Spectral Reconstruction in X-Ray Computed Tomography

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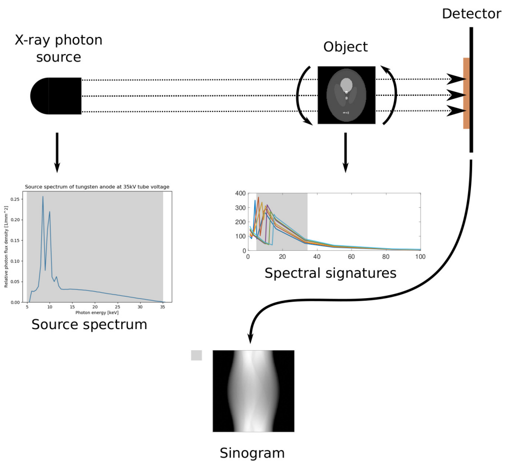
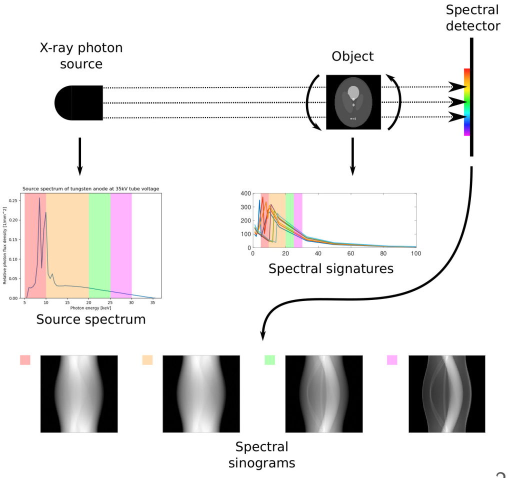
Spectral Imaging

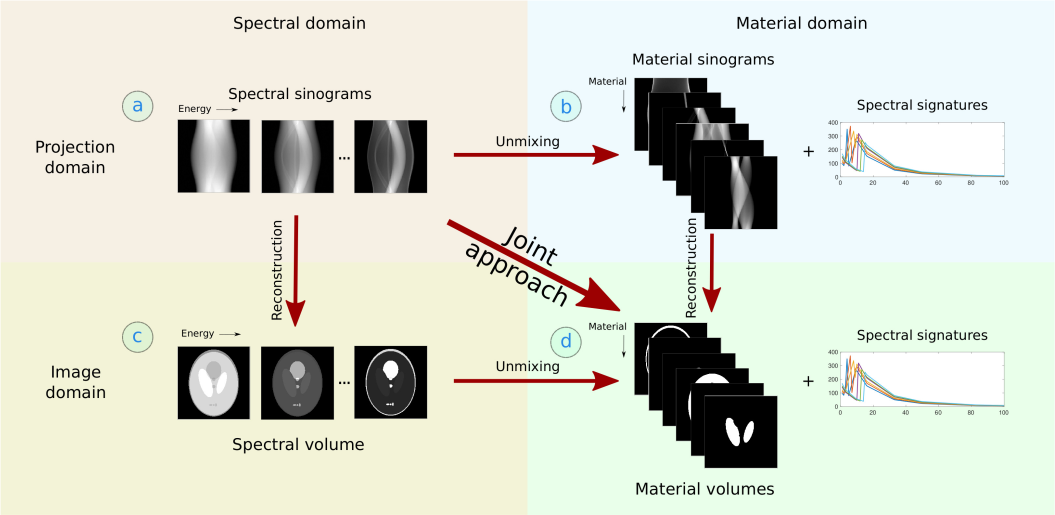
Conventional X-ray CT



Spectral X-ray CT



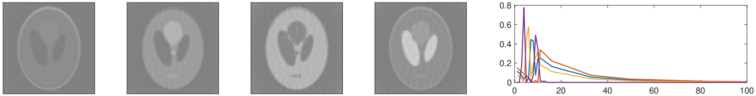
Spectral Imaging Methods



Performance

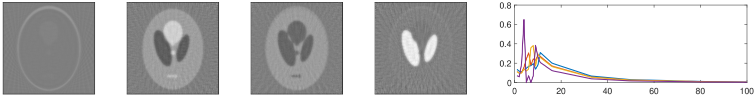
True

Reconstruction then Unmixing



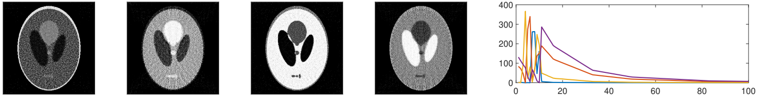
✗ ill-posed

Unmixing then Reconstruction



✗ ill-posed

Vanilla Joint



✗ill-conditioned

ADJUST

A **D**ictionary-based **J**oint **U**nmixing of **S**pectral **T**omographic projections

$$\begin{aligned}
\underset{\boldsymbol{A},\boldsymbol{R}}{\text{minimize}} \quad & \mathcal{J}(\boldsymbol{A},\boldsymbol{R}) \triangleq \tfrac{1}{2}\left\|\boldsymbol{Y}-\boldsymbol{WART}\right\|_F^2 && \text{(least-squares misfit)}\\
\text{subject to} \quad & \boldsymbol{A} \in \mathcal{C}_A^{n\times k} \triangleq \left\{0 \le a_{ij} \le 1, \boldsymbol{A}\mathbf{1} = \mathbf{1}\right\} && \text{(spatial map)}\\
& \boldsymbol{R} \in \mathcal{C}_R^{k\times p} \triangleq \left\{0 \le r_{ij} \le 1, \boldsymbol{R}\mathbf{1} = \mathbf{1}, \boldsymbol{R}^T\mathbf{1} \le \mathbf{1}\right\} && \text{(dictionary coefficients)}
\end{aligned}$$

ADJUST: determine $(n+p)k$ unknowns from mc measurements by solving a bi-convex program

- ✔ Implicit regularization on material maps and spectral responses
- ✔ May be a well-posed & better conditioned problem: unknowns $\ll$ measurements
- ✗ Strong assumption on material spectral response

Optimization

Accelerated Alternating Proximal Updates

for $j = 0, \ldots, J-1$

$$\boldsymbol{R}_{j+1} = \mathbf{prox}_{\mathcal{C}_R^{k\times p}}\Big(\boldsymbol{R}_j - \alpha \frac{\partial}{\partial \boldsymbol{R}} \widetilde{\mathcal{J}}(\boldsymbol{A}_j, \boldsymbol{R}, \boldsymbol{U}_j)\Big) \quad \text{(update dictionary coeff)}$$

$$\boldsymbol{A}_{j+1} = \mathbf{prox}_{\mathcal{C}_A^{n\times k}}\Big(\boldsymbol{A}_j - \beta \frac{\partial}{\partial \boldsymbol{A}} \widetilde{\mathcal{J}}(\boldsymbol{A}, \boldsymbol{R}_{j+1}, \boldsymbol{U}_j)\Big) \quad \text{(update material maps)}$$

$$\boldsymbol{U}_{j+1} = \boldsymbol{U}_j + \rho\,(\boldsymbol{W}\boldsymbol{A}_{j+1}\boldsymbol{R}_{j+1}\boldsymbol{T} - \boldsymbol{Y}) \quad \text{(update running-sum-of-errors)}$$

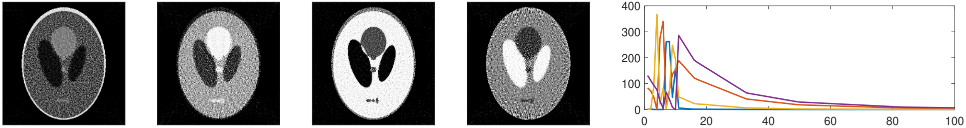
augmented functional: $\widetilde{\mathcal{J}}(\boldsymbol{A}, \boldsymbol{R}, \boldsymbol{U}) = \frac{1}{2}\|\boldsymbol{W}\boldsymbol{A}\boldsymbol{R}\boldsymbol{T} - \boldsymbol{Y}\|_F^2 + \langle \boldsymbol{U}, \boldsymbol{W}\boldsymbol{A}\boldsymbol{R}\boldsymbol{T} - \boldsymbol{Y}\rangle$

- ✔ Simple gradient computations
- ✔ Fast proximal operations
- ✔ Dynamic update of ρ
- ✗ Matrix-Matrix multiplications
- ✗ Initialization matters

Example I

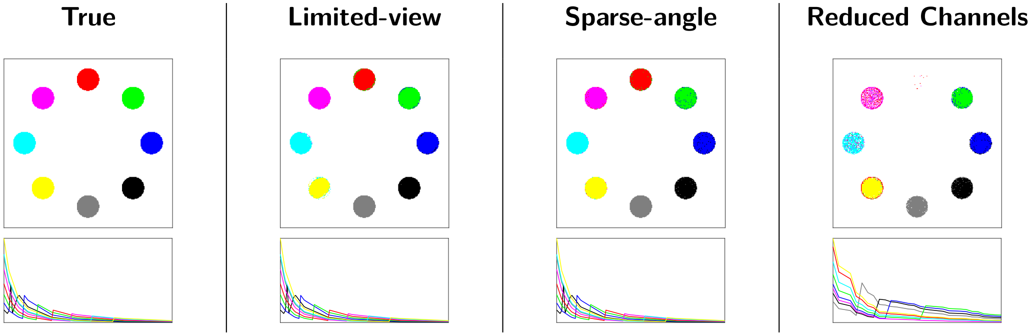
True

Joint



ADJUST

Example II



- **Limited-view**: 60 angles in $[0, \pi/2]$, 100 spectral bins
- **Sparse-angle**: 10 angles in $[0, \pi]$, 100 spectral bins
- **Reduced Channels**: 60 angles in $[0, \pi]$, 30 spectral bins

Conclusions and Outlook

- Spectral Imaging reveals the **material characterization** of the object.
- Two-step methods and vanilla joint approach suffer from **ill-posedness** and **ill-conditioning**.
- ADJUST improves the **spectral reconstructions** of materials present in the object.
- ADJUST can handle **limited measurement** scenarios, *e.g.*,
  - Sparse-angle tomography
  - Limited-view tomography
  - Low spectral resolution
- Proposed formulation is **generic**: can be applied to other unmixing problems.

Future work:

- Testing on real spectral/hyperspectral X-ray datasets
- Experiments on more advanced phantoms, various spectral settings