X a variety $/k$ alg. closed

$Z(X)$ = free ab gp on
subvar of codim 1

If X is normal ($\mathcal{O}_{X,Z}$ are all DVR's), then we can take

$$\operatorname{div}\phi = \sum_Z \operatorname{mult}_Z \phi\, [Z].$$

$\rightsquigarrow$ equiv. relation on Weil divisors

$D \sim D' \iff \exists\, \phi \in k(X)^*$ s.t.

$D - D' = \operatorname{div}(\phi)$

$Z(X)/\sim \; =: \operatorname{WDiv}(X).$

Ex.

$X = \mathbb{P}^n$; $Z = V(F) \subset \mathbb{P}^n$.
$\uparrow$ irred.

Def. $\deg Z := \deg F$

$\deg\left(\sum n_Z Z\right) := \sum n_Z \deg Z$

e.g. $\deg(2L) = 2 = \deg(C)$.

Observation. $D \sim D' \iff \deg(D) = \deg(D')$

Pf. To each $D = \sum n_Z Z$, assoc. $\prod F_Z^{n_Z}$ ...

$\mathrm{WDiv}(X) \xrightarrow{\deg} \mathbb{Z}$ surjective

$E \subset \mathbb{P}^2$

$y^2 z = x(x-z)(x-2z)$

<u>Rmk.</u> $E \to Z_0(X) \longrightarrow Z_0(X)/\sim$

(bijective)

C <u>non-singular curves</u>.

$\mathbb{P}^1 \xrightarrow{\text{constant}} \cdot$

$E \xrightarrow{\text{bijection}} Z_0(X)/\sim$

$p \longrightarrow p - p_0$

$C \hookrightarrow Z_0(C)/\sim = J(C)$