Improved Space-Time Trellis Codes for Correlated MIMO Channels

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***Abstract*— Space-time code designs commonly rely on the assumption of independent and identically distributed (i.i.d.) Rayleigh channels. However poor scattering conditions can have detrimental effects on the performance of space-time codes. In this paper, we derive new Space-Time Trellis Codes that are robust in the presence of a large variety of propagation conditions. First, robust space-time trellis codes are derived when the transmitter has no channel knowledge. Then, trellis codes exploiting some statistical knowledge of the channel at the transmitter are presented. Codes satisfying those criteria are shown to perform much better in real-world channels than codes designed only for i.i.d. channels.**

I. Introduction

Space-Time signaling for Multiple-Input Multiple-Output (MIMO) systems has drawn considerable attention in the communication society due to the significant gains such systems provide both from the spectral efficiency and link reliability points of view [1]. A common assumption in the design of such signaling is to consider the fading coefficients between the pairs of transmit-receive antennas as independent and identically Rayleigh distributed (i.i.d.). This is however an idealistic situation. In practice, the fading coefficients are correlated and these effects highly influence the performance of the space-time processing.

In a first part of the paper, we design new Space-Time Trellis Codes (STTCs) [2] that are robust in the presence of a large variety of propagation conditions and that do not require any information about the channel at the transmitter. Few papers have addressed this problem [3], [4]. In [3], a phase randomization was introduced in order to achieve good performance of full-rank space-time codes in line-of-sight scenarios. In [4], based on a virtual channel representation, precoding matrices are optimized in order to guarantee the robustness of spatial multiplexing in correlated channels. In this paper, based on the design criterion developed in [6], the robustness of several space-time trellis codes are discussed and new codes are proposed.

In order to improve the performance of space-time codes on correlated channels, it has been proposed to exploit some statistical channel information at the transmitter. However, this has been limited, up to now, to the cases of Spatial Multiplexing, Orthogonal Codes and Delay Diversity [4], [5], [7]–[9]. For other kind of codes, no efficient solution has been proposed. A common way of dealing with the optimization of codes when the transmit correlation is available at the transmitter, is to optimize a precoder designed such that the minimum distance code error matrix is matched to the channel eigenvectors [8], [9]. However, from [10], this way of working is not advised. It only works for orthogonal codes or delay-diversity codes where the code error matrix responsible for the increase of the pairwise error probability in correlated channels is the same as the minimum distance code error matrix. In the second part of the paper, based on the non-linear signal constellations developed in [7], we design non-linear constellations based STTCs that exploit the knowledge of the phase of the transmit correlation coefficient.

II. System Model

We consider a MIMO system with n_t transmit and n_r receive antennas, communicating through a frequency flat-fading channel. Over T symbol durations, a codeword $\mathbf{C} = [\mathbf{c}_1 \ldots \mathbf{c}_T]$ of size $n_t \times T$ is transmitted through n_t transmit antennas. At the k^{th} time instant, transmitted and received signals are related by the following relationship

$$\mathbf{r}_k = \sqrt{E_s}\mathbf{H}\mathbf{c}_k + \mathbf{n}_k \tag{1}$$

where $\mathbf{r}_k$ is the $n_r \times 1$ received signal vector, $\mathbf{H}$ is the $n_r \times n_t$ channel matrix, $\mathbf{n}_k$ is a $n_r \times 1$ zero mean complex additive white Gaussian noise (AWGN) vector with[1] $\mathcal{E}\{\mathbf{n}_k\mathbf{n}_l^H\} = \sigma_n^2 \mathbf{I}_{n_r}\delta[k-l]$, $\mathbf{I}_{n_r}$ is the identity matrix of dimension $n_r \times n_r$. E_s is a energy normalisation factor. The channel is considered as constant over T symbols durations. We assume that the instantaneous channel realizations are unknown at the transmitter and perfectly known at the receive side. Maximum-likelihood (ML) decoding is performed so that the receiver computes the estimates of the transmitted codeword according to

$$\hat{\mathbf{C}} = \arg\min_{\mathbf{C}} \sum_{k=1}^{T} \left\| \mathbf{r}_k - \sqrt{E_s}\mathbf{H}\mathbf{c}_k \right\|^2 \tag{2}$$

where the minimization is performed over all possible codeword vectors $\mathbf{C}$.

For a given channel realization $\mathbf{H}$, the conditional pairwise error probability (PEP), i.e. the probability of decoding another

[1] In this paper, $\mathcal{E}$ stands for expectation, ^{T} for transposition, * for elementwise conjugation, ^{H} for conjugate transpose, $|.|$ is the absolute value, $\|.\|_F$ is the Frobenius norm, $\text{Tr}\{\mathbf{A}\}$ is the trace of matrix $\mathbf{A}$.

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codeword $\mathbf{E}$ when $\mathbf{C}$ is transmitted, can be written as

$$P(\mathbf{C} \rightarrow \mathbf{E}|\mathbf{H}) = Q\left(\sqrt{\frac{E_s}{2\sigma_n^2}\left\|\mathbf{H}(\mathbf{C}-\mathbf{E})\right\|_F^2}\right) \quad (3)$$

Design of trellis codes for both the unknown and known channel statistics cases are described next.

III. Unknown Channel Statistics

In [6], several code design criteria were derived for the case when the transmitter does not have any information about the channel. In this section, a brief summary of some criteria are presented and new space-time trellis codes are designed based on those criteria.

A. Design Criteria

Let us first derive how $\left\|\mathbf{H}(\mathbf{C}-\mathbf{E})\right\|_F^2$ evolves in the presence of poor scattering conditions at the transmitter side. Rich scattering environments at the transmitter are obtained when the scatterers are located so that the transmit angle spread is large. Situations become problematic when the transmit angle spread decreases, leading to detrimental interactions between the channel and the code. Since robust codes should perform well whatever the channel conditions, they should be robust even with very small angle spreads. In the extreme case of very small angle spread in a direction θ, it was shown in [6] that

$$\|\mathbf{H}(\mathbf{C}-\mathbf{E})\|_F^2 = \left\|(\mathbf{C}-\mathbf{E})^T\mathbf{a}_t(\theta)\right\|^2 \sum_{n=1}^{n_r}\left|\sum_{l=1}^{L}\mathbf{H}_l(n,1)\right|^2 \quad (4)$$

where $\mathbf{a}_t(\theta)$ is the transmit array response in the direction of departure θ.

Equation (4) expresses that in poor scattering conditions at the transmit side, the MIMO channel degenerates into a SIMO channel where the $1 \times T$ transmitted codeword is given by $\mathbf{a}_t^T(\theta)\,\mathbf{C}$. Note that $\left\|(\mathbf{C}-\mathbf{E})^T\mathbf{a}_t(\theta)\right\|^2$ is nothing else than the Euclidean distance between the $1 \times T$ codewords $\mathbf{a}_t^T(\theta)\,\mathbf{C}$ and $\mathbf{a}_t^T(\theta)\,\mathbf{E}$. Since a space-time code designed for i.i.d. channels is only concerned with $\mathbf{C}$ and $\mathbf{E}$, its interaction with $\mathbf{a}_t(\theta)$ is not taken into account. So, there is no guarantee that a space-time code designed for i.i.d. channels will perform well on correlated channels.

The following definition will be extensively used in the sequel

Definition 1: The *error matrix-related array factor* $G_{\mathbf{C}-\mathbf{E}}(\theta|\mathcal{C}_{\text{MIMO}}, \mathbf{a}_t(\theta))$ of a code $\mathcal{C}_{\text{MIMO}}$ is defined as the following quantity

$$G_{\mathbf{C}-\mathbf{E}}(\theta|\mathcal{C}_{\text{MIMO}}, \mathbf{a}_t(\theta)) = \min_{\substack{\mathbf{C},\mathbf{E}\\ \mathbf{C}\neq\mathbf{E}}} \left\|(\mathbf{C}-\mathbf{E})^T\mathbf{a}_t(\theta)\right\|^2 \quad (5)$$

Focusing on the minimization of the maximum PEP (i.e. optimization of the worst case), based on equation (4), the following design criterion was proposed in [6]

Code design criterion 1: Over the space of candidate codes $\mathcal{C}$ of interest, choose the code $\mathcal{C}_{\text{optimal}} \in \mathcal{C}$ such that

$$\mathcal{C}_{\text{optimal}} = \arg\max_{\mathcal{C}}\min_{\theta} G(\theta|\mathcal{C}, \mathbf{a}_t(\theta)) \quad (6)$$

i.e. choose the code with the largest minimum error matrix related array factor over all directions of departure.

The code design criterion 1 requires the knowledge of the signal constellation, the inter-element distance and the geometry of the transmit antenna array.

From a transmit diversity point of view, the design criterion 1 was shown to maximize the coding gain in correlated channels. It corresponds for correlated channels what the determinant criterion [2] or the trace criterion [12] are for i.i.d. Rayleigh channels.

It was also shown that the minimum of the error matrix related array factor over all directions of departure θ is lower bounded by $n_t\lambda_{\text{min}}$

$$\min_{\theta} G(\theta|\mathcal{C}, \mathbf{a}_t(\theta)) \geq n_t\lambda_{\text{min}} \quad (7)$$

with λ_{min} the smallest eigenvalue of $(\mathbf{C}-\mathbf{E})^*(\mathbf{C}-\mathbf{E})^T$ evaluated over all pairs of codewords $\mathbf{C}$, $\mathbf{E}$ with $\mathbf{C} \neq \mathbf{E}$.

Thanks to this property, it is possible to design robust space-time codes independently of the inter-element spacing and the shape of the transmit antenna array.

Code design criterion 2: Over the space of candidate codes $\mathcal{C}$ of interest, choose the code $\mathcal{C}_{\text{optimal}} \in \mathcal{C}$ such that

$$\mathcal{C}_{\text{optimal}} = \arg\max_{\mathcal{C}}\min_{\substack{\mathbf{C},\mathbf{E}\\ \mathbf{C}\neq\mathbf{E}}}\min_{i=1\ldots n_t}\lambda_i(\widetilde{\mathbf{E}}) \quad (8)$$

i.e. that maximizes the smallest eigenvalue of $\widetilde{\mathbf{E}} = (\mathbf{C}-\mathbf{E})^*(\mathbf{C}-\mathbf{E})^T$ evaluated over all pairs of codewords $\mathbf{C}$, $\mathbf{E}$ with $\mathbf{C} \neq \mathbf{E}$.

B. Design Examples

In i.i.d. channels, assuming a large number of receive antennas in the design of STTCs leads to the trace criterion [11], [12]. Codes satisfying this criterion have been shown to outperform previously developed codes based on the determinant criterion [2]. Hence full-rank codes performing well in terms of the trace criterion have been chosen and a unitary precoder has been optimized in the sense of design criterion 1.

In Fig. 1, we give the error matrix related array factor for 3 QPSK 4-state codes: "TSC" proposed in [2], "CYV" proposed in [12], and a new proposed code consisting of the "CYV" code combined with an optimized unitary precoder. We also present 4 QPSK 8-state codes: "TSC" proposed in [2], "CYV" proposed in [12], a new code performing as well as the "CYV" code on i.i.d. channels and the new code combined with an optimized unitary precoder. Unitary precoders have been randomly generated and the one performing best in terms of design criterion 1 has been chosen. For the 8-states case, a new code has been proposed and a precoder optimized because the "CYV" combined with a precoder could not lead to large error matrix related array factor. The 4 and 8-states "TSC" present a good error matrix related array factor in all directions of departure. On the other hand, the 4 and 8-states "CYV" codes have the drawback that for some directions of departure, the error matrix related array factor can be quite low. The proposed code combining the "CYV"/new code and the unitary precoder present a much better error matrix related array factor compared to the "CYV" code. Its performance on correlated channels should then be better. By combining a code optimized following the trace criterion and an unitary precoder designed following design criterion 1, the code should present good performance on i.i.d. channels and also be robust against fading correlations, as it will be confirmed below.

In Fig. 2 and Fig. 3, we simulate the frame error rate of the trellis codes in the presence of i.i.d. channels and correlated channels for 4 receive antennas. Each frame consists of 130 symbols out of each transmit antennas. For generating a correlated channel, we use the channel model detailed in [13]. A inter-element spacing of 20λ at the base station, and a spacing of 0.5λ at the mobile station in poor scattering conditions has been considered. Linear horizontal arrays are considered. The scatterers are mainly situated in the direction corresponding to an angle of departure equal to $\pi/4$ and 2.7π for Fig. 2 and Fig. 3 respectively. Those directions are such that the simulated codes present their worse performance. We see that the "CYV" code, while performing well on i.i.d. channels, highly suffers from correlations. On the other hand the degradations undergone by the "TSC" code are much more limited. The precoded 4-state "CYV" code and the new precoded 8-state code offer the best performance on i.i.d. and correlated channels. This motivates the design of space-time codes based on two criteria: one for ideal channels and one for correlated channels.

It is worth noting that the 4-state and 8-state "CYV" codes as well as the 8-state new code achieve the lower bound (7) for some directions of departure. While this lower bound is not affected by the use of a precoder, the error matrix related array factor is highly affected. This explains why even a code with small lower bound (7) can perform well in correlated channels. The 8-state "TSC" has a much larger lower bound than the 8-state new code, but its performance on correlated channels is worse for the considered antenna array.

Space-time codes that are robust whatever the shape of the transmit antenna array should be designed based on criterion 2. However, such codes should also maintain good performance in terms of minimum distance. Simulations have shown that in practice it is difficult to find codes that have large minimum distance and large $\lambda_{\min}$. Therefore, designing codes based on criterion 1 should always be preferred.

As explained in section *A*, the MIMO transmission degenerates into a SIMO transmission on highly correlated channels. A SIMO transmission performs thus better[2] than a MIMO one if the minimum euclidean distance of the SIMO transmission is larger than the error matrix related array factor of the the MIMO transmission. Comparison with a QPSK based uncoded SIMO transmission, presenting a minimum squared Euclidean distance equal to 2, leads to the conclusion that the uncoded QPSK SIMO transmission will only be outperformed by the precoded 8-state new code with precoder. The others codes present error matrix related array factor equal, lower and even much lower than 2.

[2]To be more rigorous, the distance spectrum should be taken into account.

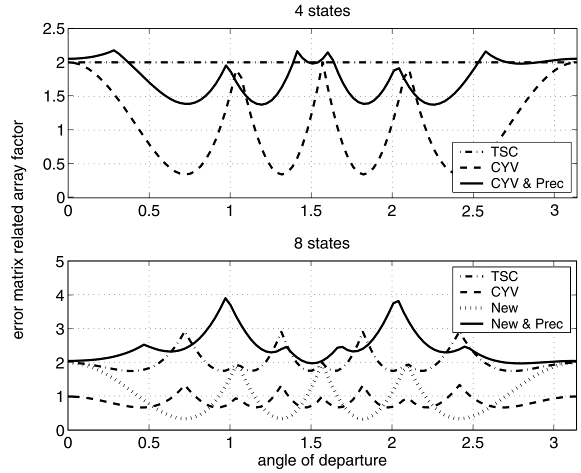

Fig. 1. Error matrix related array factor of several 4 and 8 states STTC as a function of the angle of departure θ [rad].

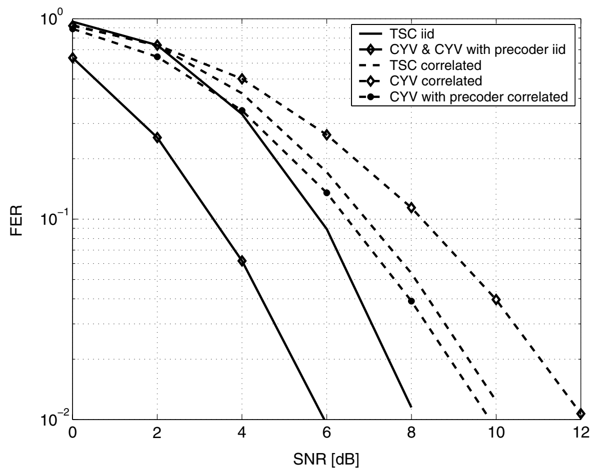

Fig. 2. Frame error rate of several 4 states STTC in i.i.d. and correlated channels with $n_t = 2$ and $n_r = 4$.

IV. KNOWN CHANNEL STATISTICS

In this section we exploit the non-linear constellations developed in [7] to derive new space-time trellis codes that exploit the transmit diversity and also the transmit array gain offered by the knowledge of the phase of the transmit correlation.

As explained in [7], the non-linear signal constellations consists in introducing some dependencies between the simultaneously transmitted symbols: when a symbol $S_m \in \mathbf{S}$ is transmitted on antenna 1, a symbol $Q_{mn} \in \mathbf{Q}_m$ will be transmitted on antenna 2. The symbol transmitted on the second antenna is thus chosen in a constellation $\mathbf{Q}_m$ that

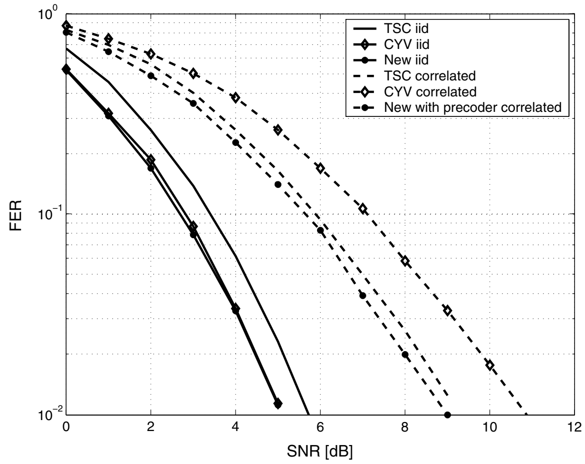

Fig. 3. Frame error rate of several 8 states STTC in i.i.d. and correlated channels with $n_t = 2$ and $n_r = 4$.

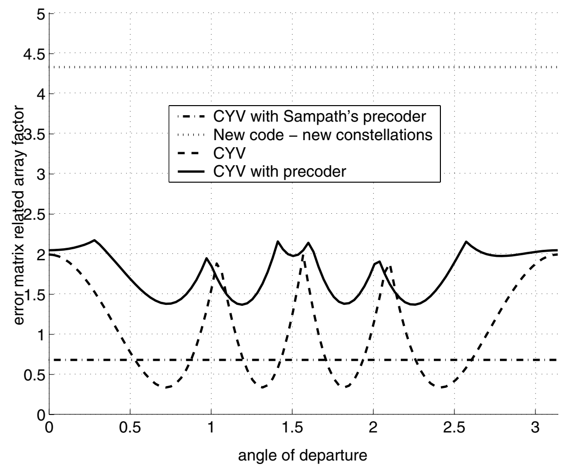

Fig. 4. Error matrix related array factor of several 4 and 8 states STTC as a function of the angle of departure θ [rad].

depends on the symbol transmitted on the first antenna. At the k^{th} time instant, the transmitted vector consists in

$$\left[\begin{array}{c} S_m \\ Q_{mn} \end{array}\right] = \underbrace{\left[\begin{array}{cc} 1 & 0 \\ 0 & t/|t| \end{array}\right]}_{\text{precoder } \mathbf{P}} \left[\begin{array}{c} S_m \\ K_{mn} \end{array}\right] \tag{9}$$

where t stands for the transmit correlation coefficient. Constellations $\mathbf{S} = \{S_m\}$ and $\mathbf{K}_m = \{K_{mn}\}$ have been optimized in order to minimize an estimate of the error probability averaged over a Rayleigh channel with transmit correlation. Such constellations allow to keep the same spectral efficiency than a Spatial Multiplexing scheme while, at the same time, they benefit from some array gain by transmitting along the main eigenvector of the transmit correlation matrix.

In [11], [12], it has been shown that assuming a large number of receive antennas in the design of space-time trellis codes results in improved performance compared to codes developed based on the rank-det [2] criterion. We will therefore also use this assumption to design the trellis code.

Assuming a large number of receive antennas,

$$\lim_{\substack{n_r \to \infty \\ n_t \text{ finite}}} \frac{1}{n_r} \mathbf{H}^H \mathbf{H} = \mathcal{E}_{\mathbf{H}} \left\{ \mathbf{H}^H \mathbf{H} \right\} = \mathbf{R}_t \tag{10}$$

and minimizing the PEP (3) comes to maximize a modified euclidean distance d^2 that takes into account the transmit correlation

$$d^2 = \mathrm{Tr} \left\{ \mathbf{R}_t \left(\mathbf{C} - \mathbf{E} \right) \left(\mathbf{C} - \mathbf{E} \right)^H \right\} \tag{11}$$

Let $\mathbf{C_K}$ be a codeword of a code designed for the constellations $\mathbf{S}$ and $\{\mathbf{K}_m\}$, the physically transmitted codeword $\mathbf{C}$ is then given by $\mathbf{C} = \mathbf{PC_K}$. Thanks to the presence of the precoder $\mathbf{P}$, d^2 can be re-expressed as

$$d^2 = (1 - |t|) \left\| \mathbf{C_K} - \mathbf{E_K} \right\|_F^2 + |t| \left\| (\mathbf{C_K} - \mathbf{E_K})^T \left[\begin{array}{cc} 1 & 1 \end{array} \right] \right\|^2 . \tag{12}$$

Focusing on the minimum distance, we see that the minimum euclidean distance as seen by the receiver is decomposed into two terms: the first is the minimum euclidean distance of the code based on constellations $\mathbf{S}$ and $\{\mathbf{K}_m\}$, the second is the error matrix related array factor of the same code, based on the same constellations, and evaluated in the direction of departure corresponding to the phase of t equal to 0.

The optimal way of treating the problem of designing a space-time trellis codes for correlated channels when the statistical behavior is known at the transmitter would be to jointly adapt the code and the constellations depending on the channel conditions. However it would require to adapt the constellations and the code each time the environment changes. That can be unrealistic to implement. Therefore, since the non-linear constellations have been shown to perform well on i.i.d. and correlated channels, we resorted to choose a full-rank code that has large minimum euclidean distance and large error matrix related array factor. It is important to note that a compromise has to be found since it is impossible to find a code that is optimal for both quantities we want to maximize. The so designed codes have the advantage of being very simple to implement, while still offering gains that are are far from being negligible. A huge difference with the beamforming in the dominant mode, in which the same symbol is transmitted on all antennas, is that the proposed scheme transmits different symbols on each antenna, while still exploiting the transmit array gain. For a same spectral efficiency, on highly correlated channels, the proposed trellis codes exploit the array gain and the coding gain (coding inherent from the trellis code) while the beamforming approach only exploits the array gain. The difference between these two scheme should increase as the number of states in the trellis increases.

The difference with the unknown case is that now, since we know the phase of the transmit correlation coefficient, we actually know, somehow, the direction where the scatterers are located. The error matrix related array factor can then be maximized in that direction.

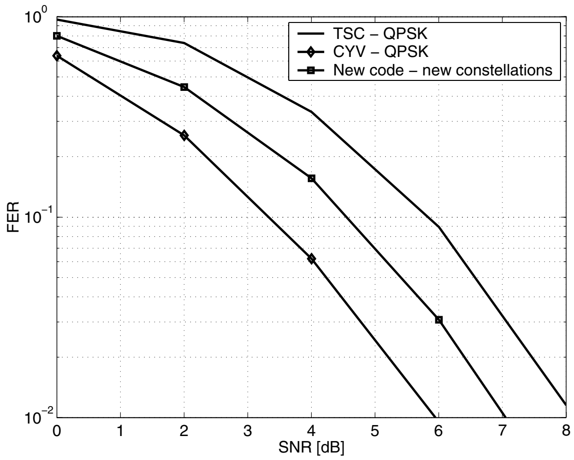

Fig. 5. Frame error rate of several 4 states STTC in i.i.d. channels with $n_t = 2$ and $n_r = 4$.

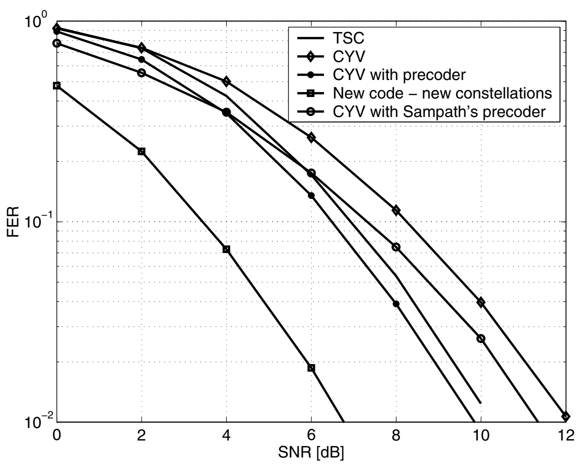

Fig. 6. Frame error rate of several 4 states STTC in correlated channels with $n_t = 2$ and $n_r = 4$.

In Fig. 4, comparisons between a 4-states non-linear constellations based STTC and other 4-states codes are presented. We see that the error matrix related factor of the non-linear constellations based STTC is a least twice as large as the error matrix related array factor of the other codes, thanks to the efficient exploitation of the knowledge of the phase of the transmit correlation coefficient required by the non-linear constellations. Comparisons with the "CYV" combined with a precoder optimized as in [9] are also displayed. As explained in [10], such a precoder is not optimal since it matches the minimum distance error matrix to the channel eigenvectors. We see that the gain offered by this precoder is very limited.

In Fig. 5 and Fig. 6, the performance has been evaluated in the presence of i.i.d. and correlated channels. As expected from the error matrix related array factor, the gain on correlated channels is quite important. On i.i.d. channels, since our code exploits the non-linear constellations, the performance of the new code is not the best since the non-linear constellations are not optimal on i.i.d. channels. However its performance is quite good and the code satisfies a good compromise between performing well on i.i.d. and on correlated channels with a limited complexity. Indeed, the only parameter to be known at the transmitter is the phase of the transmit correlation coefficient. No precoder has to be optimized based on a waterfilling solution.

V. Conclusions

Space-time codes designs commonly assume independent and identically Rayleigh distributed channels. Real-world channels are however correlated and code performance is affected by the non-ideality of those propagation conditions. Robust space-time trellis codes for the case of unknown and known channel statistics at the transmitter have been designed and shown to perform much better on correlated channels than codes designed only for i.i.d. channels.

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