Space-Time Codes For Wireless Communication: Code Construction

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ABSTRACT

We consider the design of channel codes for improving the data rate and/or the reliability of communications over fading channels using multiple transmit antennas. Here, data is encoded by a channel code and the encoded data is split into n streams that are simultaneously transmitted using n transmit antennas. The received signal at each receive antenna is a linear superposition of the n transmitted signals. We review the performance criteria for designing such codes under the assumption that the fading is slow and frequency non-selective established in [3]. Performance is determined by *diversity gain* quantified by ranks and *coding gain* quantified by determinants of certain matrices that are constructed from the code sequences.

The performance criterion is then used to design trellis codes for high data rate wireless communication. These codes are easy to encode and decode. They provide the best trade-off between data rate, diversity gain, constellation size and trellis complexity. Simulation results are provided for 4 and 8 PSK signal sets with data rates of 2 and 3 bits/symbol, demonstrating excellent performance that is within 2-3 dB of the outage capacity for these channels.

1. INTRODUCTION.

In recent years, there has been an explosion of interest in providing high speed wireless data services. Current cellular standards such as IS-136 provide approximately 28.8 kb/sec using 30 kHz channels. In order to provide internet access, video conferencing, simultaneous voice and data or similar services, it is desirable to have data rates in the range of 64-128 kb/sec. One approach to higher data rates over 30 kHz channels is to expand the signal constellation. However, this results in a significant SNR penalty.

In this work we consider the use of multiple transmit antennas to provide diversity and/or higher data rate. In particular, we consider the problem of coding in conjunction with multiple transmit antennas.

We assume that the transmitter is equipped with n antennas and the mobile is equipped with m antennas. At any time interval n signals are transmitted each from a different transmit antenna. Signals arriving at different receive antennas undergo independent fades. The signal at each receive antenna is a superposition of the faded version of the n transmitted signals.

Assuming that the transmitted signal from the i-th antenna at transmission time t is c_t^i, and the received signal at transmission time t at antenna j at the receiver is d_t^j, we have

$$d_t^j = \sum_{i=1}^{n} \alpha_i^j c_t^i + \eta_t^j. \tag{1}$$

The coefficients α_i^j are modeled as independent samples of a stationary complex Gaussian stochastic process with mean zero and variance 0.5 per dimension. Also, η_t^j are independent samples of a zero mean complex white Gaussian process with two sided power spectral density $N_0/2$ per dimension.

In [3], we considered the probability that the receiver decides erroneously in favor of a signal

$$\mathbf{e} = e_1^1 e_1^2 \cdots e_1^n e_2^1 e_2^2 \cdots e_2^n \cdots e_l^1 e_l^2 \cdots e_l^n$$

assuming that

$$\mathbf{c} = c_1^1 c_1^2 \cdots c_1^n c_2^1 c_2^2 \cdots c_2^n \cdots c_l^1 c_l^2 \cdots c_l^n$$

was transmitted. The analysis performed in [3] led us to the following design criteria.

Design Criteria For Rayleigh Space-Time Codes:

- **The Rank Criterion:** In order to achieve the maximum diversity mn, the matrix

$$B(\mathbf{c},\mathbf{e}) = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & \cdots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \ddots & \vdots & e_l^3 - c_l^3 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & \cdots & e_l^n - c_l^n \end{pmatrix}$$

  has to be full rank for any codewords $\mathbf{c}$ and $\mathbf{e}$. If $B(\mathbf{c},\mathbf{e})$ has minimum rank r over the set of two tuples of distinct codewords, then a diversity of rm is achieved.

- **The Determinant Criterion:** Suppose that a diversity benefit of rm is our target. The minimum of r-th roots of the sum of determinants of all $r \times r$ principal cofactors of $A(\mathbf{c},\mathbf{e}) = B(\mathbf{c},\mathbf{e})B^*(\mathbf{c},\mathbf{e})$ taken over all pairs of distinct codewords $\mathbf{e}$ and $\mathbf{c}$ corresponds to the coding gain, where r is the rank of $A(\mathbf{c},\mathbf{e})$. Special attention in the design must be paid to this quantity for any codewords $\mathbf{e}$ and $\mathbf{c}$. The design target is making this sum as large as possible. If a diversity of nm is the design target, then the minimum of the determinant of $A(\mathbf{c},\mathbf{e})$ taken over all pairs of distinct codewords $\mathbf{e}$ and $\mathbf{c}$ must be maximized.

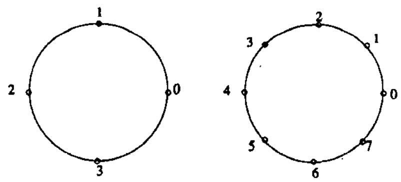

Figure 1: 4-PSK And 8-PSK Constellations

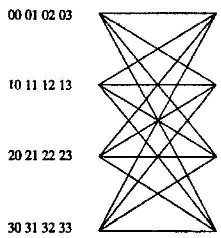

Figure 2: 2-Space-Time Code, 4-PSK, 4-States, 2 bits/sec/Hz

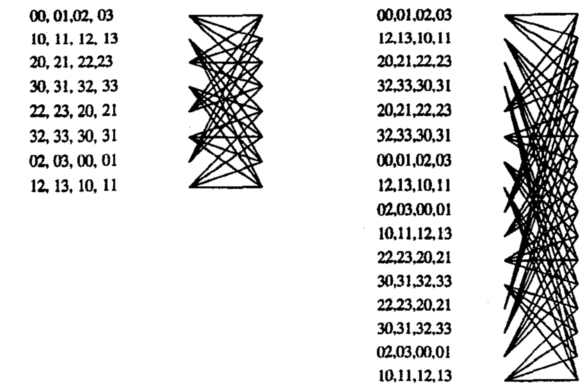

Figure 3: Space-Time Codes, 4-PSK, 8 and 16-States, 2 bits/sec/Hz

It was proved in [3, 5] that the above criteria remains valid in the presence of channel estimation error for codes designed using signal constellations with equal energy elements as well as in multi-ray propagation environments. Furthermore, if a code is designed using the above criteria, it's performance can only be improved in Rician environments [3]. In this work, we will design codes using the above criteria. For full details, we refer the reader to [3].

2. CODE DESIGN

In this section, we use the criteria derived in the previous section to design codes for a wireless communication system that employs n transmit antennas and (optional) receive antenna diversity. These codes are *Space-Time* codes as they combine spatial and temporal diversity techniques. We begin with a simple example.

Example 1: Here the signal constellation is 4-PSK, where the signal points are labeled by $0, 1, 2,$ and 3 (See Figure 1). We consider the 4-states trellis code shown in Figure 2. The edge label $x_1 x_2$ indicates that signal x_1 is transmitted over the first antenna and that signal x_2 is transmitted over the second antenna. It was proved in [3] that the above code provides diversity gain 2 (assuming one receive antenna), and has minimum determinant 2. For fixed rate and diversity gain, we can increase coding gain by increasing the number of trellis states.

Example 2: We provide 4-PSK trellis codes with 8, 16 and 32 states in Figures 3 and 4. The minimum determinants of these codes are respectively 12, 20 and 28.

The **design rules** that guarantee the diversity in Figures 6 and 7 are:

- **Design Rule 1:** Transitions departing from the same state differ in the second symbol.
- **Design Rule 2:** Transitions arriving at the same state differ in the first symbol.

In Figure 5 and 6, we provide simulation results for the performance of these codes with 2 transmit and 1 and 2 receive antennas where each frame corresponds to 130 transitions in the trellis diagram. In [3], we compared the performance of these codes to the the outage capacity given in [1]. It was observed that, at the frame error rate of 0.10, the codes perform within 2.5 dB of the outage capacity. In any case, simulation results prove that the codes we constructed, perform very well.

In fact, using the design criterion established in this paper, we have designed space-time trellis codes with number of states less or equal to 64 for 8-PSK, and 16-QAM constellations. We include the 16-state 16-QAM code as well (Figure 10), but for brevity, we avoided including the rest of these codes. We provide simulation results for the 8-PSK case demonstrating that the performance of 8-PSK space-time codes for two and one receive antenna is also very close to the outage capacity. A similar statement is true for the 16-QAM space-time codes. The Design rules 1 and 2, again guarantee the diversity in the construction of all these codes.

Again, we observed in [3], that these codes perform within 2-3 dB of the outage capacity.

An important problem arising in the design of space-time codes is the trade-off between diversity gain and transmission rate. In [3], we examined this problem. There, we established the trade-off between diversity gain, data rate, constellation size and trellis complexity. It was proved that the space-time codes designed here provide the best possible data rate, diversity gain and constellation size. Achievable lower bounds on trellis complexity, as a function of diversity gain and data rate were also given in [3]. It is possible to design space-time codes for use with more than 2 transmit antennas. At present time, these may not be practical because of the cost of transmitter at the base station. To demonstrate the theoretical effectiveness of the techniques developed here, we construct a space-time code for a 4 transmit antenna mobile communication system. The limit on transmission rate is 2 bits/sec/Hz [3]. Thus the trellis complexity of the code is bounded below by 64. The input to the encoder is a block of length 2 of bits a_1, b_1 corresponding to an integer $i = 2a_1 + b_1$. The 64 states of the trellis correspond to set of all three tuples (s_1, s_2, s_3) with $0 \leq s_j \leq 3$ for $j = 1, 2, 3$. At state (s_1, s_2, s_3) upon input data i, the encoder outputs (i, s_1, s_2, s_3) elements of 4-PSK constella-

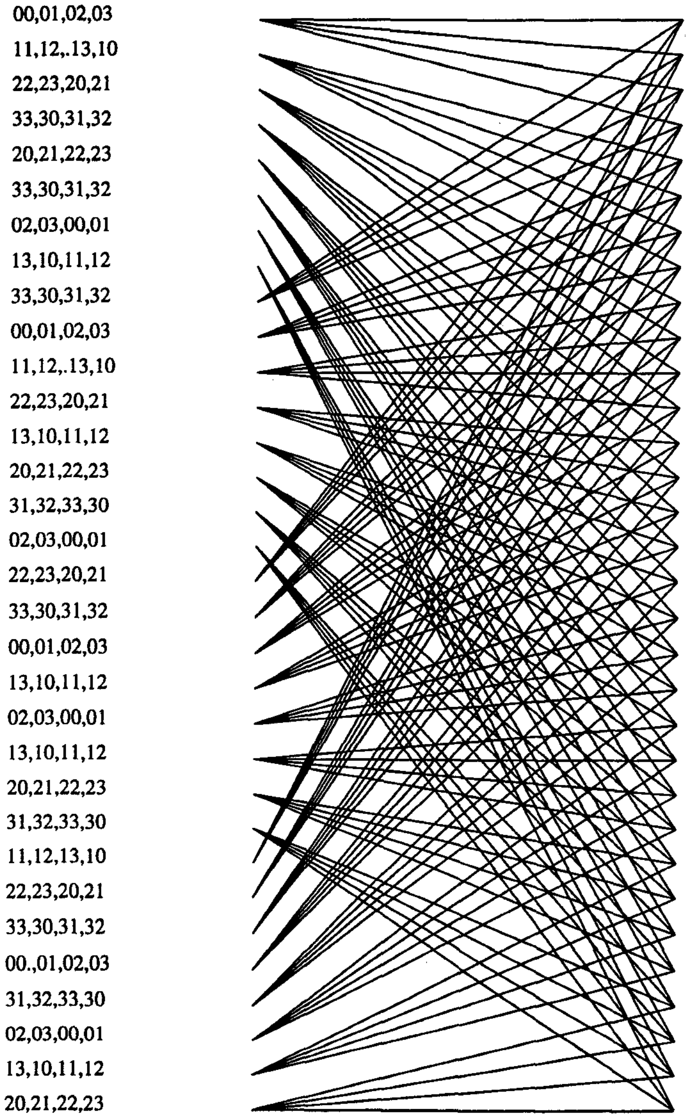

Figure 4: Space-Time Codes, 4-PSK, 32-States, 2 bits/sec/ Hz

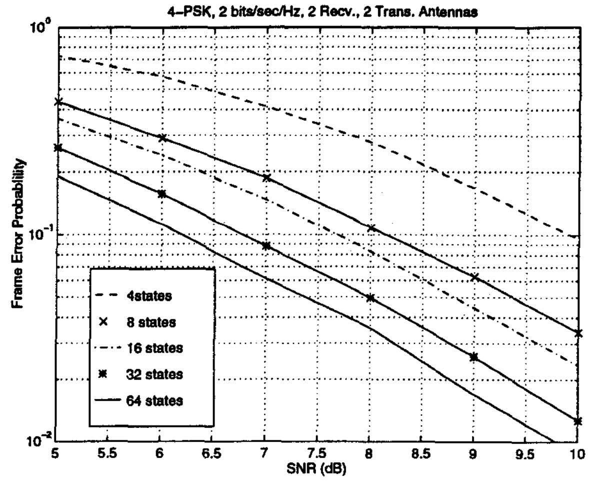

Figure 5: 4-PSK Codes Achieving Diversity 4, 2 Received and 2 Transmit Antennas,2 bits/sec/Hz

tion (see Figure 1) and moves to state (i, s_1, s_2). It is proved in [3], that given one receive antenna a diversity gain of 4 using this code in conjunction with 4 transmit antennas can be achieved.

3. DELAY DIVERSITY CODES

The delay diversity scheme of [2] are a subset of the family of space-time codes. In [2] a rate $1/2$ (repetition) block code is used in conjunction with two transmit antennas. At each time instance t a block of bits are mapped into a codeword $\tilde{c}_t^1\tilde{c}_t^2$. The symbols $\tilde{c}_{t-1}^1$ and $\tilde{c}_t^2$ are respectively sent from antennas one and two. This can be viewed as a space-time code by defining

$$c_t^1 = \tilde{c}_{t-1}^1$$
$$c_t^2 = \tilde{c}_t^2,$$

where c_t^1 and c_t^2 are the symbols of the equivalent space-time code at time t.

Using this notation, the performance analysis carried out in this paper applies to the delay diversity scheme of [2]. Suppose that the 8-PSK signal constellation is used and the encoder maps a sequence of three bits $a_k b_k c_k$ at time k to ii with $i = 4a_k + 2b_k + a_k$. This code is given in Figure 11 and has minimum determinant $(2-\sqrt{2})^2$.

Next, we consider the block code

$$C = \{00, 15, 22, 37, 44, 51, 66, 73\}$$

of length 2 defined over the alphabet 8-PSK. This block code is the best in the sense of product distance [2] amongst all the codes of cardinality 8 and of length 2 defined over the alphabet 8-PSK. This means that the minimum of the product distance $|c_1 - e_1||c_2 - e_2|$ between pairs of distinct codewords $\mathbf{c} = c_1c_2 \in C$ and $\mathbf{e} = e_1e_2 \in C$ is maximum amongst all such codes. The delay diversity code in this case is identical to the space-time code given by trellis diagram of Figure 12. The minimum determinant of this delay diversity code is thus 2.

The 16-state code for the 16-QAM constellation given in Figure 10, is obtained from the block code

$$\{0\,0, 1\,11, 2\,2, 3\,9, 4\,4, 5\,15, 6\,6, 7\,13, 8\,8,$$
$$9\,3, 10\,10, 11\,1, 12\,12, 13\,7, 14\,14, 15\,5\}$$

using the same delay diversity construction. Again, this block code is the best in the sense of product distance.

The delay diversity code construction can also be generalized to systems having more transmit antennas than 2. For instance, the 4-PSK 4-space-time code given before is a delay diversity code. The corresponding block code is the repetition code. By applying the delay diversity construction to the 4-PSK block code

$$\{0\,0\,0\,0, 1\,2\,3\,1, 2\,1\,2\,3, 3\,3\,1\,2\}$$

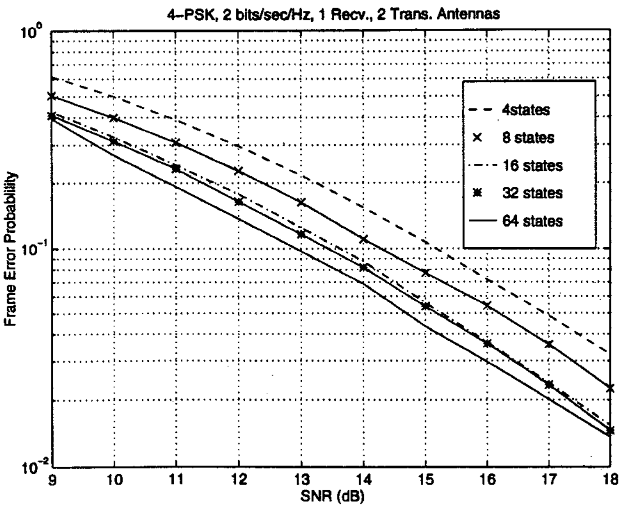

Figure 6: 4-PSK Codes Achieving Diversity 2, 1 Received and 2 Transmit Antennas,2 bits/sec/Hz

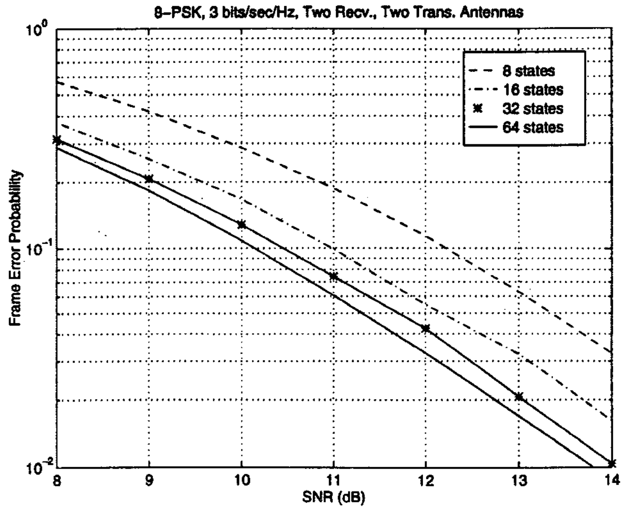

Figure 7: 8-PSK Codes Achieving Diversity 4, 2 Receive and 2 Transmit Antennas, 3 bits/sec/Hz

one can obtain a more powerful 4-PSK 4-space-time code having the same trellis complexity.

It is an interesting open problem whether it is possible to construct good space-time codes of a given complexity using coding in conjunction with delay diversity.

4. MULTI-LEVEL CODING

When the number of transmit antennas is significantly greater than 2, multi-level coding is a convenient way to construct space-time codes of low complexity [4]. Although, this topic is studied extensively in [4], we will briefly provide an explicit construction of a delay diversity multi-level code here. Delay diversity multi-level coding to multi-level space-time coding is like delay diversity codes to space-time codes.

Consider a scheme using $n = 3$ transmit antennas and 8-PSK constellation. Suppose that a data rate of 5 bits/sec/Hz is desired. We construct a multi-level scheme that has this data rate and provides diversity gain 2. If trellis space-time coding is employed, at least $2^5 = 32$ states is required with 32 transitions leaving each state of the trellis. Instead, we employ a multi-level code with multi-stage decoding.

At each time t the input to the encoder is 5 bits of information $b_t^1 b_t^2 b_t^3 b_t^4 b_t^5$. The input sequence b_t^5 is encoded using a repetition code of rate $1/3$ giving the out put sequence $b_t^5 b_t^5 b_t^5$. The pair of bits $b_t^1 b_t^2$ and $b_t^3 b_t^4$ are encoded using a parity check code of rate $2/3$ yielding sequences $b_t^1 b_t^2 b_t^6$ and $b_t^3 b_t^4 b_t^7$. Let

$$\begin{aligned} c_t^1 &= 4b_t^1 + 2b_t^3 + b_t^5, \\ c_t^2 &= 4b_{t-1}^2 + 2b_{t-1}^4 + b_{t-1}^5, \\ c_t^3 &= 4b_{t-2}^6 + 2b_{t-2}^7 + b_{t-2}^5, \end{aligned}$$

be elements of the 8-PSK constellation using the labeling given in Figure 3. The transmitted signal from antenna $1 \le i \le 3$ at time t is c_t^i.

At the decoder multi-stage decoding is performed. At first a decision on b_t^5 is made. A trellis diagram for b_t^5 has only 4 states where the states depend on b_{t-1}^5 and b_{t-2}^5. In such a trellis diagram each branch has 15 parallel branches. There are 32 branches leaving each state. It is easy to use the criterion developed in this paper and observe that a diversity gain of 3 on deciding the bits $b_1^5, b_2^5, \cdots, b_l^5$ is guaranteed.

Assuming that $b_1^5, b_2^5, \cdots, b_l^5$ are determined, the multistage decoder performs decoding to determine $b_t^3 b_t^4$. Here, the states at time t are given by the triplet $(b_{t-1}^3, b_{t-1}^4, b_{t-2}^7)$, thus there are 8 states in the trellis diagram. There are 4 parallel transitions between any two connected states. The criteria for diversity can be used to observe that assuming correct decisions in the first stage of decoding, a diversity gain of two is achieved in the second stage.

In the third stage, the multi-level decoder determines $b_t^1 b_t^2$ using a trellis. The states at time t are given by the triplet

$$(b_{t-1}^1, b_{t-1}^2, b_{t-2}^6),$$

thus there are 8 states in the trellis diagram. There are no parallel transitions between any two connected states. Assuming correct decisions in the first and second stage of decoding, a diversity gain of two is achieved in the third stage.

The total number of branches visited in decoding this multi-level scheme is almost half as much as the one given by the trellis space-time code having 32 states. Thus it is natural to expect that multi-level coding is the way to produce powerful space-time codes for various high bit rate applications if the number of antennas at the base station is high [4]. This increases the base-station cost. For this reason, at present time multi-level space-time coding is merely of theoretical interest. Nevertheless, we have thoroughly studied this subject in [4].

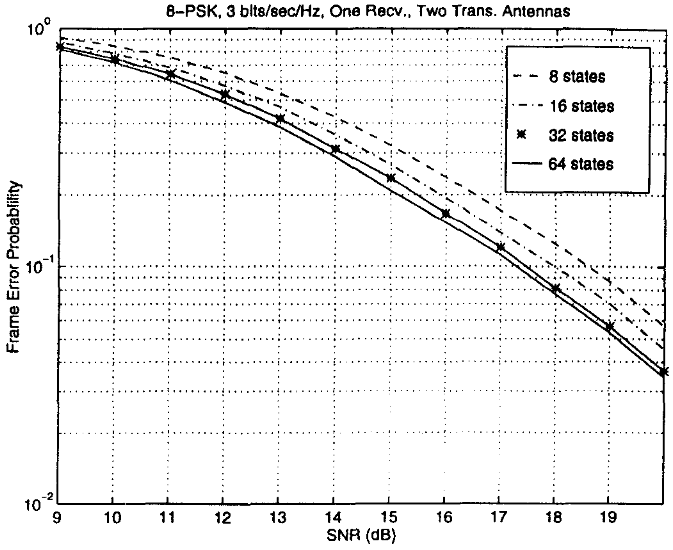

Figure 8: 8-PSK Codes Achieving Diversity 2,
1 Receive and 2 Transmit Antennas,
3 bits/sec/Hz

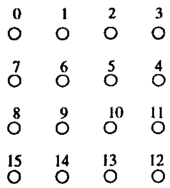

Figure 9: The QAM Constellation

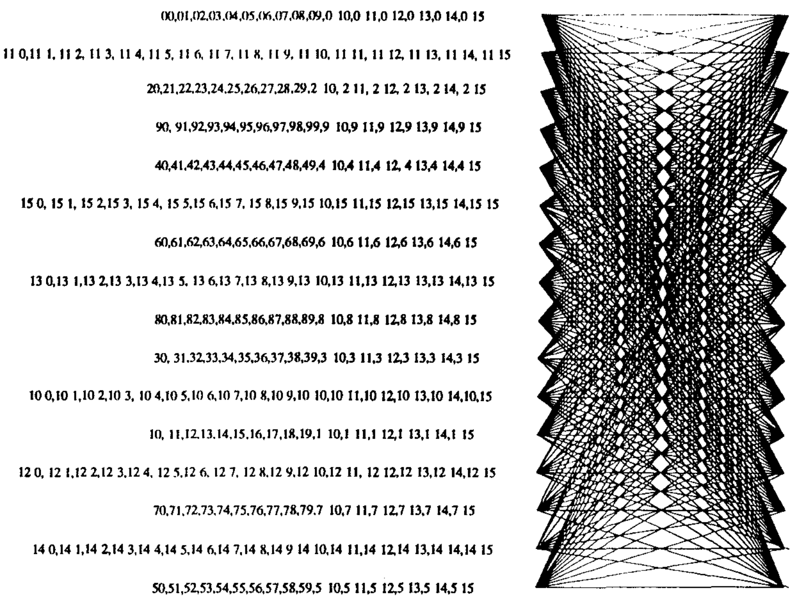

Figure 10: 2-Space-Time 16 QAM Code,
16 states, 4 bits/sec/Hz

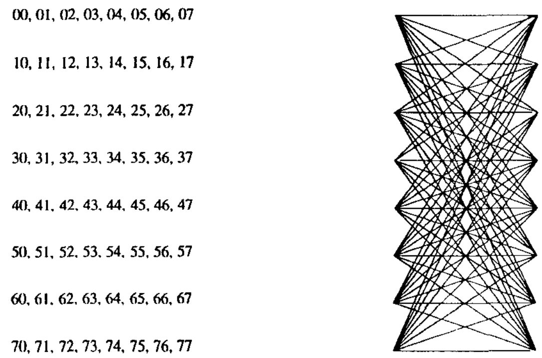

Figure 11: Space-time realization of a delay diversity 8-PSK code constructed from a repetition code.

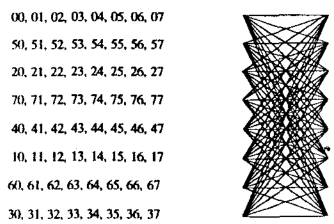

Figure 12: 2-Space-Time Code, 8-PSK, 8-States, 3 bits/sec/Hz

5. REFERENCES

[1] G.J. Foschini, Jr. and M.J. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Communication*, submitted for publication.

[2] N. Seshadri and J.H. Winters, "Two Signaling Schemes for Improving the Error Performance of Frequency-Division-Duplex (FDD) Transmission Systems Using Transmitter Antenna Diversity", *International Journal of Wireless Information Networks,* Vol. 1, No. 1, 1994.

[3] V. Tarokh, N. Seshadri and A.R. Calderbank, "Space-Time Codes For High Data Rate Wireless Communication: Performance Criterion and Code Construction," submitted to *IEEE Trans. Info. Theory.*

[4] V. Tarokh, N. Seshadri and A.R. Calderbank, "Multi-level Space-Time Codes For High Data Rate Wireless Communication," in preparation.

[5] V. Tarokh, A. Naguib, N. Seshadri and A.R. Calderbank, "Practical Consideration For An implementation of Space-Time Codes," submitted to *IEEE trans. on Communications.*