Problem 1 (20 Points).

(a) What is the DFT of $(1, 0, 0, 0)$? What is the appropriate choice of ω in this case? What sequence is $(1, 0, 0, 0)$ the DFT of?

(b) Repeat for $(1, 0, 1, -1)$

Problem 2 (30 Points).

(a) Say we want to multiply two polynomials $x+1$ and x^2+1 using the FFT. Choose an appropriate power of two, find the FFT of the two sequences, multiply the results componentwise, and compute the inverse FFT to get the final result.

(b) Repeat for the pair of polynomials $1 + x + 2x^2$ and $2 + 3x$.

Problem 3 (20 Points).

(a) What is the sum of the nth roots of unity?

(b) If n is odd, what is the product of the nth root of unity?

(c) What if n is even?

Problem 4 (30 Points). Let $(a_0, a_2, ..., a_{n-1})$ be a sequence, and let $(A_0, ..., A_{n-1})$ be its DFT.

(a) Suppose we construct a new input sequence

$$(a'_0, a'_1, ..., a'_{n-1}) = (a_k, a_{k+1}, ..., a_{n-1}, a_0, a_1, ..., a_{k-1})$$

obtained by rotating the original by k spots. What is the DFT of this sequence in terms of $(A_0, ..., A_{n-1})$, the DFT of the original sequence.

(b) What input sequence would yield the DFT

$$(A'_0, ..., A'_{n-1}) = (A_k, A_{k+1}, ..., A_{n-1}, A_0, ..., A_{k-1}) \ ?$$

(c) What is the DFT of

$$(a_{n-1}, a_{n-2}, ..., a_0)$$

in terms of the A_js?

Total points: 100