Mathematics for Science — cleaner notes & corrections (surds, roots, operations)

Nice work — you've captured lots of key ideas. I cleaned and corrected the notes, explained the mistakes, and added a few practice problems with answers so you can check your skill.

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1. Quick definitions / corrections

- **Rational numbers** are numbers that **can** be written as pq\dfrac{p}{q}qp where p,qp,qp,q are integers and q≠0q\neq0q□=0.
  Examples: 34, −2, 0.5 \dfrac{3}{4},\; -2,\; 0.543,−2,0.5.
- **Irrational numbers** are numbers **that cannot** be written as pq\dfrac{p}{q}qp.
  Examples: 2, π, e\sqrt{2},\; \pi,\; e2,π,e.
- **Surd**: a root that cannot be simplified to a rational number (e.g. 2,3\sqrt{2},\sqrt{3}2,3).
  More generally an **nth root** an \sqrt[n]{a}na can be written n□a \;{}^{n}\!\sqrt{a}na or a1/na^{1/n}a1/n.
  If you use the symbol \sqrt{\;} with no index, it means **square root** (index 2).
- When you wrote "order": that means the **index** of the root.
  Examples:
  - 3\sqrt{3}3 has index (order) **2** (square root).
  - 113\sqrt[3]{11}311 (cube root of 11) has index **3**.
  - 74 \sqrt[4]{7}47 has index **4**.
    Notation: nan\sqrt{a}na often means "nnn times the square root of aaa" (coefficient), not the nth root — be careful. Use an \sqrt[n]{a}na if you mean the nth root.

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2. Addition & subtraction of surds

- You **can only add/subtract like surds**: they must have the **same radicand** (the number under the root) **and the same index**.
  - Example: 23+53=732\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}23+53=73.
  - Example: 2+23\sqrt{2} + \sqrt[3]{2}2+32 — cannot combine (different index).
- Always simplify surds first (factor perfect powers out).

Your example:

27+73−243\sqrt{27} + 7\sqrt{3} - \sqrt{243}27+73−243

Simplify:

- 27=9·3=33\sqrt{27} = \sqrt{9\cdot3} = 3\sqrt{3}27=9·3=33
- 243=81·3=93\sqrt{243} = \sqrt{81\cdot3} = 9\sqrt{3}243=81·3=93
  So: 33+73−93=(3+7−9)3=13=33\sqrt{3} + 7\sqrt{3} - 9\sqrt{3} = (3+7-9)\sqrt{3} = 1\sqrt{3} = \sqrt{3}33+73−93=(3+7−9)3=13=3. ☑

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3. Multiplication & division of surds

- **Multiplication**: multiply coefficients, multiply radicands, and then simplify:

  (am)(bn)=(ab)mn.(a\sqrt{m})(b\sqrt{n})=(ab)\sqrt{mn}.(am)(bn)=(ab)mn.

  Example: 25×37=6352\sqrt{5}\times 3\sqrt{7}=6\sqrt{35}25×37=635. ☑

- **Division**: treat the roots appropriately. If indices are both 2 (square roots), you can use:

  ambn=abmn\frac{a\sqrt{m}}{b\sqrt{n}}=\frac{a}{b}\sqrt{\frac{m}{n}}bnam=banm

  but **better** simplify if one radical is a perfect square.

  Example you wrote: 614129 \dfrac{6\sqrt{141}}{2\sqrt{9}}296141.

  Since 9=3\sqrt{9}=39=3, the denominator 29=2×3=62\sqrt{9}=2\times3=629=2×3=6. So

  614129=61416=141.\dfrac{6\sqrt{141}}{2\sqrt{9}}=\dfrac{6\sqrt{141}}{6}=\sqrt{141}.296141=66141=141.

  (That's the simplest form.) ✔

  Alternatively, if you wanted to keep radicals:

  621419=3473,\frac{6}{2}\sqrt{\frac{141}{9}}=3\sqrt{\frac{47}{3}},269141=3347,

  but 141\sqrt{141}141 is simpler.

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4. Binomial surds & conjugates

- **Conjugate** of a+ba+ba+b is a−ba-ba−b. For surds the conjugate of x+yx+\sqrt{y}x+y is x−yx-\sqrt{y}x−y.

- Multiplying conjugates removes the surd (difference of squares): (a+b)(a−b)=a2−b2(a+b)(a-b)=a^2-b^2(a+b)(a−b)=a2−b2.

Examples:

- Rationalizing: 13+2=3−2\dfrac{1}{\sqrt{3}+\sqrt{2}}=\sqrt{3}-\sqrt{2}3+21=3−2 because denominator becomes 3−2=13-2=13−2=1.
- Your expansion: (4+25)(3−37)(4+2\sqrt{5})(3-3\sqrt{7})(4+25)(3−37) — expand term-by-term:

  4(3−37)+25(3−37)=12−127+65−635.4(3-3\sqrt7)+2\sqrt5(3-3\sqrt7) =12-12\sqrt7+6\sqrt5-6\sqrt{35}.4(3−37)+25(3−37)=12−127+65−635.

  That is correct. ☑

- Your conjugate example: (5−3)(5+3)=25−3=22(5-\sqrt3)(5+\sqrt3)=25-3=22(5−3)(5+3)=25−3=22. ☑

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5. Common mistakes & tips

- “Irrational numbers are numbers that can be expressed in terms of p/q” — **wrong**. Fix: irrationals **cannot** be written as p/qp/qp/q.
- Be careful with notation: 3113\sqrt{11}311 means 3×113\times\sqrt{11}3×11, not the cube root of 11. Use 113\sqrt[3]{11}311 for cube root.
- Always simplify radicals before combining like terms.
- When dividing, check if any radicand is a perfect square (like 9=3\sqrt{9}=39=3) — this often simplifies the whole expression.

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6. Practice (try these; answers below)

1. Simplify: 52−28+185\sqrt{2} - 2\sqrt{8} + \sqrt{18}52−28+18.
2. Simplify: 41523\dfrac{4\sqrt{15}}{2\sqrt{3}}23415.
3. Rationalize and simplify: 23+5\dfrac{2}{3+\sqrt{5}}3+52.
4. Multiply out and simplify: (6+2)(6−2)(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})(6+2)(6−2).

Answers

1. 52−28+18=52−2(22)+32=52−42+32=42.\;5\sqrt{2}-2\sqrt{8}+\sqrt{18} = 5\sqrt{2}-2(2\sqrt{2}) + 3\sqrt{2} = 5\sqrt{2}-4\sqrt{2}+3\sqrt{2} = 4\sqrt{2}.52−28+18=52−2(22)+32=52−42+32=42.

2. 41523=42153=25.\dfrac{4\sqrt{15}}{2\sqrt{3}} = \dfrac{4}{2}\sqrt{\dfrac{15}{3}} = 2\sqrt{5}.23415=24315=25. (Also note 15/3=5\sqrt{15}/\sqrt{3}=\sqrt{5}15/3=5).
3. Multiply top & bottom by conjugate 3−53-\sqrt53−5:

   23+5⋅3−53−5=2(3−5)9−5=2(3−5)4=3−52.\frac{2}{3+\sqrt5}\cdot\frac{3-\sqrt5}{3-\sqrt5}=\frac{2(3-\sqrt5)}{9-5}=\frac{2(3-\sqrt5)}{4}=\frac{3-\sqrt5}{2}.3+52⋅3−53−5 =9−52(3−5)=42(3−5)=23−5.

4. (6+2)(6−2)=6−2=4.(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})=6-2=4.(6+2)(6−2)=6−2=4.