Improved Codes for Space–Time Trellis-Coded Modulation

Stephan Bäro, Gerhard Bauch, and Axel Hansmann

***Abstract*—Space–time coded modulation has been shown to efficiently use transmit diversity to increase spectral efficiency. In this letter we propose new trellis codes found through systematic code search. These codes achieve the theoretically maximal diversity gain and improved coding gain compared to known codes.**

***Index Terms*—Diversity, space–time coded modulation, wireless communications.**

I. Introduction

RECEIVE antenna diversity is a widely applied technique to reduce the detrimental effects of multipath fading in wireless communications. However, it is hard to efficiently use receive antenna diversity at the remote units since they should remain relatively simple, inexpensive and small. Therefore, receive diversity has been used nearly exclusively at the base station. In order to enable high data rate transmission over wireless fading channels, recently different transmit diversity techniques have been introduced to benefit from antenna diversity also in the downlink while putting the diversity burden on the base station. In [1] Tarokh *et al.* introduced space–time trellis-coded modulation (STTCM) proposing a joint design of coding, modulation and transmit diversity for flat Rayleigh fading channels. By avoiding destructive superposition after combination of the signals transmitted simultaneously from different antennas STTCM achieves the same, theoretically optimal, diversity advantage as receive diversity.

In [1] codes were presented that achieve the maximal possible diversity gain, but not necessarily full coding gain. In this letter we introduce a new description of space–time codes using a generator matrix and present results of a systematic code search. This search yielded space–time trellis codes which perform better than those in [1] at the same complexity. The results give rise to the hope that for other constellation sizes and complexities improved codes can be found as well.

II. System Model

We consider a mobile communications system where the transmitter is equipped with n_T antennas and the receiver is equipped with n_R antennas. The signals on the matrix channel, i.e., the $n_T \cdot n_R$ transmission paths between transmitter and receiver, are supposed to undergo independent flat Rayleigh fading. It is assumed that the path gains are constant during one frame and change independently from one frame to another (quasi-static fading). The block length is described by the l symbols that are transmitted sequentially out of each antenna. We restrict ourselves to M-PSK and the case that the redundancy is spread in space rather than in time, i.e., during each use of the matrix channel we assume that $m = \log_2 M$ information bits are transmitted.

III. Code Search and Results

Space–time codes can be represented and analyzed in a number of fashions. In [1] most codes were presented in their trellis form. In order to perform a systematic code search it is useful to transform the code representation from the appealing trellis form into a more tractable generator matrix form. Assuming a stream of information bits $\mathbf{u} = (u_1, \cdots, u_n)$ to be transmitted, $n = l \cdot \log_2 M$, we consider each bit u_i to be 0 or 1.

Let $\mathbf{G}$ be a generator matrix with n_T columns and $m+s$ rows, s referencing the number of memory elements in the encoder, each entry being between 0 and $M-1$. Let $\mathcal{M}$ be a mapping function that maps integer values to the M-PSK constellation, $\mathcal{M}(x) = \exp(2\pi jx/M)$.

We then obtain the stream of coded complex phase-shift keying (PSK) symbols $\mathbf{c} = (c_1^1 c_1^2 \cdots c_1^{n_T} \cdots c_l^1 c_l^2 \cdots c_l^{n_T}) = (\mathbf{c}_1, \cdots, \mathbf{c}_l)$ by applying the mapping function $\mathcal{M}$ to the following matrix multiplication:

$$\mathbf{c}_t = \mathcal{M}(\mathbf{u}_t \cdot \mathbf{G} \pmod M) \tag{1}$$

$\mathbf{u}_t$ denotes those input bits

$$\mathbf{u}_t = (u_{mt+(m-1)} \cdots u_{mt+1} u_{mt} \cdots u_{mt-s})$$

influencing the coded symbols at time t.

The simplest example would be the generator matrix for the 4-PSK space–time code with four states proposed in [1] $(m = s = 2)$. This is delay diversity which was also included in proposals by Wittneben [2]. Here the two current information bits are grouped, mapped on an M-PSK symbol and transmitted over the first antenna, whereas the two preceding bits are grouped and transmitted simultaneously over the second antenna

$$\mathbf{G}_{\mathrm{DD}} = \begin{pmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{pmatrix}.$$

This representation permits us to search systematically all matrices $\mathbf{G}$.

Manuscript received June 7, 1999. The associate editor reviewing this letter and approving it for publication was Prof. A. Haimovich.

The authors are with the Institute for Communications Engineering, Munich University of Technology, 80290 Munich, Germany (e-mail: Stephan.Baero@ei.tum.de).

Publisher Item Identifier S 1089-7798(00)01303-X.

TABLE I
COMPARISON OF KNOWN AND NEW 4-PSK
TRELLIS CODES, WITH THEORETICAL CODING GAINS

States	Ref. [1]	Gain	New	Gain	Improvement
	0 2		2 2		
	0 1		0 2		
4	2 0	2	1 0	$\sqrt{8}$	1.50 dB
	1 0		3 1		
	0 2		2 2		
	0 1		0 1		
8	2 0	$\sqrt{12}$	2 0	4	0.62 dB
	1 0		1 0		
	2 2		2 2		
	1 0		1 2		
	2 0		2 0		
16	2 1	$\sqrt{12}$	2 1	$\sqrt{20}$	1.11 dB
	0 2		0 2		
	0 2		0 2		
	2 0		2 0		

Tarokh *et al.* have put forward design criteria for space–time trellis codes in [1]. According to those criteria the pairwise error probability must be minimized, that is the probability for detecting an erroneous code sequence instead of the transmitted code sequence.

Thus for each possible code all possible pairs of code and error sequences must be generated. An error sequence

$$\mathbf{e} = (e_1^1 e_1^2 \cdots e_1^{n_T} \cdots e_l^1 e_l^2 \cdots e_l^{n_T})$$

would be a valid code sequence detected erroneously by the maximum-likelihood decoder in favor of the transmitted sequence $\mathbf{c}$. The number of codewords to compare can be reduced if space–time codes are known to be geometrically uniform. The codes proposed in [1] fulfill this condition. But as we found counter examples during our code search we cannot generally assume geometrical uniformity.

It has been shown in [1] that the pairwise error probability can be bound above by

$$P(\mathbf{c} \to \mathbf{e}) \leq \left(\prod_{i=1}^{R} \lambda_i \right)^{-n_R} (E_s/4N_0)^{-Rn_R} \qquad (2)$$

where $R \leq n_T$ is the rank of the difference matrix $\mathbf{B}$,

$$\mathbf{B}(\mathbf{c}, \mathbf{e}) = \begin{pmatrix} e_1^1 - c_1^1 & \cdots & c_\ell^1 - c_\ell^1 \\ \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & \cdots & e_\ell^{n_T} - c_\ell^{n_T} \end{pmatrix}$$

and λ_i are the nonvanishing eigenvalues of the distance matrix $\mathbf{A} = \mathbf{B}\mathbf{B}^*$. Either bit-error or frame-error probability can be approximated from (2) by taking a weighted sum over dominant error events. Asymptotically, error events corresponding to distance matrices with minimum rank and minimum product of eigenvalues will be dominating performance according to (2).

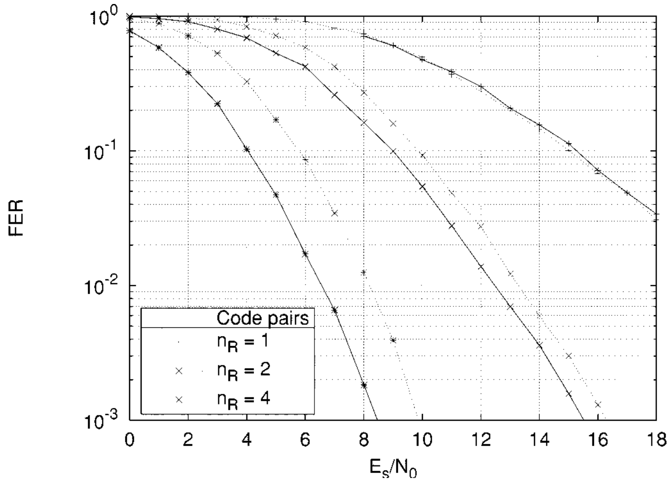

Fig. 1. Comparison of space–time codes with two transmit antennas, 4-PSK, 4 states, frame length 130 symbols.

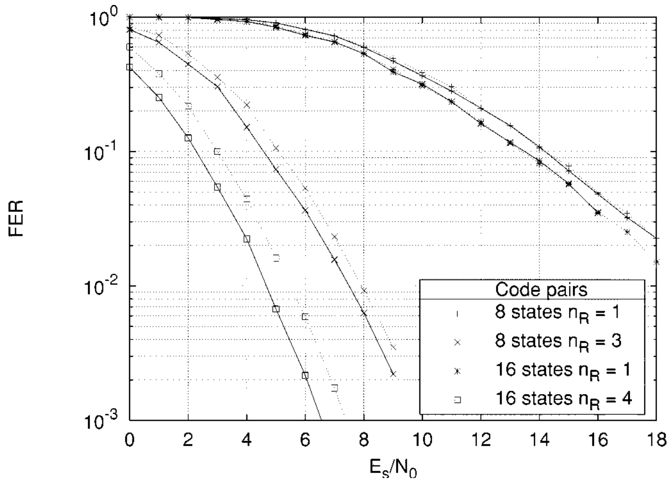

Fig. 2. Comparison of space–time codes with 2 transmit antennas, 4-PSK, 8, and 16 states, frame length 130 symbols.

From this expression it can be seen that R is crucial in determining the asymptotic slope of the resulting error rate curve when plotted on a double-logarithmic scale, and the product of eigenvalues will determine a horizontal shift on the E_s/N_0 scale. The horizontal shift in decibels is called coding gain, whereas Rn_R is called diversity gain. The rank criterion being predominant, we can stop the evaluation as soon as any pair of sequences does not achieve full rank.

All codes presented in [1] have full rank $R = n_T$ by the design rules. Nevertheless, coding gain can be improved. Through systematic code search we were able to find improved codes on quasi-static fading channels. Some of these new codes for 4-PSK and 4, 8, and 16 states are presented in Table I together with theoretical coding gains and the improvement in decibels, i.e., $10\log(\text{gain}_{\text{new}}/\text{gain}_{[1]})$ dB. In the case of 4 states the presented coding gain is optimal in the sense that an exhaustive code search was performed.

In Fig. 1 we illustrate the possible improvement for the simplest case: 4-PSK space–time codes having rank 2 for all code sequences with four states. The dotted line shows the frame-error rate (FER) for delay diversity, the full line represents the code proposed in Table I. The improvement of approximately 1.5 dB is visible, but only for more than one receive antenna.

Considering codes with 8 and 16 states, we compare their performance in Fig. 2. Once again we can see that the improvement in coding gain is only achieved if there is more than one receive antenna.

IV. CONCLUSION

Through systematic code search using the known design criteria for STTCM, optimal codes for a given complexity can be found. Three such codes were presented in this letter, improving on published codes by between 0.62 and 1.5 dB if receive diversity is used. As exhaustive search proves difficult for more complex codes when the number of states and possible transitions increases, an algorithmic approach to the code search problem would be an interesting issue.

ACKNOWLEDGMENT

The authors would like to thank A. Naguib, V. Tarokh, and N. Seshadri for valuable discussions.

REFERENCES

[1] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inform. Theory*, vol. 44, pp. 744–765, Mar. 1998.

[2] A. Wittneben, "A new bandwidth efficient transmit antenna modulation diversity scheme for linear digital modulation," in *Proc. IEEE'ICC*, 1993, pp. 1630–1634.