Design of 16-QAM Space-time Trellis Codes for Quasi-Static Fading Channels

Andrew Wong, Jinhong Yuan, Jinho Choi
School of Electrical Engineering and Telecommunications
University of New South Wales
UNSW 2052, Australia
Email: {j.yuan, j.choi}@unsw.edu.au

Seong Rag Kim, In-Keong Choi, Dong-Seung Kwon
Mobile Communications Research Lab.
Electronics Telecommunications Research Institute
South Korea
Email: {srkim, ikchoi, dskwon}@etri.re.kr

***Abstract*— We consider the design of 16-QAM space-time trellis codes (STTCs) for quasi-static Rayleigh flat fading channels in this paper. Based on the Σ_o-rank criterion and the trace criterion, new 16-state and 64-state 16-QAM STTCs were constructed. The performance of the codes were evaluated by simulations. It is shown that the new 16-state code outperforms the previous hand-designed codes by about 1 dB.**

I. Introduction

Space-time trellis codes (STTCs) have been shown to significantly reduce the effect of fading and improve the error performance of wireless transmissions [1]. The design of STTCs have been investigated in [6], [7], [8], [9]. Up to now, most of the codes in the literature use BPSK, QPSK or 8-PSK modulation [4], [5]. To improve the spectral efficiency for the future high data rate transmissions, it is highly desirable to construct STTCs with a higher order constellation, such as 16-QAM. Usually, the design of STTCs involves the use of computer search. The search space increases exponentially with the constellation size, the number of transmit antennas and the number of states in the code trellis. In this paper, we consider the design of 16-QAM STTCs for quasi-static Rayleigh fading channels. In order to make code design more tractable, we separate the code design into two phases. In the first phase of code design, we represent the STTCs in a complex ring, which can be translated into the 16-QAM constellation with a translation mapping. We use the Σ_o*-rank criterion* [3] to construct 16-QAM STTCs with full transmit diversity. In the second phase of the design, we choose the code with the maximum coding gain among the set of codes with full transmit diversity based on the trace criterion [4], [5]. New 16-state and 64-state 16-QAM STTCs are constructed. It is shown that the new 16-state code outperforms the previous hand-designed codes by about 1 dB.

II. System Model

We consider a multiple-input multiple-output (MIMO) system with n_T transmit antennas and n_R receive antennas. At each time instant t, the space-time symbol, $\mathbf{x}_t = (x_t^1\ x_t^2\ \cdots\ x_t^{n_T})^T$, is transmitted. In each frame, a sequence of L space-time symbols form a space-time codeword matrix, denoted by an n_T by L matrix $\mathbf{X}$, where L is the frame size. The transmitted symbols undergo quasi-static fading, in which the fading coefficients remain constant over the entire frame but vary independently between frames.

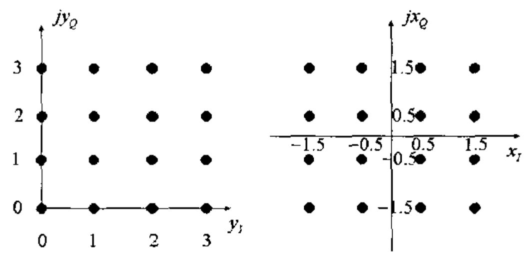

Fig. 1. Translational mapping from the complex ring $\mathbb{Z}_4(j)$ (on the left) to the 16-QAM constellation (on the right).

At the receiver, the signals are decoded with maximum likelihood sequence estimation and it may decide erroneously for the symbol $\hat{\mathbf{x}}_t = (\hat{x}_t^1\ \hat{x}_t^2\ \cdots\ \hat{x}_t^{n_T})^T$ in favour of $\mathbf{x}_t$. Suppose that the decided sequence is $\hat{\mathbf{X}}$, then the codeword difference matrix, $\mathbf{B} = \mathbf{X} - \hat{\mathbf{X}}$, and the codeword distance matrix, $\mathbf{A} = \mathbf{B} \cdot \mathbf{B}^H$, are formed [1], where H denotes the Hermitian, i.e. the conjugate transpose of a matrix. Let r denote the rank of $\mathbf{A}$ and assume that $n_R = 2$. We intend to design codes with a full transmit diversity for $n_T = 2$. In this case, we have $rn_R \geq 4$. The *trace criterion*, which is to maximize the minimum trace of $\mathbf{A}$ among all pairs of distinct codewords [2], should be used as the the design criterion.

III. Codes Representation and Σ_o-Rank Criterion

The Σ_o-rank criterion was proposed in [3] for the design of space-time codes (STCs) in quasi-static Rayleigh fading channels with 2^{2^k}-QAM constellations, such as QPSK, 16-QAM and 64-QAM etc. The main idea of the criterion is to linearize the full diversity code design problem by providing sufficient conditions to define the code generator matrices over finite rings.

To use the Σ_o-rank criterion for 16-QAM STTCs, the STTCs are represented in a finite complex-integer ring $\mathbb{Z}_4(j)$, where each element $z \in \mathbb{Z}_4(j)$ has the form $z = z_I \oplus_4 jz_Q$ and $z_I, z_Q \in \mathbb{Z}_4 = \{0,1,2,3\}$. A linear $\mathbb{Z}_4(j)$ code has symbols over the complex ring $\mathbb{Z}_4(j)$. With a translational mapping, each element can result in a symbol in the 16-QAM constellation, as shown in Figure 1.

The Σ_o-rank criterion allows the determination of the full rank of a 16-QAM STC to be carried out in the $\mathbb{Z}_4(j)$ domain rather than in the domain of the complex number field, $\mathbb{C}$. Therefore the computation can become a lot more efficient and the code design can be significantly simplified.

Let $\mathbf{G}_i$ be the generator matrix for the ith transmit antenna with outputs in $\mathbb{Z}_4(j)$ and $\mathbf{i}$ be the information sequence with elements in $\mathbb{Z}_4$. A Σ_o-coefficient set is a set of coefficients $\{\alpha_1, \cdots, \alpha_{n_T}\}$ over $\mathbb{Z}_4(j)$ such that there exists at least one coefficient with the sum of its real part and its imaginary part being odd. The Σ_o-rank criterion says that if the linear combinations of the generator matrices over all possible non-equivalent Σ_o-coefficient sets satisfy

$$\mathbf{i}\Big(\bigoplus_{i=1}^{n_T} \alpha_i \mathbf{G}_i\Big) \neq 0 \tag{1}$$

over all possible information sequences, then the code achieves full transmit diversity [3].

For a 16-state 16-QAM STTC with 2 transmit antennas, the generator matrices have the special structure

$$\begin{aligned} \mathbf{G}_i &= \begin{pmatrix} g_i^1(x) \\ g_i^2(x) \end{pmatrix} \\ &= \begin{pmatrix} a_{i,I}^1 \oplus_4 b_{i,I}^1 x \\ a_{i,I}^2 \oplus_4 b_{i,I}^2 x \end{pmatrix} \oplus_4 j \begin{pmatrix} a_{i,Q}^1 \oplus_4 b_{i,Q}^1 x \\ a_{i,Q}^2 \oplus_4 b_{i,Q}^2 x \end{pmatrix} \end{aligned} \tag{2}$$

for $i = 1, 2$, where the real part and the imaginary part of each element in the matrix is a generator polynomial of degree 1. The inputs are represented by the input vector $\mathbf{i} = (\mathrm{i}_1(x) \;\; \mathrm{i}_2(x))$, where each element of the vector is a polynomial of degree L', and L' is the length of the information sequence. For both the generator polynomials and the information sequence polynomials, the symbol x is used to denote the dummy variable of a polynomial and their coefficients are elements over $\mathbb{Z}_4$.

Based on the Σ_o-rank criterion, the problem of determining full transmit diversity can be further simplified as follows. Let $\mathbf{G}$ be a linear combination of the generator matrices

$$\begin{aligned} \mathbf{G} &= \alpha_1 \mathbf{G}_1 \oplus_4 \alpha_2 \mathbf{G}_2 \\ &= \begin{pmatrix} g_I^1(x) \\ g_I^2(x) \end{pmatrix} \oplus_4 j \begin{pmatrix} g_Q^1(x) \\ g_Q^2(x) \end{pmatrix}. \end{aligned} \tag{3}$$

If the polynomial $g_I^1(x) g_Q^2(x) \ominus_4 g_Q^1(x) g_I^2(x)$ has at least one odd coefficient for all non-equivalent Σ_o-coefficient sets $\{\alpha_1, \alpha_2\}$, then the code achieves full transmit diversity [3].

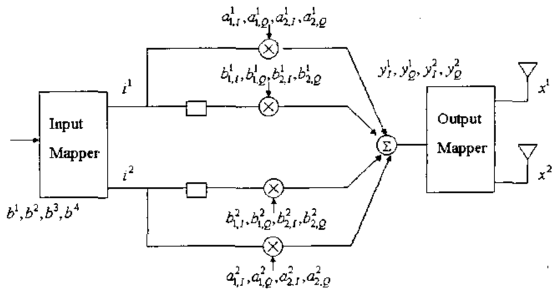

Fig. 2. Encoder structure for 16-state 16-QAM STTCs with 2 transmit antennas.

Method	Complexity
Direct computation of codeword difference matrices	$O(16^{2L})$
Direct application of the Σ_o-rank criterion	$O(16^L)$
Σ_o-rank criterion with special generator matrices	$O(1)$

TABLE I

COMPUTATIONAL COSTS FOR THE FULL TRANSMIT DIVERSITY DETERMINATION PROBLEM USING VARIOUS METHODS.

IV. CODE DESIGN

In this section, we design 16-QAM STTCs with 16 and 64 states for 2 transmit antennas. We begin with the 16-state code. The code can be represented by the generator matrices as shown in Equation (2). The block diagram of the encoder is depicted in Figure 2.

In this representation, the input mapper converts the 4 serial information bits $\mathbf{b}_t = (b_t^1 \;\; b_t^2 \;\; b_t^3 \;\; b_t^4)$ into 2 parallel information symbols $\mathbf{i}_t = (i_t^1 \;\; i_t^2)$ through the natural mapping. The generator coefficients $a_{i,I}^n, a_{i,Q}^n, b_{i,I}^n, b_{i,Q}^n$, $i = 1, 2$, $n = 1, 2$, are elements from $\mathbb{Z}_4$, where the subscript i denotes the antenna number and the superscript n denotes the input branch number. The outputs $\mathbf{y}_t = (y_{t,I}^1 \;\; y_{t,Q}^1 \;\; y_{t,I}^2 \;\; y_{t,Q}^2)$ are produced by the encoder based on the generator coefficients, the information symbols and the state values, such that

$$\begin{aligned} y_{t,I}^i &= \bigoplus_{n=1}^{2} a_{i,I}^n i_t^n \oplus_4 b_{i,I}^n i_{t-1}^n, \\ y_{t,Q}^i &= \bigoplus_{n=1}^{2} a_{i,Q}^n i_t^n \oplus_4 b_{i,Q}^n i_{t-1}^n. \end{aligned} \tag{4}$$

The output mapper takes the encoder outputs and coverts them into $\mathbf{x}_t = (x_t^1 \;\; x_t^2)$, where $x_t^i = y_{t,I}^i + j y_{t,Q}^i$ for $i = 1, 2$. After the translational mapping of x_t to elements in the 16-QAM constellation, the signals are transmitted by the 2 transmit antennas.

$\lvert\Omega\rvert$	$\lvert\Omega_m\rvert$	R_{FD}	d^2_{min}
4 294 967 296	2 147 385 345	18.4579%	4.8

TABLE II

RESULTS OF CODE SEARCH FOR THE 16-STATE 16-QAM STTC.

Code	r	d^2_{min}	relative coding gain (dB)
TSC	2	1.6	0
LFT	2	3.2	0.1
New16	2	4.8	0.94

TABLE III

COMPARISON OF NEW 16-STATE CODE WITH PUBLISHED RESULTS.

In the first phase of the code design, the Σ_o-rank criterion is used to find the set of all codes that achieve full transmit diversity. Table I shows the computational complexity of solving the full transmit diversity problem using three different methods for a frame size of L. It can be seen that the conventional method of computing the rank of every distinct pair of codeword difference matrices has a worst complexity which is an exponential of $2L$. The direct application of the Σ_o-rank criterion as shown in Equation (1) improves the complexity by exploiting the linearity of the codes. Finally, the use of the Σ_o-rank criterion as shown in Equation (3) produces a dramatic improvement in the complexity because it does not require checking all possible information sequences.

In the second phase of the code design, the trace criterion is used to maximize the minimum squared Euclidean distance between all pairs of distinct codewords within the set of full transmit diversity codes, which were found in the first phase of design. Based on the Σ_o-rank criterion and the trace criterion, a code that is optimal in terms of a full transmit diversity and a maximum coding gain is found through systematic computer search. The result is shown in Table II. The entire code search space is denoted by Ω. After exploiting symmetries and observed singularities, the reduced code search space is denoted by Ω_m. It is found that about $R_{\text{FD}} = 18.5\%$ of the codes within the search space, Ω_m, achieve full transmit diversity. For $n_T = 2$, having a full transmit diversity is equivalent to having a full rank of $r = 2$ among all possible codeword difference matrices. Among them, the largest minimum squared Euclidean distance is 12, which corresponds to a normalized value of $d^2_{\text{min}} = 4.8$.

For the 64-state code design, a new code that achieves full transmit diversity is found. It has a minimum squared Euclidean distance of 14, which corresponds to a normalized value of $d^2_{\text{min}} = 5.6$. Note that no exhaustive search has been conducted for the 64-state code.

V. SIMULATION RESULTS

We compare a new 16-state code found with two published results, the Tarokh/Seshadri/Calderbank (TSC) 16-QAM STTC [1], and the Liu/Fitz/Takeshita (LFT) 16-QAM STTC [3]. The new 16-state code is denoted as New16.

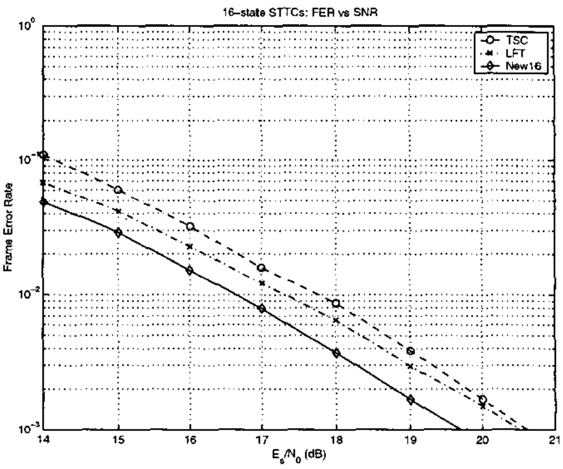

Fig. 3. Simulation results for 16-state STTCs.

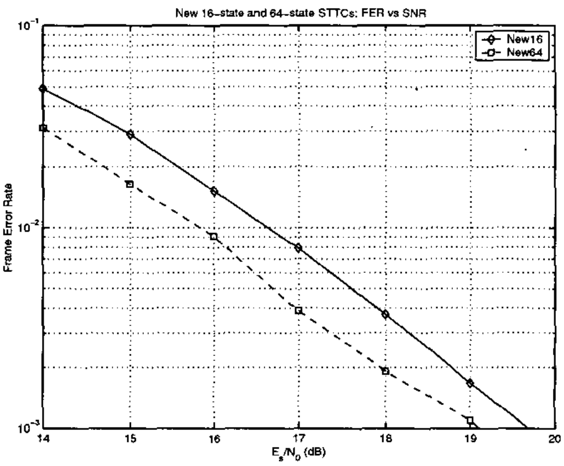

Fig. 4. Simulation results for the new 16-state and 64-state STTCs.

The generator matrices of the three codes are shown in the Appendix. All three codes achieve full transmit diversity, but the new code has the largest minimum squared Euclidean distance among the three as shown in Table III.

The performance of the three codes is evaluated by simulations. We consider two transmit and two receive antennas and the channel being modelled as a quasi-static Rayleigh fading channel. The frame length is $L = 66$. The frame error rate (FER) at various signal-to-noise ratios for the three codes are shown in Figure 3. It is found that the new code is about 1 dB better than the previous published results at the FER of 10^{-3}.

We also compare the performance between the new 16-state code and the new 64-state code. The new 64-state code is denoted by New64. Its generator matrices are shown in the Appendix. The results are depicted in Figure 4. It is found that the 64-state code outperforms the 16-state code by about 0.6 dB.

VI. CONCLUSION

In this paper, we design 16-QAM STTCs for quasi-static Rayleigh flat fading channels by using the Σ_o-rank criterion and the trace criterion. New 16-state and 64-state codes have been constructed and the performance of the codes is evaluated by simulations. It is shown that the new codes have a better error performance than the codes in [1] and [3].

APPENDIX

Generator matrices of the TSC code:

$$\mathbf{G}_1 = \begin{pmatrix} 0 \oplus_4 3x \\ 0 \oplus_4 2x \end{pmatrix} \oplus_4 j \begin{pmatrix} 0 \oplus_4 2x \\ 0 \oplus_4 3x \end{pmatrix}$$

$$\mathbf{G}_2 = \begin{pmatrix} 1 \oplus_4 0x \\ 0 \oplus_4 0x \end{pmatrix} \oplus_4 j \begin{pmatrix} 0 \oplus_4 0x \\ 1 \oplus_4 0x \end{pmatrix}$$

Generator matrices of the LFT code:

$$\mathbf{G}_1 = \begin{pmatrix} 3 \oplus_4 2x \\ 1 \oplus_4 0x \end{pmatrix} \oplus_4 j \begin{pmatrix} 3 \oplus_4 2x \\ 0 \oplus_4 2x \end{pmatrix}$$

$$\mathbf{G}_2 = \begin{pmatrix} 2 \oplus_4 3x \\ 0 \oplus_4 0x \end{pmatrix} \oplus_4 j \begin{pmatrix} 0 \oplus_4 3x \\ 2 \oplus_4 1x \end{pmatrix}$$

Generator matrices of the New16 code:

$$\mathbf{G}_1 = \begin{pmatrix} 3 \oplus_4 2x \\ 1 \oplus_4 0x \end{pmatrix} \oplus_4 j \begin{pmatrix} 0 \oplus_4 1x \\ 1 \oplus_4 0x \end{pmatrix}$$

$$\mathbf{G}_2 = \begin{pmatrix} 0 \oplus_4 1x \\ 2 \oplus_4 2x \end{pmatrix} \oplus_4 j \begin{pmatrix} 2 \oplus_4 1x \\ 1 \oplus_4 1x \end{pmatrix}$$

Generator matrices of the New64 code:

$$\mathbf{G}_1 = \begin{pmatrix} 2 \oplus_4 2x \\ 0 \oplus_4 0x \oplus_4 3x^2 \end{pmatrix} \oplus_4 j \begin{pmatrix} 1 \oplus_4 2x \\ 2 \oplus_4 3x \oplus_4 0x^2 \end{pmatrix}$$

$$\mathbf{G}_2 = \begin{pmatrix} 3 \oplus_4 0x \\ 0 \oplus_4 2x \oplus_4 2x^2 \end{pmatrix} \oplus_4 j \begin{pmatrix} 3 \oplus_4 3x \\ 1 \oplus_4 3x \oplus_4 0x^2 \end{pmatrix}$$

REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744-765, Mar. 1998.

[2] Z. Chen, J. Yuan, and B. Vucetic, "Improved space-time trellis coded modulation scheme on slow Rayleigh fading channels," *Electron. Lett.*, vol. 37, no 7, pp. 440-441, Mar. 2001

[3] Y. Liu, M. Fitz, O. Takeshita, "A Rank Criterion for QAM Space-Time Codes," *IEEE Trans. Inform. Theory*, vol. 48, no. 12 pp. 3062-3079, Dec. 2002.

[4] B. Vucetic, and J. Yuan, *SPACE-TIME CODING*, John Wiley & Sons, April 2003.

[5] J.Yuan, Z. Chen, B. S. Vucetic, and W. Firmanto, "Performance and design of space-time coding in fading channels," *IEEE Transactions on Communications*, vol. 51, no. 12, pp. 1991-1996, Dec. 2003.

[6] J. Grimm, M. P. Fitz, and J. V. Krogmeier, "Further results in space-time coding for Rayleigh fading", 36th Allerton conference on Communications, Control and Computing Proceedings, Sep. 1998.

[7] A. R. Hammons and H. E. Gammal, "On the theory of space-time codes for PSK modulation", IEEE Trans. on. Inform. Theory, vol. 46, no. 2, Mar. 2000, pp. 524-542.

[8] S. Baro, G. Bauch and A. Hansmann, "Improved codes for space-time trellis coded modulation", IEEE Commun. Lett., Vol. 4, No. 1, Jan. 2000.

[9] R. S. Blum, "New analytical tools for designing space-time convolutional codes," submitted to IEEE Trans. on Inform. Theory.