Special Days and Where to Find Them

Resources

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2. A theory of intraday patterns: Volume and Price Variability (A.R. Admati)
3. The behavior of daily stock market trading volume (B.B. Ajinkya)
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1 Forecasting Intraday Trade Volume: A Kalman Filter Approach

The authors of this paper propose a state-space model to forecast intraday trading volume via the Kalman filter and derives closed-form expectation-maximization (EM) solutions for model calibration.

Notation

$$\text{volume}_{t,i} = \frac{\text{shares traded}_{t,i}}{\text{daily outstanding shares}_t}$$
$$\text{Volume}_{t,i} = \text{daily}_t \times \text{intraday periodic}_i \times \text{intraday dynamic}_{t,i} \times \text{noise}_{t,i}$$

Algorithm 1: How to write algorithms

```
Result: Write here the result
initialization;
while While condition do
    instructions;
    if condition then
        instructions1;
        instructions2;
    else
        instructions3;
    end
end
```

2 Intradaily volume modeling and prediction for algorithmic trading (Browlees C.T)

Brownlees proposes a dynamic model for intra-daily volume forecasting that captures salient features of the series such as intra-daily periodicity and volume asymmetry.
Component Multiplicative Error Model
Each element has its own dynamic specification. The model is specified in a semiparametric fashion, thus avoiding the choice of a specific distribution of the error term. All of the parameters are estimated at once by Generalized Method of Moments. The estimated model can be used to dynamically forecast the relative intra-daily volumes.

3 Distributions to Look At

- daily (over 1 day, by hour, by number of bins)

- over 1 week
- over 1 hour
- data grouped by holiday
- data grouped by special day

4 General Ideas:

- binning the data over the course of the day into hour bins
- binning days into groups as well

5 Variables:

- trade volume
- trade price
- liquidity
- volatility
- margin size
- fraction of shares outstanding traded

6 Algorithms for Forecasting:

- Moving average, weighted moving average
- Naive (using previous day's value, baseline)
- Linear Regression
- Random forests/regression decision trees
- Kalman + EM + Regularized Intraday Forecasting
- Autoregressive Moving Average (ARMA)
- Autoregressive Integrated Moving Average (ARIMA)
- LSTM Models
- Kalman Filtering
- Exponential Smoothing
- Hidden Markov Models
- Support Vector Regression
- Using Sliding Windows

7 Clustering of Special Days Ideas:

- kmeans clustering
- correlation clustering

- kernel PCA
- It is imperative to come up with a good similarity measure.

8 Questions We're Looking to Answer:

- How do the special days' market behavior compare to normal days?
- How can we be able to identify unknown special days? The unencountered, not on the list, days?

Calibration Tool For Special Days

Background

In futures exchange markets, there is a need for calibration between regular and abnormal trading days. In algorithmic trading, trade volume of specific products is an important characteristic of the market, especially for investors to want to minimize the market impact on their execution orders.

Problem

This problem is two-pronged. The first problem is to be able to identify which special days have an effect on which products and which asset classes. We want to create a mapping from product or asset class (input) to a list of special days (output) that correspond to special days that have an impact on that product.

The second part of the problem is that for every product and asset class, we want to fit an improved calibrated model for intraday volume forecasting for normal days as well as a separate model for each of the special days. We can cluster by similar special days, so we don't have tons of models.

Part 1: Testing the Relationship Between Special Days and Products

Variable of interest: Unexpected or abnormal component

Algorithm 1: Kolmogorov-Smirnov Goodness of Fit Test

The K-S Goodness of Fit Test is a statistical test used to decide if two samples come from the same distribution. Suppose the first sample has size m with an observed empirical cumulative distribution function $F(x)$ and that the second sample has size n with observed eCDF $G(x)$. Define

$$D_{m,n} = \min_{x} |F(x) - G(x)| \tag{1}$$

$D_{m,n}$ is the difference between the two distributions, so if we can show that $D_{m,n}$ is sufficiently small, we can show that the distribution of trade volume over two different days is similar, and vice versa for differently distributed days.

The null hypothesis is H_0: both samples come from a population with the same distribution. For the K-S test for normality, we reject the null hypothesis (at significance level α) if $D_{m,n} > D_{m,n,\alpha}$ where $D_{m,n,\alpha}$ is the critical value.

$c(\alpha)$ = the inverse of the Kolmorogov distribution at α.The values of $c(\alpha)$ are also the numerators of the last entries in the Kolmogorov-Smirnov Table. The Kolmogorov Distribution has value

$$F(x) = \frac{\sqrt{2\pi}}{x} \sum_{k=1}^{\inf} e^{\frac{-(2k-1)^2 \pi^2}{8x^2}}$$

.

For every product or asset class, we will check each special day against the aggregate average daily distribution of trade volume that that product. We will run a K-S test with a high alpha to ensure confidence that the distributions differ.

Algorithm 2: Dynamic Time Warping

Dynamic time warping is an algorithm used to measure similarity between two sequences which may vary in time or speed. A non-linear alignment produces a more intuitive similarity measure, allowing similar shapes to match even if they are out of phase in the time axis. It allows for stretched and compressed sections of the sequence. This is a dynamic programming solution.

Algorithm 2: Dynamic Time Warping

Result: DTW Measure of Similarity between two series

Two time series of trade volume over the course of a day;

1. Divide the two series into K equal data points.;
2. Calculate the euclidean distance between the first point in the first series and every point in the second series. Store the minimum distance calculated. (this is the ?time warp? stage);
3. Move to the second point and repeat 2. Move step by step along points and repeat 2 till all points are exhausted.;
4. Repeat 2 and 3 but with the second series as a reference point.;
5. Add up all the minimum distances that were stored and this is a true measure of similarity between the two series.;

There are optimizations that can be performed to prune the search space of the dynamic time warping function, including restrictions on monotonicity, continuity, boundary conditions, warping window, and slope constraint.

In finding the minimum distances along the grid, you create an optimal path from the bottom left of the grid to the top right. This path is called the warping path and this path follows a function called the warping function. When the warping function is applied to both time series it transformed them to two new time series that are aligned in time.

If the time-adjusted distance between daily time series differs significantly, we flag the day as a special day with impact on this product.

Algorithm 3: Sum of Squared Differences

This is the second most simple difference identifier between distributions. It's a bit naive, but taking the mean absolute value helps correct for some of the naivety.
Given two time series (assume one for normal days and one for special days)

$$G = \{g_1, g_2, ..., g_n\}$$

$$H = \{h_1, h_2, ..., h_n\}$$

We divide the day into n bins to simplify the computation. For each series, we compute

$$D(G, H) = \sum_{i}^{n} |avg(G_i) - avg(H_i)|^2$$

If either curve crosses over the other, we split on the intersection, so as to not zero out the difference. Alternatively, the integral could by used instead of a difference in average after fitting a short linear curve to the bin.

Algorithm 4: Fit a Predictor and Check Deviation

This algorithm does a time series prediction on the hour of the day, or the bins of the day if we want to bin the day on some size k. We do some feature engineering, or just time series prediction. We use different regression models to predict the next bin's or hour's trade volume for the product. We measure the squared difference between predicted volume and actual volume.

$$D(G, H) = \sum_{i}^{n} |predictedVol(G_i) - predictedVol(H_i)|^2$$

Right off the bat, some simple models we could use are linear regression, moving average, and random forests, as well as more complex models like LSTMs. We use the best cross-validated performing regressor, measured too with a ROC curve analysis. If the difference is greater than some threshold, we can flag this special day as impactful on the product.

Alternatively, we can use percentage of unexpected/anomalous values as a measure of distance.

$$D(G, H)_{alternative} = \frac{1}{n} \sum_{i}^{n} I(G_i, H_i)$$

$$I(G_i, H_i) = \begin{cases} 1 & \text{if } |predictedVol(G_i) - predictedVol(H_i)| \geq threshold \\ 0 & otherwise \end{cases}$$

Algorithm 6: Deviation from Basic Statistical Measures

This is the most basic approach, where we take the basic statistical measures of each daily distribution: IQR, mean, medium, standard deviation, range.

$$D(G, H) = \sum_{measures} |G(i) - H(i)|^2$$

Algorithm 7: Kernel Density Estimation to Fit a Distribution and Measure Deivation from Fitted Distribution

Kernel Density Estimate approximates the probability distribution function of a dataset. KDE is a technique that let's you create a smooth distribution curve given a set of data. It essentially generates points that look like they came from a certain dataset, and this behavior can well simulate the real data.

The KDE algorithm takes a parameter, bandwidth, that affects how smooth the resulting curve is. The KDE is calculating by weighting the distances of all the data points we've seen for each location. Changing the bandwidth changes the shape of the kernel: a lower bandwidth means only points very close to the current position are given any weight, which leads to the estimate looking squiggly. A higher bandwidth means a shallow kernel where distant points can contribute.

The weighted probability distribution function is as follows:

$$\hat{f}(x) = \sum_{obs} K(\frac{x - obs}{bandwidth}) = \frac{1}{n}\sum_{i=1}^{n} K(\frac{x - x(i)}{h})$$

The kernel function for the normal distribution is

$$\frac{1}{\sqrt{2\pi}} exp(-\frac{1}{2}x^2)$$

which I will use because it's continuous and non piecewise. The rule of thumb is to use

$$\hat{h_0} = 2.78\hat{\sigma}n^{-1/5}$$

We then use 1-sample K-S testing to see if the pdf that we fit to the normal days is the same as the sample series from a special day. If it differs sufficiently, we flag this special day as having impact on this product.

Recall from Algorithm 1, Define

$$D_{m,n} = \min_x |\hat{f}(x) - G(x)| \tag{2}$$

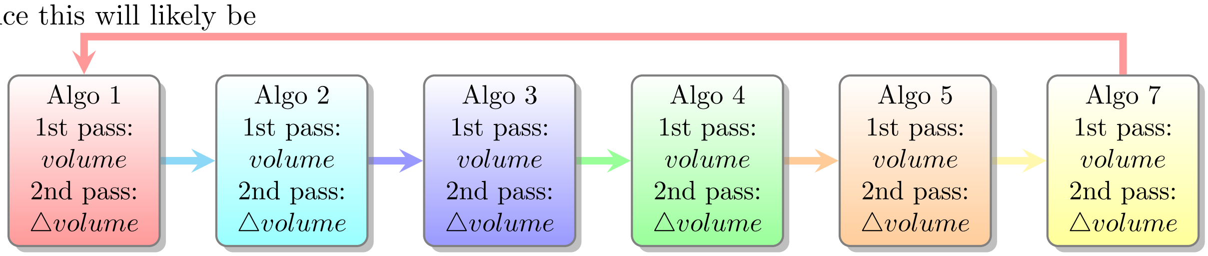
$D_{m,n}$ is the difference between the two distributions, so if we can show that $D_{m,n}$ is sufficiently small, we can show that the distribution of trade volume over two different days is similar, and vice versa for differently distributed days.

The null hypothesis is H_0: both samples come from a population with the same distribution. For the K-S test for normality, we reject the null hypothesis (at significance level α) if $D_{m,n} > D_{m,n,\alpha}$ where $D_{m,n,\alpha}$ is the critical value.

$c(\alpha)$ = the inverse of the Kolmorogov distribution at α.

Part 1 Composite Algorithm

Since this will likely be



If 6/12 or more are sufficiently, the special day is flagged, and if over 50 percent of the special day's occurrences flag an impact, then we can conclude that the special day indeed has impact on the trade volume for that product.