Fully Secure Functional Encryption Without Obfuscation

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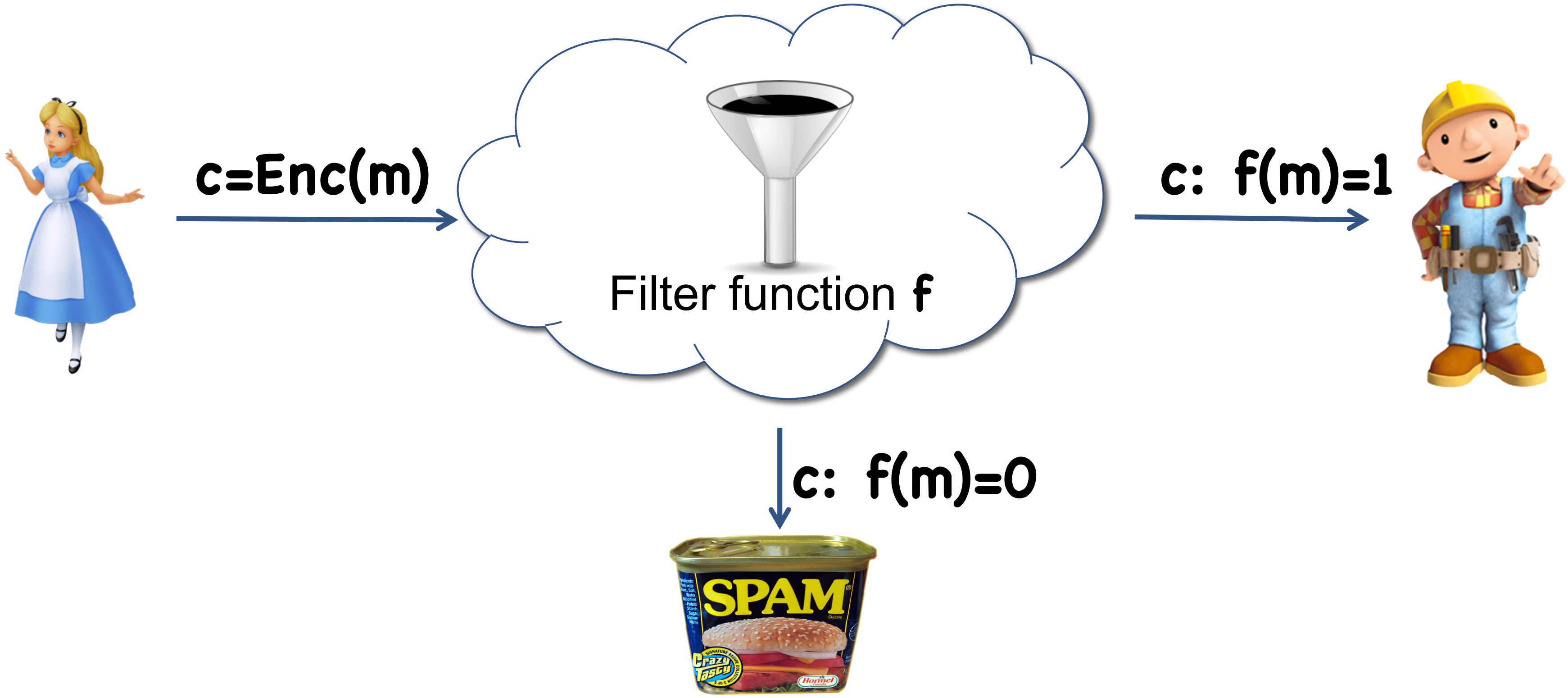
Example: Spam Filter



c=Enc(m)

Filter function **f**

c: f(m)=1

c: f(m)=0

Solution 0: Give cloud **sk** ⇒ cloud learns entire message ✗

Solution 1: Use FHE ⇒ cloud only learns **Enc(f(m))** ✗

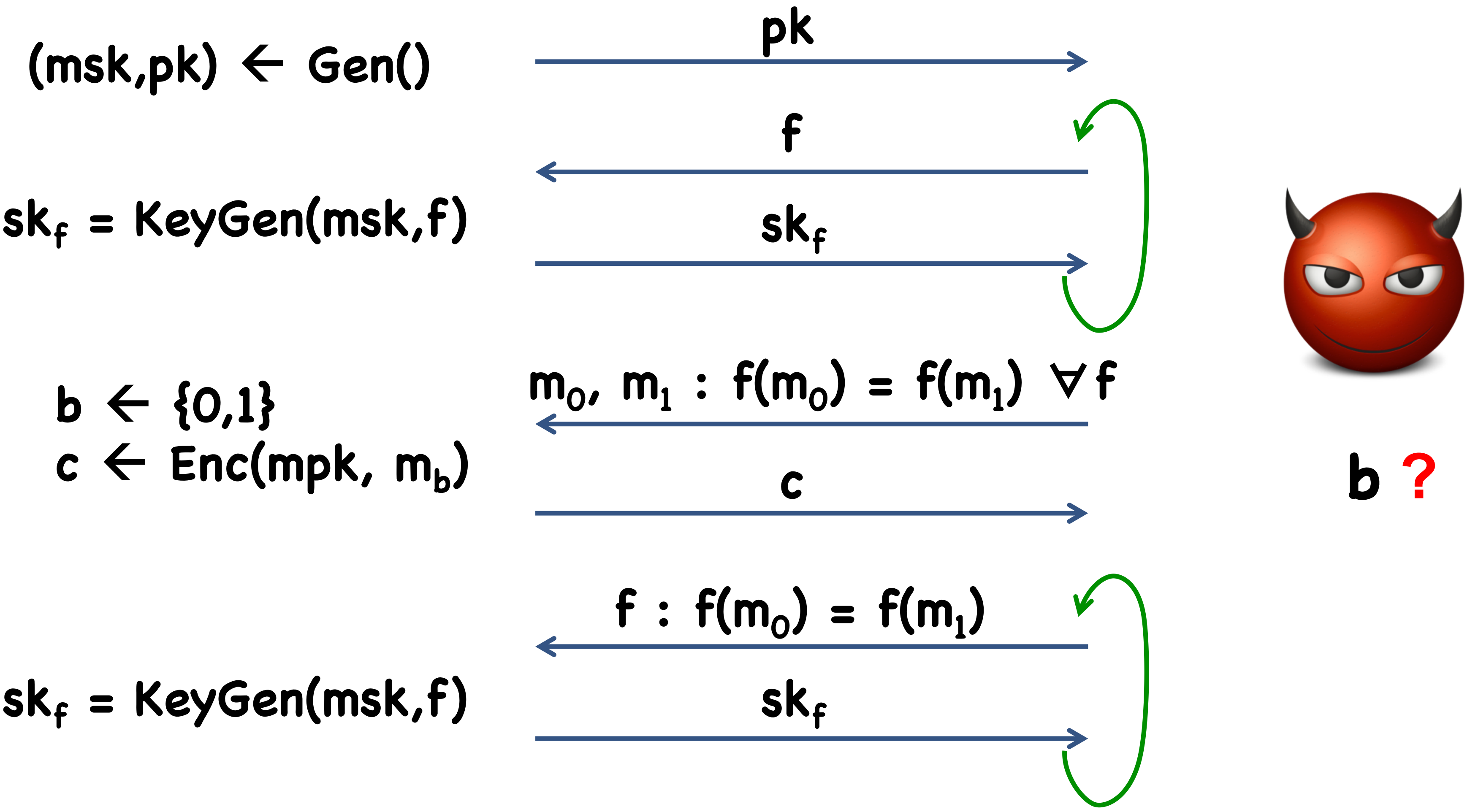
Solution 2: Functional encryption: cloud learns **f(m)**, nothing else ✓

Functional Encryption: Semantics [BSW'11]

Gen():	Output keys **(msk, pk)**
Enc(pk, m):	Output ciphertext **c**
KeyGen(msk, f):	Output decryption key **sk_f**
Dec(sk_f, c):	Output **f(m)**

Functional Encryption: Security [BSW'10,O'N'10]

Unbounded full adaptive game-based security:



$(msk,pk) \leftarrow Gen()$

pk

f

$sk_f = KeyGen(msk,f)$

sk_f

$b \leftarrow \{0,1\}$

$c \leftarrow Enc(mpk, m_b)$

$m_0, m_1 : f(m_0) = f(m_1) \ \forall f$

c

b ?

$f : f(m_0) = f(m_1)$

$sk_f = KeyGen(msk,f)$

sk_f

Before Obfuscation

Tons of work on special cases: IBE, ABE, PE…

[SW’05, BSW’10,O’N’10]: Definitions

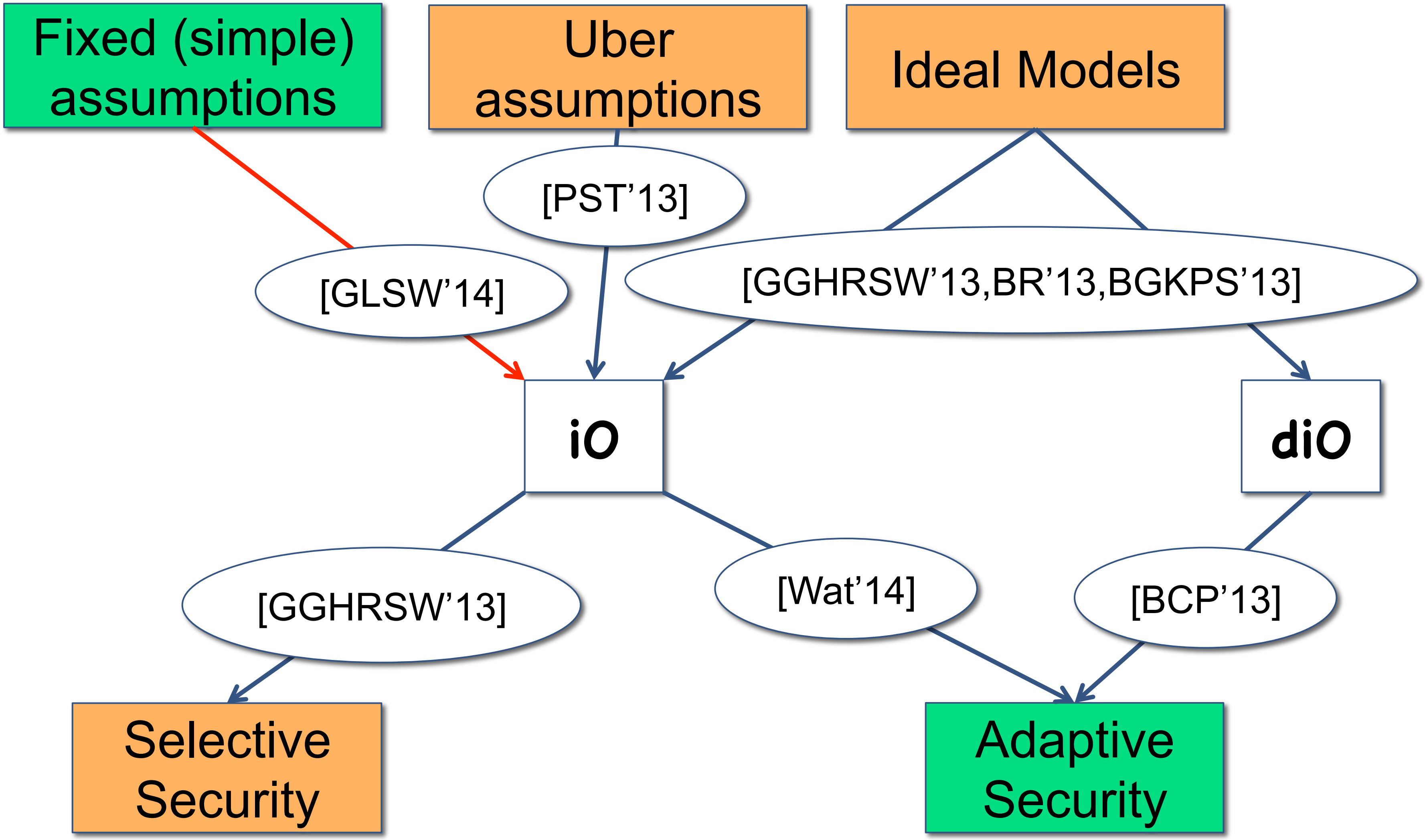
[BW’07,KSW’08,AFV’11,SSW’09]: Simple functions

[SS10,GVW’12,GKPVZ’12]: Bounded number of secret keys

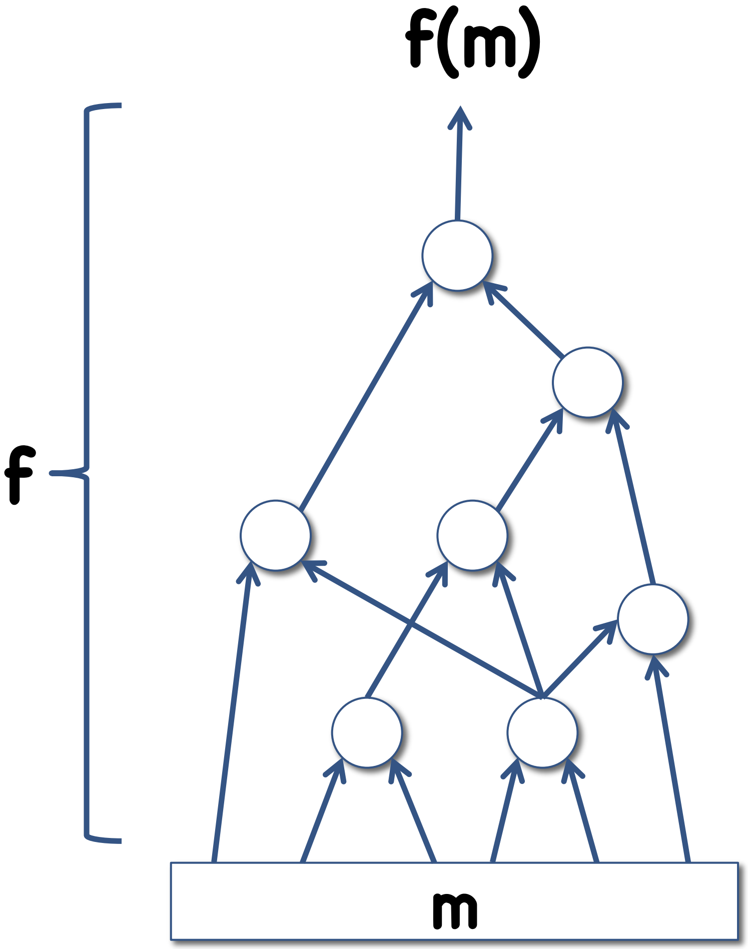
[AGVW’12]: Impossibility of unbounded simulation-based def

No unbounded constructions until…

After Obfuscation: First Unbounded Constructions

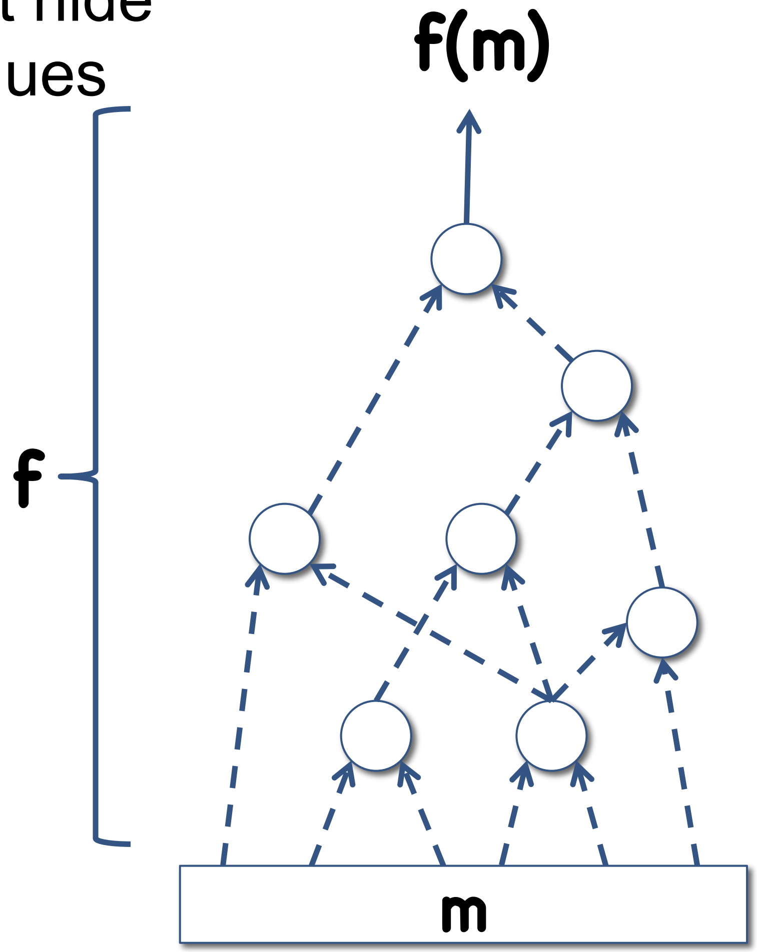


Why Obfuscation Seems Inherent



Why Obfuscation Seems Inherent

Decryption must hide intermediate values



Why Obfuscation Seems Inherent

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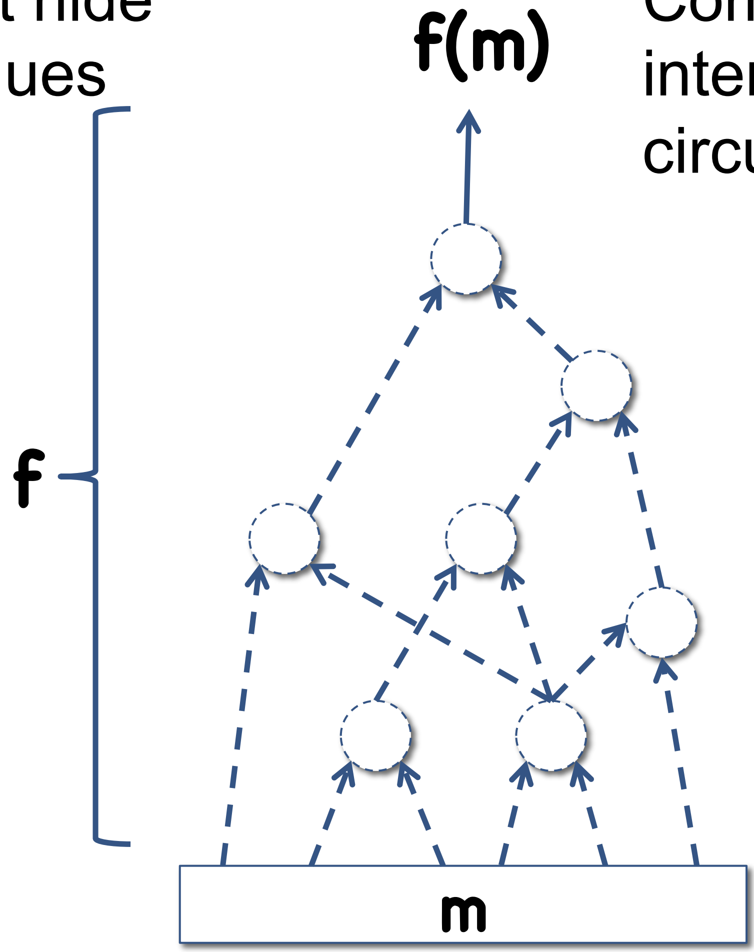


Common ways to hide intermediate values hide circuit too. E.g.

- garbled circuits
- branching progs
- obfuscation

f is now hidden

Note: [BCP'13] does **not** have function hiding

Function Hiding ⇒ IO

```
iO(C):               (msk,pk) ← Gen()
                     sk ← KeyGen(msk,C)
                     Output (pk,sk)

Eval( (pk,sk), x):   e = Enc(pk,x)
                     y = Dec(sk,e)
```

sk hides **C** → indistinguishability obfuscation

Takeaway: FE with function hiding implies iO

Question 1:

Can we build FE without iO?

Why avoid Obfuscation?

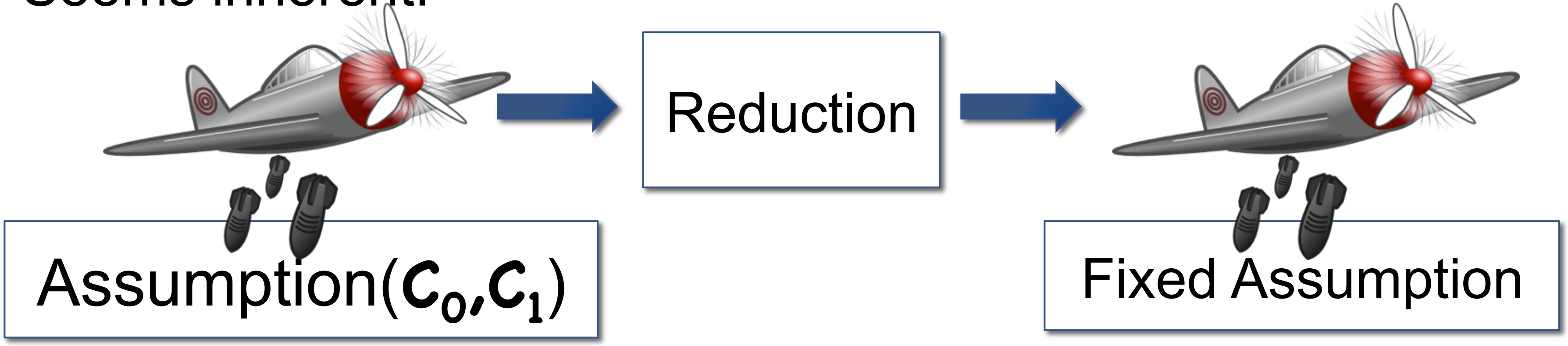
iO = exponentially many assumptions

- One per pair of circuits

Assumption(C_0,C_1):

$$iO(C_0) \approx iO(C_1)$$

Seems inherent:



Assumption(C_0,C_1)

Reduction

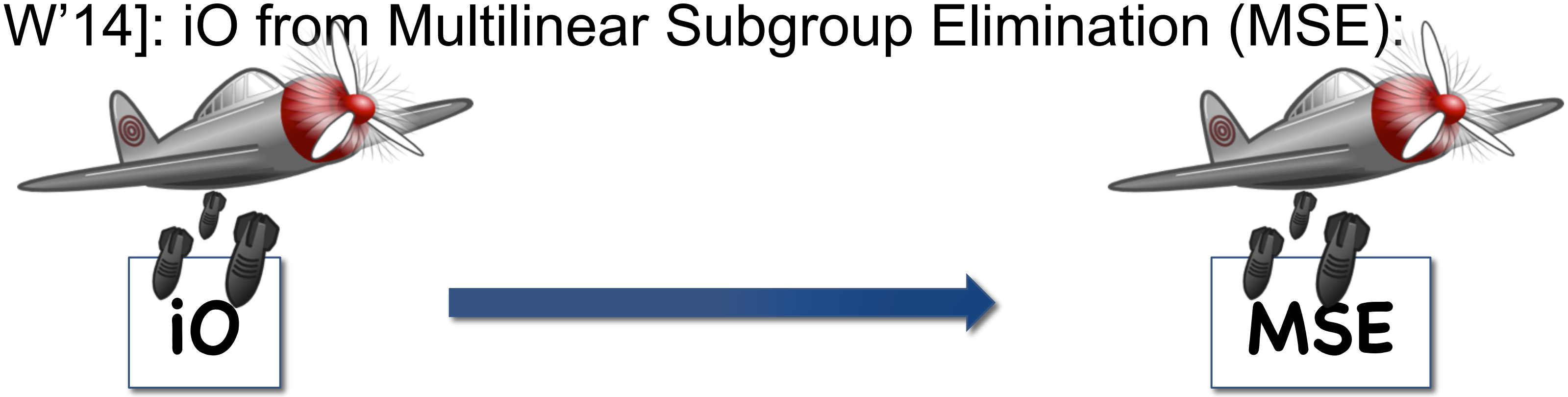
Fixed Assumption

Reduction can only work for equiv C_0, C_1

⇒ must somehow decide equivalence (NP-hard)

What about GLSW?

[GLSW’14]: iO from Multilinear Subgroup Elimination (MSE):



iO

MSE

What about GLSW?

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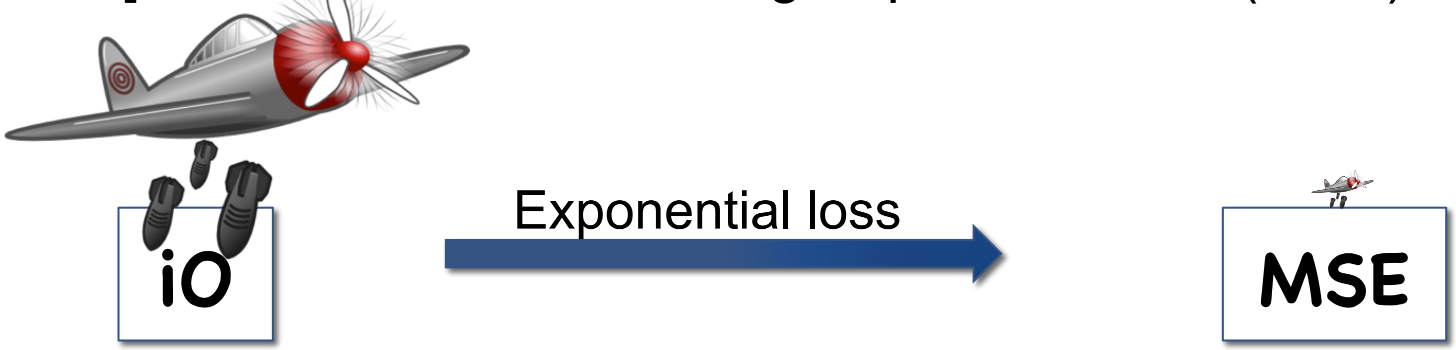


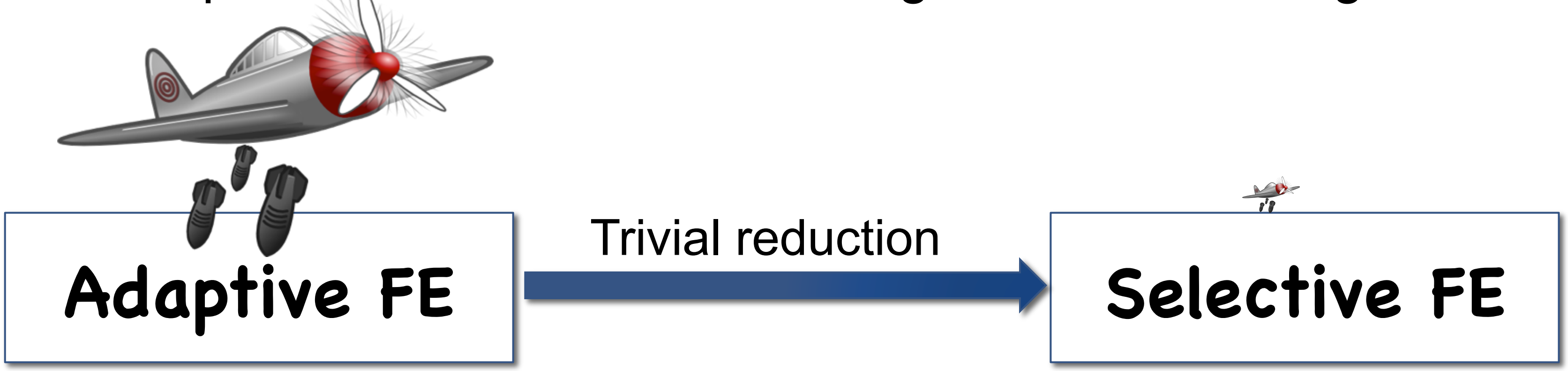
iO

Exponential loss

MSE

- Need to assume MSE **really** hard (complexity leveraging)

Note: Adaptive vs selective FE meaningless in this setting



Adaptive FE

Trivial reduction

Selective FE

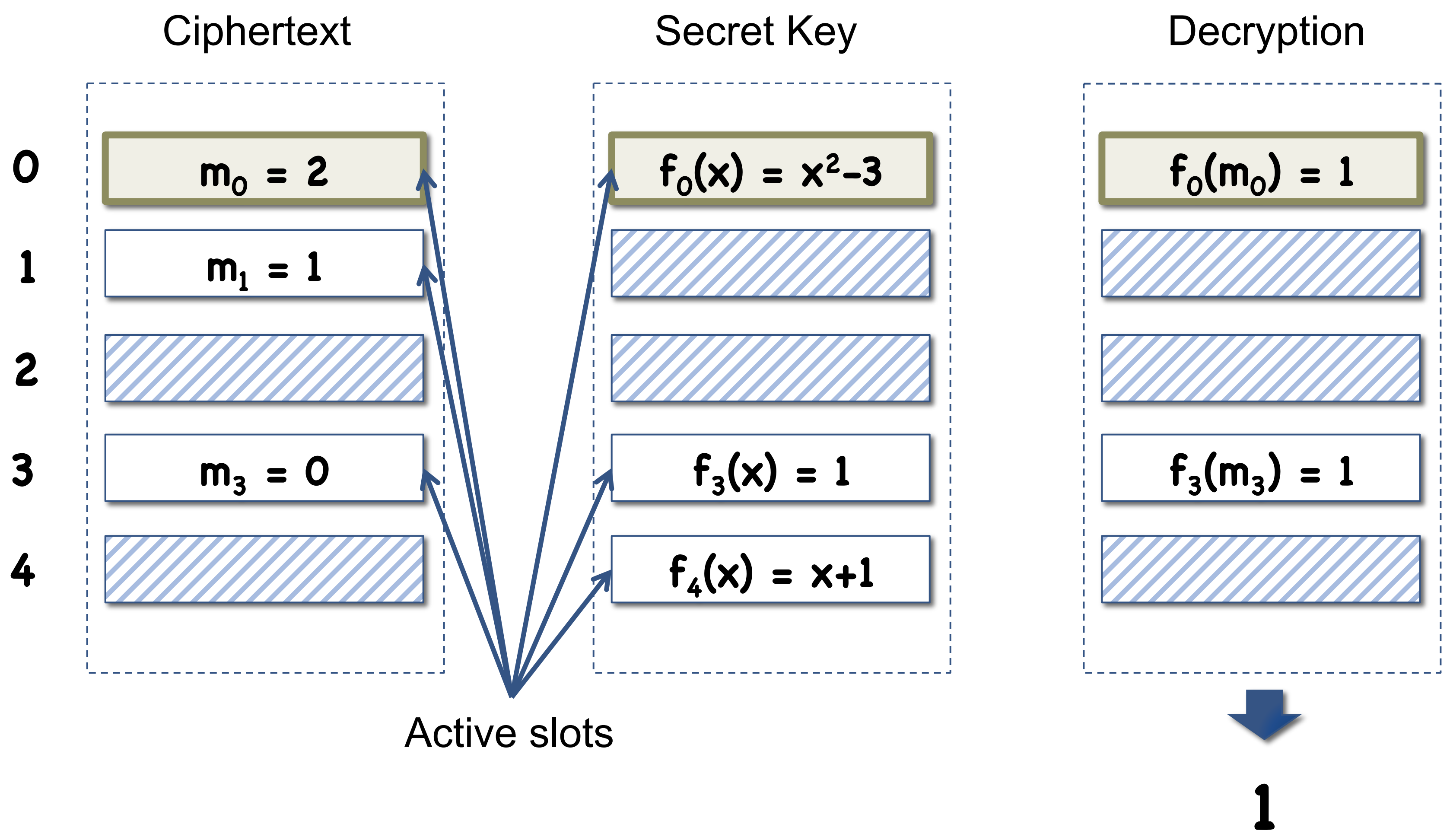
Question 2:

Can we build (adaptive) FE from fixed assumptions w/o complexity leveraging?

Our answer to questions 1 & 2:

YES!

Generalization: Slotted Functional Encryption



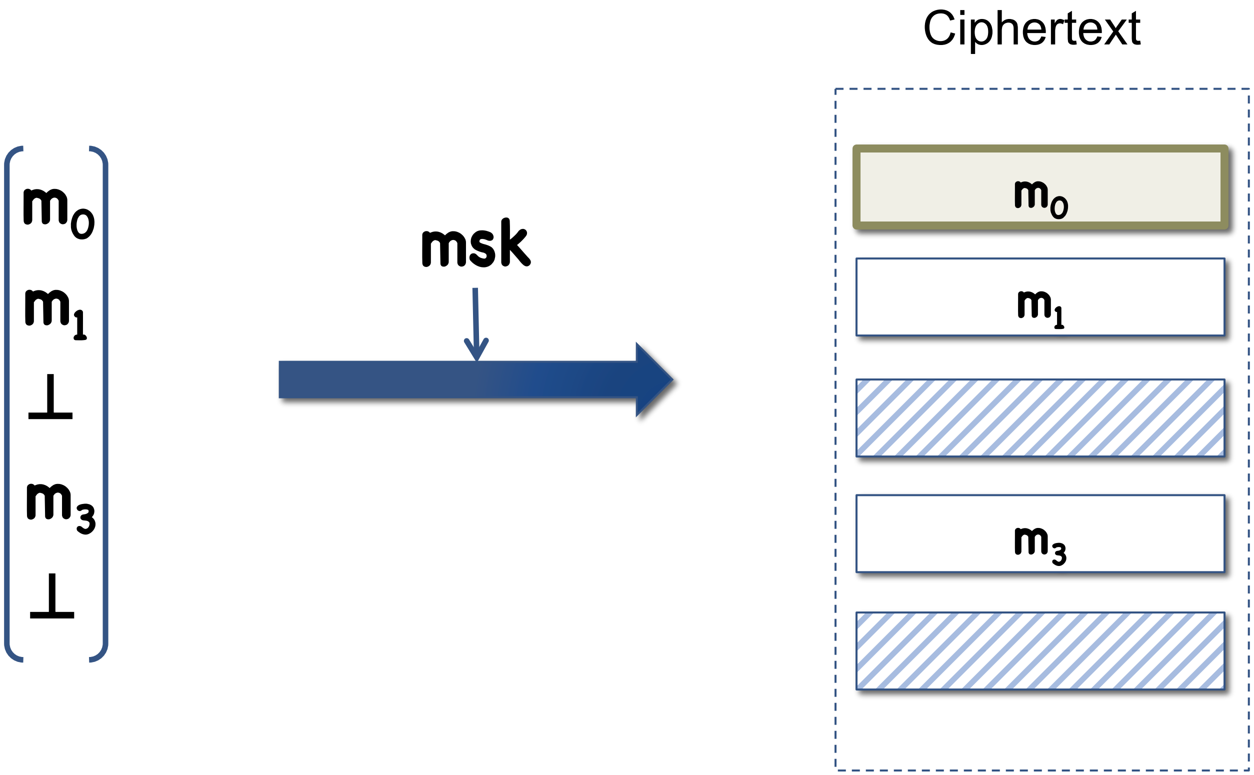
	Ciphertext	Secret Key	Decryption
0	$m_0 = 2$	$f_0(x) = x^2-3$	$f_0(m_0) = 1$
1	$m_1 = 1$	▨	▨
2	▨	▨	▨
3	$m_3 = 0$	$f_3(x) = 1$	$f_3(m_3) = 1$
4	▨	$f_4(x) = x+1$	▨

Active slots

1

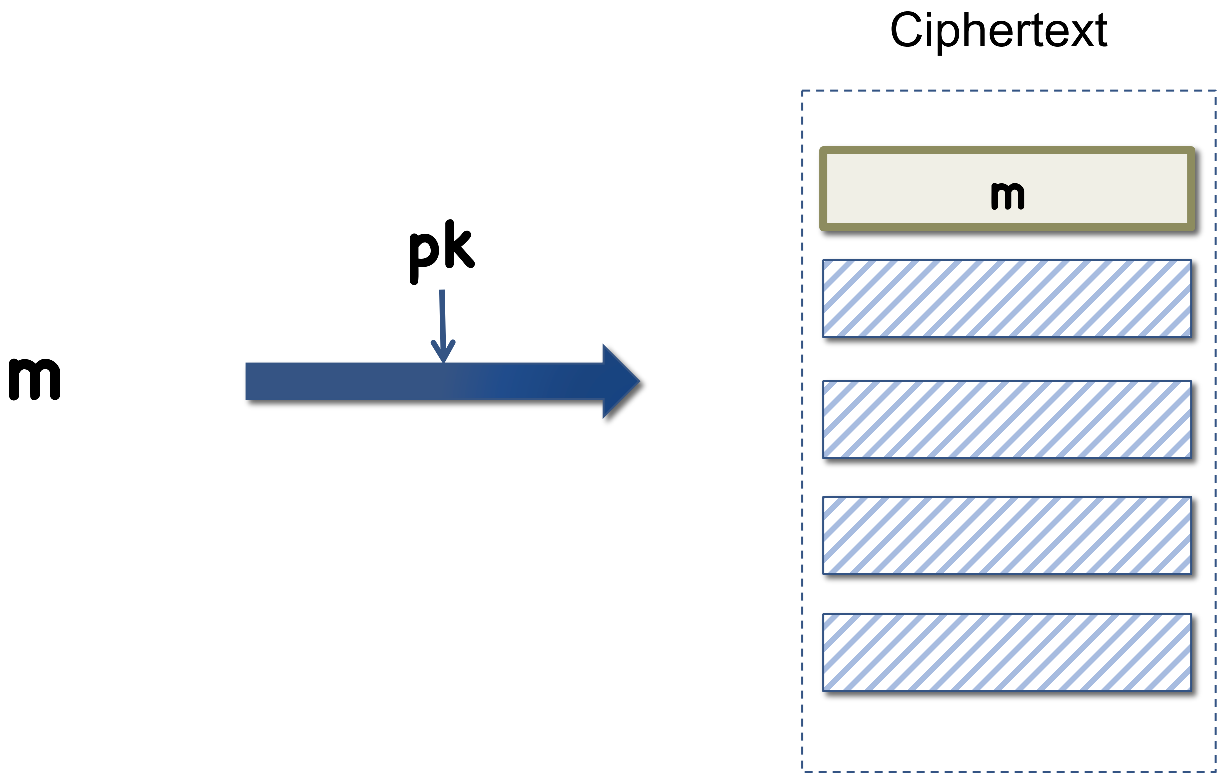
Slotted Functional Encryption

Private (slotted) encryption: encrypt in all slots


$\begin{bmatrix} m_0 \\ m_1 \\ \perp \\ m_3 \\ \perp \end{bmatrix}$
msk
Ciphertext
m_0
m_1
m_3

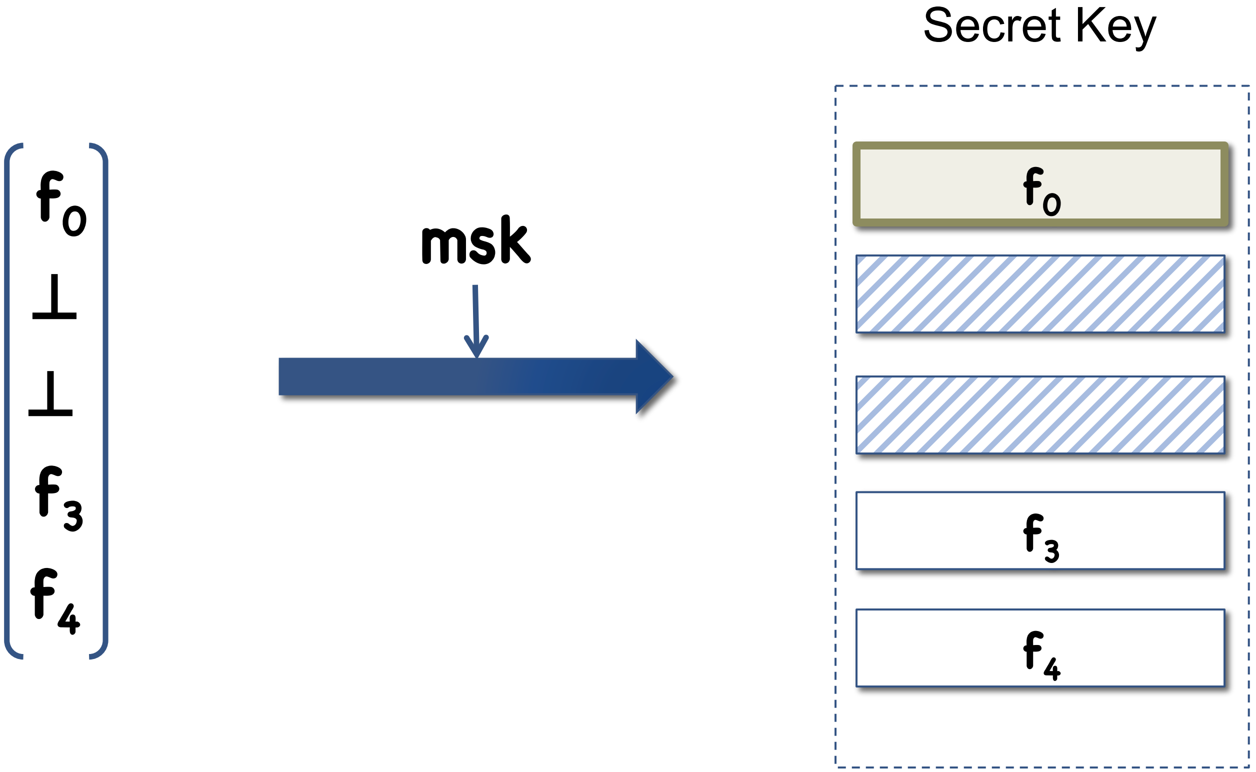
Slotted Functional Encryption

Public (unslotted) encryption: encrypt in slot 0



Slotted Functional Encryption

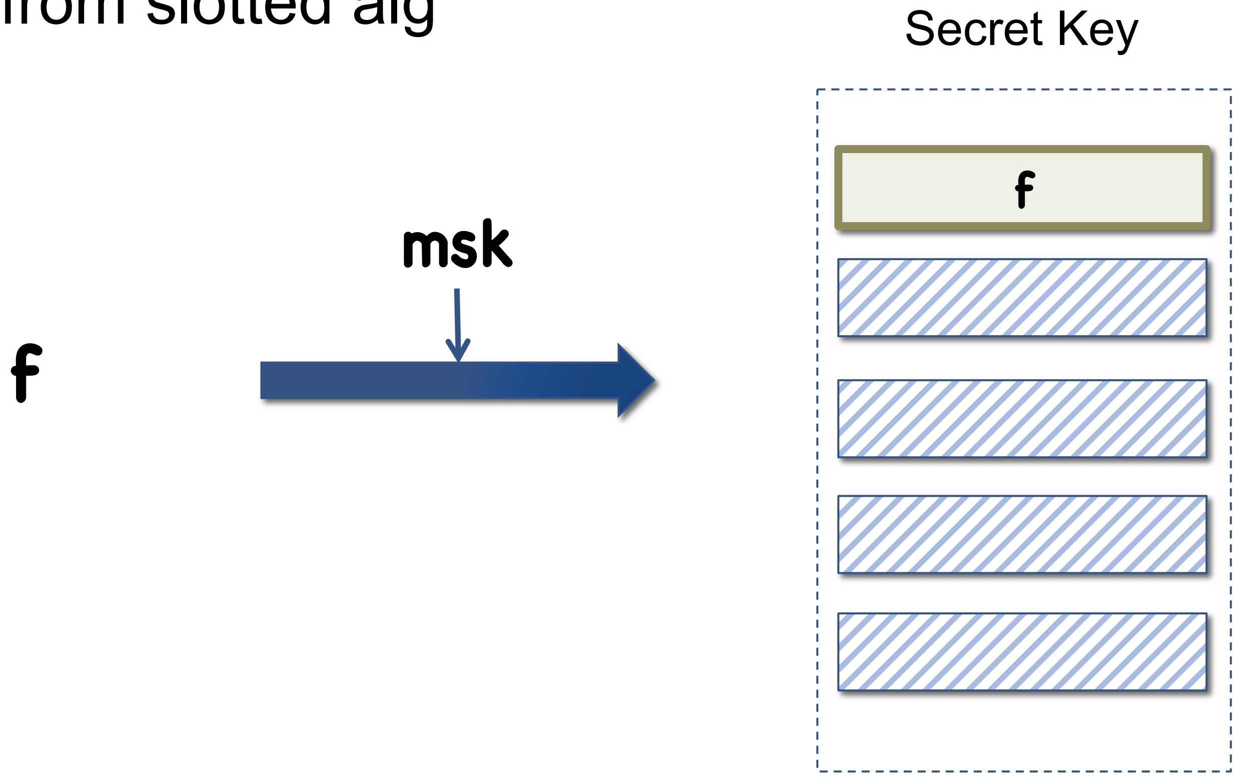
Slotted keygen: secret keys in all slots



Slotted Functional Encryption

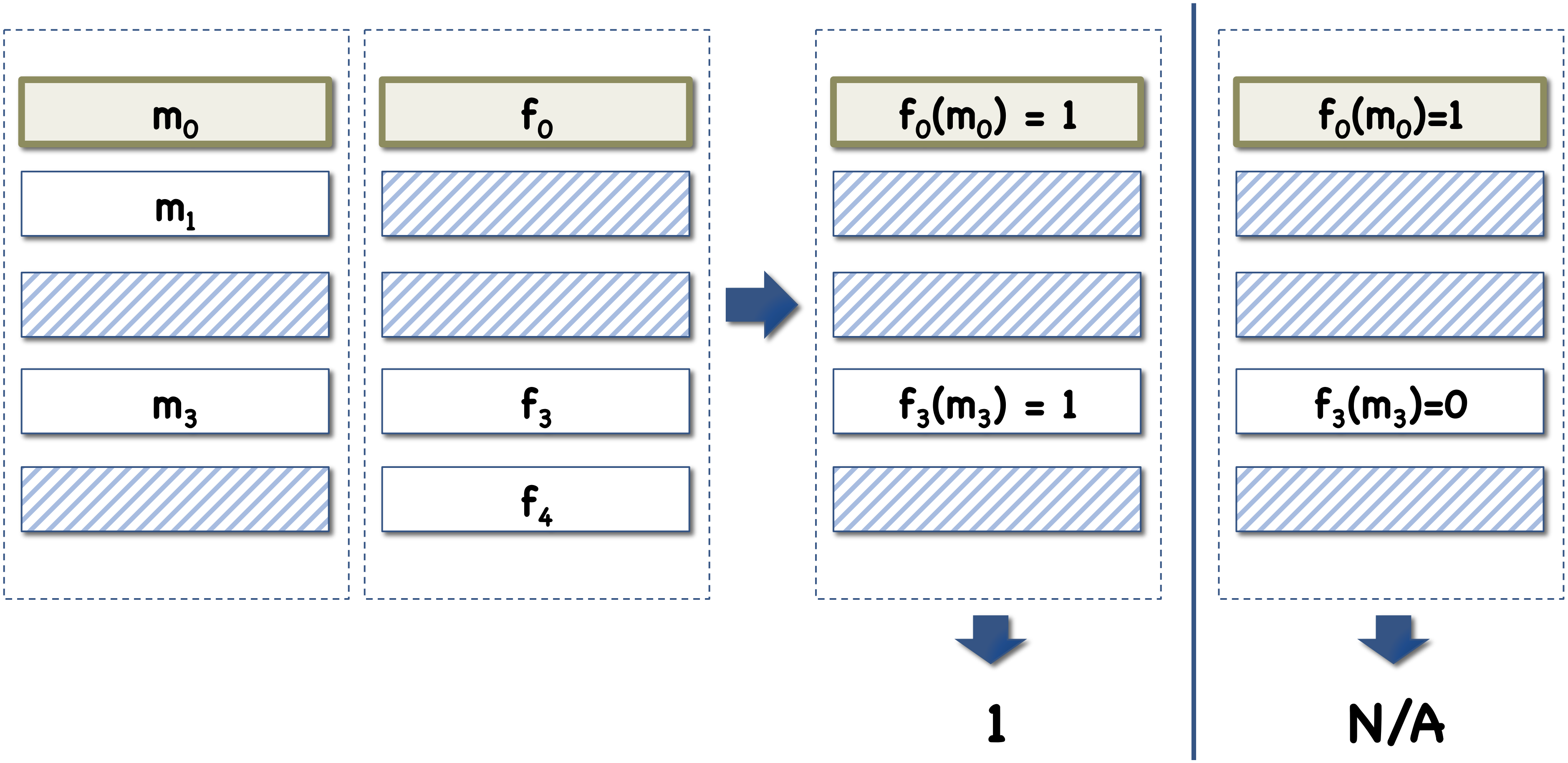
Unslotted keygen: secret keys in slot 0

- Derived from slotted alg



Slotted Functional Encryption

Decryption: decrypt all active slots, output result if agree



Slotted FE to (Unslotted) FE

Throw away slotted algorithms

Enc(msk, (m_0, m_1, m_2, …))

Enc(pk, m)

KeyGen(msk, (f_0, f_1, f_2, …)

KeyGen(msk, f)

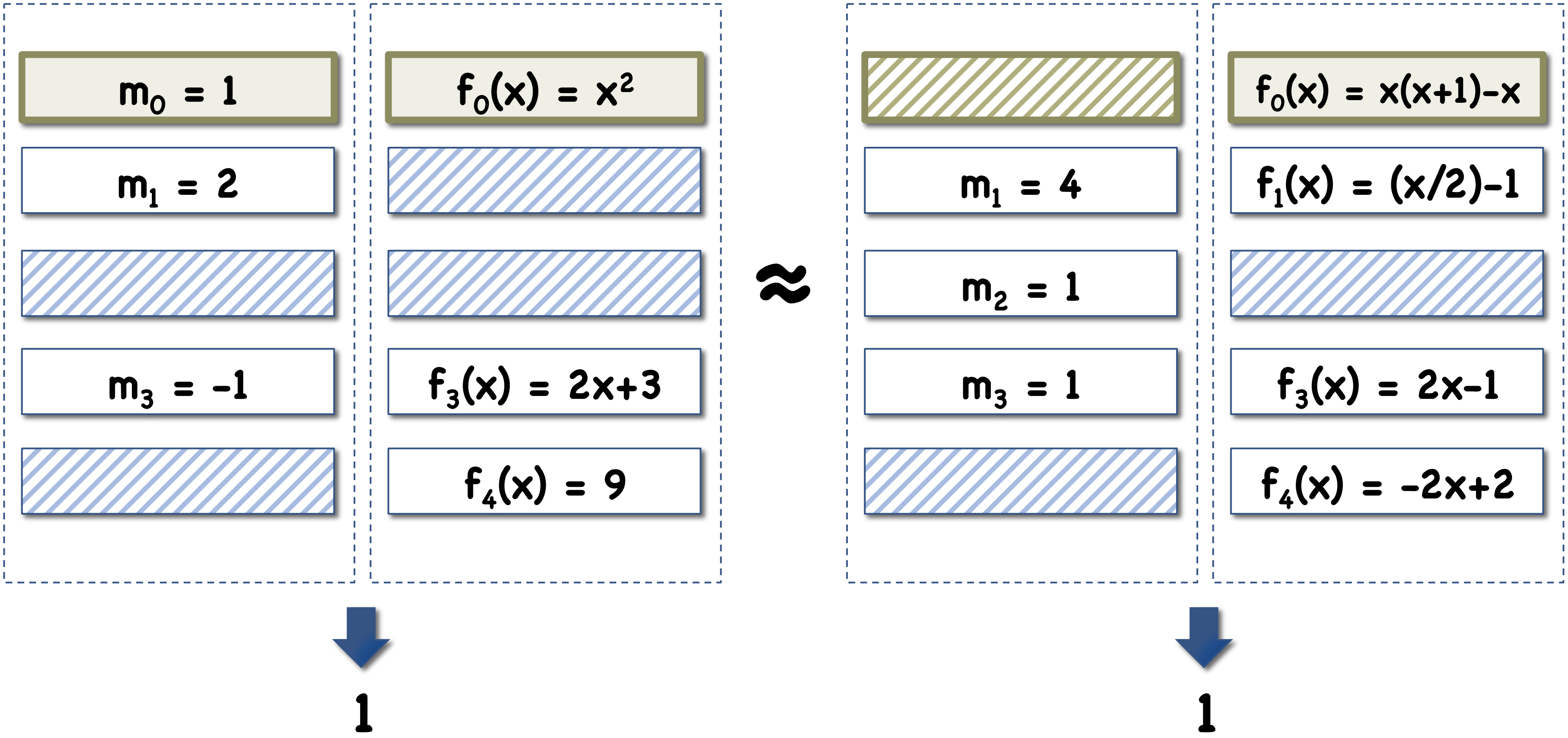
→

Enc(pk, m)

KeyGen(msk, f)

Security of Slotted Functional Encryption

Ideal: can’t learn anything except through decryption



$m_0 = 1$

$m_1 = 2$

$m_3 = -1$

$f_0(x) = x^2$

$f_3(x) = 2x+3$

$f_4(x) = 9$

$\approx$

$m_1 = 4$

$m_2 = 1$

$m_3 = 1$

$f_0(x) = x(x+1)-x$

$f_1(x) = (x/2)-1$

$f_3(x) = 2x-1$

$f_4(x) = -2x+2$

1

1

Too strong: implies function hiding in unslotted scheme

Security of Slotted Functional Encryption

Strategy: define desired property:

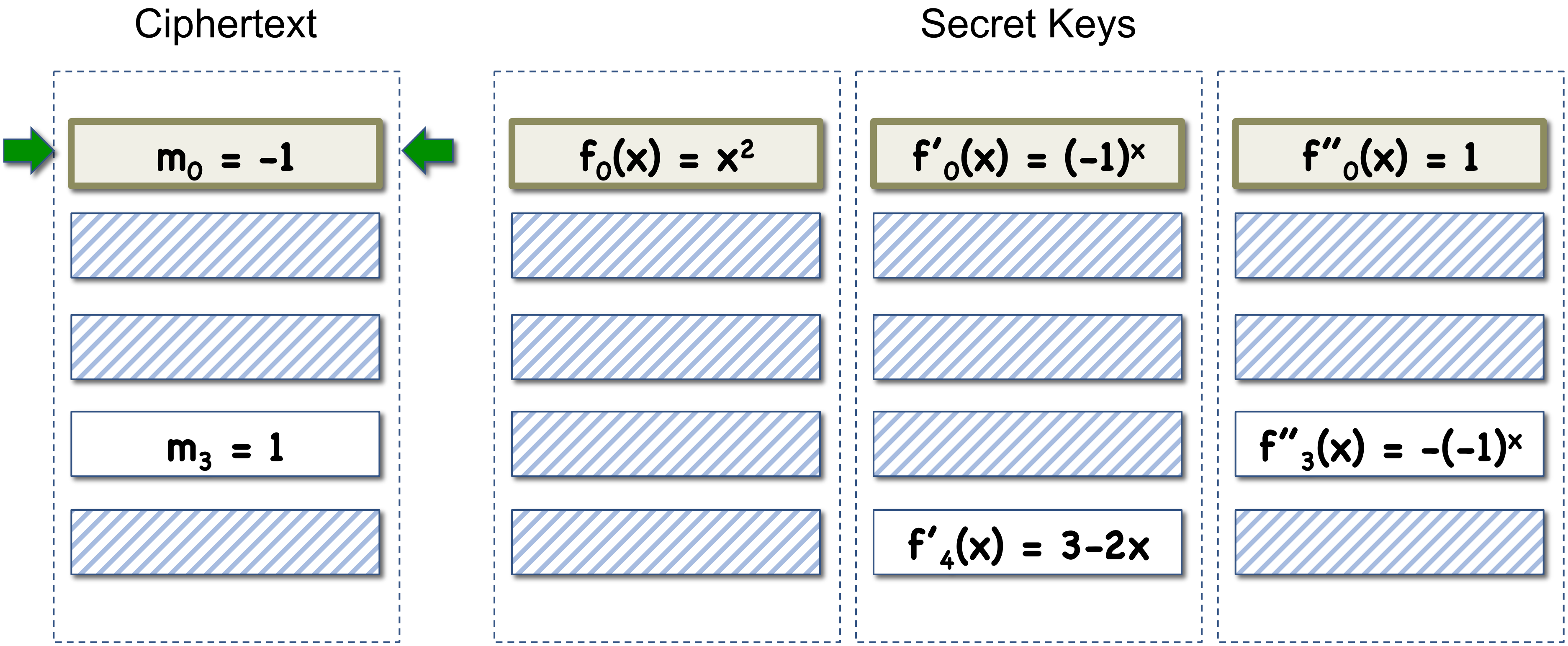
- Strong ciphertext indistinguishability

Derive from other simpler properties:

- Slot Duplication
- Slot symmetry
- Single use hiding
- Ciphertext moving
- Weak key moving
- Strong key moving
- New slot
- Weak ciphertext indistinguishability

Security of Slotted Functional Encryption

Strong Ciphertext Indistinguishability: change ciphertext slot (possibly in slot **0**) as long as decryption unaffected



Ciphertext

Secret Keys

$m_0 = -1$

$m_3 = 1$

$f_0(x) = x^2$

$f'_0(x) = (-1)^x$

$f'_4(x) = 3-2x$

$f''_0(x) = 1$

$f''_3(x) = -(-1)^x$

$m_0 = -1 \rightarrow m_0 = 1$ does not affect decryption

Security of Slotted Functional Encryption

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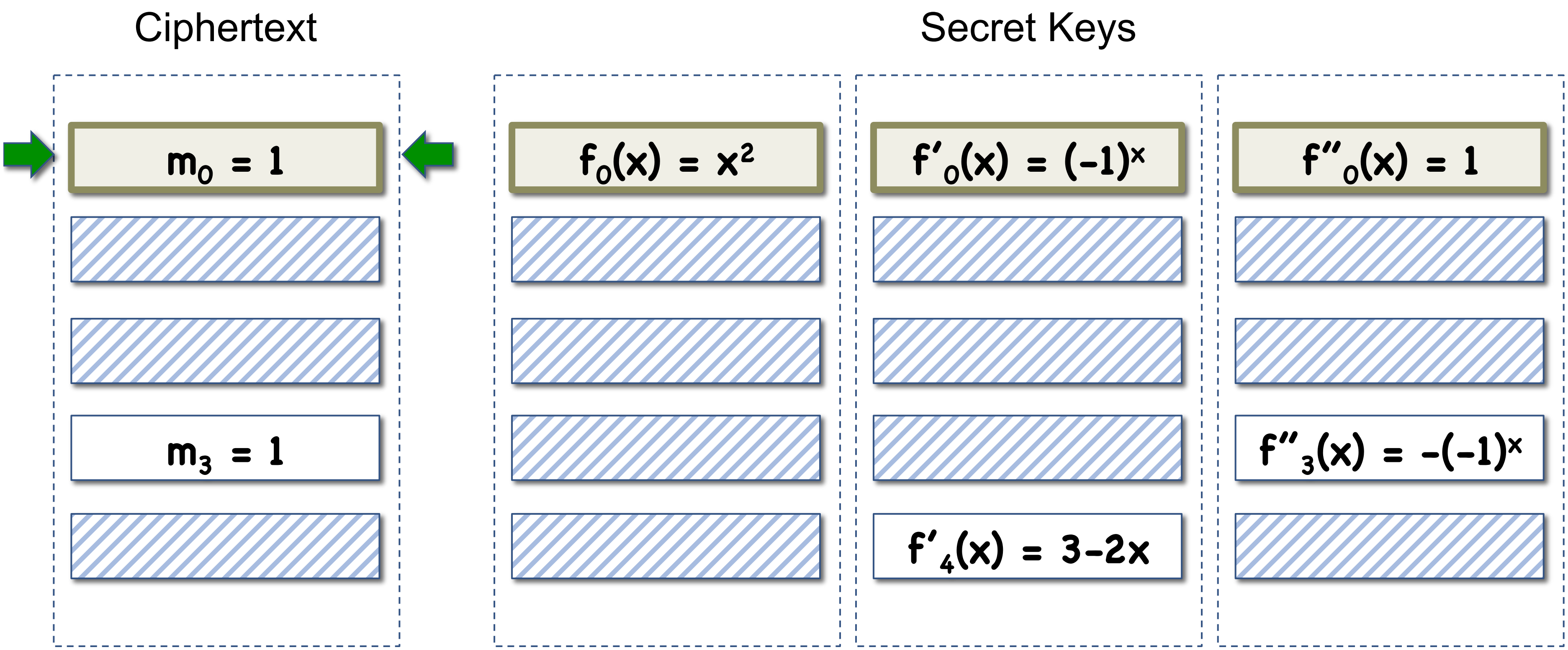

Ciphertext

$m_0 = 1$

$m_3 = 1$

Secret Keys

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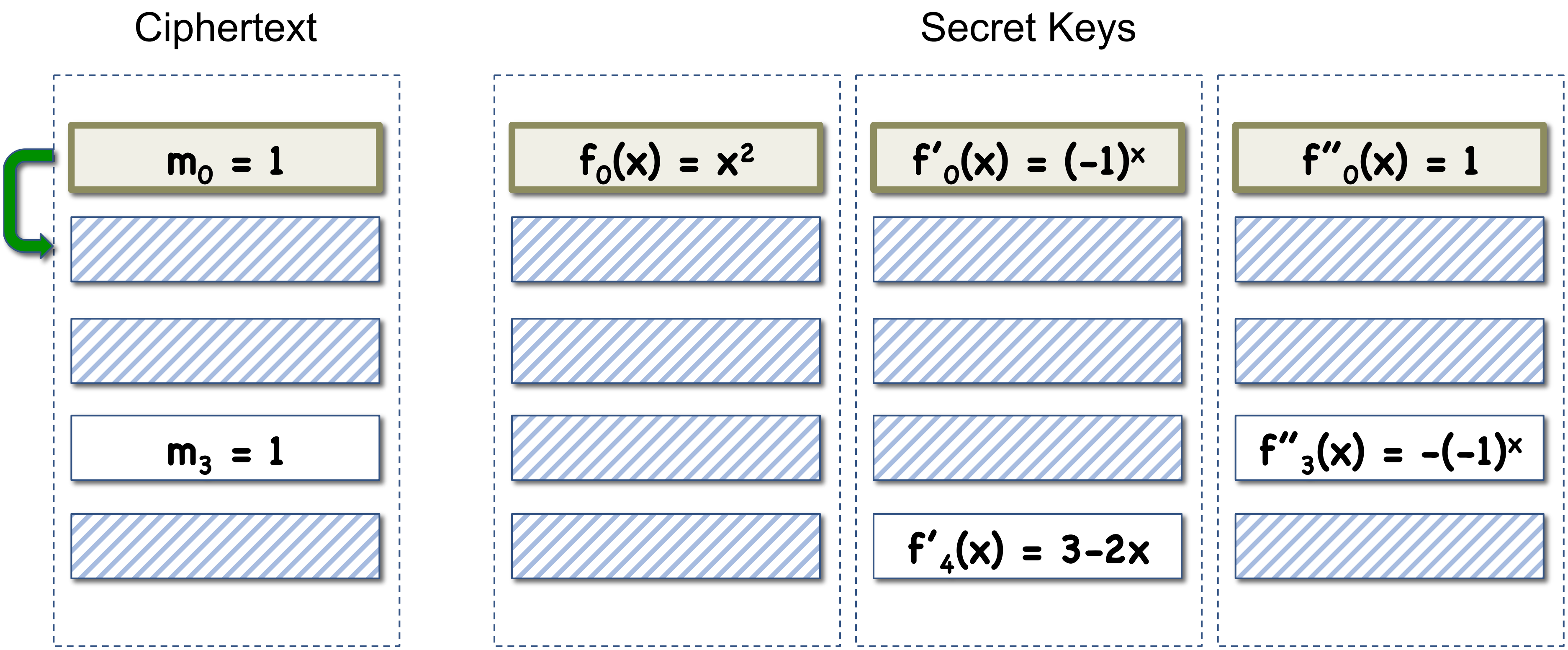
Security of Slotted Functional Encryption

Slot Duplication: Copy any slot (inc. slot **0**) into unused slot (except slot **0**) (don't have to copy everything)



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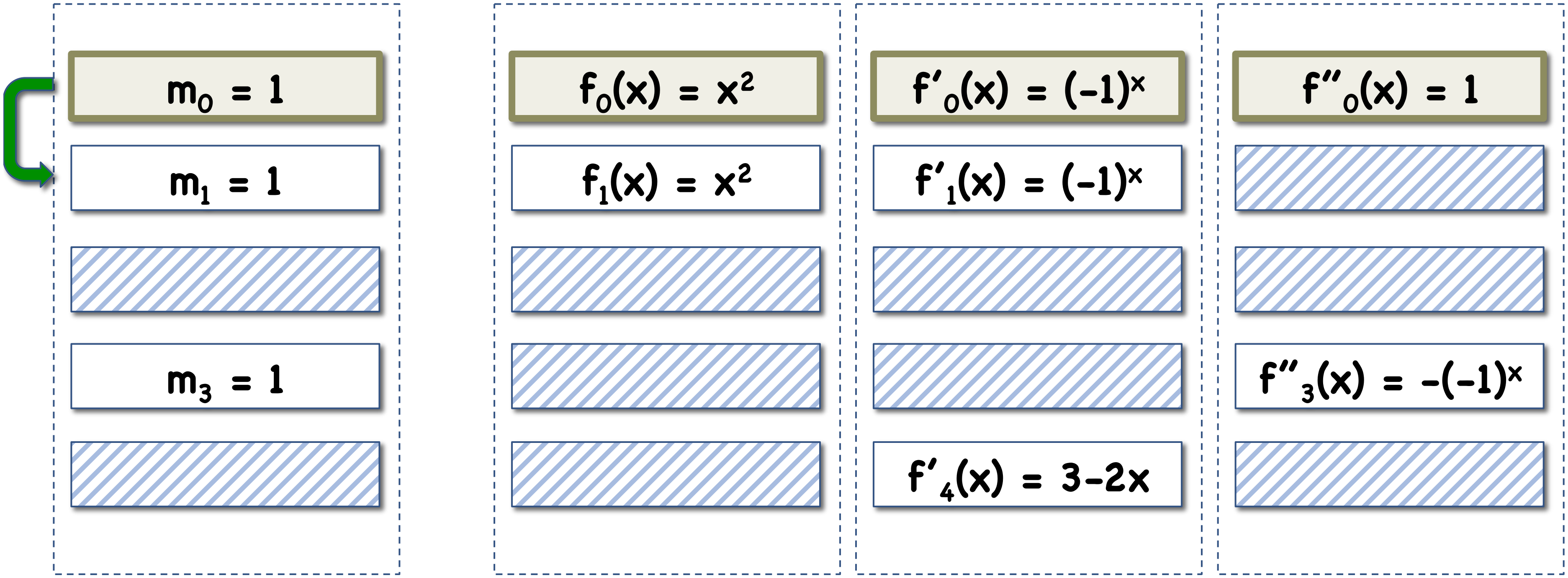

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▨	▨	▨	▨
$m_3 = 1$	▨	▨	$f''_3(x) = -(-1)^x$
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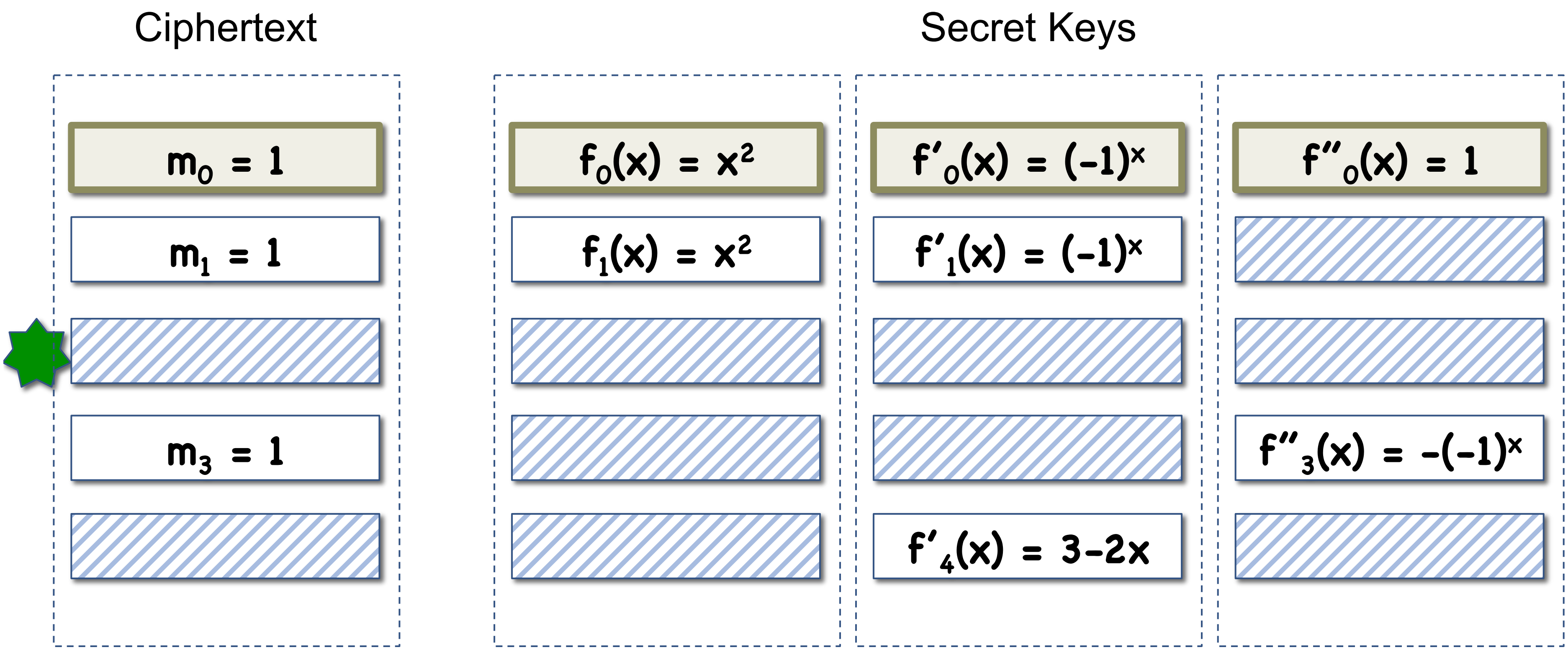
Security of Slotted Functional Encryption

New Slot: In unused slot (except slot **0**), put any ciphertext val



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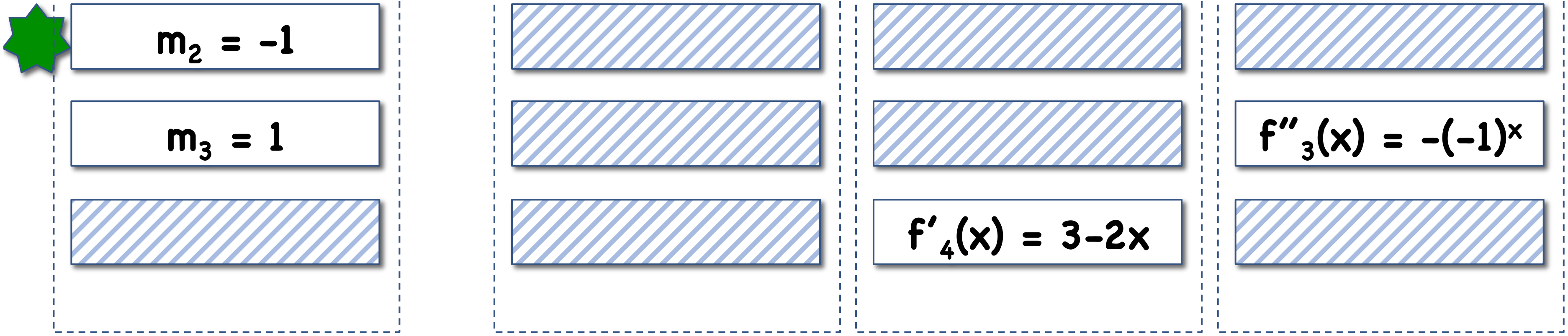

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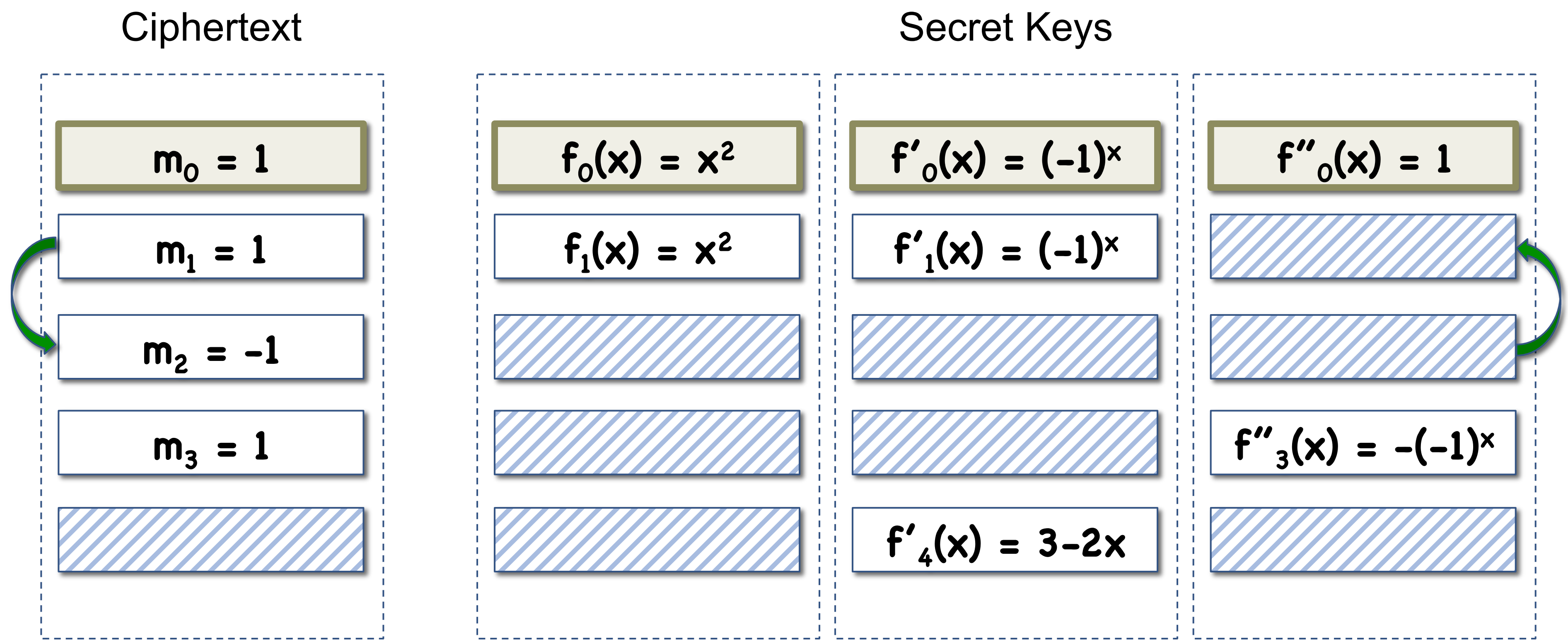
Security of Slotted Functional Encryption

Slot Symmetry: Swap two slots (except slot **0**)



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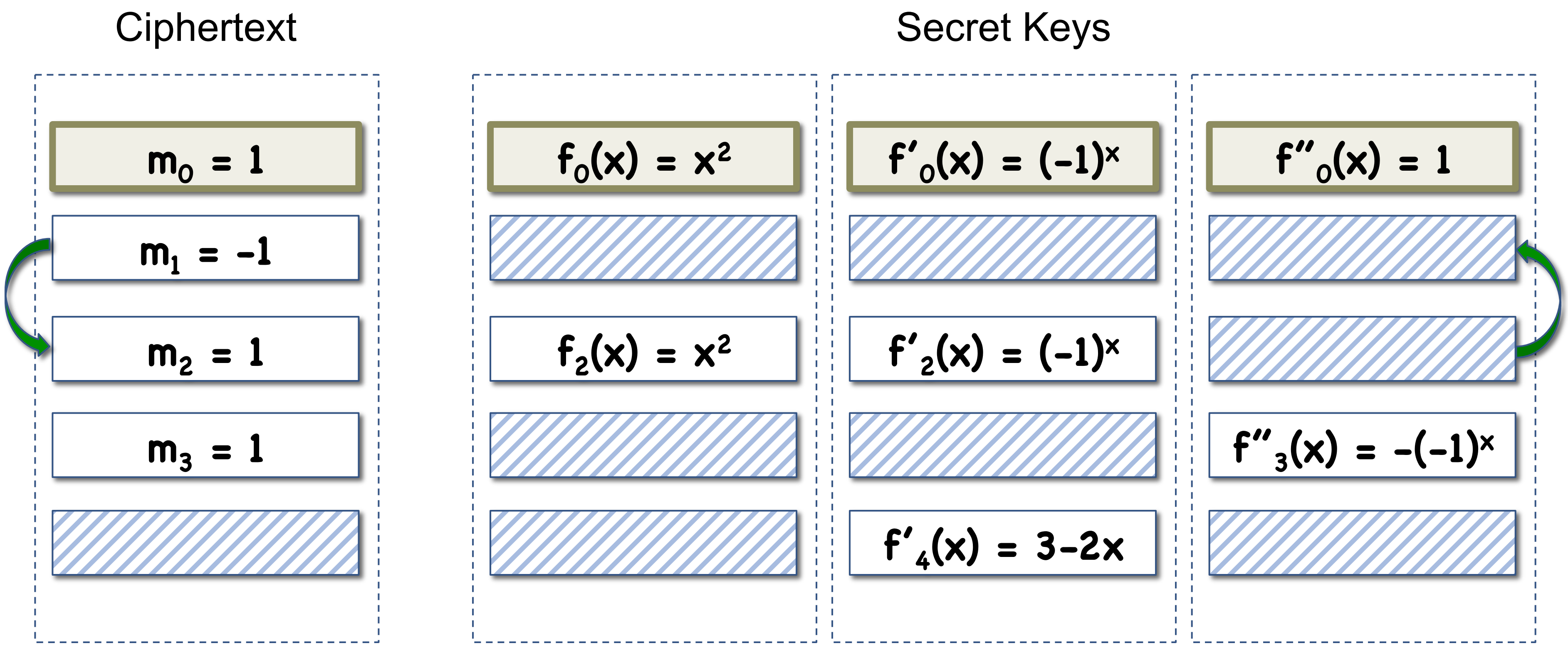

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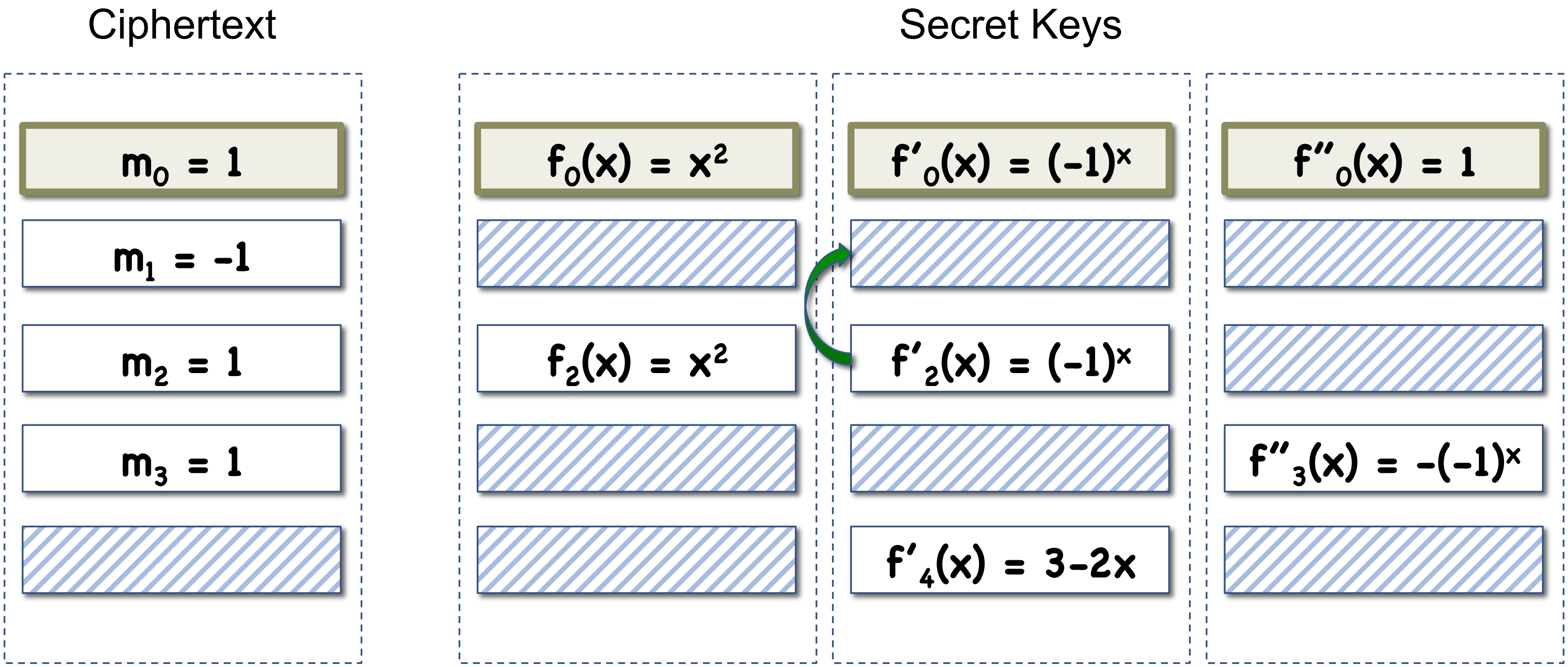
Strong Key Moving: Move any secret key slot into inactive slot (neither can be slot **0**) as long as decryption unaffected



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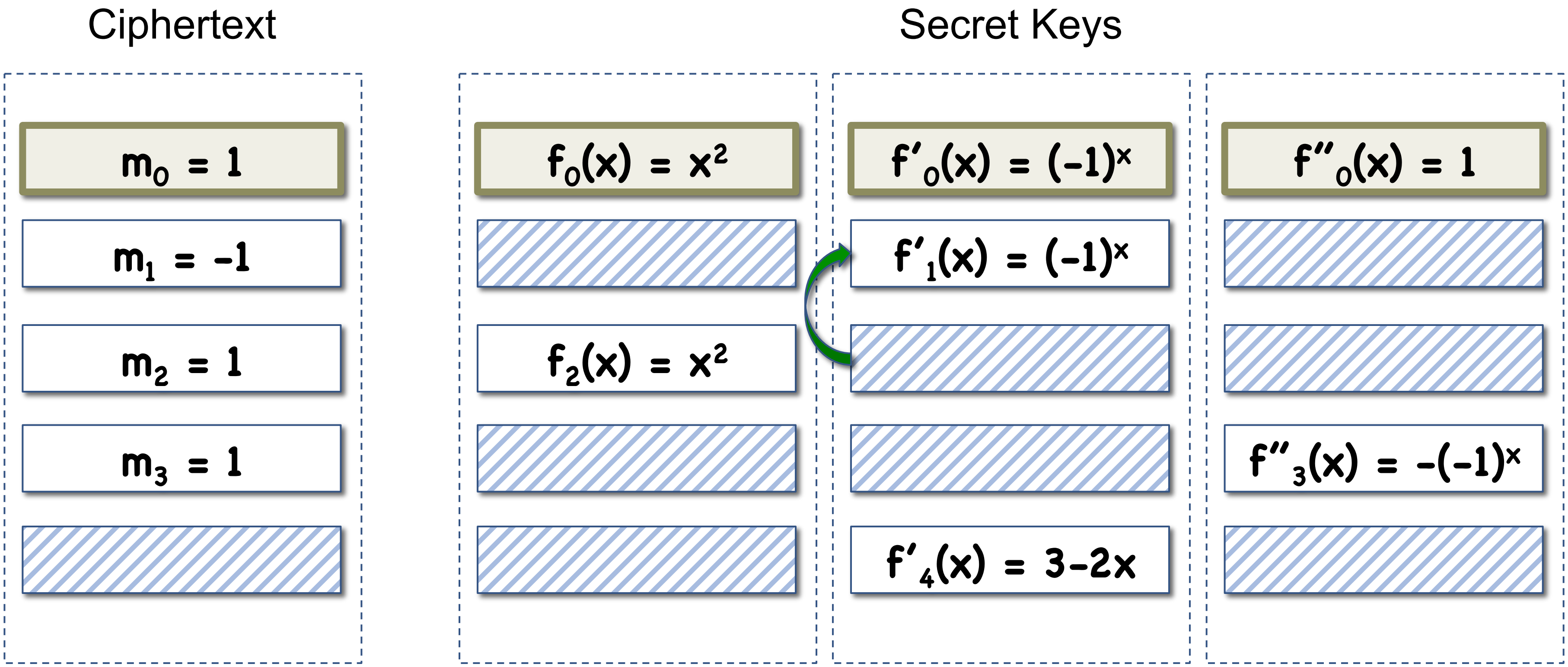

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Security of Slotted Functional Encryption

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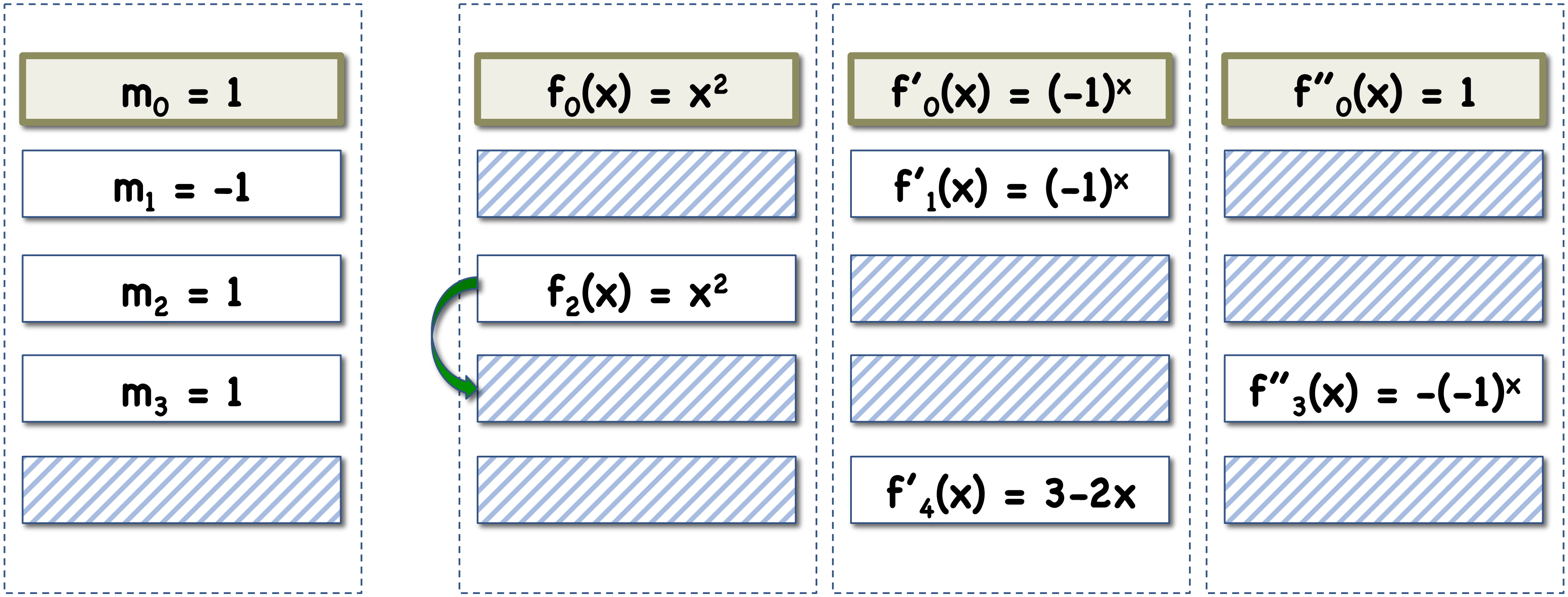

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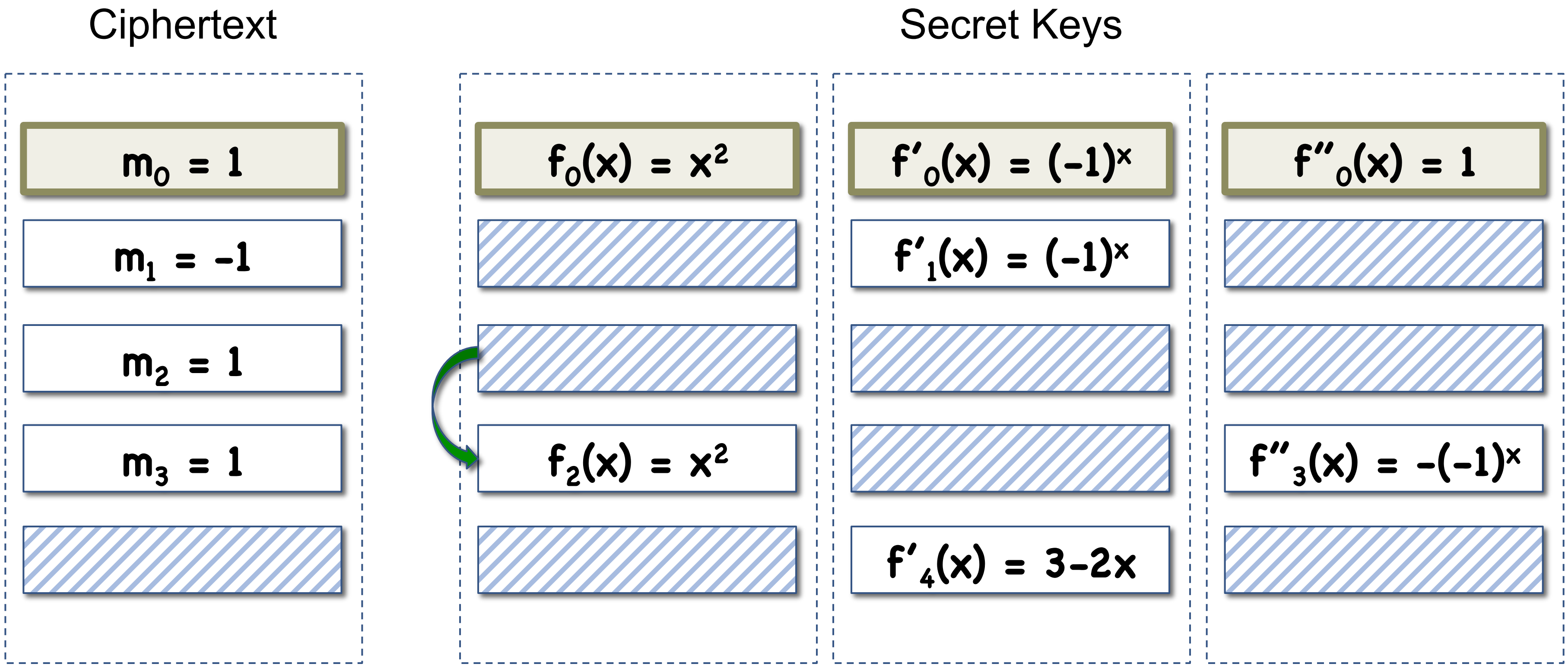

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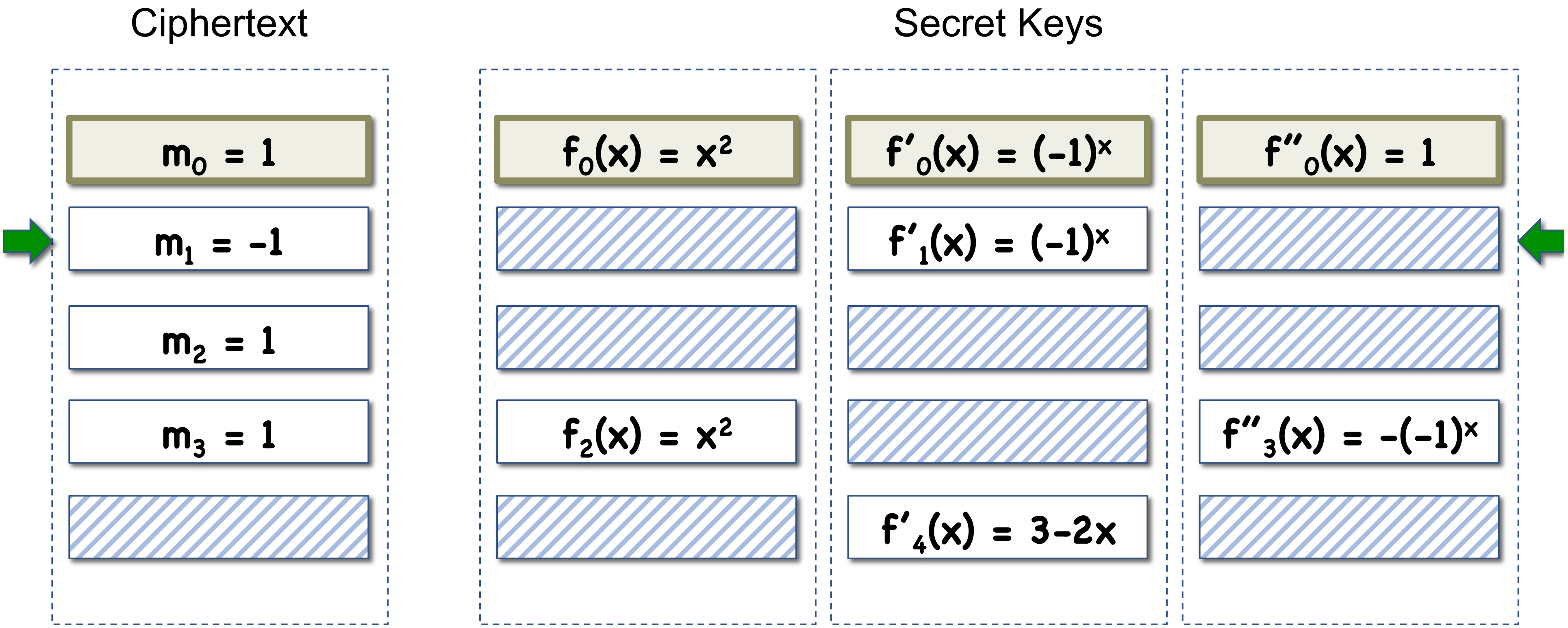
Security of Slotted Functional Encryption

Single Use Hiding: Change ctxt and 1 sk in otherwise unused slot (except slot **0**) as long as decryption unaffected



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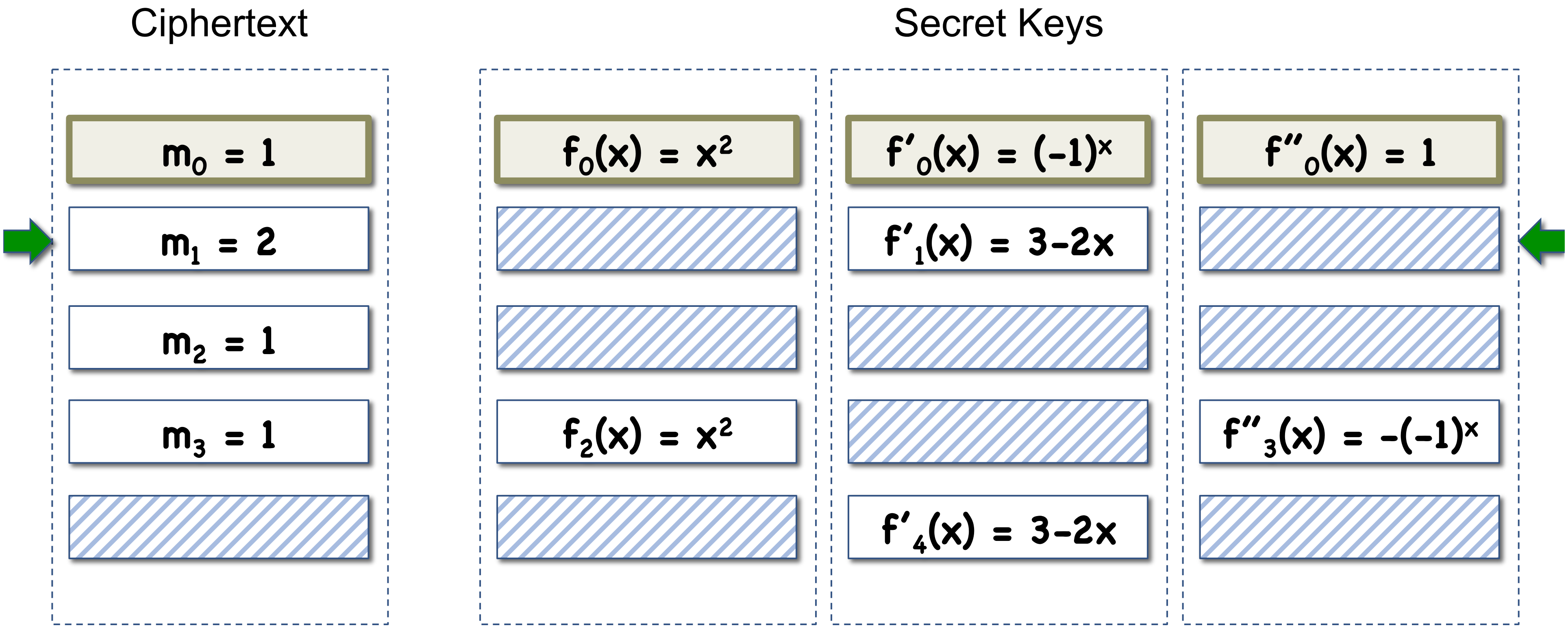

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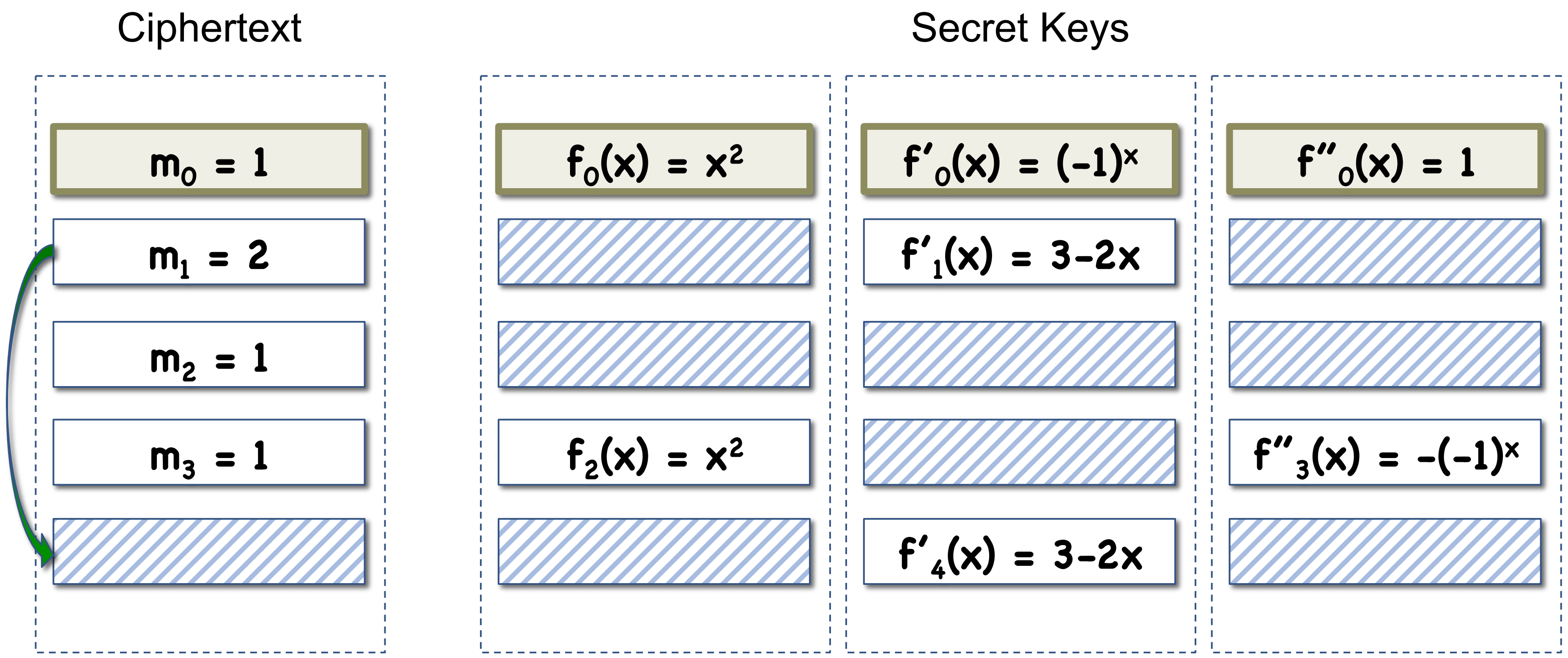
Security of Slotted Functional Encryption

Ciphertext Moving: Move ciphertext into an empty slot (possibly slot **0**) as long as secret keys are all identical



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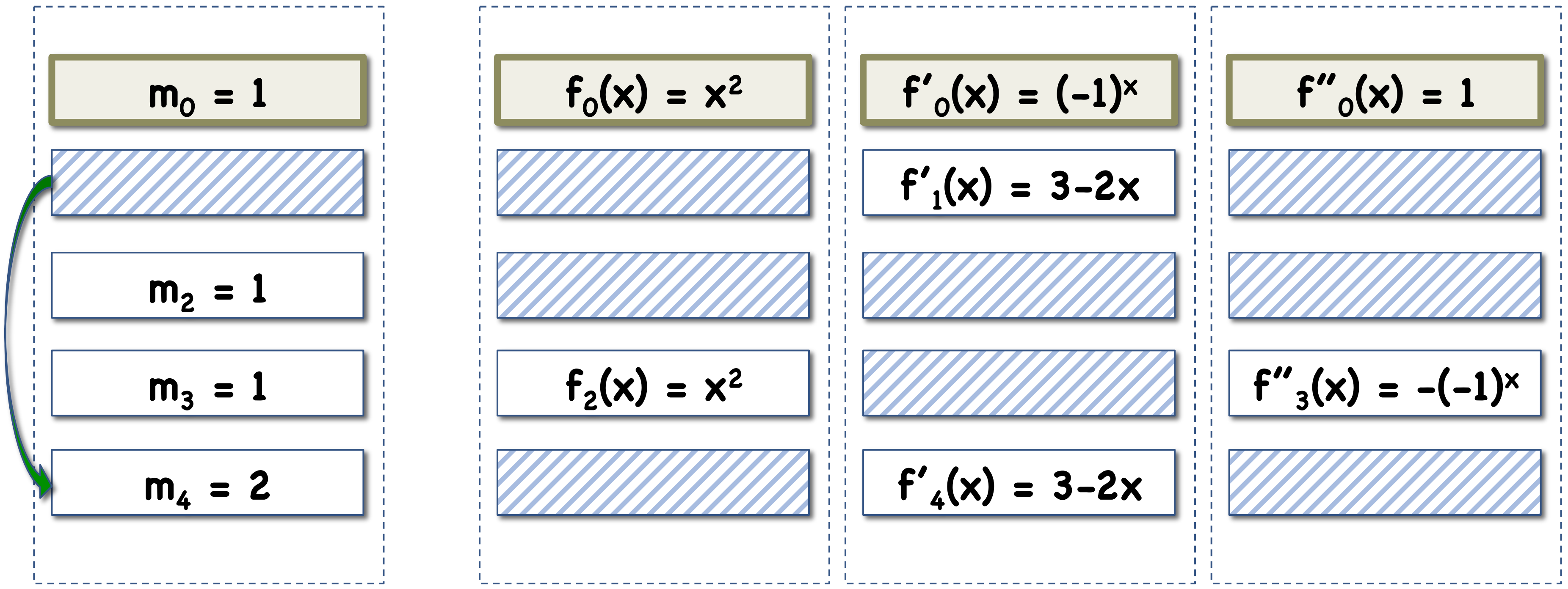
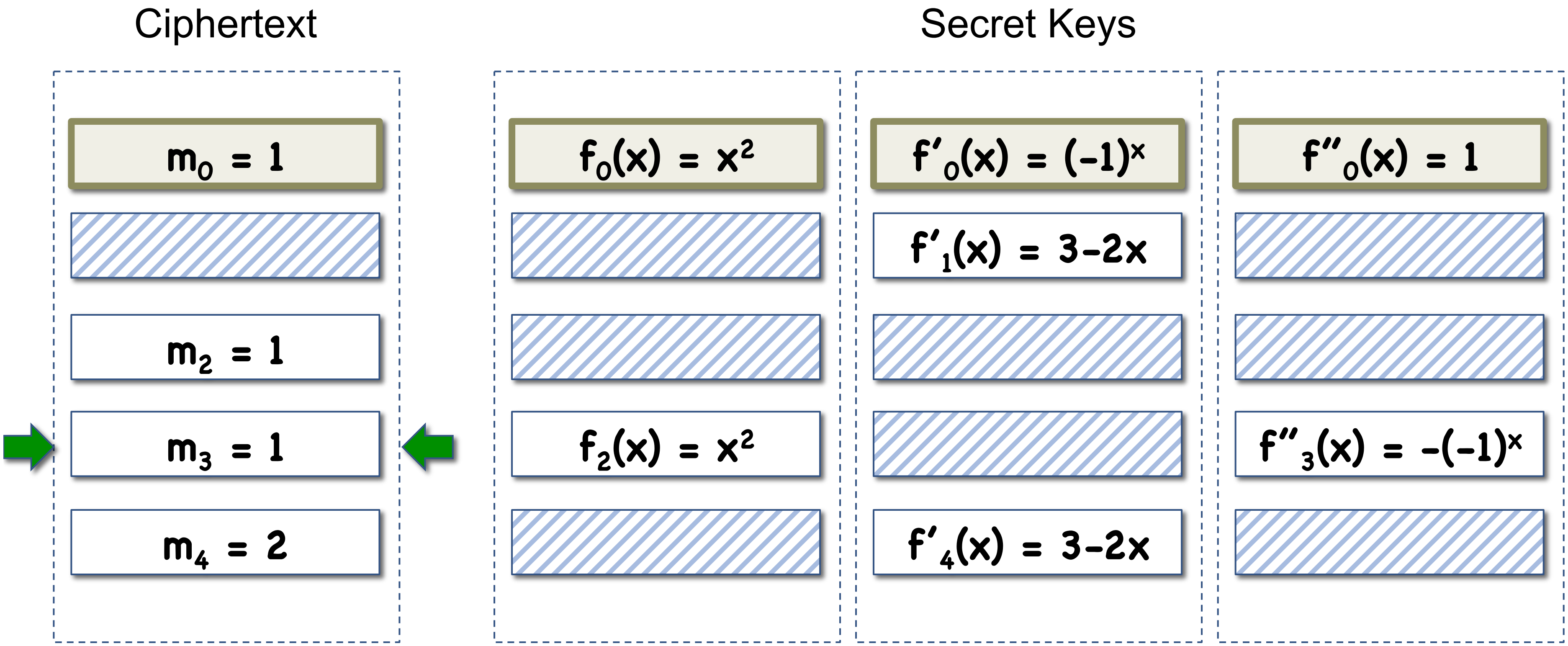


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
▨	▨	$f'_1(x) = 3-2x$	▨
$m_2 = 1$	▨	▨	▨
$m_3 = 1$	$f_2(x) = x^2$	▨	$f''_3(x) = -(-1)^x$
$m_4 = 2$	▨	$f'_4(x) = 3-2x$	▨

Security of Slotted Functional Encryption

Weak Ciphertext Indistinguishability: change ciphertext slot (except slot **0**) as long as decryption unaffected



Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
▨	▨	$f'_1(x) = 3-2x$	▨
$m_2 = 1$	▨	▨	▨
$m_3 = 1$	$f_2(x) = x^2$	▨	$f''_3(x) = -(-1)^x$
$m_4 = 2$	▨	$f'_4(x) = 3-2x$	▨

Security of Slotted Functional Encryption

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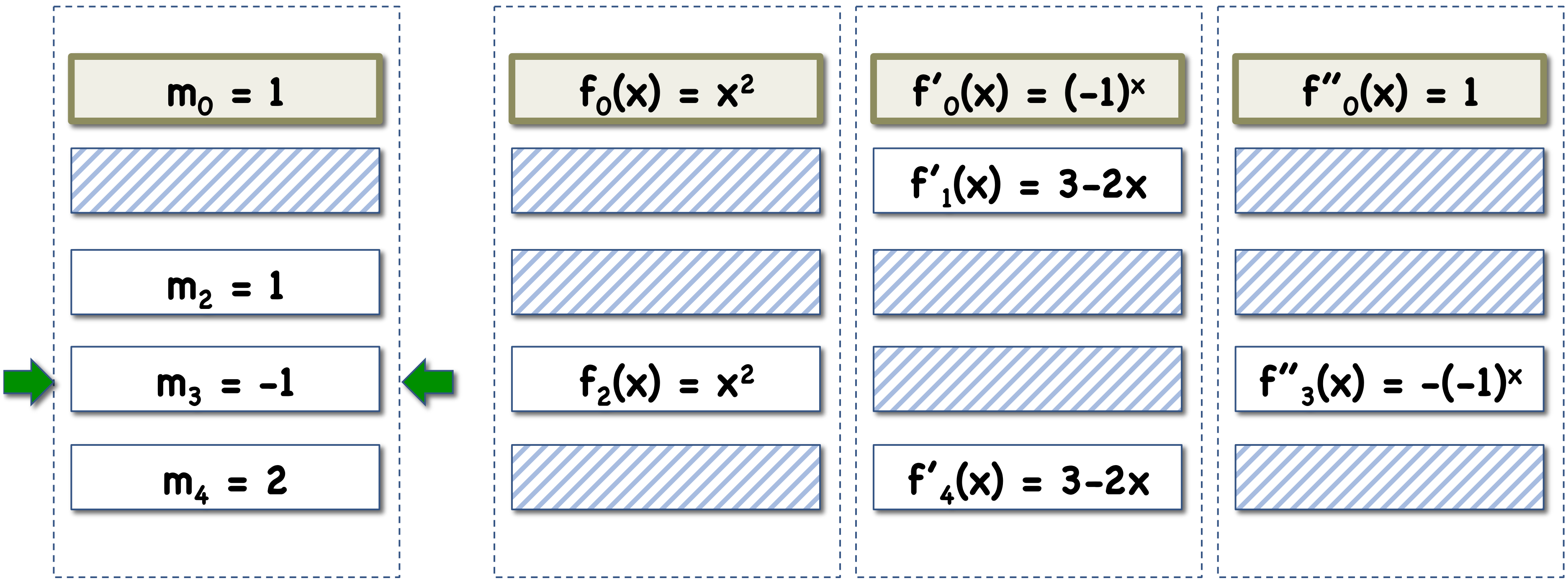
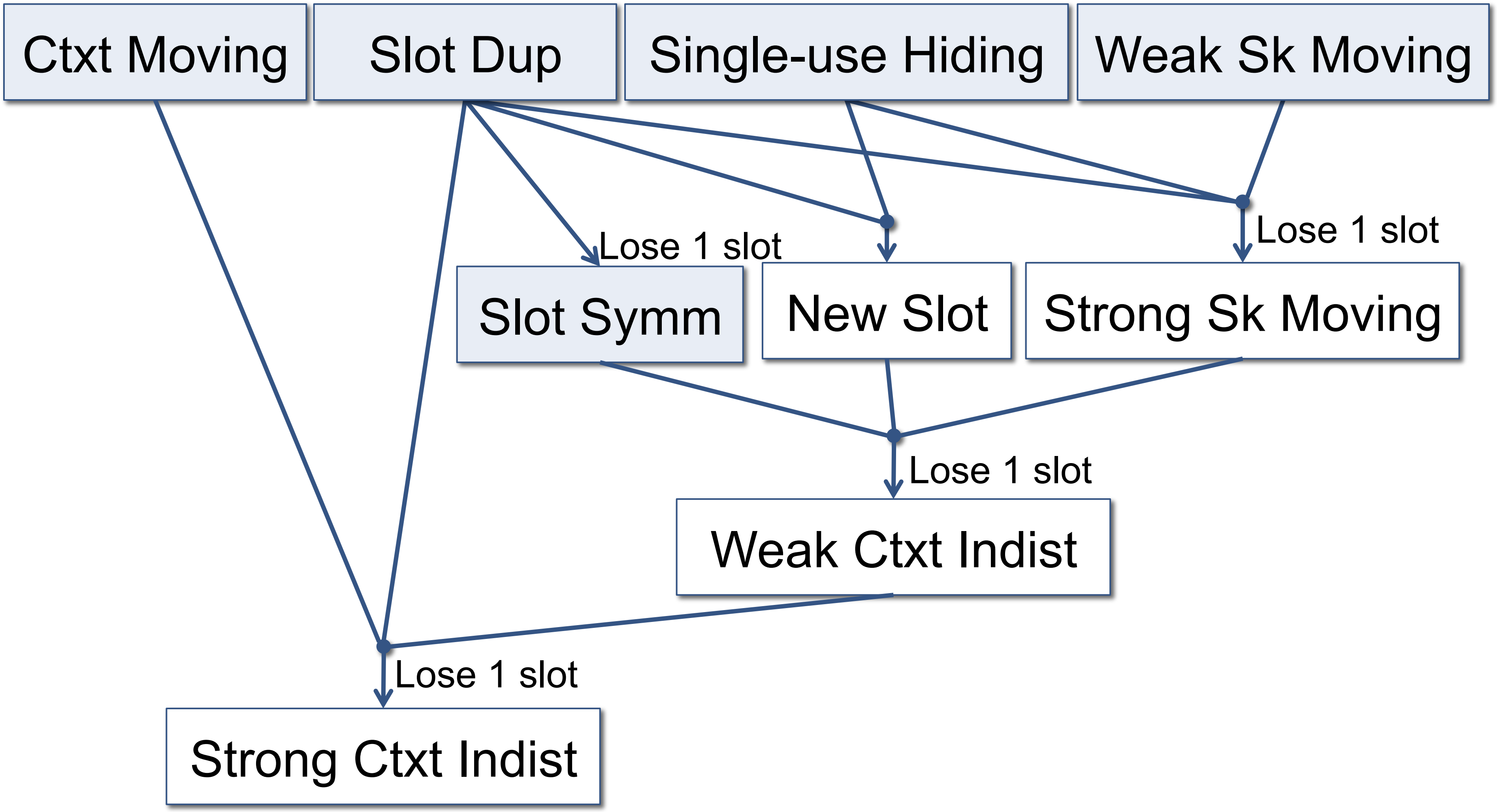


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
▨	▨	$f'_1(x) = 3-2x$	▨
$m_2 = 1$	▨	▨	▨
$m_3 = -1$	$f_2(x) = x^2$	▨	$f''_3(x) = -(-1)^x$
$m_4 = 2$	▨	$f'_4(x) = 3-2x$	▨

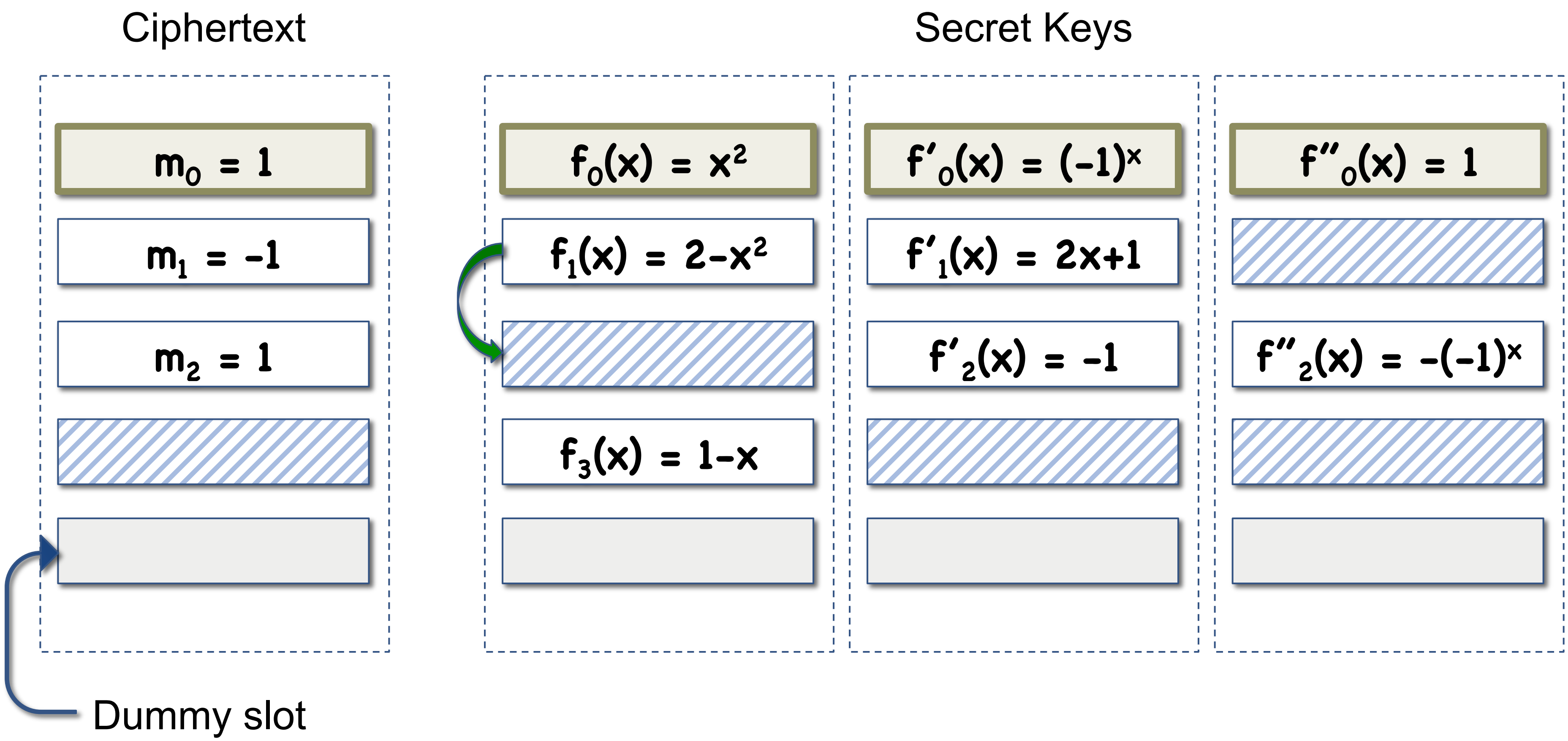
Reductions!



Sanity Check: Slot 0 in secret keys cannot change ⇒ no function hiding

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**



Ciphertext

Secret Keys

$m_0 = 1$

$m_1 = -1$

$m_2 = 1$

$f_0(x) = x^2$

$f_1(x) = 2-x^2$

$f_3(x) = 1-x$

$f'_0(x) = (-1)^x$

$f'_1(x) = 2x+1$

$f'_2(x) = -1$

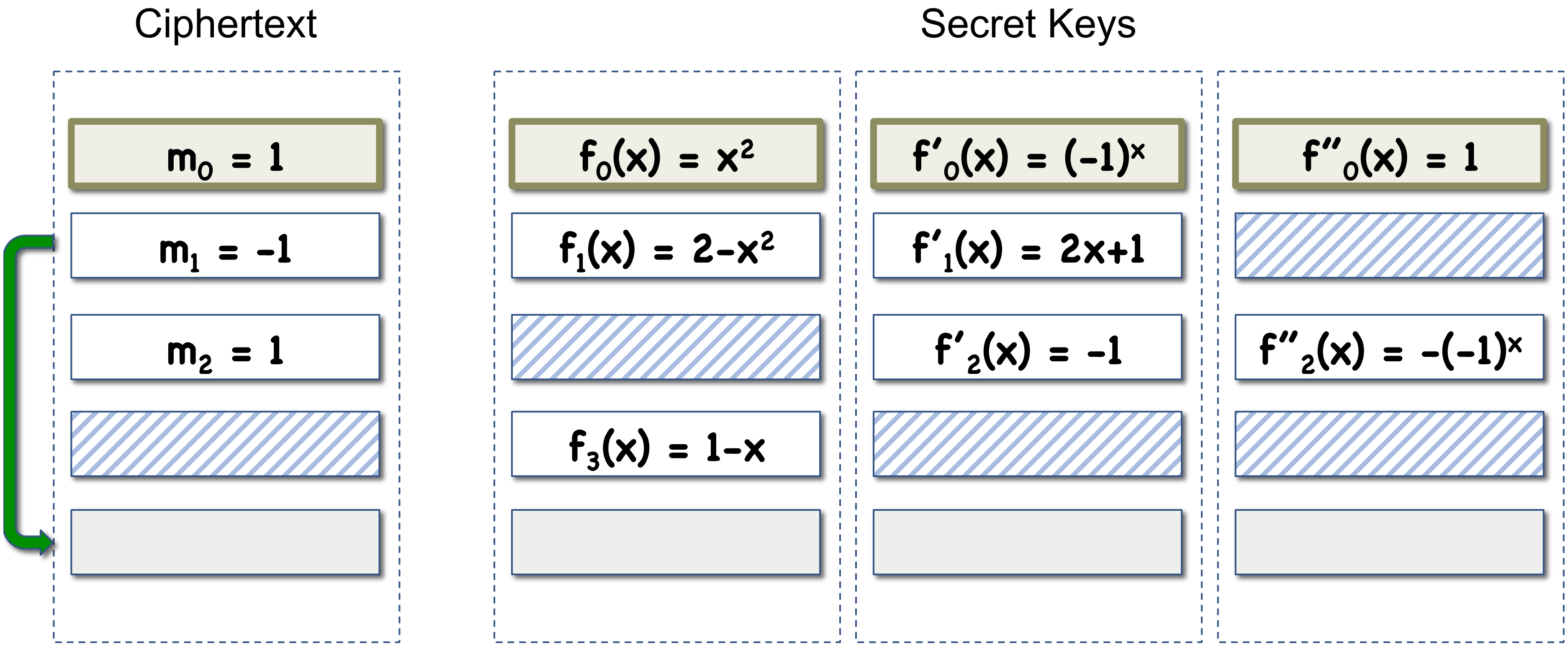
$f''_0(x) = 1$

$f''_2(x) = -(-1)^x$

Dummy slot

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

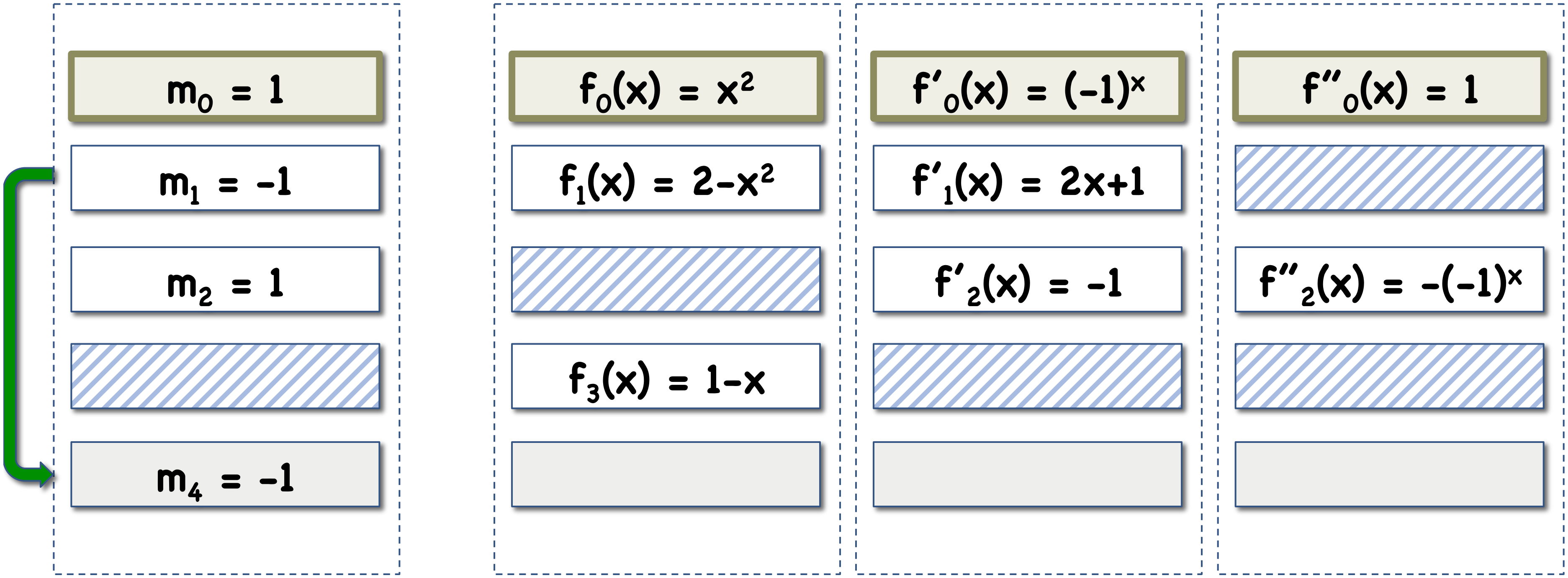


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	$f_1(x) = 2-x^2$	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Slot Duplication

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

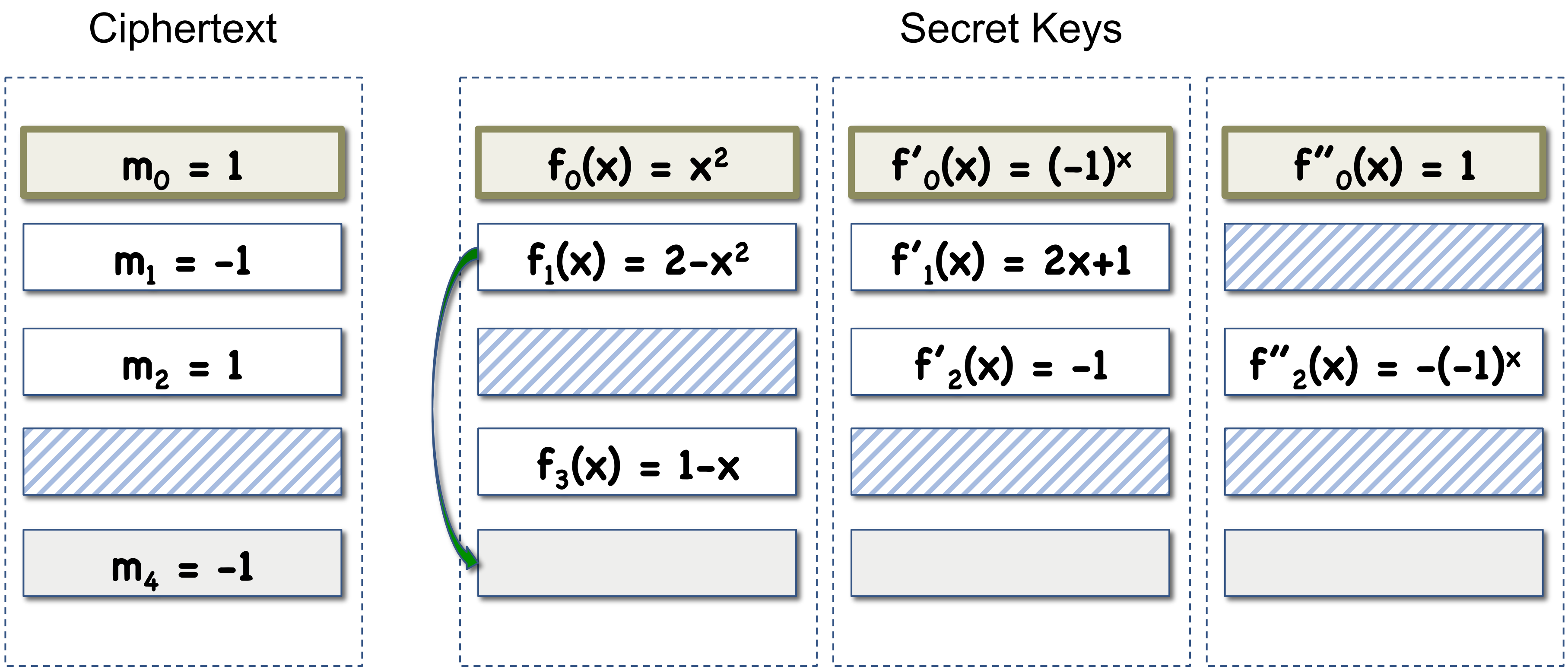


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	$f_1(x) = 2-x^2$	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$			

Slot Duplication

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

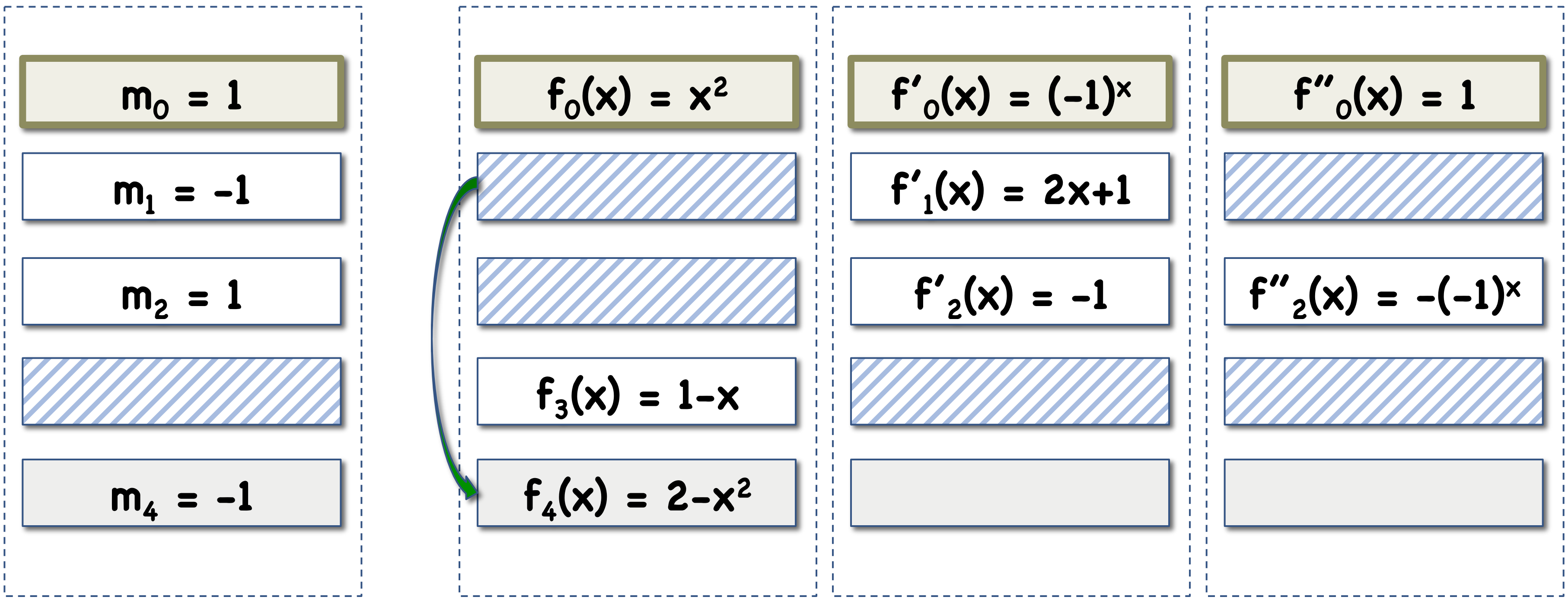


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	$f_1(x) = 2-x^2$	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$			

Weak Sk Moving

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

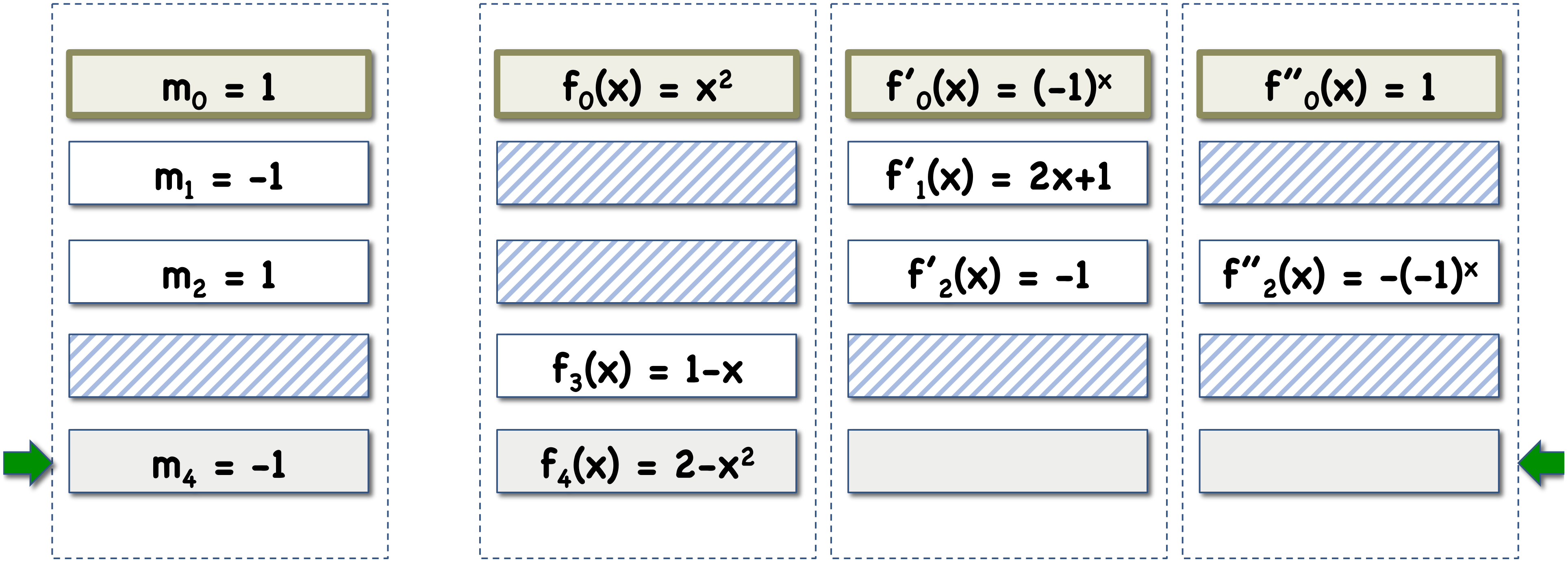


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$		

Weak Sk Moving

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

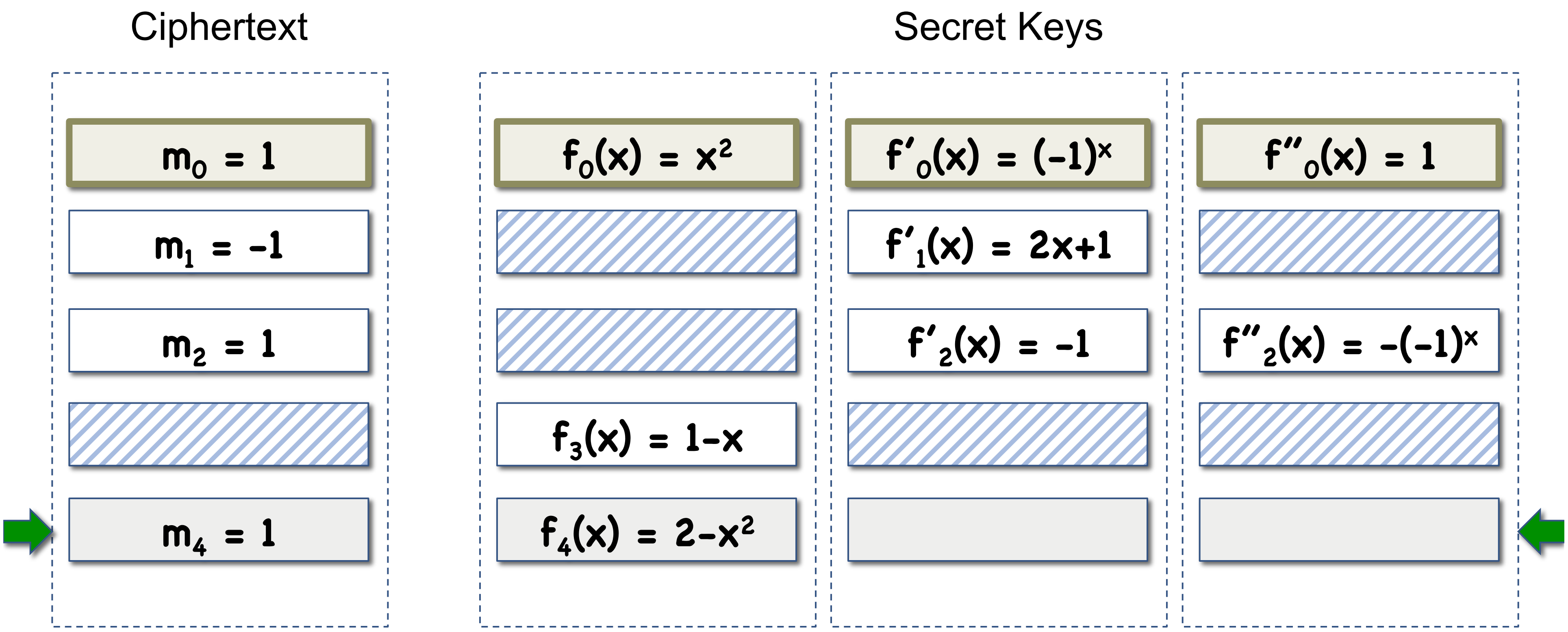


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$		

Single Use Hiding

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

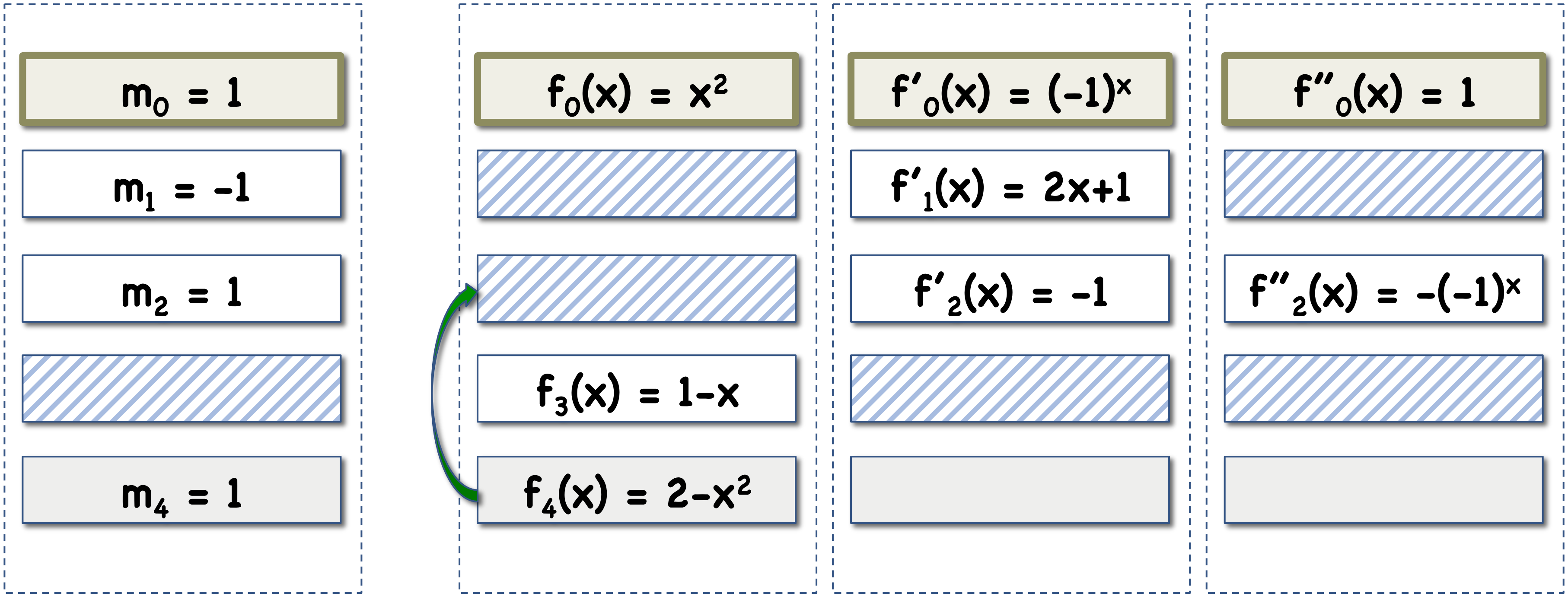


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = 1$	$f_4(x) = 2-x^2$		

Single Use Hiding

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

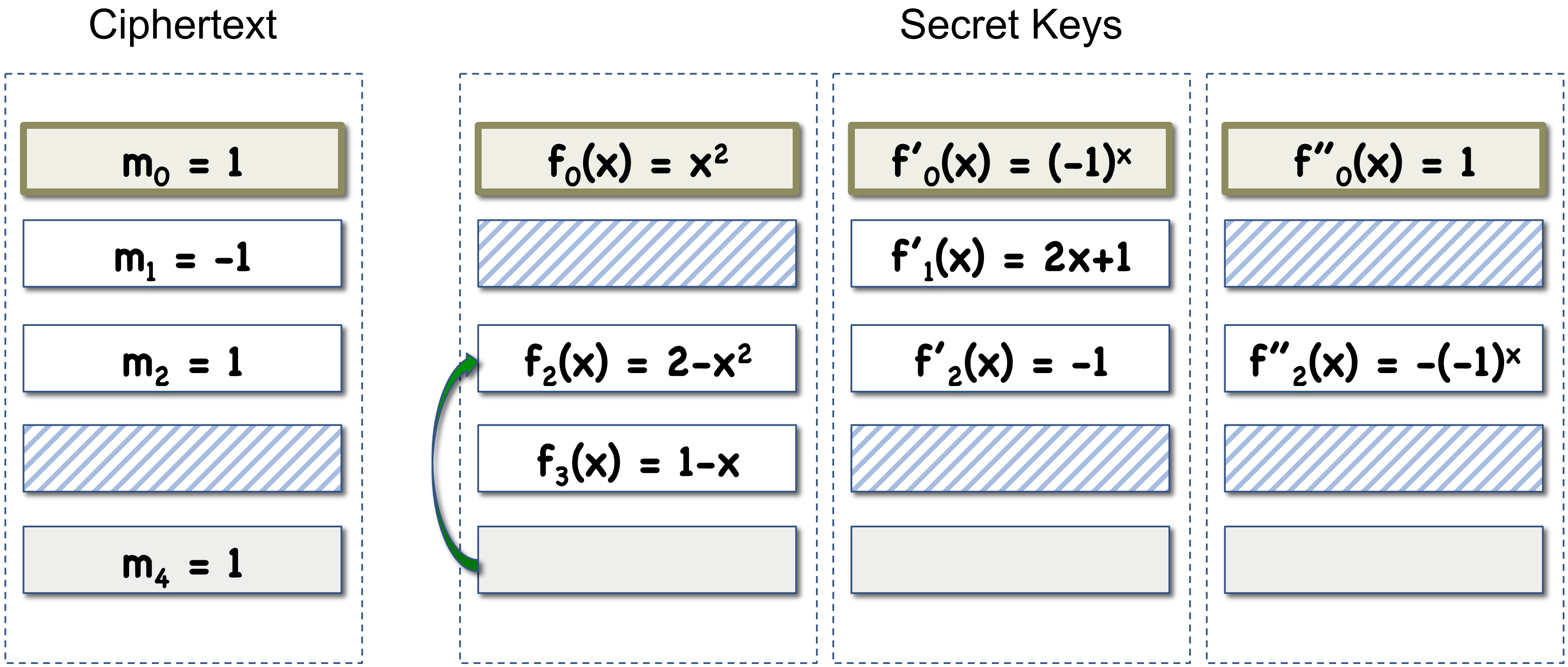


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = 1$	$f_4(x) = 2-x^2$		

Weak Sk Moving

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

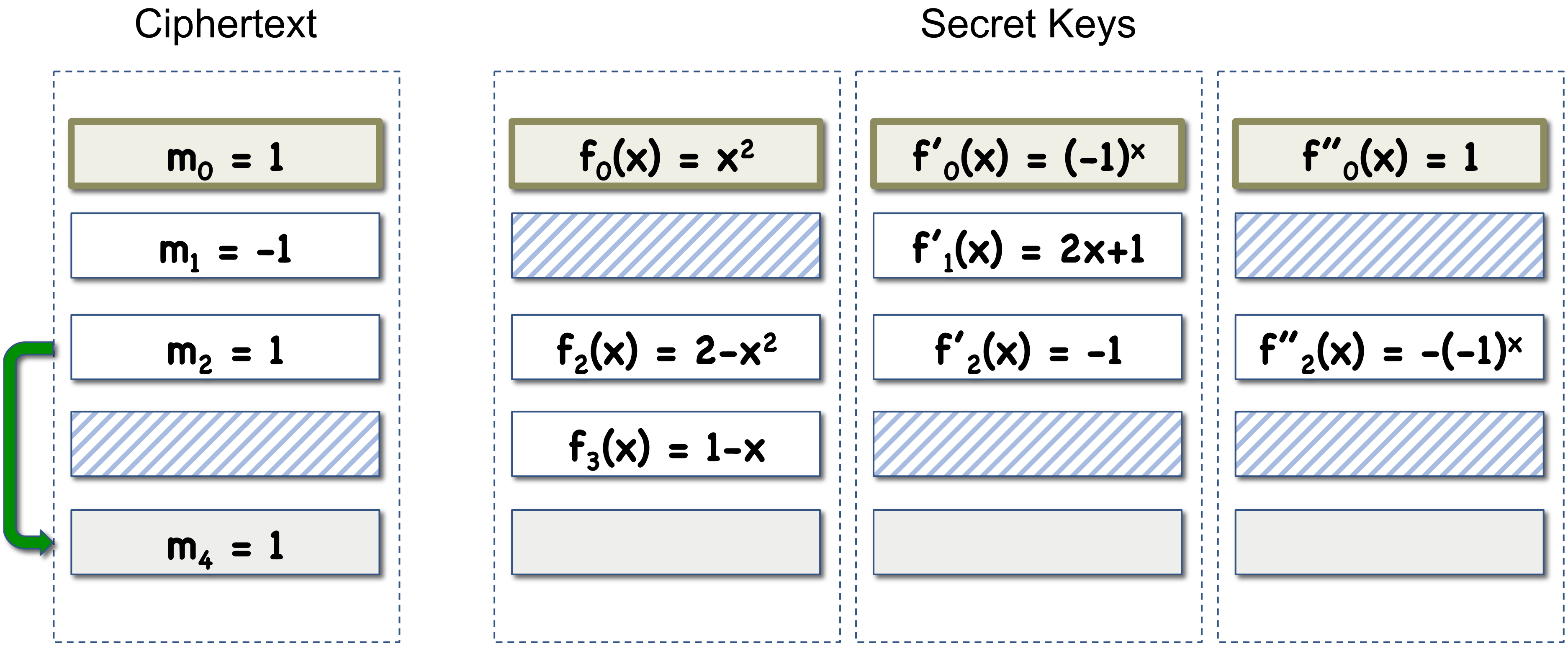


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = 1$			

Weak Sk Moving

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

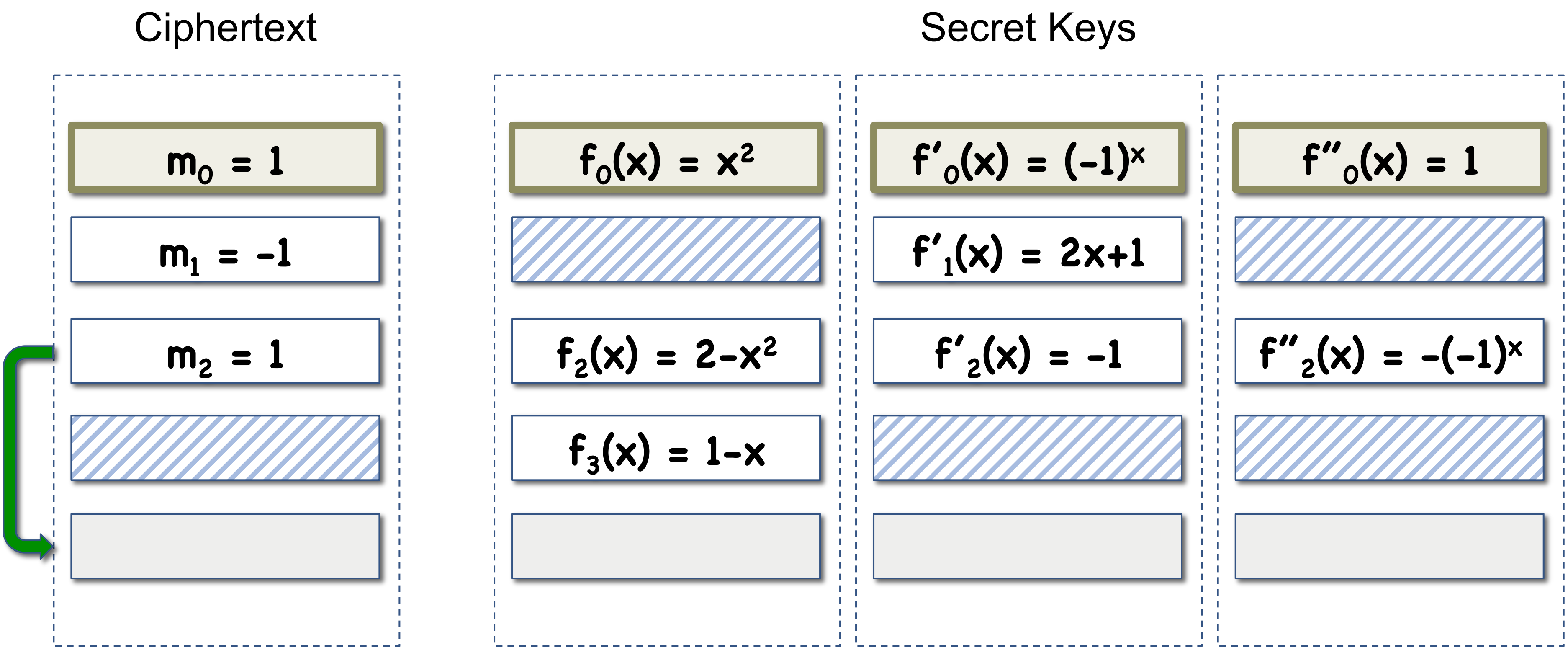


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = 1$			

Slot Duplication

Example Reduction: Strong Sk Moving

Goal: move $\mathbf{f_1}$ to slot **3**

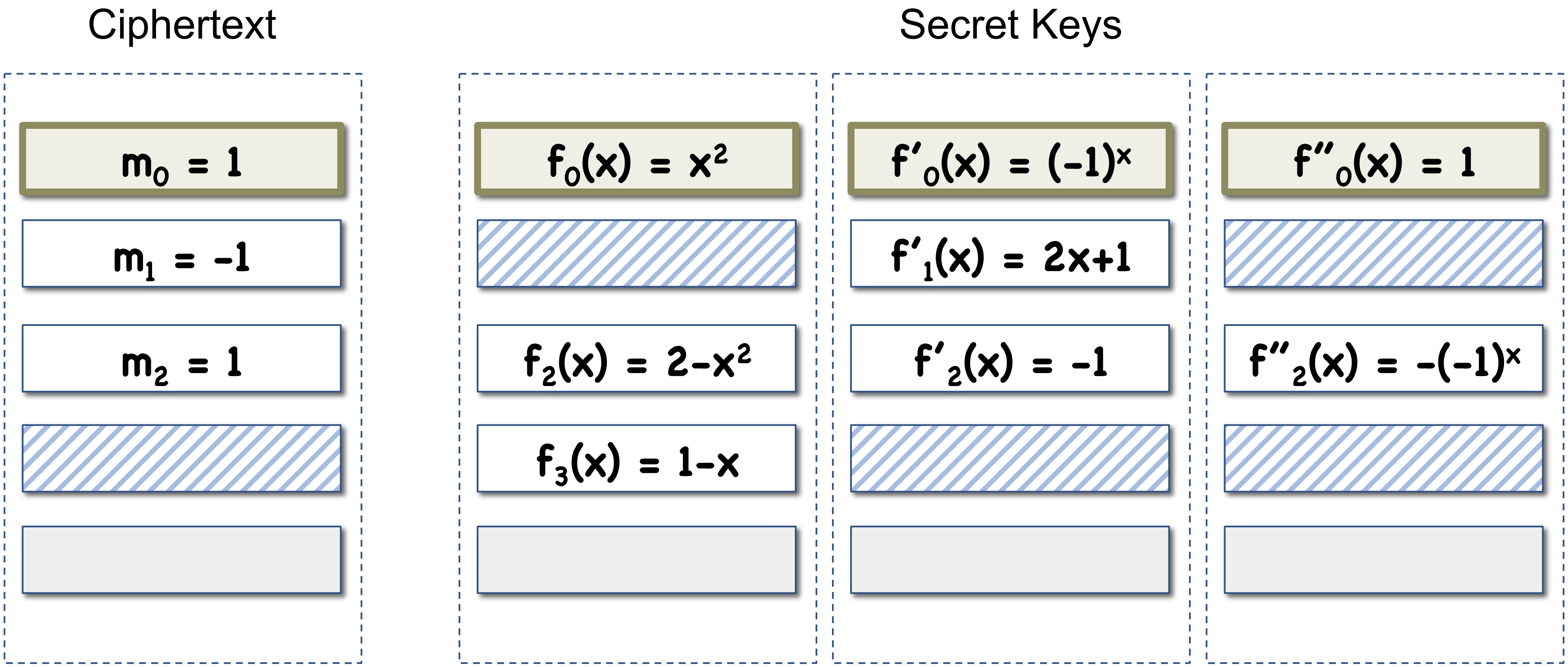


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Slot Duplication

Example Reduction: Strong Sk Moving

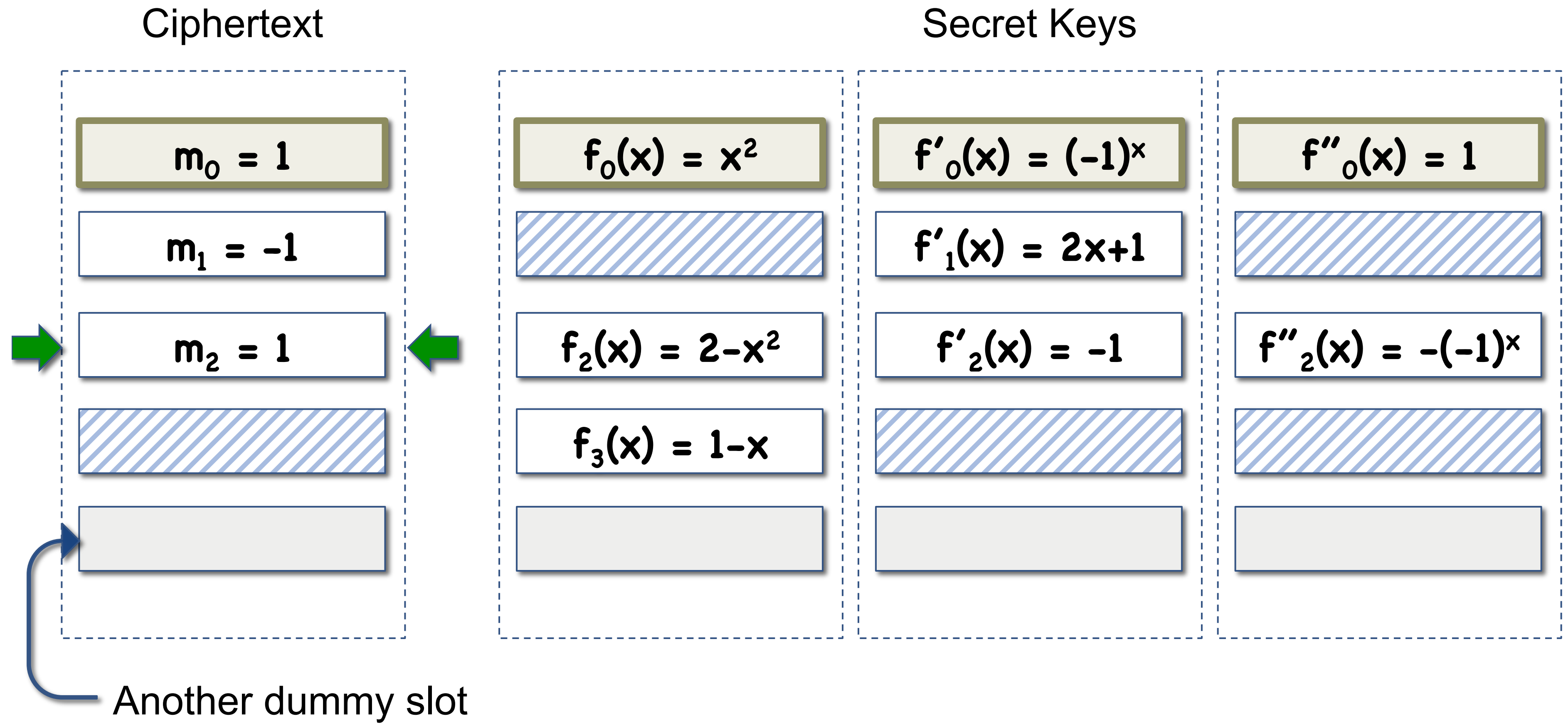
Goal: move $\mathbf{f_1}$ to slot **3**



Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

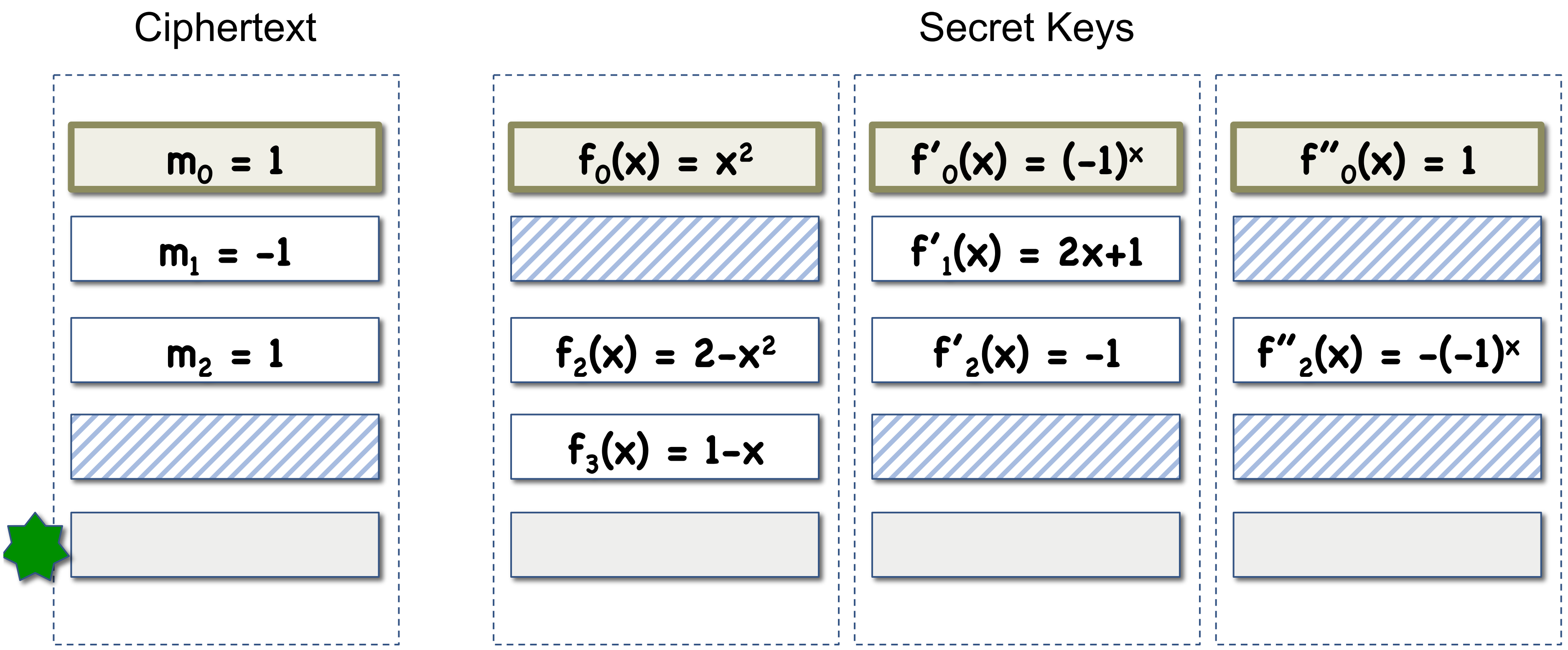


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Another dummy slot

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

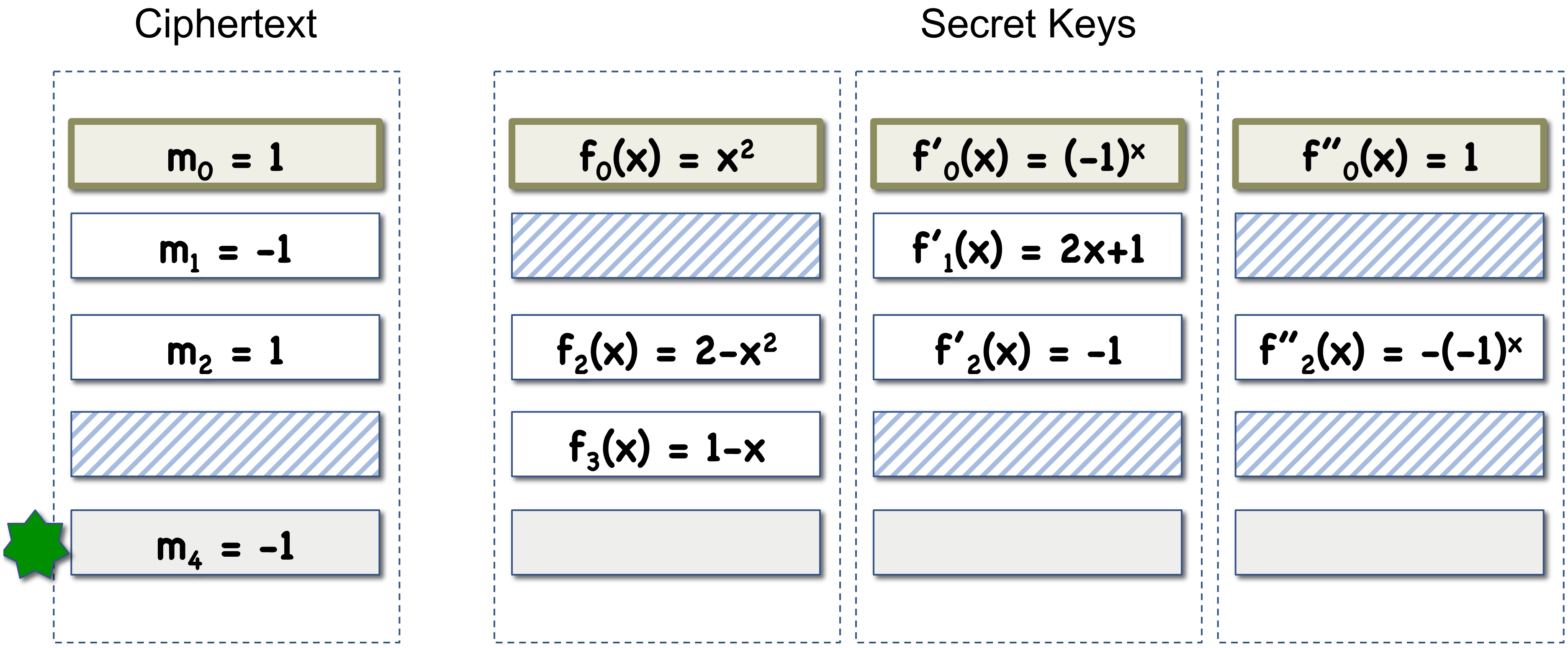


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

New Slot

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

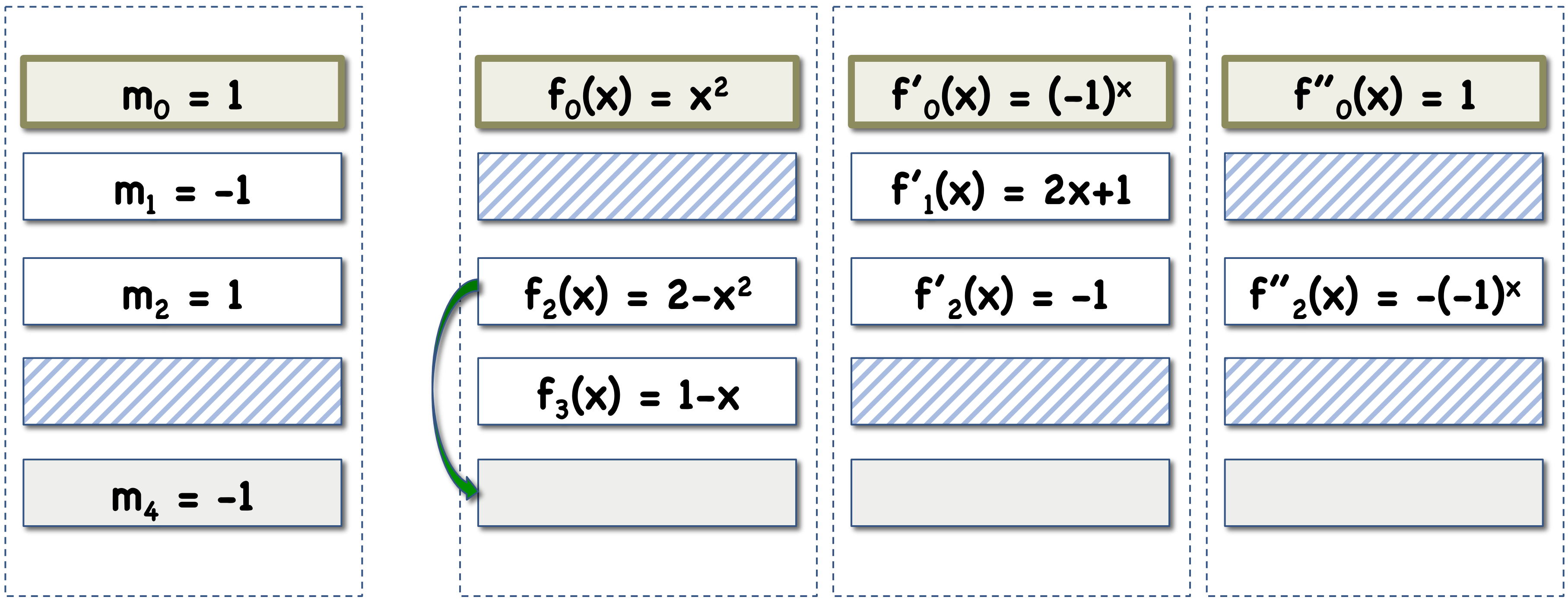


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
✶ $m_4 = -1$			

New Slot

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

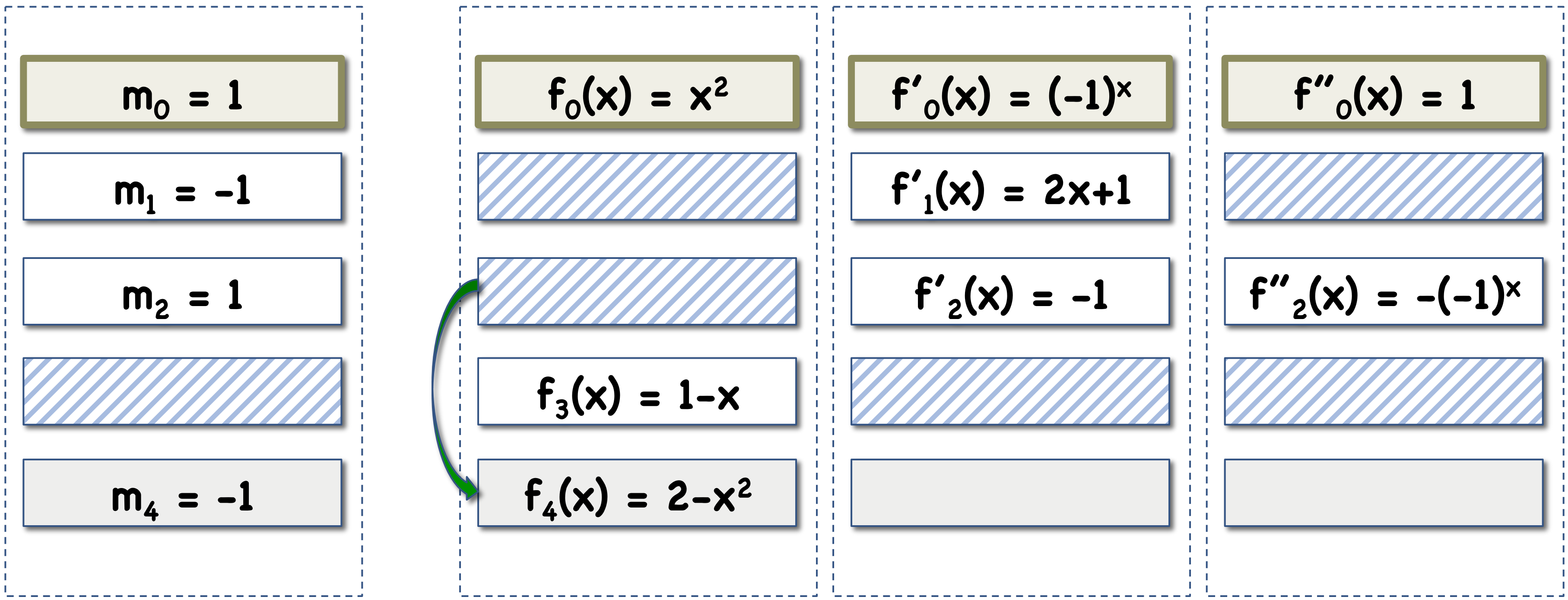


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$			

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to **-1**

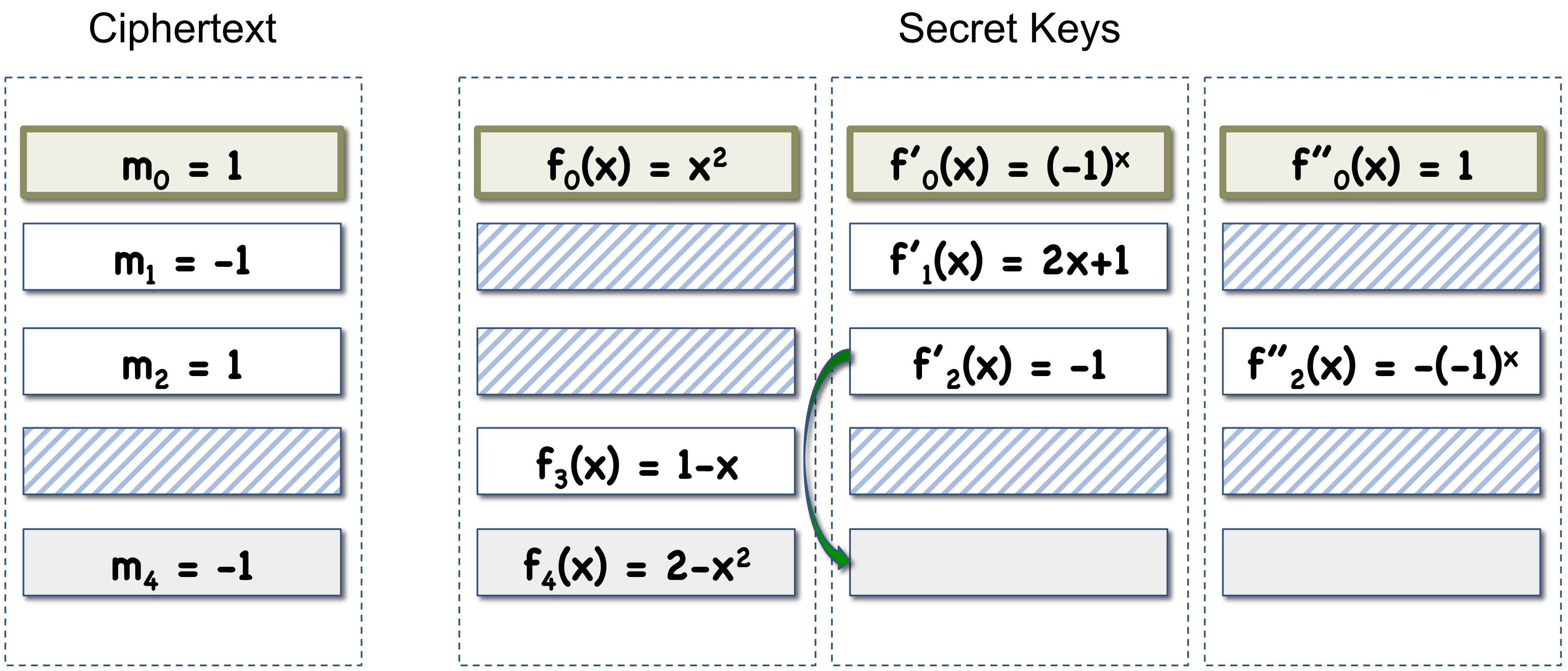


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$		

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

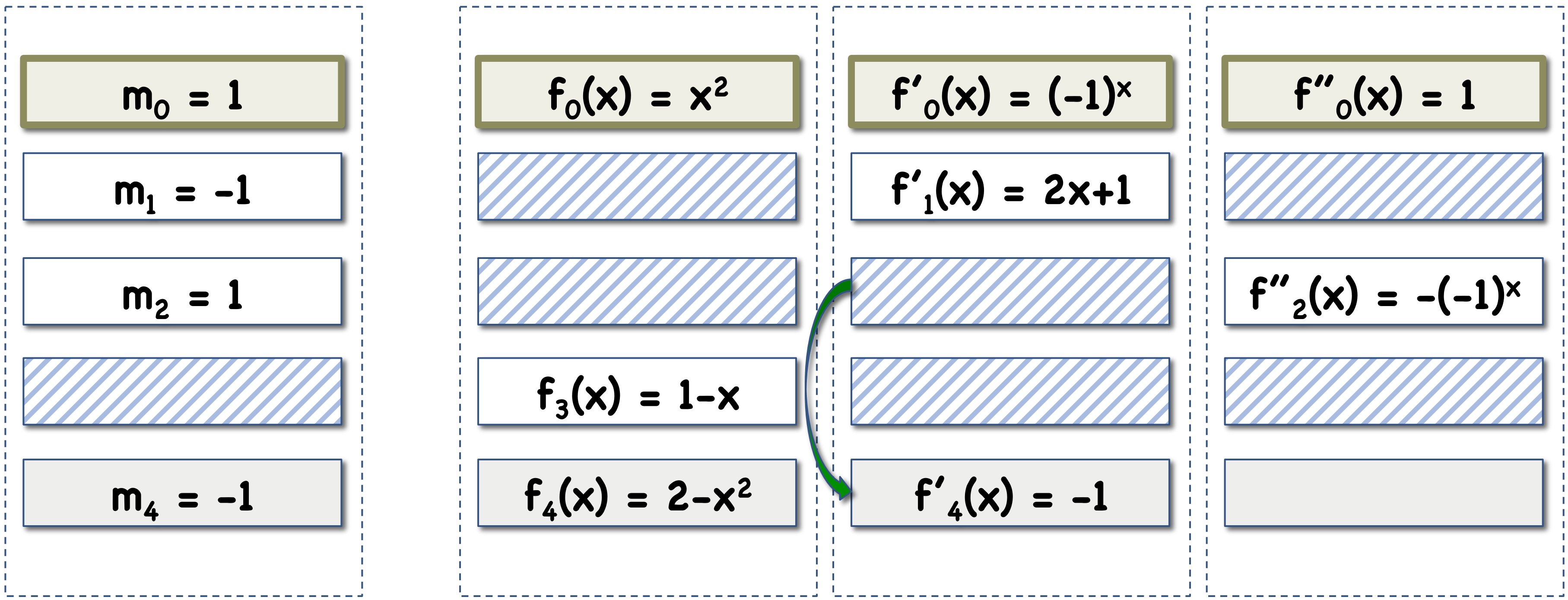


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$		

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

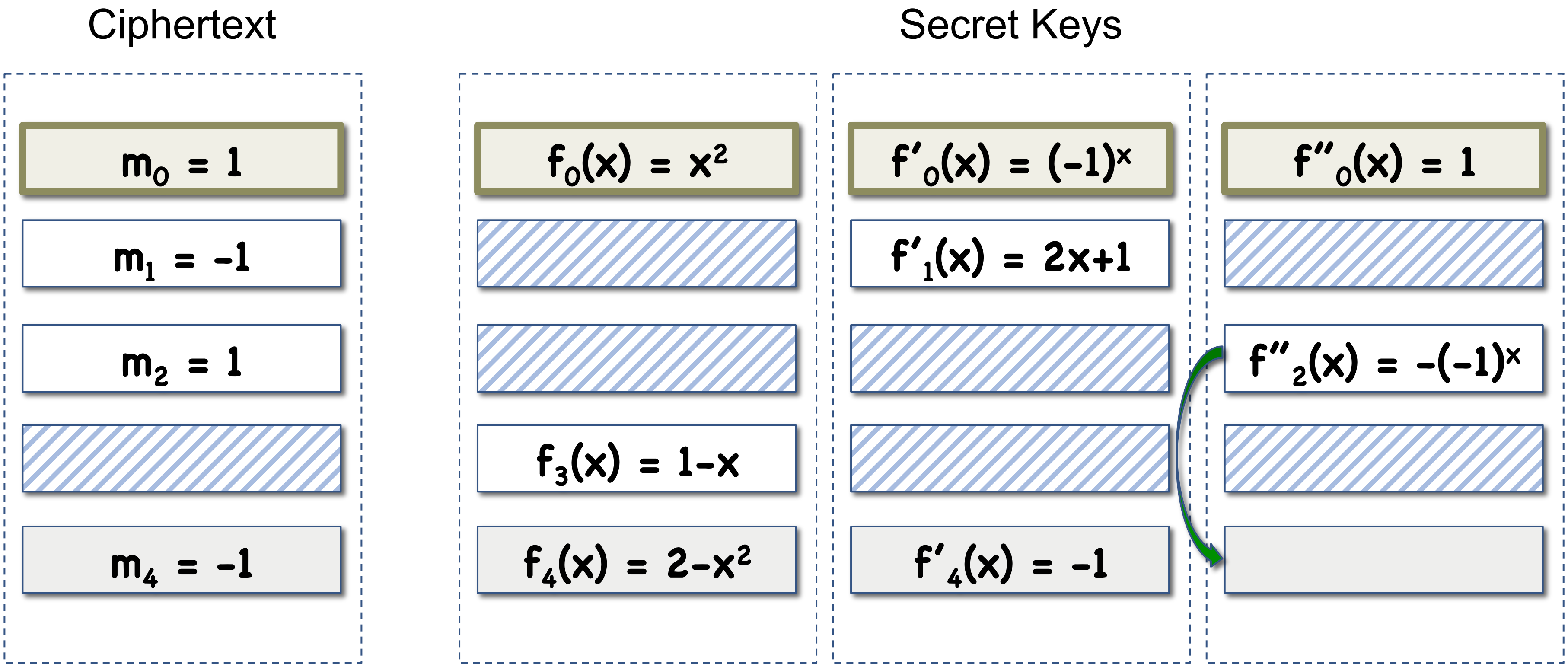


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	▨	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

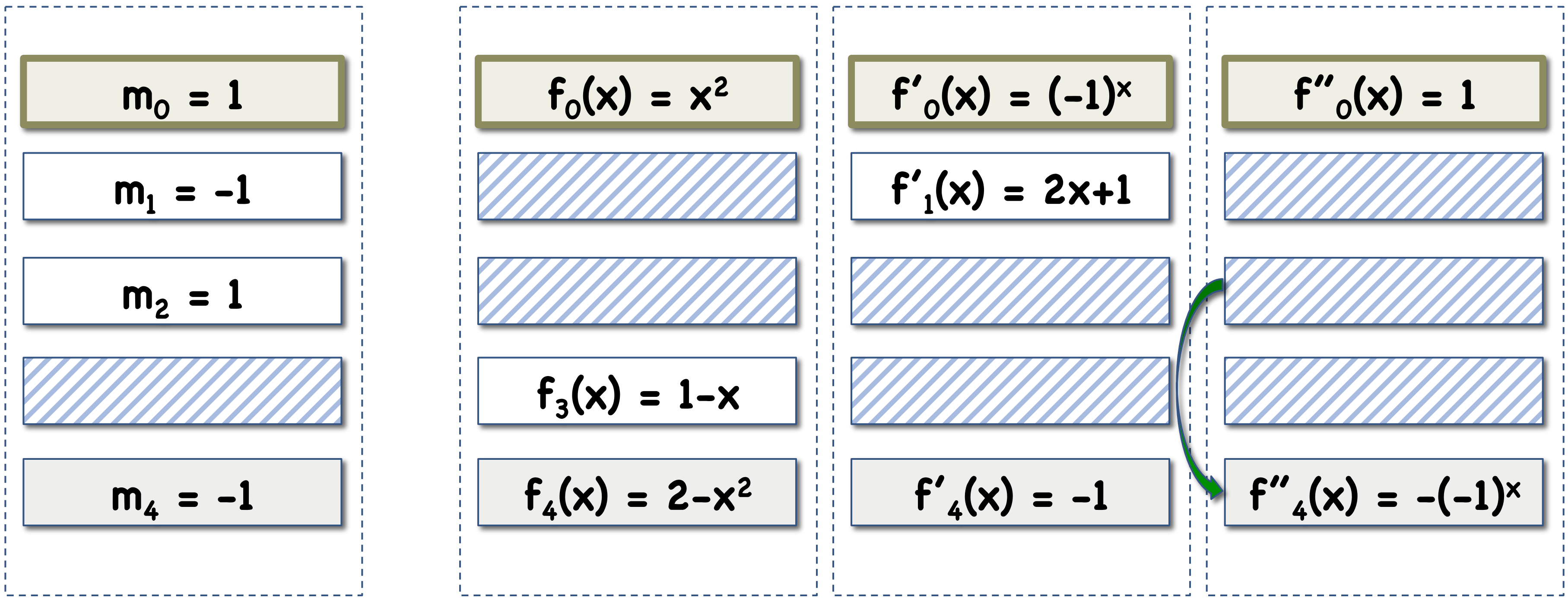


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	▨	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

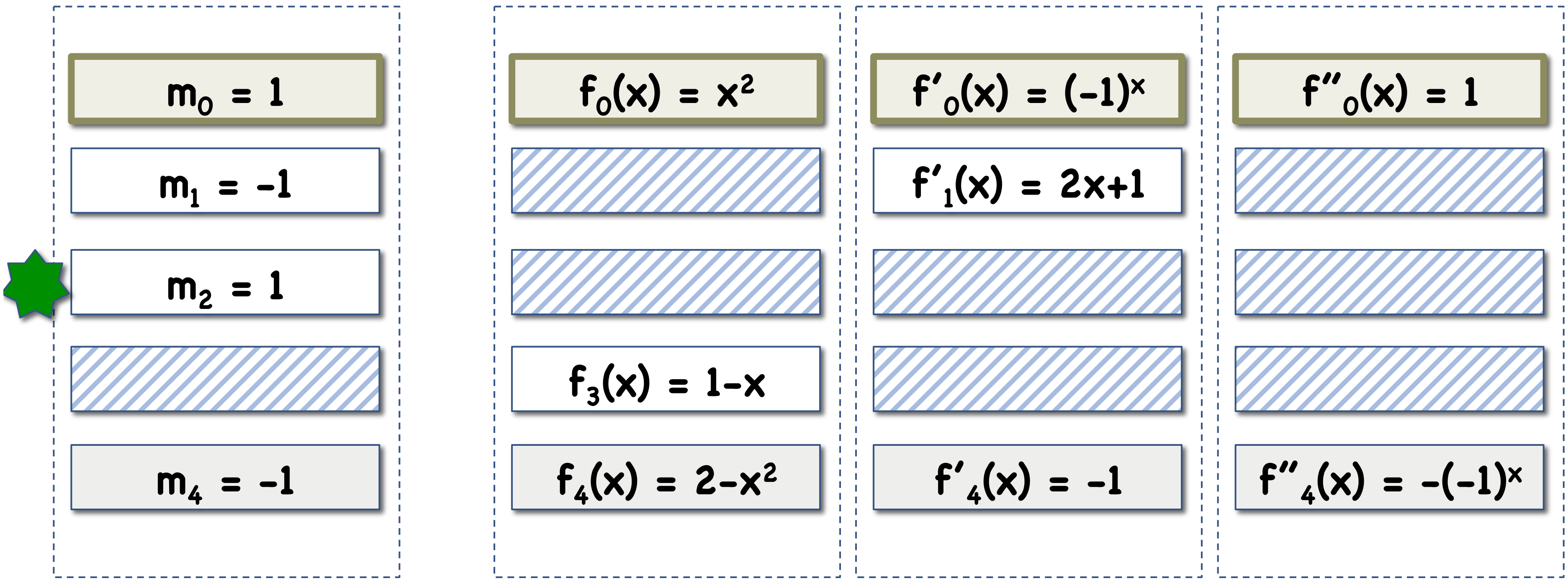


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	▨	▨
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	$f''_4(x) = -(-1)^x$

Strong Sk Moving

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

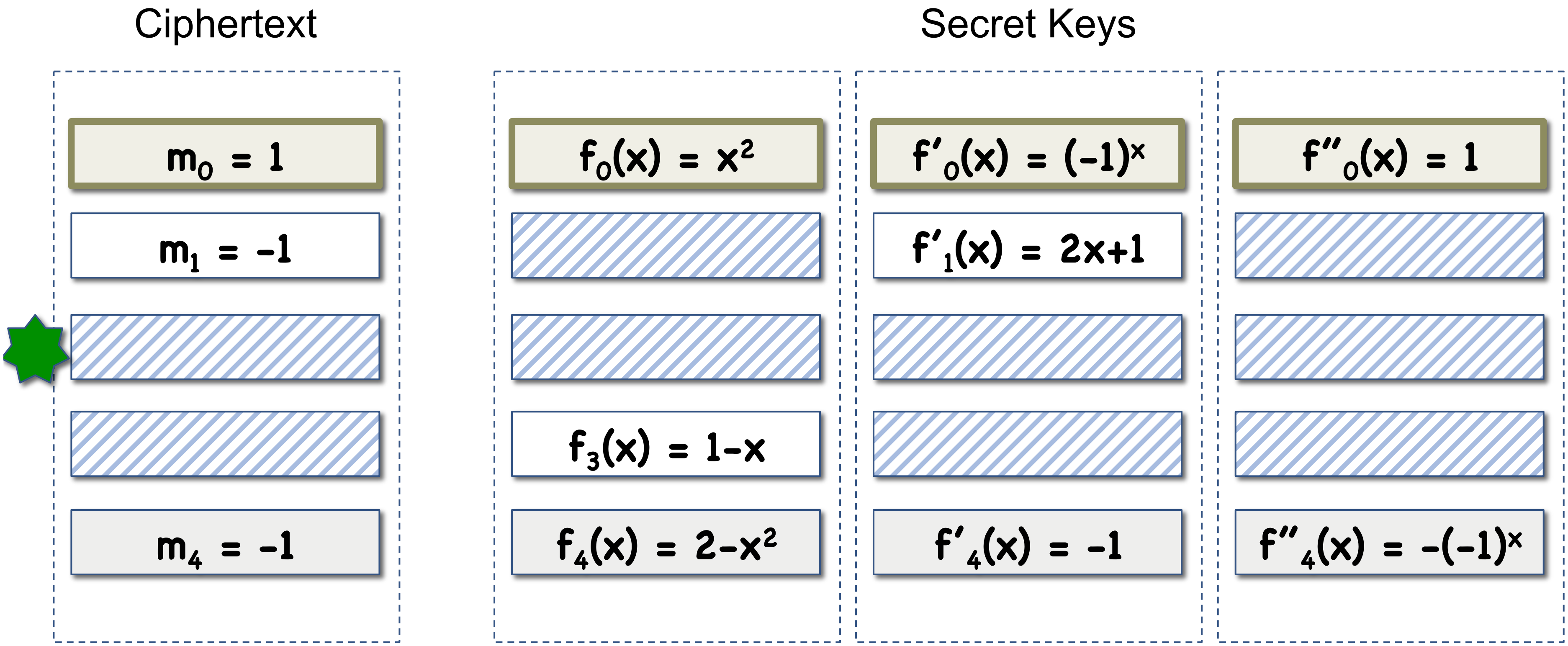


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = 1$	▨	▨	▨
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	$f''_4(x) = -(-1)^x$

New Slot

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

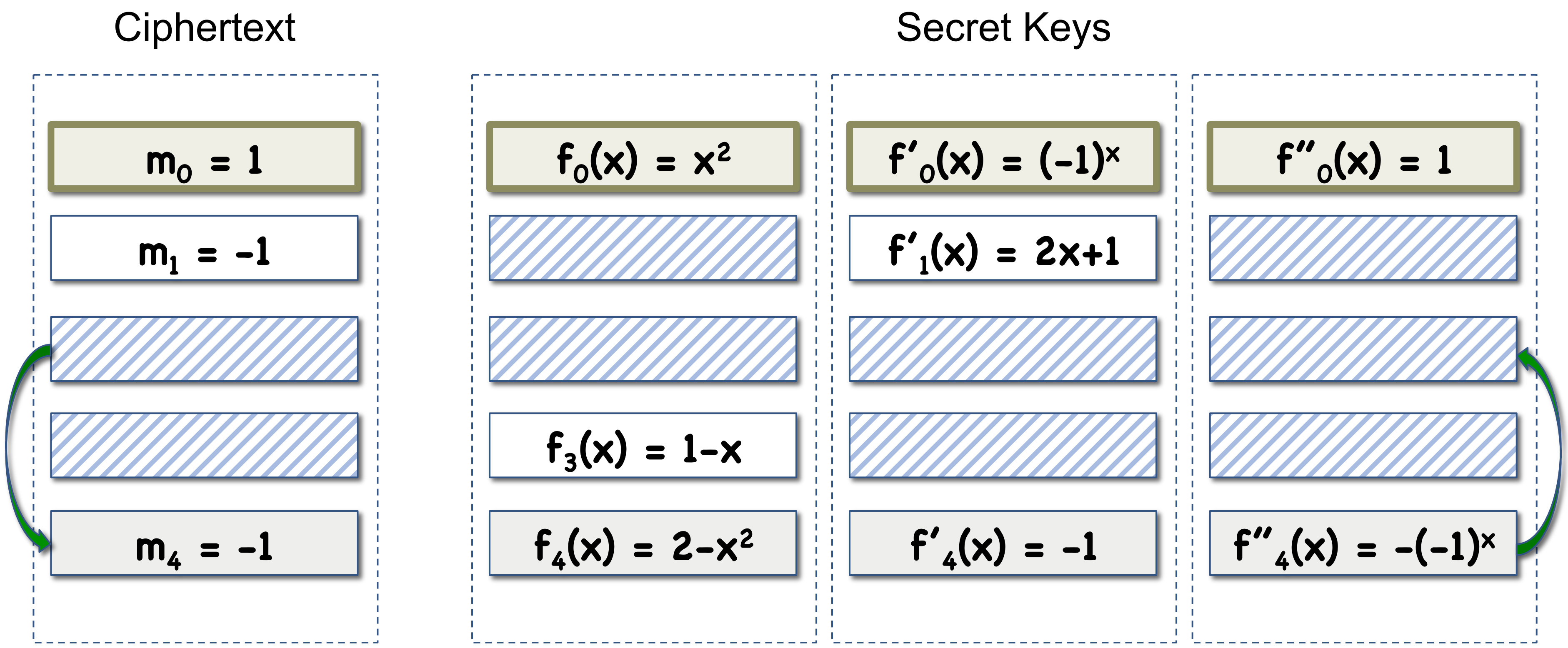


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
▨	▨	▨	▨
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	$f''_4(x) = -(-1)^x$

New Slot

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

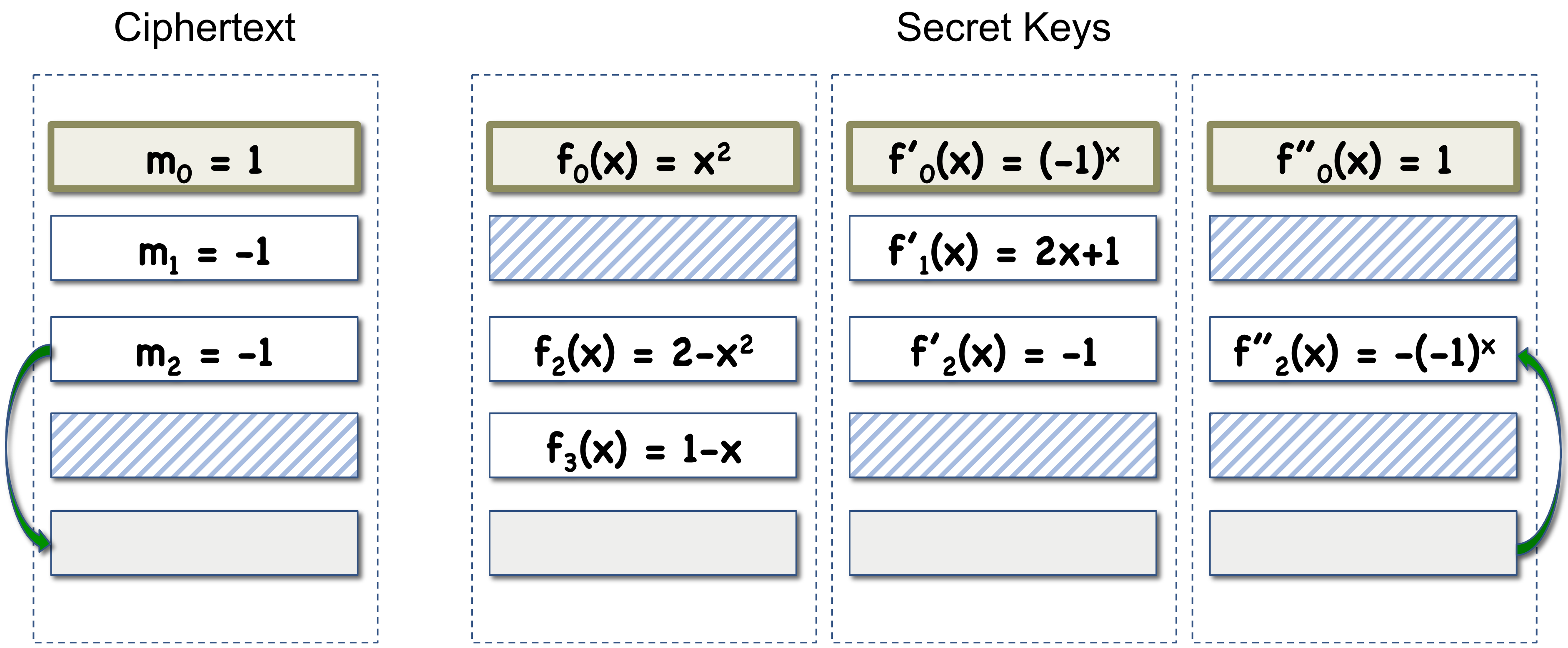


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
▨	▨	▨	▨
▨	$f_3(x) = 1-x$	▨	▨
$m_4 = -1$	$f_4(x) = 2-x^2$	$f'_4(x) = -1$	$f''_4(x) = -(-1)^x$

Slot Symmetry

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$

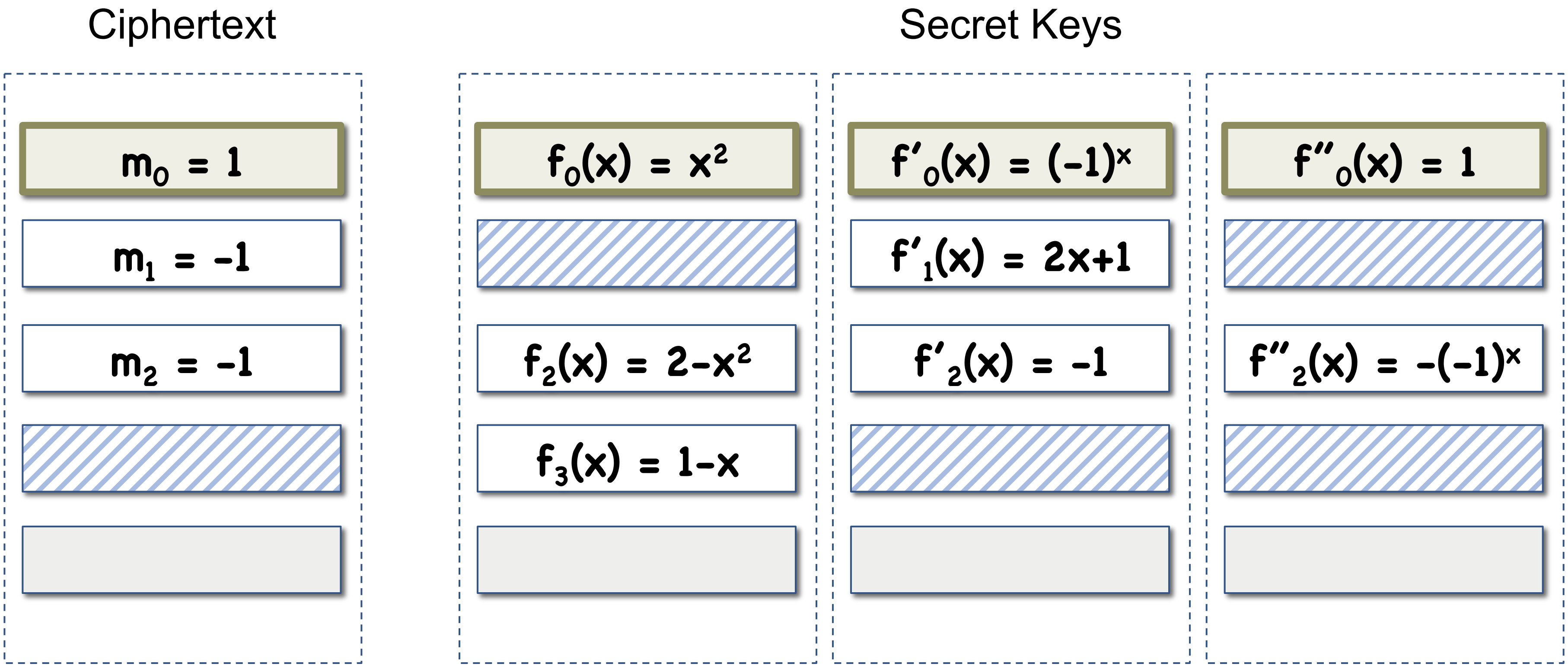


Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = -1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Slot Symmetry

Example Reduction: Weak Ctxt Indist

Goal: change $\mathbf{m_2}$ to $\mathbf{-1}$



Ciphertext	Secret Keys		
$m_0 = 1$	$f_0(x) = x^2$	$f'_0(x) = (-1)^x$	$f''_0(x) = 1$
$m_1 = -1$	▨	$f'_1(x) = 2x+1$	▨
$m_2 = -1$	$f_2(x) = 2-x^2$	$f'_2(x) = -1$	$f''_2(x) = -(-1)^x$
▨	$f_3(x) = 1-x$	▨	▨

Instantiating Slotted FE

We give construction for NC^1 circuits from composite-order graded encodings

- Slot Symmetry/Single-use Hiding: Information theoretic
- Slot Duplication/Ctxt Moving/Sk Moving: simple assumptions

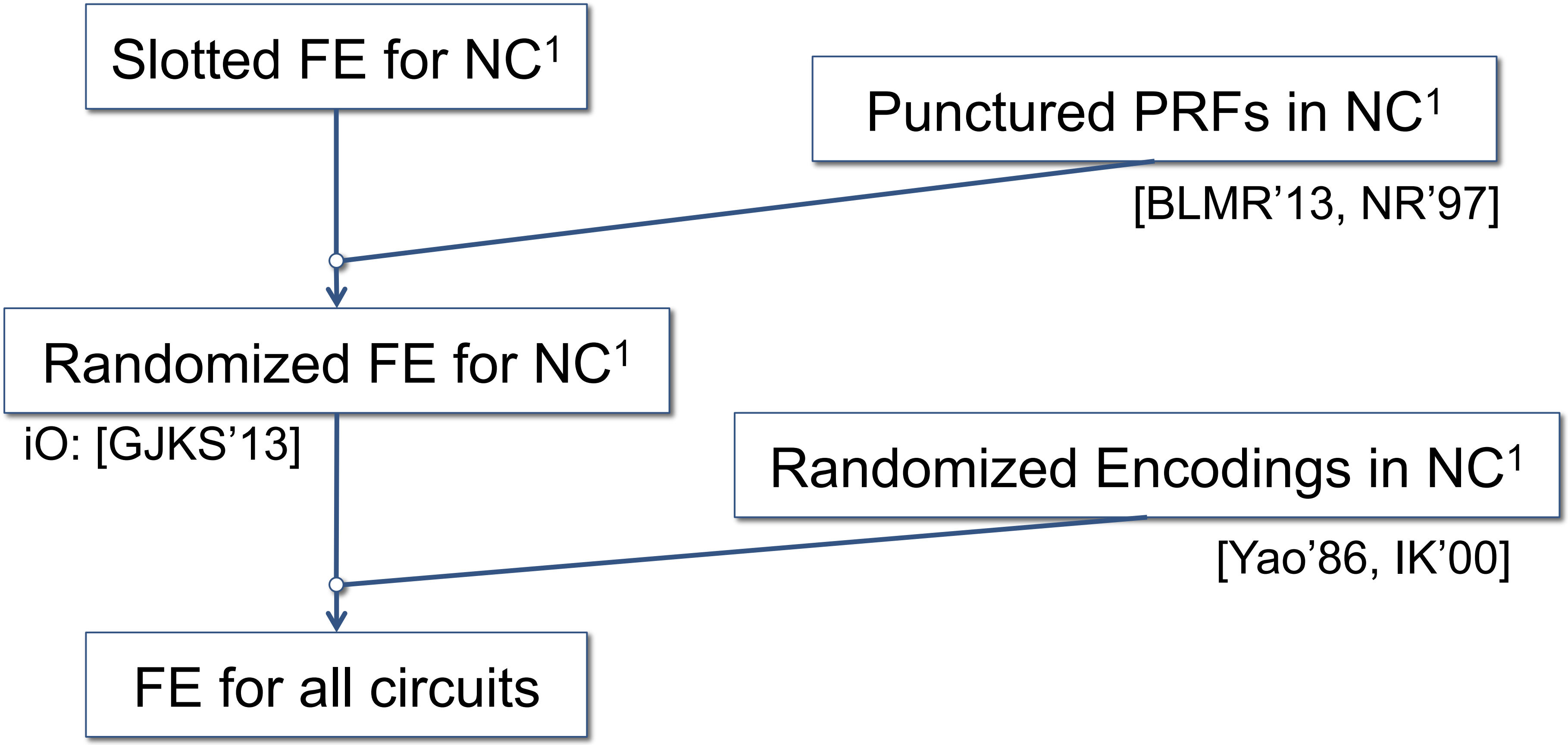
Construction requires new **extension** procedure on encodings

- bind ctxt (or sk) components together (no “mixing and matching”)
- Do not need to modify underlying encodings

Theorem: Relatively simple assumptions on mmaps
⇒ (adaptively) secure FE for NC^1

But I promised FE for all circuits…

Achieving FE for All Circuits



Slotted FE for NC^1

Punctured PRFs in NC^1

[BLMR’13, NR’97]

Randomized FE for NC^1

iO: [GJKS’13]

Randomized Encodings in NC^1

[Yao’86, IK’00]

FE for all circuits

Randomized FE for NC^1

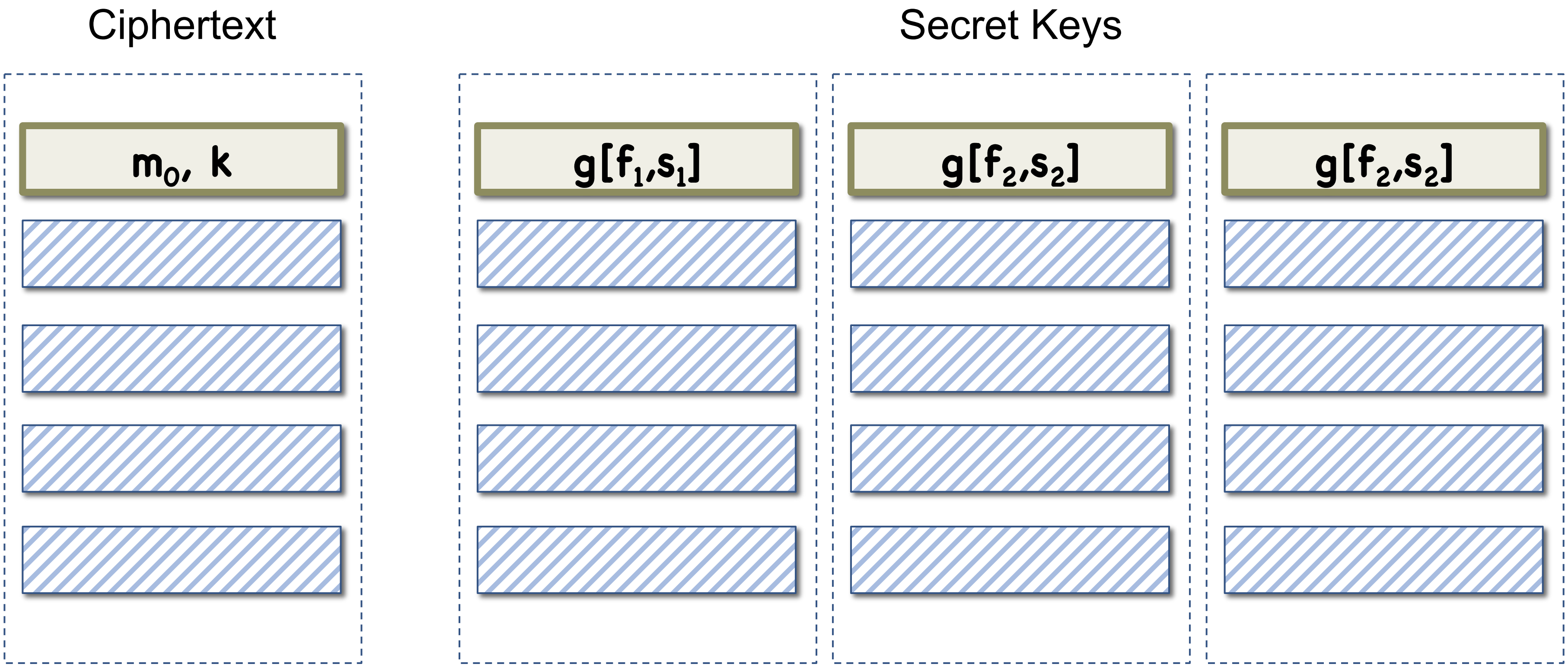
Basic idea: ctxt contains PRF key which generates randomness

```
Enc_R(mpk, m):      k ← {0,1}^λ
                    c ← Enc( mpk,  (m,k) )
                    Output c

Define:             g[f,s](m,k) := f( m ; PRF(k,s) )

KeyGen_R(msk, f):   s ← {0,1}^λ
                    sk_f ← KeyGen(msk, g[f,s])
                    Output sk_f
```

Actual scheme more complicated

Randomized FE for NC^1

Proof idea:



Ciphertext

Secret Keys

m_0, k

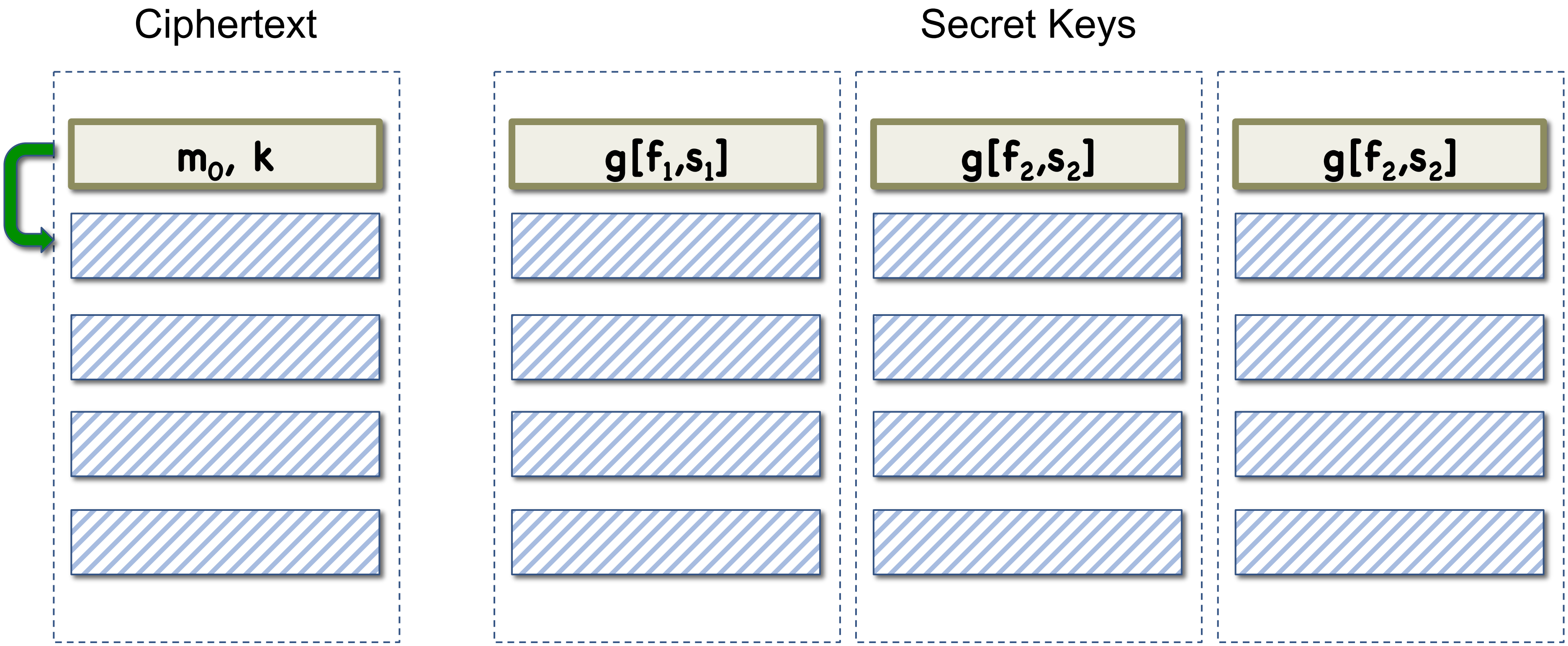
$g[f_1,s_1]$

$g[f_2,s_2]$

$g[f_2,s_2]$

Randomized FE for NC[1]

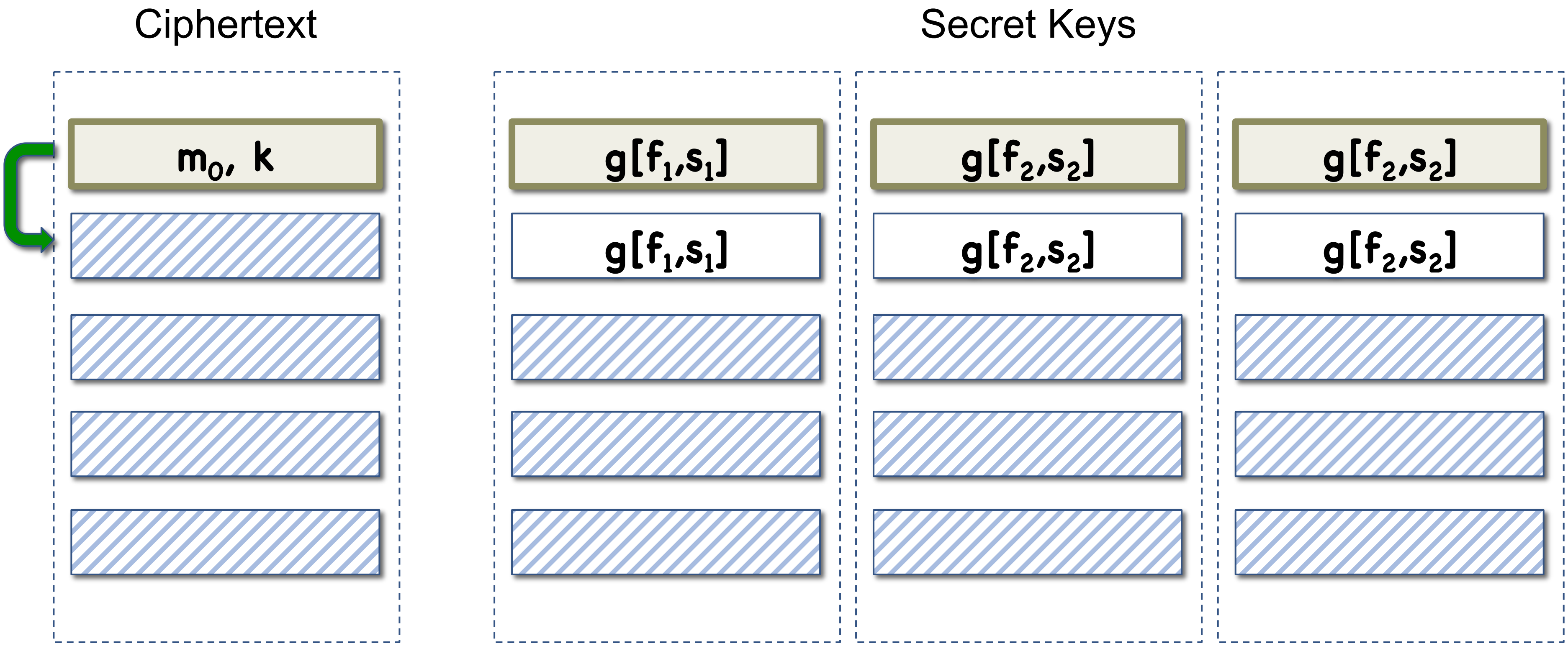
Proof idea:



Slot Duplication

Randomized FE for NC[1]

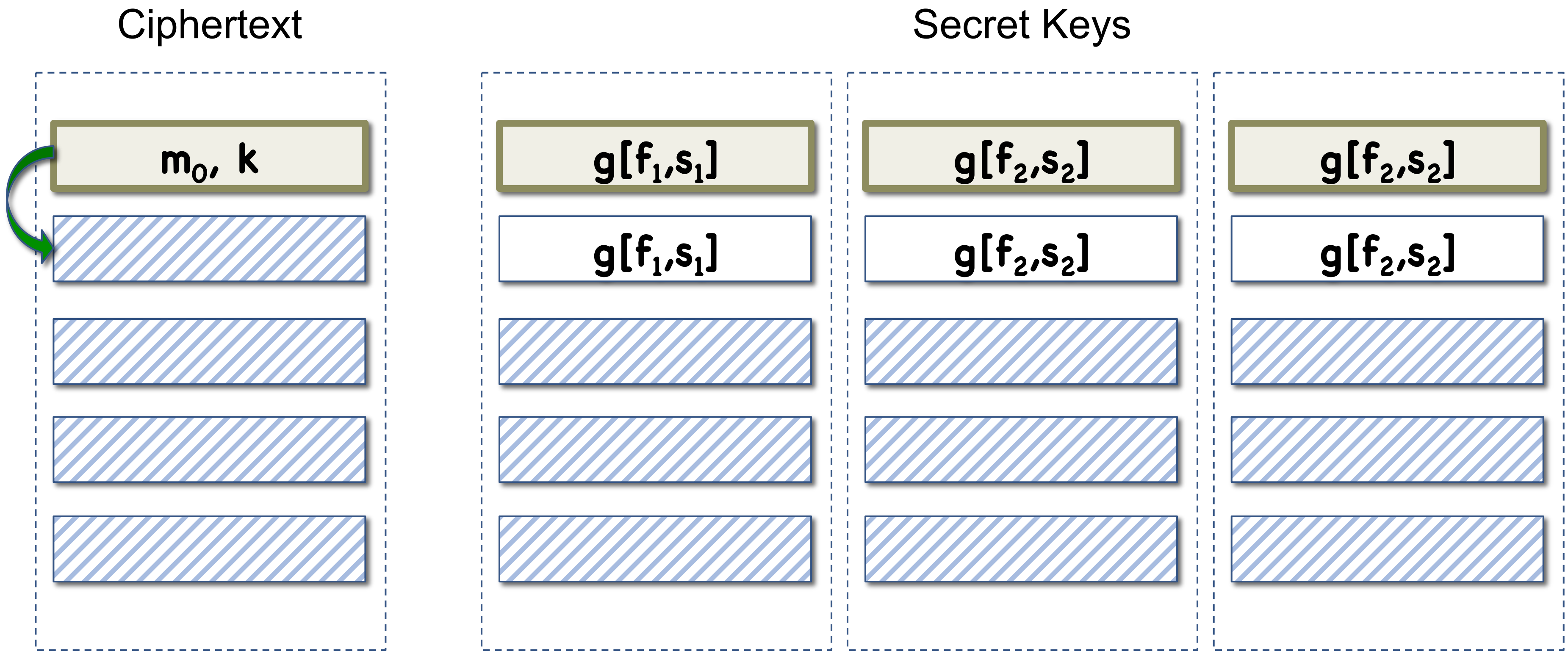
Proof idea:



Slot Duplication

Randomized FE for NC^1

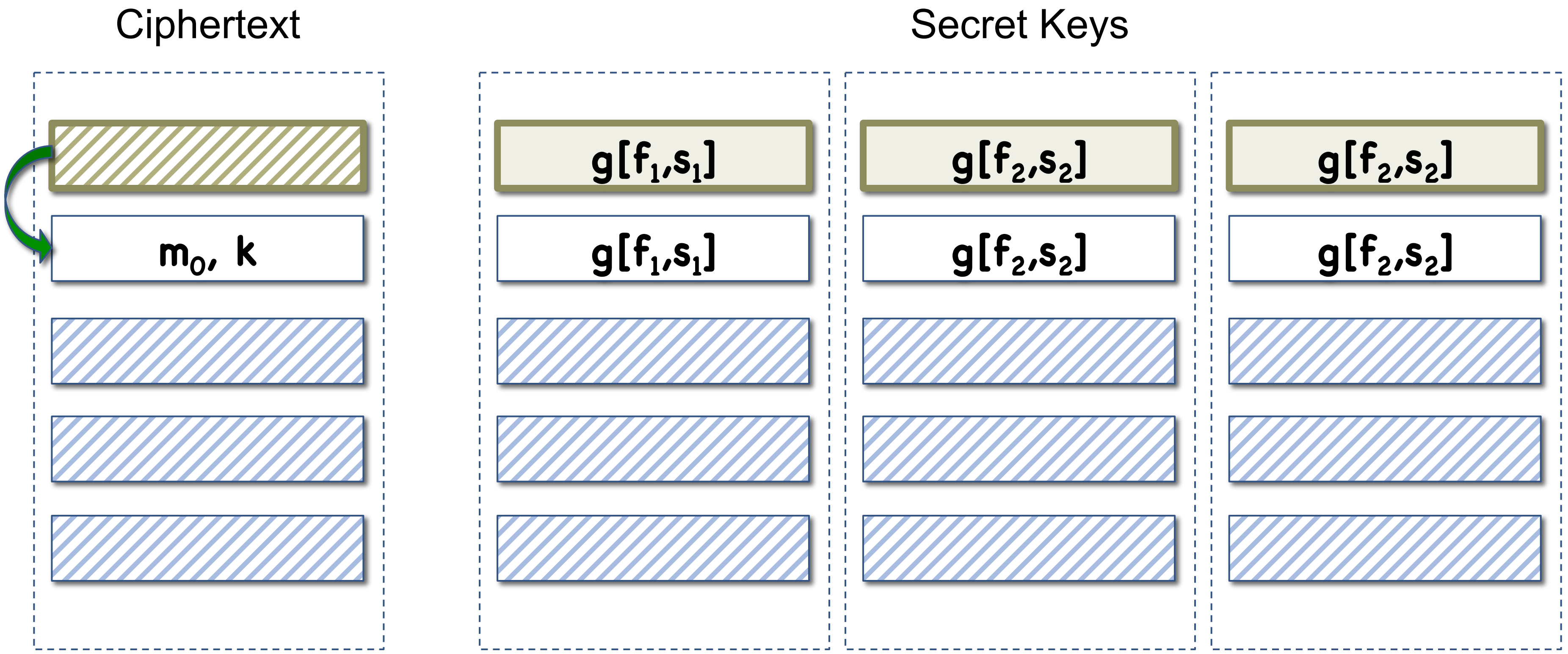
Proof idea:



Ciphertext Moving

Randomized FE for NC^1

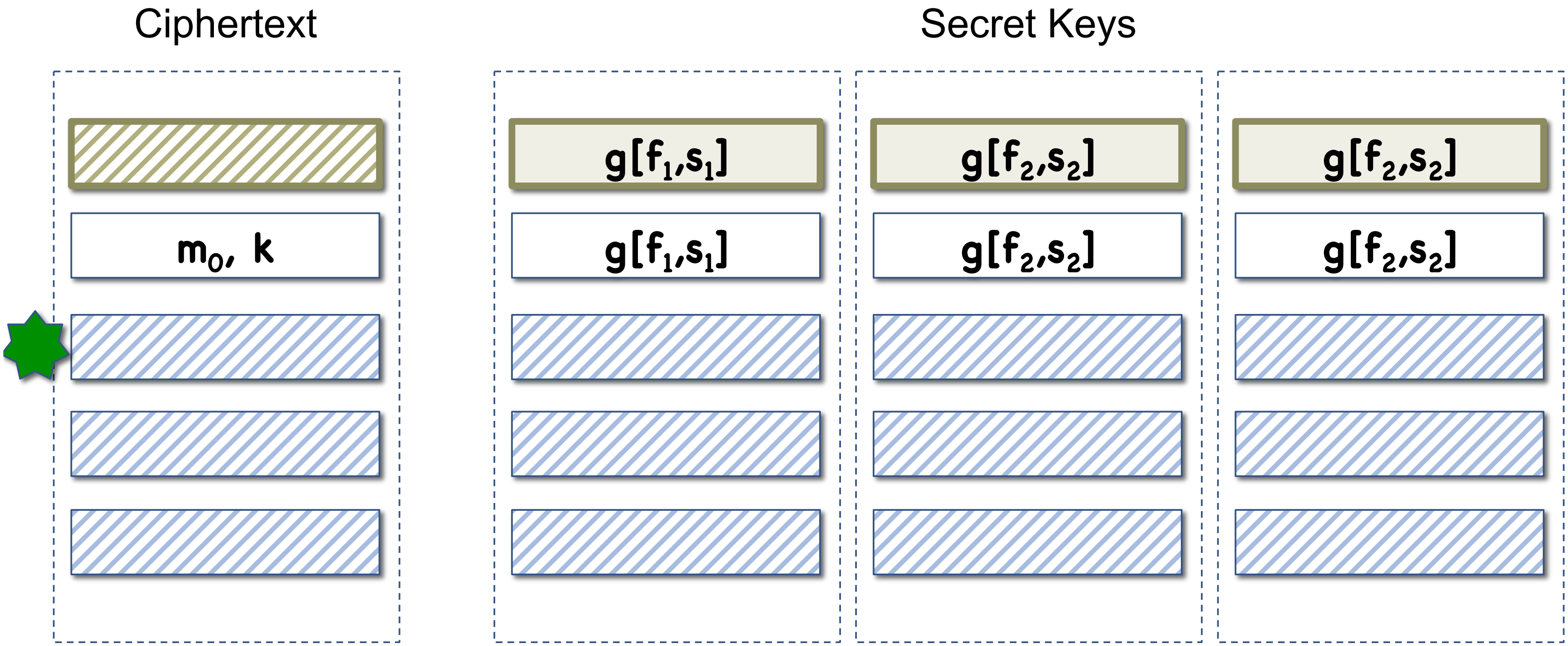
Proof idea:



Ciphertext Moving

Randomized FE for NC^1

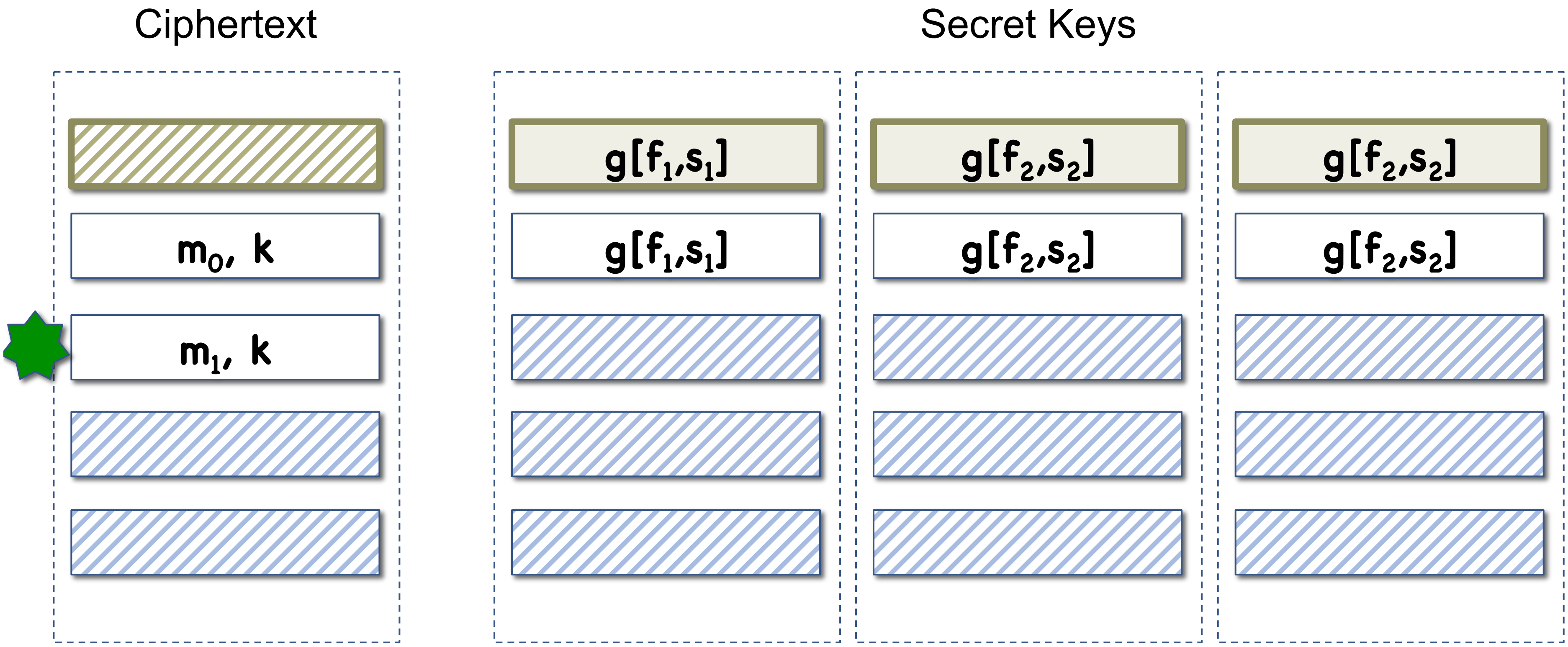
Proof idea:



New Slot

Randomized FE for NC^1

Proof idea:



Ciphertext

Secret Keys

m_0, k

m_1, k

g[f_1,s_1]

g[f_1,s_1]

g[f_2,s_2]

g[f_2,s_2]

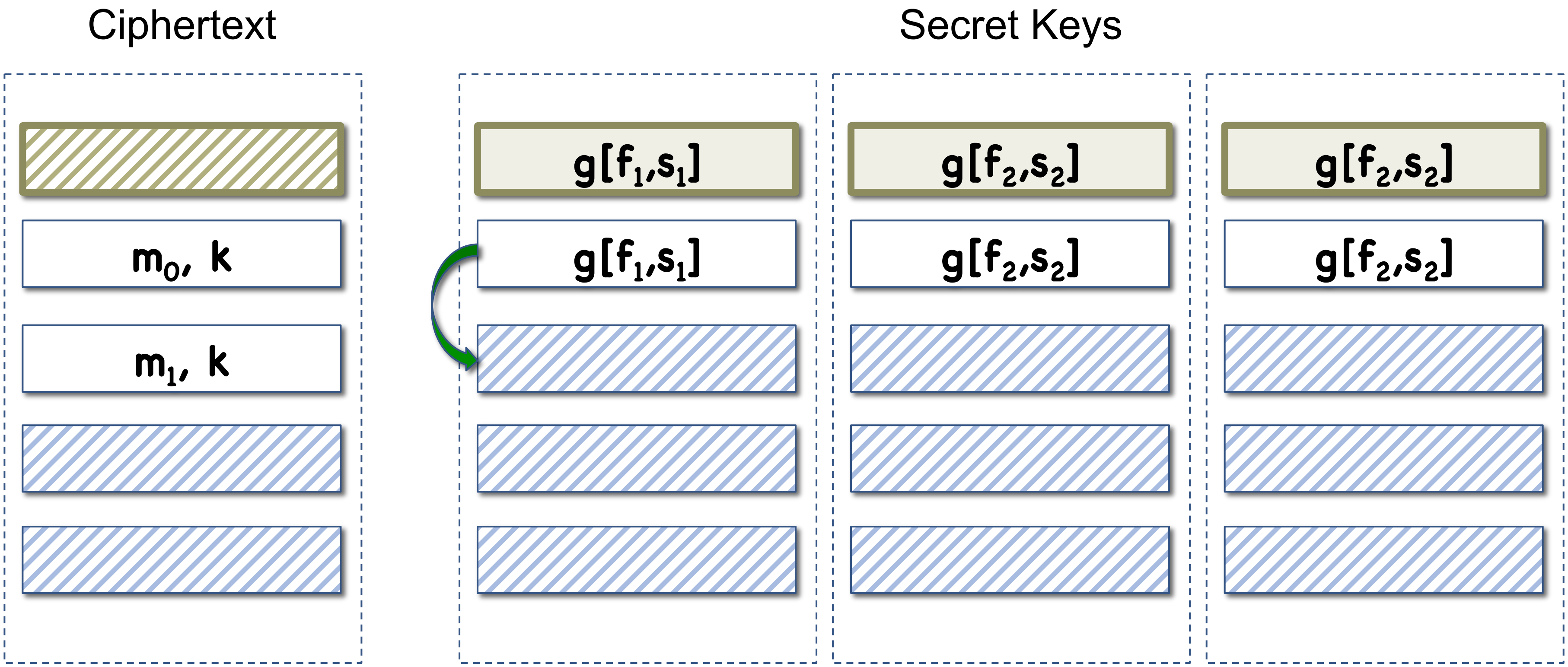
g[f_2,s_2]

g[f_2,s_2]

New Slot

Randomized FE for NC^1

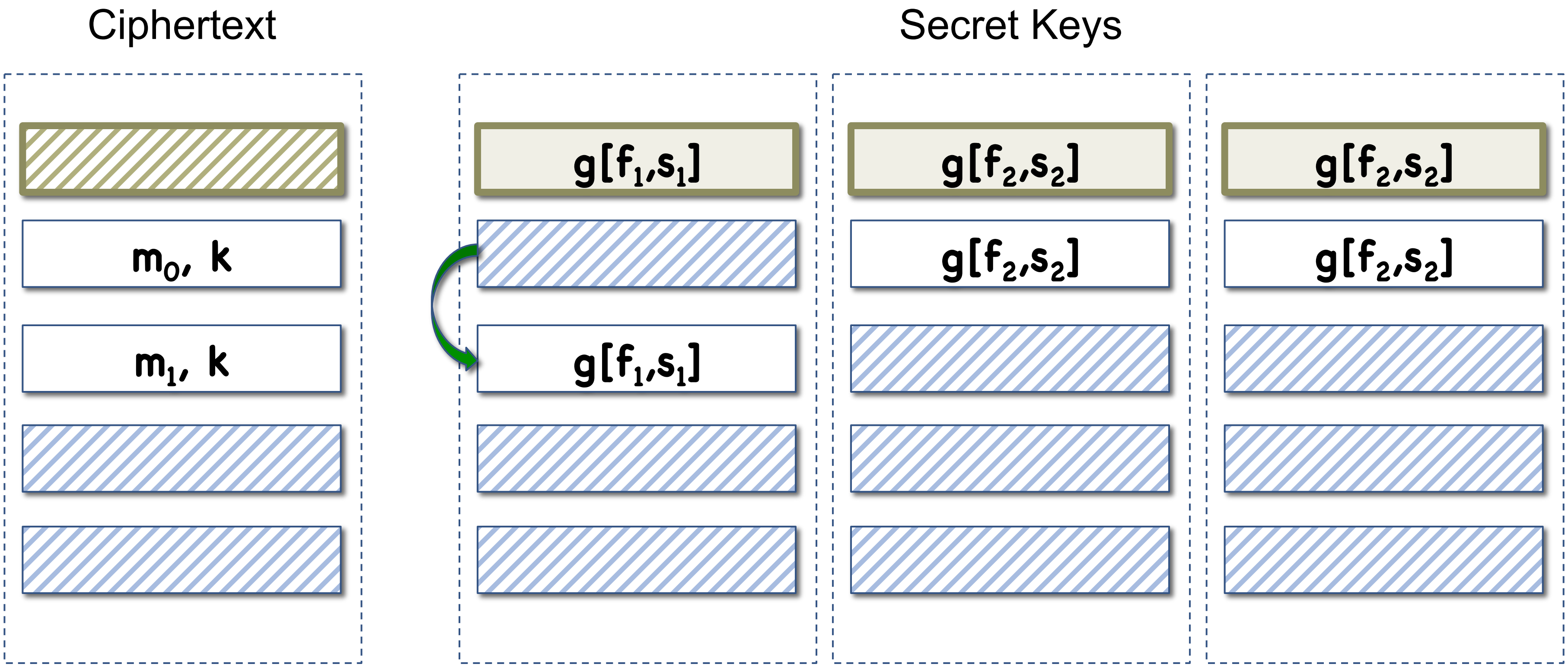
Proof idea:



“Super Strong Secret Key Moving”

Randomized FE for NC^1

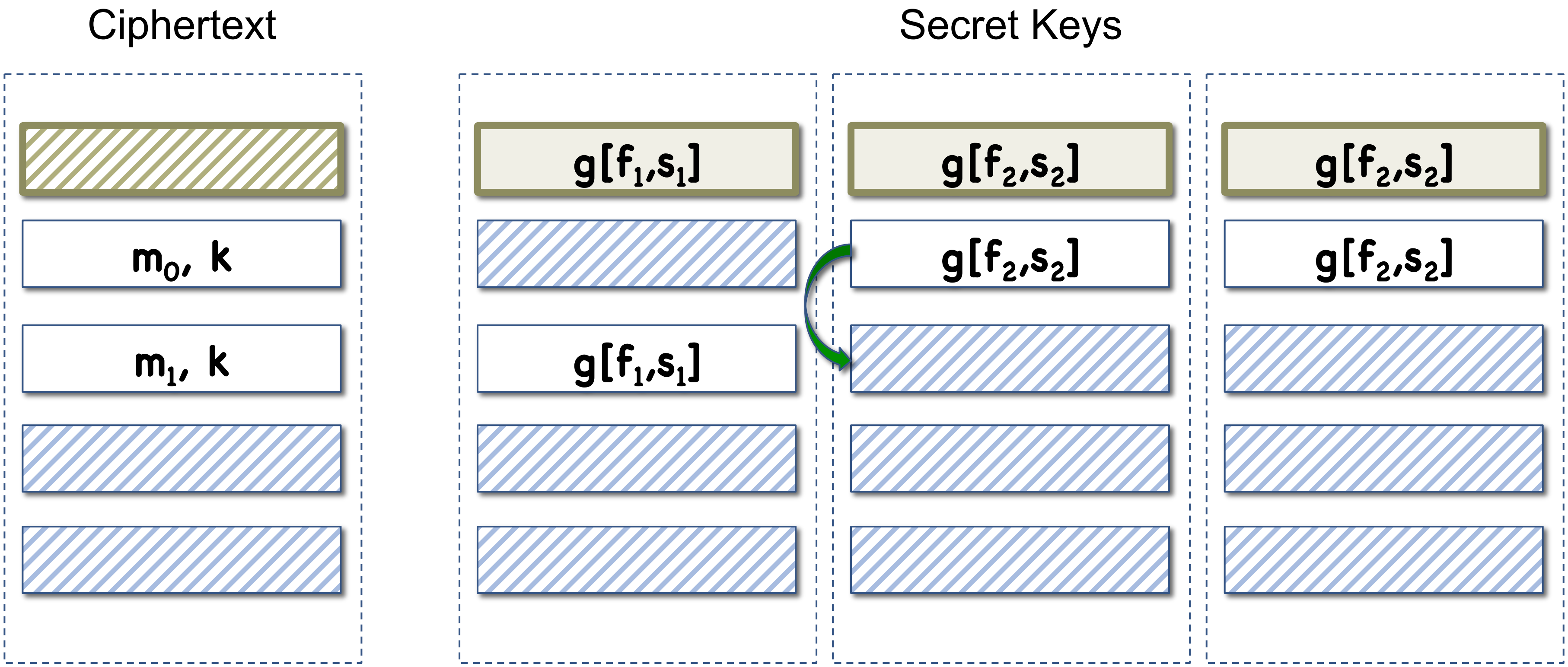
Proof idea:



“Super Strong Secret Key Moving”

Randomized FE for NC^1

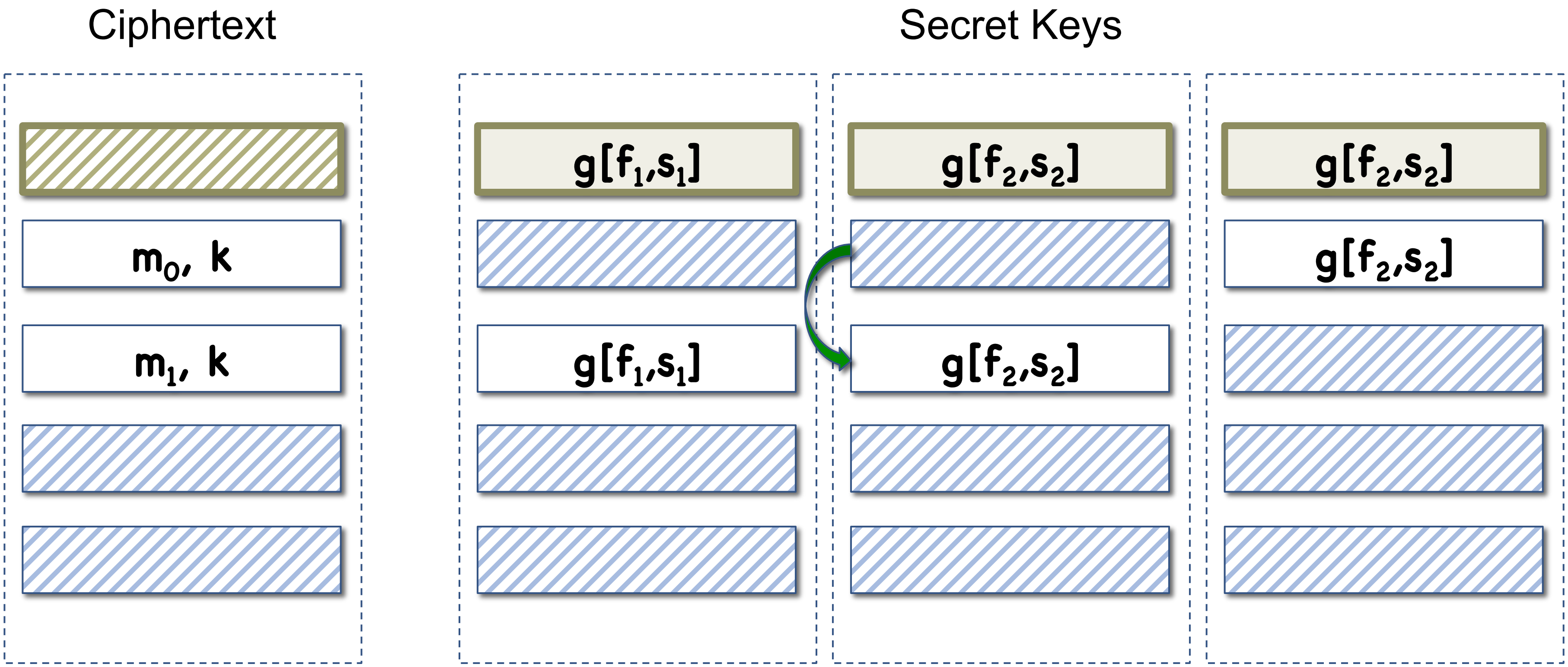
Proof idea:



“Super Strong Secret Key Moving”

Randomized FE for NC^1

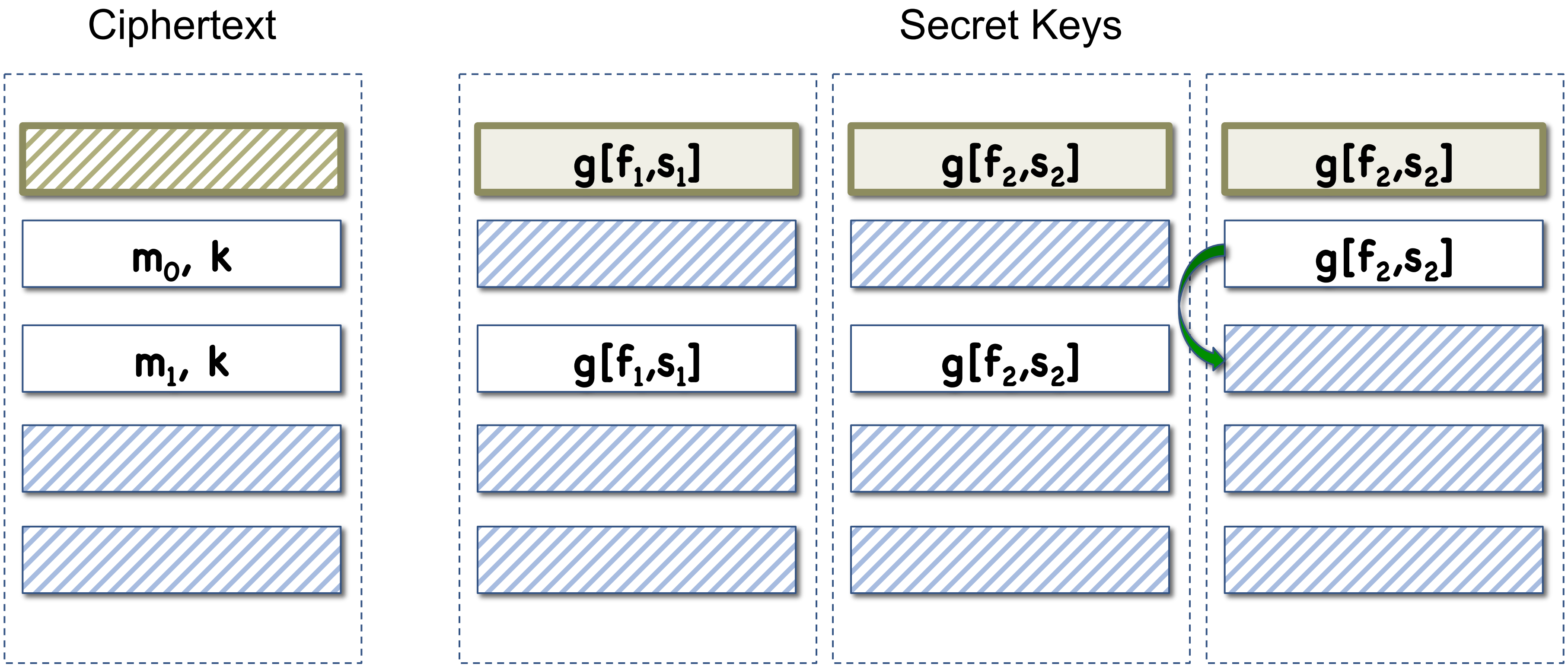
Proof idea:



"Super Strong Secret Key Moving"

Randomized FE for NC^1

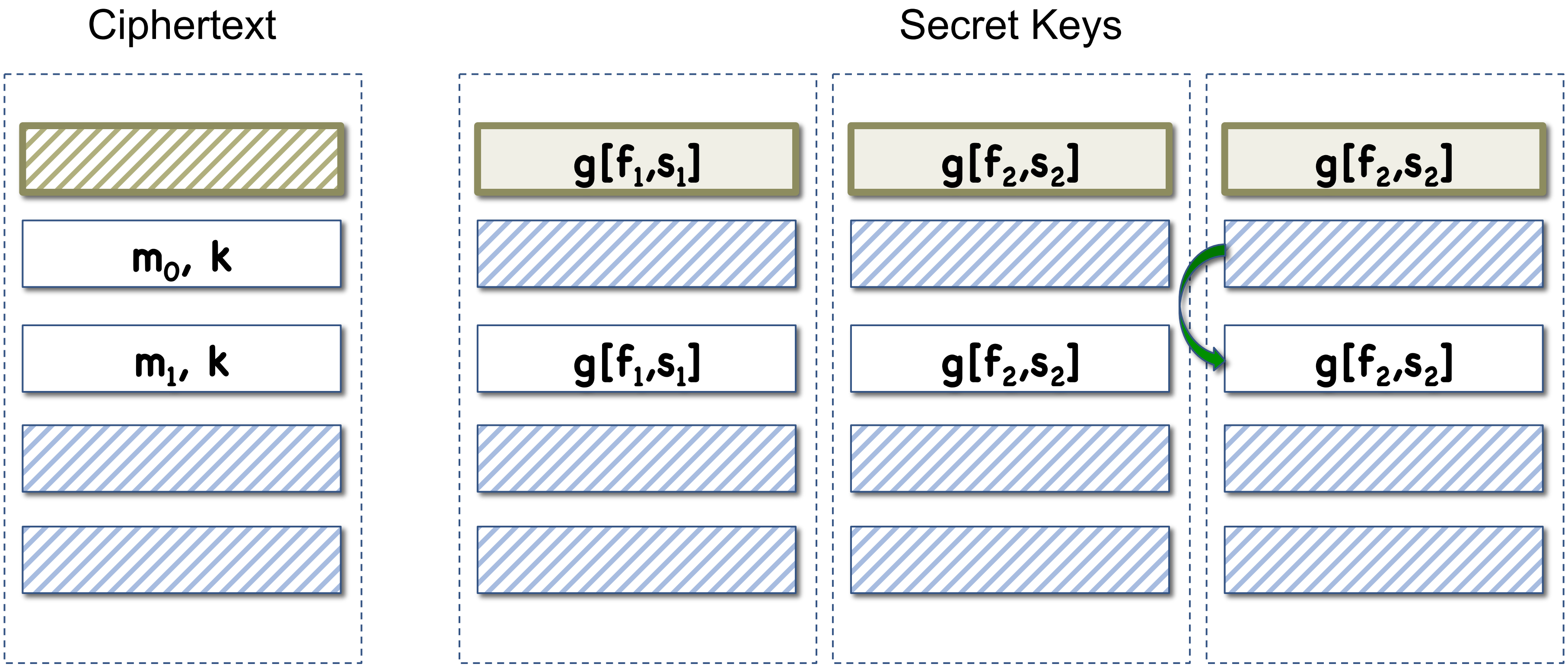
Proof idea:



"Super Strong Secret Key Moving"

Randomized FE for NC[1]

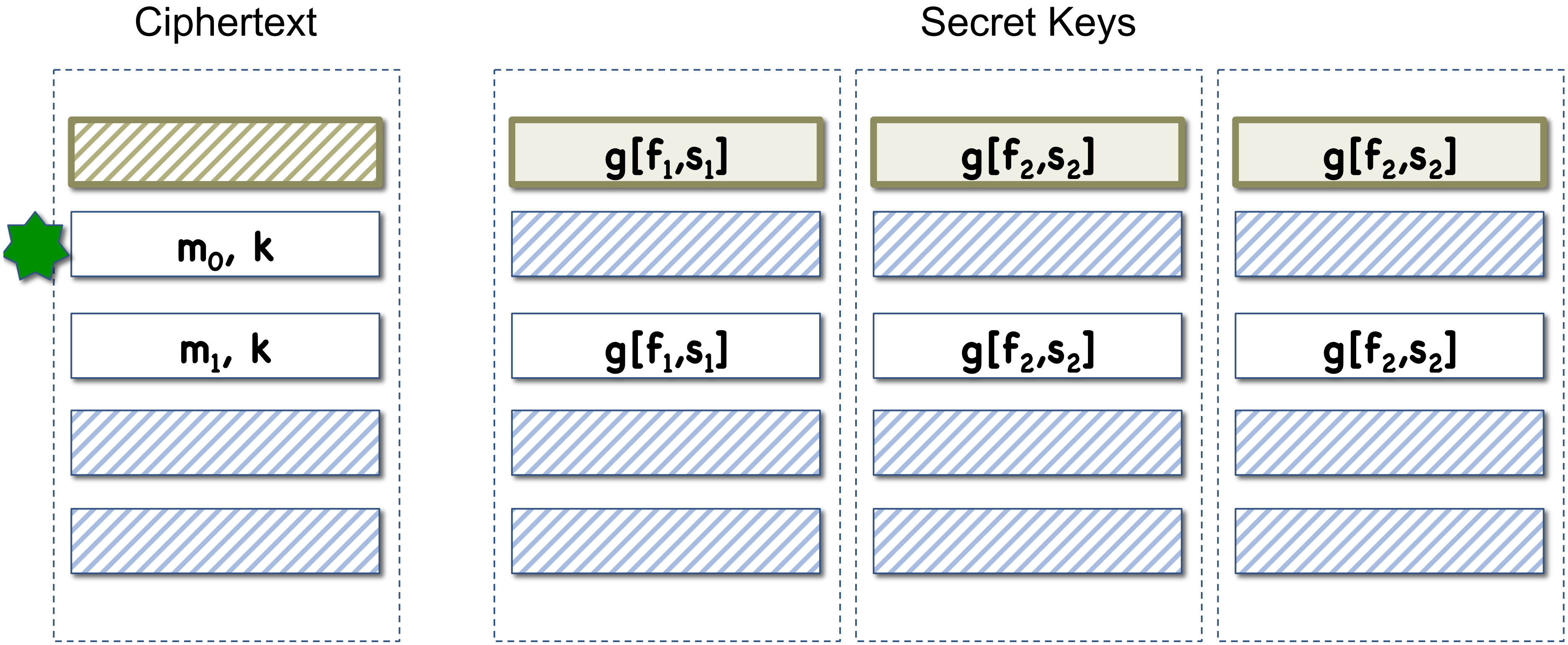
Proof idea:



“Super Strong Secret Key Moving”

Randomized FE for NC^1

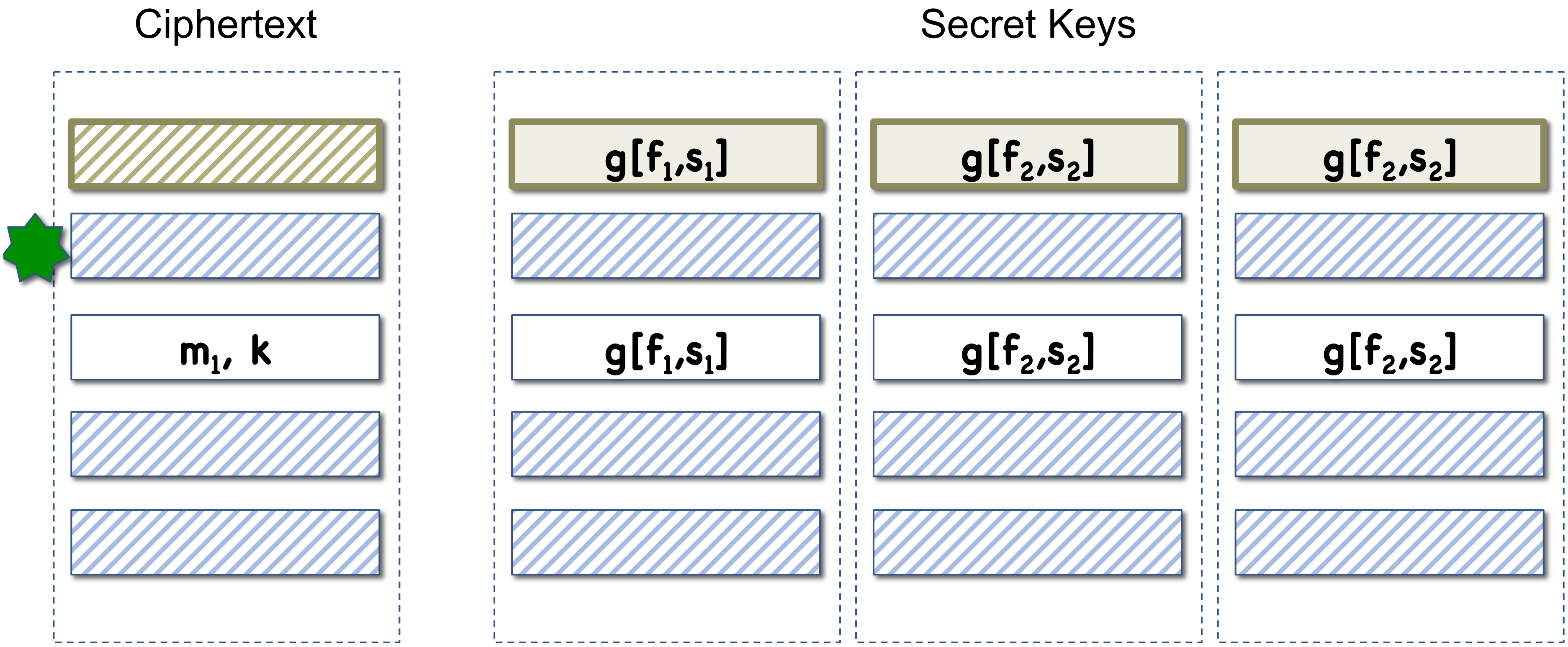
Proof idea:



New Slot

Randomized FE for NC^1

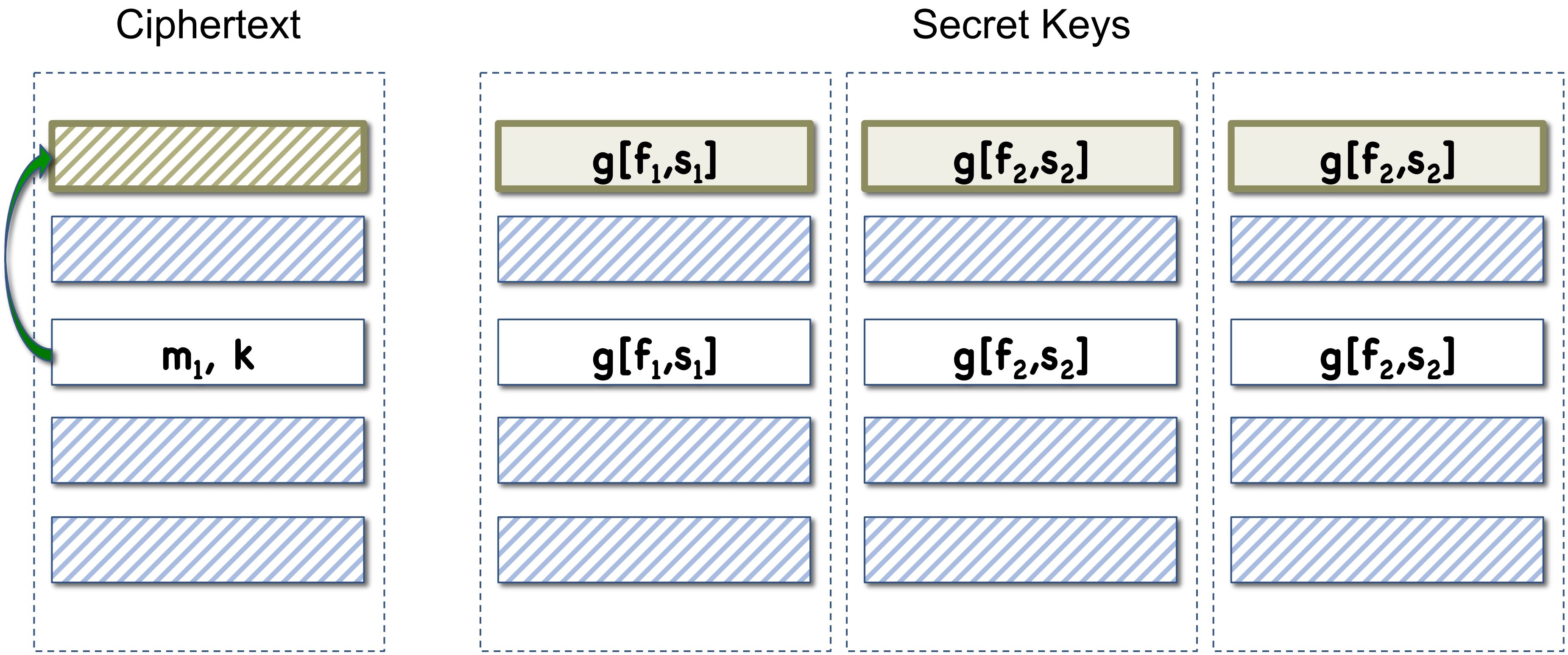
Proof idea:



New Slot

Randomized FE for NC^1

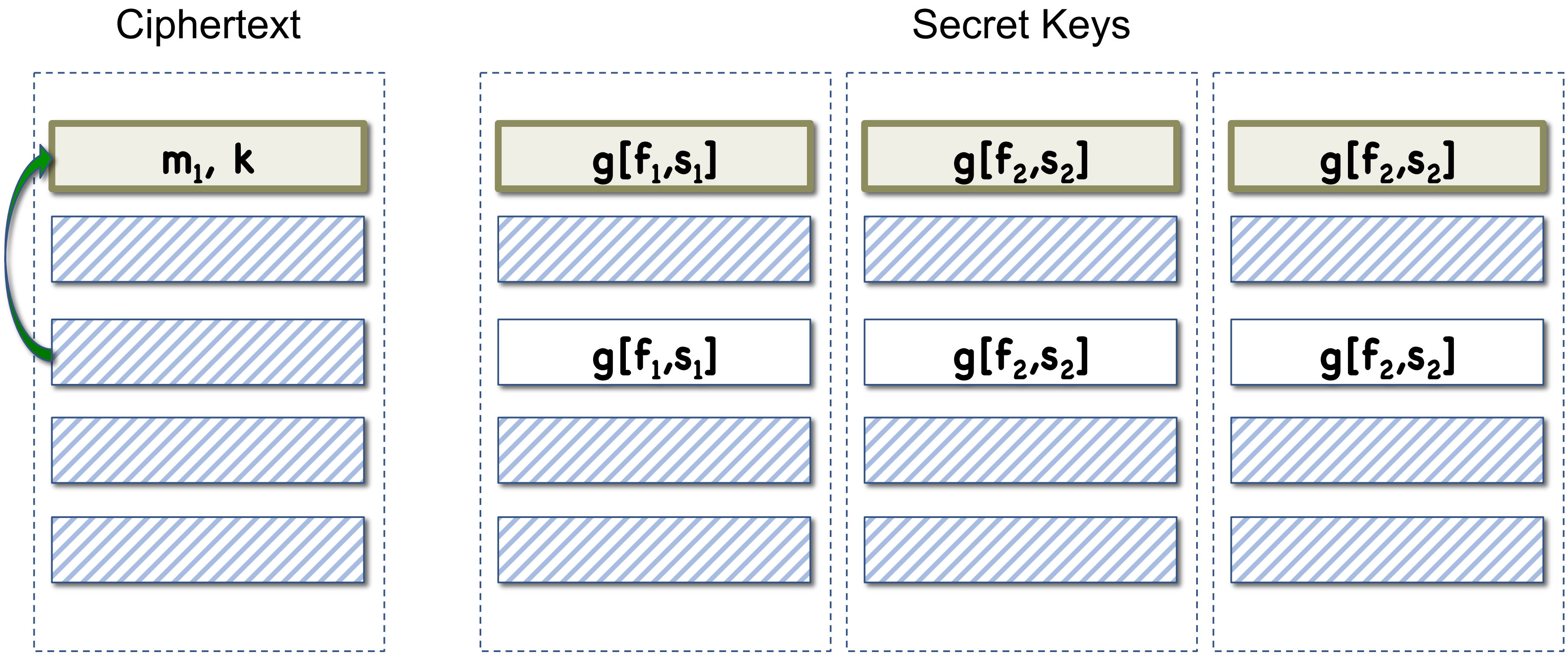
Proof idea:



Ciphertext Moving

Randomized FE for NC^1

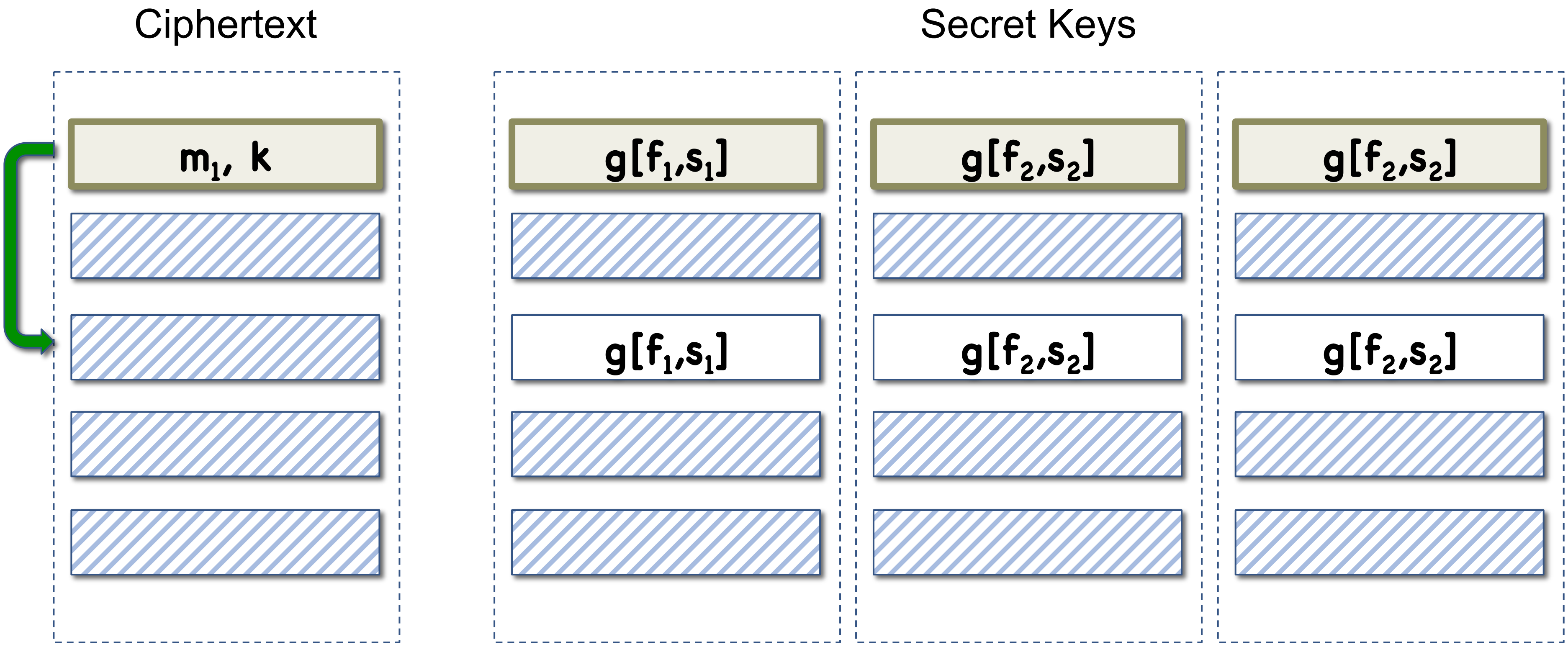
Proof idea:



Ciphertext Moving

Randomized FE for NC^1

Proof idea:



Ciphertext

Secret Keys

m_1, k

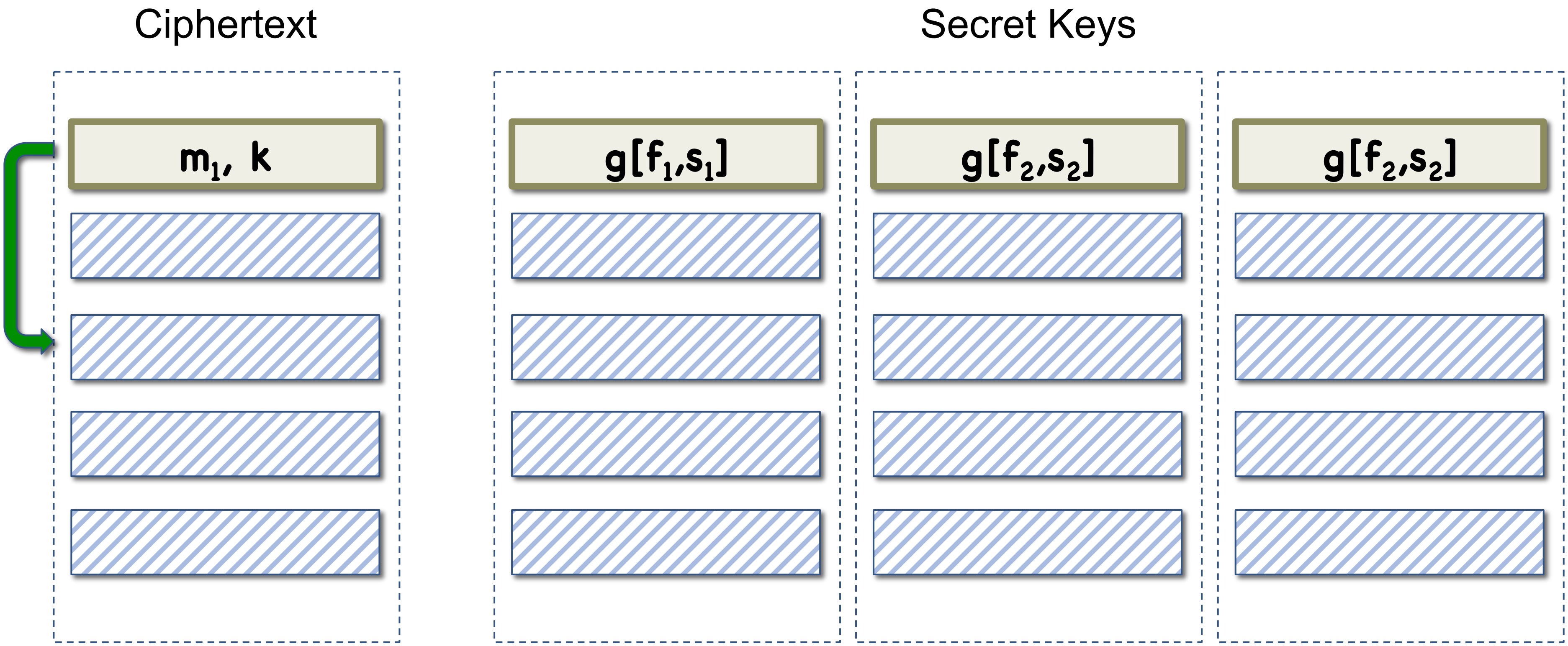
$g[f_1,s_1]$ $g[f_2,s_2]$ $g[f_2,s_2]$

$g[f_1,s_1]$ $g[f_2,s_2]$ $g[f_2,s_2]$

Slot Duplication

Randomized FE for NC^1

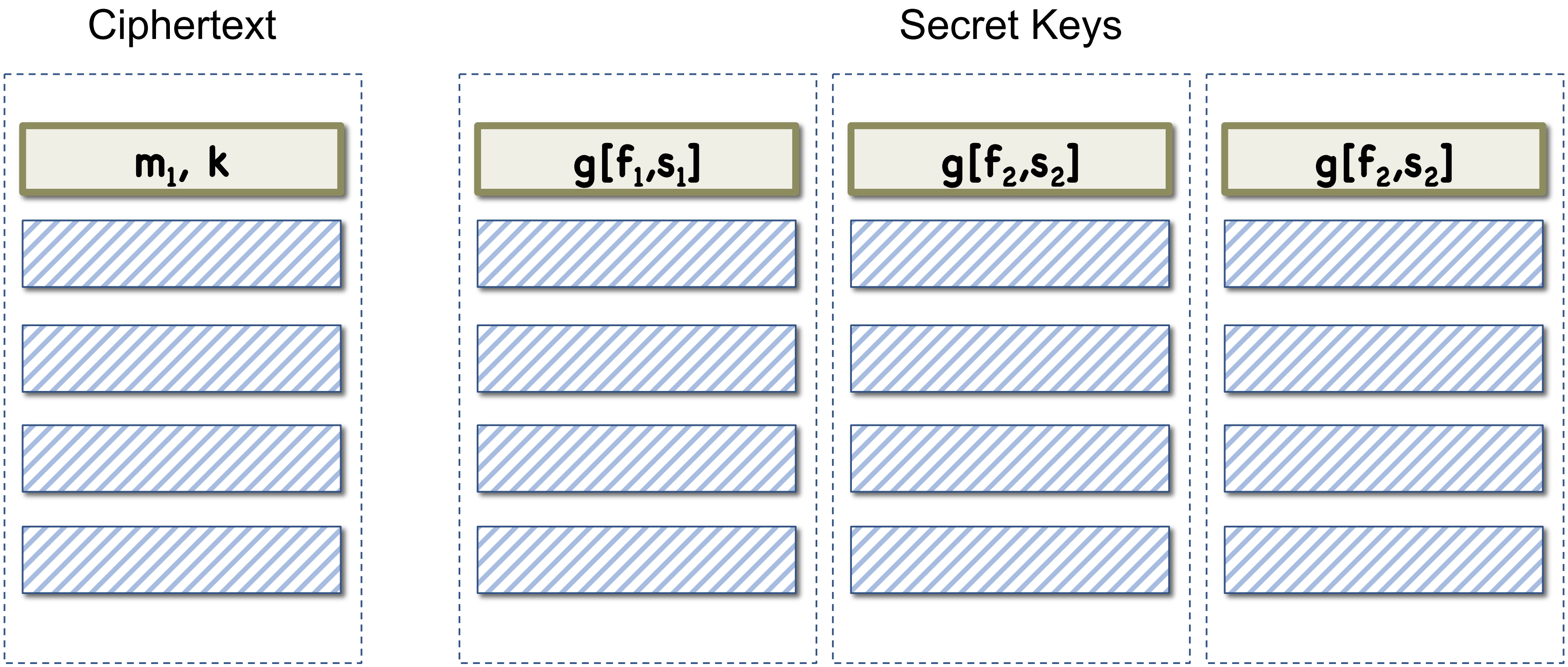
Proof idea:



Slot Duplication

Randomized FE for NC^1

Proof idea:



Ciphertext

Secret Keys

m_1, k

$g[f_1,s_1]$

$g[f_2,s_2]$

$g[f_2,s_2]$

Achieving "Super Strong Secret Key Moving"

Outputs different, even though indistinguishable

⇒ strong secret key moving not enough

More involved proof:

- Puncture **k** at **s**
- Hardcode **f(m_0, PRF(k, s))**
  - In ciphertext if secret key before ciphertext. Use ctxt indist.
  - In secret key if secret key after ciphertext. Use single-use hiding+
- Replace with **f(m_1, PRF(k, s))**
  - Using PRF security and sample indistinguishability
- Move secret key
- Un-puncture

FE for all Circuits

Basic idea: Output randomized encoding rather than actual val

$\mathbf{Enc_C(mpk, m)}$:

$c \leftarrow \mathrm{Enc}_R(\mathrm{mpk}, m)$
Output $\mathbf{c}$

$\mathbf{KeyGen_C(msk, f)}$:

$f'(m; r) := \mathrm{Encode}_f(m ; r)$
$sk_f \leftarrow \mathrm{KeyGen}_R(\mathrm{msk}, f')$
Output $\mathbf{sk_f}$

$\mathbf{Dec_C(sk_f, c)}$:

$e \leftarrow \mathrm{Dec}_R(sk_f, c)$
$o \leftarrow \mathrm{Decode}(e)$
Output $\mathbf{o}$

Conclusion and Open Problems

Simple assumptions → Slotted FE → Fully-secure unbounded FE

- iO/complexity leveraging/function hiding **not** inherent to FE

New tools on graded encodings

Open Problems:

- Other apps for slotted FE?
- Simplify: remove punctured PRFs / randomized encodings?
- Other ***iO*** apps → simple assumptions
  - Deniable encryption
  - Multiparty NIKE w/o trusted setup