Space–Time TCM with Improved Performance on Fast Fading Channels

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***Abstract*—New space–time trellis-coded modulation (STTC) which best satisfies the design criteria proposed by Tarokh *et al.* over fast fading channels are presented. STTC with feedforward structure have been constructed by a systematic and exhaustive search. Simulation results and parameter comparison show that these codes offer a better performance on fast fading channels than other known codes while maintaining a comparable performance on slow fading channels.**

***Index Terms*—Diversity, fading channels, multiple antennas, space–time codes.**

I. Introduction

RECENT developments of transmit diversity combined with coding techniques brought about a new scheme called space-time coding. Tarokh *et al.* proposed a powerful space–time trellis-coded modulation (STTC) scheme [1], and showed the performance of these codes on quasi-static fading channels. Grimm *et al.* [2] extended the work in [1], and Baro *et al.* proposed some improved codes [3] on quasi-static fading channels.

The main contribution of this paper is a set of new STTC which best satisfies the design criteria on fast fading channels, as proposed by Tarokh *et al.* A systematic and exhaustive search was conducted to find a set of codes with a feedforward structure over 4-PSK and 8-PSK signal sets.

II. System Model

The system under consideration employs space–time TCM with n_T transmit antennas and n_R receive antennas. While the transmitter has no knowledge about the channel, it is assumed that the receiver can recover the channel state information perfectly. Information bits are encoded into n_T streams of M-PSK symbols by the STTC encoder. At any given time t, an M-PSK symbol x_t^i is transmitted through the ith antenna, $i = 1, 2, \ldots, n_T$.

At the receiver, each antenna receives a noisy superposition of n_T transmitted symbols which have been subjected to independent fading. After matched filtering, assuming ideal timing information, the received signal r_t^j at the jth antenna at time t can be expressed as

$$r_t^j = \sqrt{E_s} \sum_{i=1}^{n_T} h_{i,j}(t) x_t^i + n_t^j \tag{1}$$

where $h_{i,j}(t)$ models the complex fast fading gain from transmit antenna i to receive antenna j at time t, $i = 1, 2, \ldots, n_T$, $j = 1, 2, \ldots, n_R$, and E_s is the energy per symbol. On a fast fading channel, we assume the fading coefficients change independently from symbol to symbol. The fading gains are modeled as independent samples of a complex Gaussian random variable with mean zero and variance 0.5 per dimension. The noise n_t^j at the jth antenna is modeled as independent samples of a zero-mean complex Gaussian random variable with a noise spectral density of N_0.

A. A Brief Review of Design Criteria for Fast Fading Channels

Assuming that a codeword $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_t, \ldots, \mathbf{x}_l)$ where l is the codeword length and $\mathbf{x}_t$ is a space-time symbol at time t given by $\mathbf{x}_t = x_t^1 x_t^2 \cdots x_t^{n_T}$ was transmitted, the probability that a maximum-likelihood decoder at the receiver decides erroneously in favor of another legitimate codeword $\mathbf{e} = (\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_t, \ldots, \mathbf{e}_l)$, where $\mathbf{e}_t = e_t^1 e_t^2 \cdots e_t^{n_T}$ can be expressed as [1]

$$P(\mathbf{x} \to \mathbf{e}) \le \prod_{t \in \upsilon(\mathbf{x}, \mathbf{e})} \left(\frac{\|\mathbf{x}_t - \mathbf{e}_t\|^2 E_s}{4N_0} \right)^{-n_R} \tag{2}$$

where $\upsilon(\mathbf{x}, \mathbf{e})$ denotes the set of time instances $t \in \{1, 2, \ldots, l\}$ such that the Euclidean distance between the two space-time symbols $\|\mathbf{x}_t - \mathbf{e}_t\|$ is nonzero.

If we define δ_H, the symbol Hamming distance, as the number of time instances in which two codewords $\mathbf{x}$ and $\mathbf{e}$ differ, then $\delta_H = |\upsilon(\mathbf{x}, \mathbf{e})|$, and the pairwise error bound can be expressed as

$$P(\mathbf{x} \to \mathbf{e}) \le \prod_{t \in \upsilon(\mathbf{x}, \mathbf{e})} \|\mathbf{x}_t - \mathbf{e}_t\|^{-2n_R} \cdot \left(\frac{E_s}{4N_o} \right)^{-n_R \delta_H}. \tag{3}$$

The average word error probability at high E_s/N_o is dominated by the minimum symbol Hamming distance, $\delta_{H_{\min}} = \min_{\mathbf{x}, \mathbf{e}} |\upsilon(\mathbf{x}, \mathbf{e})|$, which determines the diversity gain, taken over all codewords. Therefore, the average word error probability will be minimized if $\delta_{H_{\min}}$ is maximized.

The next important parameter which determines the coding gain at high E_s/N_o is the minimum product distance,

$$\mathrm{pd}_{\min} = \min_{\mathbf{x}, \mathbf{e}} \prod_{t \in \upsilon(\mathbf{x}, \mathbf{e})} \|\mathbf{x}_t - \mathbf{e}_t\|^2 \tag{4}$$

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TABLE I
4-PSK STTC, BANDWIDTH EFFICIENCY 2 bits/s/Hz

	$\nu = 2$	$\nu = 3$	$\nu = 4$	$\nu = 5^\dagger$
(a_0^1, a_0^2)	(3,1)	(0,2)	(0,2)	(0,1)
(a_1^1, a_1^2)	(2,1)	(1,3)	(0,1)	(2,0)
(a_2^1, a_2^2)	-	(2,0)	(2,2)	(3,0)
(a_3^1, a_3^2)	-	-	-	(1,2)
(b_0^1, b_0^2)	(2,2)	(2,2)	(2,0)	(1,3)
(b_1^1, b_1^2)	(0,2)	(1,2)	(1,2)	(1,2)
(b_2^1, b_2^2)	-	-	(0,2)	(2,1)
new $\delta_{H_{\min}}$	2	2	3	3
Baro $\delta_{H_{\min}}$	2	2	3	n/a
Tarokh $\delta_{H_{\min}}$	2	2	3	3
new $\mathrm{pd}_{\min}$	24.0	48.0	64.0	144.0
Baro $\mathrm{pd}_{\min}$	8.0	32.0	32.0	n/a
Tarokh $\mathrm{pd}_{\min}$	4.0	16.0	16.0	128.0

TABLE II
8-PSK STTC, BANDWIDTH EFFICIENCY 3 bits/s/Hz

	$\nu = 3$	$\nu = 4^\dagger$	$\nu = 5^\dagger$
(a_0^1, a_0^2)	(2,1)	(0,4)	(3,4)
(a_1^1, a_1^2)	(2,4)	(4,2)	(0,4)
(b_0^1, b_0^2)	(0,4)	(1,5)	(1,1)
(b_1^1, b_1^2)	(4,0)	(2,1)	(0,6)
(b_2^1, b_2^2)	-	(0,5)	(6,1)
(d_0^1, d_0^2)	(4,6)	(5,1)	(1,2)
(d_1^1, d_1^2)	(2,1)	(6,4)	(3,1)
(d_2^1, d_2^2)	-	-	(1,0)
new $\delta_{H_{\min}}$	2	2	2
Tarokh $\delta_{H_{\min}}$	2	2	2
new $\mathrm{pd}_{\min}$	15.51	24.0	29.66
Tarokh $\mathrm{pd}_{\min}$	2.0	8.0	13.66

along the paths with the minimum symbol Hamming distance. Maximizing the coding gain is equivalent to maximizing $\mathrm{pd}_{\min}$.

III. CODE CONSTRUCTION AND CODE SEARCH

The encoder is implemented as a feedforward shift register with a memory order of ν. For 4-PSK STTC with two transmit antennas, the input to the encoder at time t is a sequence of two binary bits (c_t^0, c_t^1). The output, x_t^1, x_t^2, is obtained as a modulo 4 sum of the linear combinations of the current and delayed binary inputs, and can be expressed algebraically as

$$x_t^1 = \sum_{j_0=0}^{\nu_0} c_{t-j_0}^0 a_{j_0}^1 + \sum_{j_1=0}^{\nu_1} c_{t-j_1}^1 b_{j_1}^1 \mod 4$$
$$x_t^2 = \sum_{j_0=0}^{\nu_0} c_{t-j_0}^0 a_{j_0}^2 + \sum_{j_1=0}^{\nu_1} c_{t-j_1}^1 b_{j_1}^2 \mod 4$$

where $\nu = \nu_0 + \nu_1$, $a_{j_i}^1$, $b_{j_i}^1$, $a_{j_i}^2$, $b_{j_i}^2 \in \{0, 1, 2, 3\}$, $i \in \{0, 1\}$, and $j_i \in \{0, 1, \ldots, \nu_i\}$. $c_{t-j_i}^i$ is the input bit c_t^i delayed by j_i time units.

Given this encoder structure, we would like to search for a set of coefficients $(a_{j_i}^1, a_{j_i}^2)$ and $(b_{j_i}^1, b_{j_i}^2)$ which will maximize $\delta_{H_{\min}}$ and $\mathrm{pd}_{\min}$. The search was conducted over all possible pairs of paths in the trellis. Table I lists the new 4-PSK STTC with 4, 8, 16 and 32-state trellises which best satisfy the design criteria. A symbol † indicates that the search for that memory order was not exhaustive. We also computed $\delta_{H_{\min}}$ and $\mathrm{pd}_{\min}$ of 4-PSK STTC proposed by Tarokh *et al.* [1] and by Baro *et al.* [3] for comparison (n/a indicates that no code of that memory order is available from the corresponding author). Table I suggests that for a given memory order, the new code and those proposed by Baro and Tarokh achieve the same diversity gains but different coding gains. It is worthwhile to point out that while on a slow fading channel each of these codes achieves a diversity gain of 2, on a fast fading channel the same codes of memory 4 and 5 achieve a diversity gain of 3, as $\delta_{H_{\min}} = 3$.

For 8-PSK STTC, the input to the encoder at time t is a sequence of three binary bits (c_t^0, c_t^1, c_t^2). With two transmit antennas, the output, x_t^1, x_t^2, is the modulo 8 sum of the linear combinations of the current and delayed binary inputs which can be expressed as

$$x_t^1 = \sum_{j_0=0}^{\nu_0} c_{t-j_0}^0 a_{j_0}^1 + \sum_{j_1=0}^{\nu_1} c_{t-j_1}^1 b_{j_1}^1 + \sum_{j_2=0}^{\nu_2} c_{t-j_2}^2 d_{j_2}^1 \mod 8$$
$$x_t^2 = \sum_{j_0=0}^{\nu_0} c_{t-j_0}^0 a_{j_0}^2 + \sum_{j_1=0}^{\nu_1} c_{t-j_1}^1 b_{j_1}^2 + \sum_{j_2=0}^{\nu_2} c_{t-j_2}^2 d_{j_2}^2 \mod 8$$

where $\nu = \nu_0 + \nu_1 + \nu_2$, $a_{j_i}^1$, $b_{j_i}^1$, $d_{j_i}^1$, $a_{j_i}^2$, $b_{j_i}^2$, $d_{j_i}^2 \in \{0, 1, \ldots, 7\}$, $i \in \{0, 1, 2\}$, and $j_i \in \{0, 1, \ldots, \nu_i\}$.

Table II lists the new 8-PSK STTC with 8, 16 and 32-state trellises. We did not search for 4-state 8-PSK STTC since any 4-state 8-PSK STTC will have parallel transitions, resulting in $\delta_{H_{\min}} = 1$. None of the new codes in Tables I and II coincides with those presented in [1] and [3]. We also computed $\delta_{H_{\min}}$ and $\mathrm{pd}_{\min}$ of 8-PSK STTC proposed by Tarokh *et al.* in [1]. (Reference [3] did not provide any 8-PSK codes.) All codes in Table II have a diversity gain of 2 but different coding gains, as indicated by different values of $\mathrm{pd}_{\min}$.

IV. CODE PERFORMANCE

In the simulations, two transmit and one receive antennas are assumed. Each frame consists of 130 symbols transmitted out of each antenna. The performance curves are plotted against the signal energy to noise ratio at the receive antenna, defined as $\mathrm{SNR} = n_T E_s / N_o$. An ML Viterbi decoder which operates on the code trellis is employed at the receiver. If we define the decoder complexity as the product of the number of trellis states and the number of branches entering each trellis node, the decoder complexity is given by $2^\nu \cdot M$. It is assumed that the receiver has a perfect knowledge of the channel. Note, however, that in a practical system a perfect channel estimation may be

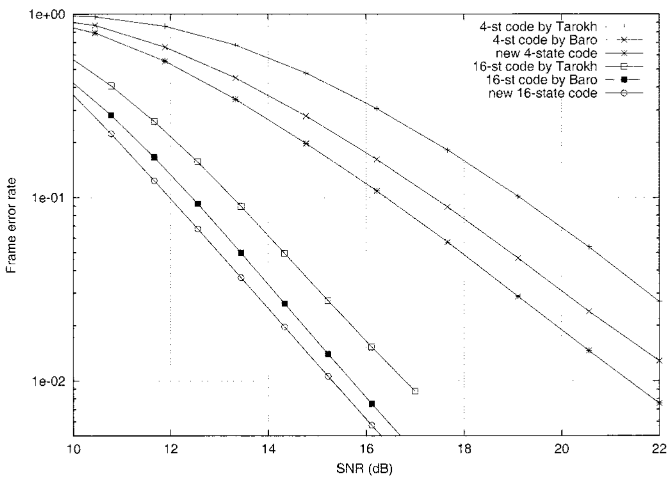

Fig. 1. FER comparison between Tarokh's, Baro's and the new 4-PSK STTC with 4 and 16 states on fast fading channels, bandwidth efficiency 2 bits/sec/Hz.

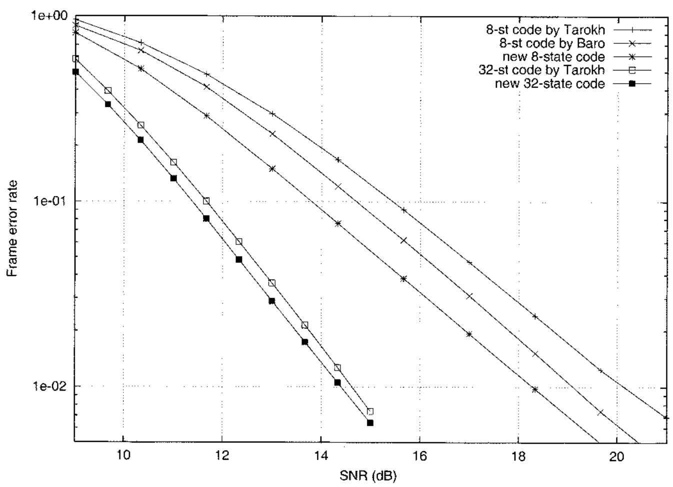

Fig. 2. FER comparison between Tarokh's, Baro's and the new 4-PSK STTC with 8 and 32 states on fast fading channels, bandwidth efficiency 2 bits/s/Hz.

difficult to achieve when the fading coefficients change in each symbol interval. This may deteriorate the performance.

Fig. 1 shows the frame error rate performance of the new 4-PSK STTC with 4 and 16 states on a fast fading channel. Their performance is compared with STTC of the same memory order proposed by Tarokh *et al.* and by Baro *et al.* The bandwidth efficiency is 2 bits/s/Hz. Fig. 2 shows the performance of 8-state and 32-state 4-PSK STTC on fast fading channels. In both figures, the performance curves of the codes with the same memory order appear parallel, as predicted by the same value of $\delta_{H_{\min}}$. Different values of $\mathrm{pd}_{\min}$ yield different coding gains which are illustrated by the horizontal shifts of the curves. Furthermore, it is worthwhile to mention that all 4-PSK STTC of 16 states have a steeper slope than those of four states. This occurs because the 16-state codes have $\delta_{H_{\min}} = 3$, while the 4-state codes have $\delta_{H_{\min}} = 2$. On slow fading channels, the performance curves appear parallel [1], [3] because the diversity gain is 2, as indicated by their full ranks. The same observation can also be made on the 8-state and 32-state 4-PSK STTC in Fig. 2.

Fig. 3 shows the performance of the new 8-PSK STTC with 8 and 32 states, in comparison with codes of the same memory

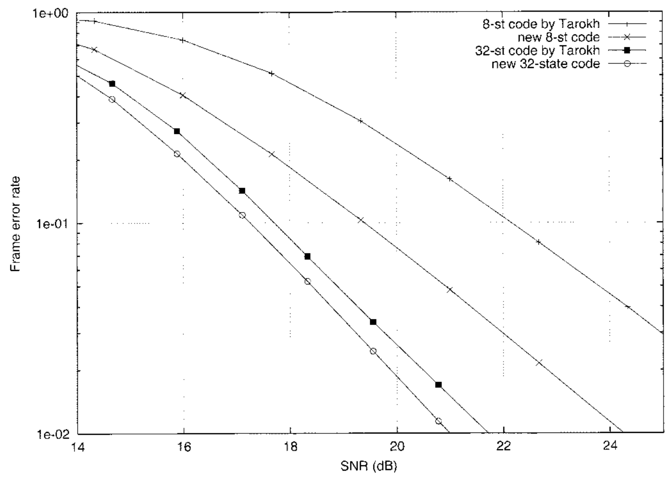

Fig. 3. FER comparison between Tarokh's codes and the new 8-PSK STTC with 8 and 32 states on fast fading channels, bandwidth efficiency 3 bits/s/Hz.

order proposed by Tarokh *et al.* on a fast fading channel. The bandwidth efficiency is 3 bits/s/Hz. The performance curves for 8-PSK STTC with 16 states are not shown to avoid clutter. All figures suggest that the new STTC offers a better performance on fast fading channels than the previously proposed codes. This claim is supported by the parameter values in Tables I and II which show that for a given memory order the new codes have larger $\mathrm{pd}_{\min}$ than all other known codes.

It is worthwhile to mention that on quasi-static fading channels where fading coefficients remain constant over a frame and change independently between frames, most of the new 4-PSK STTC maintain a comparable performance to those proposed in [1] and [3]. Further analysis shows that the new 4-PSK codes of 4, 8 and 16-states listed in Table I have full ranks, which is the most important parameter determining STTC performance on slow fading channels [1]. A further improvement can be gained when an STTC is combined with an outer code. It would be interesting to see how the new STTC performs in comparison to previously known codes in a serial concatenation structure.

V. Conclusions

New space-time trellis codes designed for fast fading channels have been presented. Simulation results and parameter comparisons confirm that these codes offer a better performance on fast fading channels than previously introduced STTC. They also offer a comparable performance on slow fading channels, as suggested by their full ranks.

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