

Iterative Maximum-Likelihood Sequence Estimation for Space-Time Coded Systems

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Abstract—In recent work on decoding space-time codes, it is either assumed that perfect channel state information (CSI) is present, or a channel estimate is obtained using pilot symbols and then used as if it were perfect to extract symbol estimates. In the latter case, a loss in performance is incurred, since the resulting overall receiver is not optimal. In this letter, we look at maximum-likelihood (ML) sequence estimation for space-time coded systems without assuming CSI. The log-likelihood function is presented for both quasi-static and nonstatic fading channels, and an expectation-maximization (EM)-based algorithm is introduced for producing ML data estimates, whose complexity is much smaller than a direct evaluation of the log-likelihood function. Simulation results indicate the EM-based algorithm achieves a performance close to that of a receiver which knows the channel perfectly.

Index Terms—Complexity, EM algorithm, performance, sequence estimation, space-time coding.

I. INTRODUCTION

THE USE of transmitter diversity has been shown to be a promising technique for increasing the capacity of wireless systems [1]. The first transmit diversity scheme was proposed by Wittneben [2]; subsequently, a delay diversity scheme was introduced by Seshadri and Winters [3]. Tarokh *et al.* proposed trellis-based space-time codes [4] that combine signal processing at the receiver and coding appropriate to multiple transmit antennas. These so-called space-time codes perform well in slowly fading channels, assuming perfect channel state information (CSI) at the receiver. In the presence of channel estimation errors, however, system performance can suffer significantly [5].

In this letter, we look at maximum-likelihood (ML) sequence estimation for space-time coded systems without assuming CSI, and, in contrast to previous work focusing on static channels, also consider nonstatic fading. The log-likelihood function and an expectation-maximization (EM) algorithm (see, for example, [6] and [7]) are derived for the sequence estimation problem. The EM-based algorithm is seen by simulations to perform close to the performance of an ML decoder that assumes perfect CSI.

The letter is organized in four sections following this introduction. Section II presents the system model, Section III includes the derivation of the ML and EM-based algorithms, and Section IV presents simulation results. Section V concludes.

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II. SYSTEM MODEL

We consider a mobile radio system where the transmitter is equipped with N transmit antennas and the mobile is equipped with M receive antennas. Data blocks of length L are encoded by a space-time encoder. The output of the encoder is arranged into N blocks, each containing L complex modulation symbols; all N blocks are transmitted via separate transmit antennas simultaneously. Let $\mathbf{D}_j = (d_j^{(1)} \ d_j^{(2)} \ \dots \ d_j^{(L)})^T$, $j = 1, 2, \dots, N$, be the $L \times 1$ column vector containing the L symbols transmitted by the j th transmit antenna, and $\mathbf{D} = (\mathbf{D}_1 \ \mathbf{D}_2 \ \dots \ \mathbf{D}_N)$ be the $L \times N$ matrix of all transmitted symbols. The total transmit energy per discrete time over all N antennas is normalized to unity; thus, $|d_j^{(i)}|^2 = 1/N$ for $j = 1, 2, \dots, N$ and $i = 1, 2, \dots, L$. To facilitate the derivations that follow, we also define the row-vectors of \mathbf{D} (the spatial domain) by $\mathbf{D}^{(l)}$, $l = 1, 2, \dots, L$, and $\mathbf{B}_i = \text{diag}(\mathbf{D}_i)$. In describing the fading channel between transmit and receive antennas, let the $L \times 1$ fading vector in the path from the i th transmit to the j th receive antenna be $\mathbf{\Gamma}_{ij} = (\gamma_{ij}^{(1)} \ \gamma_{ij}^{(2)} \ \dots \ \gamma_{ij}^{(L)})^T$. In what follows, we make the standard assumption that the fading vectors $\mathbf{\Gamma}_{ij}$ within distinct transmit/receive paths (i.e., distinct ij pairs) are independent. Within each transmit/receive path, the components of $\mathbf{\Gamma}_{ij}$ at each discrete time l , $\gamma_{ij}^{(l)}$, $l = 1, 2, \dots, L$, are complex, Gaussian random variables having covariance matrix $\mathbf{\Theta}$. As usual for fading channels, their second moment is absorbed into the signal-to-noise ratio (SNR) definition, so in the sequel we will assume that they are of unit-variance. To facilitate derivations, we also define the $N \times M$ matrix of fading gains at time l by

$$\mathbf{\Gamma}^{(l)} = \begin{pmatrix} \gamma_{11}^{(l)} & \gamma_{12}^{(l)} & \dots & \gamma_{1M}^{(l)} \\ \gamma_{21}^{(l)} & \gamma_{22}^{(l)} & \dots & \gamma_{2M}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{N1}^{(l)} & \gamma_{N2}^{(l)} & \dots & \gamma_{NM}^{(l)} \end{pmatrix}$$

and the column-vectors of $\mathbf{\Gamma}^{(l)}$ by $\mathbf{\Gamma}_j^{(l)}$, $j = 1, 2, \dots, M$.

A. Nonstatic Fading Case

The received signal at the j th receive antenna at discrete-time l for nonstatic fading is

$$y_j^{(l)} = \sum_{i=1}^N d_i^{(l)} \gamma_{ij}^{(l)} + n_j^{(l)} = \mathbf{D}^{(l)} \mathbf{\Gamma}_j^{(l)} + n_j^{(l)},$$

$$j = 1, 2, \dots, M, \quad l = 2p1, 2, \dots, L. \quad (2)$$

In matrix form, which is useful in deriving expressions in the sequel, we have

$$\mathbf{Y}_j = \sum_{i=1}^N \mathbf{B}_i \mathbf{\Gamma}_{ij} + \mathbf{n}_j = \mathbf{B} \mathbf{\Gamma}_j + \mathbf{n}_j, \quad j = 1, 2, \dots, M \quad (3)$$

where \mathbf{n}_j is an $L \times 1$ additive, independent and identically distributed (i.i.d.) Gaussian noise vector having covariance $\mathbf{\Sigma}_{\mathbf{n}_j} = \mathbf{I}/\text{SNR}$ and

$$\begin{aligned} \mathbf{\Gamma}_j &= (\mathbf{\Gamma}_{1j}^T \quad \mathbf{\Gamma}_{2j}^T \quad \dots \quad \mathbf{\Gamma}_{Nj}^T)^T \\ \mathbf{B} &= (\mathbf{B}_1 \quad \mathbf{B}_2 \quad \dots \quad \mathbf{B}_N). \end{aligned} \quad (4)$$

Let \mathbf{Y} be an $L \times M$ matrix whose M column-vectors are \mathbf{Y}_j , $j = 1, 2, \dots, M$. The covariance matrix of $\mathbf{\Gamma}_j$ is then the block-diagonal, $NL \times NL$ matrix $\mathbf{\Sigma}_{\mathbf{\Gamma}}$ with $\mathbf{\Theta}$ in its main diagonal.

B. Quasi-Static Fading Case

In the case of a quasi-static fading channel, all elements of $\mathbf{\Gamma}_{ij}$ are the same. Corresponding to (4), define the fading vector $\mathbf{\Lambda}_j = (\gamma_{1j} \quad \gamma_{2j} \quad \dots \quad \gamma_{Nj})^T$, with covariance $\text{cov}(\mathbf{\Lambda}_j) = \mathbf{I}$, where \mathbf{I} is an $N \times N$ identity matrix. Then (3) becomes

$$\mathbf{Y}_j = \mathbf{D} \mathbf{\Lambda}_j + \mathbf{n}_j. \quad (5)$$

Next, we introduce the ML receiver and its EM-based implementation.

III. ML AND EM-BASED RECEIVERS

A. ML Receiver

The conditional covariance matrix of \mathbf{Y}_j is (* denotes conjugate transpose)

$$\begin{aligned} \Phi &= \sum_{i=1}^N \left(\mathbf{B}_i \mathbf{\Theta} \mathbf{B}_i^* + \frac{\mathbf{I}}{N \cdot \text{SNR}} \right) \\ &= \sum_{i=1}^N \mathbf{B}_i \left(\mathbf{\Theta} + \frac{\mathbf{I}}{\text{SNR}} \right) \mathbf{B}_i^*, \end{aligned} \quad (6)$$

$$\Phi = \sum_{i=1}^N \left(\mathbf{B}_i \mathbf{\Theta} \mathbf{B}_i^* + \frac{\mathbf{I}}{N \cdot \text{SNR}} \right) = \sum_{i=1}^N \mathbf{B}_i \left(\mathbf{\Theta} + \frac{\mathbf{I}}{\text{SNR}} \right) \mathbf{B}_i^* \quad (6)$$

and it is the same for each of the receive antennas. Under the standard assumption that fading processes through different paths are mutually independent, it is easy to obtain the ML receiver, which is similar to the single antenna system in [7] (the ML receiver for the quasi-static channel has a similar form)

$$\arg \min_{\mathbf{B}} \sum_{j=1}^M (\mathbf{Y}_j^* \Phi^{-1} \mathbf{Y}_j + \log |\Phi|). \quad (7)$$

However, whereas in the single antenna case and phase-shift keying (PSK) signaling, the determinant $|\Phi|$ is not a function

of the data, in the multiple transmit antenna case it generally is, depending on the coding used; this further complicates implementation of the true ML receiver. We note that both the ML and EM-based receivers assume knowledge of the channel covariance matrix. We will show below that under this condition, the EM algorithm provides an efficient way to implement the ML receiver, even for long sequences.

B. The EM-Based Receiver

For a description of the EM algorithm in all its generality, the reader is urged to read the original paper [6]. For an application to fading channels, [7] may be also useful.

1) *Nonstatic Fading Case:* For our problem, we choose the complete data to be $(\mathbf{Y}, \{\mathbf{\Gamma}_j\})$. The log-likelihood function for the complete data then is (using (2) and dropping unnecessary terms)

$$\begin{aligned} l(\mathbf{Y}|\mathbf{D}, \{\mathbf{\Gamma}_j\}) &= \\ &= \sum_{l=1}^L \sum_{j=1}^M \left[\Re \left(\overline{y_j^{(l)}} \mathbf{D}^{(l)} \mathbf{\Gamma}_j^{(l)} \right) - \frac{1}{2} \mathbf{D}^{(l)} \mathbf{\Gamma}_j^{(l)} (\mathbf{\Gamma}_j^{(l)})^* (\mathbf{D}^{(l)})^* \right] \end{aligned} \quad (8)$$

where $\bar{}$ denotes complex conjugation. The expectation step of EM then yields¹

$$\begin{aligned} Q(\mathbf{D}|\mathbf{D}^k) &= \\ &= \sum_{l=1}^L \sum_{j=1}^M \left[\Re \left(\overline{y_j^{(l)}} \mathbf{D}^{(l)} (\hat{\mathbf{\Gamma}}_j^{(l)})^k \right) - \frac{1}{2} \mathbf{D}^{(l)} (\hat{\mathbf{\Omega}}_j^{(l)})^k (\mathbf{D}^{(l)})^* \right] \end{aligned} \quad (9)$$

where

$$\begin{aligned} (\hat{\mathbf{\Gamma}}_j^{(l)})^k &= E \left[\mathbf{\Gamma}_j^{(l)} | \mathbf{Y}, \mathbf{D}^k \right], \\ (\hat{\mathbf{\Omega}}_j^{(l)})^k &= E \left[\mathbf{\Gamma}_j^{(l)} (\mathbf{\Gamma}_j^{(l)})^* | \mathbf{Y}, \mathbf{D}^k \right] \end{aligned} \quad (10)$$

are, respectively, the conditional mean and second moment of the fading process, given the received data and the most recent estimate of the transmitted symbols at the k th iteration.

Using the matrix form for the received data in (3), we note that when the data \mathbf{B} is known, we have a linear set of equations for the observed data. Thus, $\hat{\mathbf{\Omega}}_j^k$ and $\hat{\mathbf{\Gamma}}_j^k$ can be obtained using standard minimum mean-square-error (MMSE) estimation [8]

$$\begin{aligned} \hat{\mathbf{\Gamma}}_j^k &= \mathbf{\Sigma}_{\mathbf{\Gamma}} (\mathbf{B}^k)^* \left(\mathbf{B}^k \mathbf{\Sigma}_{\mathbf{\Gamma}} (\mathbf{B}^k)^* + \frac{\mathbf{I}}{\text{SNR}} \right)^{-1} \mathbf{Y}_j \\ \hat{\mathbf{\Omega}}_j^k &= \mathbf{\Sigma}_{\mathbf{\Gamma}_j | \mathbf{Y}, \mathbf{D}^k} + \hat{\mathbf{\Gamma}}_j^k (\hat{\mathbf{\Gamma}}_j^k)^* \end{aligned} \quad (11)$$

where $\mathbf{\Sigma}_{\mathbf{\Gamma}_j | \mathbf{Y}, \mathbf{D}^k}$ is the conditional covariance matrix given by

$$\begin{aligned} \mathbf{\Sigma}_{\mathbf{\Gamma}_j | \mathbf{Y}, \mathbf{D}^k} &= \mathbf{\Sigma}_{\mathbf{\Gamma}} - \mathbf{\Sigma}_{\mathbf{\Gamma}} (\mathbf{B}^k)^* \\ &\quad \cdot \left(\mathbf{B}^k \mathbf{\Sigma}_{\mathbf{\Gamma}} (\mathbf{B}^k)^* + \frac{\mathbf{I}}{\text{SNR}} \right)^{-1} \mathbf{B}^k \mathbf{\Sigma}_{\mathbf{\Gamma}}. \end{aligned} \quad (12)$$

¹Conditioning on \mathbf{D}^k is equivalent to conditioning on \mathbf{B}^k . Detection of \mathbf{D} yields the decoded codeword.

Rearranging (11), finally the conditional estimates in (9) are given by

$$\begin{aligned} ((\hat{\mathbf{r}}_j^{(l)})^k)_n &= (\hat{\mathbf{r}}_j^k)_{l+(n-1)L} \\ ((\hat{\mathbf{\Omega}}_j^{(l)})^k)_{p,q} &= (\hat{\mathbf{\Omega}}_j^k)_{l+(p-1)L, l+(q-1)L}. \end{aligned} \quad (13)$$

The maximization step of the EM algorithm then yields

$$\begin{aligned} \mathbf{D}^{k+1} &= \arg \max_{\mathbf{D}} \sum_{l=1}^L \sum_{j=1}^M \\ &\cdot \left[\Re \left(\overline{y_j^{(l)}} \mathbf{D}^{(l)} (\hat{\mathbf{r}}_j^{(l)})^k \right) - \frac{1}{2} \mathbf{D}^{(l)} (\hat{\mathbf{\Omega}}_j^{(l)})^k (\mathbf{D}^{(l)})^* \right] \end{aligned} \quad (14)$$

which can be efficiently done using the Viterbi algorithm when trellis coding is used.

2) *Quasi-Static Fading Case:* From (5) and similar to the nonstatic case, it can be shown that (9), (13), and (14) become

$$\mathcal{Q}(\mathbf{D}|\mathbf{D}^k) = \sum_{l=1}^L \sum_{j=1}^M \left[\Re \left(\overline{y_j^{(l)}} \mathbf{D}^{(l)} \hat{\mathbf{\Lambda}}_j^k \right) - \frac{1}{2} \mathbf{D}^{(l)} \hat{\mathbf{\Omega}}_j^k (\mathbf{D}^{(l)})^* \right] \quad (15)$$

$$\hat{\mathbf{\Lambda}}_j^k = \left((\mathbf{D}^k)^* \mathbf{D}^k + \frac{\mathbf{I}}{\text{SNR}} \right)^{-1} (\mathbf{D}^k)^* \mathbf{Y}_j \quad (16)$$

$$\hat{\mathbf{\Omega}}_j^k = \mathbf{I} - \left((\mathbf{D}^k)^* \mathbf{D}^k + \frac{\mathbf{I}}{\text{SNR}} \right)^{-1} (\mathbf{D}^k)^* \mathbf{D}^k + \hat{\mathbf{\Lambda}}_j^k (\hat{\mathbf{\Lambda}}_j^k)^* \quad (17)$$

$$\begin{aligned} \mathbf{D}^{k+1} &= \arg \max_{\mathbf{D}} \sum_{l=1}^L \sum_{j=1}^M \\ &\cdot \left[\Re \left(\overline{y_j^{(l)}} \mathbf{D}^{(l)} \hat{\mathbf{\Lambda}}_j^k \right) - \frac{1}{2} \mathbf{D}^{(l)} \hat{\mathbf{\Omega}}_j^k (\mathbf{D}^{(l)})^* \right]. \end{aligned} \quad (18)$$

In both the nonstatic and the quasi-static fading cases, to initialize the EM algorithm, rough channel estimates are extracted from pilot symbols.

C. Approximation of the EM-Based Algorithms

For the quasi-static case, as the SNR increases in (17), the \mathbf{I}/SNR term can be approximated by zero. Then, the first two terms in (17) cancel each other and $\hat{\mathbf{\Omega}}_j^k \approx \hat{\mathbf{\Lambda}}_j^k (\hat{\mathbf{\Lambda}}_j^k)^*$, resulting in significant simplification. Similarly for the nonstatic case, $\hat{\mathbf{\Omega}}_j^k \approx \hat{\mathbf{r}}_j^k (\hat{\mathbf{r}}_j^k)^*$ in (11). Simulation results show there is practically no distinguishable difference in performance between the optimum and the approximate EM-based receivers for the SNR range used in the simulations.

IV. PERFORMANCE

We use the rate-1, trellis-based eight- and four-state, quadrature PSK (QPSK), space-time codes introduced in [4] to study the performance of the EM-based algorithms introduced above. The trellis encoder starts and ends at the zero state. First, we deal with the quasi-static fading case. To set the initial fading estimate needed by the EM algorithm, pilot symbols are inserted into the modulated data stream every J symbols. In making decisions, the receiver has available a vector of data of length K

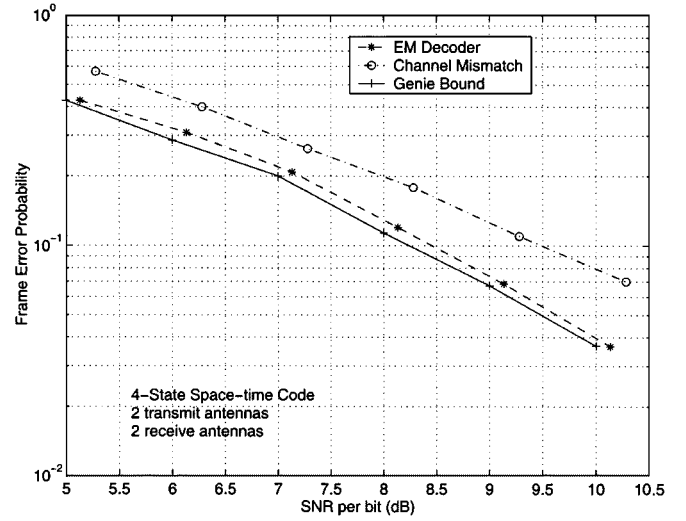


Fig. 1. Frame-error probability comparison between the EM and “genie” decoders: $N = 2$, $M = 2$, four-state space-time code, 128 QPSK symbol frame size, quasi-static fading.

blocks, each of length J (i.e., $L = J \cdot K$ symbols), for some integer K . Of the J symbols in a block, the first is a pilot symbol. The pilot symbol sequences on each antenna are chosen such that they are orthogonal [9]. In the simulations, an initial MMSE estimate of the fading is obtained based on the pilot symbols to initialize the algorithm.

The maximization step of the EM algorithm is efficiently performed using the Viterbi algorithm. In all the simulations of the EM-based algorithm, we limit the number of iterations to three. As mentioned above, the approximate EM receiver has a performance indistinguishable from the exact, so only one set of simulation results is plotted in the figures. Fig. 1 shows simulation results for the frame-error probability for the EM-based algorithm. The frame size is 128 QPSK data and four pilot symbols ($J = 33$, $K = 4$, $L = 132$) and the results account for the rate-loss due to pilot symbols. For comparison, we include simulation results for the “genie” receiver, i.e., the ML decoder with perfect CSI, and results for a receiver that first produces a channel estimate and then uses it as if it were perfect to extract symbol estimates. In the simulations for the channel estimation errors case, eight pilot symbols were inserted in each frame to estimate the channel. It is clear that the EM decoder performs very close to the “genie bound,” while the suboptimal receiver that first estimates the channel incurs about 1-dB loss at a frame error rate of 0.1. At higher SNR, performance loss becomes even larger.

Fig. 2 shows simulation results for the symbol-error probability at various frame sizes and for the quasi-static fading case. It is seen that large frame size improves the performance of the EM decoder, which approaches that of the ML decoder with perfect CSI. Fig. 3 shows simulation results for a non-static mobile radio channel for a normalized Doppler bandwidth $BT = 0.005$. Similar results were obtained for other values. It is seen from the figure that when the frame size is small ($K = 1$ and $K = 2$), there is an error floor at high SNR. As the frame size increases, the EM-based decoder makes better use of the channel memory and the performance floor sets in at

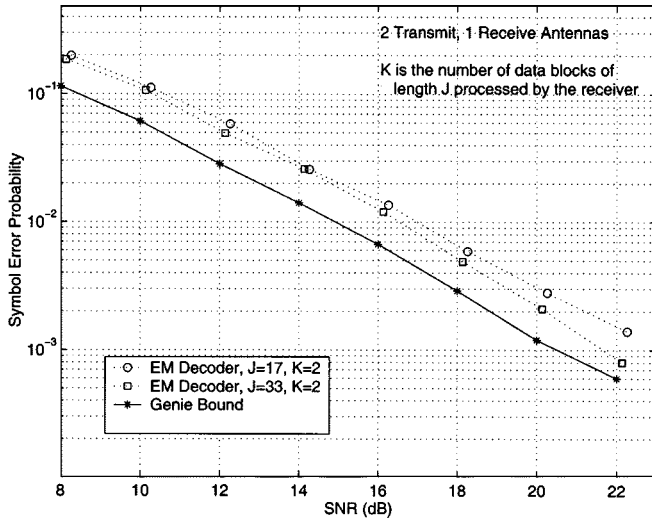


Fig. 2. Symbol-error probability of the "genie" and EM-based decoders as a function of block length: $N = 2$, $M = 1$, eight-state space-time code, quasi-static fading.

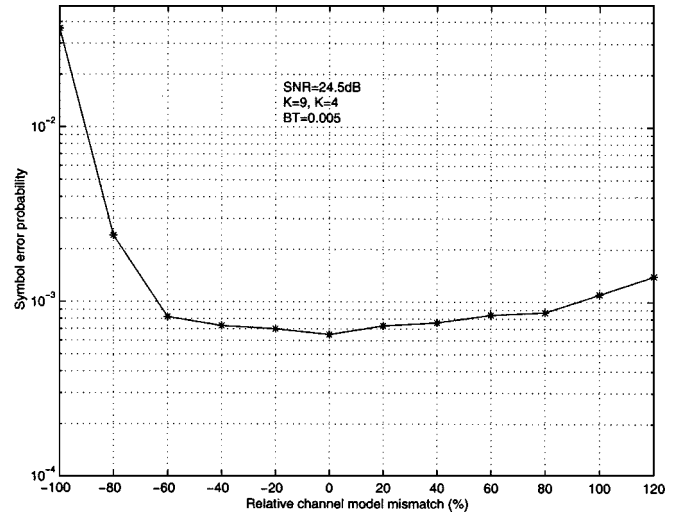


Fig. 4. Symbol-error probability for channel model mismatched case: $N = 2$, $M = 1$, $BT = 0.005$, eight-state space-time code.

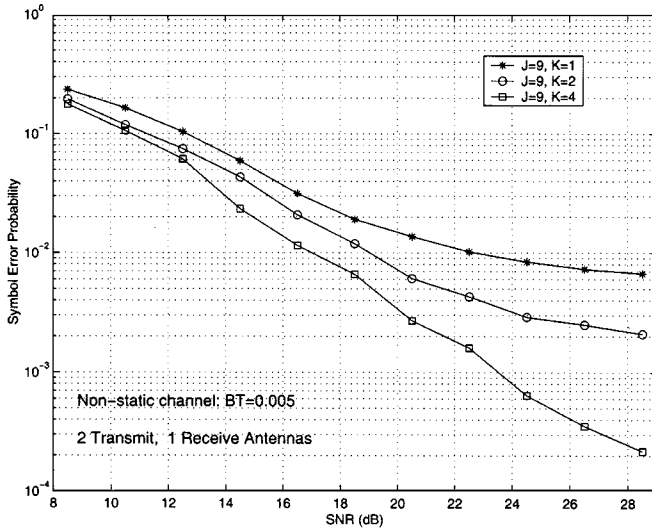


Fig. 3. Symbol-error probability for a nonstatic fading channel: $N = 2$, $M = 1$, $BT = 0.005$, eight-state space-time code.

higher SNRs. To test the robustness of the EM-based receiver to nonperfect knowledge of the Doppler frequency, Fig. 4 plots symbol-error performance as a function of the mismatch. The relative channel mismatch η is defined as $\eta = (\hat{BT} - BT)/BT$, where BT is the true normalized Doppler offset and \hat{BT} the estimated value the receiver uses. It can be seen the EM algorithm is quite robust to channel mismatch. Symbol-error probability degrades only slightly when relative channel mismatch is within 60%. The plot also shows that there is an asymmetry and that it is better to overestimate than underestimate.

V. CONCLUSION

We investigated the application of the EM algorithm to sequence estimation for space-time coded systems. Simulation results show that the EM-based algorithms converge quickly for both the quasi-static and nonstatic fading channels and has a

performance that approaches that of an ML receiver with perfect CSI.

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