(s) = \frac{1}{(s-4)(s+6)} + \frac{2}{(s+6)}$$

$$Y(s) = \frac{1+2s-8}{(s-4)(s+6)}$$

$$2s-7 = \frac{A}{s-4} + \frac{B}{s+6}$$

$$2s-7 = A(s+6) + B(s-4)$$

$$\text{let } s=4$$

$$\text{let } s=-6$$

$$1 = 10A$$

$$-19 = -10B$$

$$A = \frac{1}{10}$$

$$B = \frac{19}{10}$$

$$y(t) = \frac{1}{10} \mathcal{D}^{-1} \left\{ \frac{1}{s-4} \right\} + \frac{19}{10} \mathcal{D}^{-1} \left\{ \frac{1}{s+6} \right\}$$

$$y(t) = \frac{1}{10} e^{4t} + \frac{19}{10} e^{-6t}$$

$$\text{Ans 34) } y' - y = 2 \cos 5t, \quad y(0) = 0$$

$$\mathcal{L}\{y'\} - \mathcal{L}\{y\} = 2 \mathcal{L}\{\cos 5t\}$$

$$sY(s) - y(0) - Y(s) = 2 \left\{ \frac{s^2}{s^2 + 25} \right\}$$

$$(s-1)(Y(s)) = 2s$$

$$(s^2 + 25)$$

$$Y(s) = \frac{2s}{(s-1)(s^2 + 25)}$$

$$2s = \frac{A}{s-1} + \frac{Bs + C}{s^2 + 25}$$

$$2s = A(s^2 + 25) + (Bs + C)(s-1)$$

$$\text{let } s=1 \quad \text{let } s=0$$

$$2 = 26A$$

$$25A - C = 0$$

$$A = \frac{1}{13}$$

$$\frac{25}{13} = C$$

$$\text{let } s = -1$$

$$-2 = 26A + 2B - 2C$$

$$-1 = 2 + 2B - \frac{25}{13}$$

$$13$$

$$B = \frac{-1}{13}$$

$$B = -\frac{1}{13}$$



$$Y(s) = \frac{1}{13} \cdot \frac{1}{s-1} + \frac{24s+25}{13(s^2+25)}$$

$$y(t) = \frac{1}{13} e^t + \frac{1}{13} \mathcal{L}^{-1} \left\{ \frac{24s+25}{s^2+25} \right\}$$

$$y(t) = \frac{1}{13} e^t + \frac{1}{13} \mathcal{L}^{-1} \left\{ \frac{24s+25}{s^2+25} \right\}$$

$$Y(s) = \frac{1}{13} \cdot \frac{1}{s-1} + \frac{-s+25}{13} \cdot \frac{1}{s^2+25}$$

$$y(t) = \frac{1}{13} \mathcal{L}^{-1} \left\{ \frac{1}{s-1} \right\} + \frac{1}{13} \mathcal{L}^{-1} \left\{ \frac{-s+25}{s^2+25} \right\} + 5 \cdot \frac{5}{s^2+25}$$

$$y(t) = \frac{1}{13} e^t - \frac{1}{13} \cos 5t + \frac{5}{13} \sin 5t$$

$$\text{Ans 35) } y'' + 5y' + 4y = 0 ; \quad y(0) = 1, y'(0) = 0$$

$$[s^2 Y(s) + s Y(0) - y'(0)] + 5[s Y(s) - y(0)] + 4Y(s) = 0$$

$$s^2 Y(s) - s - 0 + 5s Y(s) + 5 + 4Y(s) = 0$$

$$Y(s) [s^2 + 5s + 4] = s + 5$$

$$Y(s) = \frac{s+5}{s^2 + 5s + 4}$$

$$Y(s) = \frac{s+5}{(s+1)(s+4)}$$

$$\frac{s+5}{(s+1)(s+4)} = \frac{A}{s+1} + \frac{B}{s+4}$$

$$s+5 = A(s+4) + B(s+1)$$

$$\text{let } s = -1 \quad \text{let } s = -4$$

$$4 = 3A$$

$$1 = -3B$$

$$A = \frac{4}{3}$$

$$\frac{4}{3}$$

$$B = -\frac{1}{3}$$

$$B = -\frac{1}{3}$$

$$-2 \int e^{-t} \left\{ \frac{1}{s+3} \right\} + 3 \int e^{-t} \left\{ \frac{1}{s+4} \right\}$$

$$-2e^{-t} + 3e^{-4t}$$

S

~~char & S~~

~~while (C >= 88 & C <= 96)~~

~~{ String S;~~

24+

~~S += C;~~

Q

$$A = \frac{4}{3}$$

$$B = -\frac{1}{3}$$

$$y(t) = \frac{4}{3} S^{-1} \left\{ \frac{1}{S+1} \right\} - \frac{1}{3} S \left\{ \frac{1}{S+1} \right\}$$

$$y(t) = \frac{4}{3} e^{-t} - \frac{1}{3} e^{-4t}$$

$$\text{Ans 36) } y'' - 4y' = 6e^{3t} - 3e^{-t} \quad y(0)=1, y'(0)=-1$$

$$\{s^2Y(s) - sY(0) - y'(0)\} - 4\{sY(s) - y(0)\} = 6\{s^2e^{3t}\} - 3\{e^{-t}\}$$

$$s^2Y(s) - s + 1 - 4sY(s) + 4 = 6\left\{\frac{1}{s-3}\right\} - 3\left\{\frac{1}{s+1}\right\}$$

$$Y(s)[s^2 - 4s] - s + 5 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$Y(s) = \frac{6(s+1) - 3(s-3) + s - 5}{(s-3)(s+1)(s^2 - 4s)}$$

$$Y(s) = \frac{6(s+1) - 3(s-3) + (s-5)(s-3)(s+1)}{(s-3)(s+1)(s^2 - 4s)}$$

$$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = \frac{A}{s-3} + \frac{B}{s+1} + \frac{Cs - D}{s^2 - 4s}$$

$$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = A(s+1)(s^2 - 4s) + B(s-3)(s^2 - 4s) + (Cs + D)(s-3)(s+1)$$

$$\text{let } s=3 \quad \text{let } s=-1$$

$$24 = -12A$$

$$TA = -2$$

$$12 = -20B$$

$$B = \frac{3}{5}$$

$$\text{let } s=5$$

$$30 = -60 - 6 + 60C + 120$$

$$96 = 60C + 120$$

$$8 = 5C + D$$

$$D = 8 - 5C$$

let $s=1$

$$34 = 12 - \frac{18}{5} - 4C - 4D$$

$$34 = 8 \cdot 4 - 4(C+D)$$

$$\underline{-84} \quad C + 8 - 5C$$

$$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = A + \frac{B}{s-3} + \frac{C}{s+1} + \frac{D}{s-4}$$

$$6(s+1) - 3(s-3) + (s-5)(s-3)(s+1) = A(s-3)(s+1)(s-4) + B(s)(s+1)(s-4) \\ + C(s)(s-3)(s-4) \\ + D(s)(s-3)(s+1)$$

let $s=0$

$$s=3$$

let $s=4$

$$6+9+5 = 20A$$

$$24 = -12B$$

$$\boxed{A = \frac{5}{2}}$$

$$\boxed{B = -2}$$

$$22 = 20D$$

$$\boxed{D = \frac{11}{10}}$$

let $s=-1$

$$\cancel{34} - 12 = -20C$$

$$\boxed{C = -\frac{3}{5}}$$

$$\frac{5}{2} \delta^{-1} \left\{ \frac{1}{s} \right\} - 2 \delta^{-1} \left\{ \frac{1}{s-3} \right\} - \frac{3}{5} \delta^{-1} \left\{ \frac{1}{s+1} \right\} + \frac{11}{10} \left\{ \frac{1}{s-4} \right\}$$

$$\boxed{\frac{5}{2} e^{3t} - \frac{3}{5} e^{-t} + \frac{11}{10} e^{4t}}$$

Ans 3#) $y'' + y = \sqrt{2} \sin \sqrt{2} t$ $y(0) = 10, y'(0) = 0$

$$s^2 Y(s) - s y(0) - y'(0) + Y(s) = \sqrt{2} \left\{ \frac{\sqrt{2}}{s^2 + 2} \right\}$$

$$Y(s) [s^2 - 10s + 10] - 0 = \frac{2}{s^2 + 2}$$

$$\cancel{Y(s)} = \frac{2}{s^2 + 2}$$

$$Y(s) [s^2 + 1] - 10s - 0 = \frac{2}{s^2 + 2}$$

$$Y(s) [s^2 + 1] = \frac{2}{s^2 + 2} + 10s$$

$$Y(s) [s^2 + 1] = \frac{2}{(s^2 + 2)(s^2 + 1)} + \frac{10s}{s^2 + 1}$$

$$Y(s) = \frac{2 + 10s(s^2 + 2)}{(s^2 + 2)(s^2 + 1)}$$

$$Y(s) = \frac{2 + 10s}{(s^2+2)(s^2+1)}$$

$$2 = \frac{A}{s^2+2} + \frac{B}{s^2+1}$$

$$2 = A(s^2+1) + B(s^2+2)$$

$$\text{let } s=0 \quad \text{let } s=1$$

$$2 = A + 2B$$

$$2 = 2A + 3B$$

$$A = 2 - 2B$$

$$2 = 4 - 4B + 3B$$

$$-2 = -B$$

$$A = -2$$

$$B = 2$$

$$Y(s) = \frac{-2}{s^2+2} + \frac{2}{s^2+1} + \frac{10s}{s^2+1}$$

$$y(t) = -\mathcal{F}^{-1}\left\{\frac{2}{s^2+2}\right\} + 2\mathcal{F}^{-1}\left\{\frac{1}{s^2+1}\right\} + 10\mathcal{F}^{-1}\left\{\frac{s}{s^2+1}\right\}$$

$$y(t) = -2 \frac{\sin \sqrt{2}t}{\sqrt{2}} + 2 \sin t + 10 \cos t$$

$$\text{Ans 38) } y'' + 9y = e^t ; y(0) = 0, y'(0) = 0$$

$$s^2 Y(s) - sY(0) - y'(0) + 9Y(s) = \mathcal{L}\{e^t\}$$

$$Y(s)[s^2 + 9] - 0 - 0 = 1$$

$$s-1$$

$$Y(s) = \frac{1}{(s-1)(s^2+9)}$$

$$1 = \frac{A}{s-1} + \frac{Bs+C}{s^2+9}$$

$$1 = A(s^2 + 9) + (Bs + C)(s - 1)$$

$$\text{let } s = 1$$

$$\text{let } s = 0$$

$$\text{let } s = -1$$

$$1 = 10A$$

$$1 = \frac{9}{10} - C$$

$$1 = \frac{10}{10} + 2B - 2C$$

$$A = \boxed{\frac{1}{10}}$$

$$\boxed{\frac{1-1}{10} = C}$$

$$0 = 2B + \boxed{\frac{1}{5}}$$

$$\boxed{B = -\frac{1}{10}}$$

$$Y(s) = \frac{1}{10} \left\{ \frac{1}{s-1} \right\} - \frac{1}{10} \left\{ \frac{1}{s^2+9} \right\} - \frac{1}{10} \left\{ \frac{s+2}{s^2+9} \right\}$$

$$Y(t) = \frac{1}{10} e^t - \frac{1}{30} \sin 3t - \frac{1}{10} \cos 3t$$

$$\text{Ans 39) } 2y''' + 3y'' - 3y' - 2y = e^{-t}, \quad y(0)=0, \quad y'(0)=0, \quad y''(0)=1$$

$$2[s^3Y(s) - s^2y(0) - sy'(0) - y''(0)] + 3[s^2Y(s) - sy(0) - y'(0)] - 3[sY(s) - y(0)] - 2Y(s) = \frac{1}{s+1}$$

$$2[s^3Y(s) - 1] + 3[s^2Y(s)] - 3sY(s) - 2Y(s) = \frac{1}{s+1}$$

$$2s^3Y(s) - 2 + 3s^2Y(s) - 3sY(s) - 2Y(s) = \frac{1}{s+1}$$

$$Y(s)[2s^3 + 3s^2 - 3s - 2] - 2 = \frac{1}{s+1}$$

$$Y(s)[(s+2)(s-1)(s+\frac{1}{2})] = \frac{1}{s+1} + 2$$

$$Y(s) = \frac{1+2s+2}{(s+1)(s+2)(s-1)(s+\frac{1}{2})}$$

$$Y(s) = \frac{3+2s}{(s+1)(s+2)(s-1)(s+\frac{1}{2})}$$

$$3+2s = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{s-1} + \frac{D}{2s+\frac{1}{2}}$$

$$3+2s = A(s+2)(s-1)(s+0.5) + B(s+1)(s-1)(s+\frac{1}{2}) + C(s+1)(s+2)(s+0.5) + D(s+1)(s+2)(s-1)$$

$$\text{let } s = -1$$

$$1 = A$$

$$\text{let } s = -2$$

$$-1 = -\frac{9}{2}B$$

$$B = -\frac{1}{18}$$

$$3+2s = \frac{A}{(s+1)} + \frac{B}{(s+2)} + \frac{C}{(s-1)} + \frac{D}{(2s+1)}$$

$$3+2s = A(s+2)(s-1)(2s+1) + B(s+1)(s-1)(2s+1) \\ + C(s+1)(s+2)(2s+1) + D(s+1)(s+2)(s-1)$$

$$\text{let } s = -1 \quad \text{let } s = -2 \quad \text{let } s = 1 \quad \text{let } s = -\frac{1}{2}$$

$$1 = 2A \quad -1 = -4B \quad 5 = 18C \quad \frac{1}{8} = -\frac{9}{8}D$$

$$A = \frac{1}{2} \quad B = \frac{1}{9} \quad C = \frac{5}{18} \quad D = -\frac{8}{9}$$

$$y(t) = \frac{1}{2} e^{-t} \left\{ \frac{1}{s+1} \right\} + \frac{1}{9} e^{-2t} \left\{ \frac{1}{s+2} \right\} + \frac{5}{18} e^{t} \left\{ \frac{1}{s-1} \right\} - \frac{8}{9} e^{-\frac{1}{2}t} \left\{ \frac{1}{2s+1} \right\}$$

$$y(t) = \frac{1}{2} e^{-t} + \frac{1}{9} e^{-2t} + \frac{5}{18} e^t - \frac{8}{9} e^{-\frac{1}{2}t}$$